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RIVER MODELS

by

B.G. Krishnappan

Hydraulics Division
National Water Research Institute
Canada Centre for Inland Waters
Burlington, Ontario, Canada
L7R 4A6

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MANAGEMENT PERSPECTIVE

Mathematical models of river flows have gained wide acceptance as a tool for many different types of investigations, e.g., flood routing, flow distribution and reservoir control. There are many models currently available and they differ in their degree of sophistication and applicability.

This report, written by the Task Force on River Models of the Canadian Society for Civil Engineering under the chairmanship of Dr. Krishnappan, compares ten of the more popular models by examining their theoretical formulations and underlying assumptions. The strengths and weaknesses of each model are compared so that potential users can have a solid basis on which to select the tool which they need.

A/Chief
Hydraulics Division

PERSPECTIVE-GESTION

Les modèles mathématiques de débits fluviaux sont maintenant largement utilisés pour de nombreux types de recherches, notamment le calcul de la propagation des crues, la distribution du débit et le contrôle des réservoirs. Il existe actuellement de nombreux modèles; ceux-ci diffèrent quant à leur degré de perfectionnement et leur applicabilité.

Cette étude, rédigée par le Groupe de travail sur les modèles fluviaux de la Société canadienne de génie civil, présidée par B.G. Krishnappan, compare les dix modèles les plus répandus en examinant leurs fondements théoriques et leurs hypothèses sous-jacentes. Elle dégage les forces et les faiblesses de chaque modèle pour donner aux usagers éventuels des informations fiables qui les guideront dans le choix de l'outil qui leur convient.

Le chef par intérim,

Division de l'hydraulique

RÉSUMÉ

Ce document présente une évaluation d'un certain nombre de modèles d'écoulement fluvial unidimensionnels. L'évaluation a porté sur les fondements théoriques des modèles. Pour ce faire, on a établi un ensemble d'équations générales décrivant les débits fluviaux unidimensionnels, lesquelles ont ensuite servi d'étalon. Ainsi, il a été possible non seulement d'évaluer les modèles en question, mais aussi de cerner les domaines dans lesquels il nous manque les connaissances de base nécessaires pour pousser plus loin l'élaboration de modèles réalistes. Il s'agit du premier jalon des travaux d'évaluation en deux étapes qui ont été confiés au groupe de travail. En seconde étape, des séries de données communes seront utilisées pour comparer des modèles de même calibre d'après leur rendement.

RIVER MODELS

The CSCE Task Group on River Models[1]

ABSTRACT

In this paper, an evaluation of a number of existing one-dimensional river flow models is presented. The evaluation is based on theoretical formulations of the models. To perform this evaluation, a set of general governing equations describing river flows in one dimension was formulated and was used as a "yardstick". By doing so, it was possible not only to effect the model evaluation but also to identify areas where basic knowledge is needed for further development of realistic models. This work forms the first part of a two-stage model evaluation work that is being carried out by the Task Group. In stage-two evaluation, models of similar calibre will be compared based on their performances for common data sets.

[1] Members of the Task Group are:

1) Dr. B.G. Krishnappan, Chairman, National Water Research Institute, Burlington, Ontario
2) Mr. L. Alexander, Secretary, McLaren Plansearch, Toronto
3) Dr. Alex Harrington, Ontario Ministry of Natural Resources, Toronto
4) Dr. Peter Ko, Shawinigan Consultants Inc., Montreal
5) Dr. Alex McCorquodale, Univesity of Windsor, Windsor
6) Mr. S. Moin, Water Planning and Management, IWD, Burlington
7) Mr. M. Sydor, Water Planning and Management, IWD, Ottawa

INTRODUCTIO

Mathematical models of river flows are very useful for solving a variety of hydrotechnical problems related to river engineering. They are economical and less time-consuming in comparison to the traditional approach of physical models. Recent developments of mini and micro computers make mathematical models even more attractive. Mathematical models of river flows, in general, are numerical solutions of differential equations describing the flow and often require simplifying assumptions and closure approximations. When applying the models to practical problems, the model user has to be aware of these limitations.

River models that are currently available differ widely in complexity and applicability. There are steady-state and unsteady-state flow models treating flows in rigid and mobile boundary channels. The accuracy of a river model depends not only on the numerical technique used to solve the governing equations, but also on the adequacy of the auxiliary relationships that are used to evaluate the friction factor and the sediment transporting capacity of river flows. The number of available sediment transport rate predictors and the fricion factor predictors are many. Therefore, selection of a river flow model for a particular application becomes a problem for a practising engineer.

To overcome this problem, the Research Activities Committee of the Hydrotechnical Divison of CSCE established a Task Group to examine a number of river models in public domain and perform a comparative evaluation, which can serve as a valuable guide for model users in selecting appropriate model for a particular need. The Task Group is carrying out this model evaluation in two stages. In stage one, the group is examining the theoretical base and assumptions of each model in order to bring out the strengths and weaknesses of the models. In stage two, the group will actually run the models with common data sets and compare the models' results with measured data from both laboratory and field. The results of stage one evaluation is presented in this paper.

The group had selected ten models for the evaluation. These models are listed below in a tabular form in different categories.

Type of Models	Rigid Boundary	Mobile Boundary
Steady State	HEC-2; RIVER	HEC-6, IALLUVIAL
Unsteady State	DWOPER; DAMBRK; 1D; FERNS	MOBED; FLUVIAL11

In performing the stage one evaluation of the above models, the group had formulated a general set of governing equations describing river flows in one dimensions and the governing equations of the individual models were then compared with this general set. By doing so, it was possible not only to make a comparative statement about the model, but also to identify areas where basic knowledge is needed for further development of river models. The details of the general equation set is given first, followed by the description of individual models and a model comparison.

GENERAL FORM OF GOVERNING EQUATIONS

The general one-dimensional form of equations governing the unsteady flows in natural channels are listed below:

<u>Momentum Equation</u>

$$[1] \quad \frac{\partial Q}{\partial t} + 2\beta \frac{Q}{A}\frac{\partial Q}{\partial x} - \beta B\frac{Q^2}{A^2}\left(\frac{\partial y}{\partial x}\right) + gA\frac{\partial y}{\partial x} + \frac{Q^2}{A}\frac{\partial \beta}{\partial x} = gA\{S_x - (S_{ks} + S_{fd} + S_{bend} + S_{fp} + S_{ice} + S_{ec})\} + q_1\left(U_q - \frac{Q}{A}\right) + \beta\frac{Q^2}{A^2}A_x^y$$

<u>Continuity Equation</u>

$$[2] \quad \frac{\partial Q}{\partial x} + B\frac{\partial y}{\partial t} + P\frac{\partial z}{\partial t} = q_{\ell}$$

<u>Sediment Mass Balance Equation</u>

$$[3] \quad \frac{\partial Q_s}{\partial x} + P_o\left(\frac{\partial z}{\partial t}\right)p + B\,Cav\frac{\partial y}{\partial t} + A\frac{\partial\, Cav}{\partial t} = q_s$$

where
- x is the longitudinal coordinate axis (see Figs. 1 and 2)
- t is the time axis
- Q is the flow rate
- y is the flow depth
- z is the vertical distance between a fixed datum and the mean bed level within a control volume
- P is the wetted parameter
- B is the top width

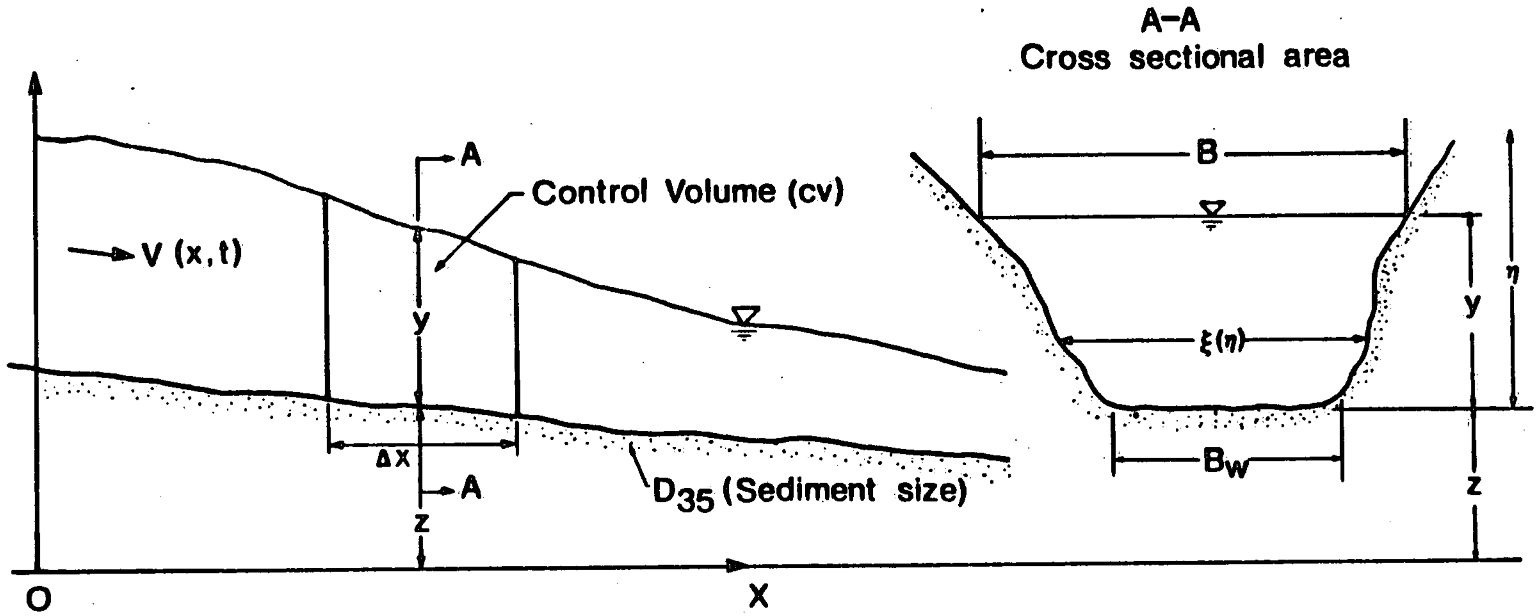

Fig. 1. Schematic representation of the longitudinal profile and a flow cross section in a river.

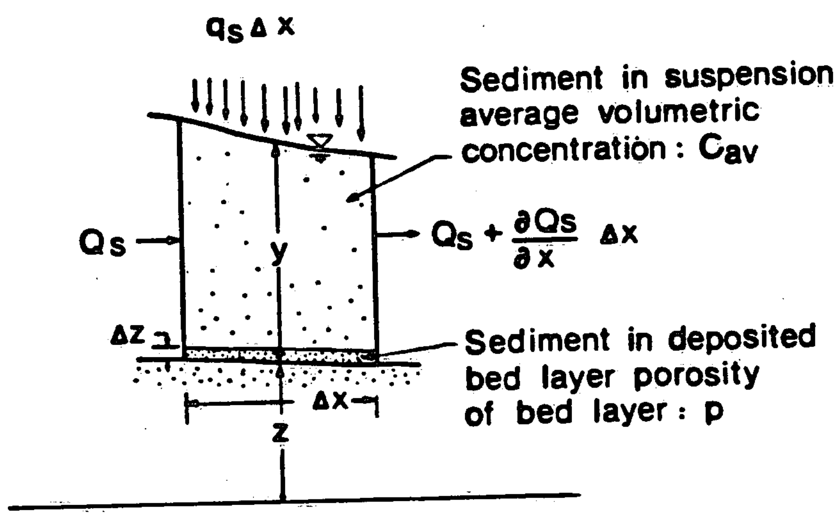

Figure 2 CONTROL VOLUME TO DERIVE THE SEDIMENT CONTINUITY EQUATION.

A is the flow cross-sectional area
g is the acceleration due to gravity
A_x^y is the rate of change of A with respect to x when y is is held constant
q_l is the lateral flow input rate
U_q^l is the x component of the lateral inflow velocity
S_x is the slope of the river bed at the location of the control volume
S_{ks} is the energy loss per unit weight of fluid and unit river length due to skin friction
S_{fd} is the energy loss per unit weight of fluid and unit river length due to form drag caused by sand waves
S_{bend} is the energy loss per unit weight of fluid and unit river-length due to meander bends
S_{fp} is the energy loss per unit weight of fluid and unit river-length due to interaction of main channel flow and floodplain flow
S_{ice} is the energy loss per unit weight of fluid and unit river-length due to ice-cover
β is the momentum correction factor
S_{ec} is the energy loss per unit weight of fluid and unit river length because of sudden expansion or contraction
Q_s is the volumetric sediment transport rate (total bed material load)
P_o is the portion of the wetted perimeter over which the sediment is in motion
C_{av} is the average volumetric concentration of the suspended bed material within a control volume
p is the volume of sediment on the bed per unit volume of bed layer
q_s is the sediment input rate entering the stream from tributaries (size fractions are assumed to be similar to that of main stream sediment)

The momentum correction factor, β is defined as follows:

[4] $$\beta = \left(\frac{Q_c^2}{A_c} + \frac{Q_f^2}{A_f}\right) / \frac{Q^2}{A}$$

where Q_c is the flow rate in the main channel
Q_f is the flow rate in the flood plains
A_c is the cross-sectional area of the main channel
and A_f is the cross-sectional area of the flood plains.

β can be evaluated by the following two relations

[5] $$Q = Q_c + Q_f$$

$$[6] \quad \frac{Q_f}{Q_c} = \frac{\frac{A_f}{\kappa} R_f^{1/2} [\ln(\frac{y_f}{K_{sf}}) + R_f^{1/2} A_f \cdot B_{sf}]}{\frac{A_c}{\kappa} R_c^{1/2} [\ln(\frac{y_c}{K_{sc}}) + A_c R_c^{1/2} B_{sc}]}$$

where K_{sf} is the equivalent sand grain roughness of flood plains
K_{sc} is the equivalent sand grain rougness of the main channel bed
B_{sf} is the additive constant of the logarithmic velocity distribution corresponding to the flood plain flow
B_{sc} is the same corresponding to the main channel flow
R_f is the hydraulic radius of the flood plain section
R_c is the hydraulic radius of the main channel section
and κ is the von-Karman constant

Note that the evaluation of β by the above method is on a time lag basis

The energy loss terms, S_{ks}, S_{fd}, S_{bend}, S_{fp}, S_{ice} and S_{ec} are, in principle, determinable when the bulk flow, sediment and the geometric characteristics of the river are known. A set of general relationships for these terms is listed below:

$$[7] \quad S_{ks} = \phi_{ks} (\frac{V_* D_{65}}{\nu} ; \frac{D_{65}}{y})$$

$$[8] \quad S_{fd} = \phi_{fd}(\frac{\Delta_d}{\Lambda_d} ; \frac{\Delta_d}{y})$$

$$[9] \quad S_{bend} = \phi_{bend}(\frac{\Lambda_m}{H} ; \frac{H}{B} ; \text{etc.})$$

$$[10] \quad S_{fp} = \phi_{fp} (\frac{B_f}{B_c} ; \frac{y_{fp}}{y_c} ; \text{etc.})$$

$$[11] \quad S_{ice} = \phi_{ice} (\frac{V_* K_{s,ice}}{\nu} ; \frac{K_{s,ice}}{y} ; \text{etc.})$$

$$[11a] \quad S_{ec} = \phi_{ec} \Delta(Q/A)^2/2g\Delta x$$

where V_* is the shear velocity
D_{65} is the sediment size for which 65% of the material by weight is finer
ν is the kinematic viscosity of fluid
Δ_d is the sand wave height

Λ_d	is the sand wave length
Λ_m	is the meander wavelength
H_m	is the meander amplitude
B_f	is the width of the flood plain
B_c	is the width of the main channel
y_{fp}	is the flow depth in the flood plain
y_c	is the flow depth in the main channel
$K_{s,ice}$	is the equivalent sand size roughness height of the ice cover
ϕ_{ec}	is an empirical constant varying between 0 and ±1 (+ for contraction and - for expansion)
Δ	is an operator which signifies a change between adjacent nodes

Among the relationships listed above, only ϕ_{ks} in Equation 7 has a reasonably well established functional form (family of curves of Moody Diagram). Determination of the form of other functions in Eq. (8 to 11) has been and is still a subject of extensive research in the area of river hydraulics. For example, the works of Einstein and Borbarossa (1952), Engelund (1966), Garde and Ranga Raju (1966), Alam and Kennedy (1969), Vanoni and Hwang (1967), Raudkivi (1967), Smith (1968), Simons and Richardson (1966), Yalin (1964), Kishi and Kuroki (1974) and Van Rijn (1984) have attempted to determine the form of the function ϕ_{fd}. The works of Knight and Demetriou (1983), Myers and Elsawy (1975), Wormleaton, Allen and Hadjipanos (1982), Wormleaton and Hadjipanos (1985) and Ervine and Baird (1982) have focussed on the interaction of flood plain and main channel flows which will help to determine the form of the function ϕ_{fp}. Information on ϕ_{bend} and ϕ_{ice} is scanty and lot more research is needed to fully understand the form of these two functions.

In the system of equations listed above, there are altogether ten unknowns, namely, Q, y, z, Q_s, β, S_{ks}, S_{fd}, S_{bend}, S_{fp} and S_{ice}. Therefore, to close the system, one more equation is required. This is provided by the sediment transport relationship to evaluate Q_s. There are a number of sediment transport relationships that can be found in the literature. The following is a list of relationships that are frequently used:

1. Ackers and White equation (1973)
2. Meyer Peter and Müeller equation (1949)
3. Einstein's bed load function (1950)
4. Bagnold's equation (1966)
5. Yang's stream power equation (1973)
6. Yalin's equation (1963)

None of the above equations is capable of predicting sediment transport rate for a whole range of flow and sediment conditions. For a particular flow and sediment characteristic some formulae are better suited than the others. Therefore, selection of a particular relationship for a given application requires some understanding of the sediment transport mechanism on the part of a model user.

The three governing equations describing the unsteady flows in mobile boundary channels are coupled equations and require a simultaneous solution procedure. However, all the currently existing models dealing with mobile boundary channels assume that the term $\partial z/\partial t$ in Equation 2 is small in comparison to the term $(\partial y/\partial t)$ and uncouple the momentum and continuity equations from the sediment mass balance equation by dropping the $(\partial z/\partial t)$ term from Equation 2. As a result, the solution procedures used in these models are comparatively simpler. First, the flow characteristics at a particular time step are predicted by solving Equations 1 and 2 and then the sediment mass balance equation is solved making use of the predicted flow characteristics to evaluate Q_s and C_{av}. The solution of Equation 3 is then used to correct the flow characteristics before proceeding to solve Equations 1 and 2 for the next time step. The process continues until the required number of time steps is reached.

INITIAL AND BOUNDARY CONDITIONS

Initial conditions

The governing equations (1) and (2) require that Q and y be specified all along the river at time zero. The bed elevation as a function of x is normally specified as part of geometrical data of the river. As pointed out earlier, the variables Q_s and C_{av} are usually computed from the flow conditions resulting from Equations 1 and 2 and hence there is no need to specify initial conditions for these two variables.

Boundary conditions

Most of the models under consideration are applicable to subcritical flows only. For such models, conditions at both upstream and downstream boundary sections need to be specified. Since both Q and y are independent variables, a total of nine combinations of boundary conditions are possible. These combinations are summarized in Table 1. Although there are nine combinations in Table 1, the diagonal combinations are not recommended since they do not involve both the dependent variables.

The values of Q_s at the upstream boundary have to be specified when dealing with the non-equilibrium problems. Under equilibrium conditions, the Q_s values at the extreme boundaries can be equated to those prevailing in the adjacent sections.

TABLE 1

Nine Possible Combinations of Boundary Conditions

		Upstream Boundary Type		
		y(t)	Q(t)	Q(y)
Downstream Boundary Type	y(t)	y(t) u/s y(t) d/s	Q(t) us y(t) d/s	Q(y) u/s y(t) d/s
	Q(t)	y(t) u/s Q(t) d/s	Q(t) u/s Q(t) d/s	Q(y) u/s Q(t) d/s
	Q(y)	y(t) u/s Q(y) d/s	Q(t) d/s Q(y) d/s	Q(y) u/s Q(y) d/s

DESCRIPTION OF INDIVIDUAL MODELS

MOBED (National Water Research Institute Model)

MOBED is an unsteady, mobile boundary flow model which solves the full St. Venant equations, i.e., Equations 1 and 2 except that it assumes β to be unity and S_{bend}, S_{fp}, S_{ice} and S_{ec} are negligible. It also solves the sediment mass balance equation (Equation 3) with the assumption that the movable wetted perimeter P_o is equal to the total wetted perimeter, P.

MOBED takes into account the energy loss components, S_{ks} and S_{fd} and groups them together into a single term S_f which is expressed in a general form as shown below.

[12] $$S_f = \mathrm{CONST}\left(\frac{R}{D_{65}}\right)^m \left(\frac{Q^2}{gRA^2}\right)^n \qquad (12)$$

The values of CONST, m and n depend on the type of bed form geometry and the sediment and flow characteristics.

The general form for S_f as given in Equation 12 allows the adaptation of different friction factor relationships covering both rigid boundary and mobile boundary channel flows into the model. The version of MOBED that is chosen for model evaluation uses the friction factor relation of Kishi and Kuroki (1974) for mobile boundary channel flows.

Kishi and Kuroki's relations consider the bed form configurations in seven different regimes. These are: Dune I, Dune II, Flatbed, Transition I, Transition II, antidunes and chute and Pool.

The sediment transport rate, Q_s is evaluated using the equations of Ackers and white (1973) in MOBED.

MOBED accepts six different types of boundary conditions, namely, the six non-diagonal combinations shown in Table 1.

MOBED uses a four point, implicit, finite difference scheme developed by Priesmann (1960 to discretize the governing equations. The discretized equations are linearized and solved using a Double-Sweep Method. This scheme has been shown to be unconditionally stable for the values of the weight factor, θ between 0.5 and 1.0. the model uses a value for θ of 0.67 as recommended by Cunge (1961). With this value of θ, the scheme is first-order accurate.

The model can treat tributary inflows of water and sediment and storage basin within modelled reach. The model is fully documented in a users' manual (Krishnappan, 1981) and in an update (Krishnappan, 1983). Recently, the model has been updated to include an armouring routine.

The model is also available in PC version.

FLUVIAL 11 (San Diego State University Model)

Like MOBED, FlUVIAL 11 is an usteady, mobile boundary flow model. It solves the set of equations 1 to 3 with the following assumptions:

1) $\beta = 1$.

2) $(S_{fd} + S_{bend} + S_{fp} + S_{ice} + S_{ec})$ is negligible and $S_{ks} = n^2v^2/R^{4/3}$ (where n is Manning's roughness coefficient, v is average flow velocity and R is hydraulic radius).

3) $C_{av} = 0$.

The governing equations were solved using the four-point implicit finite difference scheme analyzed by Fread (1974) and Amien and Chu (1975).

The model simulates channel width variation using the concept of minimum stream power at each time step. It also calculates the changes in channel-bed profile and the changes in cross-sectional profile due to lateral migration of channel bends. It updates the bed-material composition at every time step.

Input data to the model consists of initial cross-section geometry, channel roughness and bed-material composition. the data can be in HEC-2 format. The output of the model consists of bed-material composition water surface profile, channel width, flow depth, flood discharge, velocity, energy gradient, median sediment size and bed material load as a function of time and space. Cross-sectional profiles can also be printed out at specified time intervals.

For further details on the capabilities of this model, the reader can refer to H.H. Chang (1982).

IALLUVIAL (Iowa Alluvial River Model)

IALLUVIAL is a steady, mobile boundary flow model. It solves the steady-state version of Equations (1) and (2) and a simplified version of Equation (3). The model assumes that the momentum correction factor, β, is unity and the energy-loss slopes, S_{bend}, S_{fp}, S_{ice} and S_{ec} are negligible. The lateral inflows of water and sediment (q_ℓ and q_s) are not considered and only the bed load transport of sediment is taken into account (i.e., C_{av}, is neglected). The movable wetted perimeter, P_o is assumed to be equal to the total wetted perimeter, P. Therefore, the governing equations of the model take the following simplified forms:

[13] $$\frac{\partial}{\partial x}\left(y + z + \frac{Q^2}{2gA^2}\right) = (S_{ks} + S_{fd})$$

[14] $$\frac{\partial Q_s}{\partial x} + P\frac{\partial z}{\partial t}p = 0$$

The energy loss due to the bed form geometry S_{fd} is considered in the model; but, it is done in an indirect way. The two energy loss terms, S_{ks} and S_{fd}, are combined together into a single slope S which is evaluated from a friction-factor equation developed at the Iowa Institute of Hydraulic Research by Karim and Kennedy (1981). This equation contains the sediment discharge, Q_s, as one of the independent variables and therefore, reflects the influence of the bed form geometry that is generated because of the sediment movement. The sediment discharge itself is computed from another equation developed by Karim and Kennedy (1981). The friction-factor and the sediment discharge relations are coupled and hence an iterative scheme is adopted in the model to solve for these quantities simultaneously.

In IALLUVIAL, the armouring of the stream bed is simulated. The effects of armouring on sediment transport and friction factor are taken into account. The grain size distribution of the stream bed is predicted as a function of time. The details of the armouring algorithm and the evaluation of the grain size distribution can be found in the model's users manual (Karim and Kennedy, 1982).

The model considers only the subcritical flows and therefore the solution of Equation (13) begins at a downstream station and proceeds in the upstream direction. The water surface elevation at the extreme downstream station is the boundary condition for the model. Equations of friction factor and sediment transport are also solved along with Equation (13) using the standard step method. The sediment mass balance equation is solved independently starting from the upstream station and proceeding downstream. The sediment input rate at the extreme upstream station is the other boundary condition required for the model.

The space and time steps selected for the model have great influence on the stability of the model. The ratio between the time step and the space step cannot exceed a certain maximum value which depends on the mean flow velocity, the celerity of the bed sediment wave and/or the time required for the complete development of the armour layers. Karim and Kennedy (1982) have suggested some relationships to evaluate the maximum time step for a given value of space step. The computer code of the IALLUVIAL incorporates a trial and error procedure that gives allowable maximum time steps for given space steps from which an appropriate time step can be selected. For more information, readers can refer to Karim and Kennedy (1982).

HEC-6 (U.S. Army Corps of Engineers Model)

Like IALLUVIAL, HEC-6 is also a steady state, mobile boundary flow model. The governing equations for the model is identical to those of IALLUVIAL. It also assumes that the energy slope components, S_{bend}, S_{fp} and S_{ice} are negligible and uses Manning's equation to evaluate the friction slope neglecting the influence of bed forms. The energy loss due to sudden expansion or contraction is included. The lateral inflows of water and sediment are not considered in the model and only bed load transport of sediment is taken into account. A movable bed width, different from the total wetted perimeter can be specified to the model by the user.

HEC-6 has an armouring routine to predict the changes in bed material size distribution. The model has four built in sediment transport rate relationships. These are:

1) Toffaleti's equation,
2) Madden's modification of Laursen's equation,
3) Yang's stream power, and
4) DuBoy's equation

The description of the above sediment transport rate equations can be found in ASCE's manual on sedimentation engineering edited by Vanoni (1975).

HEC-6 treats sediment of all size ranges starting from clay (dia: .004 m) to very coarse gravel (dia: 64.0 mm). It considers these sediments in three groups, namely, CLAY, SILT and SAND and GRAVEL. For the case of silts and clays, the model is not able to simulate scour and resuspension. Therefore, once they become deposited, they will not be remobilzed during high flows.

The model calculates the flow conditions using the standard step method. Both subcritical and super critical flows can be handled by the model. The stability criterion for the model is similar to that of the IALLUVIAL. The model is fully documented in a users' manual (1977).

FERNS (Water Planning and Management, Ontario Region Model)

FERNS (Finite Element River Network Simulator) is an unsteady flow model for rigid boundary open channels. The model solves equations (1) and (2) with the assumptions that β is unity and (S_{fd} + S_{bend} + S_{fp} + S_{ice} + S_{ec}) is negligible. The model employs the Manning's equation to evaluate the friction slope. The model uses the finite element solution procedure employing linear basis function and linear spatial elements and a forward finite difference discretization for temporal derivatives. The resulting algebraic equations are non-linear and are expressible in the form of a bi-tridiagonal matrix. The equations are solved by a predictor-corrector method.

For a single reach or a tree-type river system, the solution is effected by an efficient double sweep technique. However, for a looped network (like flow around islands) the double sweep technique cannot be used. In such situations, the final form of equations is

[15] $[A]\ \{x\} = \{B\}$

Where [A] is a 2 m x 2 m coefficient matrix (m is the number of nodes); {x} is the column vector containing the unknowns y and Q; {B} is another column vector containing the known values. The operation on the full 2 m x 2 m matrix is inefficient because in a typical network only a few nodes are connected (junction-nodes) and hence it is possible to create a different matrix with a central band of non-zero elements. The central band width of M can be computed as:

[16] $M = 4L + 3$

Where L is the maximum node difference across a junction. FERNS creates this banded matrix of size 2 m x M by shifting matrix element and solves the matrix using the Gauss Elimination Technique. FERNS is an implicit model with optimal results for a temporal weighting coefficient of 0.5. In FERNS, this value is read in . A value in excess of 0.55 is recommended to achieve numerically stable results.

Initial conditions: Initial conditions to FERNS could be of the following types:

a) known steady state solution for y and Q for all nodes,
b) computed steady-state solution from known flows and downstream depth;
c) known unsteady flow conditions from a previous FERNS run.

Boundary Conditions: The upstream boundary condition can either be a stage hydrograph or a flow hydrograph. For the downstream boundary the model has the following options:

1) stage hydrograph,
2) flow hydrograph,
3) single-value rating curve,
4) looped rating curve, and
5) stage-discharge power function relationship.

FERNS also allows a variety of internal boundaries that can be defined by the user. These are at the junctions with tributaries, branching around islands, sudden expansion or contractions in the channel, bridges and flow over weirs. Details on these can be found in the users manual (1978).

Channel Geometry: FERNS uses three different specifications of channel geometry. These are:

1) Cross-section definition by a series of space-elevation co-ordinates similar to HEC-2.
2) Cross-section definition by power function such as

$$A_i = C_i y_i^{P_i} + T_i y_i + Ao_i \qquad [16]$$

where y_i is the depth of flow at node (i)
A_i is the cross-sectional area
C_i, P_i and T_i are the regression coefficients, and
Ao_i is the channel area below datum.

3) Cross-section definition by a stage vs top width table. Channel properties are then interpolated and computed. This option is similar to that employed in DWOPER and DAMBRK. FERNS also has the option to include off-channel storage where non-conveying cross-sections exist.

ID: (One Dimensional Unsteady Flow Model Developed at MIT)

ID model is similar to FERNS as it also simulates unsteady flows in rigid boundary river networks. It solves the same equations, i.e., equations (1) and (2) with β taken as unity and with the assumption that (S_{fd} + S_{bend} + S_{fp} + S_{ec}) is negligible. There is a provision in the model to consider the full or partial ice-cover and the ice-cover growth and decay. The model can account for the reduction in cross-sectional area caused by ice-cover thickness. The friction slope is computed using the Manning's equation or Chezy's equation. The governing equations are solved by a numerical scheme developed by Gunaratnam and

Perkins (1970). The scheme was obtained by applying a weighted residual method of optimization to a simplified linearized version of the governing equations and the discrete approximations. Appropriate local and temporal adjustments were used in order to account for the non-linear characteristics of the governing equations. The following criteria were proposed for the convergence of the scheme:

[17] $100 \geq \lambda/\Delta x$

[18] $\Delta T \leq 5.5 \dfrac{\Delta x}{V+|c|}$

where λ is the wavelength of the flood wave; V is the average flow velocity; and c is the celerity of the flood wave.

Initial conditions: Initial conditions are generated by running the model under steady state conditions. For this purpose, approximate initial conditions are specified and the model is run for sufficiently long periods of time. The hydraulic conditions obtained at the end of this steady state simulation are compatible with the governing equations and can be used as initial conditions without problems under transient flow simulations.

Boundary conditions: External boundary conditions such as discharge or water surface elevation hydrographs can be specified at the upstream extremities of the network. In the case of downstream boundaries, a relationship between water levels and discharges representing natural controls or hydrulic structures can be used.

Internal boundary conditions such as small rapids or falls, bridges, weirs, gates and spillways, represented by a relationship or a family of curves between water elevations upstream and downstream of the structure and the discharges can be handled by the model.

Channel geometry: The network topology or schematization must be defined by the user with the minimum number of reaches and nodes. Each reach is described by a length, one or more Manning's coefficient of roughness, an estimate of the space increment and a table of hydraulic parameters (total top width, core top width, core area, wetted perimeter, and total area at different water depth or elevation) for each input cross-section. These parameter tables can be manually generated. However, the Water Planning and Management Branch, IWD, Ottawa (the operator of the model) has developed front-end computer programs to generate these tables from survey data.

<u>DWOPER (Dynamic Wave Operational Model, NWS, USA)</u>

DWOPER belongs to the same category of models as FERNS and ID. It solves equations (1) and (2) with β taken as unity and (S_{fd} + S_{bend} + S_{fp} + S_{ice}) considered negligible. The energy loss due to sudden expansion or contraction is included. The friction slope is evaluated using the Manning's equation. The governing equations are discretized using the four-point, implicit, finite-difference scheme (Fread, 1974). The resulting non-linear system of algebraic equations are solved by Newton-Raphson method which is an iterative procedure.

Initial conditions: Initial conditions for stage and discharge values at each node can be read in or can be obtained from a previous DWOPER simulation. The model has an option to compute the initial conditions from steady flow calculations.

Boundary conditions: Stage or discharge hydrographs can be used as upstream and downstream conditions. At the downstream boundary, a single valued or a loop rating curve or a weir equation can also be specified.

River geometry: Geometry of flow cross-sections are specified by a table of values of elevations and top-widths. There is a provision in the model to consider the off-channel storage. Roughness coefficients in terms of Manning's n can be specified as functions of either stage or discharge for each nodal section. The model can also perform an automatic calibration, i.e., it can adjust the values of Manning's n by computing the root mean square error between the model prediction and specified measurement values.

DAMBRK (Dam Break, NWS, USA)

This model has two components. In the first component the breach formation and the resulting reservoir-outflow hydrograph is calculated.

The second component deals with the flow routing through the valley downstream.

Reservoir-outflow hydrograph: The total outflow resulting from a dam breach is calculated as the sum of broad-crested weir flow through the breach and flow through spillway outlets, i.e.,

[19] $Q = Q_b + Q_s$

where Q_b is the breach flow computed as

[20] $Q_b = c_1(h - h_b)^{1.5} + c_2(h - h_b)^{2.5}$

in which c_1 is a factor accounting for the flow through the rectangular portion of the breach and c_2 for the triangular ends; h is the water surface elevation behind the reservoir and h_b is the elevaton of the breach bottom.

Equation (2) is for a breach forming at the top and working downwards. If the dam failure is due to piping, then Q_b is computed differently as a flow through a trapezoidal orifice.

The term Q_s in eqn. (19) accounts for flow over spillways and turbines. This term covers the uncontrollable spillway discharge, gated spillway discharge and the flow over the crest of the dam if it is topped. The water balance in the reservoir can be handled by either the hydrologic storage routing method or by the complete solution of the St. Venant equations.

Downstream routing: The unsteady flow equations in rigid boundary, i.e., equations (1) and (2) are solved in this portion of the model. Since the flow resulting from a dam breach could occupy a significant portion of overbank and floodplain regions, the momentum correction factor β is allowed to take values other than one and it can vary as a function of time and space. The energy loss components, S_{fd}, S_{bend}, S_{fp} and S_{ice} are neglected. The energy loss due to sudden expansion or contraction is included. The friction slope is evaluated using the Manning's equation.

The governing equations are solved using the same numerical method as that of DWOPER. The initial and boundary condition requirements are the same as those for DWOPER except at the upstream boundary where breach generated or user specified flow hydrograph is used. DAMBRK can handle supercritical flows. The operational problems that can be handled by DAMBRK can be summarized as follows:

1) Reservoir storage routing to compute outflow hydrograph from reservoir with subcritical dynamic routing of outflow hydrograph through entire length of downstream valley.
2) Same as above (item 1) but with supercritical dynamic routing through the entire length of the downstream valley.
3) Same as above (item 2) but with an allowance for subcritical flow routing through the latter half of the downstream valley.
4) Same as item 1, but with dynamic routing through the reservoir to compute outflow hydrograph from reservoir.
5) Same as item 2 but with dynamic routing through the reservoir.
6) Same as item 3 but with dynamic routing through the reservoir.
7) Subcritical dynamic routing of input hydrograph through a channel valley.
8) Supercritical dynamic routing of input hydrograph through a channel valley.
9) Sequential method - same as item 1, but there is a channel reservoir with a dam that may fail.
10) Sequential method - same as item 2, but there is a channel reservoir with a dam that may fail.
11) Simultaneous method. For a single dam or bridge with dynamic routing upstream and downstream and internal boundary at the structure.
12) Same as item 11, but with multiple dams and bridges.

The model is fully documented in its user manuals (1978) and (1984).

RIVER: (McMaster University Model)

RIVER is a steady-state model and predicts steady flows in rigid boundary channel networks. The governing equation is the steady-state version of equation (1) and it is of the same form as the flow equation solved by IALLUVIAL and HEC-6. The central method for advancing the solution is by the Ezra method. The model makes extensive use of a set of specialized subroutines in the CIVILIB library developed at the McMaster University.

The computation proceeds from the most downstream point of the network along the main stem. The flow is divided at the junctions and balanced around the network. This balancing proceeds until the elevation difference between iterations meets a preset criteria.

The program is developed for micro-computers and runs on an interactive mode. The current version of the program operates on TI professional computer and IBM-PC. TI version is equipped with graphics capabilities.

The program operates through a series of commands which allow the user to review results, make modificatins and rerun the program. These commands are described in a users' manual.

The program computes the water level, energy grade line and the flow regime for the main channel and tributaries or branches between a specified upstream limit and a section where the downstream water level was set to initiate the computation. If required, the program computes critical depths at sections to ascertain that the solution is proceeding in the right flow regime.

The cross-section which is described by a series of horizontal and vertical coordinates can be redefined either by displaying the section graphically or adjusting the tabular form. The results can also be plotted for the profiles.

The flow resistance law in the program is user specified and can be one of the following:

1) Chezy's equation,
2) Manning's equation,
3) Strickler's equation,
4) Colebrook-White equation, or
5) Nikuradse's smooth and rough relations.

HEC-2: (U.S. Army Corps of Engineers Model)

Like RIVER, HEC-2 is a steady-state model and predicts water surface elevations in a single reach of rigid boundary channels. The governing equation is the same as that of RIVER. However, the solution technique is different. HEC-2 uses the standard step method and it can handle both subcritical and supercritical flows. The friction slope is computed using the Manning's equation. The energy loss due to sudden expansion and contraction, S_{ec} is considered in the model. The model has provision to account for the presence of bridge piers, flow over a roadway and for the flow spilling over the top of bridges.

Channel geometry: Cross-sectional shapes are described in terms of coordinate points (lateral distance vs elevation). Manning's n-values can be prescribed at the first cross-section and held constant throughout the study area or they can be changed at any or all cross-sections. Each cross-section can be divided into three subsections and different n value can be prescribed for each subsection. The overbank subsections can be further subdivided if necessary. A maximum of 20 subdivisions are possible for the overbank sections. Manning's n values can also be varied over the vertical.

Calibration: Manning's n values can be computed using the model by providing the cross-sectional data and the observed water surface elevations and the discharge. In such cases, it is necessary to specify the ratio between channel and overbank n values. Once the calibration of the model is completed, a minor restructuring of the input data is all that is necesary to proceed with the water surface profile predictions.

COMPARISON OF MODELS

The capabilities of the models described in the last section are summarized in Table 2 which gives a comparison of the models based on their formulation. The models are compared under five major headings and a total of thirty-one subheadings (see Table 2). This table gives, at a glance, an idea of the capabilities and the applicability of a particular model. For example, if we are looking for a model to predict flows in a network of channels, then there are only three models that would be suitable for the problem. These are: RIVER, FERNS and 1D. If, in the same example, the flow conditions do not change significantly with time, then the steady-state model RIVER can be selected. If the flow conditions change rapidly with time, then either FERNS or 1D can be selected. At present, a single model which would predict flows in networks of mobile boundary channels is not available.

From Table 2, it can also be seen that only two models namely, IALLUVIAL and MOBED consider the energy loss due to bed forms and none of the currently available models consider the energy loss due to bends, interactions of main channel and flood plain flows and the ice-cover effects. DAMBRK considers the momentum correction due to overbank flow; but it does not include the energy loss due to the interaction of main channel and flood plain flows.

All the mobile boundary flow models would predict armouring of stream bed; but only FLUVIAL 11 and MOBED can predict unsteady flows. Between FLUVIAL 11 and MOBED, only FLUVIAL 11 can predict changes in channel

		HEC-2	RIVER	HEC-6	IALLUVIAL	FERNS	ID	DWOPER	DAMBRK	FLUVIAL 11	MOBED
Type of Model	Steady State - Rigid Boundary	X	X								
	Steady State - Mobile Boundary			X	X						
	Unsteady State - Rigid Boundary					X	X	X	X		
	Unsteady State - Mobile Boundary									X	X
Governing Equations	Energy Equation	X	X								
	Energy Equation + Sediment Mass Banance Eqn.			X	X						
	Complete St. Venant Equation					X	X	X	X		
	Complete St. Venant Equation + Sediment Mass Balance Eqn.									X	X
Energy Loss Components	Friction Slope - S_{ks}	X	X	X	X	X	X	X	X	X	X
	Bed Form Drag - S_{bend}				X						X
	Bend Energy Loss - S_{bend}										
	Floodplain - Main Channel Interaction - S_{fp}										
	Energy Loss Due to Ice Cover - S_{ice}										
	Energy Loss Due to Sudden Expansion and Contraction S_{ec}	X		X		X					
Solution Procedure	Finite Element Technique					X	X				
	Finite Difference Method	X	X	X	X			X	X	X	X
	Non-Linear Algebraic Equations					X		X	X	X	
	Linear Algebraic Equations						X				X
	Stability Criterion Investigated					X	X	X	X	X	X
Special Features	Momentum Correction for Overbank Flow								X		
	Off-Channel Storage					X	X	X	X		X
	Tributary Inflow of Water					X	X	X	X	X	X
	Tributary Inflow of Sediment									X	X
	Channel Network		X			X	X				
	Ice-Cover Effects						X				
	Bridge, Weir and Spillways	X				X	X	X	X		
	Armouring			X	X					X	X
	Generalized Energy Equation										X
	Automatic Calibration							X	X		
	Documentation	X	X	X	X	X	X	X	X		X
	Channel Width Adjustment + Lateral Migration of Bends									X	

width and lateral migration of a stream bend. Between these models, only MOBED considers the alluvial bed roughness. MOBED also has a generalized energy equation and hence is applicable to different types of rivers such as gravel bed rivers, alluvial rivers, rigid boundary channels and for laminar flows.

SUMMARY AND CONCLUSIONS

A number of existing one-dimensional river models are compared based on the theoretical formulation of the models. A set of general governing equations was derived which facilitates the model comparison. Descriptions of individual models are outlined along with a summary table of model capabilities. This summary table can be used to get an idea of the capability and the applicability of a particular model. This table also indicates that there is still a lot more room for further refinement of one-dimensional river flow models.

REFERENCES

Ackers, P. and White, W.R. 1975. Sediment Transport - New Approach and Analysis. Journal of Hydraulics Division, ASCE, Vol. 99, No. HY11.

Alam, M.A. and Kennedy, J.F. 1969. Friction Factors for Flow in Sand Bed Channels. Journal of Hydraulics Division, ASCE, Vol. 95, No. HY6.

Amein, M., and Chu, H.L. 1975. Implicit Numerical Modelling for Unsteady Flows. Journal of the Hydraulics Division, ASCE, Vol. 101, No. HY6, pp. 717-732.

Bagnold. R.A. 1966. An Approach to the Sediment Transport Problem From General Physics. Geological Survey, Professional Paper 422-I, U.S. Government Printing Office, Washington.

Chang, H.H. 1982. Modelling of River Channel Change. Journal of Hydraulic Engineering, Vo. 110, No. 2, pp. 157-172.

Cunge, J.A. 1961. Étude d'un schéma de différences finies appliqué à l'intégration numérique d'un certain type d'équation hyperbolique d'écoulement. Thesis presented to the Faculty of Science of Grenoble University.

Einstein, H.A. 1950. The Bed Load Function for Sediment Transportation in Open Channel Flows. U.S. Dept. Agriculture, Soil Conservation Service, Tech. Bull. 1026.

Einstein, H.A. and Barbarossa, L. River Channel Roughness. Trans. ASCE, Vol. 117.

Engulund, F. 1961. Hydraulic Resistance of Alluvial Streams. Journal of the Hydraulics Division, ASCE, Vol. 92, No. HY 4.

Ervine, D.A. and Baird, J.I. 1982. Rating Curves for Rivers with Overbank Flow. Proc. Institution of Civil Engineers, Part 2, Vol. 73, June.

Fread, D.L. 1974. Numerical Properties of Implicit Four-Point Finite Difference Equations of Unsteady Flow. National Weather Service, NOAA, Technical Memorandum, NWS Hydro-18.

Garde, R.J. and Ranga Raju, K.G. 1966. Resistance Relationshps for Alluvial Channel Flow. Journal of Hydraulics Division, ASCE, Vol. 92, No. HY4.

Gunaratnam, D.J. and Perkins, P.E. 1970. Numerical Solution for Unsteady Flows in Open Channels. Hydrodynamics Laboratory Report No. 127, MIT.

Karim, M.F. and Kennedy, J.F. 1981. Computer-Based Predictors for Sediment Discharge and Friction Fctor of Alluvial Streams. Iowa Institute of Hydraulic Research, Report. No. 242, University of Iowa, Iowa City, Iowa.

Karim, M.F. and Kennedy, J.F. 1982. IALLUVIAL: A Computer-Based Flow and Sediment Routing Model for Alluvial Streams and Its Application to the Missouri River. IIHR Report No. 250, Iowa Institute of Hydraulic Research, The University of Iowa, Iowa City, Iowa.

Kishi and Kuroki. 1974. The Bed Configuration and Roughness of Alluvial Streams. By Task Committee on the Bed Configuration and Hydraulic Resistance of Alluvial Streams, Committee on Hydraulics and Hydraulic Engineering, the Japan Society of Civil Engineers.

Knight, D.W. and Demetriou, J.D. 1983. Flood Plain and Main Channel Flow Interaction. Journal of Hydraulic Engineering, Vol. 109, No. 8.

Krishnappan, B.G. 1981. Users Manual: Unsteady, Non-Uniform, Mobile Boundary Flow Model - MOBED. Hydraulics Division, National Water Research Institute, Burlington, Ontario.

Krishnappan, B.G. 1983. MOBED Users Manual Update I. Hydraulics Division, National Water Research Institute, Burlington, Ontario.

Meyer-Peter, E and Müller, R. 1949. Eine Formed zur Berechnung des Geschiebetriebes. Schweiz. Bauzeitung. 67 Jg., Zürich.

Meyers, W.R.C. and Elsawy, E.M. 1975. Boundary Shear in Channel With Flood Plain. Journal of the Hydraulics Division, ASCE, Vol. 101, No. HY7.

Moin, Syed M.A. 1978. Unsteady Flow River Network Simulation by Finite Element Method. Water Planning and Management Branch, IWD, Burlington, Ontario, Canada.

Priesmann, A. 1961. Propagation des intumescences dans les canaux et rivières, le congrès de l'association française de calcul, Grenoble.

Raudkivi, A.J. 1967. Analysis of Resistance in Fluvial Channels. Journal of Hydraulics Division, ASCE, Vol. 93, No. HY5.

Simons, D.B. and Richardson, E.V. 1966. Resistance to Flow in Alluvial Channels. Geological Survey of Professional Paper, 422J, U.S. Government Printing Office, Washington, D.C.

Smith, K.V.H. 1968. Alluvial Channel Resistance Related to Bed Forms. Journal of Hydraulics Division, ASCE, Vol. 94, No. HY1.

U.S. Army Corps of Engineers. 1977. HEC-6: Scour and Deposition in Rivers and Reservoirs. Users' Manual, The Hydrologic Engineering Centre, Davis, Calif.

U.S. Army Corps of Engineers. 1973. HEC-2: Water Surface Profiles. Users' Manual, The Hydrologic Engineering Centre, Davis, Calif.

Van Rijn, L.C. 1984. Sediment Transport, Part III; Bed Forms and Alluvial Roughness. Journal of Hydraulic Engineering, ASCE, Vol. 110, No. 12.

Vanoni, V.A. and Hwang, L.S. 1967. Relation Between Bed Forms and Friction in Streams. Journal of Hydraulics Divison, ASCE, Vol. 93, No. HY3.

Vanoni, V.A. (editor). 1975. Sedimentation Engineering. Prepared by ASCE Task Committee for the preparation of the manual on sedimentation, New York, NY.

Wormleaton, P.R., Allen, J. and Hadjipanos, P. 1982. Discharge Assessment in Compound Channel Flow. Journal of the Hydraulics Division, ASCE, Vol. 108, No. HY9.

Wormleaton, P.R. and Hadjipanos, P. 1985. Flow Distribution in Compound Channels. Journal of Hydraulic Engineering, ASCE, Vol. 111, No. 2.

Yalin, M.S. 1964. On the Average Velocity of Flow Over a Mobile Bed. La Houille Blanche, No. 1.

Yalin, M.S. 1963. An Expression for Bed Load Transportation. Proc. SCE, Vol. 89, No HY3.

Yang, C.T. 1973. Incipient Motion and Sediment Transport. Journal of the Hydraulics Division, ASCE, Vol. 99, No. HY10.