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CAN MEANDERS FORM IN LAMINAR FLOWS?

by

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Management Perspective

The management of rivers requires the ability to predict changes in river morphology which may be caused by human interference, including changes in depth, width and plan form. A prerequisite to achieving this capability is the understanding of the forces controlling the formation of meanders. This study is able to eliminate certain theories concerning meander formation so that further research can be directed towards the more probable causes of meander formation.

Perspective-gestion

La gestion des cours d'eau nécessite la capacité de prévoir des changements de la morphologie du cours d'eau qui peuvent être causés par des interférences anthropiques, y compris des changements de profondeur, de largeur et de forme du plan d'eau. Cette aptitude nécessite la compréhension préalable des forces qui composent la formation des méandres. La présente étude peut éliminer certaines théories concernant la formation étude peut éliminer certaines théories concernant la formation des méandres, de sorte que les recherches ultérieures puissant porter sur des causes plus probables de la formation des méandres.

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Running Head:

Abstract

Some of the earlier studies on formation of meanders suggest that the turbulence-driven secondary currents that are present in a straight channel with non-circular flow geometry may be responsible for the development of meanders in natural streams. The present study was undertaken to test this hypothesis. Laboratory experiments in mobile boundary channels with glycerine as a flowing medium showed that meanders do form in laminar flows. Since laminar flows are free from such secondary currents it becomes evident from this study that the presence of turbulence is not a necessary condition for the formation of meanders. Therefore, attention should be directed to other mechanisms such as fluvial instability for the explanation of meander formation.

Keywords: Meanders, laminar flow, secondary-circulation, turbulence stability.

Résumé

Certain des études antérieures portant sur la formation de méandres suggèrent que des courants secondaires générés par des turbulences, qui sont présents dans un canal droit à géométrie d'écoulement non circulaire, peuvent être responsables du développement de méandres dans les cours d'eau naturels. La présente étude a été entreprise afin de vérifier cette hypothèse. Des essais en laboratoire de canaux à limites mobiles, utilisant de la glycérine comme milieu liquide mobile, ont indiqué que de tels méandres se forment dans les zones d'écoulement laminaire. Etant donné que les zones d'écoulement laminaire sont exemptes de courants secondaires, il est évident, d'après cette étude, que la présence de turbulences n'est pas une condition nécessaire pour la formation de méandres. Par conséquent, il faut examiner d'autres mécanismes comme l'instabilité des cours d'eau pour expliquer la formation des méandres.

Mots clés: Méandres, écoulement laminaire, circulation secondaire, stabilité des turbulences.

Introduction

The mechanism controlling the formation of meanders in natural streams has been the subject of intense investigation over the past few decades. Yet, our understanding of the reasons behind the formation of meanders is still incomplete. Existing studies on meander formation can be grouped into three different categories based on the approach. The first group of studies such as those of Einstein and Li (1958), Einstein and Shen (1964), Shen and Kumura (1968), Quick (1974), Yalin (1971) and Yang (1971) offered explanations that are based on the physical characteristics of channel flows. The second group of studies such as those of Callendar (1969), Sukegawa (1973), Engulund and Skovgaard (1973), Hayashi (1973), Hansen (1967), Fredsoe (1978), Parker (1975), (1975), (1976) and Ikeda et al. (1981) adopted an approach based on stability analysis in which the governing equations were used to show that the possibility exists of a small perturbation to the bed level leading to the development of alternate bars and eventually to meander patterns. The third group of studies such as those of Von Schelling (1951), Langbein and Leopold (1966), Scheidegger (1967), Thakur and Scheidegger (1969), Peschke (1973), Chang and Toebes (1970), Surkon and VanKan (1969) and Ghosh and Scheidegger (1971) treated the river meandering as a stochastic process and applied the tools of statistics and probability to describe the meander patterns.

Among the studies in the first group, the one by Einstein and Li (1958) contained an expression for the streamwise vorticity in uniform flows and showed that the gradients of turbulent stresses that may be present in some portion of the flow field can generate secondary currents even in straight channels. Using this finding, Einstein and Shen (1964), Shen and Kumura (1968) and Quick (1974) postulated that the turbulence induced secondary circulation could be one of the controlling factors for the formation of meanders. Yalin (1971) argued that the meanders are the horizontal version of the sand dunes and attributed the large-scale turbulence eddies for their formation. All these studies imply that only turbulent flows can initiate the formation of meanders. In other words, according to these studies, a laminar flow which is free from the streamwise vorticity or turbulent eddies should not produce meanders. (The absence of streamwise vorticity in laminar flows has been theoretically proved by Einstein and Li (1958) and Brendrett and Baines (1964)).

A good test of the hypothesis that only turbulent flows produce meanders will be to investigate whether meanders will form in laminar flows or not and the present study is aimed at such a test. If meanders do form in laminar flows, then one can conclude that the presence of turbulence is not a necessary condition and attention should be directed to other factors. There is no reported study on meander formation in laminar flows in the literature. Therefore, it was decided to carry out a series of experiments with laminar flows in an erodible channel in the laboratory. The details of the experiments and the results are outlined in this paper.

Experimental Setup and Procedure

Experiments were carried out in a recirculating tilting flume, 7.3 m long, 30 cm wide and 15 cm deep. The flume was filled with sand to a thickness of 5 cm and a straight initial channel of trapezoidal cross section of top width 5 cm, depth 2.5 cm and side slop of 45° was cut in the sand. The sand is of near uniform size with a median diameter, D_{50} of 0.88 mm. Glycerene was used as the fluid medium. It was circulated through the channel using an oil pump. The glycerene was first pumped into a constant head tank. From the constant head tank, it flowed into an inlet reservoir with baffles which facilitated a smooth entry into the channel.

At the downstream end of the channel, the glycerene emptied into a tank from where it entered the suction pipe of the pump. A tailgate at the downstream end of the channel was used to adjust the flow depth in the channel.

The flume was set at a particular slope and the flow was started. The flowrate and the tailgate were adjusted until the flow in the channel was uniform. This condition was checked by measuring flow depths at two sections in the working section of the channel. The uniform flow established in the channel was capable of transporting the sand and it was determined using the extended Shield's diagram presented by Yalin and Karahan (1979). The flowrate of glycerene was determined by collecting the glycerine at the downstream end for a known period of time.

Attempts to feed sand at the upstream end of the channel failed as the sand dropped from a sand feeder did not sink in the glycerine and stayed in floatation for the whole length of the channel. Therefore, the flow was allowed to scour the bed in the upstream reach during a run. The transported sand emptied into the downstream tank and it was removed at the end of a run.

Altogether, three runs with different flow rates were carried out. In each run uniform flow was established and the flow was allowed to run for a period of four to five days. The average Reynolds number ranged between 1.79 and 2.64 for these three runs (see Tables 1-3). Changes in plan-form were observed by taking photographs every two hours using a still camera. The camera was mounted on a frame at a height directly above the channel so that the field of view covered the working section of the flume.

Results and Discussion

Meanders did form in all three runs tested. The photographs taken for a run before and after the formation of meanders is shown in Fig. 1. Initially, the channel width increased and then the alternating pattern of erosion and deposition of channel banks started to develop. Signs of full-fledged meanders began to form after about two days of continuous flow. At the end of a run, the glycerene is drained out of the channel and the cross-sectional shapes of the channel at a number of sections were measured. The glycerene level at these sections was also noted prior to shutting off the flow. From

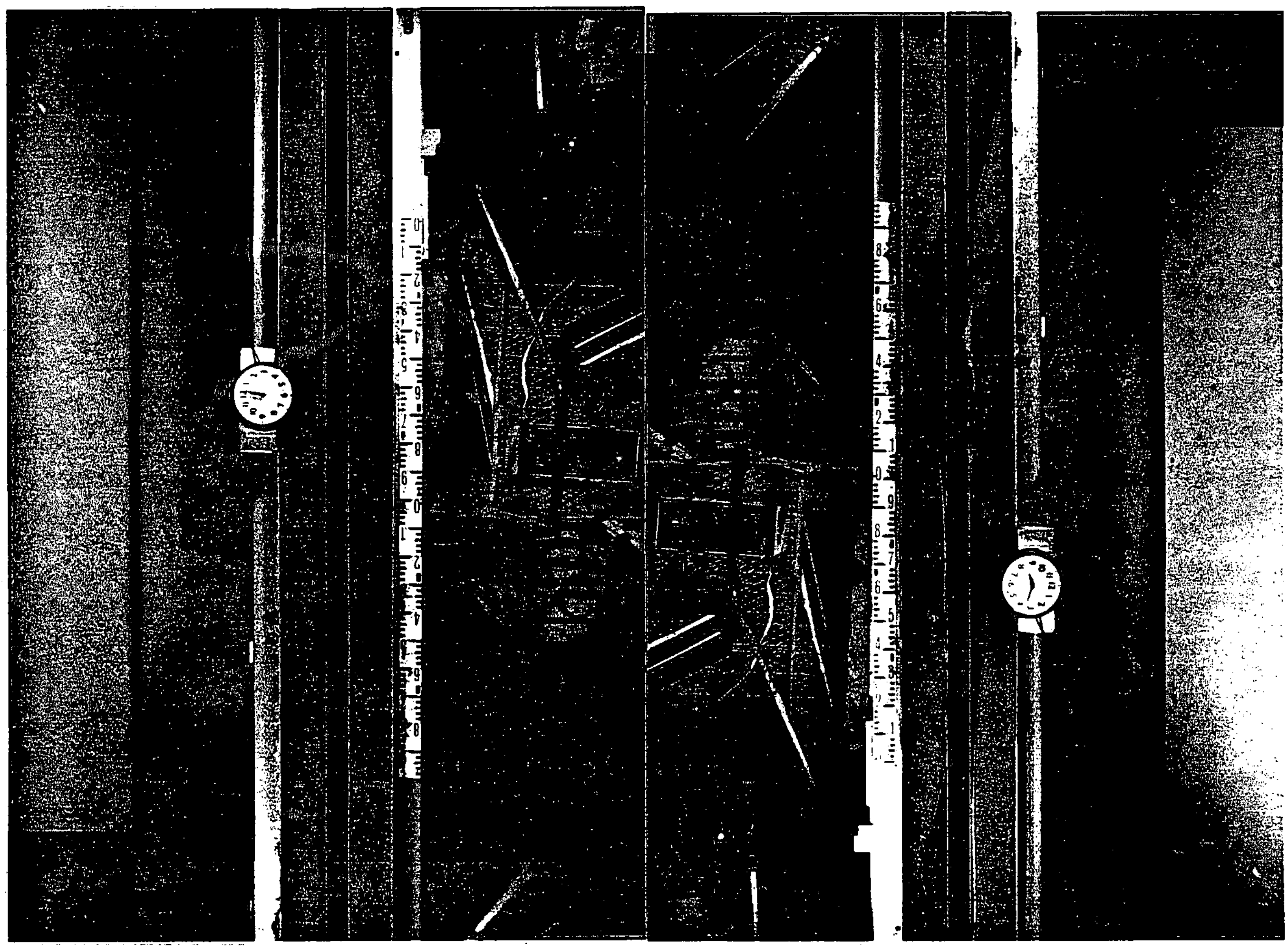

Fig. 1 Photographs showing the development of meanders for Run #1.

these, the hydraulic characteristics such as flow cross-sectional area, wetted perimeter, hydraulic radius, average flow velocity and Reynolds number were computed for all the measurement sections. These data are shown in Tables 1 to 3. Note also, the reach average values for these parameters at the bottom row of each table. The Reynolds number for these three runs range from 1.79 to 2.64.

Based on the present experiments, it can be concluded that the meanders do form in laminar flows and that the presence of turbulence is not an essential requirement for the formation of meanders. The possibility of meander development in laminar flows can be predicted using the stability analysis adopted by the second group of studies mentioned earlier. For example, the analysis carried out by Callendar (1969) which was originally developed for turbulent flows can be extended to laminar flows since the governing equations used can also describe laminar flows and the relationships for sediment transport rate and friction factor are of a general nature. The analysis of Callendar indicated that all feasible channels are unstable. The criterion for the stable channel derived by Callendar takes the following form:

$$[1] \qquad \frac{m_1 V}{t_o} > (\frac{1}{2} N-1) + \sqrt{(N-1)}\ \mu^2\ Fo^2 + (\frac{1}{2})\ N^2$$

TABLE 1: Hydraulic Data of Run No. 1

Flow Rate: 187.2 cc/sec

Slope: .0015

Density of Glycerene: 1.261 gm/cc

Kinematic Viscosity of Glycerene (ν): 3.40 cm^2/sec

Station Number	Flow Cross-Sectional Area, A in cm^2	Wetted Perimeter P in cm	Hydraulic Radius R in cm	Flow Velocity V in cm/s	Reynolds Number Re= VR/ν
1	15.2	23.7	0.64	12.32	2.32
2	16.1	17.9	0.90	11.63	3.08
3	16.0	21.3	0.75	11.70	2.60
4	16.5	18.3	0.90	11.35	3.00
5	16.0	24.9	0.65	11.70	2.24
6	17.9	20.2	0.89	10.46	2.74
7	19.5	23.9	0.81	9.60	2.29
8	15.7	18.2	0.87	11.92	3.05
9	16.7	22.9	0.73	11.21	2.41
Reach average Values	16.6	21.3	0.79	11.32	2.64

TABLE 2: Hydraulic Data of Run No. 2

Flow Rate: 175.3 cc/sec

Slope: .0015

Density of Glycerene: 1.261 gm/cc

Kinematic Viscosity of Glycerene (ν): 3.40 cm^2/sec

Station Number	Flow Cross-Sectional Area, A in cm^2	Wetted Perimeter P in cm	Hydraulic Radius R in cm	Flow Velocity V in cm/s	Reynolds Number Re= VR/ν
1	18.3	20.4	0.90	9.58	2.54
2	16.9	18.0	0.94	10.37	2.87
3	18.7	22.9	0.81	9.37	2.23
4	18.8	17.8	1.06	9.32	2.91
5	19.8	25.4	0.78	8.85	2.03
6	18.5	30.6	0.60	9.47	1.67
7	19.7	25.2	0.78	8.90	2.04
8	17.6	17.8	0.99	9.96	2.90
9	19.8	24.0	0.82	8.85	2.13
Reach average Values	18.7	22.5	0.85	9.41	2.37

TABLE 3: Hydraulic Data of Run No. 3

Flow Rate: 108.6 cc/sec

Slope: .0015

Density of Glycerene: 1.261 gm/cc

Kinematic Viscosity of Glycerene (ν): 3.40 cm^2/sec

Station Number	Flow Cross-Sectional Area, A in cm^2	Wetted Perimeter P in cm	Hydraulic Radius R in cm	Flow Velocity V in cm/s	Reynolds Number Re= VR/ν
1	14.7	21.1	0.70	7.39	1.52
2	14.1	16.1	0.88	7.71	1.99
3	14.0	20.4	0.70	7.76	1.60
4	13.9	16.9	0.82	7.82	1.89
5	12.3	19.0	0.65	8.83	1.69
6	13.3	15.1	0.88	8.17	2.11
7	14.3	19.7	0.73	7.60	1.63
8	12.5	15.2	0.82	8.69	2.10
9	14.4	19.8	0.73	7.54	1.62
Reach average Values	18.7	18.1	0.77	9.41	1.79

where

m_1 is the gradient of the shear stress vs veloctiy relationship

V is the unperturbed flow velocity in the longitudinal direction

τ_o is the bed shear stress of unperturbed flow

N is the exponent in the relationship between sediment transport rate and the flow velocity. (Callendar expressed the sediment transport rate as veloctiy raised to Nth power).

μ = (nπho/2bS_o, where h_o is the unperturbed flow depth, S_o is the slope of the channel, b is the half width of the channel and n is an integer taking odd number such as 1,3,5,...,etc., and

$F_o = U/\sqrt{gh_o}$, the Froude number of the unperturbed flow.

For laminar flows, the left side of the inequality (1) takes a value of unity and hence if the value of N is greater than two then the inequality is not fulfilled implying that the channel will be unstable, i.e. a perturbation will grow in time leading to meander formation.

At present, data on sediment transport rate for laminar flows are not available in the literature. Therefore, it is difficult to draw conclusions as to whether the exponent N would exceed the value of two or not. The values of N for turbulent flows range between 3 and 5.94 (Meyer-Peter and Müller bed-load equation gives a value of 3;

Bagnold's total load equation gives a value of 4 and the value deduced by Callendar is 5.94). Intuitively, one would expect a higher value of N for laminar flow than for the turbulent flow. No attempt was made in the present study to establish the value of N for laminar flow as the main thrust of the study was to find out whether meanders will form in laminar flows or not in order to draw conclusions regarding the role of turbulence in formation of meanders.

Since the work of Callendar, several papers on the stability of mobile boundary flows have appeared in the literature. A partial list of such studies has already been given in the earlier section. The most noteworthy contribution is the analysis of Parker (1976) in which he had shown that the instability is not inherent in the flow alone and that the presence of sediment transport is a necessary condition for its occurrence. Parker has also shown that the instability analysis can be applied to explain the meandering of supraglacial melt streams (Parker, 1975) and of the oceanic currents such as gulf stream (Stommel, 1965). For the meandering of supraglacial melt streams, Parker (1975) had shown that the differential freezing and melting is a necessary condition for instability similar to the role of sediment transport in alluvial rivers. The above results led Parker to postulate that for meandering a "third effect" is required in addition to potential (inertial and gravitational) and frictional effects. He identified this third effect for various meandering processes as follows: sediment transport for alluvial stream meandering, heat transfer for supraglacial meltwater streams, Coriolis force for gulf stream meandering and the surface tension for Gorycki's (1973) stream of a few millimeters wide on inclined plastic plates.

Even though the instability concept offers a convincing argument for the formation of meanders, it does not provide answers for such important practical questions as time scales for the lateral and longitudinal migration of meander patterns, and the shape of equilibrium plan form if such a plan form exists. Answers to such questions require further research on bank erosion and, in general, a better understanding of the sediment-flow interaction.

Summary and Conclusions

An attempt has been made in this study to answer the question: "Can meanders form in laminar flows?". Laboratory experiments were carried out using glycerene as fluid medium to produce laminar flows in a straight channel excavated in a sand layer. The experiments showed that meanders do form in laminar flows. Such a finding is in direct contradiction with some of the earlier studies which suggested that the turbulence-driven secondary circulation are responsible for the initiation of meander formation. Other mechanisms such as fluvial instability should be examined more thoroughly for explaining the formation of meanders.

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List of Symbols

A : flow cross-sectional area

b : half-width of the channel

F : Froude number

h : flow depth

m_1: gradient of the shear stress V_s velocity relationship

N : exponent of a relationship between sediment transport rate and velocity

P : wetted perimeter

Q : flow rate

R : hydraulic radius

Re: Reynolds number

S : slope of the channel

V : flow velocity

ν : kinematic viscosity

τ : shear stress

Subscript o refers to unperturbed flow conditions.

FIGURE CAPTION

LIST OF TABLES