Control No. AP268

MIXING IN RESERVOIRS

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February 1987
NWRI Contribution #87-33

MANAGEMENT PERSPECTIVE

This paper is the first reporting of Task Force on Reservoir Mixing of the Canadian Society for Civil Engineering commissioned by the Society's Hydrotechnical Committee and is intended to be preliminary in nature. This review draws heavily from the results of previous Departmental studies of the Great Lakes and lakes of the Pacific and Yukon Region particularly those concerned with the simulation of the temperature, dissolved and suspended solids distributions in lakes over long time periods. This paper points out that mixing processes near the lake surface and bottom are sufficiently well understood to provide reliable specifications of the mixing process but in the lake interior away from boundaries mixing formulations are largely ad hoc or at best semi-empirical.

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ABSTRACT

Knowledge of mixing processes in reservoirs, lakes and cooling ponds is vital to the accurate prediction of water quality both in the main body of the reservoir and in regions downstream of the reservoir particularly over time scales spanning several days or longer. An overview of the current state of knowledge of mixing processes in reservoirs is presented. The physical principles for mixing in various classes of reservoirs are described and an application of several reservoir mixing models to a large lake is compared with field observations over a long-term simulation period.

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NWRI Control No. 87-33

MÉLANGE DANS LES RÉSERVOIRS

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SOMMAIRE

Il est essentiel de connaître les processus de mélange dans les réservoirs, les lacs et les étangs de refroidissement pour être en mesure de prévoir avec précision la qualité de l'eau dans les réservoirs aux-mêmes ou dans les régions en aval des réservoirs, surtout s'il s'agit d'une période de temps qui s'étend sur les plusieurs jours ou plus. Nous présentons donc ici un tableau général de l'état actuel des connaissances sur les processus de mélange dans les réservoirs, une description des principes physiques à la base des processus de mélange dans diverses catégories de réservoirs et enfin, une comparaison entre l'application dans un grand lac de plusieurs modèles de mélange et l'observation sur le terrain d'une simulation d urant une longue période.

INTRODUCTION

In lakes and reservoirs the transport of pollutants through the aquatic system is of major concern. The distribution of nutrients and contaminants is determined by such physical processes as lake circulation, ambient light levels, thermal stratification and mixing processes. Of these physical processes the last process has perhaps the most influence particularly when there is concern for long-term effects. Attention is focussed on this last factor which will be dealt with in the report of the task group on mixing processes in reservoirs. This report is intended to be preliminary in nature with a more in depth state-of-the-art review to follow in the future.

Besides reviews of mixing processes in closely related fields such as river mixing (Elhadi et al. 1984), an excellent introduction and a review of turbulent mixing processes in reservoirs is given in Fischer et al. 1979 and also Turner (1981). A more recent review has been published by Blumberg (1986). In contrast to these more general reviews the present report will attempt to concentrate on mixing processes in the vertical direction as it is these processes that are vital to the one-dimensional simulation models presently within computational feasibility for long-term studies and on the evaluation of the sensitivity of the prediction of the vertical distributions of temperature, salt and suspended sediment in a large lake to vertical mixing formulations.

REVIEW OF RESERVOIR MIXING

In the case of molecular diffusion it is well known (Fischer et al. 1979) that the diffusive flux of a substance is given by Fick's law which expresses the flux in terms of the product of molecular diffusivity and the spatial concentration gradients. In environmental application, however, this concept may no longer be valid since the flow is generally turbulent. Essentially two approaches have evolved to deal with engineering problems involving environmental flows, the eddy diffusion hypothesis and the integral or mixed layer approach.

Integral Mixing Models

The underlying hypothesis is that quantities in a turbulent flow may be expressed in terms of a time-averaged component plus a deviation. One half of the sum of the squares of the deviations of turbulent velocity components is termed the turbulent kinetic energy and may be expressed explicitly in terms of correlations between velocity and turbulent pressure and density fluctuations as well as the viscous dissipation of turbulent kinetic energy. A detailed derivation of this equation is given in standard texts such as

Tennekes and Lumley (1972). It is frequently observed that a homogeneous surface layer forms in lakes and reservoirs by the action of vigorous wind stirring or by convective cooling. The integration of the turbulent kinetic energy equation over this uniform "mixed layer" has been described in detail in Fischer et al. (1979) and more penetratingly by Imberger and Patterson (1981). When the integrated turbulent kinetic equation is combined with the conservation of heat equation and the momentum equation, prognostic equations for the thickness and temperature of the mixed layer result in terms of such near surface processes as stirring by wind shear and penetrative or convective cooling, adsorption of short wave radiation, shear induced stirring at the base of the mixed layer and the decay of the turbulent kinetic energy by molecular dissipation. Parameterizations of the above processes leading to the production and decay of turbulent kinetic energy typically result in a set of four constants of proportionality characterizing the efficiencies of the conversions of energy from one form to another. These unknown coefficients are supplied from either laboratory or oceanographic observations and are thus not usually considered to be adjustable for reservoir applications. Mixed layer models have worked well in lakes and reservoirs in temperate climates (Patterson et al. 1984), but of course are not valid outside of the mixed layer which may occupy only a small fraction of the water column during the stratified season in deep lakes. In a unique study Ivey and Patterson (1984) attempted to apply the mixed layer concept to a near bottom layer in Lake Erie referred to as the benthic boundary layer. As yet this model has not been developed to the practical point where independent estimates of the bottom stress from direct measurements of bottom flows are not required.

Another drawback of the mixed layer approach is that momentum equations must be solved concurrently with the mixed layer equations in order to supply the shear required for production of turbulent kinetic energy within the mixed layer. To date the momentum equations have been solved only for two idealized cases, one for the open ocean (Pollard et al. 1973) who ignored the presence of horizontal pressure gradients but retained Coriolis forces and by Spigel and Imberger (1980) who ignored the earth's rotation but allowed for horizontal pressure gradients required by no mass flux through the reservoir boundaries over the first one quarter period of the internal seiche. More work needs to be done to estimate the shear at the base of the mixed in water bodies intermediate in size between these two extremes.

Another problem frequently encountered in Canadian lakes and reservoirs is that the mixed layer does not exist during the most stratified period of the year. This occurs in reservoirs in basins containing glaciers where the main inflow is delayed well into the stratified season instead of in early spring. The vertical velocities induced near the surface by the insertion of more dense river inflows at intermediate depths often exceed the mixed layer entrainment velocities until either flows drop or vertical entrainment is augmented by penetrative convection in the autumn.

Another aspect of mixed layer dynamics of special interest in Canada is mixing under ice cover in late winter caused by the warming of near surface waters below the temperature of maximum density of 4°C by the adsorption of short wave radiation. This process has been successfully simulated by the mixed layer approach in Babine Lake by Patterson and Hamblin (1985).

Finally it is possible theoretically to have internal mixed layers caused by insertion of sediment ladden river inflows into a water column near 4°C. Near this temperature realistic concentrations of suspended sediment can overwhelm the thermal stratification and cause density instabilities which could lead to an internal mixed layer. Since we know of no field observations supporting this phenomenon we demonstrate in Figure 1 the development of such a layer by a series of staggered suspended sediment profiles in which a flow containing suspended sediments is permitted to settle out at a velocity of 10^{-4} m/s. In the mixing model unstable layers are combined volumetrically with the adjacent lower layer until a stable

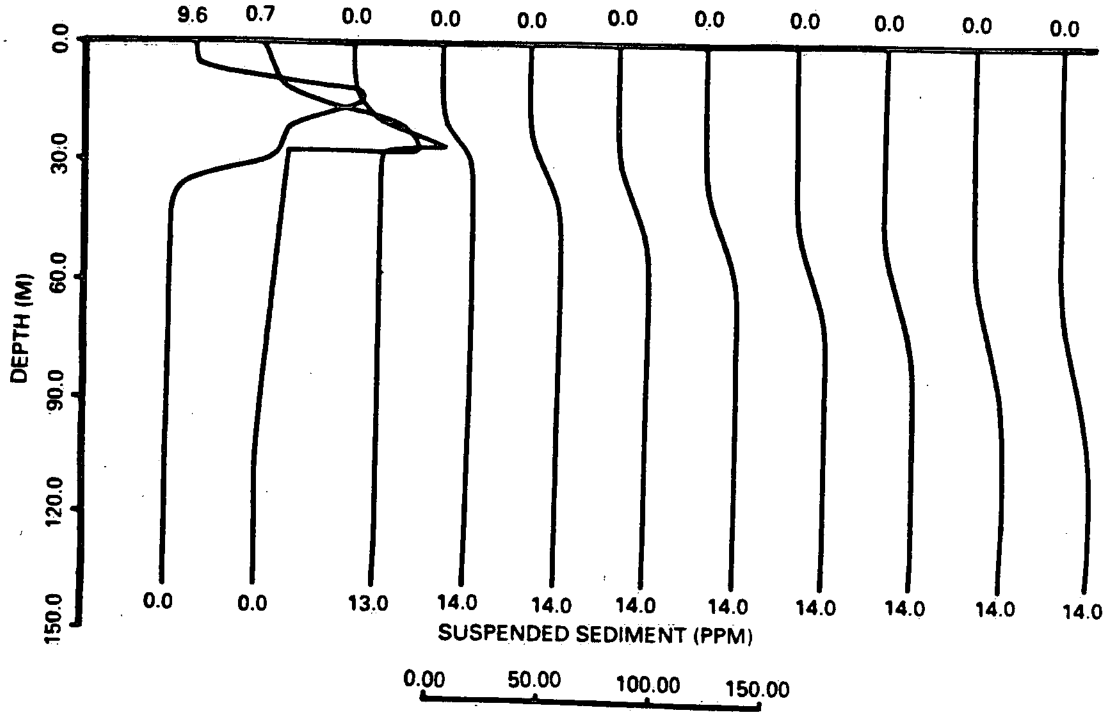

Figure 1 Evolution of daily profiles of suspended sediment for near surface inflow of suspended sediment on the initial day for a vertical settling velocity of 10^{-4} m/s.

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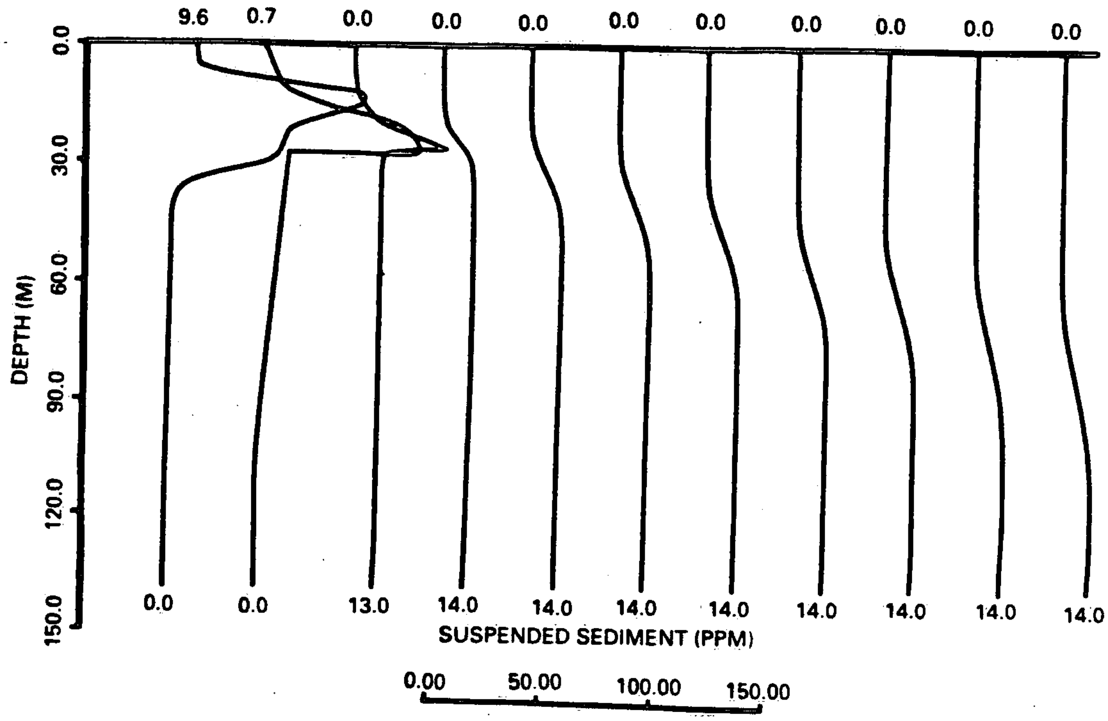

Figure 1 Evolution of daily profiles of suspended sediment for near surface inflow of suspended sediment on the initial day for a vertical settling velocity of 10^{-4} m/s.

profile is obtained. This simple mixing strategy is often termed convective adjustment in the literature. Wei and Hamblin (1986) added suspended sediment concentrations to other components of water quality in a mathematical model.

Due to the limitations of mixed layers discussed above but principally because they account for mixing over a small portion of the stratified water column, other methods are required to specify mixing in the interior of the water column. Before discussing mixing in the interior of the water column there is one interior mixing process, mixing due to interfacial shears, that is usually included with mixed layers. When the shear between the mixed layer and the underlying layer increases the flow becomes unstable when the minimum gradient Richardson number falls below 1/4. Mixing of the water column proceeds by this instability or billowing process over a depth interval sufficiently large to keep the Richardson number greater than 1/4 for the shear and density stratification appropriate to the base of the mixed layer. While this mixing process probably occurs at other locations in the interior from time to time, normally the shear is not known. Usually the billowing length scale is small so this process does not account for much mixing.

Empirical Methods

Empirical methods are based upon the analogy with molecular diffusion except that the diffusivity becomes an eddy diffusivity for turbulent flows. It is instructive to point out the range of vertical eddy diffusivities that have been inferred from field measurements of the thermal structure and introduced tracers. Near the surface mixing coefficients range from 10^{-2} to 10^{-3} m^2/s and in the hypolimnion range from molecular up to 10^{-5} m^2/s (Patterson et al. 1984). Various empirical representations of the vertical eddy diffusivity have been used, for example Kilworth and Carmack (1979). In small and shallow reservoirs these arbitrary specifications of eddy diffusivity have worked well. In these reservoirs either the near surface mixing is determined by the mixed layer dynamics and the bottom mixing by the inflow or vertical advection caused by surface inflows coupled with bottom withdrawal dominates vertical diffusion (Harleman 1986). The less than satisfactory results from these ad hoc specifications of mixing in deeper and larger water bodies have lead to the development of semi-empirical parameterizations of mixing.

In one such approach prescription for the eddy coefficient is in terms of the conventional mixing length hypothesis and a turbulent velocity scale. In turn, these properties are obtained from the solution of equations based on parameterization of higher order correlations in terms of primary variables. Similar to the mixed layer approach this method relies upon the determination of a variety of empirical constants. One member of the family of closure schemes, the so called K-ε model, has been applied to the seasonal development of thermal stratification in a lake by Svensson (1978). In this method prognostic equations for the turbulent kinetic energy, K, and the dissipation of the turbulent energy, ε, are developed. The near surface layer was well represented in his simulations and as such the K-ε method could be an alternative to the mixed layer approach. However, the K-ε model failed to account for the vertical flux of heat across the thermocline region and in the hypolimnion or deeper layer. Perhaps a major weakness of this type of mixing model is that the unknown coefficients have been determined for boundary layer flows mainly in the laboratory and so may not represent the nature of turbulent mixing away from boundaries in the environment.

Another semi-empirical method has been advanced by Fischer et al. 1979 who state that from a number of field studies correlating the vertical diffusivity with the stability frequency, $N=\sqrt{g/\rho \; \partial\rho/\partial z}$, have shown a dependency of the the form $N^{-.5}$ to N^{-4} depending on the water body considered. Here g is gravitational acceleration and ρ water density. They present an argument for N^{-2} dependence based on the hypotheses that a fraction of the turbulent kinetic energy dissipated goes to increase the potential energy during the mixing process and that the dissipation is independent of depth. They also suggest an energy conversion efficiency of about 5%. When the stability frequency is small ε becomes unrealistically large. Therefore, it may be necessry to cut the diffusivity off at an upper limit of 10^{-5} m^2/s. In summary, the vertical diffusivity, ε_z,

[1] $$\varepsilon_z = .05\,(\tau/\rho)^{1.5}\,A_s/VN^2 \text{ if } \varepsilon \text{ is } < 10^{-5}\ m^2/s.$$

where τ is the wind stress, A_s the surface area and V the basin volume.

While the physical basis for this mixing formulation first postulated by Ozmidov (1965) appears to be reasonable the weakness is the assumption of uniform dissipation of wind stirring energy plus potential energy released from plunging river inflows. In the ocean Caldwell and Dillon (1979) found that dissipation of turbulence decreased by at least an order of magnitude from 10 to 40 m depth. It seems reasonable to expect a similar decrease in lakes and reservoirs.

Harleman (1986) proposes a parameterization of the hypolimnetic vertical diffusivity which captures the episodic wind mixing events similar to the above Ozmidov formulation of Fischer et al. (1979) but excludes the dependency on local stratification. His proposal may be viewed as the assumption that the local dissipation correlates with N^2. Instead of the local stratification the effect of the stratification of the entire lake is included in terms of the potential energy of the stratification, P_s. Harleman's expression is

[2] $$\varepsilon_z = \frac{C\ \rho\ (\frac{\tau}{\rho})^{1.5}\ As\ h^2}{P_s}$$

where $P_s = \int_o^h (\overline{\rho} - \rho(z))\ gz\ Adz$

where $\overline{\rho}$ is the volumetric mean density and C is dimensionless constant which was not specified by Harleman. Harleman has pointed out that if P_s is small or zero it is necessary to specify an upper bound for ε_z. In a section to follow simulations based on the hypolimnetic mixing according to expressions (1) and (2) are compared to field observations in a deep lake.

Other Mixing Processes

Mixing by buoyancy driven bottom inflows is discussed in depth by Fischer et al. (1979) which takes place through the entrainment of lake water into the more turbulent inflowing plume. They caution that for rivers with high entrance Froude numbers mixing at the plunge point becomes appreciable and that this type of inflow mixing has not been successfully parameterized. Formulae for low Froude number inflow mixing are presented as a function of bottom drag coefficient, bed slope and the base angle of the triangular river cross section. Vertical mixing by possible shear induced by the inflow intrusion has been ignored.

In the vicinity of outflows the turbulence induced by flow converging into outflows is implicitly taken into account in outflow dynamics discussed by Fischer et al. (1979) and by Imberger (1980). Since in general outflows are dominated by advection and stratification outflow induced mixing is relatively weak in reservoirs but may be more important in lakes where the outflows interact with bottom bathymetry.

In the Canadian context the influence of ice and snow cover on reservoir mixing is worthy of mention. The principal effect of the cover is to cut off the action of the wind stirring and penetrative convection. It is likely that mixing rates approach molecular values in lakes with complete ice cover except for those having strong winter through flows. If the snow cover is not too deep short wave radiation under certain circumstances induces mixing beneath the ice cover as discussed in Babine Lake by Patterson and Hamblin (1985).

Another kind of mixing found in Canadian water bodies is the convective instability that depends on the density extremum of fresh water at 4°C. When two water masses meet with temperature differences straddling the temperature of maximum density, the 4°C water will sink forming large vertical eddies adjacent to the sinking zone. A dramatic example of this phenomenon known as the thermal bar is discussed by Carmack (1979) which originates from horizontal temperature gradients maintained by river inflow. In connection with winter thermal gradients established by heated discharges into lakes, Marmoush et al. (1984) have reproduced the thermal bar in the laboratory.

Carmack and Farmer (1982) have pointed out the importance of the decrease in the temperature of maximum density due to hydrostatic pressure in the mixing of deep cold lakes with particular reference to observations in Babine Lake. The mixing mechanism due to this effect is not understood at present and is unlikely to be parameterized in terms of a simple one-dimensional mixing model.

APPLICATION

In this section we compare the application of the two hypolimnetic mixing formulations with selected field observations over a one year simulation period for a large European lake. The details of the field observation of Lake Constance and the simulation model are given in Hollan et al. (1987). The coefficient C in equation [2] has been set to unity, the depth h is taken as the mean depth and the total diffusivity is the sum of the eddy diffusivity either given by equation [1] or [2] and the appropriate molecular diffusivity for each of the three water quality constituents, heat, salt and suspended sediment.

Figure 2 shows the simulated profiles 110 days after the initialization of the model on February 1, 1979. It is evident that even though the diffusivity given by equation [1] is much larger than by equation [2], more heat has diffused downward near the surface resulting in better agreement. This must have occurred earlier when the potential energy of the stratification was less. There is an indication that the downward fluxes while good near the surface are somewhat too large near the bottom in the Harleman formulation. No attempt was made to limit the diffusivity as suggested by Harleman.

Somewhat later, on June 19, 1979, the Ozimidov formulation results in much larger eddy coefficients but not large enough to make up the heat deficit in the mesolimnion. It is notable that the inflows are inserted at lower levels in Figure 3(b) than in Figure 3(a). This allows more suspended sediment to be settled out and be diluted by lake water before insertion. Thus the concentrations of sediment are less in Figure 3(b).

Again on August 20, the eddy diffusivity according to Harleman's expression is much less than the Ozmidov expression but the heat deficit in Figure 4(a) continues to increase whereas Figure 4(b) demonstrates excellent agreement. Also of interest is that both

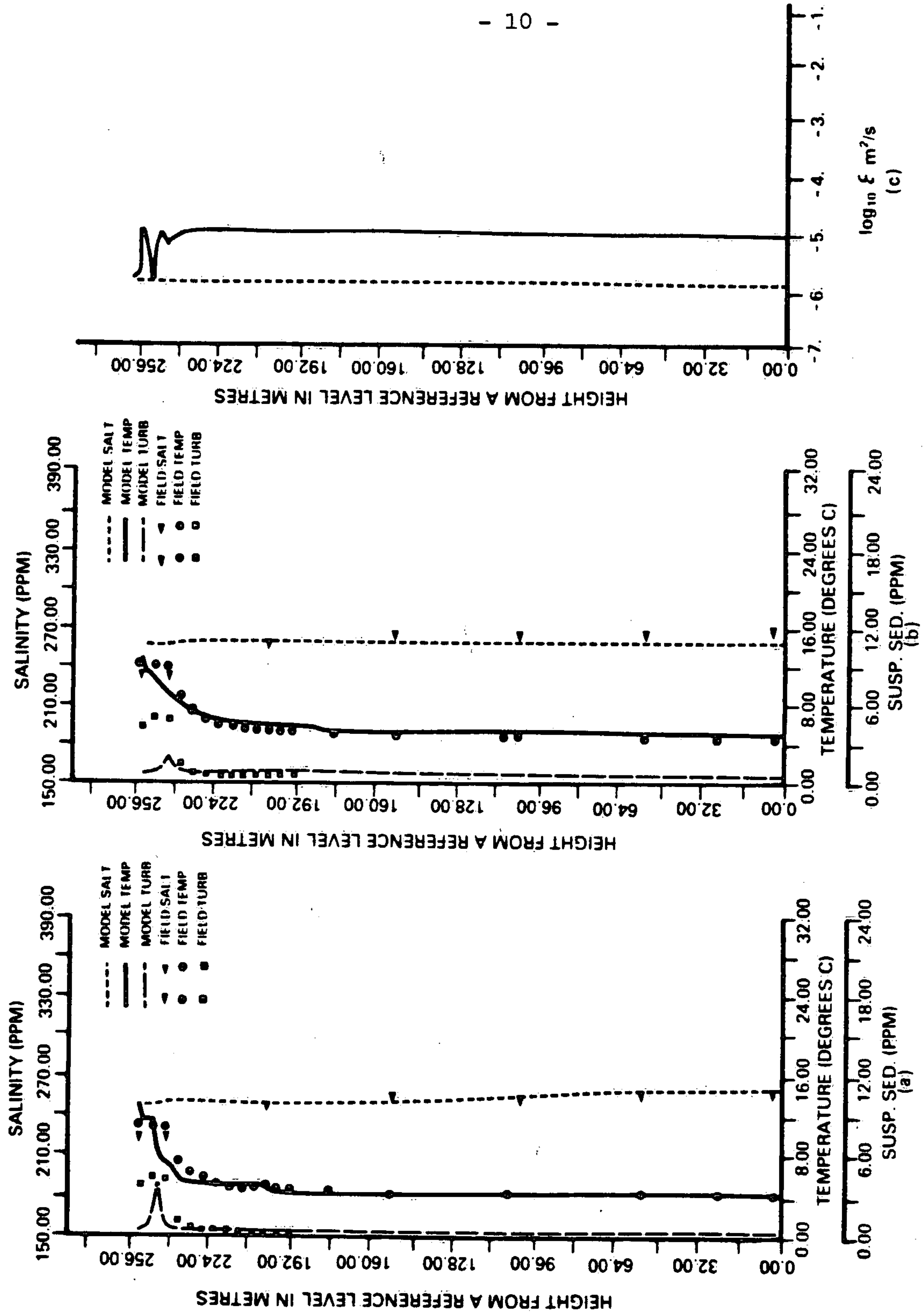

Figure 2 Calculated and observed distribution of temperature, salinity and suspended sediment for May 21, 1979 (a) mixing given by equation (1), (b) mixing given by equation (2), (c) vertical eddy diffusivity given by equation (1) solid line, equation (2) dashed line.

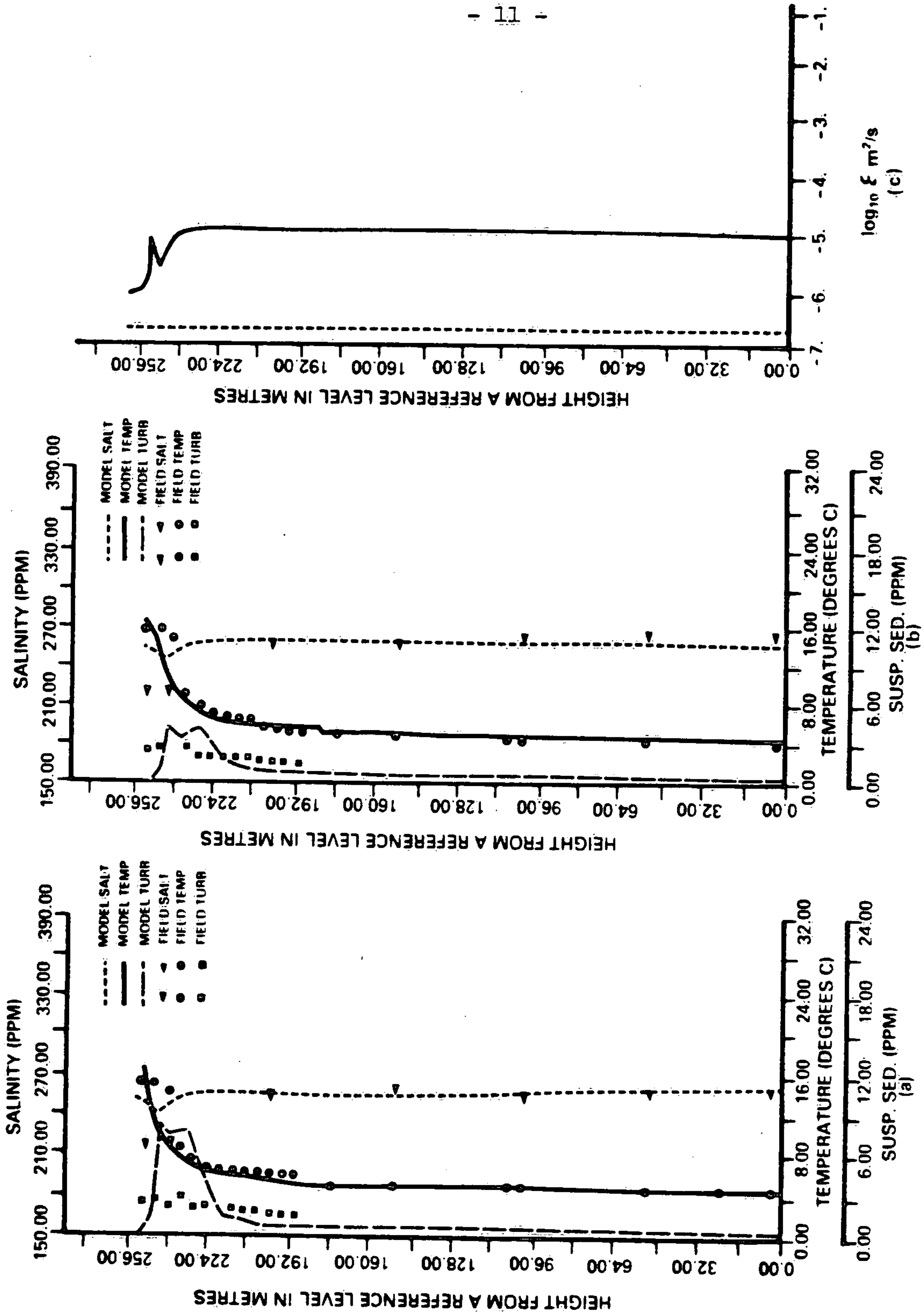

Figure 3 Calculated and observed distribution of temperature, salinity and suspended sediment for June 19, 1979 (a) mixing given by equation (1), (b) mixing given by equation (2), (c) vertical eddy diffusivity given by equation (1) solid line, equation (2) dashed line.

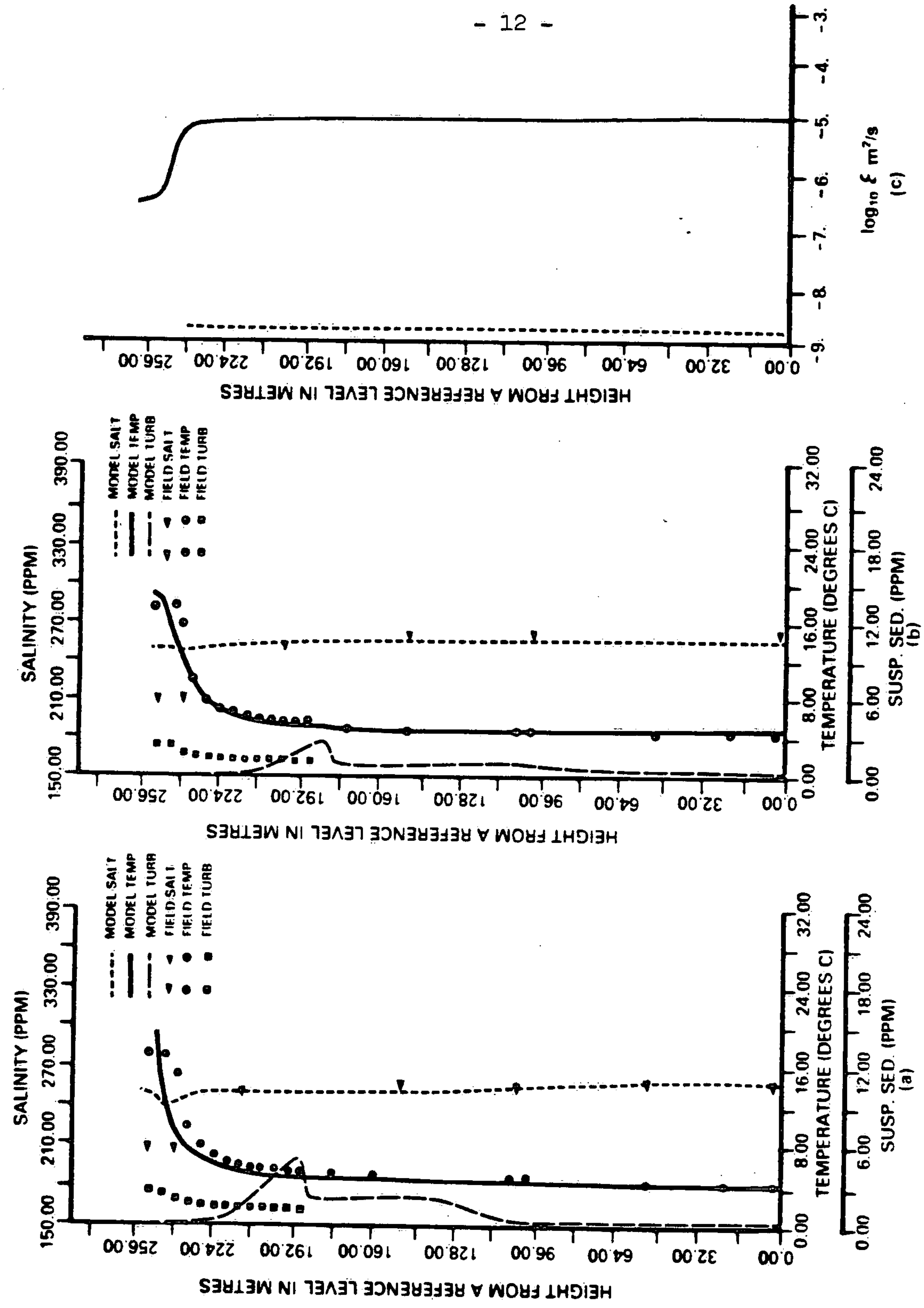

Figure 4 Calculated and observed distribution of temperature, salinity and suspended sediment for August 20, 1979 (a) mixing given by equation (1), (b) mixing given by equation (2), (c) vertical eddy diffusivity given by equation (1) solid line, equation (2) dashed line.

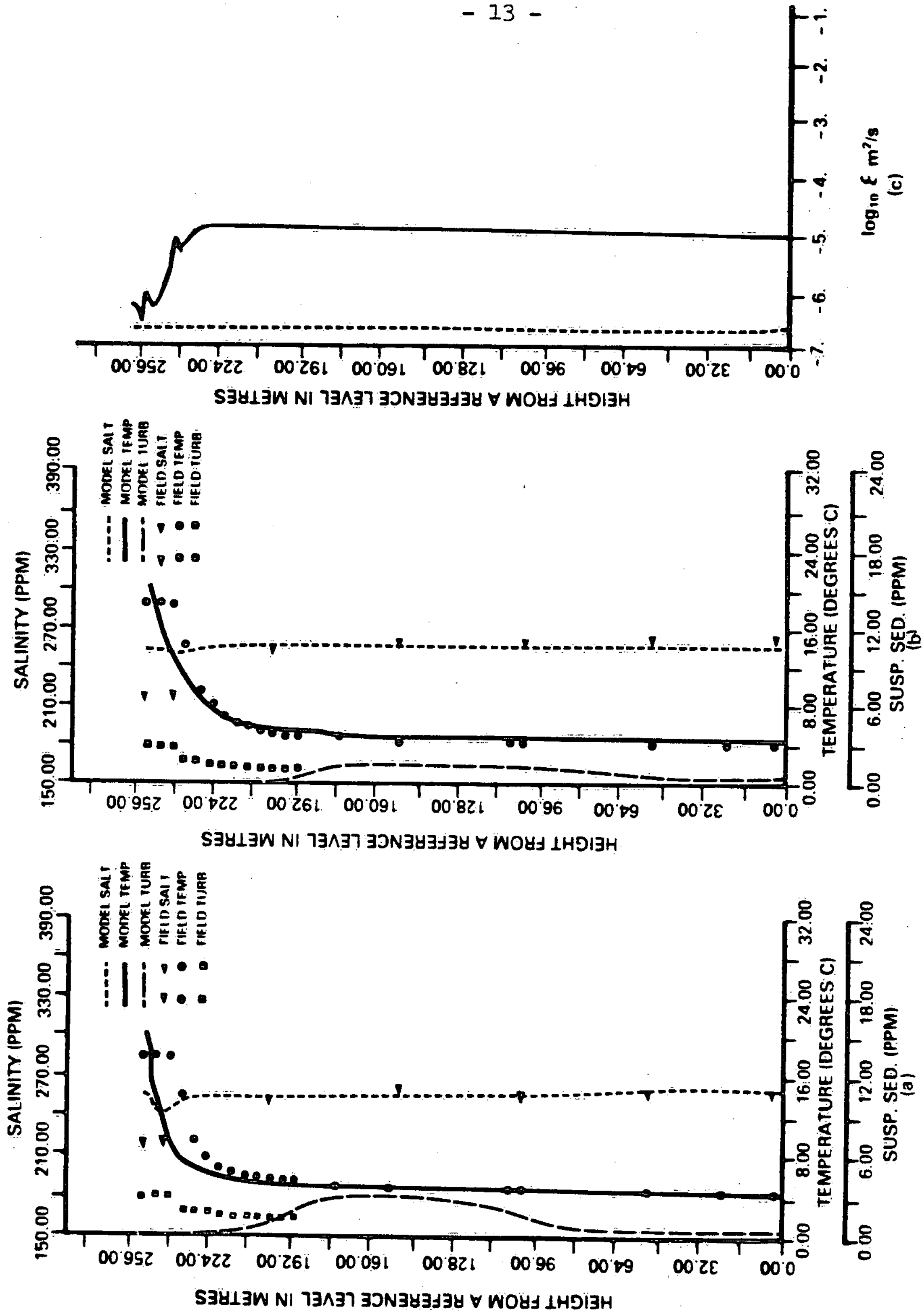

Figure 5 Calculated and observed distribution of temperature, salinity and suspended sediment for September 17, 1979 (a) mixing given by equation (1), (b) mixing given by equation (2), (c) vertical eddy diffusivity given by equation (1) solid line, equation (2) dashed line.

simulations demonstrate an internal mixed layer of the type illustrated in Figure 1.

In Figure 5 both mixing formulations result in an underestimation of heat near the surface and fail to account for the shallow mixed layer near the surface. This may be due to the use of shore-based winds which may not have captured the overlake wind field accurately.

In Figure 6(b) the autumnal mixed layer is accurately modelled whereas it is too shallow in Figure 6(a). As well, the underestimation of heat is larger in Figure 6(a).

In the case of March 20, 1980, the vertical diffusivity is larger in Figure 7(c) for the Harleman formulation unlike the previous cases. More than a year after the specification of the initial conditions and without any adjustment to model coefficients, both simulations closely match the field observations although the Harleman formulation is better. Over the entire simulation period consisting of 29 field observations, the volumetrically weighted rms temperature error for the specification of equation [1] is 0.89°C compared to 0.45°C for equation [2]. Similarly, the average error in heat content is 9% for equation [1] compared to 0.2% for equation [2]. Clearly, the specification of vertical mixing below the surface mixed layer is sensitive in water bodies with deep hypolimnia. Further study on the mixing of weakly stratified water bodies below the mixed layer is strongly recommended.

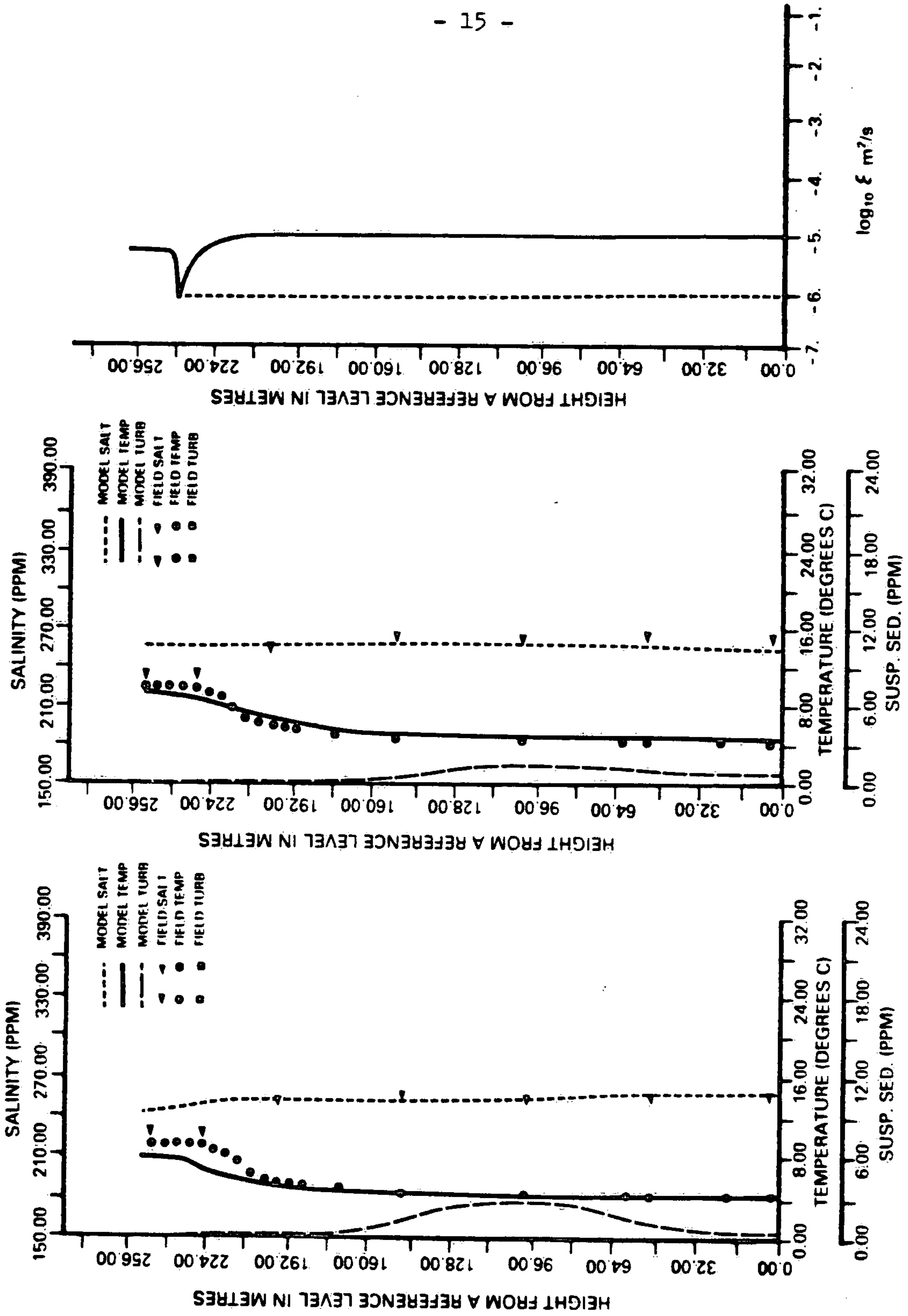

Figure 6 Calculated and observed distribution of temperature, salinity and suspended sediment for November 13, 1979 (a) mixing given by equation (1), (b) mixing given by equation (2), (c) vertical eddy diffusivity given by equation (1) solid line, equation (2) dashed line.

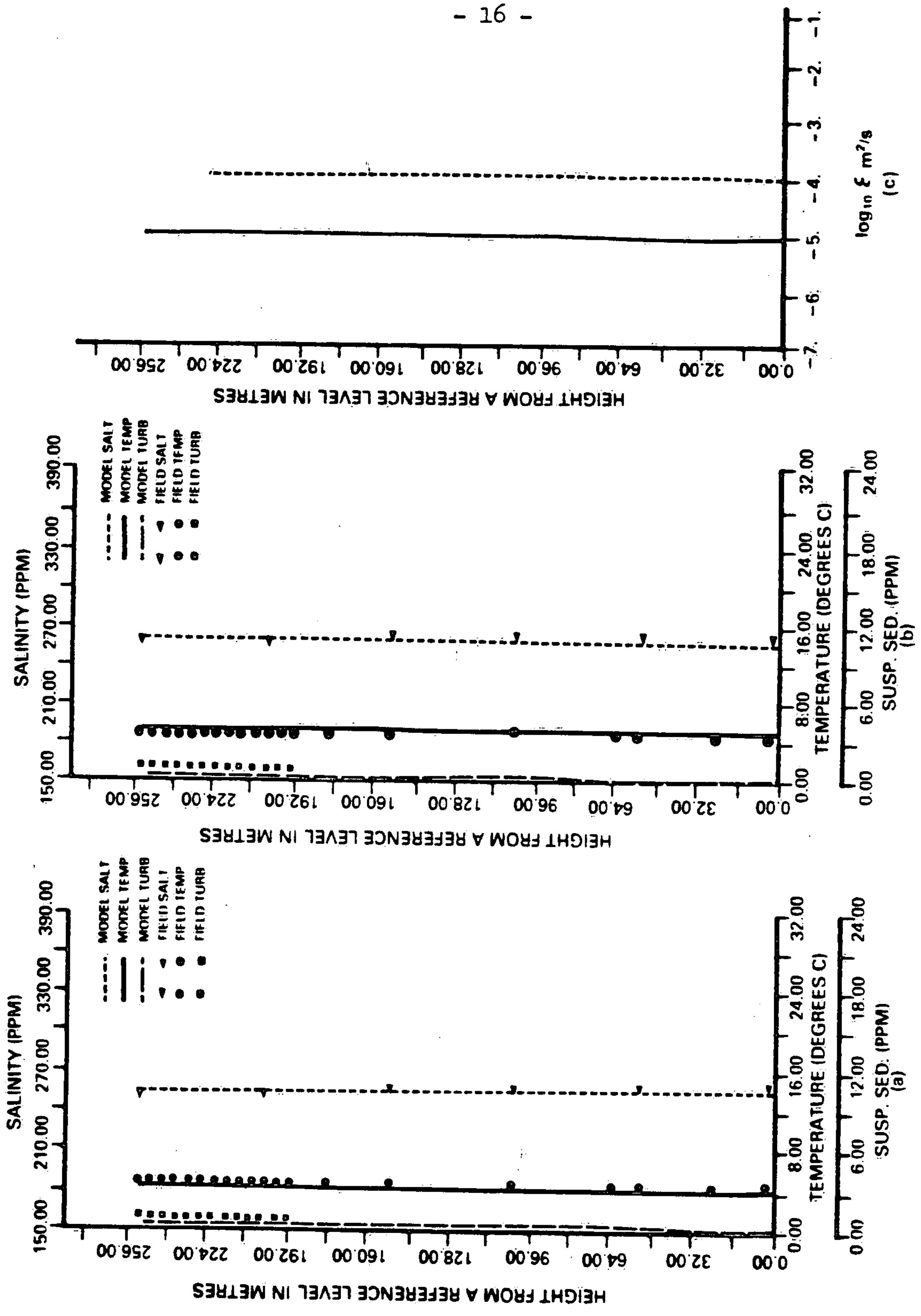

Figure 7 Calculated and observed distribution of temperature, salinity and suspended sediment for March 20, 1980 (a) mixing given by equation (1), (b) mixing given by equation (2), (c) vertical eddy diffusivity given by equation (1) solid line, equation (2) dashed line.

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