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HYDRAULIC RESISTANCE OF RIPPLES

by

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MANAGEMENT PERSPECTIVE

The prediction of flows in a movable bed river is complicted because of the interaction between the flow and the sand waves on the bed. The flow modifies the sand waves which in turn offers a changing resistance to the flow. Relationships for the hydraulic resistance of various types of sand waves are required in order that one can predict what the flow stage and velocity will be, given a certain discharge of water. Such information is very basic for all river management studies.

Resistance relationships for sand waves in the ripple regime has hitherto not been available. This report provides such information and will be useful to all those engaged in the modelling of river flows.

PERSPECTIVE - GESTION

Il est compliqué de prévoir l'écoulement des eaux sur un lit mouvant à cause de l'interaction qui joue entre l'écoulement et les rides de sable formées sur le lit. En effet, l'écoulement modifie les rides, si bien que la résistance qu'elles opposent change. Pour prévoir le mode et la vitesse d'écoulement d'une quantité d'eau donnée, il faut connaître les relations représentant la résistance hydraulique de divers types de rides de sable. Ce genre de renseignements est essentiel dans les études d'aménagement de cours d'eau.

Jusqu'ici, on n'avait jamais formulé de relations représentant la résistance dans les cours d'eau à lit de sable ridé. C'est le genre de renseignements qu'on trouve dans ce rapport, ce qui sera utile à tous ceux qui travaillent à la modélisation de l'écoulement dans les cours d'eau.

KEY WORDS: Flow resistance; Ripples; Bed forms; Shear stress; Form losses; Friction factor

ABSTRACT: Experiments were conducted to investigate the division of the total shear stress into a friction loss and a form loss component for flows with ripples on the bed. Four different sizes of glass beads were used as bed material, each representing a different value of the dimensionless variable Ξ. The experimental evidence indicates that Y', the mobility number related to the frictional shear, is dependent only upon Y, the mobility number related to the total shear. The variable Ξ appears to have an effect only in so far as it determines the value of Y_{cr}, the critical mobility number for the initiation of sediment motion. Inspection of other published data revealed much the same dependence.

SUMMARY: The division of the total shear stress into a friction loss and a form loss component was investigated for flows with ripples. The experimental evidence was used together with other data to obtain an empirical relationship for the resistance of ripples.

MOTS-CLEFS : Résistance à l'écoulement; rides; formes du lit; contrainte au cisaillement; déformations; facteur de friction

RÉSUMÉ

On a fait des expériences pour étudier la répartition de la contrainte au cisaillement totale en perte par friction et en déformation dans des cours d'eau dont le lit est ridé. Pour constituer le lit des cours d'eau expérimentaux, on s'est servi de billes de verre de quatre grosseurs différentes; à chaque type de bille correspondait une valeur différente de la variable - (sans dimensions). D'après les résultats des expériences, Y', la mobilité en relation avec le cisaillement de friction, ne varie qu'en fonction de Y, la mobilité en relation avec le cisaillement total. La variable - ne semble avoir d'influence que dans la mesure où elle détermine la valeur de Y_{cr}, la mobilité critique à laquelle les sédiments commenceront à se déplacer. À l'examen des autres données publiées sur le sujet, on a trouvé le même genre de relation.

SOMMAIRE

On a étudié la répartition de la contrainte au cisaillement totale en perte par friction et en déformation dans des cours d'eau à lit ridé et l'on a combiné les résultats des expériences avec d'autres données pour définir une relation empirique représentant la résistance des rides.

HYDRAULIC RESISTANCE OF RIPPLES

By Y. Lam Lau[1]

INTRODUCTION

In an open channel flow with sand size materials on the bed, the total shear stress can be considered to be made of three components. One of these is the true frictional shear stress on the bed surface and the other two are "apparent stresses" which account for the resistance of the sand waves and for the drag forces experienced by the grains which are carried in suspension. The relative contribution of these three components to the total shear vary depending on the conditions of the flow.

The depth and velocity in such flows are affected by the changes in bed form roughness which in turn are modified by the changing flow conditions. Calculations of the hydraulic conditions and sediment transport require some relationship between the different components of the total shear. Many of the existing relationships for flow resistance have been reviewed by Yalin (8). One example of these is the empirical relationship between Y and Y' developed by Engelund (4) in which Y is the overall mobility number and Y' is the mobility

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number pertaining to the purely frictional component of the total shear. Given the water discharge, the bed slope and the sediment size, this relationship allows one to calcualte the flow depth and velocity, as well as the frictional shear stress, which is that portion of the total shear responsible for the transport of bed sediment.

Although many relationships for flow resistance have been put forward they are applicable only to flows with dunes and other higher flow regimes corresponding to plane bed, standing wave and anti-dunes. A relationship which is applicable for ripples is not yet available and will form the subject of this paper.

ANALYSIS

In the flow range applicable to ripples, the influence of drag forces on suspended particles is negligible. The total bed shear can therefore be written as

$$\tau_o = \tau_o' + \tau_o'' \qquad (1)$$

in which $\tau_o=\rho grS$=the total bed shear, ρ=the fluid density; g=gravitational acceleration; r=the hydraulic radius; S=the energy slope, $\tau_o'=\rho gr'S$=the frictional component of the total shear and τ_o''=the shear due to the form drag of the ripples. It is necessary to determine the relationship between τ_o and τ_o' or between u_* and u_*' where u_* and u_*' are the corresponding shear velocities. Physical reasoning indicates that, given the kinematic viscosity, ν, the fluid density, ρ, the median grain size, D_{50}, the submerged specific weight, γ_s, the flow depth, h, and the shear velocity, u_*, the value of u_*' should be completely determined. Dimensional analysis then shows that

$$Y' = f\,(Y, X, Z) \qquad (2)$$

in which $Y=\rho u_*^2/\gamma_s D_{50}$ is the mobility number, $Y'=\rho u_*'^2/\gamma_s D_{50}$ is the corresponding mobility number for the frictional component; $X=u_* D_{50}/\nu$ is the grain size Reynolds number and $Z=h/D$ is the relative depth.

Because the formation of ripples is not dependent on the depth of flow, is reasonable to assume that the relative depth, Z, can be left out of Eq. 2. For experimental investigation it is also more convenient to consider the variable $\Xi=X^2/Y=\gamma_s D^3_{50}/\rho\nu^2$ in order to isolate the effect of u_* and write

$$Y' = f(Y,\Xi) \qquad (3)$$

For bed forms in the dune regime, Engelund (4) presented Y' as a function of Y only. For ripples, however, it is obvious that both Y and Ξ should be taken into consideration. The purpose of this paper is to investigate the influence of Y and Ξ on Y'.

EXPERIMENTS

The experiments were conducted in a flume 57.3 m in length, 0.756 m in width and 0.29 m in depth. The flume slope could be adjusted with a pair of motor-driven screw jacks. The flow depth was controlled by a set of louvred gates located at the downstream end of the flume. The flume discharged into a large basin where all the sediments transported by the flow could settle out. The water was returned to the sump at the end of the basin through a triangular weir from which the discharge was measured. A variable speed pump delivered water from the sump to the flume headbox. The water temperature was constant at about 20°C.

Flex-O-Lite glass beads having a specific gravity of 2.5 were used as bed material. Four sizes of beads were used, BT5 with D_{50}=0.40 mm; BT6 with D_{50}=0.30 mm; BT10 with D_{50}=0.15 mm and BT13 with D_{50}=0.082 mm. A 6 cm thick layer of a particular size of beads was laid in the flume and was levelled at the beginning of each experiment.

The experiments were conducted without sediment being added at the upstream end of the flume. The flow was allowed to erode the upstream section and extract the sediment that it was able to carry. This created a short section of nonuniform flow near the upstream end but this nonuniform region was only several metres long. There was always a uniform flow section of at least 45 m in length. At the relatively small sediment transport rates which occurred, this method was probably more accurate than the trial and error feeding of sediments to obtain equilibrium conditions.

At the start of an experimental run the flume was set to the desired slope and the bed was levelled. The downstream gates were closed and water was gradually introduced into the flume. Discharge was gradually increased to the desired value and the gate was then slowly opened. As the bed conditions changed from plain bed to ripples and as the bed forms developed, the louvred gates were adjusted to maintain uniform flow. As the ripples grew in size, flow depth increased until equilibrium conditions were reached. The flow conditions were continually monitored to ensure that the ripples had reached equilibrium conditions and that no more changes were taking place. Depending on the slope and discharge that were set, this took several hours to several days. The equilibrium flow depth was recorded and the bedform length and height were measured. The bed was then relevelled for another experiment.

With the fluid properties being constant, each size of glass bead represents a constant value for the variable Ξ. By varying the discharge and slope but using one type of bead, the effect of Y on Y' can be observed. By changing to a different size of bead, the effect of the variable Ξ can be investigated.

A total of 35 experimental runs were made using the four sizes of beads.

DATA ANALYSIS

To obtain the values of u_*' and Y' the concept introduced by Einstein (3) was used.

The average velocity, U_m, is assumed to satisfy the logarithmic velocity distribution, i.e.

$$\frac{U_m}{U_*'} = \frac{1}{\kappa} \ln \left[0.368\, e^{\kappa B_s} \frac{r'}{k_s}\right] \tag{4}$$

in which κ is the von-Karman constant; k_s is the equivalent sand grain roughness and B_s is the constant in the logarithmic distribution. B_s is equal to 8.5 for fully rough turbulent flow but in the lower ranges of grain size Reynolds numbers in which ripples occur, it is a function of the Reynolds number.

The variation of B_s with Reynolds number is incorporated into Eq. 4 by using the following expression which was empirically fitted to the data of Nikuradse by Yalin (personal communication):

$$B_s = [5.5 + 2.5\, R]\, e^{-.217R^2} + 8.5\, [1 - e^{-.217R^2}] \tag{5}$$

where

$$R = \ln \left(\frac{u_*'\, k_s}{\nu}\right) \tag{6}$$

The equivalent sand grain roughness, k_s, for plain movable beds has been investigated by many authors. Based on the analysis of van Rijn (7), a value of $k_s = 3D_{90}$ was adopted for these calculations.

From Eq. 4, one obtains

$$U_m = \frac{u_*'}{\kappa} \ln \left[0.368 e^{\kappa B_s} \frac{r'}{k_s}\right] \qquad (7)$$

The only unknown in Eq. 7 is r' and it can be obtained by a trial and error solution, after which u_*' and Y' can be calculated.

RESULTS

The measured and calculated hydraulic variables for the 35 experimental runs are listed in Table 1.

Figure 1 is a plot of Y vs Y' for all the runs. As Y' represents the frictional component of the total shear, it is quite obvious that, prior to the initiation of sediment motion, there is no energy loss due to bed forms and that Y' is in fact equal to Y. Therefore, when Y is less than Y_{cr}, the relationship between Y and Y' should follow the 45 degree line in Fig. 1. As the critical mobility number for the initiation of sediment transport is exceeded and ripples are formed, the flow begins to experience form losses in addition to the frictional shear stress. The Y vs Y' relationship should then move above the 45 degree line. For each size of glass bead, i.e. each constant value of Ξ, there is a different value of Y_{cr}. Therefore, for the four sizes of beads, with four different values of Y_{cr}, one should expect curves departing from the 45 degree line at four different locations. An inspection of the data in Fig. 1 shows four such curves. It also reveals that as soon as Y' reaches the critical value, there is a very rapid increase in Y, corresponding to the form loss brought about by the bedforms. The most notable result is that after the initial increase in Y, the data for the four different beads fall on more or less the same line, indicating that the variable, Ξ, has very little effect on Y' except in determining the location where the curve should spring from the 45 degree line.

The data for the four different Ξ values can be collapsed onto a single curve by plotting Y/Y_{cr} versus Y'/Y_{cr}, as shown in Fig. 2. The present data fall very well on one single curve. The data show Y/Y_{cr} to increase with Y'/Y_{cr} up to Y/Y_{cr} values of about ten. However, it should be expected that as Y/Y_{cr} increases above this value, the ripples will begin to be washed out and the stress due to form losses should decrease. The curve for Y/Y_{cr} should then decrease towards the 45 degree line.

Data for ripples obtained by Alexander (1), Barton (2), Vanoni and Hwang (6), and Znamenskaya (9) have also been analysed using the present method to obtain values of Y and Y'. These data, having Ξ values ranging from 19 to 197, are also plotted in Fig. 2. The scatter is much larger than the data from the present experiments but no effect of Ξ can be detected either. There are 135 data points from ref. (1) and they are spread out through the shaded area in Fig. 2. Inspection of these sets of data for possible effects of the relative depth parameter, Z, failed to reveal any dependence.

Given the water discharge, the bed slope and the sediment properties, the curve in Fig. 2 can be used to calculate the flow depth and velocity, using a trial-and-error procedure. Assuming a value for the flow depth, one can calculate Y and then obtain a value for Y' from Fig. 2. This gives a value for r' and allows the mean velocity to be calculated using Eq. (7). The discharge calculated from this mean velocity should be equal to the given discharge. Otherwise, a different value for the depth can be assumed and the procedure repeated. Finally, the Reynolds number, X, should be calculated to verify that it is in the range where the sand waves will be ripples.

CONCLUSIONS

Analysis of the data from the present experiments indicates that, for bed forms in the ripple range, the variable Y'/Y_{cr} depends only on Y/Y_{cr} and that the variable Ξ has very little effect. The geometry of these ripples does depend on Ξ and has been shown to agree with data from other sources (5). Therefore, if Ξ has any effect of Y'/Y_{cr}, it should have shown up in the present data.

Data from other sources also indicate that Y'/Y_{cr} can be considered as a function of Y/Y_{cr} only. Although the scatter in some of the data is fairly large, no systematic dependence on Ξ can be detected. The curve of Y/Y_{cr} vs Y'/Y_{cr} must necessarily start from the origin in Fig. 2 and should approach the 45 degree line when the mobility number increases to the range where ripples begin to be washed out, as shown in Fig. 2.

ACKNOWLEDGMENTS

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APPENDIX I. - REFERENCES

1. Alexander, L.J.D., "On the Geometry of Ripples Generated by Unidirectional Open Channel Flows," thesis submitted to the Department of Civil Engineering, Queen's University, Kingston, Ontario, in 1980, in conformity with the requirements for the degree of Master of Science.

2. Barton, J.R., and Lin,, P.N., "Sediment Transport in Alluvial Channels," Report No. 55 JRB 2, Civil Engineering Dept., Colorado A & M Coll., Fort Collins, Colorado, 1955.

3. Einstein, H.A., "The Bed-Load Function for Sediment Transportation in Open Channel Flows," Technical Bulletin 1026, U.S. Dept. of Agriculture, Soil Conservation Service, 1950.

4. Engelund, F., "Hydraulic Resistance of Alluvial Streams," Journal of the Hydraulics Division, ASCE, Vol. 92, No. HY2, Mar., 1966, PP 315-326.

5. Lau, Y.L., Discussion of "On the Determination of Ripple Geometry," by M.S. Yalin, Journal of the Hydraulics Division, ASCE, Vol. 113, No. 1, Jan., 1987, pp. 128-132.

6. Vanoni, V.A., and Hwang, L.D., "Relation Between Bed Forms and Friction in Streams," Journal of the Hydraulic Engineering, ASCE, Vol. 93, No. HY3, May, 1967, pp 121-144.

7. van Rijn, L.C., "Equivalent Roughness of Alluvial Bed," Journal of the Hydraulics Division, ASCE, Vol. 108, No. HY10, Oct., 1982, pp 1215-1218.

8. Yalin, M.S., "Mechanics of Sediment Transport," 2nd Ed., Pergammon Press, Inc., London, England, 1977.

9. Znamenskaya, N.S., "Experimental Study of the Dune Movement of Sediment, "Transactions of the State Hydrologic Institute, (Trudy GGI) No. 108, 1963, pp 89-111. Translated by L.G. Robbins.

APPENDIX II. - NOTATION

The following symbols are used in this paper:

B_s = constant in the logarithmic velocity distribution;

D_{50} = grain size for which 50% of the sediment mixture is finer;

g = gravitational acceleration;

h = flow depth;

k_s = equivalent sand grain roughness;

r = r'+r"= hydraulic radius;

S = slope;

U_m = average velocity;

u_* = shear velocity;

u_*' = shear velocity related to true frictional stress;

X = grain size Reynolds number;

Y = mobility number;

Y_{cr} = critical mobility number;

Y' = mobility number corresponding to the frictional shear;

Z = relative depth;

γ_s = submerged specific weight of particles;

κ = von Karman constant;

ν = kinematic viscosity;

Ξ = dimensionless number reflecting nature of particles and fluid;

τ_o = total bed shear;

τ_o' = frictional component of the total shear; and

τ_o'' = apparent shear stress due to form losses

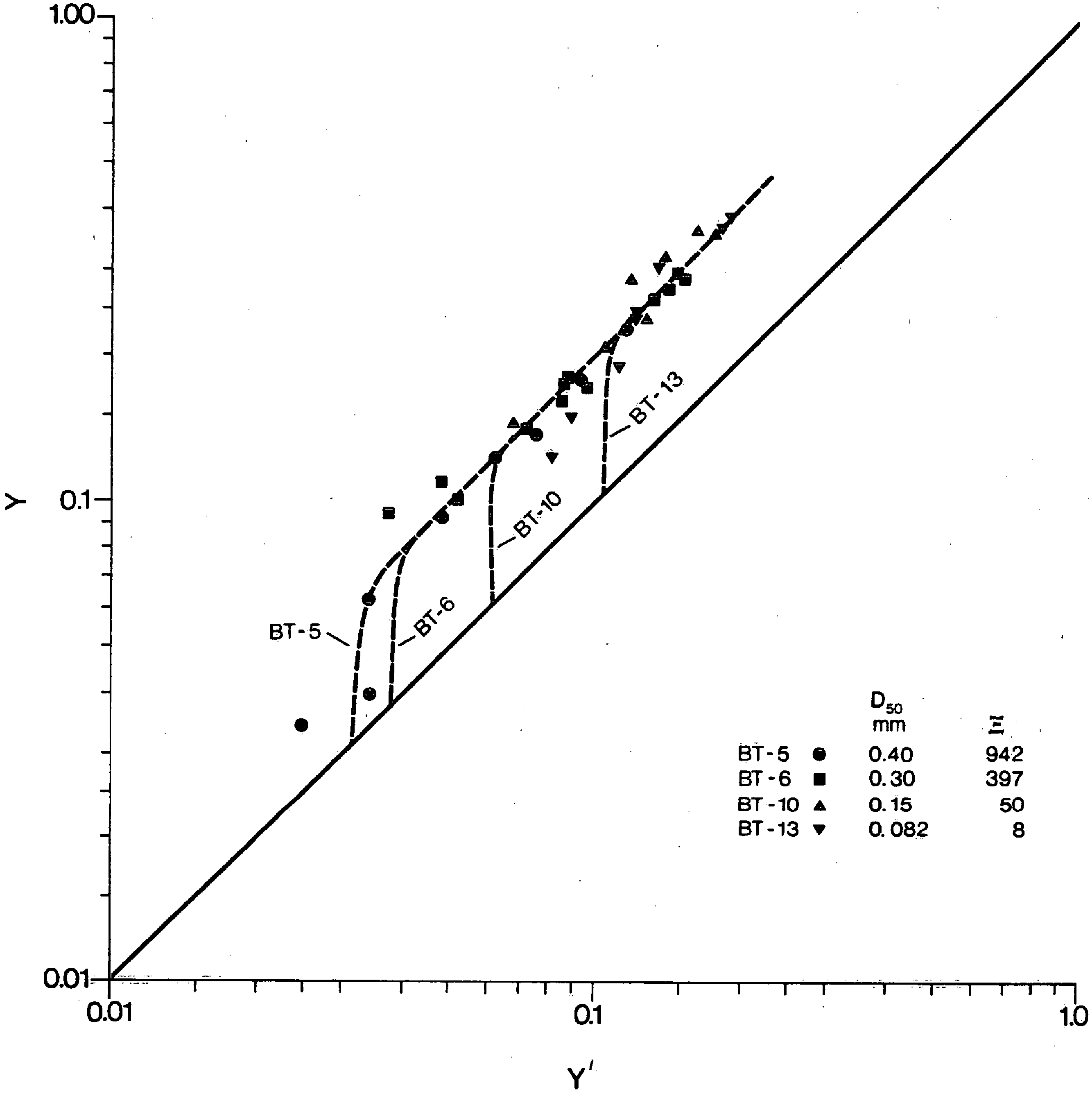
TABLE 1. - Summary of Hydraulic Data

Average Velocity (m/s) (1)	Slope (2)	Flow Depth (m) (3)	r (m) (4)	r' (m) (5)	X (6)	Y (7)	Y' (8)	Y/Y_{cr} (9)	Y'/Y_{cr} (10)
(a) BT-5 D_{50}=0.40 mm; D_{90}=0.42 mm; Y_{cr}=0.031; Ξ=942									
0.243	.00026	.1233	.0930	.0802	6.11	.0398	.0344	1.28	1.11
0.348	.00158	.0822	.0675	.0356	12.94	.1778	.0939	5.74	3.03
0.381	.00217	.0746	.0623	.0323	14.55	.2253	.1168	7.27	3.77
0.223	.00400	.1262	.0946	.0502	7.72	.0631	.0335	2.04	1.08
0.261	.00070	.1008	.0796	.0420	9.35	.0928	.0490	2.99	1.58
0.205	.00020	.1410	.1027	.0743	5.66	.0342	.0248	1.10	0.80
0.321	.00114	.0895	.0724	.0402	11.37	.1375	.0764	4.44	2.47
0.279	.00114	.0780	.0647	.0324	10.78	.1229	.0615	3.97	1.98
(b) BT-6 D_{50}=0.30 mm; D_{90}=0.33 mm; Y_{cr}=0.036; Ξ=397									
0.359	.00217	.0625	.0536	.0274	10.13	.259	.132	7.18	3.67
0.305	.00114	.0880	.0714	.0345	8.47	.181	.088	5.02	2.43
0.386	.00217	.0731	.0613	.0306	10.84	.295	.148	8.21	4.10
0.223	.00070	.0868	.0706	.0307	6.59	.110	.048	3.05	1.33
0.190	.00070	.0717	.0603	.0240	6.10	.094	.037	2.61	1.04
0.309	.00100	.1002	.0792	.0388	8.35	.176	.086	4.89	2.40
0.239	.00059	.0959	.0765	.0388	6.08	.100	.051	2.79	1.41
0.307	.00100	.0900	.0727	.0384	7.86	.162	.085	4.49	2.37
0.382	.00200	.0738	.0617	.0320	10.24	.274	.142	7.62	3.95
0.400	.00200	.0793	.0655	.0344	10.75	.291	.153	8.09	4.25
0.276	.00100	.0771	.0640	.0325	7.08	.142	.072	3.95	2.01
0.331	.00100	.0980	.0778	.0432	8.27	.173	.096	4.80	2.67
(c) BT-10 D_{50}=0.15 mm; D_{90}=0.185 mm; Y_{cr}=.061; Ξ=50									
0.202	.00059	.0650	.0555	.0261	2.38	.145	.068	2.38	1.42
0.265	.00059	.1000	.0791	.0395	3.20	.207	.104	3.40	1.70
0.305	.00059	.1190	.0905	.0491	3.44	.237	.129	3.89	2.11
0.265	.00100	.0775	.0643	.0262	3.76	.286	.117	4.69	1.91
0.299	.00100	.0890	.0720	.0316	3.99	.320	.141	5.25	2.30
0.327	.00100	.1020	.0803	.0363	4.21	.357	.162	5.85	2.65
0.345	.00100	.1020	.0803	.0395	4.20	.357	.176	5.85	2.88
(d) BT-13 D_{50}=0.082 mm; D_{90}=.11 mm; Y_{cr}=.105; Ξ=8									
0.284	.00059	.1030	.0809	.0402	1.77	.388	.193	3.70	1.84
0.175	.00030	.0593	.0513	.0341	1.00	.123	.082	1.17	0.78
0.224	.00059	.0772	.0641	.0282	1.57	.308	.135	2.93	1.29
0.208	.00059	.0587	.0508	.0254	1.39	.244	.122	2.32	1.16
0.185	.00030	.0736	.0616	.0372	1.08	.148	.089	1.41	0.85
0.215	.00030	.0991	.0785	.0462	1.22	.188	.111	1.79	1.06
0.206	.00059	.0571	.0946	.0251	1.39	.238	.121	2.27	1.15
0.274	.00059	.0955	.0762	.0379	1.89	.366	.182	3.48	1.73

FIGURE CAPTIONS

Fig. 1 Variation of Y with Y' for present data.

Fig. 2 Variation of Y/Y_{cr} with Y'/Y_{cr}.


1.00
0.1
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BT-5
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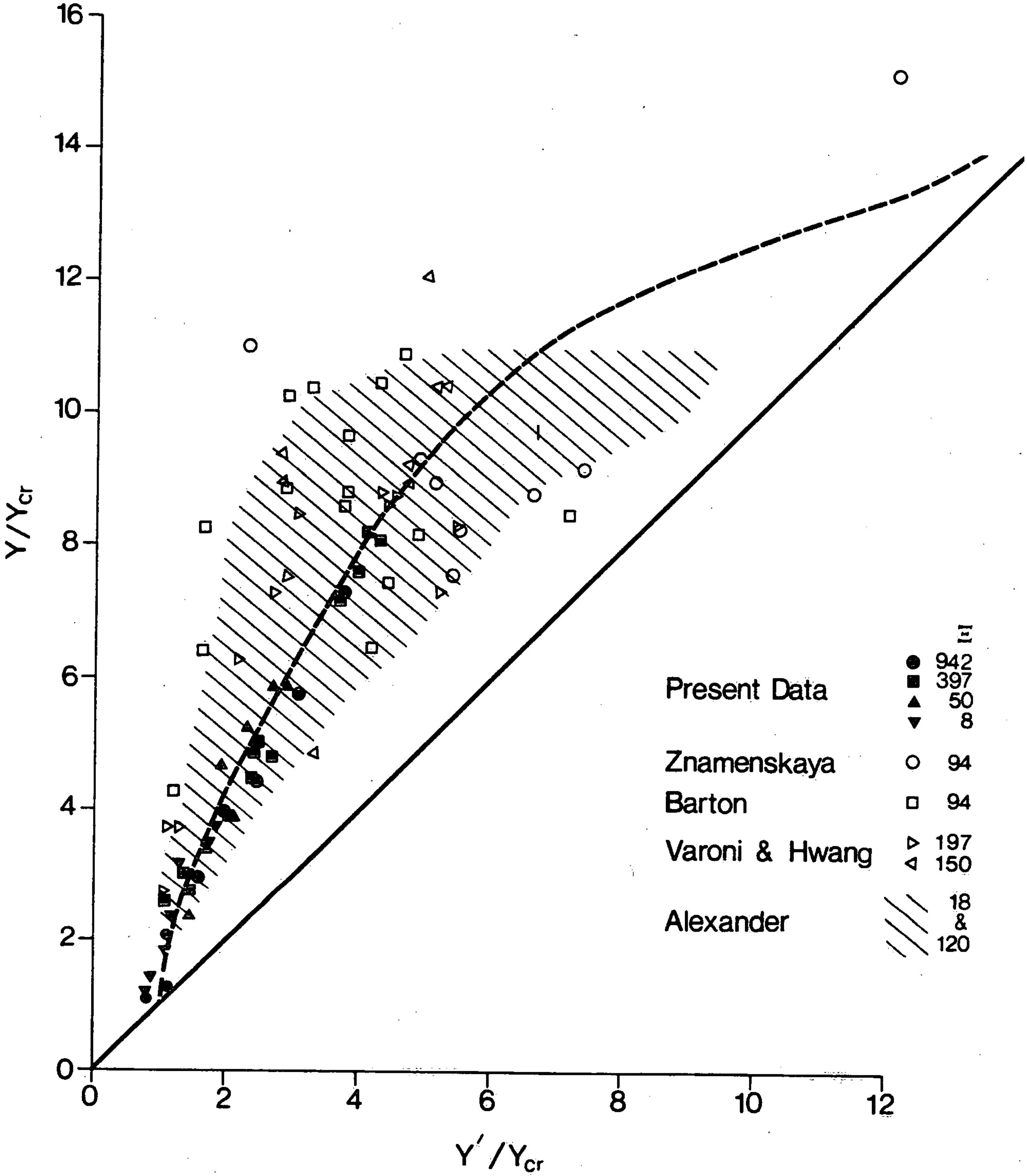
BT-6 ■ 0.30 397
BT-10 ▲ 0.15 50
BT-13 ▼ 0.082 8



16
14
12
10
8
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4
2
0
Y/Y_cr
0
2
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Y′/Y_cr
Present Data
942
397
50
8
Znamenskaya
94
Barton
94
Varoni & Hwang
197
150
Alexander
18
&
120