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ON THE RATE OF TRANSFER OF HEAT BETWEEN A LAKE AND AN ICE SHEET

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MANAGEMENT PERSPECTIVE

This study attempts to derive estimates of the sensible heat transfer between lake ice and water based on directly observed or easily modelled primary variables, namely the speed and temperature of the water one metre below the ice cover from field data collected from 1983 to 1985 in the headwater lakes of the Yukon River Basin. This quantity is required for studies on the thermal simulation of northern lakes and reservoirs. The estimation of the extent and location of open water areas in an ice sheet is important for the operation of intakes for hydroelectric power generation, for the evaluation of suitable overwintering habitat for waterfowl and the possible disruption of migratory routes for game.

DANS UNE PERSPECTIVE DE GESTION

Cette étude s'efforce d'obtenir des estimations du transfert de la chaleur sensible entre la glace de lac et l'eau basées sur les principaux variables directement observés ou facilement simulés au moyen de modèles, à savoir la vitesse et la température de l'eau à un mètre de profondeur sous le manteau glaciel obtenus à l'aide de données recueillies sur le terrain de 1983 à 1985 dans les lacs situés à la source de la rivère du Yukon. Cette quantité est requise pour les études sur la simulation thermique des lacs et réservoirs du Nord. Il est important d'évaluer l'étendue et l'emplacement des zones d'eau libre dans un glacier continental pour assurer le bon fonctionnement des prises d'eau dans les installations hydro-électriques, pour juger si l'habitat où hiverne la sauvagine est convenable et pour veiller à ce que les routes migratoires du gibier ne soient pas perturbées.

ABSTRACT

The flux of heat between water and ice is either ignored or set to a constant value in present sea ice models. More accurate treatment of this flux may be desirable in the vicinity of polynyas and ice margins, and for more accurate thermal simulation of lakes and reservoirs.

Observations of currents and temperatures under the ice in three lakes of the headwater region of the Yukon River permit the determination of the coefficient of sensible heat transfer between water and ice by various indirect methods. At a depth of 1 m and an estimated surface roughness of less than 0.01 m, the coefficient of sensible heat transfer between water and ice was found to be $(0.8 \pm .3)*10^{-3}$. This value is within the limits of smooth and rough ice conditions found in laboratory investigations and appears to be somewhat less than that found in recent sea ice studies.

SUR LE TAUX DE TRANSFERT DE LA CHALEUR ENTRE UN LAC ET UN GLACIER CONTINENTAL

P.F. Hamblin et E.C. Carmack

RÉSUMÉ

Dans les modèles actuels de la glace de mer, le flux de chaleur entre l'eau et la glace est ou bien ignoré ou bien gardé à une valeur constante. Il serait souhaitable de procéder à un traitement plus précis de ce flux dans le voisinage des clairières et des bordures de glace et d'obtenir une simulation thermique plus précise des lacs et réservoirs.

Grâce à des observations des courants et des températures sous la glace dans trois lacs situés près de la source de la rivière du Yukon, on a pu déterminer le coefficient de transfert de chaleur sensible entre l'eau et la glace par diverses méthodes indirectes. À une profondeur de 1 m et avec une rugosité de surface approximative de 0,01 m, le coefficient de transfert de chaleur sensible était de $(0,8 \pm .3)* 10^{-3}$. Cette valeur se situe dans les limites des conditions de la glace lisse et rugueuse observées au cours des recherches en laboratoire et semble être quelque peu inférieure à celle observée au cours des études récentes sur la glace de mer.

The transfer of heat between water and ice is usually ignored or set to a constant value in the thermodynamic modelling of lake and ocean ice cover (cf. Maykut and Untersteiner, 1971). However, recent studies related to through-flow in ice-covered lakes (Patterson and Hamblin, 1987), have demonstrated the sensitivity of such water bodies to the exchange of heat between water and ice. In this study we attempt to establish a bulk parameterization of the transfer of heat between water and ice suitable for numerical modelling purposes.

The heat transfer between rivers and their ice covers has been theoretically treated by Baines (1961) and modelled in the laboratory by Acres Consulting Services (1979); these authors quantified heat flux in terms of a Reynolds number which may be appropriate for channel flow, but is not meaningful for lakes or oceans. A more useful viewpoint for lake and ocean modelling is the parameterization of the turbulent vertical heat flux in terms of a sensible heat transfer coefficient and the temperature and flow at some distance from the boundary. Gilpin _et_ _al_. (1980) pioneered this approach in their laboratory study, and found a transfer coefficient that varied between (0.6 - 1.0) x 10^{-3} depending on the surface roughness of the ice sheet. However, problems with laboratory methods are that the heat flux is referenced to a cross-sectionally averaged velocity, and that scales of turbulent motion representative of lake and ocean settings are not usually obtained in the laboratory setting.

We are not aware of quantitative determinations of thermal transfers between water and ice in lakes, despite the early interest in ice covered lakes by Scandinavian workers (Liljequist, 1941) who collected observations of winter ice conditions and temperatures in Lake Vetter in Sweden. Melin (1947, 1948) drew attention to regulated waterflow and its effect on black ice thickness, particularly late in the season and showed convincingly the upwelling of warmer water near the outflow of Swedish lakes. More recently, Stigebrandt (1978) discussed the dynamics of through-flow in ice-covered lakes. However, none of the above authors have attempted to quantify the heat transfer process.

Few field studies on thermal transfers in ice-covered seas have been reported. Lewis and Perkin (1986) have summarized the results of Josberger and Meldrum (1985) and Bogorodskiy and Sukhorukov (1983) concerning the melting of sea ice in terms of the melting rate per degree Celcius versus under-ice current speed. In terms of the sensible heat transfer coefficient, the oceanic studies yield heat transfer coefficients ranging between $(0.8 \text{ to } 1.4) \times 10^{-3}$. Unfortunately, neither of these studies reported the characteristics of the under-ice thermal boundary layer. Lewis and Perkin (1986) conjecture that the above range of values represents the variation in the heat transfer coefficient from smooth to rough ice conditions.

In the following we discuss field measurements taken in three headwater lakes of the Yukon River Basin during the late winter

$$C_s = \left(L\rho_I \frac{dH}{dt} + \frac{K_I \, T_{SI}}{H} \right) / \rho_w \, C_p \, T_I \, U_I \qquad (3)$$

not 3

In this equation the surface temperature of the ice, T_{SI}, is assumed to be the average air temperature of -9.3°C during the interval between the two successive measurements. By this assumption the effect of a thin (~5 cm) wind compacted layer of snow on the surface temperature of the ice is taken to be negligible. The daily averaged air temperatures were observed at the Whitehorse airport. The distribution of ice thickness across the outflow of Marsh Lake (Figure 4) shows ablation of ice in the centre of the outflow where the velocities are hightest and growth of ice at the extremities. Based on 19 observations of ice growth and thickness and under-ice flow and temperatures, C_s is $(0.63 \pm .25) \times 10^{-3}$. Further, in contrast to the two sea ice studies, the effect of heat flux through the ice and snow cover may not be neglected as it accounts for at least one half of the upward total heat flux supplied by the water on the average.

This method like the previous one, is subject nonetheless to large uncertainties. The ice thickness is probably known only to within 5 cm, the field temperatures were not measured on the same day as the currents which themselves are close to the threshold of the current meter. Finally, the effect of snow cover and radiation on the seasons of 1985 and 1986 (Figure 1) with a view to the determination of the coefficient of sensible heat transfer between water and ice sheets on fresh water.

SENSIBLE HEAT TRANSFER

According to the bulk formula, the sensible heat flux, H, in a turbulent boundary layer is given by

$$H = C_s\rho \quad C_p \, \Delta TU \qquad (1)$$

where C_s is the sensible heat transfer coefficient, ρ is the density, C_p, the thermal heat capacity, ΔT is the difference over a height h, from the boundary, and U is the speed of flow at height h. In standard atmospheric applications H is taken as 10 m, and C_s, has the values 1.5×10^{-3} for neutral conditions, 0.8×10^{-3} for very stable conditions and 1.6×10^{-3} for extremely unstable conditions (Fischer et al. 1979).

From the requirement for the continuity of stress across the air-water boundary and the proportionality of boundary layer thickness to the friction velocity, the analogous height in the water should be of the order $(\rho_a/\rho_w)^{-1/2}$ of the height where ρ_a and σ_w are the densities of air and water, respectively. This leads to a height of order 1 m. Although this boundary layer height will be adopted throughout this study, it is noteworthy that this reference depth has not been standardized as in the case of the atmospheric boundary layer. Josberger and Meldrum (1985) use 2.55 m as a reference depth for C_s whereas Langleben (1982) uses 1 m as the reference depth for the under-ice drag coefficient. Of course, it is assumed that there is sufficient flow under the ice to maintain a turbulent boundary layer. In the case of lakes with little or no winter flow-through, Patterson and Hamblin (1987) have proposed that a molecular heat transport term be added to equation (1).

In the present study the sensible heat transfer coefficient on the underside of the lake ice is inferred by a number of approaches employing the field data collected during the headwater lakes study.

EVALUATION OF C_s FROM FIELD DATA

Marsh Lake

A temperature section parallel to the outflow in Marsh Lake (Figure 2) shows that the temperature at 1 m depth, T_1, (0.5°C) is essentially independent of horizontal position. Thus, at this depth, only the vertical heat flux need be considered. Since the vertical heat flux is composed of a diffusive term $C_p\rho$, Kv $\frac{\partial T}{\partial Z}$, and an advective term, C_p ρ W T, where Kv is the vertical eddy conductivity and W is the vertical velocity, it follows that from equating this flux to that of equation (1) that

$$C_s = \frac{(WT + Kv\frac{\partial T}{\partial z})}{U_1 T_1} \qquad (2)$$

The horizontal velocity at a depth of one metre, U_1, was measured directly by a mechanical current meter to be 6 cm/s at station 4/11. Now, assuming that flow lines are parallel to the isotherms, the vertical velocity approaching the outflow can be estimated from continuity considerations. Using a radio-tracked, under-ice drogue (see Hamblin, 1987), a horizontal flow of 0.006 m s^{-1} was measured at a depth of 6 m and 100 m further upstream from station 16; the slope of the corresponding isotherm (0.75°C) at this location and depth was 4.2×10^{-3}. Hence, a vertical velocity W of 2.4×10^{-5} ms^{-1} is found. A two-dimensional numerical model of the throughflow not described herein supports the assumption that the flow lines are parallel to the isotherms under the steady winter conditions.

The vertical eddy conductivity was estimated by the dissipation method employing the overturning scales of motion as observed from temperature profiles taken near the outflow. It is supposed that temperature inversions of the type shown in Figure 3 represent shear induced events and are consequently amenable to the analysis described by Dillon (1982). The first step of the analysis is to reconstruct the original temperature profile by an objective method such as that described by Papadakis (1981) here shown by the dashed line in Figure 3. The root mean square vertical displacement of each point from its original position is 0.9 m for this event. In turn, this is related to the Ozmidov length scale (Dillon, 1982) and then to the turbulent kinetic energy dissipation when combined with the local stability frequency. Based on the assumption of a typical mixing efficiency for lakes of 5% (Fischer _et al._, 1979), we arrive at a vertical eddy conductivity of 1×10^{-4} m^2/s. The resulting value of C_s from (2) is 1.2×10^{-3}. This calculation of the vertical flux of heat suggests that the diffusive flux is twice that due to upwelling. The principal shortcomings of this method arise from possible errors in the estimation of Kv and W.

A second method of determining the sensible heat transfer coefficient is motivated by readings of ice thickness at two times (1 March and 9 March) as well as under-ice temperatures and velocities at a depth of 1 m. Now, if the ice thickness is H, the thermal conductivity of ice is K_I, the density of the ice is ρ_I, and the latent heat of fusion is L, then the ice growth equation may be written in terms of C_s as

$$C_s = \left(L\rho_I \frac{dH}{dt} + \frac{K_I\ T_{SI}}{H} \right) / \rho_w\ C_p\ T_1\ U_1 \qquad (3)$$

In this equation the surface temperature of the ice, T_{SI}, is assumed to be the average air temperature of −9.3°C during the interval between the two successive measurements. By this assumption the effect of a thin (~ 5 cm) wind compacted layer of snow on the surface temperature of the ice is taken to be negligible. The daily averaged air temperatures were observed at the Whitehorse airport. The distribution of ice thickness across the outflow of Marsh Lake (Figure 4) shows ablation of ice in the centre of the outflow where the velocities are highest and growth of ice at the extremities. Based on 19 observations of ice growth and thickness and under-ice flow and temperatures, C_s is $(0.63 \pm .25) \times 10^{-3}$. Further, in contrast to the two sea ice studies, the effect of heat flux through the ice and snow cover may not be neglected as it accounts for at least one half of the upward total heat flux supplied by the water on the average.

This method like the previous one, is subject nonetheless to large uncertainties. The ice thickness is probably known only to within 5 cm, the field temperatures were not measured on the same day as the currents which themselves are close to the threshold of the current meter. Finally, the effect of snow cover and radiation on the surface temperature of the ice have been ignored. Nevertheless, the agreement between the two methods is encouraging.

Tagish Lake

The approach used here was to estimate C_s as the residual of the divergence in the horizontal heat flux. To do this, temperature profiles were measured along three sections across the north end of Tagish Lake, each separated from one another by approximately 500 m (Figures 5a, 6a and 7a). Drogue measurements from 6 m depth supplied the reference level current for the thermal wind relation. The corresponding northward component of flow or outflow for each line (Figures 5b, 6b and 7b) yield cross-sectionally averaged transports of 136, 140 and 137 m^3/s for each section, in good agreement with the nominal winter flow through rate of 140 m^3/s measured by the Water Survey of Canada (M.E. Alford, personal communication). The heat transport through section (14-12) is 7.85×10^5 kJ s^{-1}, through section B (2-7), is 7.36×10^5 kJ s^{-1} and through section C (13-11) is 7.63×10^5/kJ s^{-1}. A least squares best fit passing through the heat flux value of line A yields an average rate of heat loss to the ice of 3.72×10^4 kJ km^{-1}. With an average width of 1.9 m, and an average current speed of 1 cm/s and temperature of 0.8°C at a depth of one meter, the sensible heat transfer coefficient is $(0.58 \pm .3) \times 10^{-3}$. Errors in this calculation arise from the close separation of the cross sections and the necessity to subtract large numbers from one another to establish a rate of heat flux to the ice. Unfortunately, due to unsafe ice conditions elsewhere on Tagish Lake, it was not possible to measure the cross-sectional heat transports at larger horizontal separations.

Observed Heat Fluxes

Ice thickness temperature and current profiles were measured along a section across the outflow region of Lake Laberge in March 1986 (Figures 8a and b). An attempt made to determine C_s from these data proved unsuccessful due to the lack of knowledge of the flow field and outflow bathymetry across the outflowing region as well as an appropriate value of the vertical eddy conductivity. From the measured flows and temperatures at 1 m below the ice the sensible heat flux has been calculated in Figure 8d employing equation (1) with $C_s = 0.8 \times 10^{-3}$, $\rho = 1000$ Kg m^{-3} and $C_p = 4194$ JKg^{-1}°C^{-1}.

In addition, the current profiles on the underside of the ice for the three outermost stations yield a aerodynamic roughness of 0.01 m when fitted by a log-wall boundary layer. The heat flux at the ice edge approaches a maximum value of 240 Wm^{-2} close to that observed in Tagish Lake. A plot of temperature at 1 m below the ice or the water surface reveals in Figure 8c a distinct peak about midway across the ablating portion of the ice edge where due to the constricted depth upwelling does not supply heat at a rate sufficient to maintain the heat flux to the ice whereupon the outflow begins to cool.

From the measured and inferred flows and temperatures at a depth of 1 m, the sensible heat transfer has been determined and plotted for Tagish lake in Figure 9a. In general, the associated ice thickness shown in Figure 9b is strongly correlated with the heat flux from water to the ice. Ice tends to be thinner on the western side of the lake where the upwelled warmer water is closer to the surface (Figures 5 to 7) and towards the outflow. The maximum heat flux measured in the vicinity of the permanent polynya on Lakes Tagish and Laberge is 230-240 Wm^{-1}. This lower bound on the heat flux required to maintain a permanent polynya may be reasonably compared to mean heat loss over the Dundas Island polynya in the Canadian Archipeligo of 329 Wm^{-2} (Der Hartog et al., 1983), 380 Wm^{-2} for the heat flux over the St. Lawrence Island polynya (Pease, 1985), or 340 Wm^{-2} over an open river in the Yukon basin (Alford and Carmack, 1986).

CONCLUSIONS

Observations of ice thickness, current and temperature under the ice in three lakes of the headwater system of Yukon River Basin have been used to infer a coefficient of sensible heat transfer between water and ice of $(0.8 \pm .3) \times 10^{-3}$ at a depth of one meter below the ice. It is difficult to quantitatively compare this value with either the laboratory values or the sea ice values because these studies generally did not report the observation depth and roughness height required to make such a comparison. In the case of the sea ice study of Josberger and Meldrum (1985), if a standard sea ice roughness of 10^{-3} m (Langleben, 1982) is assumed which may be compared to our value in Lake Laberge of .01 m, then their value of 0.8×10^{-3} at a depth of 2.55 m becomes 1.0×10^{-3} at one meter which suggests that the heat transfer in lakes may be somewhat less than for oceanic ice sheets but in both cases less than the neutral atmospheric boundary layer. It is recommended that in future investigations the depth and the roughness height be reported in the study or that a common reference depth for bulk transport formulae be adopted as in the case in the atmospheric boundary layer.

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LIST OF FIGURES

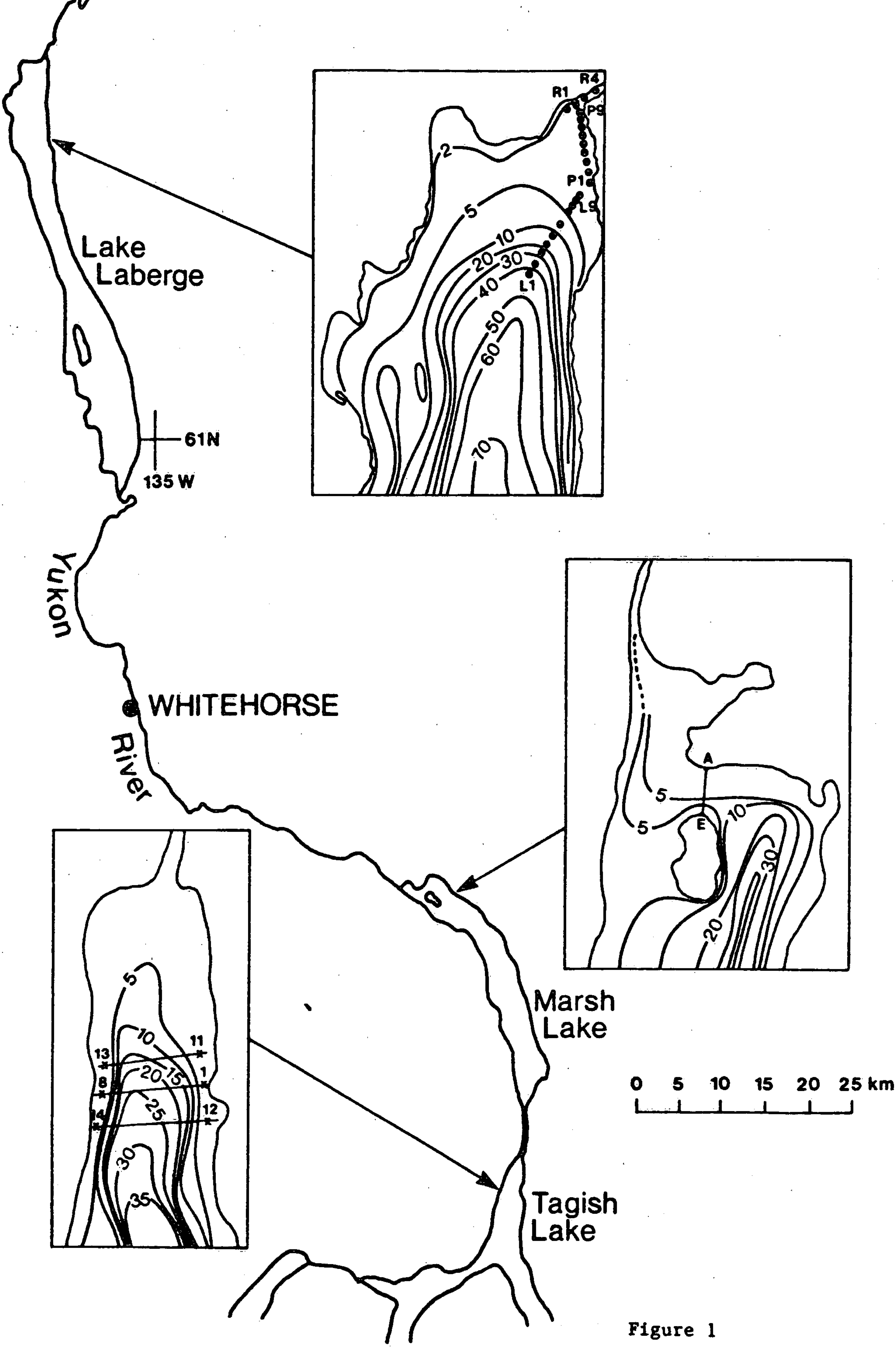
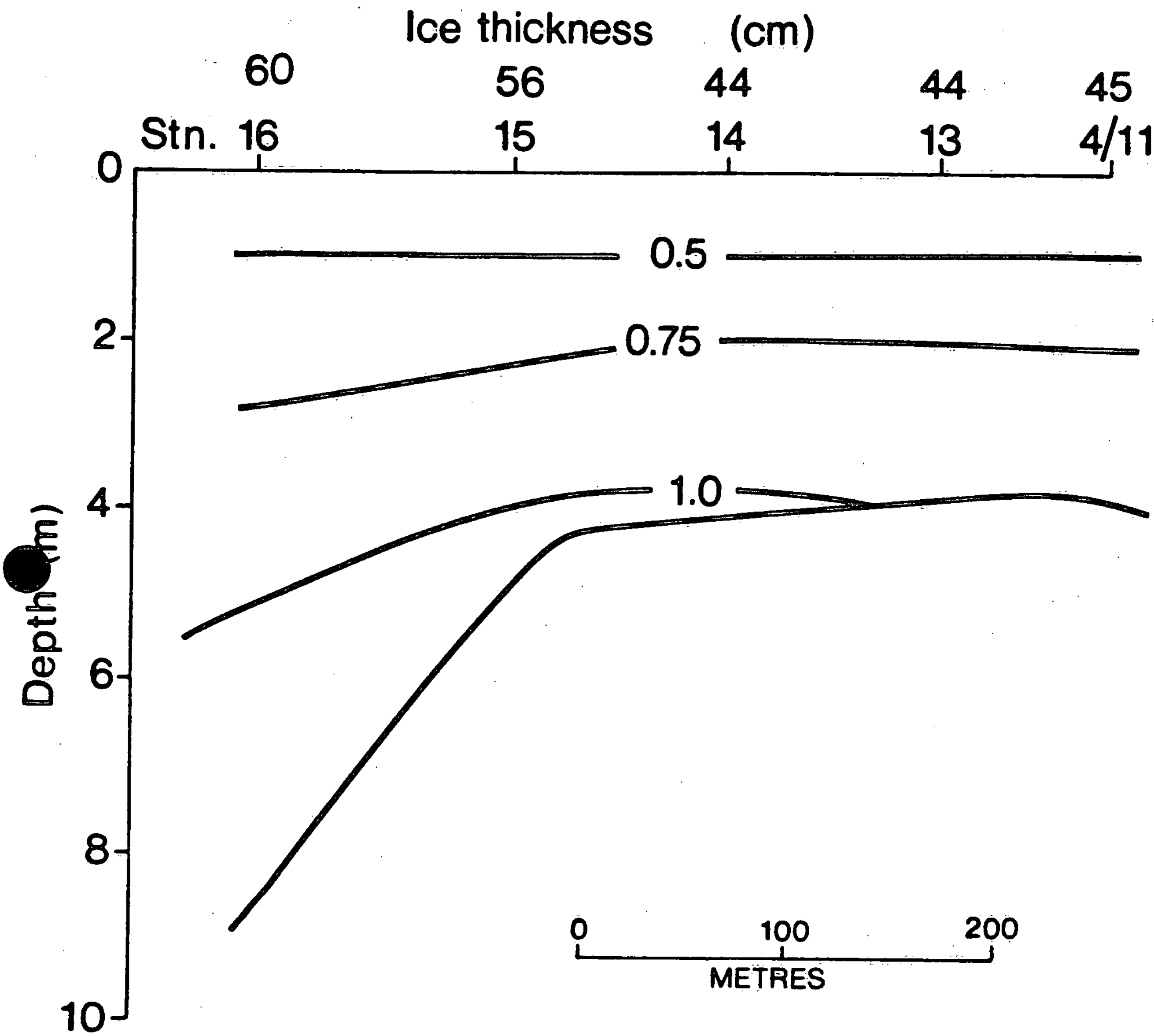

Lake Laberge
61N
135 W
Yukon
WHITEHORSE
River
Marsh Lake
Tagish Lake
0 5 10 15 20 25 km
R4
R1
P9
P1
L9
L1
2
5
10
20
30
40
50
60
70
A
E
5
10
20
30
5
10
15
20
25
30
35
13
11
8
1
14
12

Figure 1



Ice thickness (cm)
60
56
44
44
45
Stn. 16
15
14
13
4/11
0
2
4
6
8
10
Depth (m)
0.5
0.75
1.0
0
100
200
METRES

Figure 2

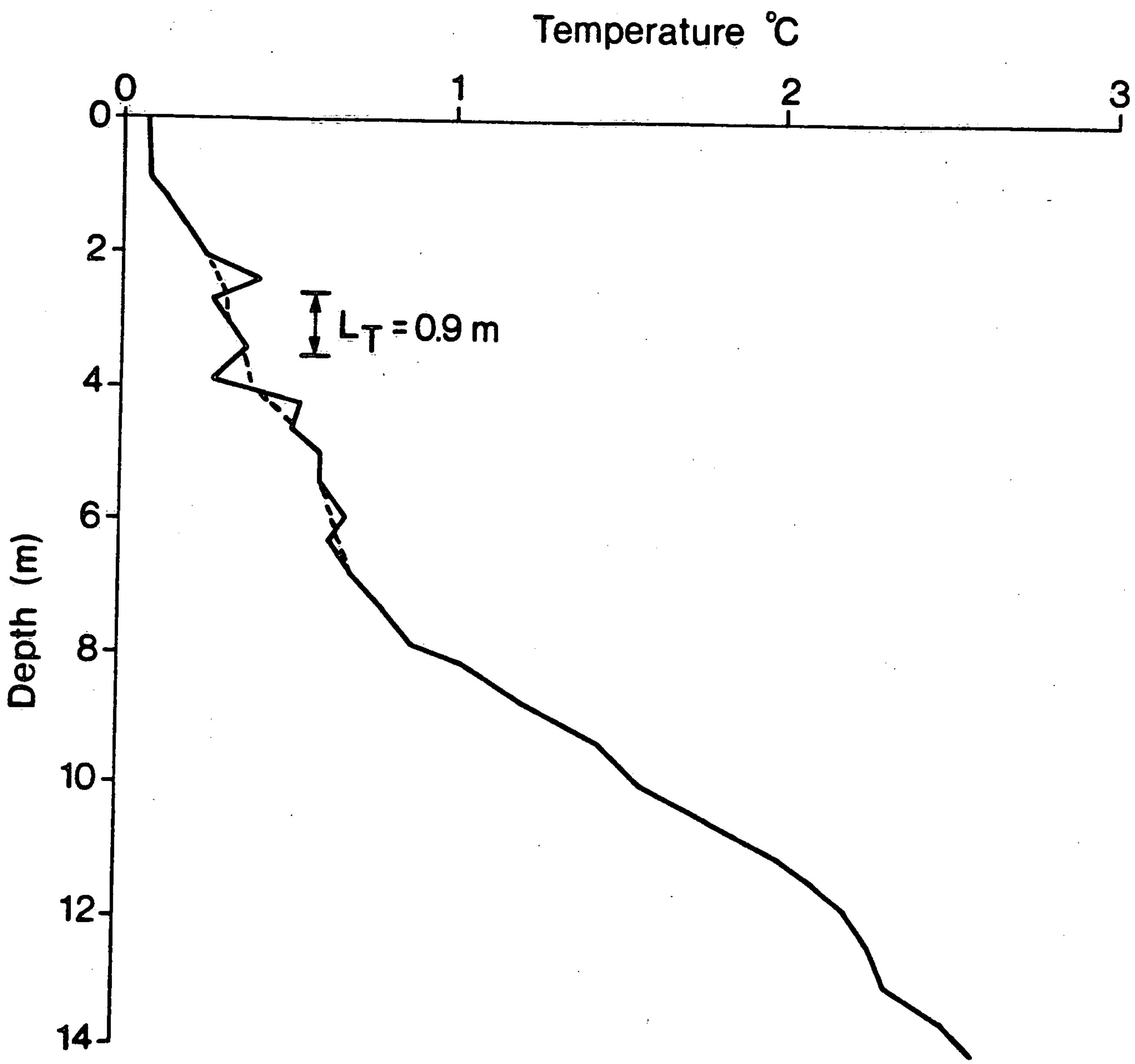


Figure 3

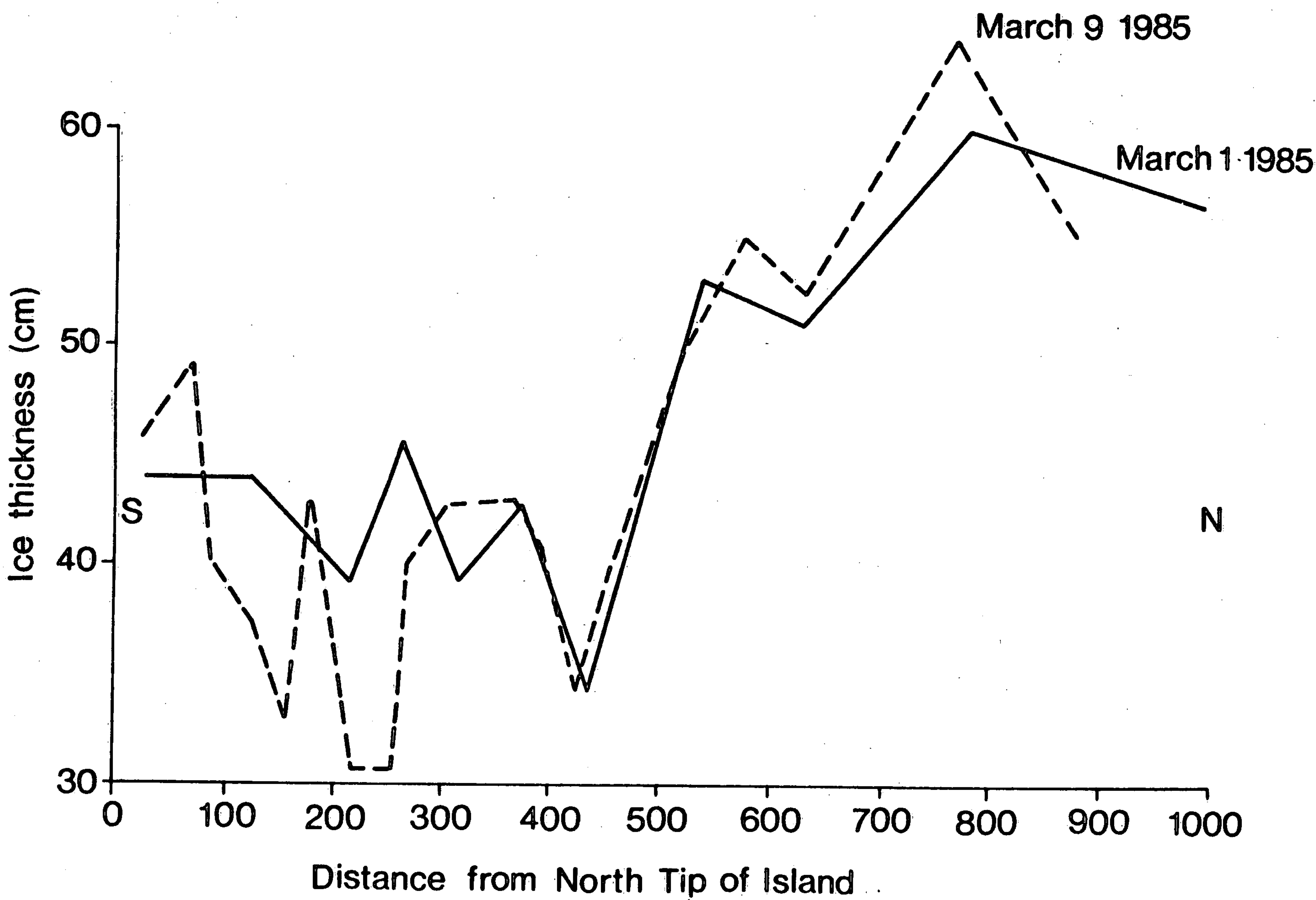
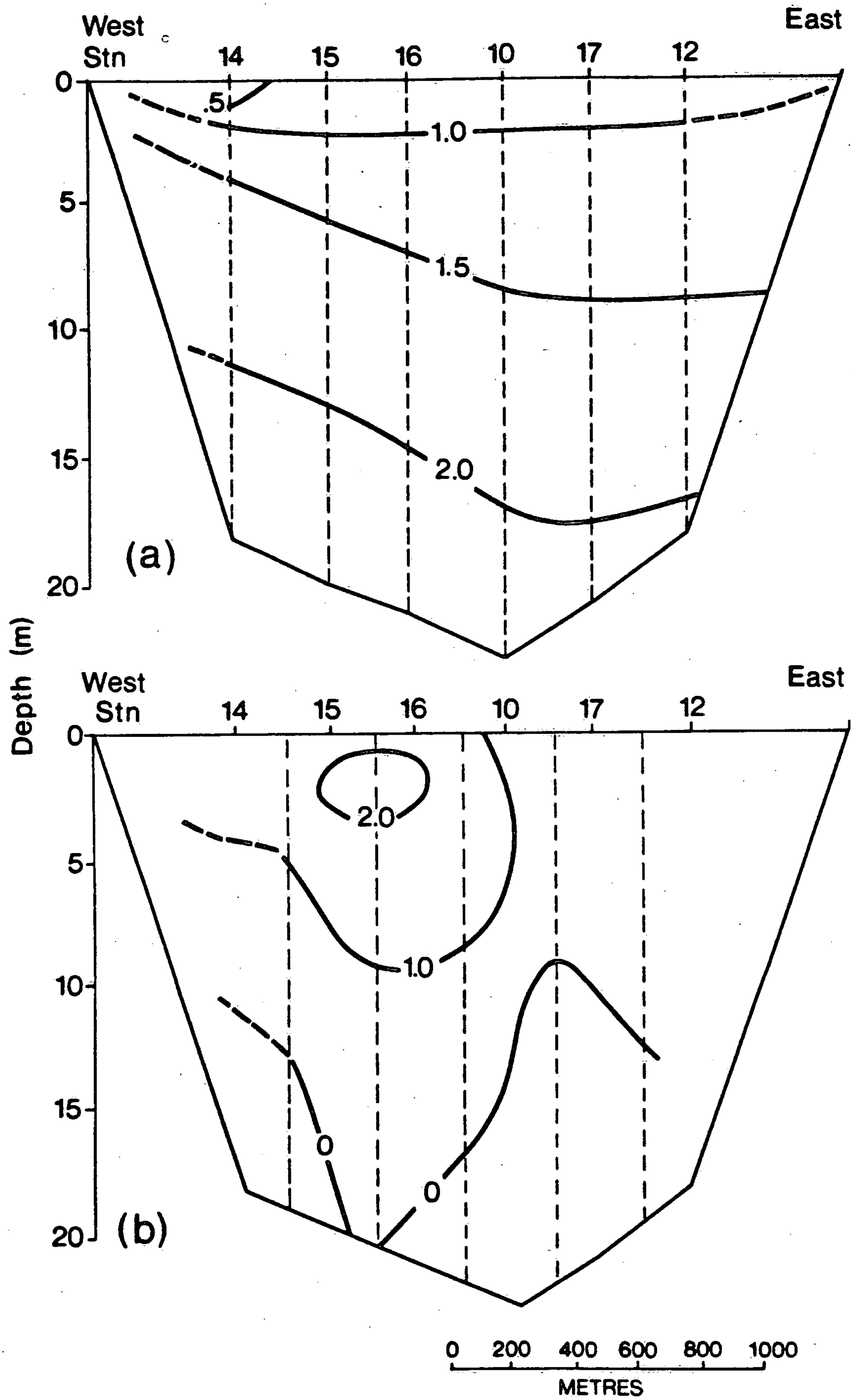

March 9 1985
March 1·1985
60
50
40
30
Ice thickness (cm)
S
N
0
100
200
300
400
500
600
700
800
900
1000
Distance from North Tip of Island

Figure 4



West
Stn
14
15
16
10
17
12
East
0
5
10
15
20
.5
1.0
1.5
2.0
(a)
Depth (m)
West
Stn
14
15
16
10
17
12
East
0
5
10
15
20
2.0
1.0
0
0
(b)
0
200
400
600
800
1000
METRES

Figure 5

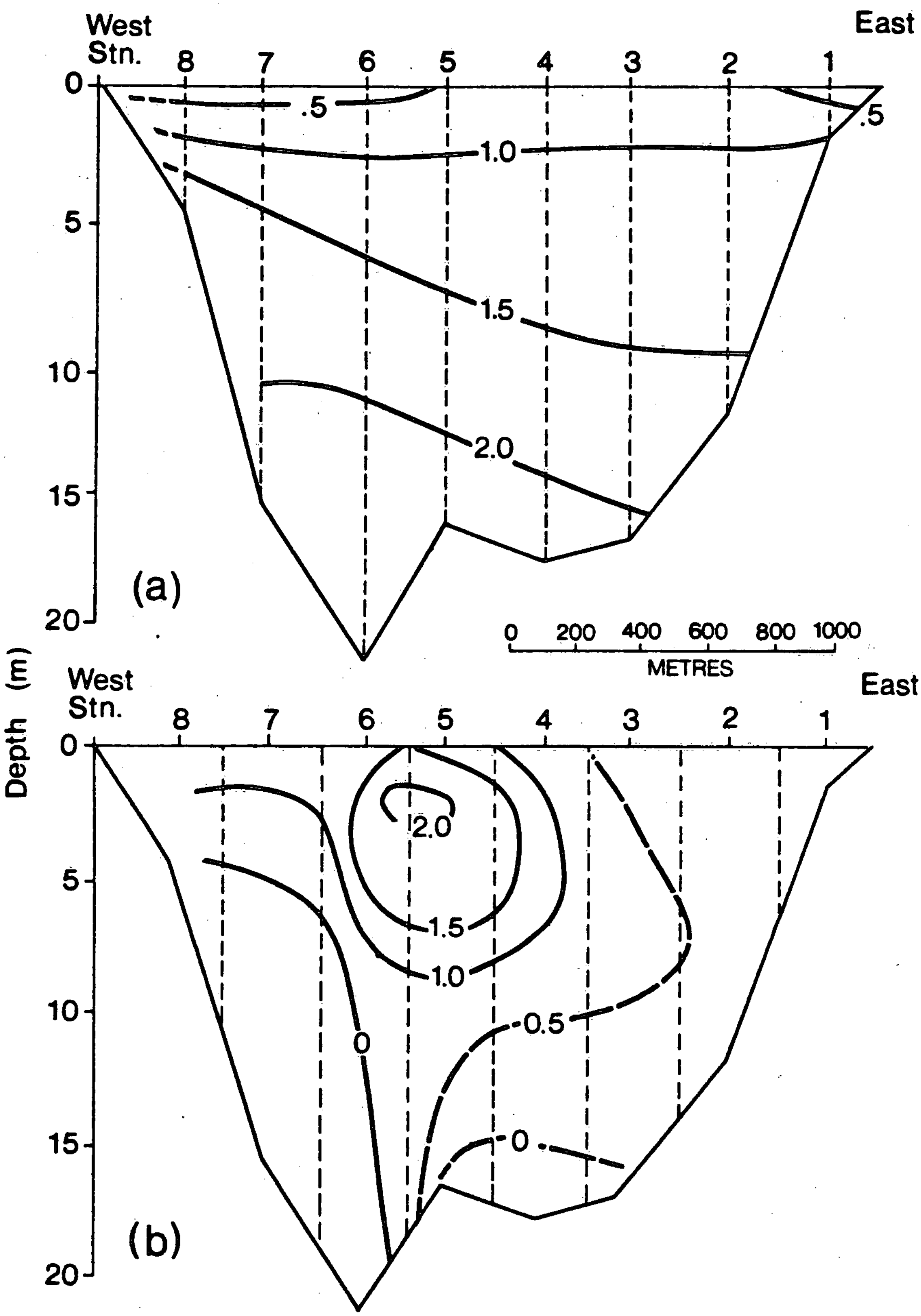
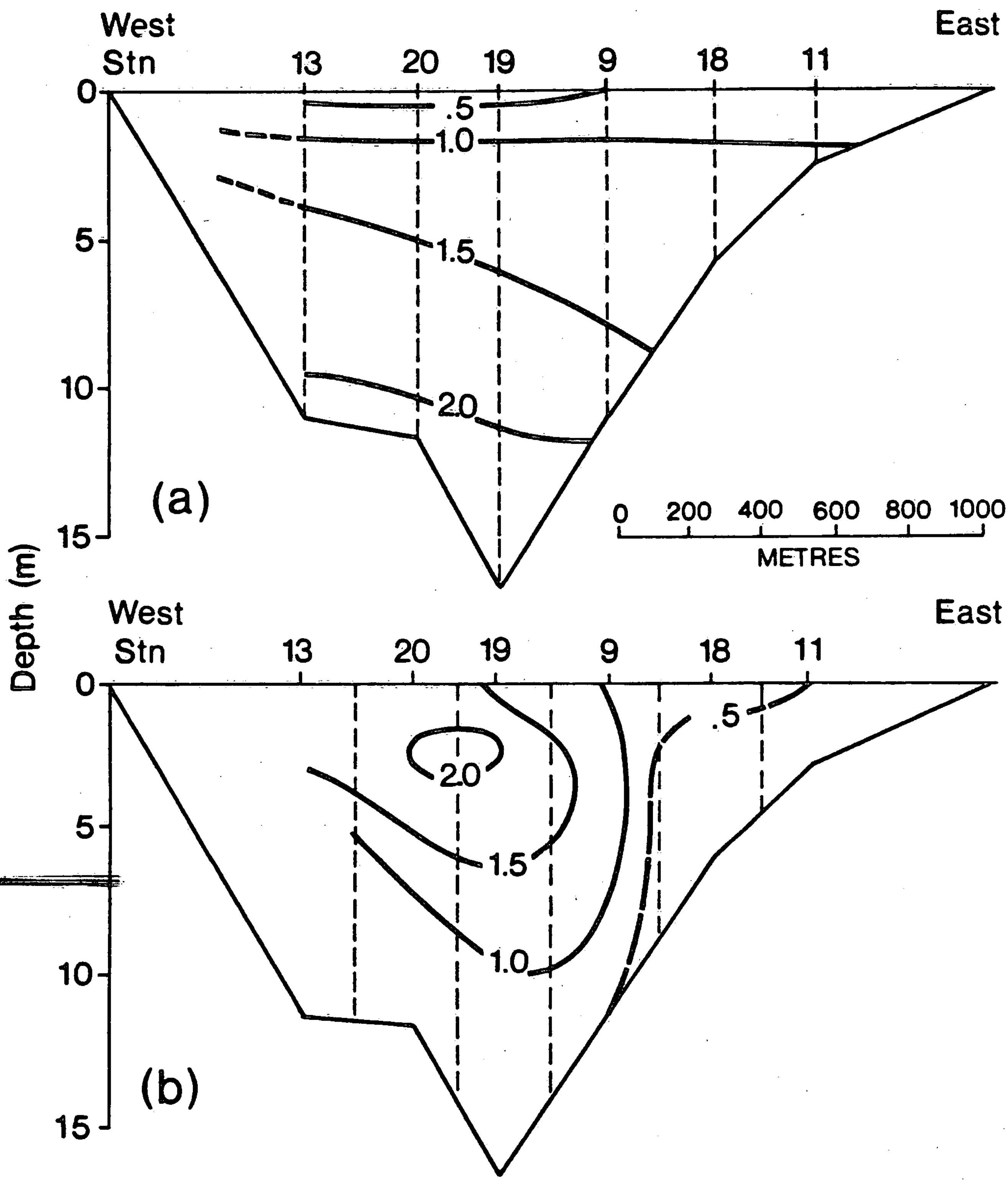
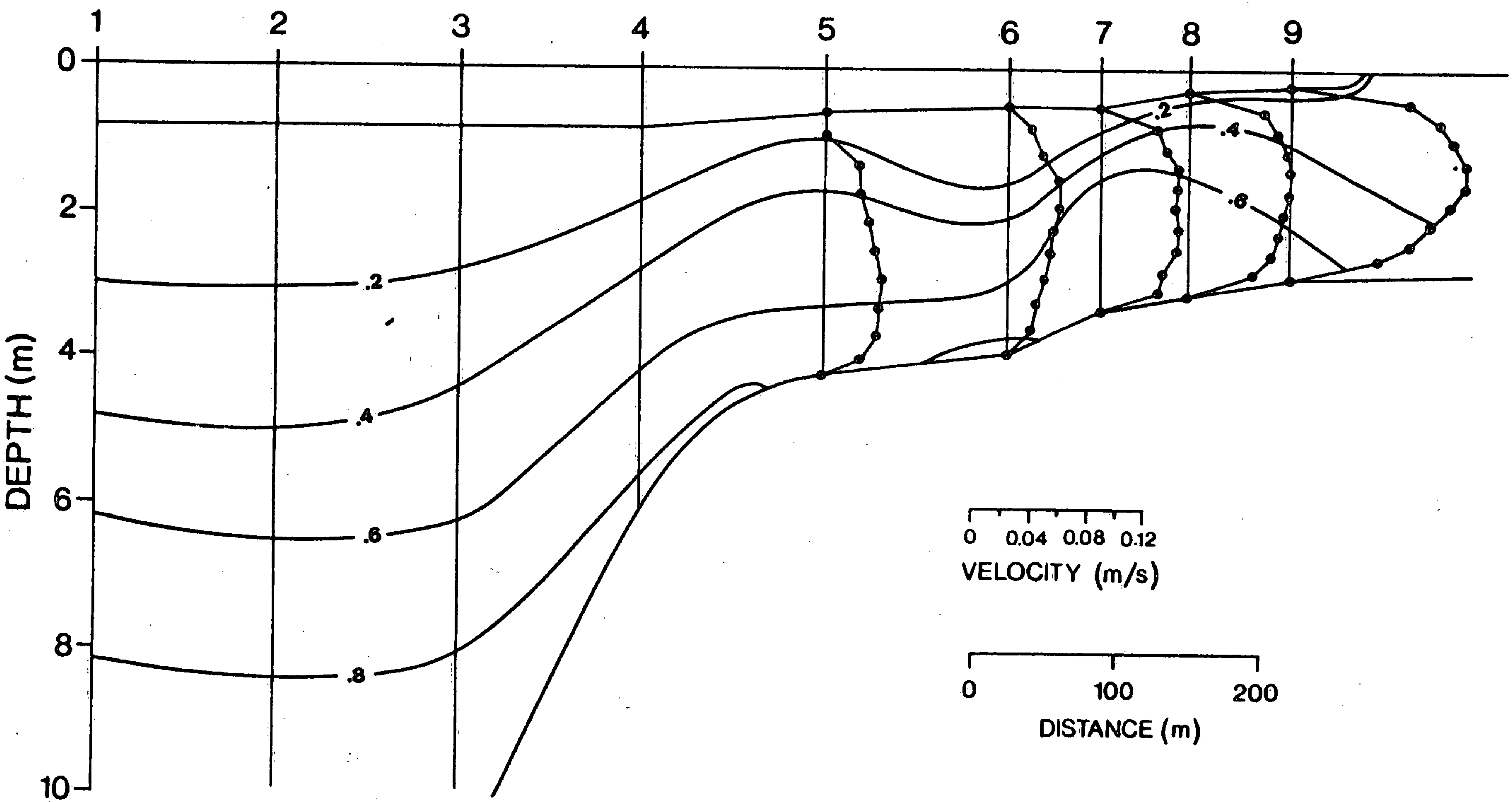

West
Stn.
East
8
7
6
5
4
3
2
1
0
5
10
15
20
.5
1.0
1.5
2.0
(a)
0
200
400
600
800
1000
METRES
Depth (m)
West
Stn.
East
2.0
1.5
1.0
0
0.5
(b)

Figure 6



West
Stn
13
20
19
9
18
11
East
0
5
10
15
.5
1.0
1.5
2.0
(a)
0
200
400
600
800
1000
METRES
Depth (m)
West
Stn
13
20
19
9
18
11
East
0
5
10
15
2.0
1.5
1.0
.5
(b)

Figure 7



1
2
3
4
5
6
7
8
9
0
2
4
6
8
10
DEPTH (m)
.2
.4
.6
.8
0
0.04
0.08
0.12
VELOCITY (m/s)
0
100
200
DISTANCE (m)

Figure 8a

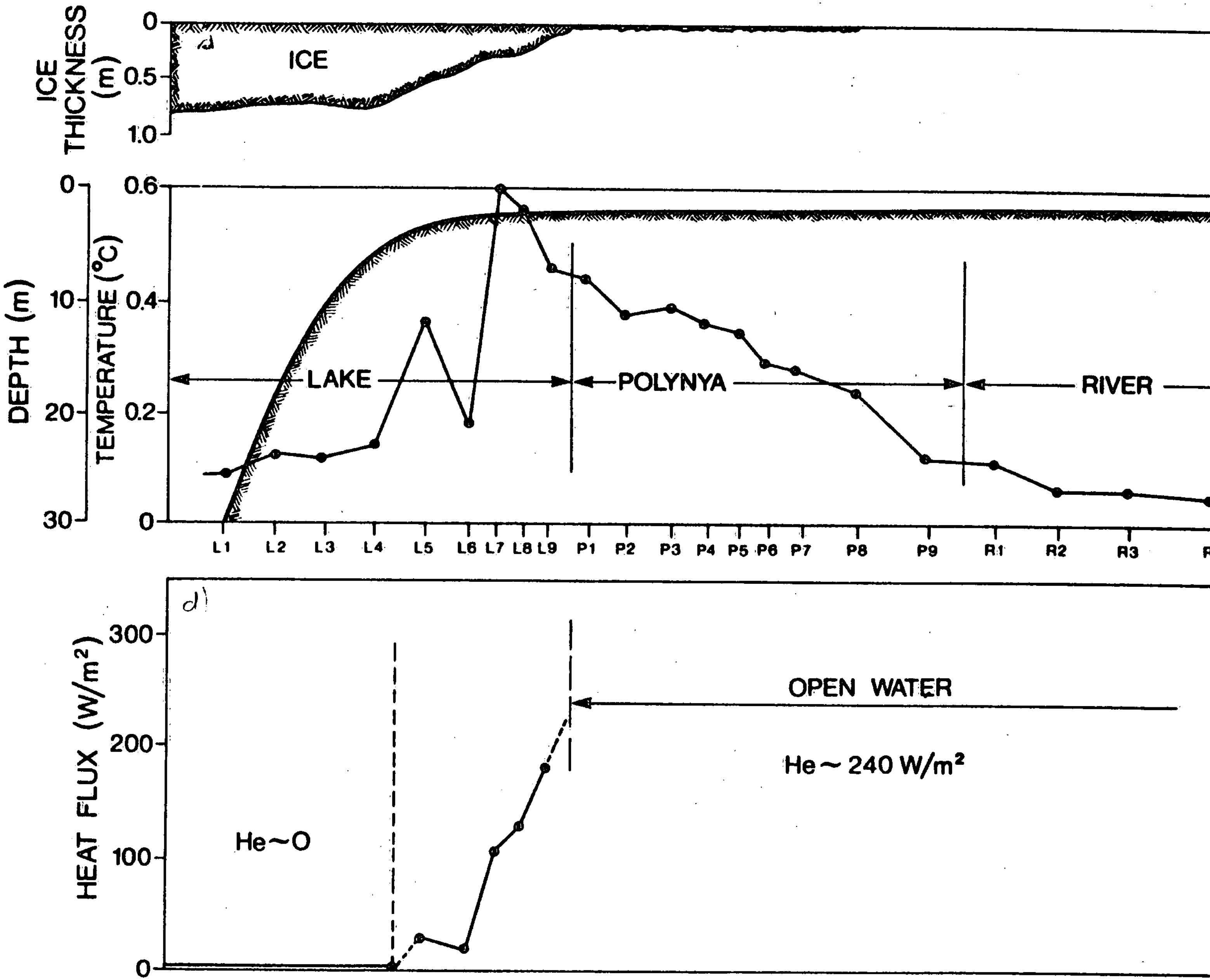


Figure 8b, 8c and 8d

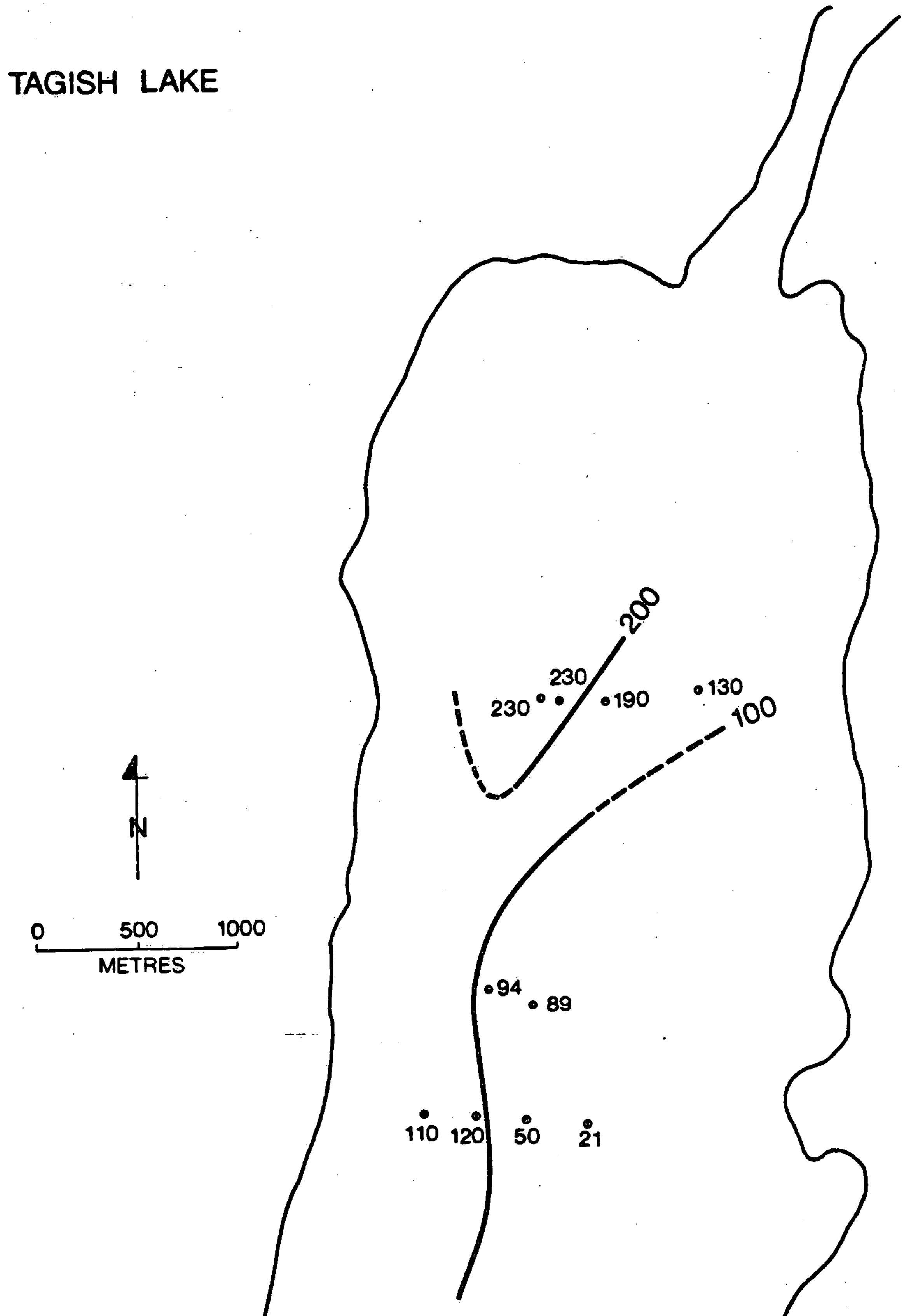

TAGISH LAKE
200
230
230
190
130
100
0
500
1000
METRES
N
94
89
110
120
50
21

Figure 9a

TAGISH LAKE

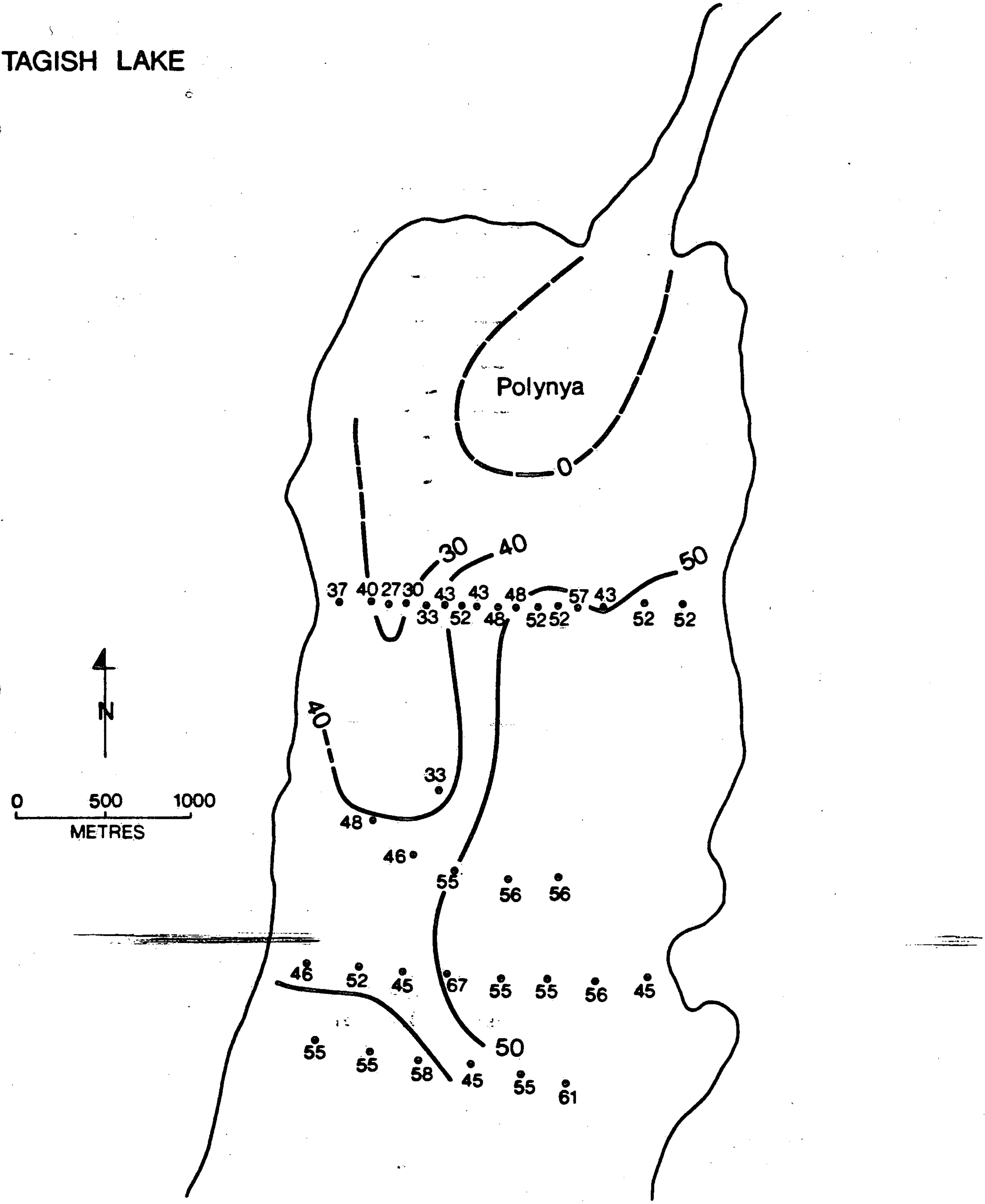
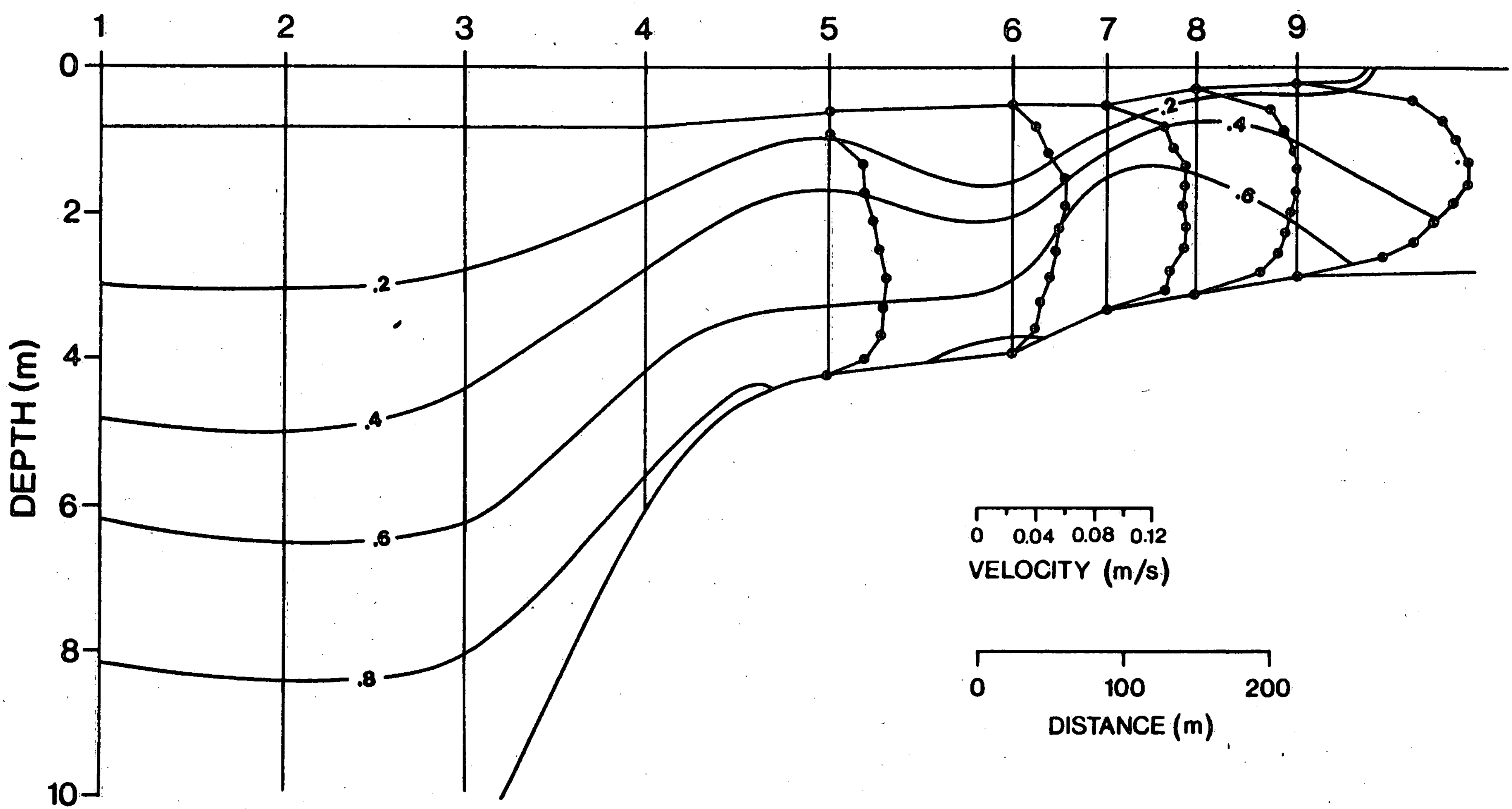


Figure 9b



1
2
3
4
5
6
7
8
9
0
2
4
6
8
10
DEPTH (m)
.2
.4
.6
.8
.2
.4
.6
0
0.04
0.08
0.12
VELOCITY (m/s)
0
100
200
DISTANCE (m)

Figure 8a

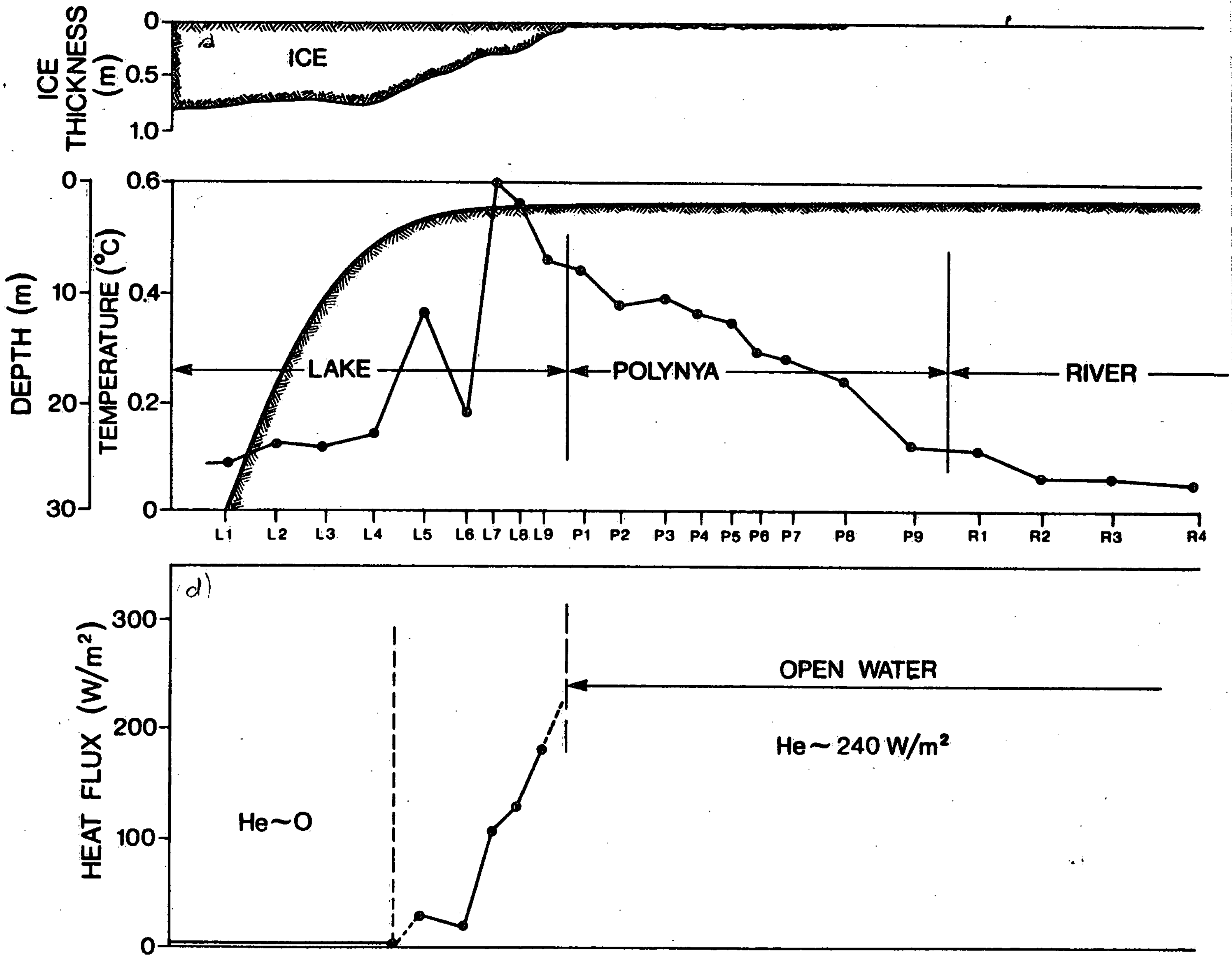


Figure 8b, 8c and 8d