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HYDRAULICS RESEARCH DIVISION

Technical Note

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TITLE: A Bed Load Equation Using Average Departures about the Mean Bed Elevation and Dune Speed

AUTHOR: P. Engel

REASON FOR REPORT:

This report is one in a series to assist the Sediment Survey Section of the Applied Hydrology Division in Developing a Method to Compute Bed Load from River Bed Profiles

CORRESPONDENCE FILE NO:

2242-1, Study File H 78 062

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1.0 INTRODUCTION

The Sediment Survey Section of the Applied Hydrology Division, Department of the Environment, have developed a hydrographic survey system (Hydac 100) (Wiebe 1972), for conducting surveys of reservoirs and river estuaries. The system obtains bed profiles using echo sounders on a rapidly moving boat the horizontal control of which is maintained with telemetric units on shore.

In this report, the method of computing bed load, given by Engel (1977a), by using the absolute average departures about the mean bed elevation as characteristic bed form height together with bed form speed, is examined further. The bed load is computed and compared with previous results as well as the results from the "Hydrographic Method" (Engel (1978)).

This report is one in a series prepared as continuing support of the Sediment Survey Section under HRD Study 78 062.

2.0 THEORETICAL DEVELOPMENT

Engel (1977a), with reference to Figure 2.1, found from continuity principles that the average volumetric bed load transport at a given point x can be given as

$$\bar{q}_{sx} = U_w (\eta - \eta_o) \qquad 2.1$$

where:

$\bar{q}_{sx}$	=	volumetric transport rate at point x per unit width including the voids
U_w	=	dune speed
η	=	bed elevation at any given point
η_o	=	elevation of zero transport

The transport over a full bed form is obtained from

$$\bar{q}_{s_\Lambda} = U_w \frac{1}{\Lambda} \int_0^\Lambda (\eta - \eta_o)\, dx \qquad 2.2$$

where:

$\bar{q}_{s_\Lambda}$	=	average volumetric transport rate over a bed form per unit width including the voids
Λ	=	length of bed form (figure 2.1)

When considering a profile record with many bed forms, the overall transport rate can be obtained by averaging over the whole length of record instead of just one wave length.

$$\bar{q}_{st} = U_w \frac{1}{L} \int_0^L (\eta - \eta_o)\, dx = U_w \overline{(\eta - \eta_o)} \qquad 2.3$$

where $\overline{(\eta-\eta_o)}$ is the average value of $(\eta-\eta_o)$ over the whole length of record. In his previous analysis, Engel (1977a), assumed that

$$\overline{(\eta - \eta_o)} = K_\varepsilon \bar{\varepsilon} \qquad 2.4$$

where:

$\bar{\varepsilon}$	=	the average absolute departure of the bed elevations about the mean bed elevation
K_ε	=	a coefficient

The coefficient K_ε is essentially a correction factor to allow the use of ε instead of $(\eta-\eta_o)$. It has to be determined from experimental measurements of bed load transport for different flow conditions. It is now proposed to obtain an alternate method to evaluate $(\overline{\eta-\eta_o})$, using published information on the location of η_o, whereby the experimental determination of a coefficient is unnecessary.

From figure 2.1, it can be seen that

$$\eta_o - \eta_t = a \qquad 2.5$$

where: η_t = elevation of the lowest point in the trough in the lee of a bed form

One may thus write

$$(\overline{\eta - \eta_o}) = (\overline{\eta - \eta_t}) - a \qquad 2.6$$

Willis and Kennedy (1977) stated that the assumption of triangular bed form shape is sufficiently accurate for computation of bed load. For triangular bed forms $(\overline{\eta-\eta_t}) = 0.5\,\Delta$ and thus equation 2.6 may be written as

$$(\overline{\eta - \eta_o}) = (0.5\ \bar{\Delta}) - \bar{a} \qquad 2.7$$

In order to determine values of "a", the position of zero transport must be known. Crickmore (1970) states that the plane passing through the elevation η_o also passes through the point of reattachment of the flow (i.e. point c in figure 2.2). Jonys (1973) conducted very detailed measurements on bed form formation and his data indicates the point of zero transport occurs at about $0.17\,\Delta$ above the elevation of the trough η_t. This means, then, that for the data of Jonys (1973) $a = 0.17\,\Delta$. Therefore $(\overline{\eta-\eta_o}) = 0.33\,\bar{\Delta}$ and from equation 2.3

$$\bar{q}_{st} = 0.33\ \bar{\Delta}\ U_w \qquad 2.8$$

The value of 0.33 varies considerably from the value of 0.5 used by Simons et al (1965), Williams (1967), Yalin (1972) and others. This constant varies only with "a" since $(\overline{\eta-\eta_t})$ is always equal to $0.5\ \bar{\Delta}$ if the bed forms are assumed triangular. Equation 2.8 can also be written in terms of $\bar{\varepsilon}$, the average absolute departure of bed elevations about the mean bed elevation. It is more convenient to use $\bar{\varepsilon}$ instead of $\bar{\Delta}$ in the computations. For triangular bed forms $\bar{\varepsilon} = \bar{\Delta}/4$. Therefore

$$\bar{q}_{st} = 1.32 \, \bar{\varepsilon} \, U_w \tag{2.9}$$

Converting to submerged weight, the bed load can be computed from

$$G_s = 1.32 \, \gamma_s (1 - \rho) \, \bar{\varepsilon} \, U_w \tag{2.10}$$

where: G_s = average submerged weight of sediment transported/unit width

γ_s = submerged unit weight of sediment

ρ = porosity of sediment

It is interesting to note that the coefficient of 1.32 is identical to the one obtained by Crickmore (1970) from characteristic bed form shapes given by Mercer (1964) and Raudkivi (1963) which agreed very well with his experimental measurements resulting in errors of only -4.4% and 3.9 % in computing G_s for the two runs conducted by him. This indicates that the chosen position of zero transport (i.e.=definition of η_o) and the adoption of triangular shapes for the bed form is realistic. However, it is not clear how the flow conditions from the tests of Crickmore (1970) compared with those of Jonys (1973). Therefore, nothing can be said about the variation of the numerical coefficient for different flow conditions

The bed load equation developed (i.e. equation 2.10) was tested with available flume data.

3.0 COMPUTATION OF BED LOAD

To compute the bed load discharge using equation 2.14, flume data from Engel (1978) was used and this together with the computed results is given in Table 3.1. Values of the computed bed load were plotted versus measured bed load in figure 3.1. Except for the case of highest transport rate, the plotted points are quite closely distributed about the line of perfect agreement. Relative errors E in percent were then computed from

$$E = \frac{\bar{G}_s - G_{sm}}{G_{sm}} \times 100\% \qquad 3.1$$

where: G_{sm} = measured bed load as submerged weight per unit width

The percent errors are also given in Table 3.1. The largest error was found to be -29.4% whereas the smallest was 6.3%. The average absolute error was 15.2%. Out of ten runs, six runs give an error of less than 15% and for three runs the estimated bed load is within 10%.

The uncertainty in computing the bed load was obtained from

$$S_E = \sqrt{\frac{\Sigma (E_i - \bar{E})^2}{N_E - 1}} \qquad 3.2$$

where:

	E_i	=	the error in computing bed load in the ith run
	$\bar{E}$	=	the average error in computed bed load over all the runs
	N_E	=	number of runs
	S_E	=	standard deviation of E

The computed standard deviation was found to be 16.6%. This value of S_E means that the accuracy of estimating the bed load discharge is

$$-22.6\% < E < 10.7\% \qquad 68\% \text{ of the time}$$
$$-39.2\% < E < 27.3\% \qquad 95\% \text{ of the time}$$

3.1 Discussion of Results

The ten runs of flume data cover a rather narrow range of Froude number defined as $F=U/\sqrt{gh}$ (U=mean flow velocity, h=depth, g=gravitational acceleration). Froude numbers varied from .399 to .480. In view of this, the results can only be considered as being preliminary. However, they are quite encouraging showing a 95% confidence of having computed the bed load with an error of less than 40%, which is considerably better than can be obtained with present sampling methods.

The computation of the bed load depends on two assumptions, namely, that bed forms are triangular and that the level of zero transport coincides with the point of reattachment of the flow (i.e. end of eddy zone in lee of bed form). In view of the results, both assumptions seem to be justified.

The method exemplified by equation 2.14 also compares well with the "Hydrographic Method" given by Engel and Lau (1978) and tested by Engel (1978). The relative accuracies obtained with the two methods are given in table 3.2. Equation 2.14 tends toward underestimating the true bed load whereas the "Hydrographic Method" tends to overestimate. However, overall, the spread of error about the mean error is practically the same. This means that the method of using departures $\bar{\varepsilon}$ as bed form height provides a viable alternative to the "Hydrographic Method". From a procedural standpoint, the proposed method is the simpler of the two methods to apply, since the bed load can be computed directly once $\bar{\varepsilon}$ and U_w have been obtained as shown by Engel (1977a). This method also has a distinct advantage over the "Hydrographic Method" since one does not need to define a bed load discharge distribution as shown by Engel and Lau (1978). The proposed bed load equation (i.e. equation 2.10) gives the average bed load directly and shows that a relatively simple model can be used to obtain bed load discharge from profile records.

The proposed method has the limitation, at present, in that it is based on the value of $a=0.17\Delta$ obtained by Jonys (1973) and it is not certain how this relationship will hold up over a wider range of flow conditions.

4.0 CONCLUSIONS

4.1 A bed load equation has been derived which does not require the determination of any unknown coefficients.

4.2 The results suggest that the assumption of triangular shaped bed forms is sufficiently accurate to determine bed load discharge.

4.3 The results also imply that the assumption of zero transport level coinciding with the level of the point of flow reattachment is justified. Future efforts should be made to confirm this.

4.4 The method of using average departures $\bar{\varepsilon}$ and bed form speed U_w is comparable in accuracy with the "Hydrographic Method" given by Engel and Lau (1978) over the range of flows tested.

4.5 More tests are needed to evaluate the proposed method over a wider range of flow conditions.

4.6 Separate, small scale experiments should be conducted to measure values of "a" for different flow conditions and bed form shapes.

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TABLE 3.1 DATA FROM ENGEL (1977) AND RESULTS

Run	m	$\frac{U}{\sqrt{gh}}$	K_ε	U_w m/s	G_{sm} K_g/s/m	G_{sc} K_g/s/m	% E
1	.0201	.434	1.51	.000602	.0166	.0145	-12.7
3	.0191	.412	1.49	.000625	.0161	.0143	-11.2
4	.0164	.414	1.11	.0005374	.0089	.0106	18.6
5	.0175	.450	1.71	.0006516	.0177	.0137	-22.8
6	.0185	.478	1.84	.001087	.0336	.0237	-29.4
7	.0165	.399	1.70	.0004199	.0107	.00830	-22.4
8	.0207	.480	1.24	.0007327	.0171	.0182	6.3
13	.0147	.427	1.42	.000675	.0128	.0119	- 7.1
14	.0179	.419	1.21	.0007789	.0153	.0167	9.2
15	.0179	.440	1.18	.009087	.0174	.0195	12.0

|E| 15.2

Average K_ε = 1.44

Average % error = -8.9%

TABLE 3.2 ACCURACY OF COMPUTED BED LOAD

% of Time	Equation 2.14 $G_s=1.32\ \gamma_s\ (1-\rho)\ \bar{\varepsilon}\ U_w$	Hydrographic Method (Engel, 1978)
68	$-22.6\% < E < 10.7\%$	$-9.1\% < E < 22.9\%$
95	$-39.2\% < E < 27.3\%$	$-25.1\% < E < 38.9\%$

E = error in computing bed load

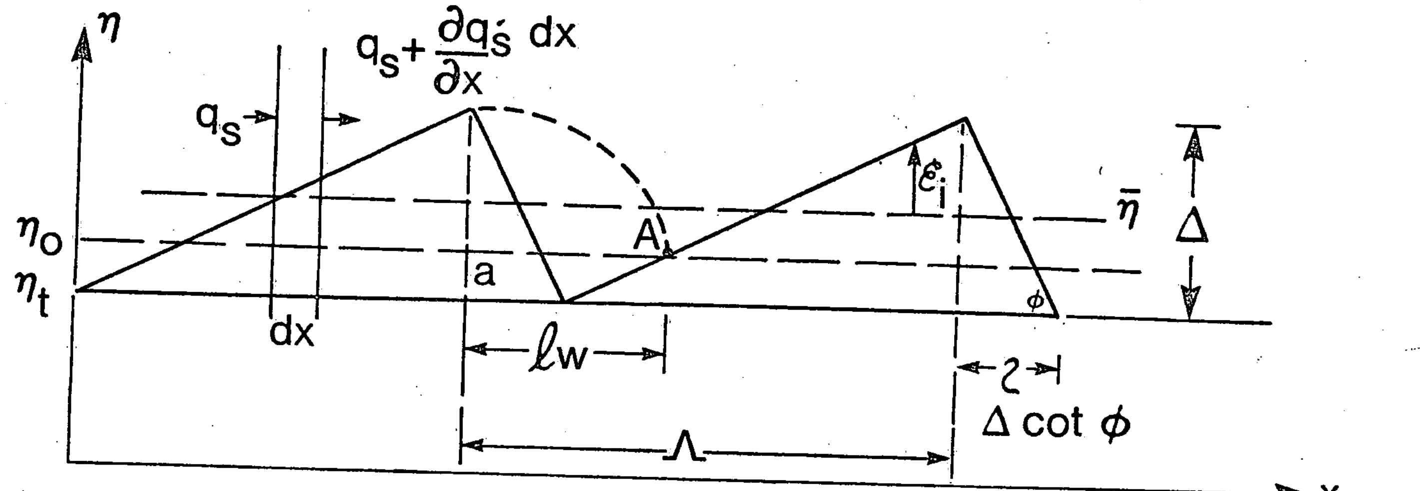

FIGURE 2.1 TRIANGULAR DUNE PROFILE DEFINITION SKETCH

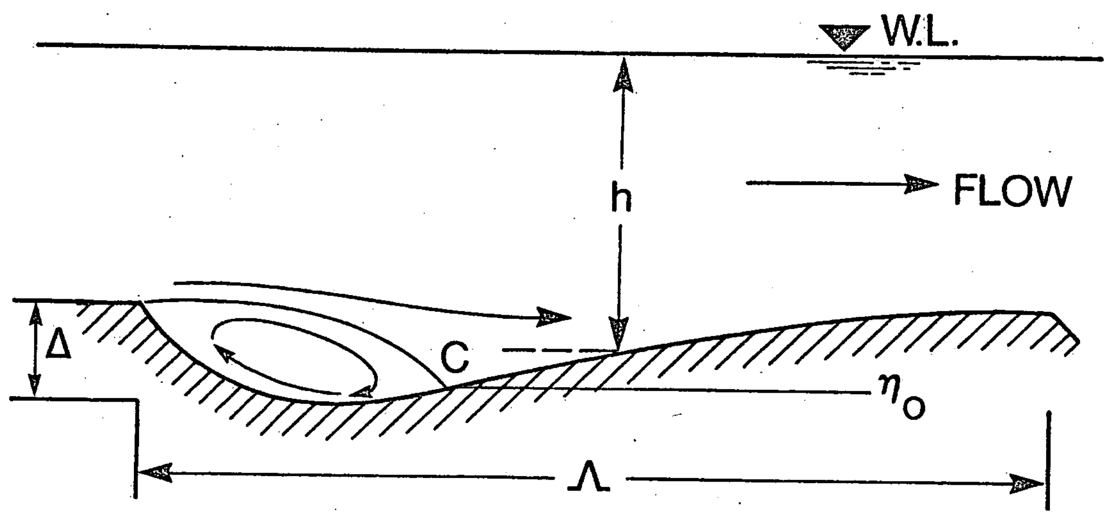

FIGURE 2·2 ELEVATION OF ZERO TRANSPORT.

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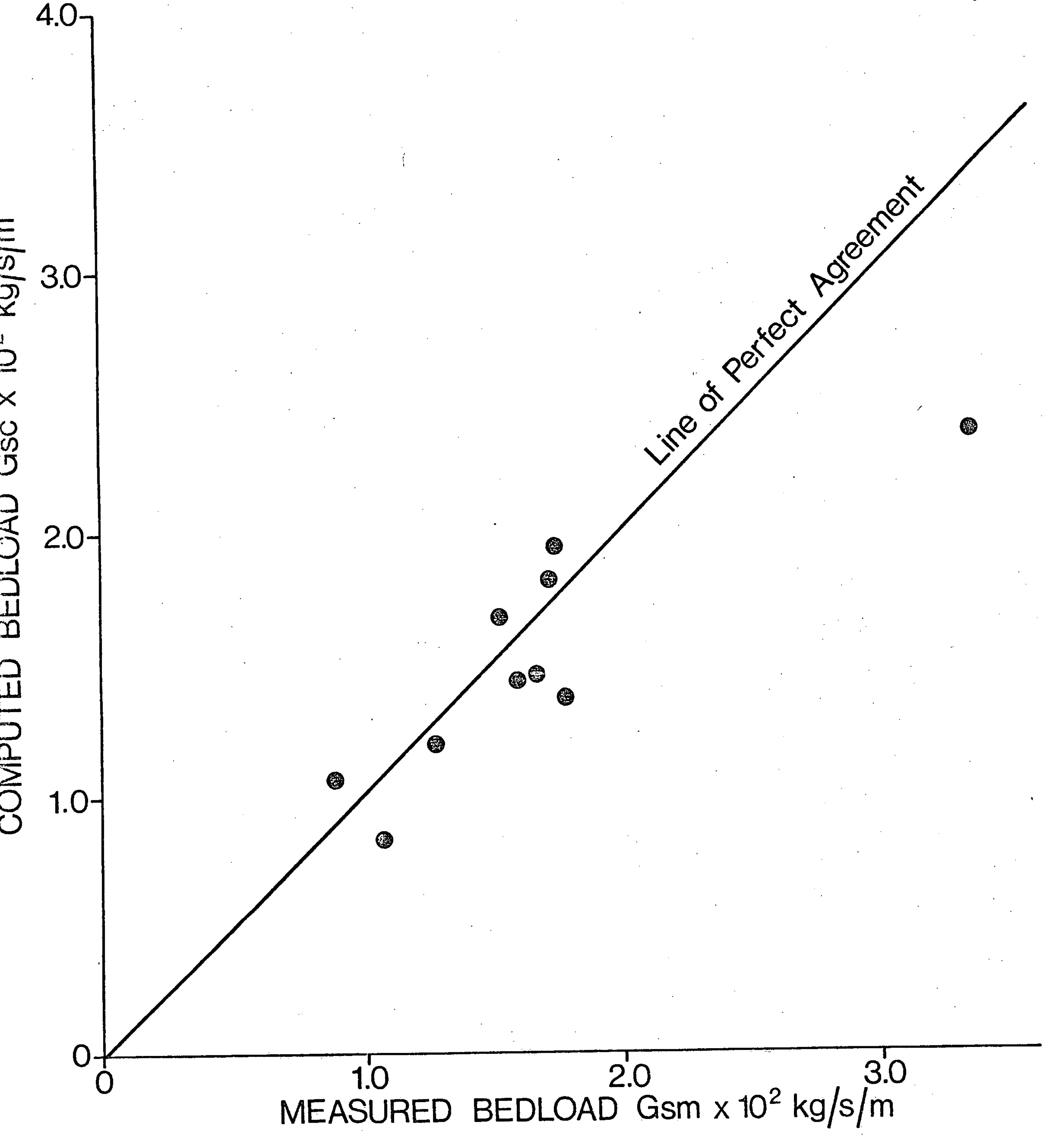

FIGURE 3.1 COMPARISON OF COMPUTED AND MEASURED BEDLOAD.