NATIONAL WATER RESEARCH INSTITUTE.

HYDRAULICS RESEARCH DIVISION

Technical Note

DATE: May 1979 **REPORT NO:** 79-13

TITLE: Terminal Velocity for Multiband Transmittance Temperature Profiler

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REASON FOR REPORT:

This report was prepared at the request of the Scientific Support Division at the National Water Research Institute.

CORRESPONDENCE FILE NO:

1371

1.0 INTRODUCTION

The Engineering Section of the Scientific Support Division at the National Water Research Institute is developing an instrument package called "Multiband Transmittance Temperature Profiler", to measure vertical temperature profiles and turbidity in the Great Lakes. The profiler gathers data while dropping from the water surface to the lake bottom. In order to have some idea of the length of profiling transit time in "free fall", and the required weight to achieve a certain descent rate, the Hydraulics Research Division was requested to determine the terminal velocity of the instrument package for two descent modes. In the first mode, the profiler descends with its longitudinal axis perpendicular to the direction of travel and, in the second mode, the longitudinal axis is parallel to this direction.

In this report the terminal velocity of a free-falling body is derived in general terms from basic principles. The results are used to examine the two descent modes of the profiler and recommendations regarding the required weight of the instrument package are made.

2.0 DESCRIPTION OF THE INSTRUMENT PACKAGE

The body of the multiband transmittance temperature profiler is a cylinder 22 cm in diameter and 56 cm in length. Attached to this cylinder is a small nose cylinder 12.5 cm in diameter and 21.6 cm in length. The wall of the nose cylinder is perforated with four slots, two of which may be approximated by a rectangle 5 cm wide and 11 cm long and the other two slots by a rectangle 7.75 cm wide and 11 cm long. The instrument cylinder and nose is surrounded by a cage which protects the unit on its traverse to the lake bottom. The cage is composed of aluminum bars 1.3 cm in diameter and 80 cm long of which there are eight equally-spaced on rings around the instrument cylinder at a radius of 28 cm. The three components, namely the main cylinder, the nose and the cage, can be seen in Figure 2.1. The mass of the total instrument package (all three components) is approximately 23 Kg.

3.0 DETERMINATION OF TERMINAL VELOCITY

3.1 General Case

The velocity of a body falling freely in a Newtonian fluid medium is given by,

$$v = \sqrt{\frac{2W(1-\alpha)}{\rho C_D A}} \left[\frac{1 + e^{-\sqrt{\frac{2\rho g^2 C_D A (1-\alpha)}{W}}\, t}}{1 - e^{-\sqrt{\frac{2\rho g^2 C_D A (1-\alpha)}{W}}\, t}} \right] \qquad 3.1$$

where
v = velocity of descent
W = gravitational force on the body
α = ratio of buoyant force to gravitational force
ρ = density of fluid
C_D = drag coefficient
A = bluff body area of the body
g = acceleration of gravity
t = time in seconds

The derivation of Equation 3.1 is given in detail in Appendix A. For the given values of W, A and the expected range of C_D examination of the exponents in Equation 3.1 indicates that for values of $\alpha < .95$ ($\alpha = 1.0$ is neutral buoyancy), the terminal velocity of the body will be reached very quickly (less than a second), since even for t=1 sec, the exponent will be large enough to make the exponential terms of the equation practically equal to zero. Therefore, the fall velocity of the body can be closely approximated by

$$v_t = \sqrt{\frac{2W(1-\alpha)}{\rho C_D A}} \qquad 3.2$$

where v_t = terminal velocity.

If the size and shape of the profiler is specified, then the only variable affecting the terminal velocity is its weight W. Therefore Equation 3.2 can be simplified to

$$v_t = \sqrt{\frac{2(W - B)}{\rho C_D A}} \qquad 3.3$$

where B = buoyant force

Equation 3.2 shows that the terminal velocity depends on W, B, ρ, A and C_D. Therefore, if it is desired to design a free-falling body for a particular terminal velocity, consideration must be given to its shape, size and weight.

In the case of the profiler, shape and size are already specified. Each of the three components, namely, the cylindrical body, cylindrical nose with perforations and the cage composed of round bars, exerts a drag force. Since no data is available from drag tests, its is assumed that the drag forces on the three components can be considered to act independently. That is, the total drag force is the sum of the component drag forces. Equation 3.2 can be adapted for this in the form,

$$v_t = \sqrt{\frac{2\,(W - B)}{\rho\,\left[(C_D A)_b + (C_D A)_n + (C_D A)_c\right]}} \qquad 3.4$$

where b, n and c denote body, nose and cage respectively. Values of A were obtained from direct measurements whereas values of C_D were obtained from the literature, Table 3.1(Rouse, 1960). In this analysis, terminal speeds up to 1 m/s are considered.

3.2 Profiler with Longitudinal Axis Perpendicular to the Direction of Travel (Mode 1)

Values of the areas A and coefficients of drag C_D are summarized in Table 3.2. The drag coefficients were obtained from Table 3.1. The values of $(C_D A)$ together with the value of ρ=1000 Kg/m^3 were substituted into Equation 3.4 to give

$$v_t = .097\ \sqrt{W - B}\ \text{m/s} \qquad 3.5$$

The total volume of displacement of the profiler from Table 3.2 is .022 m^3 and therefore the buoyant force is B=1000x.022x9.8=216N. Values of terminal velocity over a range of W for the fixed value of buoyant force pertaining to the profiler were computed and plotted in Figure 3.1 as a function of mass. The curve shows that close to the point of neutral buoyancy the terminal velocity is very sensitive to a change in mass and that this sensitivity gradually decreases as the mass increases. The additional mass necessary to obtain a particular terminal velocity for the profiler can be obtained from Figure 3.1.

3.3 Profiler with Longitudinal Axis Parallel to the Direction of Travel (Mode 2)

Since the body cylinder and the nose cylinder are concentric and since the diameter of the body is 1.8 times that of the nose and since the nose is short in relation to the body, the effect of the nose was ignored. Therefore, Equation 3.4 is reduced to

$$v_t = \sqrt{\frac{2\,(W - B)}{\rho\,[(C_D A)_b + (C_D A)_c]}} \tag{3.6}$$

Values of the areas A and drag coefficients C_D are summarized in Table 3.3. The drag coefficients were obtained from Table 3.1. Substituting the values of $(C_D A)$ into Equation 3.6 then results in the relationship

$$v_t = .192 \sqrt{W - B} \quad \text{m/s} \tag{3.7}$$

Direct comparison of Equation 3.7 with Equation 3.5 shows that for the same mass the profiler will descend approximately twice as fast in Mode 2 as in Mode 1. Values of the terminal velocity were again plotted as a function of mass and this is also given in Figure 3.1. Once again, the necessary mass to attain a particular terminal velocity can be obtained from this curve.

4.0 EFFECT OF ERROR IN C_D ON TERMINAL VELOCITY

The geometric parameters as well as W and B can be determined quite accurately. However, in the absence of drag tests, the drag coefficient must be estimated. The relative error of terminal velocity as a function of the relative error in selecting the drag coefficient can be expressed by

$$\frac{dv_t}{v_t} = -\frac{1}{2}\,\frac{dC_D}{C_D} \qquad 4.1$$

Where dv_t=the error in v_t and dC_D=the error in C_D. Equation 4.1 shows that any error in the value of C_D will result in an error half that size in the terminal velocity. For example, if the selected value of C_D was in error by 20%, the error in terminal velocity is only 10%.

5.0 CONCLUSIONS

5.1 When its longitudinal axis is parallel to the flow direction, the terminal velocity of the Multiband Transmittance Temperature Profiler is about twice the terminal velocity when its alignment is perpendicular to the flow for the same submerged weight.

5.2 At the present mass of 22.7 Kg, as reported by SSD, the profiler will reach a terminal velocity of about 25 cm/s in Mode 1 and 50 cm/s in Mode 2.

5.3 In order for the profiler to reach a terminal velocity of 1 m/s in Mode 1 about 11 Kg of mass must be added and in Mode 2 about 3 Kg of mass must be added. The mass must be added without changing the external geometry of the profiler.

5.4 The results presented are only approximate since the interference between various components has to be neglected and the drag coefficients were obtained from tables and applied with simplifying assumptions. If more precise results are required, drag tests must be conducted in the two alignments referred to as Mode 1 and Mode 2.

REFERENCES

Rouse, Hunter, 1960. "Elementary Mechanics of Fluids", John Wiley and Sons Inc., New York.

TABLE 3.1 DRAG COEFFICIENTS

Form of Body	L/D	$\frac{UD}{\nu}$	C_D
Circular Cylinder (axis $\parallel$ to flow)	0	$>10^3$	1.12
	1		0.91
	2		0.85
	4		0.87
	7		0.99
Circular Cylinder (axis $\perp$ to flow)	1	10^5	0.63
	5		0.74
	20		0.90
			1.20
	5	$>5 \times 10^5$	0.35
			0.33

L = Length of body

D = Diameter of body

ν = Kinematic viscosity

TABLE 3.2 DATA FOR PROFILER WITH AXIS PERPENDICULAR TO FLOW

Component	L/D	C_D	A m^2	$\frac{v_t D}{\nu}$	$C_D A$ m^2	Displ. Volume m^3
Cylinder Body	2.4	.80	.123	10^5-3 x 10^5	.098	.021
Cylinder Nose	1.7	.80	.018	5×10^4 - 2×10^5	.014	.000
Cage Bars	50	1.10	.083	4×10^3 x 1.2×10^4	.091	.001
End Rings		.80	.011		.009	.000
				TOTAL	.212	.022

TABLE 3.3 DATA FOR PROFILER WITH AXIS PARALLEL TO FLOW

Component	L/D	C_D	A m^2	$\frac{v_t D}{\nu}$	$C_D A$ m^2	Disp. Volume m^3
Cylinder Body	2.4	0.85	.038	10^5 - 3×10^5	.032	.021
Cylinder Nose	-	-	-	-	-	0.000
Cage Bars	50	1.20	.001	4×10^3 - 1.2×10^4	.001	.001
End Rings	22	0.90	.023		.021	.000
				TOTAL	.054	.022

APPENDIX A

DERIVATION OF GENERAL TERMINAL VELOCITY EQUATION

Net force=weight of body in air - buoyant force - drag force.

The definition sketch showing the forces is given in Figure A-1. The net balance of forces may be written as

$$m \frac{dv}{dt} = mg - \alpha mg - Kv^2 \qquad \text{A–1}$$

where:

	m	=	mass of the body
	v	=	velocity of descent
	t	=	time
	g	=	acceleration due to gravity
	α	=	ratio of buoyant force to weight in air
	K	=	a coefficient

dividing through by m:

$$\frac{dv}{dt} = g - \alpha g - \frac{K}{m} v^2 \qquad \text{A–2}$$

and

$$\frac{dv}{dt} = g(1 - \alpha) - \frac{K}{m} v^2 \qquad \text{A–3}$$

Let $g(1-\alpha)=k^2$ and $K/m=b^2$:

$$\therefore \frac{dv}{dt} = k^2 - b^2v^2 \qquad \text{A–4}$$

and

$$\frac{dv}{b^2v^2 - k^2} = -dt \qquad \text{A–5}$$

rearranging w.r.t. b^2:

$$\frac{dv}{v^2 - \frac{k^2}{b^2}} = -b^2 dt \qquad \text{A–6}$$

setting $k^2/b^2=a^2$:

$$\frac{dV}{v^2 - a^2} = -b^2\, dt$$

integrating:

$$\int \frac{dv}{v^2 - a^2} = -b^2 \int dt + c \qquad \text{A-7}$$

$$\frac{1}{2a} \ln \left[\frac{v - a}{v + a}\right] = -b^2 t + \ln C_2 \qquad \text{A-8}$$

and

$$\ln \frac{v - a}{v + a} = -2\, ab^2 t + \ln C_2 \qquad \text{A-9}$$

$$\therefore \frac{v - a}{v + a} = C_2\, e^{-2\, ab^2 t} \qquad \text{A-10}$$

when $t=0$, $v=0$, $\therefore C_2 = -1$

and

$$\frac{v - a}{v + a} = -e^{-2ab^2 t} \qquad \text{A-11}$$

solving for v gives

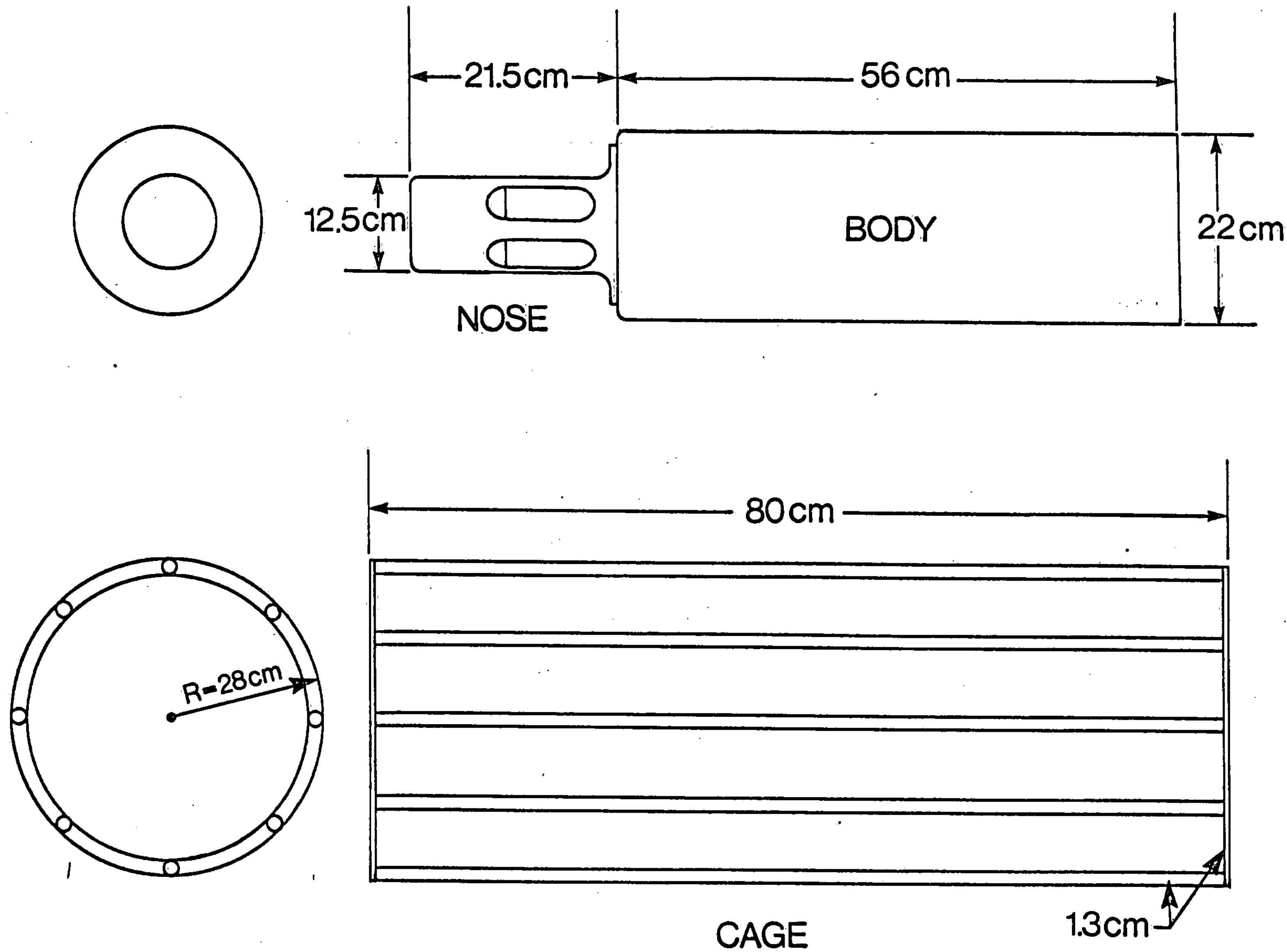
$$v = a \left[\frac{1 - e^{-2\, ab^2 t}}{1 + e^{-2\, ab^2 t}}\right] \qquad \text{A-12}$$

substituting $a=k/b$, $b=\sqrt{K/m}$ and $k=\sqrt{g(1-\alpha)}$ and $K=1/2\rho C_D A$.

$$v = \sqrt{\frac{2mg\,(1-\alpha)}{\rho C_D A}} \left[\frac{1 - e^{-\sqrt{\frac{2\rho C_D A g\,(1-\alpha)}{m}}\, t}}{1 + e^{-\sqrt{\frac{2\rho C_D A g\,(1-\alpha)}{m}}\, t}}\right] \qquad \text{A-13}$$

also, since W=mg

$$v = \sqrt{\frac{2W(1-\alpha)}{\rho C_D A}} \left[\frac{1 - e^{-\sqrt{\frac{2\rho g^2 C_D A (1-\alpha)}{W}}\, t}}{1 + e^{-\sqrt{\frac{2\rho g^2 C_D A (1-\alpha)}{W}}\, t}} \right] \qquad \text{A–14}$$



EIGHT BARS AT 1.3cm ϕ EQUALLY SPACED.

FIGURE 2.1 DETAILS OF THE TRANSMISSIVITY–TEMPERATURE PROFILER.

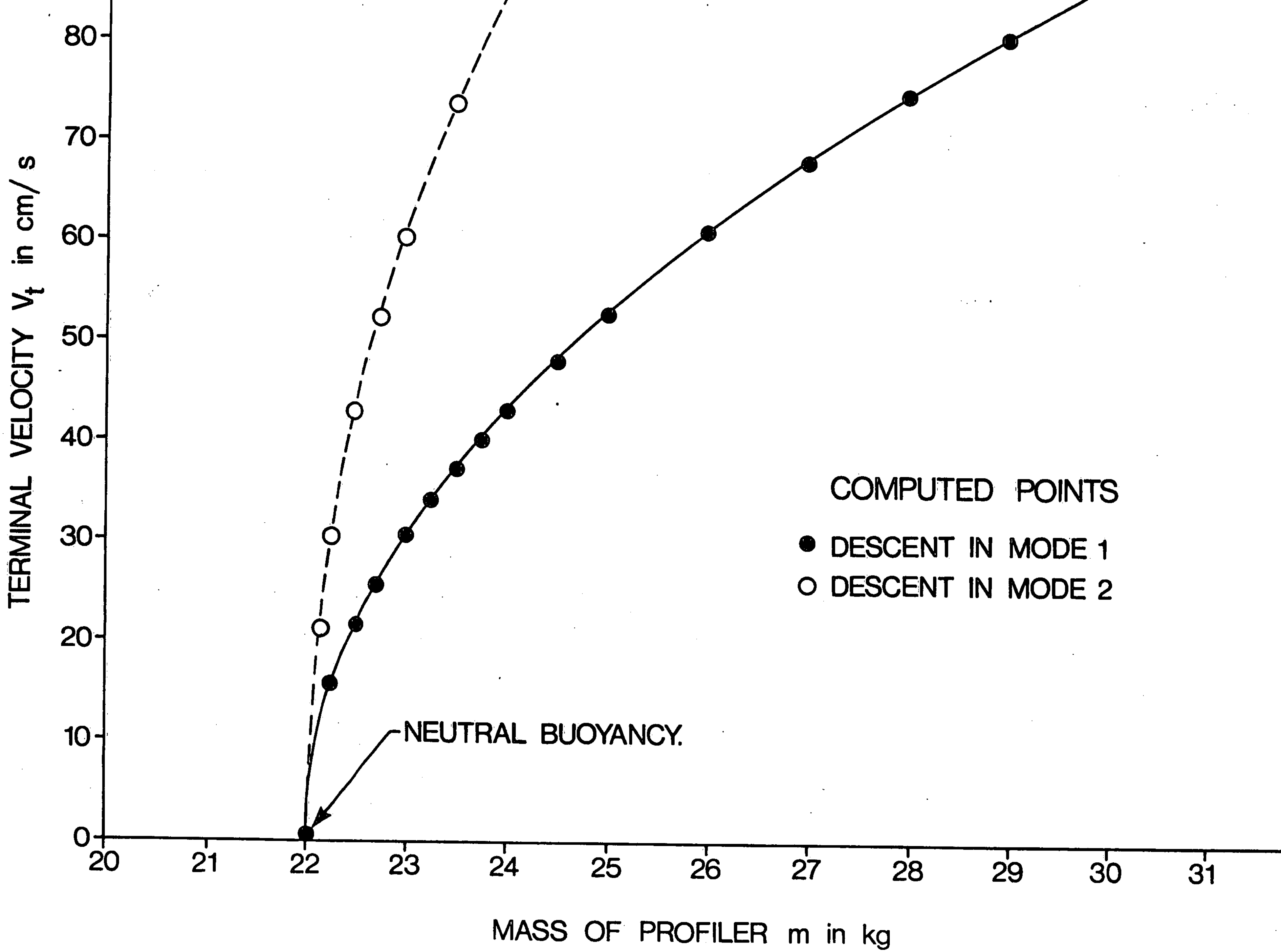


FIGURE 3.1 VARIATION OF TERMINAL VELOCITY WITH BODY MASS.

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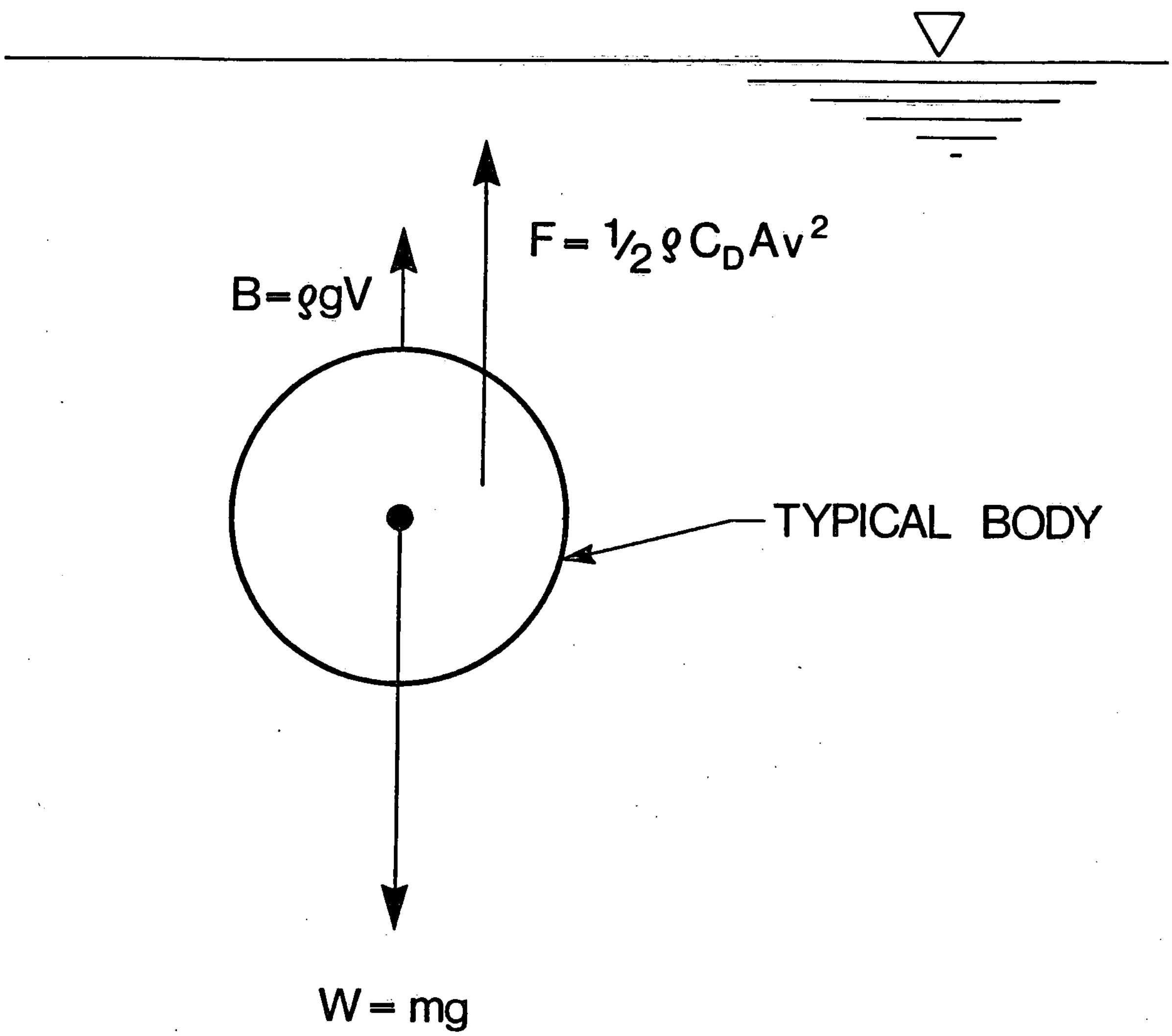

FIGURE A-1 DEFINITION SKETCH