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Evapotranspiration of Groundwater

A Time Series Analysis of the Hydrologic Regimen of a Groundwater Discharge Area

Richard E. Jackson

SCIENTIFIC SERIES NO. 17

INLAND WATERS DIRECTORATE,
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To John. Many thanks for your help on this. Dick.

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Abstract

Methods of time-series analysis were employed to examine climatological and hydrogeological variables associated with a groundwater discharge area in Manitoba, Canada. The hydrogeological variables, daily groundwater evapotranspiration and inflow rate, were adequately modelled by a first-order Markov process. Nonrandom fluctuations associated with the weather and circulation of the North American summer climate were identified in the time series of mean daily temperature and groundwater evapotranspiration, while daily precipitation was random. Statistical filtering of the hydrogeological time series showed that two processes were associated with seasonal maxima in the groundwater evapotranspiration series; one being due to the propagation of groundwater recharge through the flow system, the other to climatic effects on the discharge area, in particular the Lisse effect. Finally it is explained why the water table fluctuates within a small range of depth in groundwater discharges areas.

Notations

Major Abbreviations

acf	= autocorrelation function
acvf	= autocovariance function
Var	= variance

Greek Symbols

α	= confidence level probability
β	= limiting value of error in the corrected filter
μ	= mean
ξ	= some point on a real line
σ	= standard deviation
$\rho(\ell)$	= theoretical acf
ΣH	= sum of filter values
Δs	= daily drawdown of hydrograph
$\chi^2_{T_m}(\alpha)$	= chi-square value
ℓ	= lag
ω, ω'	= angular frequency

Other Symbols

a	= fitted acf coefficient
B( )	= theoretical acvf
b( )	= the filter
b′( )	= the corrected filter
c	= fitted acf coefficient
C( )	= estimated acvf
C*( )	= modified apparent acvf
D( )	= lag window
E/ /	= expected value
F/ /	= distribution function
$G(\ell)$	= smoothed estimate of spectral density
g( )	= function of
H	= cumulative distribution function of variance
H′	= probability density function of variance
H( )	= cumulative distribution function
h( )	= theoretical spectral density
k	= time lag
K(s, t)	= covariance kernal
m	= lag
n	= sample size
n(k)	= noise (sample)
N(k)	= noise (population)
P	= probability
$\overline{P}$	= daily precipitation in inches
p	= filter coefficient
q	= filter coefficient
Q	= groundwater evapotranspiration in feet/day
r_1	= lag one acf coefficient
r_k^*	= acf significance test coefficient
R	= groundwater inflow rate in feet/hour
R_k	= coefficient in chi-square test
R( )	= estimated acf
S( )	= signal (population)
s( )	= signal (sample)
S_y	= specific yield
t	= time
T	= set containing all possible t
$\overline{T}$	= mean daily temperature in degrees Fahrenheit
T_m	= maximum lag
t_p	= student's variable at probability level p
V( )	= raw estimate of spectral density
v, v′	= degrees of freedom
(X( ))	= random variable or stochastic process (population)
(x( ))	= sample of X(t)
(y( ))	= sample of (X( ))
z	= period

Time Series

$\overline{P}(t)$	= time series of $\overline{P}$
Q(t)	= time series of Q
R(t)	= time series of R
$\overline{T}(t)$	= time series of $\overline{T}$

Introduction

1.1 OBJECTIVES

The application of time-series analysis to the study of hydrogeological processes is relatively new. Julian (1967) suggested that its use in hydrogeology would be in the analysis of "groundwater fluctuations in an effort to study natural regulatory processes." The natural regulatory processes considered herein are concerned with the characteristic behaviour or regimen of the water table in a groundwater discharge area.

The objectives of the study were to use time-series analysis to:

(a) identify nonrandom components in the climatological variables affecting the groundwater flow system and in the hydrogeological variables associated with evapotranspiration from the discharge area;

(b) seek a stochastic model to describe these hydrogeological variables; and

(c) develop a statistical filter to extract the hydrogeological signals from the attendant noise.

1.2. TIME-SERIES ANALYSIS

Several statistical concepts need explaining before these objectives may be explored. A time series is an ordered sequence generated by taking observations on a phenomenon which is changing in time. Therefore an observation-well hydrograph may be defined as a time series of groundwater levels. Furthermore a time series may be assumed to be composed of two components. The first is a nonrandom or deterministic part that may be a trend and/or a fluctuation with a known period and amplitude. The second is a random component that develops due to some random mechanism. It is assumed that a time series which contains a random component and which contains but is not dominated by deterministic functions such as an annual cycle, is generated by a stochastic process, that is, under the control of probability laws. This stochastic process may be so complex that it becomes profitable to treat it as a random process, and then to extract the nonrandom components from it. Such an approach is pursued in this study.

A random process may be analysed by two approaches, that of the time domain and that of the frequency domain. Time-domain analysis shows by the use of correlation functions to what extent points on the hydrograph are related to each other in time. Frequency-domain analysis, more familiarly known as power-, or preferably, variance-spectrum analysis, assigns the observed variance of the random component of a variable to an infinite number of oscillations with a continuous distribution of frequencies. Gilliland in Canada and Eriksson in Sweden were the first hydrogeologists to employ these methods, derived from statistical communications theory, in analyzing hydrogeological time series.

Gilliland (1973) demonstrated that correlation functions could be used to determine whether small fluctuations in groundwater hydrographs were due to barometric or evapotranspiration effects.

Eriksson (1970) pioneered the application of variance-spectrum analysis in hydrogeology. Using groundwater levels from a riparian esker, a glaciofluvial deposit, and local streamflow, air temperature and precipitation records, Eriksson found strong persistence in all records except daily precipitation, which was almost randomly distributed in time. Furthermore he developed a stochastic model of the water balance of the esker, assuming that the groundwater reservoir was in steady state, to give estimates of the mean annual infiltration of the esker.

The hydrologic regimen of a groundwater discharge area is largely a response to climatological stimuli, which have their own periodic structure and causes. Of particular interest to hydrologists is the analysis of precipitation and temperature time series, as they are important in controlling recharge to and discharge from the flow system.

The various scales of atmospheric motion which have an observable effect on the weather include the mesoscale circulation, the synoptic-scale circulation, the Rossby waves and the semi-permanent centers of action (J. Nibler, personal communication). The mesoscale circulation is of a few hours duration and is manifested by thunderstorms, while the synoptic-scale circulation, such as frontal waves and the short waves in the westerlies, occur over periods of $^1/_2$ to 2 days. The Rossby waves, or the long waves in the

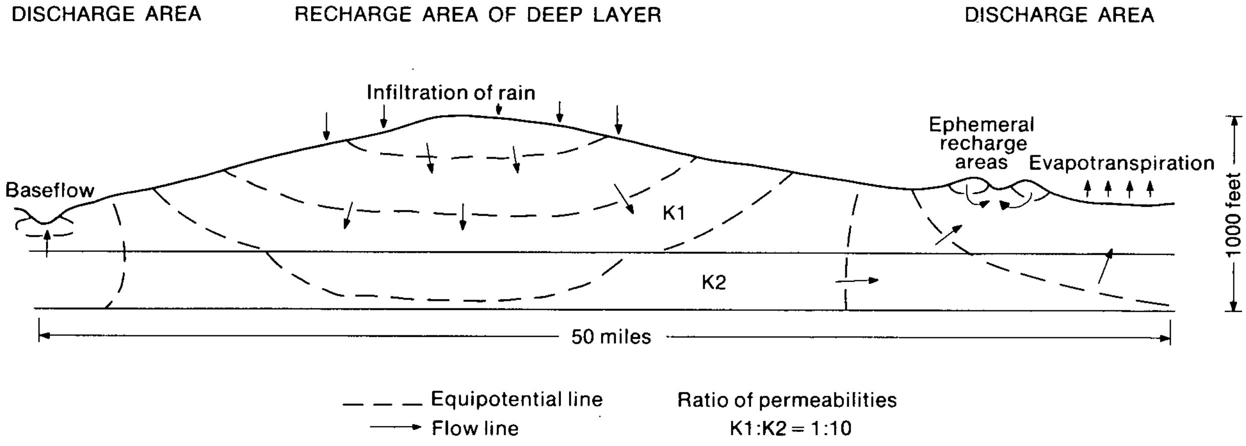


Figure 1.1 Modified prairie profile

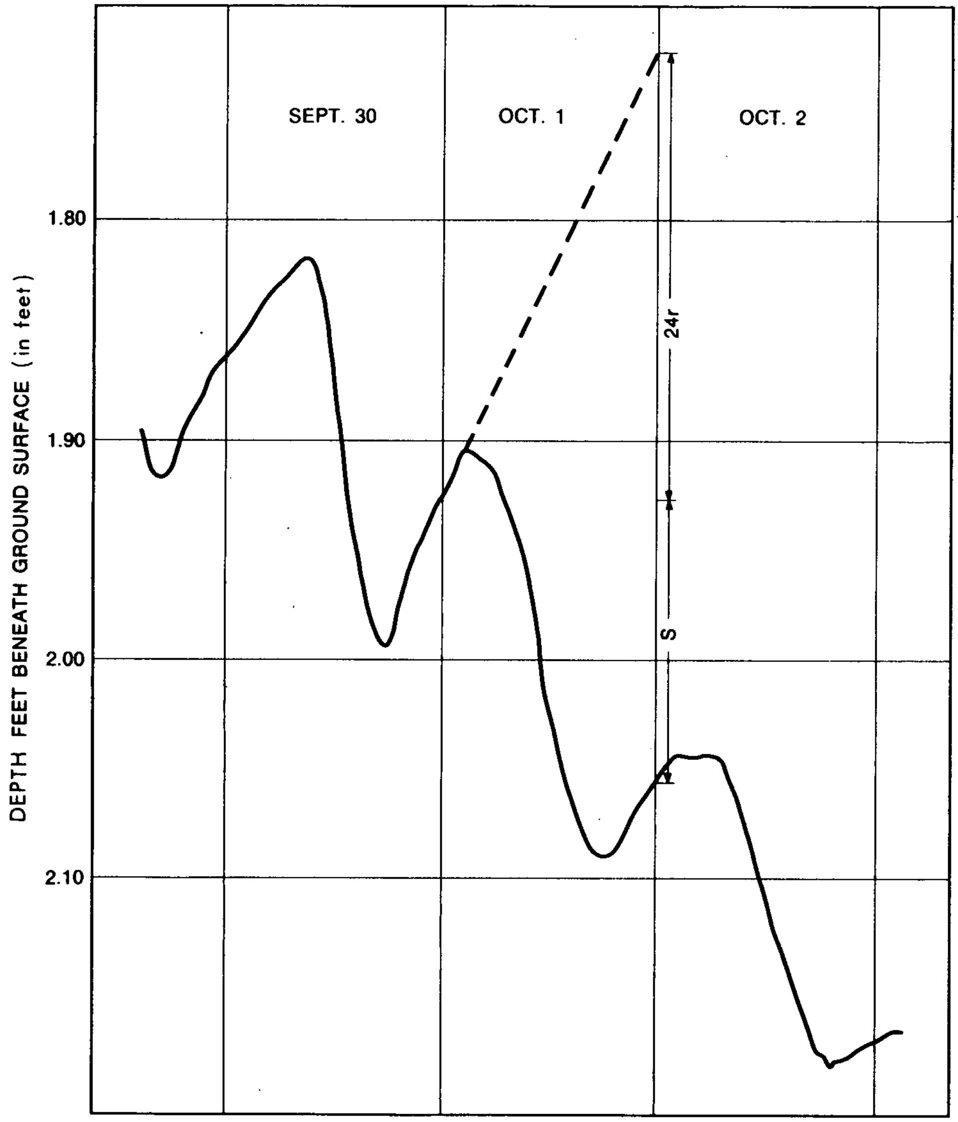


Figure 1.2 Diurnal fluctuations due to evapotranspiration at K-1

westerlies, have periods ranging between 2 and 10 days and have been observed in a previous climatological spectral analysis by Landsberg, Mitchell, and Crutcher (1959). The semi-permanent centers of action are seasonal phenomena and will appear as trends in time-series analysis. Dickson (1971) noted that a strong, summer, upper-level anti-cyclone over North America tends to concentrate variance in the long-period part of the spectrum by deflecting the westerlies northward, causing drought in the U.S.A.

1.3 HYDROLOGIC REGIMEN OF GROUNDWATER DISCHARGE AREAS

Canadian hydrogeologists were preoccupied during the 1960's with the analysis of regional, groundwater flow systems. Toth (1962) developed an analytical model, Meyboom (1963) an empirical one, and Freeze (1969) a numerical one. Meyboom's (1963) Prairie Profile (Figure 1.1) will suffice as a representative example of a regional, groundwater-flow system. The central topographic high, which forms the recharge area, is bounded on each side by a lower area in which groundwater discharge occurs - mainly by baseflow and evapotranspiration. Groundwater flow is downward in the recharge area and upward in the discharge

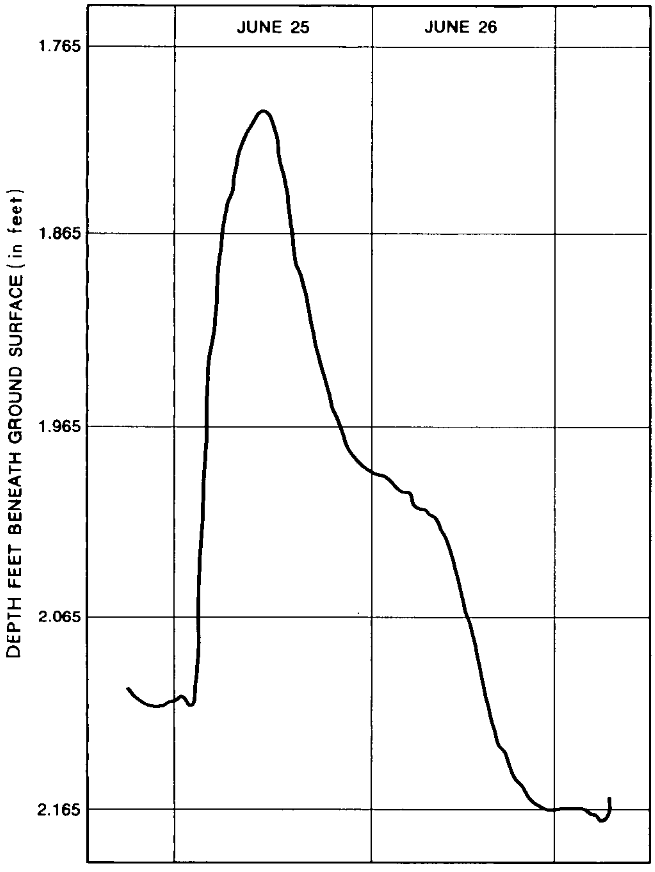


Figure 1.3 Lisse effect at D-1

area. As Freeze (1969) noted the recharge to and the discharge from the flow system keeps the water table in an almost steady-state condition. It is therefore an example of a system in dynamic equilibrium and his assumption of a steady-state water table enabled Freeze to treat the regional, groundwater-flow system as a steady-state, mathematical model.

Meyboom (1967) ascribed the natural fluctuations of water tables on the Canadian Prairies during the summer to barometric pressure, air temperature, rain, and evapotranspiration. Barometric pressure and the direct effects of temperature are insignificant from the water-balance aspect and attention is focused on the roles of rain and evapotranspiration on the water balance of a groundwater discharge area.

The diurnal fluctuations of water tables due to evapotranspiration by phreatophytes, plants that obtain water from the zone of saturation, are of particular interest because they signify the discharge of vast quantities of water from a flow system (Figure 1.2). White (1930) noted that the daily drawdown of the water table began in the morning and continued until late afternoon when transpiration ceased and the water table began to rise due to upward groundwater flow. The recovery would continue until the next morning when transpiration would recommence and the water table would decline. White concluded that the slope of the hydrograph during its nightly recovery was the hourly groundwater-inflow rate, R. If the specific yield of the aquifer S_y is known, it is possible to compute the daily evapotranspiration of groundwater Q by White's formula:

$$Q = S_y (24R \pm \Delta s) \qquad [1.1]$$

Where Δs is the daily change of the water table. Negative values of R indicate the water table is draining, while positive values indicate upward flow.

Daily groundwater losses by evapotranspiration are primarily a function of air temperature and the depth to the water table. Of the two, Meyboom (1967) found the latter more important. This being so it is natural to determine what the factors are that produce a shallow water table. Apart from upward groundwater flow, rain can alone cause a substantial rise in the water table, either by infiltration or by the Lisse effect (Figure 1.3). This latter phenomena occurs in soils in which the water table lies within three feet of the surface. Commonly an inch of infiltrated rain will cause approximately an eighteen-inch rise of the water table. This is due to the infiltrated rain compressing the air above the capillary fringe. This pressure increase causes a rise in the water table so as to re-establish the equilibrium of the soil-moisture system (Meyboom 1967).

In order to identify nonrandom components in a time series it is necessary to compute the sample autocorrelation and/or spectral density functions of that time series. Here the sample autocorrelation function is also the basis for the development of the stochastic model and the statistical filter.

Mathematical Methods

2.1 THE NATURE OF STOCHASTIC PROCESSES

Let T be some arbitrary infinite set and let X(t) be some function on T. In mathematical terms, therefore, $(X(t) : t \epsilon T)$ is a random or stochastic process "in which the set T is the region of time over which the process is defined and the time parameter t belongs to T, the index set of all possible values" (Kisiel 1969).

If the sample values $x(t_1), x(t_2), \ldots, x(t_n)$ of the stochastic process from the populations $X(t_1), X(t_2), \ldots, X(t_n)$ are statistically independent of one another, the sequence is described as pure random. However if the sequence is internally dependent, that is, serially correlated, then it is described as nonpure random or stochastically dependent. As Kisiel has suggested "we note a spectrum of different processes is possible in nature, ranging from a pure deterministic process to a pure random process."

If F/x(t)/ is defined as the distribution function of X(t), that is

$$F/x(t)/ = P(X(t) < x(t)) \qquad [2.1]$$

then the joint distribution function of a n-dimensional random variable may be defined as

$$F/x(1), x(2), \ldots, x(n)/ = P[X(t_1) < x(1), X(t_2) < x(2), \ldots, X(t_n) < x(n)] \qquad [2.2]$$

The left-hand side of [2.1] expresses the probability that X(t) takes on a value less than or equal to x(t).

If F(x) is a function defined on an interval (a,b) of a real line, and if (a,b) is divided up into n equal intervals such that $a = x_0 < x_1 \ldots < x_n = b$, where $\Delta x = x_i - x_{i-1}$ for i = 1, . . . n then the Stieljes integral with respect to F(x) of a function g(x), as x varies from a to b, is defined as

$$\int_a^b g(x)\ dF(x)$$

which in discrete terms may be written

$$\lim_{\Delta x \to 0} \sum_{i=1}^{n} g(\xi_i)/F(x_i) - F(x_{i-1})/$$

where ξ_i is a point on (a,b) so that $x_{i-1} < \Sigma_i < x_i$.

From the Stieljes integral the expected value or mean is defined as

$$E / X(t) / = \mu_t = \int_{-\infty}^{\infty} x(t)\ dF/x(t)/ \qquad [2.3]$$

and the covariance kernel as

$$K(s,t) = E /[X(s) - \mu_s] [X(t) - \mu_t] / =$$

$$\int_{-\infty}^{-\infty} [x(s) - \mu_s] [x(t) - \mu_t]\ dF / x(s), x(t) / \qquad [2.4]$$

where s and t are particular times and μ_s and μ_t the mean values of X(s) and X(t) respectively.

As Kisiel has pointed out, if $\mu_s = \mu_t = 0$ then for $s \neq t$ [2.4] yields the autocorrelation function and its graphical expression the correlogram. Also, irrespective of the values of μ_s and μ_t, if s = t then [2.4] yields the variance of the process.

If K(s,t) depends only on the difference s-t, i.e., the lag, and if E/X(t)/ is also constant for all t, then the process X(t) is said to be covariance stationary and the autocovariance function is denoted by $B(\ell)$, where (ℓ) is the lag. It is assumed henceforth that all stochastic processes will be normalized by subtracting their means so that $\mu_s = \mu_t = 0$. This, of course, does not interfere with their stationarity.

2.2 VARIANCE-SPECTRUM ANALYSIS

Variance spectrum analysis may be considered as the application of generalized harmonic analysis to the study of a stochastic process. Panofsky (1955) has stated that the difference between variance spectrum analysis and ordinary Fourier analysis is that the former does not ascribe "the observed variation to a finite number of cycles with discrete periods, but to an infinite number of small oscillations with a continuous distribution of periods." It is assumed in the following discussion, that dominant periodicities, for example annual or diurnal cycles, have been withdrawn from the time series, or do not contribute as much to the total variance of the time series as do the random and nonpure random components.

If $(X(t) : t \epsilon T)$ is a stochastic process which is sampled in the frequency range 0 to ω', then the total variance of the process in this frequency range is given by $H(\omega')$. Obviously $H(\omega')$ is a cumulative distribution function.

If the function $g(\omega)$ is now formed, resulting in $g(\omega_0) = H'(\omega_0)$, where $H'(\omega_0)$ is the derivative of $H(\omega)$ at ω_0 then $g(\omega)$ is a probability density function and

$$H(\omega_0) = \int_0^{\omega_0} g(\omega) d\omega \qquad [2.5]$$

Since it is convenient to work in positive values of frequency, the following even-valued function is defined

$$h(\omega) = \begin{cases} g(\omega)/2 & \text{if } \omega \geqslant 0 \\ g(-\omega)/2 & \text{if } \omega < 0 \end{cases} \qquad [2.6]$$

which is called the spectral density function. Therefore

$$H(\omega_0) = \int_0^{\omega_0} g(\omega) d\omega = 2 \int_0^{\omega_0} h(\omega) d\omega \qquad [2.7]$$

The relationship between the theoretical autocorrelation function $\rho(\ell)$ and the theoretical spectral density is given by the following Fourier integral transform pair:

$$h(\omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \rho(\ell) \exp(-i\ell\omega) d\ell$$

$$= \frac{1}{\pi} \int_0^{\infty} \rho(\ell) \cos\omega\ell d\ell \qquad [2.8]$$

$$\rho(\ell) = 2 \int_0^{\infty} h(\omega) \cos\omega\ell d\omega$$

The plot of $h(\omega)$ versus ω is called the variance or power spectrum, while the plot of $p(\ell)$ versus ℓ is called the correlogram. An oscillation of period z or frequency $\omega^* = z/2T_m$ where T_m is the maximum value of the lag, will have a peak on the correlogram at z and on the spectrum at ω^*. If the spectrum is constant for all ω, that is the stochastic process is truly random, it is called a white noise spectrum; if the spectrum has a concentration of variance in the low frequencies then the process is called red noise.

2.3 THE ESTIMATION OF SPECTRAL DENSITY

Before estimates of any or the necessary functions which are needed in time-series analysis may be made, it is necessary to call upon the theory of ergodicity and, in fact, make the assumption that the sample time series available is representative of all the possible time series for that time period and spatial area. Hannan (1960) has shown, based on the ergodic theorem, that if $(X(k))$ is covariance stationary and if $(y(k))$ is a sample of $(X(k))$ taken on K at $k = 1, \ldots n$, then for large n the estimate of the mean of $(y(k))$ is given by

$$\bar{y} = (1/n) \sum_{k=1}^{n} y(k) \qquad [2.9]$$

and its estimated autocovariance function (acvf), that is the estimated covariance kernel, is

$$C(\ell) = [1/(n-\ell)] \sum_{k=1}^{n-\ell} [y(k+\ell) - \bar{y}][y(k) - \bar{y}] \qquad [2.10]$$

It is much simpler to calculate $C(\ell)$ by first forming $x(k) = [y(k)-\bar{y}]/\sigma$, $k = 1, \ldots n$, where σ is the standard deviation of $(y(k))$ so that $\bar{x} = 0$ and $\sigma^2(x) = 1$ (Jenkins and Watts, 1968). This procedure is called standardization.

If the autocorrelation function (acf) of $(X(k))$ at lag ℓ is defined as

$$\rho(\ell) = B(\ell) / B(0) \qquad [2.11]$$

then the estimated acf is

$$R(\ell) = C(\ell) / C(0) \qquad [2.12]$$

where B(0) and C(0) are, respectively, the theoretical and estimated variances of $(X(k))$.

Therefore the estimate of the acf, $R(\ell)$, is a function which assumes values not greater than +1 and not less than -1, and is a measure of how points lying ℓ units apart influence each other. It is simply an adaptation of the product-moment correlation coefficient for the analysis of points in time.

A raw estimate of the spectral density at lag ℓ is given by Blackman and Tukey (1958) as:

$$V(\ell) = C(0) + 2 \sum_{j=1}^{T_m - 1} (C(j) \cos(\ell j\pi/T_m)) + C(T_m) \cos(\pi\ell) \qquad [2.13]$$

Here $1 < T_m < n$ is the maximum value of the lag and $V(\ell)$ is the raw estimate of the variance spectrum at the frequency $f = \ell/2T_m$. Equation [2.13] is simply the discrete form of the expression for $h(\omega)$ in [2.8].

However, these raw estimates do not give a satisfactory estimate of the true variance spectrum because of the small sample size, and so it is necessary to form a "modified apparent acvf", $C^*(\ell)$, (Blackman and Tukey, 1958) and to use this in calculating a smoothed version of the true variance spectrum.

Therefore define

$$C^*(\ell) = C(\ell)\,D(\ell) \qquad [2.14]$$

where $D(\ell)$ is a lag window weighting function such that $D(0) = 1$, and $D(\ell) = 0$ for all $\ell > T_m$, where T_m is the maximum lag. Kisiel (1969) has suggested that the optimal maximum lag is 10% of the record length.

A commonly used lag window is the "hanning" window:

$$\begin{aligned} D(\ell) &= {}^1/_2[1 + \cos(\pi\ell/T_m)] \quad 0 \leqslant \ell < T_m \\ &= 0 \qquad\qquad \ell \geqslant T_m \end{aligned} \qquad [2.15]$$

Therefore the "modified apparent acvf" is

$$\begin{aligned} C^*(\ell) &= {}^1/_2\, C(\ell)[1 + \cos(\ell\pi/T_m)] \quad 0 \leqslant \ell < T_m \\ &= 0 \qquad\qquad \ell \geqslant T_m \end{aligned} \qquad [2.16]$$

The smoothed spectral density estimate is

$$G(\ell) = C(0) + \sum_{j=1}^{T_m-1} C(j)\cos(\ell j\pi/T_m)\,[1+\cos(j\pi/T_m)] \qquad [2.17]$$

Agterberg and Banerjee (1969) noted;

"The individual values $G(\ell)$ can be considered as estimators of a smoothed version of the underlying true (variance) spectrum $P_t(\ell)$ with

$$\text{Average } P(\ell) = \int_{-\infty}^{\infty} Q(\ell + \ell_1)\, P_t(\ell)\, d\ell_1$$

where $Q(\ell)$ is the bell-shaped hanning response function with a main lobe that is $2/T_m$ frequency units wide."

2.4 THE METHOD OF FILTERING

The derivation of the linear, statistical filter used in this study is due to Agterberg and Banerjee (1969) and is based on the theory of Yaglom (1962, Chapter 5).

It is assumed that a signal-plus-noise model represents the structure of hydrological observations. This means that the hydrological variable X(k) is assumed to be composed of two components – a signal, S(k), and a pure random component that carries either no information or undesired information, N(k) – the noise. This process is represented by (X(k)) = (S(k)) + (N(k)).

The problem at hand here is for a given sample (x(k), k = 1, ... n) to filter out the noise n(k) for all k to leave the signal s(k). The noise may be a sampling error and/or some random fluctuation in sampling that makes the observed values something other than the true signal. The observed values are simply the sum of these two.

In the model it is assumed that (X(k)), (S(k)), and (N(k)) are all covariance stationary with zero means. Also it is assumed that the signal and the noise are uncorrelated, i.e., E/S(k)N(k)/ = 0. Therefore, if $B(\ell)$ represents the theoretical acvf, then

$$\begin{aligned} B_X(\ell) &= B_S(\ell) \qquad /\ell/ > 0 \\ B_X(0) &= B_S(0) + B_N(0) \end{aligned} \qquad [2.18]$$

Given a sample (x(k), k = 1, . . . n) for which the sample acvf $C_x(\ell)$ and the sample acf $R_x(\ell)$ for $\ell = 0,1, \ldots T$, are known, denote the sample acvf for the signal and noise as $C_s(\ell)$ and $C_n(\ell)$ respectively, then $C_s(\ell) = C_x(\ell)$ for $/\ell/ > 0$.

To design the filter however, it is necessary to know $R_s(0)$, the sample zero-lag acf, and $C_s(0)$ which are estimated as follows. Calculate the sample, signal acf by $R_s(\ell) = C_s(\ell)/C_x(0)$, where $C_x(0) = C_s(0) + C_n(0)$ for all $/\ell/>0$, that is, assume $C_n(\ell) = 0$ for all $/\ell/>0$; then $R_s(0) = C_s(0)$ times a constant. $R_s(0)$ is estimated by finding the least-squares fit to the points $R_s(\ell)$ or, in fact, $R_x(\ell)$.

It is now necessary to consider a model for the curve of the correlogram, $R_s(\ell)$. Considering what was said in Section 2.2 on the minor importance of periodic components in this context, there is reason to believe that a simple and adequate model of nonpure, random, hydrological and climatological time series is the first-order Markov process. Amongst the geophysical time series that have been successfully modelled by a first-order Markov process are daily precipitation (Kisiel, 1969), the water balance of an esker (Eriksson, 1970), annual minimum streamflow discharges (Matalas, 1963) and evaporation from and the air temperature above a lake (Yu and Brutsaert, 1969).

The acf of the first-order Markov process is an exponential function, and the following form used by Agterberg and Banerjee (1969) allows an efficient curve-fitting approach

$$R(\ell) = c \exp(-a/\ell/) \qquad [2.19]$$

The curve is fitted by least-squares technique to $R_x(\ell)$, $\ell = 1, \ldots, m$ where m is that lag such that $R(\ell) > 0$ for all $\ell = 1, \ldots, m$ and $R(m+l) \leqslant 0$. If all values of the acf are positive then $m = T_m$, the maximum lag.

Therefore $R(0) = c$, and it is the estimate of $R_s(0)$.

The equations to derive c are developed by taking the natural log of [2.19] and performing a simple linear regression,

$$\ln|R(\ell)| = \ln c - a|\ell| \qquad [2.20]$$

then

$$a = \left(\sum_{\ell=1}^{m} [\ln R(\ell)\ell] - m\overline{\ell}\,\overline{\ln R(\ell)}\right) / \left(\sum_{\ell=1}^{m} (\ell^2) - m\overline{\ell}^2\right) \qquad [2.21]$$

and

$$c = \exp\left(\overline{\ln R(\ell)} - a\overline{\ell}\right) \qquad [2.22]$$

To summarize, a least-squares line is fitted to the acf $R_x(\ell)$ and extrapolated back to lag 0 where $R_s(0) = c$, $R_n(0) = 1\text{-}c$, and $R_x(0) = R_s(0) + R_n(0)$. $R_n(0)$ is a measure of the sampling error and $R_s(0)/R_n(0)$ is called the signal-to-noise ratio.

The filter $b(\ell)$ is a function of ℓ where for all k

$$s(k) = \int_{-\infty}^{\infty} b(\ell)x(k+\ell)d\ell \qquad [2.23]$$

The problem now is to find s(k) in terms of x(k) by the discrete form of [2.23]

$$s(k) = \sum_{\ell=-\infty}^{\infty} b(\ell)x(k+\ell) \qquad [2.24]$$

so that the sum of the squares of the residuals n(k) is minimized. In the words of Yaglom (1962, p. 128) "we may finally reduce the solution of the linear filtering problem to the solution of the following system of linear algebraic equations:"

$$\sum_{\ell=-\infty}^{\infty} b(\ell)R_x(k-\ell) = R_s(k) \qquad [2.25]$$

where $k = -h, -h+1, \ldots 0, \ldots, h$.

If a and c of [2.19] are known then the solution for [2.24] is given by Agterberg and Banerjee (1969) as

$$b(\ell) = q\exp(-p|\ell|)$$

where

$$p = \sqrt{a^2 + 2ac/(1-c)} \qquad [2.26]$$

and

$$q = ac/(1-c)p$$

Since [2.23] is really a sum of the x(k)'s with the weights $b(\ell)$, then for some lag, $\ell = \alpha$, so that $b(\ell) < \beta$ for $|\ell| > \alpha$, where $\beta > 0$ is arbitrarily small, then

$$s(k) = \sum_{\ell=-\alpha}^{\alpha} b(\ell)x(k+\ell) \qquad [2.27]$$

To find α consider $\Sigma H = \sum_{\ell=\infty}^{\infty} b(\ell)$ and choose $\beta > 0$ and let α be a value so that

$$\left|\Sigma H - \sum_{\ell=-\alpha}^{\alpha} b(\ell)\right| < \beta \qquad [2.28]$$

This $b(\ell)$ is a symmetrical, bilateral exponential filter making $b(\ell) \geqslant 0$ for all ℓ. Then, assuming that [2.28] holds true, for all $|\ell| > \alpha$ then at least $0 < h(\ell) < \beta/2$. This regulates the error term.

The existance of ΣH is proven from [2.26]

$$\Sigma H = q\sum_{\ell=-\infty}^{\infty} \exp(-p|\ell|) = q + 2q\sum_{\ell=1}^{\infty} \exp(-p|\ell|) \qquad [2.29]$$

Since $|\exp(-p)| < 1$, then ΣH calculated from the sum of a geometric series is

$$\Sigma H = q + 2q(\exp(-p)/(1-\exp(-p))) \qquad [2.30]$$

A correction is necessary to keep the total sum of the weights constant in going from [2.24] to [2.27]. To accomplish this introduce $b'(\ell)$ as

$$b'(\ell) = \Sigma H \cdot b(\ell) / \left[\sum_{\ell=-\alpha}^{\alpha} b(\ell)\right] \qquad [2.31]$$

Therefore $\Sigma H = \sum_{\ell=-\alpha}^{\alpha} b'(\ell)$ and the total weight is unchanged. Consequently, the final estimate of the signal is

$$s(k) = \sum_{\ell=-\alpha}^{\alpha} b'(\ell)x(k+\ell) \qquad [2.32]$$

2.5 THE STOCHASTIC MODEL OF THE HYDROLOGICAL SIGNALS

It has been assumed that the model which will best describe the signal is the first order autogressive or Markov process

$$X(\ell) = \rho_1 X(\ell - 1) + \eta(\ell) \qquad [2.33]$$

whose theoretical acf is given by Bartlett (1966, p. 306) as

$$\rho(\ell) = \rho_1^{/\ell/} \quad /\ell/ = 0,1, \ldots . T_m \qquad [2.34]$$

where ρ_1 is the lag one acf coefficient, and $\eta(\ell)$ is a stationary random function.

The theoretical spectral density is

$$h(\omega) = \frac{[\mathrm{Var}(X(\ell))](1-\rho_1^2)}{\pi(1+\rho_1^2 - 2\rho_1 \cos\omega)} \quad (0 \leqslant \omega \leqslant \pi) \qquad [2.35]$$

This has a maximum at $\omega = 0$ given by

$$h(0) = \frac{[\mathrm{Var}(X(\ell))](1+\rho_1)}{\pi(1-\rho_1)} \qquad [2.36]$$

By converting from angular frequency co-ordinates and by standardizing the data so that $\mathrm{Var}[X(\ell)] = 1$ the numerical expression for [2.35] is

$$h(f) = (1-\rho_1^2)/[1+\rho_1^2 - 2\rho_1 \cos 2\pi f] \qquad [2.37]$$

for $-1 \leqslant \rho_1 \leqslant +1$.

The Experimental Site

An experimental site was instrumented at Delta, Manitoba (Figure 3.1) during the summers of 1970 and 1971 to study diurnal fluctuations of groundwater hydrographs and their climatological causes and effects. The site had been used previously by Gilliland who studied the cause of small fluctuations of the water table.

3.1 GEOLOGY AND HYDROGEOLOGY

The community of Delta lies in the middle of the Manitoba plain, whose physiography has been described by Bostock (1970). The surface of the Manitoba plain has an elevation of about 800 feet and is very gently undulating to flat. It is largely covered by lakes and includes most of Lake Winnipeg. In the southern part its features have been smoothed over by the deposition of the clays and silts of glacial Lake Agassiz (Bostock, 1970). The surficial geology of the Delta area (Figure 3.2) is comprised of silty clays of Lake Agassiz which are overlain by more recent alluvial

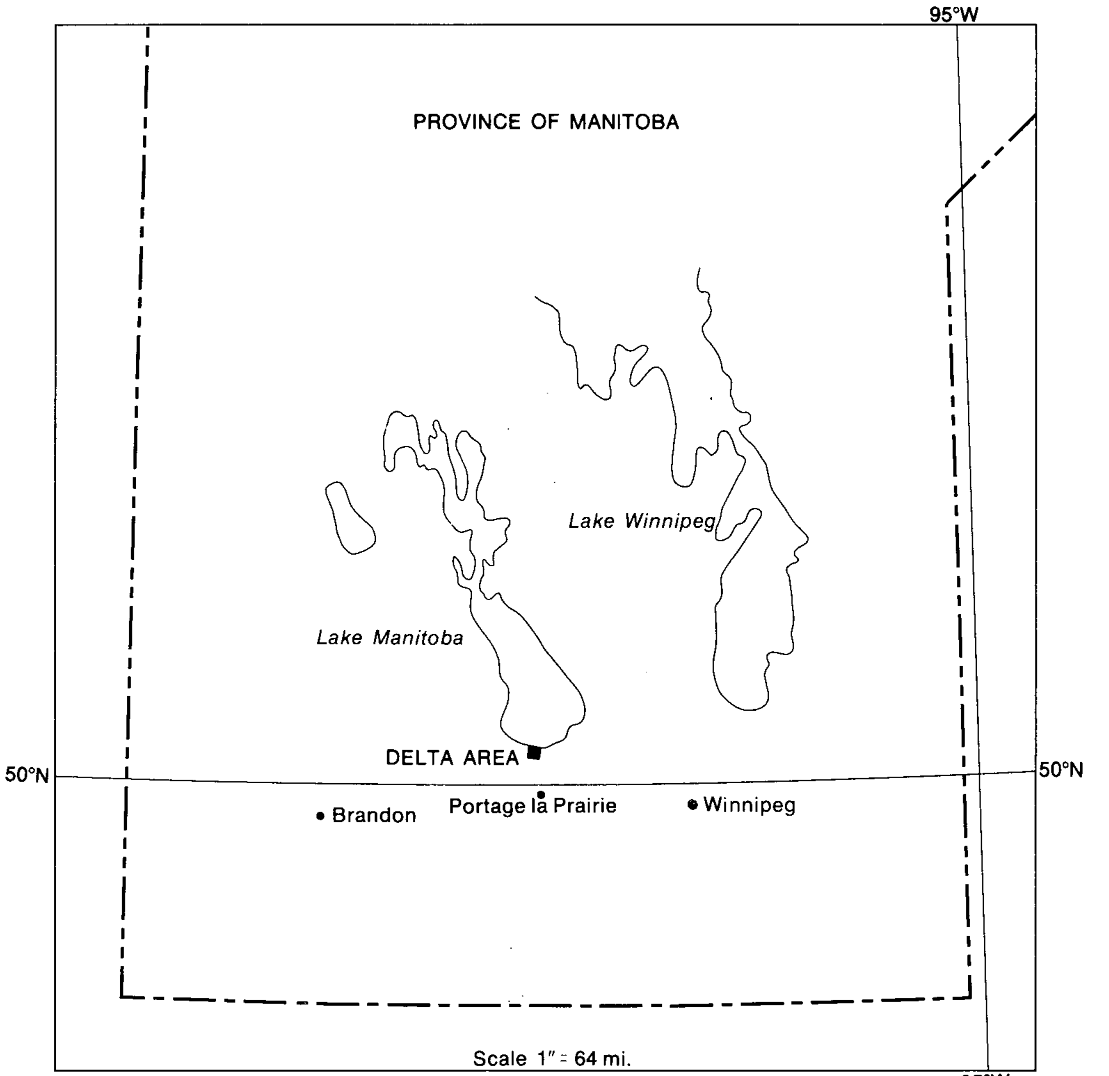


Figure 3.1 Location of Delta area, Manitoba

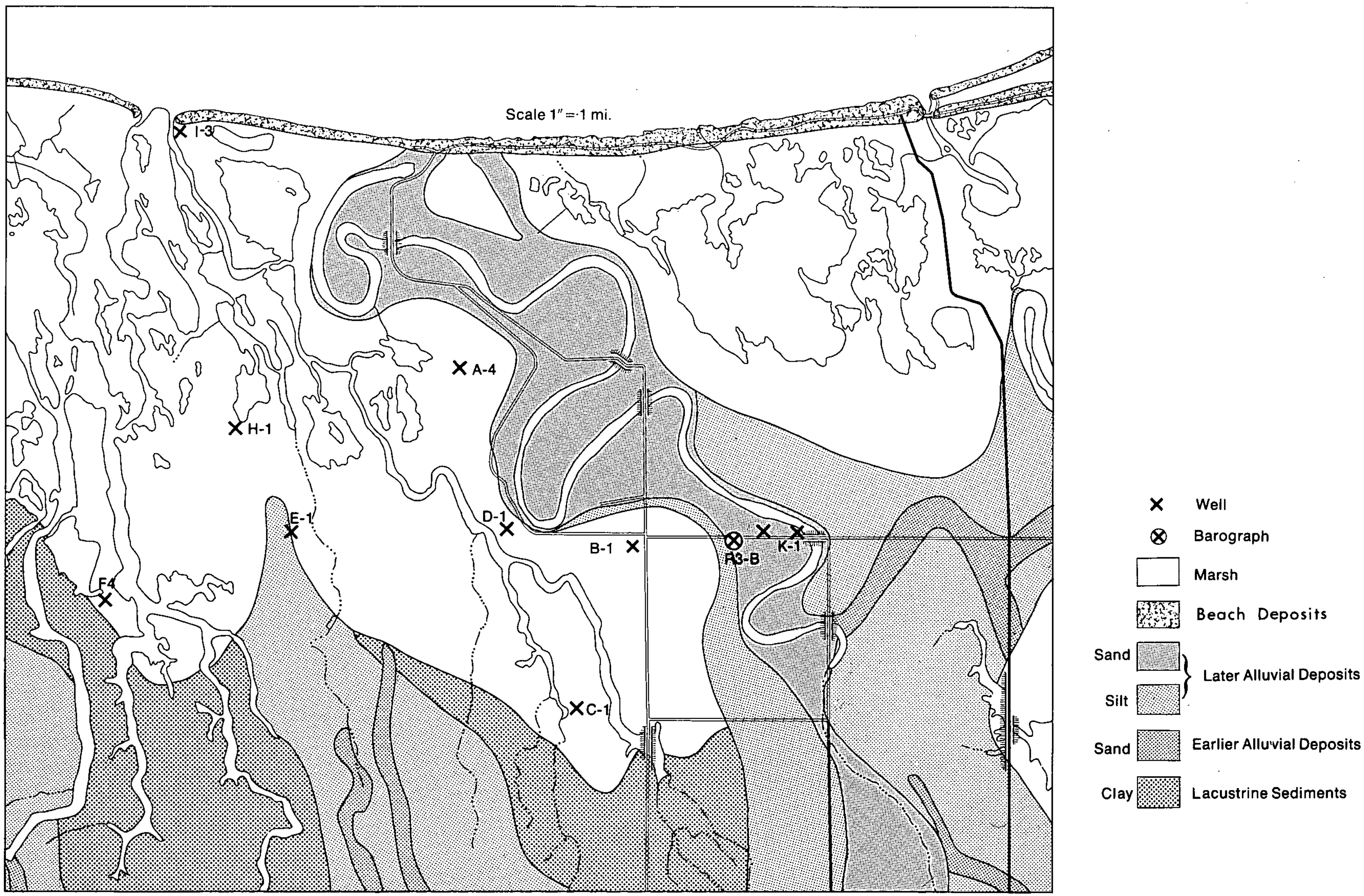


Figure 3.2 Geology and well locations

deposits which grade sharply from sands in the old river channels to silts and clayey silts outside the channels. The alluvial silty deposits are widespread and, to the eye, uniform (Gilliland, 1973).

The Delta area is a groundwater discharge area in which the water table lies beneath both the mean level of Lake Manitoba and the piezometric surface of the underlying bedrock aquifer (Gilliland, 1973). The area is drained by several channels which lead to Lake Manitoba and which are ephemeral, flowing only after snowmelt in the spring. The regional flow pattern and hydrochemistry is discussed in Cherry *et al.* (1971).

Each summer since 1967 various existing observation wells have been instrumented with Leupold Stevens type F water-level recorders. Lithological logs and details of the well construction are given in Appendix A; a brief description of each is presented.

Well K-1 sits in an ephemeral stream channel composed of alluvial sands resting on clays and silts (Figure 3.2) Well D-1 stands on relatively high ground between two ephemeral stream channels and penetrates seven feet of alluvial silts and clays which sit on top of the Lake Agassiz clays. Although D-1 is in the groundwater discharge area of the regional flow system, it probably forms an ephemeral recharge area for a local flow system during wet spells (Figure 1.1).

In his time-series analysis of water-level fluctuations at Delta, Gilliland (1973) assumed that each groundwater hydrograph H(t) was composed of a sinusoidal, periodic signal S(t) and a Gaussian random noise N(t). By cross-correlating the hydrographs with a diurnal, unit impulse, i.e., one with a period of the day, and then computing theoretical and actual, signal-to-noise ratios, Gilliland was able to identify which wells were fluctuating due to diurnal, evapotranspirative effects. Although the evapotranspirative signal was indeed sinusoidal, Gilliland found that the noise was not Gaussian and attributed it in part to natural recharge by rainstorms. Those wells with sinusoidal amplitudes of the same order as the product of their barometric efficiency and the amplitude of the fluctuation of the barometric pressure were assumed to be responsive to barometric rather than evapotranspirative effects. Both K-1 and D-1 were identified as fluctuating due to evapotranspiration.

Furthermore Gilliland showed that for wells with water levels fluctuating due to evapotranspiration, the phase and the amplitude of the fluctuations were linear functions of the depth, indicating that the transport of water to the surface from the water table is a quasi-linear process when long (approximately 100 days) time periods are considered.

3.2 CLIMATOLOGY

The climate of Southern Manitoba is typically continental – warm and humid in the summer, cold and dry in the winter. The summer climate is summarized in Table 1.

TABLE 1. Summer Climate of Southern Manitoba, 1967

	June	July	Aug.	Sept.
Rainfall (inches)*	1.08	3.24	1.39	0.40
Mean Daily Temp.* (°F)	62.6	67.9	64.7	60.3
Class A Pan Evaporation (in.)**	7.11	8.06	7.47	6.39

*Measured at Portage la Prairie
**Measured at Gimli
Source: Monthly Records, Meteorological Observations in Canada, Canada Department of the Environment, Meteorological Branch, Toronto.

There are various ways of demonstrating this continentality. Barry and Chorley (1968) showed that by use of Conrad's index, which depends on the annual temperature range and the latitude, that Manitoba has the strongest continentality of anywhere in North America south of the 60th parallel. Polowchak and Panofsky's (1968) spectral analysis of daily temperatures showed that the geographical distribution of summer and winter variance in North America is also a continentality index, in that the variance maxima are concentrated from Ontario to Alberta. In this case the spectra are expressing the pronounced fluctuations in daily climate that is the essence of a continental climate.

The precipitation and moisture balance regime of the prairie provinces is dictated by the summer precipitation, which accounts for 40% of the annual total. According to Barry and Chorley (1968, p. 201) the wettest period is "commonly in late summer or autumn when depression tracks are in higher middle latitudes."

At K-1 a climatological station was established to continuously record barometric pressure, relative humidity, air temperature, and precipitation. The instruments employed were, respectively, a Short and Mason microbarograph, a Lambrecht hygrothermograph and a Kahlsico Hellman rain gauge. Furthermore a Class "A" U.S. Weather Bureau Evaporation Pan was installed to measure hourly, free-surface evaporation using an electronic sensor.

3.3 VEGETATION

Grasses of the genuses Carex and Scripus are predominant at the Delta site.

Data Processing and Analysis

4.1 DATA PROCESSING

Observation-well hydrographs for the summers of 1967-1971 at Delta were examined for continuity and quality. This involved selecting only those periods of record showing diurnal fluctuations, which lacked gaps due to instrument malfunction and for which the pen-trace quality permitted accurate estimates of R and Δs. On this basis the following "evapotranspiration seasons" were chosen for analysis: June 1 to September 21, 1968 at K-1, June 16 – September 10, 1970 at D-1, and June 15 – September 3, 1971 at D-1.

Values of R and Δs of [1.1] were taken from the hydrographs and used as input to computer program WHITE (Appendix B), which computed daily values of Q, the groundwater evapotranspiration. On those days on which the hydrograph rose due to rainfall infiltration or to a Lisse Effect and thereby obscured the true value of R, the groundwater inflow rate, R was set equal to zero. This involved 17% of the values at K-1 in 1968 and 12% of the values for 1970 and 1971 at D-1.

Values for the specific yield of the aquifers, S_y at K-1 and D-1 were taken from data suggested by Meyboom (1967). K-1 being a fine-grained alluvial sand, was given a specific yield of 15%, while D-1, being an alluvial clayey silt, was given a value of 10%.

The climatology station at K-1 provided the data for the calculation of mean daily temperature ($\bar{T}$) and daily precipitation ($\bar{P}$) for the summers of 1970 and 1971. For the summer of 1968 values of $\bar{T}$ and $\bar{P}$ for the Delta Wildfowl Research Station, two miles north of K-1, were taken from the Meteorological Service's Monthly Record (Canada Department of the Environment).

4.2 IDENTIFICATION OF NONRANDOM COMPONENTS

Time series of Q, R, $\bar{T}$, and $\bar{P}$ for the three seasons (Appendix C) were first standardized by subtracting their means and dividing by their standard deviations. Henceforth these shall be referred to as Q(t), R(t), $\bar{T}(t)$, and $\bar{P}(t)$, respectively. This necessarily made the variance of each variable C(0), equal to unity, with a mean of zero. The sample acf of these series was computed from [2.10], which is strictly the acvf; however, since C(0) = 1 by standardization, then $R(\ell) = C(\ell)$ [2.12]. Estimates of the spectral density were computed from [2.17]. The theoretical acf, $\rho(\ell)$, was computed from [2.34], while the theoretical spectral density, h(f), was computed from [2.37]. These computations were performed by program SPECTRA (Appendix D).

A test of significance for $R(\ell)$ has been given by Eagleson and Lariviere (1970) as

$$r_k^* = t_p/(v - t_p^2)^{1/2} \qquad [4.1]$$

where v is the number of degrees of freedom = n-ℓ-2, t_p is the student's variable at exceedence probability, 1-P, r_k is the value of the acf at lag k which must be exceeded for significance at the chosen level P. For a nonrandom time series v is replaced by v':

$$v' = (n-\ell-2)/(1+2r_1^2+2r_2^2+\ldots\ldots) \qquad [4.2]$$

However if the time series is assumed to be generated by a first-order Markov process then

$$v' = (n-\ell-2)\,(1-r_1^2)/(1+r_1^2) \qquad [4.3]$$

The results of that part of the thesis directly concerned with the identification of nonrandom components in the time series are shown in Figures 4.1 to 4.8. Each figure shows either the sample correlograms or spectra of a particular variable over the three seasons. The correlograms, Figures 4.1 to 4.4, show $R(\ell)$ and $\rho(\ell)$, with the aforementioned significance test calculated for various lags by [4.1]. The equation of the stochastic model [2.33] is also listed, with its acf, $\rho(\ell)$, shown by the dashed line.

The spectra, Figures 4.5 to 4.8, show values of $G(\ell)$ and h(f). The x-axis of the spectrum is calibrated in both frequency units (cycles per day) and in periodic units (days per cycle).

4.3 STOCHASTIC MODELLING OF Q(t) AND R(t)

In order to test the adequacy of the proposed stochastic models of the signals of Q(t) and R(t) [2.33], it is necessary

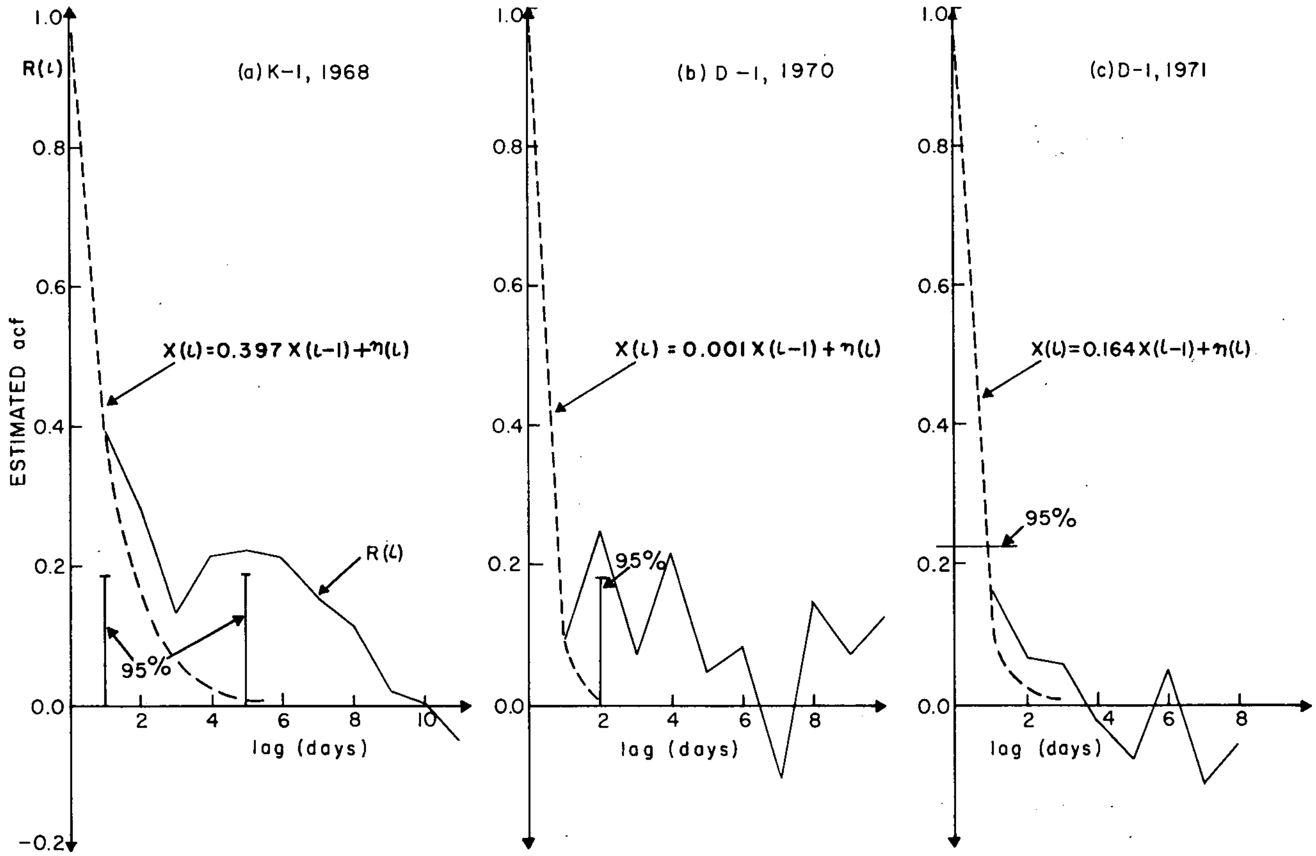


Figure 4.1 Correlograms of groundwater evapotranspiration

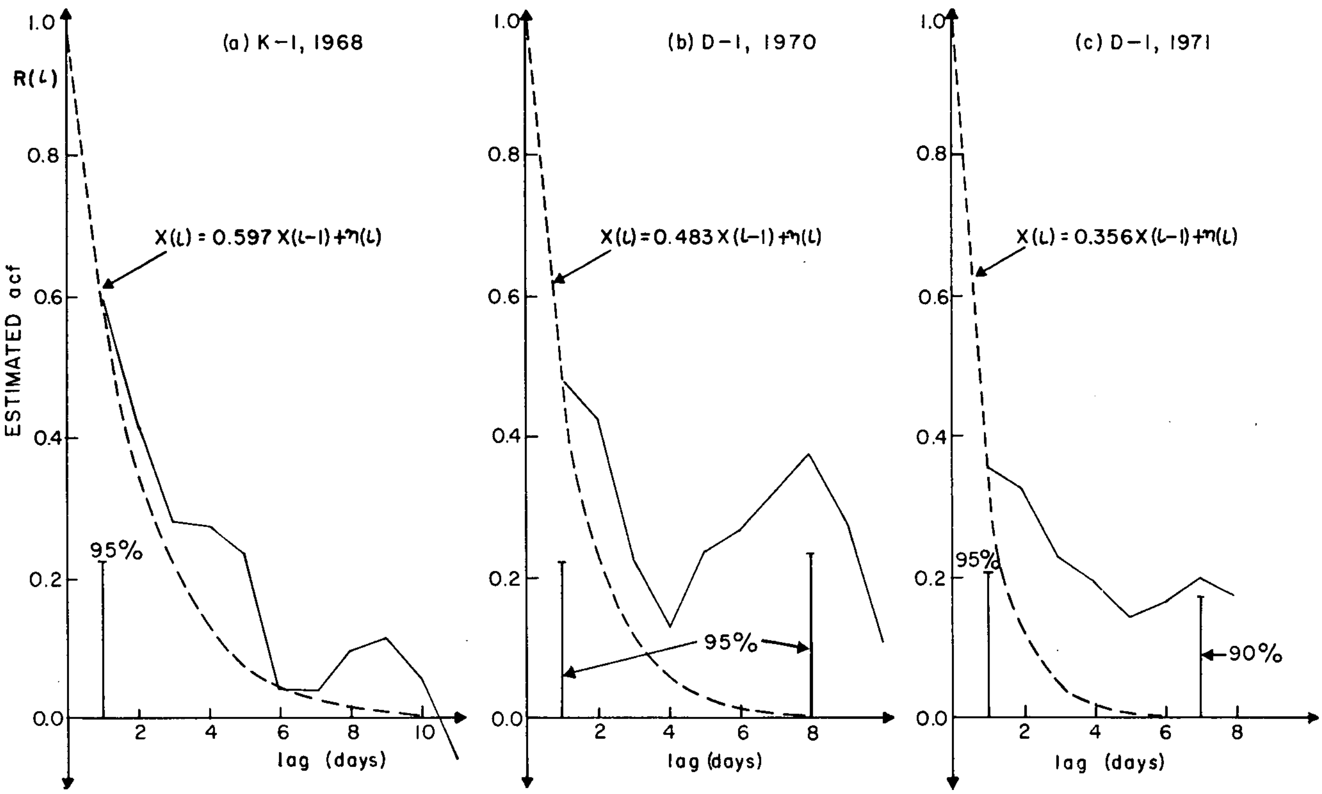


Figure 4.2 Correlograms of groundwater inflow rate

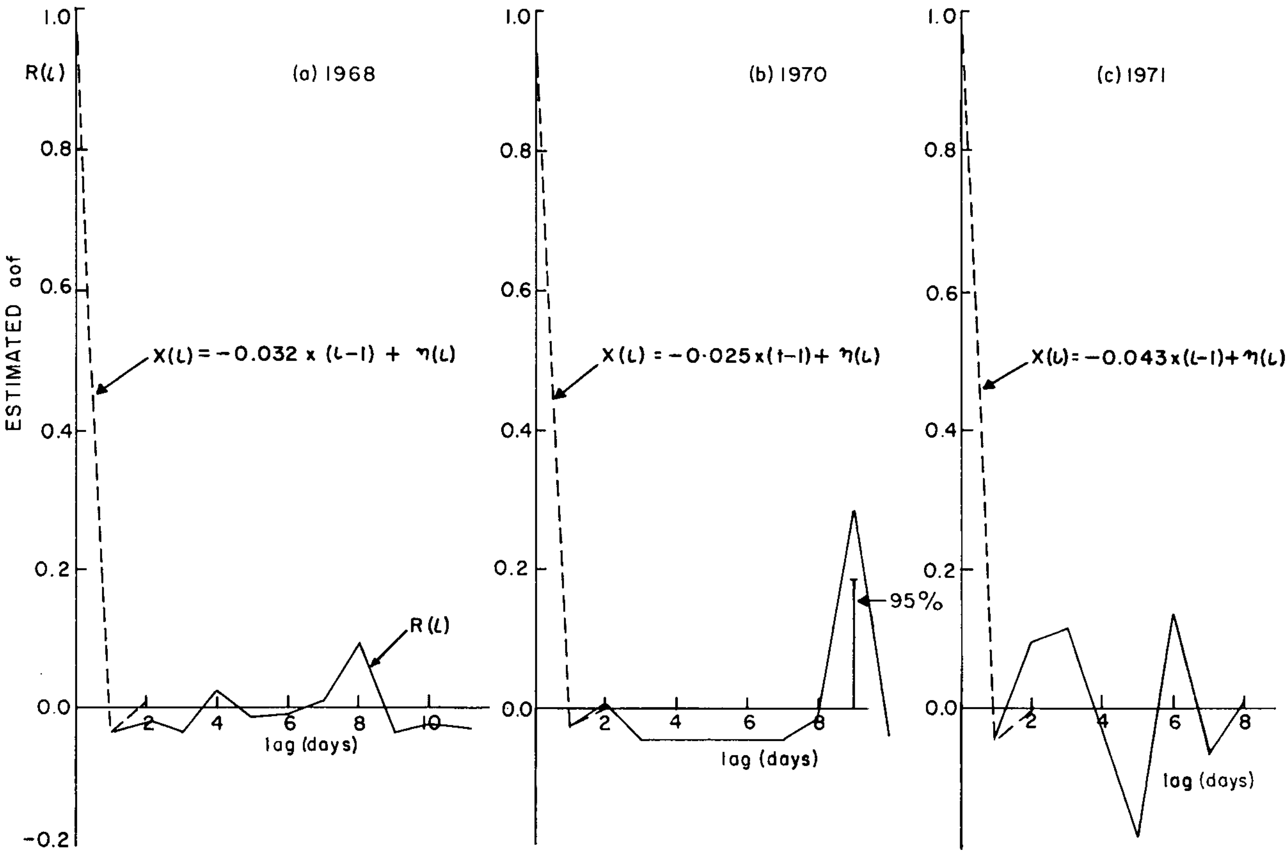


Figure 4.3 Correlograms of precipitation

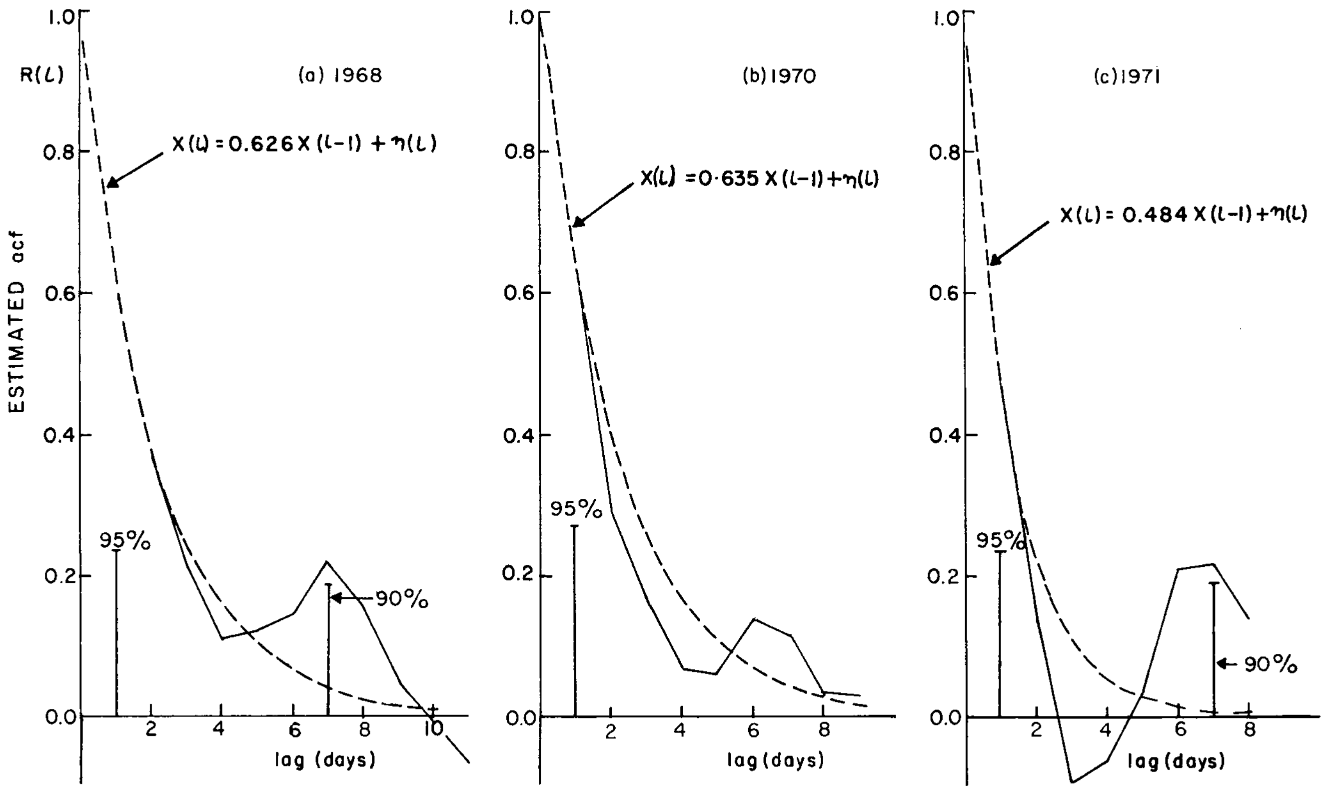
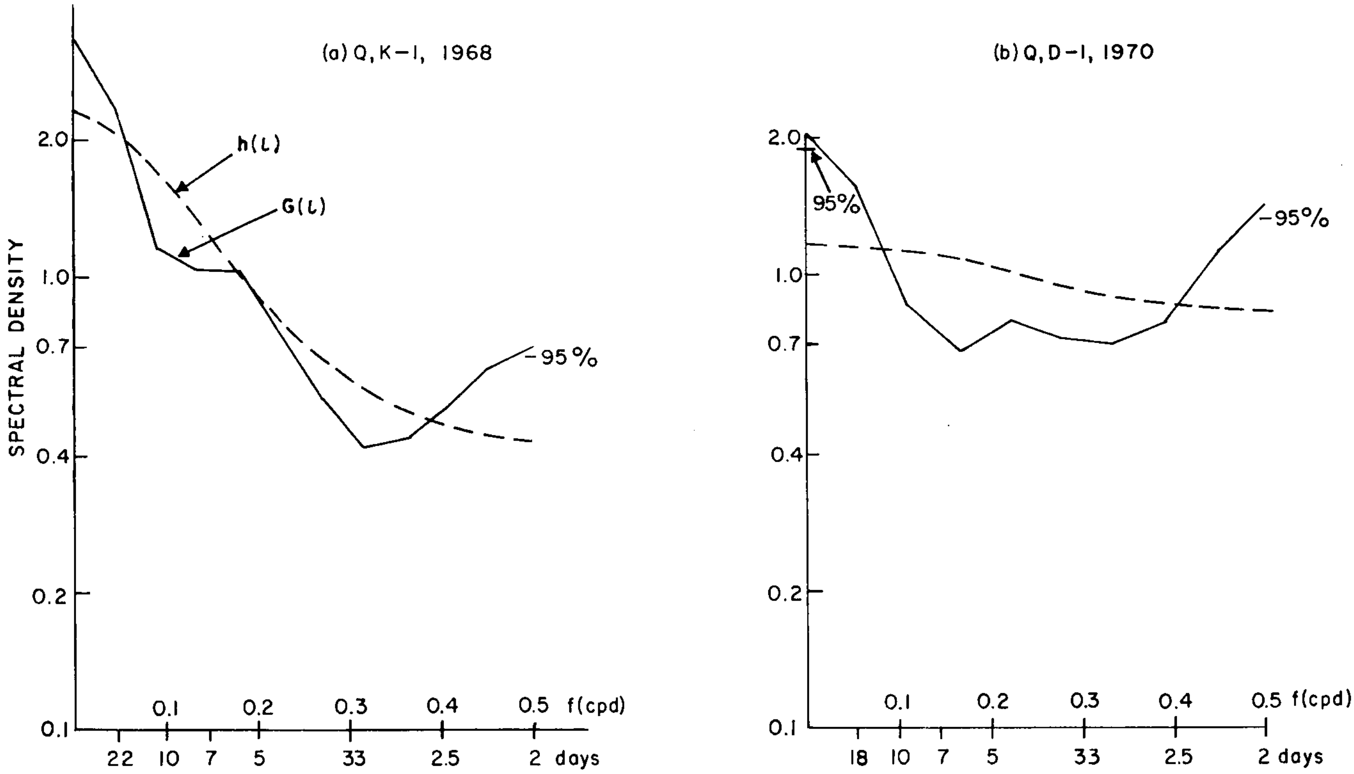


Figure 4.4 Correlograms of temperature



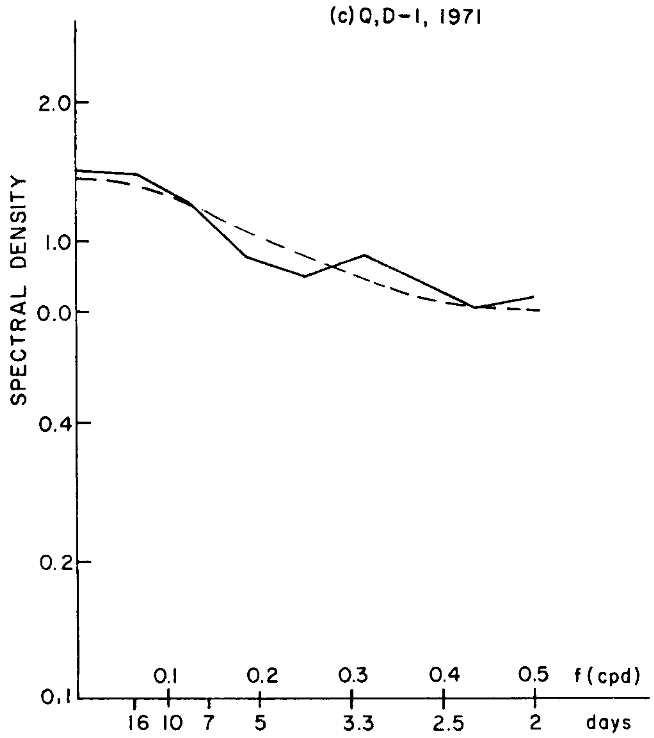
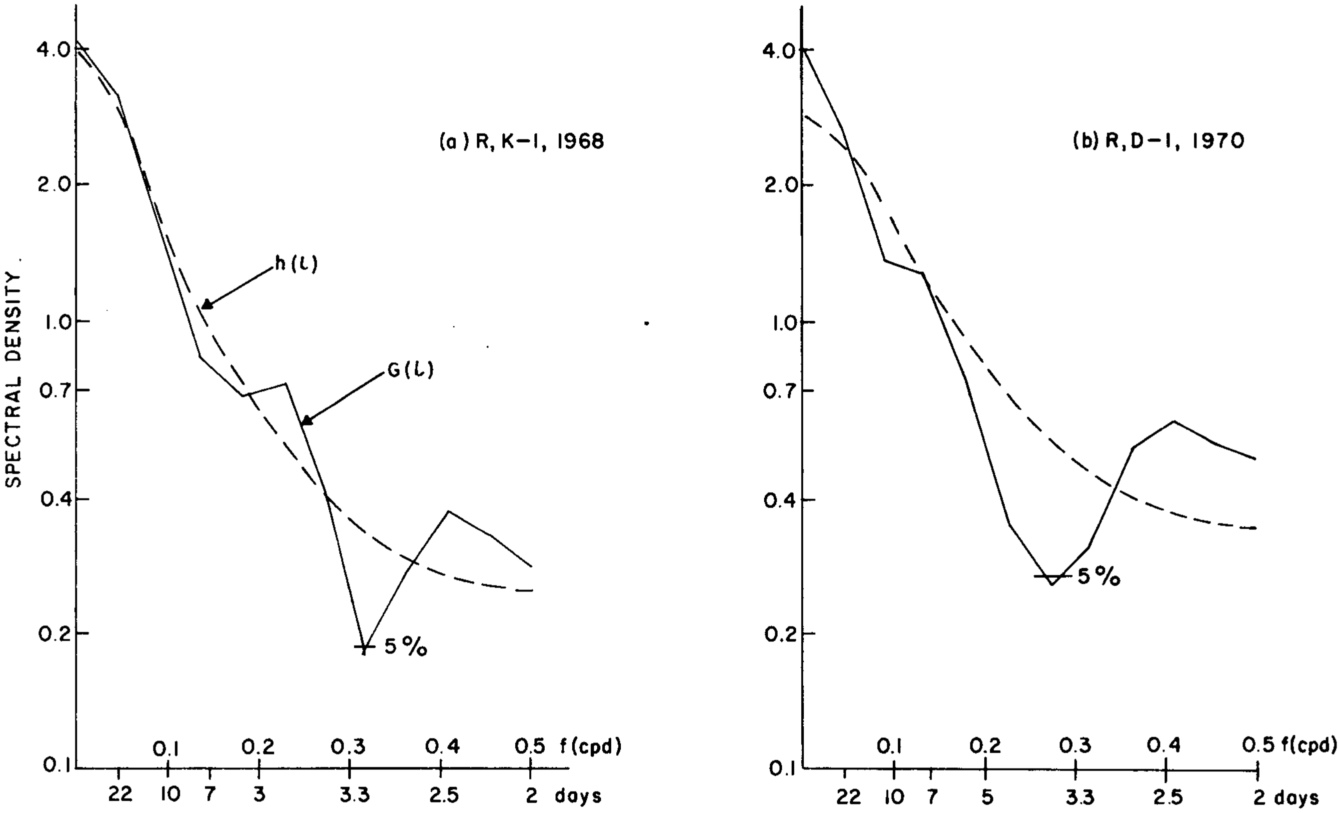


Figure 4.5 Spectra of groundwater evapotranspiration



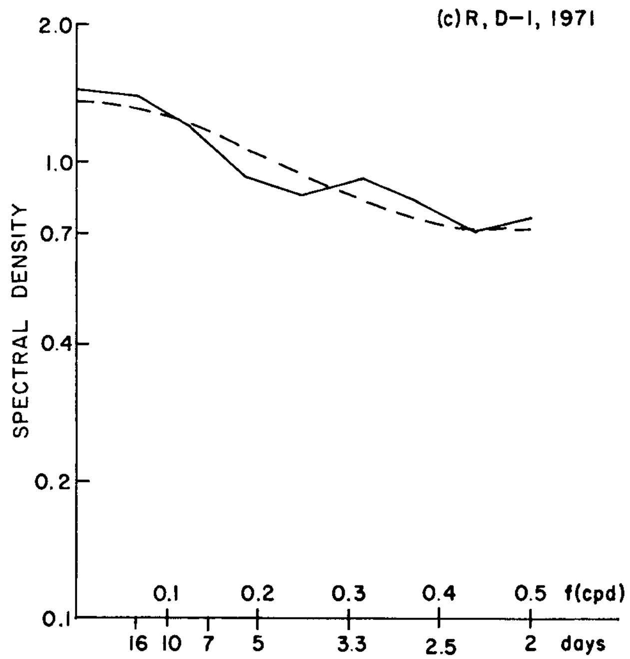


Figure 4.6 Spectra of groundwater inflow rate

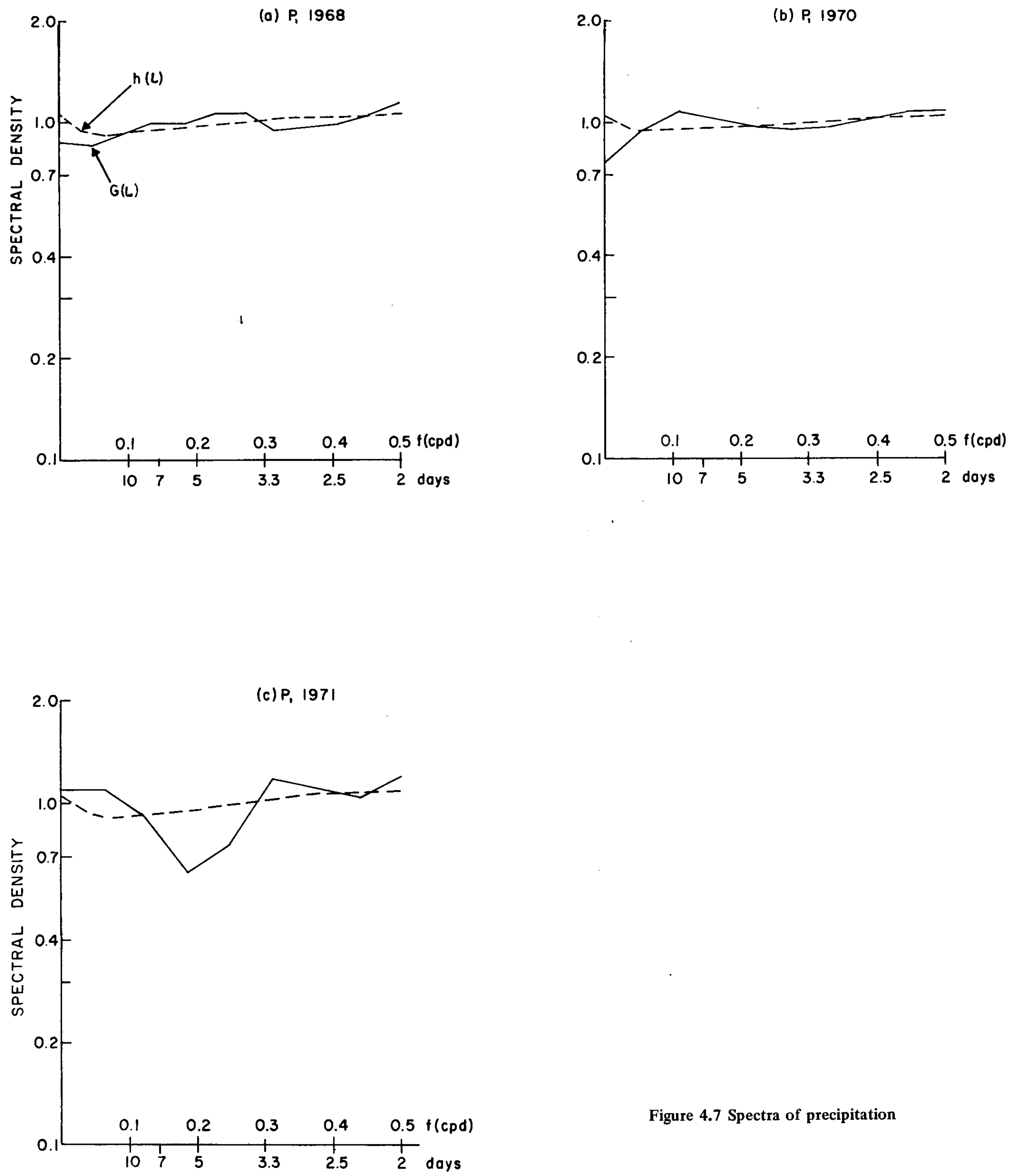


Figure 4.7 Spectra of precipitation

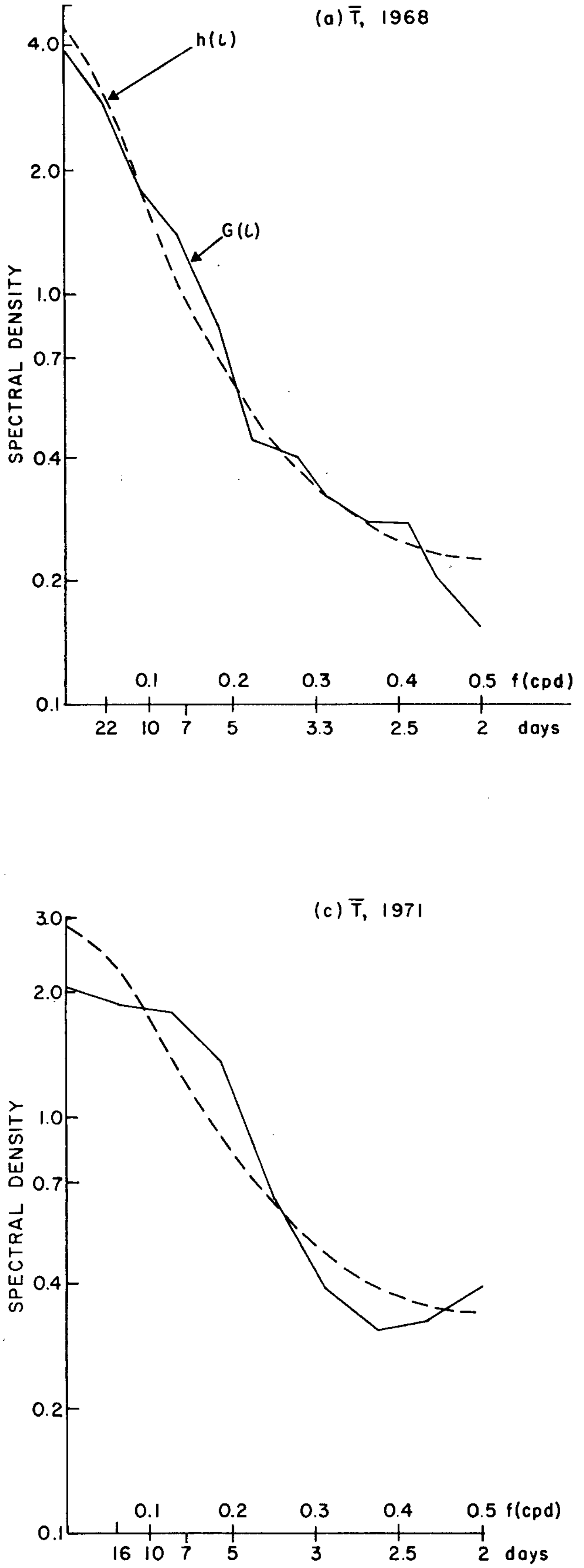


Figure 4.8 Spectra of temperature

TABLE 2. Goodness-of-Fit Test for Proposed Stochastic Model

Time Series	$\sum_{k=1}^{T_m} \chi_k^2$	$\chi^2_{T_m}$ (0.1)	Results at 90% level
Q,K-1, 1968	25.17	17.28	rejected
Q,D-1, 1970	12.72	14.68	accepted
Q,D-1, 1971	4.47	13.36	accepted
R,K-1, 1968	50.02	17.28	rejected
R,D-1, 1970	33.89	14.68	rejected
R,D-1, 1971	15.79	13.36	rejected

to apply statistical tests to both the correlograms and the spectra.

The small sample size (n) of most geophysical records leads to less damping in the sample correlogram than in the theoretical one, because the sample autocorrelation coefficients are inflated by sampling errors which would approach zero as n approaches infinity (Matalas 1963). This disadvantage is one of the reasons why the statistical testing of correlograms is less satisfactory than that of spectra.

Quenouille (1947) has suggested the following goodness-of-fit test for [2.33], the stochastic model of the signals, for which the value of χ^2 associated with the order autocorrelation coefficient is given by

$$\chi_k^2 = (n-k)R_k^2/(1-R(1)^2)^2 \qquad [4.4]$$

where $R_k^2 = R(k) - 2R(1)R(k-1) - R(1)^2 R(k-2)$

If

$$\sum_{k=1}^{T_m} \chi_k^2 > \chi^2 T_m(\propto) \qquad [4.5]$$

where the right-hand term is the value of the χ^2 with confidence level $\propto$ and T_m degrees of freedom (T_m = the maximum lag of the acf), then the first-order Markov process is rejected as a possible model. Table 2 shows the results of this goodness-of-fit test.

A further test of the suitability of the proposed stochastic model may be carried out by using Tukey's sampling theory (Kisiel 1969). This test involves determining whether any of the sample spectrum estimates are significantly different from the proposed theoretical spectrum value of the first-order Markov process [2.37]. Confidence limits for χ^2 distributed values of $G(\ell)$ have been given by Panofsky and Brier (1968). By using the computed 95% confidence level it is possible to identify significant spectral peaks, and by using the 5% confidence level it is possible to pick out significant spectral gaps. There is only a 5% probability that a sample spectrum estimate will be larger than the 95% confidence level and smaller than the 5% level. The six significant spectral estimates are shown in Figures 4.5 to 4.8 by horizontal dashes with the accompanying confidence level.

4.4 DEVELOPMENT OF THE STATISTICAL FILTER

Program SPECTRA was written so that after analyzing the time series of Q and R for nonrandom components it would then subject each to the method of statistical filtering described in Section 2.4. Values of a and c, the regression coefficients in [2.19], were computed from [2.20], [2.21] and [2.22]. In this way the acf of the signal was built into the filter, which was then derived from [2.26]. The error term, β [2.28], was fixed at 0.05. Table 3 lists values of a, c, p, and q for each time series of Q and R.

By filtering the standardized values of Q(t), and R(t), the signal s(k) of each series is given. Figures 4.9 to 4.11 show the seasonal pattern of the filtered values of Q and R, with $\overline{T}$, $\overline{P}$ and weekly water table depths. Periods of infiltration and Lisse effects are noted by 'I' and 'L' respectively.

TABLE 3. Groundwater Evapotranspiration and Groundwater-Inflow Rate-Time Series

Time Series	Sample Size (n)	Mean	Variance	a	c	p	q
Q,K-1, 1968	113	0.012	0.00007	0.224	0.487	0.692	0.309
Q,D-1, 1970	87	0.010	0.00005	0.120	0.163	0.248	0.095
Q,D-1, 1971	81	0.004	0.00001	0.511	0.239	0.763	0.211
R,K-1, 1968	113	0.000	0.00001	0.259	0.608	0.933	0.431
R,D-1, 1970	87	0.000	0.00001	0.673	0.374	0.291	0.138
R,D-1, 1971	81	0.000	0.00000	0.129	0.366	0.408	0.183

Discussion

In the procedure described in the previous chapter it was assumed that the trend and periodic components of the time series of Q and R were absent. That is, it was assumed Q(t) and R(t) to be random, which was then checked by the computation of correlograms and spectra and found to be incorrect. The tools by which the nonrandomness was identified were then used to filter out this nonrandom signal, and were further used to determine whether the proposed stochastic model was suitable.

5.1 ASSUMPTIONS

Two assumptions were made by White (1932) in deriving [1.1]. In the first case he assumed that R, the groundwater inflow rate, remained constant throughout the day and, second, he implied that there must be instantaneous discharge of Q, the evapotranspiration from the groundwater body. Hydrograph analysis of K-1, 1971, showed that while there is persistence between values of R one day apart it was quite common for the rising limb of the hydrograph in the early morning to have a different R than that of the rising limb late that night (Figure 1.2).

Similarly the second assumption is not strictly fulfilled since the vertical transport of water from the water table to the plant stomata takes place in a finite time (McDonald and Hughes, 1968).

Nevertheless the nonfulfillment of these two assumptions is not of critical importance since the quantities of Q and R are not the primary interest in the study. All that is required is a consistent method to permit analysis of their time distribution.

There are several assumptions concerning Agterberg's filter which merit discussion. One of these is conceptual, that of the validity of the signal-plus-noise model, while the other are concerned with the stationarity, normality, and ergodicity of the data.

The validity of the signal-plus-noise model, which has already been touched upon in Section 2.4, may be confirmed because of the errors just discussed which are introduced in extracting the data from the hydrographs. These errors may then be considered as white noise which is uncorrelated with the signal and must be filtered out. Furthermore, there are the actual sampling errors incurred at the observation well.

Covariance stationarity may usually be approximated by standardizing the time series. Furthermore the dynamic equilibrium model of the hydrologic regimen of the discharge area presented in Section 5.6 suggests that the data is at least homogeneous. This model states that the water table will fluctuate in a zone of limited depth beneath the ground surface, consequently extreme nonstationarity in the Q and R time series, both of which depend on depth, will be avoided.

The normality of the data is generally regarded as important in the significance and the goodness-of-fit tests. While Q, 1968 and Q, 1971 of the hydrogeological variables were both skewed, McDonald (1960) has presented convincing evidence that the climatologist and hydrologist interested in correlation analysis need not be overly concerned when their data does not approximate a bell-shaped curve.

Ergodicity, as Kisiel (1969) has noted, is impossible to establish since the ensemble of samples of any hydrologic process is never available. Consequently an attempt to satisfy ergodicity was made by using three evapotranspiration seasons from two sites in the discharge area. Such an approach should minimize the likelihood of purely local effects having a dominant effect on the results.

5.2 NONRANDOM CLIMATOLOGICAL COMPONENTS

The two climatological variables responded quite differently to the weather and atmospheric circulation associated with each of the three summer periods. As Landsberg and his co-workers (1959) noted the three precipitation seasons each have their own spectral signature which are all basically random (Figure 4.7). The correlograms (Figure 4.3) have only one statistically significant value, R(9) in 1970, and this is a chance correlation due to the occurrence of two of the summer's major storms being nine days apart.

In contrast to this strong randomness, the temperature correlograms (Figure 4.4) showed one- and two-day persistence, which can be viewed as a measure of the local

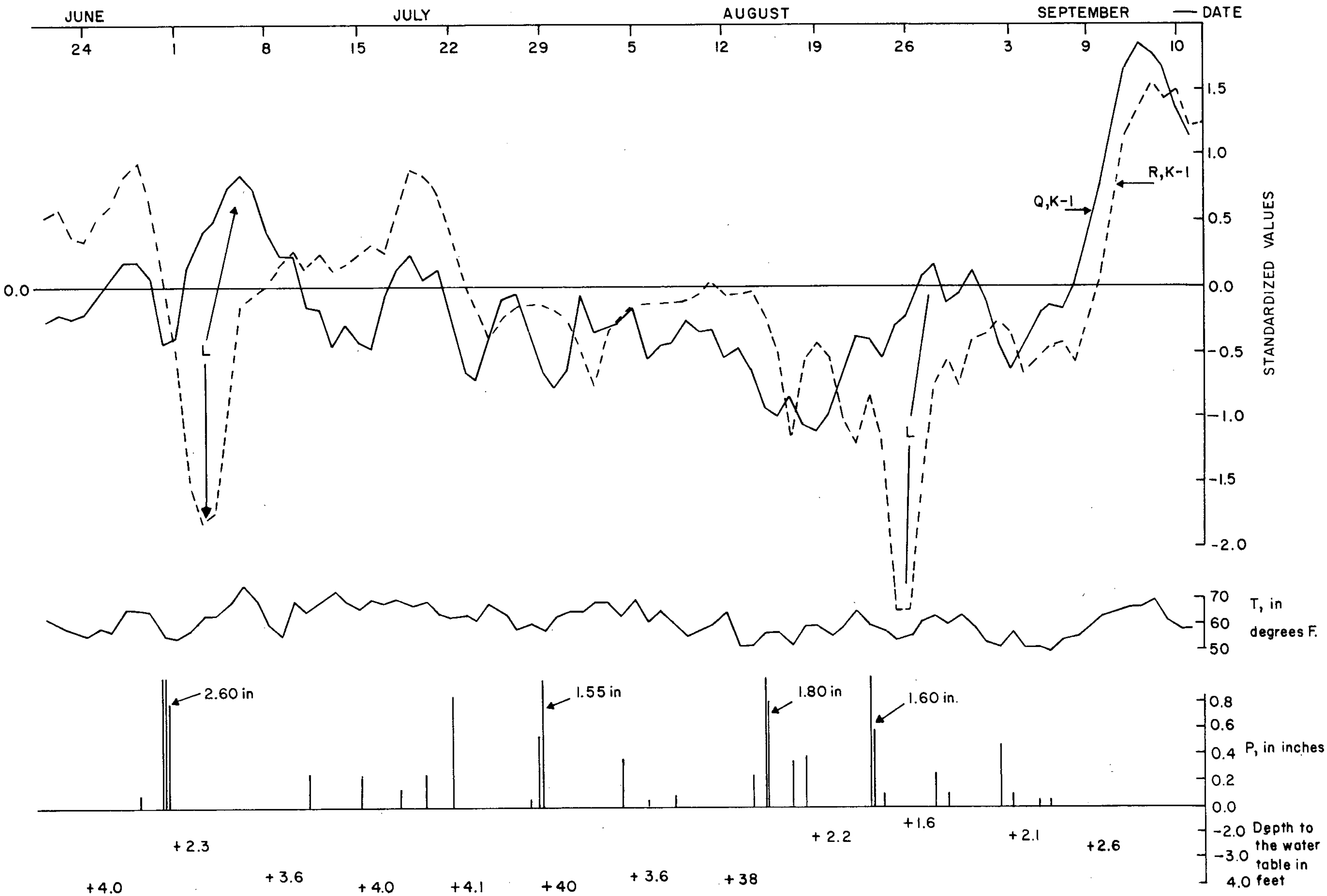


Figure 4.9 Evapotranspiration season, 1968

residence time of air masses, and significant 6 to 7 day cycles, due to the aforementioned waves in the westerlies. The temperature spectra (Figure 4.8) are all rather similar, especially 1968 and 1970, with most variance concentrated in the low-frequency or long-period range of greater than ten days, the so-called red noise spectrum.

The summer of 1968 was a particularly interesting one with strong westerly flow from June to mid-August which weakened with the growth of high-latitude blocking leading first to a trough (Sept. 3-7) and then to a dominant ridge over central North America in mid-September (Andrews, 1968; Posey, 1968). The westerly flow and the high-latitude blocking account for the significance of R(7) in Figure 4.4a, while the anticyclone of September gave the fine hot weather which produced the large values of Q (Figure 4.9). The dominance of the low frequencies in the spectrum is probably due to a trend, which in reality is the summer portion of the annual temperature cycle however since the cycle is not complete it is ascribed to a zero frequency oscillation.

The persistent, continental, anticyclonic conditions of late July and August, 1970 (Stark, 1970) resulted in two-day persistence at Delta (Figure 4.4b) and a concentration of variance in the low-frequency portion of the spectra (Figure 4.8b) as predicted by Dickson (1971).

Based on what has been said in the preceding sections and due to the absence of published analyses of this summer's climate at the time of writing (September 1971), it is possible and desirable to speculate briefly on the nature of the climate of summer 1971 by analyzing the appropriate correlogram (Figure 4.4c) and spectrum (Figure 4.8c). Unlike 1970 the correlogram exhibits one-day and not two-day persistence and a stronger 6 to 7 day peak. The spectrum has a damped low-frequency component. These factors suggest generally cool and very changeable weather dominated by the westerlies associated with the noticeable absence of a strong continental, summer anticyclone.

5.3 NONRANDOM HYDROGEOLOGICAL COMPONENTS

It is now opportune to recall the concept of dynamic equilibrium associated with Freeze in Section 1.3. The groundwater flow system is in dynamic equilibrium and responds to changes in input and output, and therefore storage, by adjusting the piezometric head distribution throughout the flow system. This response is manifested by daily fluctuations in the values of Q and R.

Considering Q first, the correlograms (Figure 4.1) show differing responses to temperature. Only in 1968, a year of strong persistence in $\overline{T}$, was there significant persistence in Q. Again only in 1968 was there a significant peak in both Q and $\overline{T}$ in the 5 to 7 day periodicity. The lack of coherence between the peaks, 5 days for Q and 7 for $\overline{T}$, probably reflects the importance of the depth to the water table. A 7-day cycle of $\overline{T}$ will progressively lower the water table such that, as Gilliland (1973) pointed out, the amplitude and phase lag of the groundwater hydrograph undergoing evapotranspiration will be significantly altered.

It should be pointed out here that most of the significant acf estimates for 1970 are intimately connected with the two Lisse effects which dominated Q and R that summer. The two- and four-day peaks in Figure 4.1b are due to the chance occurrence of two major storms eight days apart. Generally, with one exception, the correlograms for Q and R, 1970 do little to help develop a more general understanding of the role of climate in a groundwater discharge area.

The exception noted above refers to the persistence of the first three autocorrelation coefficients of R. This is permitted by the large storage of the flow system and is due to daily extractions of evapotranspiration which is replaced by upward seepage, that is, positive values of R. These positive R values reflect a disturbance in the equilibrium of the flow system, that is previous recharge and/or discharge events; therefore, an adjustment on the part of the flow system is required to offset the increase in the head difference between recharge and discharge areas. The adjustment transfers groundwater to the discharge area. An excellent example of this is the period of high R values in September, 1968 following the heavy rains of August (Figure 4.9).

The negative values of R are due to the drainage of the water table after Lisse effects or infiltration periods.

5.4 THE STOCHASTIC MODEL OF THE SIGNALS

To determine the suitability of the proposed stochastic model of the signals, it was necessary to use two kinds of statistical tests.

The results of the chi-square goodness-of-fit test to the correlograms of Q and R gave disappointing results (Table 2). Only those time-series with R(1) less than 0.3 were accepted. Similar results were experienced by Matalas (1963) with streamflow discharges. However, the key to the results may be the fact that Quenouille (1947) designed the test for large sample sizes, by which he meant several hundred values and not just one hundred.

Much more encouraging results were derived when all four variables were subjected to the Tukey sampling theory.

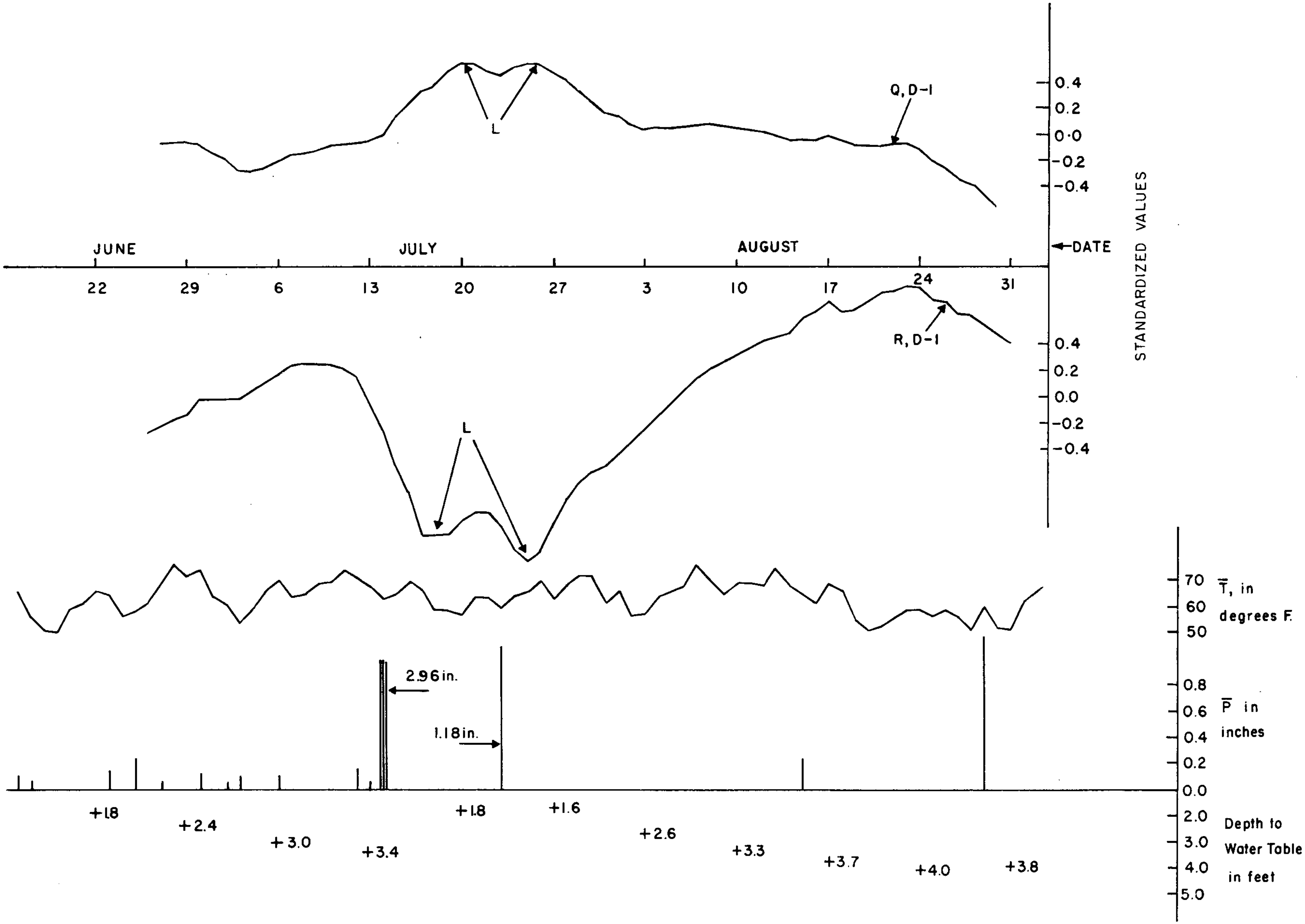


Figure 4.10 Evapotranspiration season, 1970

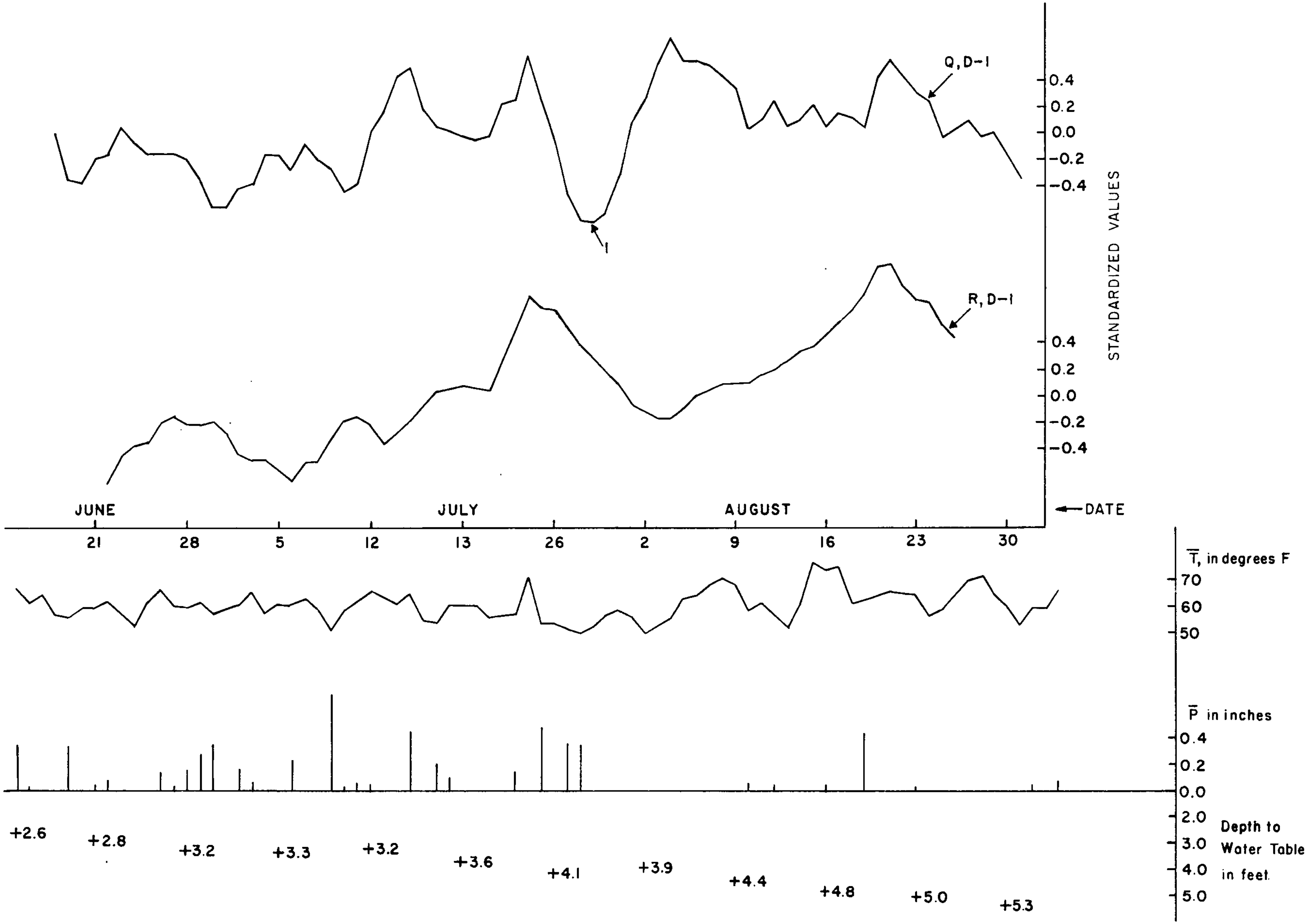


Figure 4.11 Evapotranspiration season, 1971

As was noted in Section 4.3 only six values in the twelve spectra were significantly different at the 90% confidence level from the spectrum of a first-order Markov process.

In summary the results of the Tukey sampling theory seem to be both more reliable and more positive. It is concluded that, as a visual inspection of the spectra will confirm, the first-order Markov process adequately models Q(t), R(t) and $\overline{T}(t)$, while $\overline{P}(t)$ is obviously random.

5.5 STATISTICAL FILTERING OF THE SIGNALS

The results of the statistical filtering show that two processes act to give seasonal maxima in the Q time series, both occurring in the presence of a shallow water table that is less than 3 feet, and in hot weather.

For 1968 (Figure 4.9) the maximum Q was produced in September when strong upward seepage at K-1, coupled with daily maximum temperature in the eighties and a water table between 2.6 and 3.2 feet were the principal causes. Late June and mid-July, 1968 showed similar but less important examples of this process.

This process probably accounts for the observations of McDonald and Hughes (1968) at Yuma, Arizona. They noted that the maximum daily amplitude of the diurnal fluctuations of the water table occurred in October, rather than late July as they expected, when the water table was at its deepest. The ideas developed in this thesis suggest that the large groundwater inflow rates of October were a response to the rains of August, September, and October which are a regular feature of the climatology of the U.S. Southwest. These maximum amplitudes were therefore due to the hydrogeological response of the flow system to the climatic stimuli of infiltration in the recharge area and ninety-degree heat of October causing large consumptive use in the discharge area. The amplitudes of July were small compared with those of October because of a lack of recharge due to the early summer drought of Arizona.

The observation well at D-1 showed very different responses during 1970 and 1971 than K-1 in 1968. The evapotranspiration season 1970 (Figure 4.10) was dominated by two Lisse effects which brought the water table within two feet of the surface, and consequently into the influence of a denser root zone network. Maximum temperature in the seventies and eighties then caused large daily consumptive use. The largest value of Q in 1971 (Figure 4.11) was on August 4 when infiltration brought the water table to within four feet of the surface. The difference between the response of K-1 and D-1 may reflect the existence of a local flow system at D-1, which was suggested in Chapter 3 of this study.

Therefore large values of Q are a function of hot weather and a factor that brings the water table close to the surface. In one case this factor is due to climatic effects on the recharge area producing large groundwater inflow rates. In the second case it is due to climatic elements affecting the discharge area, the Lisse effect and infiltration.

5.6 THE HYDROLOGIC REGIMEN

In the development of his mathematical model of groundwater flow systems, Freeze (1969) assumed a steady-state water table, which he defended on the grounds that the zone of fluctuation of the water table is small compared with the saturated thickness of the flow system. Toth (1962) had predicted that the zone of fluctuation would be smaller in the discharge area than in the recharge area, because of the regulating effect of the creek which was draining the flow system. Results from the present study suggest that the following more complete statement of the hydrologic regimen of groundwater discharge areas may be made.

In a groundwater discharge area the upper limit of the water table is effectively controlled by groundwater evapotranspiration and baseflow discharge to nearby streams, both of which increase with the shallowness of the water table. The lower limit is controlled by the increase in piezometric head with depth, which suggests that the groundwater-inflow rate should also increase with depth to the water table. However, the very factors that control the upper limit of the water table become less important as it is lowered by their action. In one case the rate of groundwater evapotranspiration becomes insignificant compared with the groundwater-inflow rate at depth, while in the other, baseflow discharge to a stream ceases when the water table falls beneath the base level of the stream. Although these factors become less important with depth, climatic stimuli occur which return the water table to a shallower depth. In one case these stimuli affect the discharge area in the form of the Lisse effect and infiltration, while in the other case they affect the recharge area; and due to lateral flow through the flow system, discharge increases in the discharge area.

Conclusions

Nonrandom components associated with the weather and circulation of the North-American summer climate were identified in the time series of mean daily temperature and, in much damped form, in the time series of daily groundwater evapotranspiration. Daily precipitation was random.

The groundwater variables were adequately modelled by a first-order Markov process.

Statistical filtering of the hydrogeological variables showed that seasonal maxima in the daily groundwater evapotranspiration time series occurred in the presence of hot weather and a shallow water table. Shallow water tables in the groundwater-discharge area were associated with either strong, upward groundwater flow due to a recharging of the flow system, or infiltration or the Lisse effect raising the water table.

Results from the study explain why the water table fluctuates within a small range of depth in the groundwater discharge area, thereby satisfying a major assumption of the steady-state, mathematical model of groundwater flow systems.

The methods used in this study should find ready application in suggesting sampling times for observation-well networks, and for studying nonrandom components affecting the water balance of regions.

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Observed Well Data

Well K-1
Average water level, Summer 1967: 3.34 feet below datum
Description: 4-inch casing, gravel-packed, with a 5-foot long, 4-inch screen from 4 feet to 9 feet
Lithology:

Depth from (feet)	to	Lithology	Interpretation
0	9	fine-grained sand	Alluvial
9	14	clayey silt	Alluvial
14	19	silty clay	Lacustrian
19	26	silty clay	Lacustrian
26		END OF HOLE	

Slug-test results: permeability: 2.85 x 10^{-3} cm./sec.
induced phase lag: 6 min.

Well: D-1
Average water level, Summer 1967: 3.94 feet below datum
Description: 4-inch casing slotted from 4.4 to 19.3 feet, gravel-packed
Lithology:

Depth from (feet)	to	Lithology	Interpretation
0	4	clayey silt	Alluvial
4	7	silty clay	Alluvial
7	20	silty clay	Lacustrian
20		END OF HOLE	

Slug-test results: permeability: 3.84 x 10^{-4} cm./sec.
induced phase lag: 21 min.

Program WHITE — Daily Evapotranspiration of Groundwater by White's Method

```
C
C
C   PROGRAM WHITE COMPUTES DAILY EVAPOTRANSPIRATION OF GROUNDWATER BY WHITE'S
C   METHOD.  READ IN (N) SAMPLE SIZE,(SAMP) ALFANUMERIC DATA ON THE HYDROGRAPH,
C   (R)  GROUNDWATER INFLOW IN FT/HOUR, AND (S) DAILY DECLINE IN THE WATER LEVEL
C   DUE TO EVAPOTRANSPIRATION FROM GROUNDWATER IN FT/DAY
C
C
C
      DIMENSION EVAP(200),R(200),S(200),SAMP(11)
      READ (1,1)  (SAMP(I),I=1,10),N,SY
      READ(1,2) (R(I),I=1,N)
      READ(1,2) (S(I),I=1,N)
      WRITE(3,99)  (SAMP(I),I=1,10),N,SY
C
C   USE WHITE'S EQUATION TO CALCULATE EVAPOTRANSPIRATION
C
      DO 4 J=1,N
      FAC=(24.*R(J)+S(J))
    4 EVAP(J)=SY*FAC
C
C   PRINT AND PUNCH OUT  VALUES OF DAILY EVAPOTRANSPIRATION
C
      WRITE(3,6)
      WRITE(3,7) (( J,R(J),S(J),EVAP(J)),J=1,N)
      WRITE(2,2) (EVAP(J),J=1,N)
      STOP
    1 FORMAT(10A4,I3,F4.2)
    2 FORMAT(16F5.3)
    6 FORMAT('1','                  DAILY EVAPOTRANSPIRATION',//,'            DAY
     1    R      S        VALUE',//)
    7 FORMAT(11X,I2,2X,F6.3,1X,F5.3,3X,F5.3)
   99 FORMAT('1',10A4,' N=  ',I3, ' SY = ',F4.2,//)
      END
```

Data List for Groundwater Evapotranspiration (ft./day), Groundwater Inflow Rate (ft./hr.), Daily Precipitation (in.), and Mean Daily Temperature (°F)

DATA FOR YEAR 1968

DAY	Q	R	P	T
1	0.012	-.003	0.0	58.0
2	0.017	0.001	0.0	54.5
3	0.027	0.002	0.0	74.5
4	0.027	0.003	0.0	66.0
5	0.011	0.003	0.0	64.5
6	0.019	0.004	0.0	63.0
7	0.0	0.001	0.0	55.0
8	0.009	0.004	0.05	56.0
9	0.014	0.002	0.05	56.5
10	0.028	0.008	0.0	57.5
11	0.009	0.001	0.0	60.5
12	0.004	0.001	0.55	62.0
13	0.0	0.0	0.0	61.0
14	0.003	-.005	0.0	51.0
15	0.017	0.0	0.0	54.5
16	0.017	0.0	0.0	57.5
17	0.015	0.0	0.15	62.0
18	0.016	0.002	0.0	64.0
19	0.004	0.001	0.0	59.5
20	0.021	0.004	0.0	71.5
21	0.004	0.001	0.0	63.0
22	0.012	0.003	0.0	59.5
23	0.010	0.001	0.0	58.0
24	0.008	0.001	0.0	56.5
25	0.012	0.002	0.0	59.0
26	0.012	0.002	0.0	58.5
27	0.017	0.003	0.0	66.0
28	0.016	0.004	0.10	67.0
29	0.019	0.003	0.0	65.0
30	0.0	0.001	2.60	56.5
31	0.0	0.0	0.0	55.0
32	0.019	-.006	0.0	58.0
33	0.018	-.006	0.0	63.5
34	0.015	-.007	0.0	64.0
35	0.021	-.002	0.0	67.5
36	0.022	0.002	0.0	76.5
37	0.022	0.0	0.0	71.0
38	0.013	0.0	0.0	60.5
39	0.012	0.001	0.0	56.5
40	0.020	0.002	0.0	69.5
41	0.005	0.0	0.0	65.5
42	0.016	0.002	0.25	70.5
43	0.0	0.0	0.0	73.5
44	0.015	0.001	0.0	69.0
45	0.008	0.001	0.23	67.0
46	0.0	0.002	0.0	70.0
47	0.015	0.0	0.0	69.0
48	0.014	0.002	0.12	70.0
49	0.020	0.004	0.0	68.0
50	0.008	0.003	0.25	69.5
51	0.020	0.003	0.0	64.5
52	0.012	0.002	0.85	63.0
53	0.0	0.0	0.0	63.5
54	0.0	0.0	0.0	63.0
55	0.008	-.002	0.0	68.5
56	0.016	0.0	0.0	65.0
57	0.018	0.0	0.0	57.5
58	0.010	0.0	0.08	60.5
59	0.003	0.0	1.55	58.5
60	0.0	0.0	0.0	63.5
61	0.0	0.0	0.0	66.0
62	0.026	-.001	0.0	66.0
63	0.004	-.004	0.0	69.0
64	0.009	0.0	0.0	67.5
65	0.009	0.0	0.39	64.5
66	0.018	0.0	0.0	70.0
67	0.0	0.0	0.05	62.5
68	0.009	0.0	0.0	65.5
69	0.006	0.0	0.10	59.5
70	0.015	0.0	0.0	55.5
71	0.007	0.0	0.0	58.5
72	0.013	0.001	0.06	61.0
73	0.003	0.0	0.08	65.0
74	0.011	0.0	0.0	52.0
75	0.009	0.001	0.25	51.5
76	0.0	0.0	1.80	55.5
77	0.0	0.0	0.0	57.0
78	0.011	-.007	0.35	53.0
79	0.0	0.0	0.40	60.0
80	0.0	0.0	0.47	59.5
81	0.0	0.0	0.0	55.5
82	0.006	-.004	0.0	60.0
83	0.015	-.005	0.0	65.5
84	0.011	0.0	1.60	61.0
85	0.0	0.0	0.10	59.0
86	0.012	-.011	0.0	55.0
87	0.007	-.010	0.0	56.5
88	0.016	-.004	0.0	61.5
89	0.021	0.0	0.25	68.5
90	0.004	0.0	0.10	66.0
91	0.010	-.004	0.0	63.5
92	0.020	0.0	0.0	60.5
93	0.013	-.001	0.0	54.5
94	0.007	0.0	0.48	52.5
95	0.0	0.0	0.10	57.0
96	0.007	-.003	0.0	52.0
97	0.013	-.001	0.05	52.0
98	0.014	-.001	0.05	50.0
99	0.007	0.0	0.0	55.0
100	0.010	-.003	0.0	56.0
101	0.015	0.0	0.0	59.0
102	0.018	0.0	0.0	64.5
103	0.024	0.002	0.0	66.5
104	0.031	0.005	0.0	67.0
105	0.032	0.004	0.0	67.5
106	0.029	0.006	0.0	67.0
107	0.030	0.004	0.0	70.5
108	0.022	0.006	0.0	62.5
109	0.019	0.003	0.0	58.5
110	0.024	0.004	0.0	58.0
111	0.018	0.005	0.0	59.0
112	0.024	0.003	0.0	63.5
113	0.021	0.004	0.08	62.5

DATA FOR YEAR 1970

DAY	Q	R	P	T
1	0.020	-.002	0.10	66.0
2	0.0	0.0	0.04	56.5
3	0.043	0.0	0.0	51.0
4	0.007	0.0	0.0	50.5
5	0.014	-.003	0.0	59.0
6	0.017	-.001	0.0	62.5
7	0.015	0.0	0.0	66.5
8	0.003	0.0	0.14	63.5
9	0.005	-.002	0.0	55.5
10	0.0	0.0	0.22	58.5
11	0.011	-.003	0.0	61.5
12	0.008	0.0	0.06	68.0
13	0.014	0.0	0.0	76.0
14	0.006	-.003	0.0	71.5
15	0.018	0.004	0.11	74.0
16	0.010	0.0	0.0	63.5
17	0.009	0.0	0.03	61.0
18	0.0	-.002	0.01	54.0
19	0.004	0.0	0.0	57.5
20	0.006	0.0	0.0	65.5
21	0.007	0.0	0.10	69.5
22	0.014	0.002	0.0	63.5
23	0.008	0.001	0.0	65.0
24	0.008	0.001	0.0	68.0
25	0.012	0.002	0.0	69.5
26	0.006	0.001	0.0	74.0
27	0.012	0.004	0.18	71.5
28	0.005	0.002	0.07	67.5
29	0.0	0.0	2.96	63.0
30	0.008	0.0	0.0	65.0
31	0.024	-.005	0.0	69.5
32	0.014	-.010	0.0	65.5
33	0.005	-.003	0.01	58.0
34	0.018	-.006	0.0	57.5
35	0.027	-.001	0.0	57.0
36	0.018	0.0	0.0	63.5
37	0.012	0.0	0.01	64.5
38	0.002	0.0	1.10	60.0
39	0.020	-.006	0.0	63.5
40	0.015	-.008	0.0	66.0
41	0.024	-.007	0.0	69.5
42	0.015	-.003	0.0	63.0
43	0.016	0.0	0.0	69.0
44	0.012	0.0	0.0	72.5
45	0.012	-.001	0.0	71.5
46	0.005	-.003	0.0	62.5
47	0.014	0.0	0.0	66.5
48	0.008	-.002	0.0	56.0
49	0.005	-.001	0.0	57.0
50	0.011	0.0	0.0	64.0
51	0.010	0.0	0.0	65.5
52	0.009	0.0	0.0	68.5
53	0.012	0.001	0.0	76.5
54	0.013	0.001	0.0	70.5
55	0.014	0.001	0.0	65.0
56	0.010	0.001	0.0	69.0
57	0.009	0.001	0.02	69.0
58	0.012	0.002	0.0	67.5
59	0.011	0.002	0.0	74.0
60	0.004	-.002	0.0	67.5
61	0.011	0.004	0.23	59.0
62	0.004	0.001	0.0	56.5
63	0.020	0.006	0.0	67.5
64	0.008	0.0	0.0	66.0
65	0.005	0.0	0.0	55.0
66	0.006	0.001	0.0	51.0
67	0.012	0.005	0.0	52.5
68	0.007	0.001	0.0	56.5
69	0.015	0.004	0.0	58.0
70	0.015	0.005	0.0	58.0
71	0.005	0.0	0.0	56.5
72	0.011	0.004	0.0	58.5
73	0.004	-.001	0.0	55.0
74	0.009	0.003	0.0	51.0
75	0.009	0.004	1.18	60.5
76	0.0	0.0	0.0	52.5
77	0.0	0.0	0.0	51.0
78	0.0	0.0	0.0	62.0
79	0.0	0.0	0.01	65.5
80	0.005	0.002	0.0	61.5
81	0.003	0.001	0.0	57.5
82	0.008	0.003	0.0	71.5
83	0.007	0.0	0.0	69.0
84	0.004	0.0	0.06	52.0
85	0.003	0.001	0.02	51.5
86	0.012	0.003	0.01	46.0
87	0.010	0.003	0.0	40.5

CATA FOR YEAR 1971

DAY	Q	R	P	T
1	0.0	0.0	0.34	67.0
2	0.001	-.006	0.02	62.0
3	0.013	-.003	0.0	65.0
4	0.004	-.005	0.0	57.0
5	0.0	-.002	0.33	56.5
6	0.0	0.0	0.0	60.0
7	0.005	-.002	0.04	59.0
8	0.002	-.002	0.09	62.0
9	0.007	0.0	0.0	57.0
10	0.003	-.001	0.0	53.0
11	0.003	-.002	0.0	61.5
12	0.003	0.0	0.13	66.0
13	0.004	0.0	0.02	60.0
14	0.004	-.001	0.15	59.0
15	0.003	-.001	0.29	62.0
16	0.0	0.0	0.35	57.5
17	0.0	0.0	0.0	58.5
18	0.003	-.002	0.17	60.5
19	0.001	-.002	0.06	65.0
20	0.005	0.0	0.0	57.0
21	0.004	-.002	0.0	61.0
22	0.0	-.003	0.23	61.0
23	0.006	0.0	0.0	63.0
24	0.003	-.003	0.0	58.5
25	0.004	0.0	0.74	51.0
26	0.0	0.0	0.02	58.0
27	0.0	0.0	0.07	61.5
28	0.006	0.0	0.05	65.0
29	0.004	-.003	0.0	63.5
30	0.008	-.001	0.0	60.5
31	0.009	-.001	0.46	65.0
32	0.004	0.0	0.0	55.0
33	0.003	0.0	0.20	54.0
34	0.005	0.0	0.0	61.0
35	0.004	0.0	0.01	60.0
36	0.004	0.0	0.0	60.5
37	0.002	-.003	0.0	56.5
38	0.007	0.0	0.0	57.5
39	0.003	0.0	0.14	57.5
40	0.012	0.004	0.0	70.5
41	0.005	0.0	0.48	54.0
42	0.005	0.002	0.01	54.0
43	0.0	0.0	0.37	51.0
44	0.0	0.0	0.34	49.5
45	0.0	0.0	0.0	52.0
46	0.0	0.0	0.0	57.0
47	0.002	0.0	0.0	59.0
48	0.006	-.001	0.0	56.0
49	0.005	-.001	0.0	49.5
50	0.007	-.001	0.0	53.0
51	0.010	-.001	0.0	55.5
52	0.006	-.001	0.0	64.0
53	0.007	0.0	0.0	64.0
54	0.007	0.0	0.0	67.5
55	0.007	0.0	0.0	71.0
56	0.007	0.0	0.0	67.5
57	0.002	-.001	0.06	58.5
58	0.004	0.0	0.0	62.0
59	0.008	0.0	0.0	56.0
60	0.003	0.0	0.04	51.5
61	0.004	0.0	0.0	59.5
62	0.007	0.0	0.01	77.0
63	0.003	0.0	0.04	69.0
64	0.006	0.001	0.0	69.5
65	0.005	0.0	0.0	61.0
66	0.001	0.0	0.43	61.5
67	0.008	0.003	0.0	64.0
68	0.009	0.003	0.0	66.5
69	0.006	0.0	0.0	65.0
70	0.005	0.0	0.03	63.5
71	0.007	0.002	0.0	57.0
72	0.002	0.0	0.0	59.0
73	0.004	0.0	0.0	64.0
74	0.006	0.0	0.0	69.5
75	0.003	0.0	0.0	72.0
76	0.006	0.0	0.0	64.5
77	0.003	0.0	0.0	60.0
78	0.001	0.0	0.0	53.0
79	0.0	0.0	0.04	60.0
80	0.009	0.002	0.0	59.5
81	0.002	0.0	0.08	65.5

Program SPECTRA — Spectral Analysis and Filtering for Hydrological and Climatological Analysis

```
C
C
C
C
C  PROGRAM SPECTRA
C
C      SPECTRAL ANALYSIS AND FILTERING
C  VERSION OF R.E. JACKSON FOR HYDROLOGICAL AND CLIMATOLOGICAL DATA ANALYSIS
C
C      --------------------------------------------------------------
C
C....... E X P L A N A T I O N   A N D   O R D E R   O F   I N P U T ...........
C
C FIRST CARD  -  COLUMNS  -  NAME  -  COMMENTS
C
C                  1-40      SAMP(J)    ANY ALFANUMERIC INFORMATION
C                 41-43        ID       SAMPLE NO., MUST BE DIFFERENT FROM ZERO
C                 44-46        M        MAXIMUM LAG, MUST NOT EXCEED 50
C                 47-49       ILG       CODE,AS TO WHETHER WE USE RAW DATA OR LOG
C                                       TRANSFORMED DATA.IF ILG=10 WE USE THE LOG
C                                       TRANSFORM,OTHERWISE THE RAW DATA IS USED
C                 50-52        N        NUMBER OF DATA POINTS
C
C
C  SAMPLE NUMBERS 1-10 ARE RESERVED FOR  GROUNDWATER EVAPOTRANSPIRATION (Q) AND
C  INFLOW RATE (R) IN FORMAT 16F5.3.
C  SAMPLE NOS. 11-15  ARE FOR DAILY PPT (P) IN FORMAT 20F4.2
C  SAMPLE NOS. 16-20 ARE FOR DAILY MEAN TEMP(T) IN FORMAT 20F4.1
C  EQUATION NUMBERS REFER TO EQUATIONS IN THE THESIS
C ONE BLANK CARD AFTER LAST DATA DECK
C
C NOTE IF LOG OPTION IS REQUIRED, THE LOGGING OF THE DATA SHOULD BE DONE IN DO
C  LOOP 401, I.E. CARD SPEC 205
C
C
C
C  N O T E --- C A R D   O U T P U T   C O N S I S T S   O F   S T A N D A R D I
C
C              Z E D   D A T A   A N D   F I T T E D   S I G N A L   V A L U E S
C
C
C
C      --------------------------------------------------------------

      DIMENSION X(200),VC(50),C(50)
      COMMON R(50),XX(200),SAMP(10),IN,IOUT,V(50),LH,QQ,RHO(50),H(50)
      PIE=3.14159265
      IOUT=3
      IN=1
C-----------------------------------------------------------------
C  READ IN FIRST CARD
C-----------------------------------------------------------------
```

```
 50 READ(IN,1)(SAMP(J),J=1,10),ID,M,ILG,N
    IF(ID)100,999,100
100 WRITE(IOUT,2)(SAMP(J),J=1,10),ID,M
    IF (ILG.EQ.10) GO TO 110
    GO TO 120
110 WRITE(IOUT,3)
    GO TO 130
120 WRITE(IOUT,4)
130 XM=M
    PDE=PIE/XM
    I = 1
    SUM2=0.0
    SUM=0.0
C-----------------------------------------------------------------------
C  READ IN DATA DECK
C-----------------------------------------------------------------------
    IF (ID.LE.10)   READ(IN,400) (X(I),I=1,N)
    IF(ID.GE.11.AND. ID.LE.15) READ(IN,399)  (X(I),I=1,N)
    IF(ID.GT.15)  READ(IN,398)  (X(I),I=1,N)
    WRITE(IOUT,20) (SAMP(J),J=1,10)
    WRITE(IOUT,397)  (X(I),I=1,N)
    DO 401 J=1,N
401 SUM=SUM+X(J)
    XI=N
    I=N
C-----------------------------------------------------------------------
C  COMPUTE STATISTICS OF TIME SERIES AND FORM STANDARDISED TIME SERIES
C-----------------------------------------------------------------------
    XBAR = SUM / XI
    DO 190 J=1,I
    X(J)=X(J)-XBAR
190 SUM2=SUM2+(X(J)**2)
    VAR = SUM2 / (XI - 1.0)
    SD=SQRT (VAR)
    SUM3=0.0
    DO 191 J=1,N
    X(J)=X(J)/SD
191 SUM3=SUM3+X(J)**2
    CO=SUM3/XI
C-----------------------------------------------------------------------
C  COMPUTE ACVF
C-----------------------------------------------------------------------
    MM1=M-1
    DO 210 L=1,M
    JX=I-L
    XSUM=0.0
    DO 200 JK=1,JX
    JY=JK+L
200 XSUM=XSUM+X(JK)*X(JY)
    XJ = JX
    C(L) = (1.0 / XJ) * XSUM
210 R(L)=C(L)/CO
C-----------------------------------------------------------------------
C  ESTIMATE SPECTRUM
C-----------------------------------------------------------------------
    DO 230 L=1,M
    XL = L
    SUMYC=0.0
    SUMY=0.0
    DO 220 J=1,MM1
    XJ = J
    ARG1 = COS (XL * XJ * PDE)
    ARG2=C(M)*COS (XL*PIE)
    SUMY = SUMY + C(J) * ARG1
```

```
  220 SUMYC = SUMYC + C(J) * ARG1 * (1.0 + COS (XJ * PDE))
      V(L) = CO  +  2.0 * SUMY + ARG2
  230 VC(L) = CO + SUMYC
      CJ = 0.0
      CJJ=0.0
      DO 240 JJ=1,MM1
      XJJ=JJ
      CJJ=CJJ + C(JJ)*(1.0+COS (XJJ*PDE))
  240 CJ = CJ + C(JJ)
      VO=CO +2.0*CJ +C(M)
      VOH = CO+CJJ
C-------------------------------------------------------------------------
C  CALL SUBROUTINE TO COMPUTE THEORETICAL ACVF AND SPECTRUM
C-------------------------------------------------------------------------
      RHO1=R(1)
      CALL MARKOV(RHO1,CO,PIE,M,HZERO,CHISUM,N)
      WRITE(IOUT,7)I,XBAR,VAR,SD
      WRITE(IOUT,20)(SAMP(J),J=1,10)
      WRITE(IOUT,6)CO,VO,VOH,HZERO
      WRITE(IOUT,8)
      DO 250 J=1,M
  250 WRITE(IOUT,9)J,R(J),C(J),V(J),VC(J),RHO(J),H(J)
      WRITE(IOUT,396) CHISUM
      IF(ID.GE.11)  GO TO 50
      DO 260 JJ=1,I
  260 XX(JJ)=X(JJ)
      WRITE(IOUT,20)(SAMP(J),J=1,10)
C-------------------------------------------------------------------------
C  PRINT AND PUNCH OUT STANDARDISED DATA
C-------------------------------------------------------------------------
      WRITE(IOUT,12)
      DO 290 J=1,I
  290 WRITE(IOUT,13)J,X(J)
C-------------------------------------------------------------------------
C  CALL SUBROUTINE FILTER
C-------------------------------------------------------------------------
      CALL FILTER(M,I,VAR,DUM)
      IF(DUM.LT.0.0)  GO TO 50
      WRITE(IOUT,20)(SAMP(J),J=1,10)
C-------------------------------------------------------------------------
C  PRINT AND PUNCH OUT THE SIGNAL
C-------------------------------------------------------------------------
      WRITE(IOUT,14)LH,LH
      LH1=LH+1
      LH2=I-LH
      DO 320 J=LH1,LH2
      SUM=QQ*X(J)
      IF(LH.EQ.0) GO TO 320
      GO TO 300
  300 DO 310 JJ=1,LH
      JJ1=J+JJ
      JJ2=J-JJ
  310 SUM=SUM+XX(JJ)*(X(JJ1)+X(JJ2))
  320 WRITE(IOUT,15)J,SUM
      GO TO 50
  999 STOP
    1 FORMAT(10A4,4I3)
    2 FORMAT(1H1,10A4//1H ,12HSAMPLE NO.= ,I3//1H ,13HMAXIMUM LAG= ,I3)
    3 FORMAT(//30X,30H..........LOG OPTION........../)
    4 FORMAT(//30X,40H..........LOG OPTION NOT TAKEN........../)
    5 FORMAT(8X,I3,48X,F8.3 )
    6 FORMAT(////1H ,7HC ZERO=, F12.5, 12X, 7HV ZERO=, F12.5,12X,14HHANN
     CED V ZERO=, F12.5,12X,20HTHEORETICAL V ZERO= ,F12.5/)
```

```
  7 FORMAT(///1H ,16HNO. OF SAMPLES =,I5//1H ,6HMEAN =,F12.5//1H ,10HV
   CARIANCE =,F12.5//1H ,15HSTANDARD DEVN.=,F12.5 )
  8 FORMAT(///1H ,1X,3HLAG,2X,15HAUTOCORRELATION,2X,14HAUTOCOVARIANCE,
   12X,16HSPECTRAL DENSITY,3X,15HHANNED SPECTRUM,1X,10HTHEOR. ACF,5X,
   323HTHEOR. SPECTRAL DENSITY//1H ,2X,2HL ,8X,4HR(L),10X,6HCOV(L),
   412X,4HV(L),11X,11HHANNED V(L),6X,6HRHO(L),12X,4HH(L)//)
  9 FORMAT(1H ,I3,4X,F10.7,3(5X,E13.7),3X,F10.7,5X,E13.7)
 10 FORMAT(///1H ,88HRELATIVE VARIANCE TIMES VOLUME= LAG*(VAR AT LAG V
   CALUE)/(VAR AT LAG 1).COMPUTED FROM DATA//1H ,2X,3HLAG,5X,5HVALUE/)
 11 FORMAT(1H ,I3,2X,F12.5)
 12 FORMAT(///1H ,29HPRINTOUT OF STANDARDISED DATA//1H ,1X,3HLAG,9X,
   C5HVALUE/)
 13 FORMAT(1H ,I3,5X,F10.4)
 14 FORMAT(////46H SIGNAL VALUES COMPUTED BY SUMMING FROM LAG= -,I3,2X
   C,7HTO LAG=,I3//1H ,10HSAMPLE NO.,3X,12HSIGNAL VALUE )
 15 FORMAT(1H ,3X,I4,6X,F12.6)
 20 FORMAT(1H1,10A4)
 21 FORMAT(59X,F8.4)
 22 FORMAT(8X,I3,48X,F8.4 )
396 FORMAT(////1H ,'          SUM OF CHI SQUARES    = ',F10.5)
397 FORMAT(10X,10F8.3)
398 FORMAT(20F4.1)
399 FORMAT(20F4.2)
400 FORMAT(16F5.3)
    END


    SUBROUTINE FILTER(M,N,VAR,DUM)
    DIMENSION RL (50),VV(200),RC(50),T(50),Y(50)
    COMMON R(50),XX(200),SAMP(10),IN,IOUT,V(50),LH,QQ,RHO(50),H(50)
C-----------------------------------------------------------------------
C  CARRIES OUT FILTERING OPERATION
C-----------------------------------------------------------------------
    EQUIVALENCE (VV(1),XX(1)),(RL(1),V(1)),(RC(1),V(11))
    WRITE(IOUT,1)(SAMP(I),I=1,10)
100 I=1
110 IF(R(I))130,130,120
120 I=I+1
    IF(I- M)110,130,130
C-----------------------------------------------------------------------
C  CALCULATE A AND C BY SIMPLE LINEAR REGRESSION (EQNS. 2.21 AND 2.22)
C-----------------------------------------------------------------------
130 IT=I-1
    DO 131 J=1,IT
    Y(J)=ALOG(R(J))
131 T(J)=J
    A=0.0
    B=0.0
    C=0.0
    D=0.0
    E=0.0
    DO 132 J=1,IT
    A=A+(T(J)*Y(J))
    B=B+T(J)
    C=C+Y(J)
    D=D+T(J)**2
132 E=E+Y(J)**2
    TM=B/IT
    YM=C/IT
    AA=(A-IT*TM*YM)/(D-IT*TM**2)
    CC=EXP(YM-AA*TM)
    WRITE(IOUT,2)
    WRITE(IOUT,3)CC,AA
    WRITE(IOUT,2)
    DO 150 J=1,M
```

```
      WRITE(IOUT,4)
      XJ=J
      RL(J)=CC*EXP (AA*XJ)
  150 WRITE(IOUT,5)J,R(J),RL(J)
C
C  CALCULATE P AND Q (EQN.2.26)
C
      AA=-AA
      DUM =(AA**2+2.0*(AA*CC/(1.0-CC)))
      IF (DUM.LT.0.0)  WRITE(IOUT,15)
      IF (DUM.LT.0.0) GO TO 211
      PP=SQRT(DUM)
      QQ=AA*CC/(PP*(1.-CC))
      WRITE(IOUT,9)QQ,PP
      EXSM=EXP (-PP)
C-------------------------------------------------------------------
C  CALCULATE H=SML (EQN.2.30)
C-------------------------------------------------------------------
      SML=QQ+2.*QQ*(EXSM/(1.-EXSM))
      WRITE(IOUT,10)SML
      WRITE(IOUT,11)
      WRITE(IOUT,12)QQ,QQ
      HSM=QQ
      J=0
C-------------------------------------------------------------------
C  CHECK THAT EQN.2.28 IS SATISFIED
C-------------------------------------------------------------------
      IF((SML-HSM)-.05)200,180,180
  180 DO 190 J=1,50
      XJ=J
      VV(J)=QQ*EXP (-(XJ*PP))
      HSM=HSM+2.*VV(J)
      WRITE(IOUT,13)J,VV(J),HSM
      IF((SML-HSM)-.05)200,190,190
  190 CONTINUE
  200 LH=J
C-------------------------------------------------------------------
C  CALCULATE CORRECTED FILTER (EQN.2.31)
C-------------------------------------------------------------------
      WRITE(IOUT,14)
      CORF=SML/HSM
      QQ=CORF*QQ
      WRITE(IOUT,11)
      WRITE(IOUT,12)QQ,QQ
      SUM=QQ
      DO 210 J=1,LH
      VV(J)=CORF*VV(J)
      SUM=SUM+VV(J)*2.
  210 WRITE(IOUT,13)J,VV(J),SUM
  211 RETURN
    1 FORMAT(1H1,10A4)
    2 FORMAT(1H0)
    3 FORMAT(1H0,46H DERIVED FORMULA FOR AUTOCORRELATION FUNCTION // 15X
     C,7H R(L) =,F10.7,6H*EXP (,E13.6,6H * L  ))
    4 FORMAT(///9X,24HAUTOCORRELATION,COMPUTED/1H ,1X,5HLAG  ,2X,15H(1)F
     CROM SAMPLE ,2X,16H(2)FROM EQUATION / )
    5 FORMAT(1H ,I3,2(5X,F10.7))
    6 FORMAT(///1H ,96HRELATIVE VARIANCE TIMES VOLUME= LAG*(VAR AT LAG V
     1ALUE)/(VAR AT LAG 1).COMPUTED FROM DERIVED EQN.//4X,3HLAG,5X,5HVAL
     2UE/)
    7 FORMAT(1H ,I5,3X,F10.5)
    8 FORMAT(///1H ,32HSTANDARD DEVIATION OF THE MEAN= ,F12.7 )
    9 FORMAT(///1H ,15HFILTER H(L) = (,F10.7,14H)*EXP(-ABS(L)*,E13.7,1H)
```

```
  C )
10 FORMAT(///1H ,34HSUM OF H(L) OVER ALL LAG VALUES = ,E13.7 )
11 FORMAT(///1H ,20X,20HSUM FROM -LAG TO LAG )
12 FORMAT(//1H ,8HH( 0 ) =,F10.7,6X,E13.7)
13 FORMAT(1H ,2HH(,I3,3H) =,F10.7,6X,E13.7 )
14 FORMAT(///1H ,32HCORRECTED FILTER VALUES AND SUMS )
15 FORMAT('1','     NO FILTER POSSIBLE - DUM  LT. 0.0')
   END


   SUBROUTINE MARKOV(RHO1,VAR,PIE,M,HZERO,CHISUM,N)
   COMMON R(50),XX(200),SAMP(10), IN,IOUT,V(50),LH,QQ,RHO(50),H(50)
   DIMENSION CHI(50)
C-----------------------------------------------------------------------
C  COMPUTES THEORETICAL SPECTRUM AND CORRELOGRAM OF FIRST
C  ORDER MARKOV PROCESS
C-----------------------------------------------------------------------
   IF(RHO1.LT.0.0) GO TO 2
C
C  RHO1 IS POSITIVE
C
   DO 1 I=1,M
   A=I
   RHO(I)=RHO1**A
   D=2.*PIE*A/(2.*M)
 1 H(I)=VAR*(1.-RHO1**2.)/(1.+RHO1**2.-2.*RHO1*COS(D))
   CHI(1)=(N-K)*(R(1)-2.*R(1)+(R(1)**3))**2/(1.-R(1)**2)**2
   CHI(2)=(N-K)*(R(2)-R(1)**2)**2/(1.-R(1)**2)**2
   DO 5 K=3,M
   K1=K-1
   K2=K-2
   RK=R(K)-2.*R(1)*R(K1)+(R(1)**2)*R(K2)
 5 CHI(K)=((N-K)*RK**2)/(1.-R(1)**2)**2
   CHISUM=0.0
   DO 6 K=1,M
 6 CHISUM=CHISUM+CHI(K)
   GO TO 4
C
C  RHO1 IS NEGATIVE
C
 2 RHO1=ABS(RHO1)
   B=1.
   DO 3 I=1,M
   B=-B
   A=I
   RHO(I)=B*RHO1**A
   D=2.*PIE*A/(2.*M)
 3 H(I)=VAR*(1.-RHO1**2.)/(1.+RHO1**2+2.*RHO1*COS(D))
   CHI(1)=(N-K)*(RHO1-2.*RHO1-RHO1**3)**2/(1.-RHO1**2)**2
   CHI(2)=(N-K)*(RHO1-2.*RHO1**2+RHO1**2)**2/(1.-RHO1**2)**2
   DO 7 K=3,M
   K1=K-1
   K2=K-2
   RK=R(K)-2.*R(1)*R(K1)+(RHO1**2)*R(K2)
 7 CHI(K)=((N-K)*RK**2)/(1.-RHO1**2)**2
   CHISUM=0.0
   DO 8 K=1,M
 8 CHISUM=CHISUM+CHI(K)
C
 4 HZERO=VAR*(1.+RHO1)/(1.-RHO1)
   RETURN
   END
```