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## A MEASURE OF SITE <br> ATTRACTION



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# A MEASURE OF SITE ATTRACTION 

## J. H. ROSS

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## PREFACE

This paper is concerned with the definition of the attractiveness of recreation sites, as seen by the patrons of the site themselves. It represents a novel approach to the measurement of environmental intangibles, and avoids many of the risky assumptions upon which the more traditional methods of determining site attractivity were based. In turn, it introduces biases peculiar to itself. The methodology presented here is not meant to be regarded as the final answer to the problem, but merely as a step towards a technique which will permit the definition of site attractivity on a more rational basis. The general question addressed by the paper is of critical importance to outdoor recreation researchers and planners of recreation areas as a technique which will enable them to evaluate alternative recreation areas. This paper was submitted as a dissertation to the University of Western Ontario. Its publication by the Lands Directorate is intended to $s$ imulate discussion in this important area of outdoor recreation research, a field which is becoming increasingly critical to land use planning throughout Canada.

R.J. McCormack, Director General, Lands Directorate.

## PREFACE

Cet exposé traite de la définition du degré d'attraction des lieux de loisirs, vu par les usagers eux-mêmes. Il présente une approche originale au problème de la mesure des valeurs intangibles du milieu, et évite plusieurs hypothèses hazardeuses habituellement mises de l'avant par les méthodes traditionnelles servant à identifier l'attraction du lieu. Par contre, elle introduit ses propres biais. La méthodologie qu'on présente ne doit pas être vue comme étant la réponse finale au problème, mais plutôt comme une étape vers une technique permettant de définir l'attraction du lieu sur une base plus rationnelle. L'objectif principal de cet exposé est d'importance primordiale pour les chercheurs en loisirs de plein air et aux planificateurs des lieux de loisirs, en tant que technique qui leur permettra d'évaluer et de comparer le potentiel de diverses aires de loisirs. Cet exposé fut soumis comme thèse de doctorat à 1'Université Western Ontario. Sa publication par la Direction générale des Terres veut stimuler la discussion dans ce secteur important de recherche en loisirs, de plein, air, domaine qui devient primordial au niveau de la planification de l'utilisation du sol partout au Canada.

> R.J.McCormack, Directeur Génêral, Direction Générale des Terres.

## ABSTRACT

The study seeks to deduce the attractivity of alternate service sites from data concerning the spatial interactions of individuals who patronize these sites. The data required by the model are of the form "Individual $n$, residing at $X_{n} Y_{n}$ most often patronizes alternative site $j$, situated at $X_{j} Y_{j}$, to obtain the service or good being investigated." From such data, the model produces an ordinal scale representing the attractivity of the alternative sites in the system being studied.

Four separate analyses are performed, three being on collected data and the fourth on simulated data. The attraction scales defined by the method are shown to be in relatively high agreement with the inferred judgements of the individuals sampled, the level of agreement (a statistic analogous to $\mathrm{R}^{2}$ ) varying from a low of .679 to a high of .975 with the real data, and taking the value 1.00 in the simulated trial. A rank correlation coefficient of . 795 was found between the attractions of the simulated sites and the scale recovered from the data set.

It is shown that the model is susceptible to spatial bias introduced by certain arrangements of individuals and alternative sites. Sampling procedures which avoid such bias are suggested.

The study concludes that measures of the inherent attraction of service sites may be deduced from data concerning the spatial movements of individuals weighing the attraction of various alternatives against measures of the costs of realizing these alternatives.

L'étude cherche à évaluer le degré d'attraction de diverses aires de services à partir de données concernant les interactions spatiales de leurs utilisateurs. Les données requises par le modèle décrivent 'L'individu $n$, résidant à l'endroit $X_{n} Y_{n}$ qui choisira le plus souvent le site $j$, situé à $X_{j} Y_{j}$, afin d'obtenir les services ou facilités sous étude." A partir de ces données, le modèle élabore une échelle ordinale représentant le degré d'attraction des divers sites impliqués dans le système sous étude.

Quatre analyses différentes, sont effectués, dont trois le sont à partir de données recueillies et la quatrième à partir de données simulées. Les échelles d'attraction définies par cette méthode démontrent un haut degré de compatibilité avec les opinions présumées des usagers échantillonnés, le degré de compatibilité (une mesure statistique équivalente au $\mathrm{R}^{2}$ ) varie d'un plancher .679 à un plafond .975 pour des données réelles, et prend 1 a valeur de 1.00 dans l'essai simulé. La distribution montre un coefficient de corrélation de rang, entre les degrés d'attraction des sites hypothétiques et l'échelle produite par l'ensemble des données, se situe à . 795 .

Le modèle est sensible à des biais spatiaux introduits par certains arrangements d'individus et de divers sites. On y suggère donc des techniques d'échantillonnage évitant de tels biais.

L'étude conclu qu'il est effectivement possible de mesurer le degré d'attraction inhérent aux aires de service à partir de données décrivant les mouvements spatiaux des individus, en pondérant l'attraction de diverses alternatives, comparativement aux mesures du coût de réalisation de ces mêmes alternatives.

During the pursuit of a post-graduate degree, a student benefits from almost all contact with others, regardless of whether such contacts are professional or academic, social or familial. It is, therefore, an impossible task to thank each of those who has contributed to the education process. The best one can do is to single out those without whose contributions the work accomplished would have been of much less import.

The author is most grateful to the National and Historic Parks Branch of the Department of Indian Affairs and Northern Development for permission to analyze one of their data sets, to the Central Mortgage and Housing Corporation for their generous financial support throughout the degree programme, and to his fellow students for their constant encouragement. He wishes, also, to express his thanks to the members of his advisory comnittee and to the faculty and staff of the department as a whole.

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## CHAPTER I -- INTRODUCTION

Students of many disciplines have investigated the relationships between the characteristics of service-offering sites and the number or proportion of people who choose to patronize each of these sites. It is generally conceded that an individual's satisfaction, and hence presumably his choice of site, is principally influenced by three components which may be broadly defined as "socio-economic", "site", and "distance" factors. The socio-economic factor is concerned with those characteristics of the individual which serve to classify him as a member of a certain subgroup of all individuals. The site component is comprised of a set of characteristics which express the inherent attraction of each site in the individual's choice set. Last, the distance factor is composed of a group of variables which define the geographic relationships between the individual and the service sites.

## Statement of the Problem

Geographers have traditionally been concerned with the investigation of the distance component, and in their work have advanced three major models to explain the magnitude of patronage at the level of the individual site. To these approaches the estimation of the site component has been critical, but to date no general technique
for the construction of empirically satisfactory indices of site attraction has been advanced. It is the purpose of this study to suggest such a technique, and to test its usefulness by applying it to data drawn from various geographic situations. Although the data to be used is drawn from the field of outdoor recreation, the model to be advanced is applicable to many situations in which it may be assumed that an individual weighs the attraction of alternatives against the costs of realizing those alternatives.

The remainder of this introductory chapter will consist of a brief review of the literature directly related to the topic (for excellent summaries of the development of Recreation Geography, see Mitchell, 1967 and Wolfe, 1964), a statement of the problem to be investigated and a description of the type of data upon which the model will be built and tested. Chapter II will be concerned with the methodology developed during the course of the study, while the third and fourth chapters will set forth the results of the application of the model of three different data sets. The summary, conclusions, and implications of the findings will constitute the fifth and final chapter.

## Review of Relevant Literature

In this summary of literature relevant to the study of attendance at recreation sites, three mathematical models will be discussed as they have been used in the prediction of site attendance. This is not, of course, to say that only these approaches have been used, but is merely to indicate that they have been employed most often. The models
to be discussed are broadly defined as (A) gravitational, (B) systems analytic, and (C) preference surface models. Each will be described briefly and then their common weaknesses will be discussed.
(A) The Gravitational Model

Stewart's (1941) reformulation of the social gravity concept originally advanced by Carey (1858) has had great appeal for students of all types of population flows (see, for example, 0lsson 1965). The model, in its most simple form, may be stated as:
(1) $I_{i j}=\frac{G P_{i} P_{\mathbf{j}}}{D_{i j}{ }^{X}}$ where $I_{i j}=\begin{aligned} & \text { a measure of interaction } \\ & \text { between points (or regions) } \\ & \mathbf{i} \text { and } \mathbf{j} \text { usually taken as } \\ & \text { population movements. }\end{aligned}$
$P_{i}, P_{j}=\begin{gathered}\text { measures of the mass of } \\ \\ \text { and } \\ j\end{gathered}$ their populations, or some measure of their attractiveness),
$\mathrm{D}_{\mathbf{i j}}=\underset{\text { a measure of the distance }}{\text { separating } \mathbf{j} \text { and } \mathrm{j} \text {, and }}$ $G$ and $x=$ constants to be fitted.

The intuitive simplicity and relatively good predictive power of the gravity model have made it one of the most widely used interactance models. Inspection reveals that it is most likely to be accurate when all the masses in the system being studied are identical in relative composition--although not necessarily in absolute size--and when the effects of intervening distance are a clear function of distance alone. The fact that these conditions do not generally hold in the study of human interaction has led to the addition of new terms and the modification of the input parameters. Ellis (1967, p. 2) describes a gravity model in which the constant $G$ "is made variable,
that is, assigned a specific (different) value for each ij pair, or for classes of pairs in order to fit this formula to data on an entire system." Dodd (1955) advanced a similarly flexible formula:
(2) $I_{i j}=\frac{G M_{i} P_{i} M_{j} P_{j}}{D_{i j}}$

> where $G, I, P, D, i$ and $j$ are as in Equation 1 , and $M$ represents a "molecular weight".

Stouffer (1940), on the assumption that the lapse rate observed when the lengths of interactions are plotted against frequency is attributable to the number of intervening opportunities encountered--i.e. the longer the interaction, the more likely one is to have already satisfied the original purpose of the interaction--advanced the intervening opportunities model. This model:
(3) $V_{i j}=\frac{G P_{i} O_{j}}{I_{i j}}$

$$
\begin{aligned}
& \text { where } V_{i j}=\begin{array}{l}
\text { a measure of the inter- } \\
\text { action between } i
\end{array} \\
& P_{i}=a \text { measure of the mass of } i \text {, } \\
& 0_{j}=\text { the number of opportuni- } \\
& \text { ties of satisfying the } \\
& \text { purpose of the interaction } \\
& \text { which exist at } j \text {, } \\
& I_{i j}=\text { the number of opportuni- } \\
& \text { ties of satisfying the } \\
& \text { purpose of the interaction } \\
& \text { which exist at locations } \\
& \text { closer to } \mathbf{i} \text { than point } j \text {. } \\
& \text { and } \\
& \text { G = a constant to be fitted, }
\end{aligned}
$$

is plainly a gravity model with intervening opportunities substituted for distance and site opportunity acting as a surrogate for the destination mass.

Cesario (1971), in a recent paper, has suggested the model:
(4) $t_{i j}=k U_{i} V_{j} f\left(D_{\mathbf{i j}}\right) \exp \left(e_{i j}\right)$ where $t_{i j}=\begin{aligned} & \text { to the number of trips } \\ & \text { from origin } \mathbf{i}\end{aligned}$ to destination j ,
$U_{i}=$ to the emissiveness of $i$,
$V_{j}=$ to the attractiveness
of $j$, $\begin{aligned} & f\left(D_{i j}\right)=\underset{\text { a function of the distance }}{ } \\ & \text { from to } j \text {, }\end{aligned}$
$e_{i j}=a$ random error term, and $\mathrm{k}=\mathrm{a}$ constant to be fitted.

His initial formulation allows the distance function considerably more freedom than other gravitational models do, but the later imposition of the constraint that it is a power of function of the form $f\left(D_{i j}\right)=$ $\mathrm{D}_{\mathrm{ij}}{ }^{-\mathrm{X}}$ with x taking a value greater than 1.0 negates the model's superiority by making it an ordinary gravity model.

Rodgers (1966) has used the probabilistic formulation of the gravity model in a study of campsite attendance. It has the form (5) $P_{j k}=A_{j} / D_{j k}{ }^{x} /\left[\sum_{i} A_{i} / D_{i k}{ }^{x}\right]$

$$
\begin{aligned}
& \text { where } P_{j k}=\begin{array}{l}
\text { the probability of an } \\
\text { individual from town } k
\end{array} \\
& \text { patronizing site } j \text {, } \\
& A_{j}=\underset{\text { the inherent attraction }}{\text { (mass) of site }} \mathbf{j}, \\
& D_{j k}=\begin{array}{l}
\text { a measure of the distance } \\
\text { from } j \text { to } k \text {, and }
\end{array} \\
& x=a \text { constant to be fitted. }
\end{aligned}
$$

This formulation has the advantage that sites may be added to, or deleted from, the general system being studied without assuming an infinite supply of interactants as the other gravity models do, although it still retains the property that there is a finite probability of interaction over an infinitely great distance. Indeed, all the gravitational models are based on the assumption that the level of interaction
between an origin and a destination is a rather simple function of the intervening distance. Excellent discussions of the gravity model technique may be found in Olsson (1965), Cesario (1971), Catton (1966), and Ewing (1970). The latter clearly illustrates how the gravity model, having empirical rather than theoretical underpinnings; disagrees with many intuitive ideas regarding individual spatial behaviour patterns. It is interesting to note that the probabilistic formulation of the gravity model is similar to Luce's (1959) v-scale which is developed from his choice axiom.
(B) The Systems Analytic Model

This approach is typified by the analogue model applied to recreational flows by Ellis and Van Doren (1966). Their technique can be thought of as

> an electrical analogue, where the origins act like current sources. The current (flow of campers) "sees" various paths of differing resistance and distributes itself across the network in a minimum energy fashion, eventually returning to "ground" via the park components. The flow at each park is thus determined by the relative resistances of all parks, all links in the highway network, and the relative strengths of all origin sources. (Ellis and Van Doren, 1966, p. 60)

The suggested model is comprised of two separate components, the highway link section and the park or destination component. The link equation:
(6) $F_{i}=\frac{P_{i}}{R_{i}}$

$$
\text { where } \begin{aligned}
F_{\mathfrak{i}} & =\text { the flow on any link } \mathfrak{i}, \\
P_{\mathfrak{i}} & =\text { "pressure" of people on } \\
R_{\mathfrak{i}} & =\text { resistance of link } \mathfrak{i},
\end{aligned}
$$

allows one to represent an entire transport network as a single system.

The park attraction equation,
(7) $F_{j}=A_{j} P_{j}$

$$
\text { where } \begin{aligned}
& F_{j}=\text { the flow into park } j, \\
& A_{j}=\text { attractiveness of park } j, \\
& \text { and }, \\
& P_{j}=\text { recreation pressure } \\
& \text { measured at the park, }
\end{aligned}
$$

defines attendance as a function of site attraction and social pressure. The combination of these component equations, and the solution of the simultaneous equations derived from them is accomplished by linear graph methods.

## (C) The Preference Model

Rushton $(1967,1969)$ has recently developed the Consumer Space Preference Model to extract information regarding the form and function of the relationship between the attraction of alternative sites and measures of the costs of reaching those sites. The underlying assumption of his model is that an individual in need of a certain good or service will attempt to maximize the benefit he will gain, at the same time attempting to minimize the costs he will incur. The model creates a two dimensional theoretical space in which the axes are a measure of attraction and a measure of distance. This space is then subdivided into a number of "location type" cells (by defining boundaries on the attraction and distance axes) to which alternatives are assigned on the bases of their attraction and their distance from the residences of each of the sample individuals in turn. Analysis then proceeds to the calculation of a paired comparison matrix, each cell ij containing a ratio expressing the proportion of times an alternative of location
type $i$ was patronized when an alternative in location type $j$ was available. Rushton considers the amount by which each of these ratios deviates from one-half to be an ordinal measure of the distance separating the two points $i$ and $j$ on a unidimensional preference scale. The matrix of "distances" is therefore scaled, using one of the non-metric multidimensional scaling techniques (see, for example, Kruskal, 1964), to extract an interval measure of the preference scale value for each of the location types defined. These scale values, when plotted in the original two dimensional space, define a preference surface which is assumed to be independent of the spatial arrangement of the sample individuals and their alternatives. Ewing (1970, p. 122 ff .) has shown that the height of this surface can be relatively well predicted from knowledge of the attraction and distance variables. The prediction of individual spatial behaviour patterns is accomplished by assuming that an individual faced with a choice between alternative sites will choose that which lies highest on the preference surface.

## Discussion of the Models

Two of the models outlined above involve the assumption that the attendance at a service site is a function of the distance separating the site from the origins of the individuals who might patronize that site, and the inherent attractiveness of the site itself.

Once the problem has been reduced to these simplistic terms, it is clear that it may be subdivided into three distinct components. These are:
(A) What is the true nature of site attractivity, and how may it be measured for any given site?
(B) What type of distance is involved; economic, social psychological, or real, and will the effects of varying distance be manifested in an easily definable functional relationship?
(C) How are the attractivity and distance parameters combined in the decision-making process of the individual?

Each of these questions will be addressed below, examples being drawn from the body of recreation research literature.
(A) Site Attractivity

Before this concept can be discussed, the meaning of the term attractivity must be made explicit. It is clear that two recreation sites, identical in all aspects except location, can be said to be equally attractive--in spite of the fact that attendance at the two sites may vary widely. Equal attractivity may also occur if the net results of the various site factors on unidentical sites are equal. In all cases, however, the attractiveness is a function of site factors alone and, preferably, should be derivable by a rigorous method, employing only site characteristics as inputs. Too often, however, the attraction measures derived have been empirical--i.e. those which gave the best solutions for the particular model utilized. In other cases, measures of attraction have been taken to be single site variables felt to be of importance, or combinations of such variables. Both types of measures are discussed below.

In using the conventional gravity model (Equation 1) to predict

Interurban and inter-regional population flows the most common measures of attractivity have been the populations of cities or regions. Other surrogates, such as population weighted by median income or other measures of the social well-being of the city's populace, have also been used. Indeed, there seems to be no reason why almost any measure of social mobility cannot be used, providing, of course, that the interaction being investigated is clearly related to the surrogate of mass being measured. In the modelling of recreation flows, however, there is usually no clear indication of a proxy for the "mass" of a recreation site. The use of site area is ruled out due to differing quality and quantity of facilities, while the utilization of visitation figures is invalid because they are determined in a large part by the distance between the site and the recreators' origins. Also, in many cases, the fact that a site is heavily patronized may make it less attractive to potential users (Cahn, 1968). Sometimes, as in the case of Wennergren's (1970) investigation of recreational boating, an acceptable surrogate such as lake area may be definable. Such cases are rare however. Subsequent discussion will center on attempts to define suitable indices of site attraction.

The conventional gravity model may be solved for the values of its constants by regression techniques (see, for example, Rodgers, 1966), and then indices of attraction fitted by further regression. The main problem with such an approach, however, is that the indices so derived are not completely independent of spatial effects. Rodgers (1966), for example, in utilizing the probabilistic formulation of the gravity model (Equation 5), assumed the denominator to be a constant
for each park, and then derived a separate attraction index for each origin-destination pair. He then established the final attraction index for each park by considering all indices derived for that park. This approach would yield correct indices only if the effects of distance were uniform in all cases. If this condition were fulfilled, the pairwise attraction indices would be equal from city to city. That is to say:
(8) $A_{i j}=A_{k j}$

$$
\begin{aligned}
& \text { for all values of } k \text { from } 1 \text { to } n . \\
& \text { where } A_{i j}= \text { the attractiveness of } \\
& \text { park } j \text { as seen by resi- } \\
& \text { dents of origin } i, \text { etc. }
\end{aligned}
$$

If such is not the case, the variation in the derived attraction indices will be attributable to the collection of errors resulting from incorrect estimation of the distance exponent.

Catton (1966) derived a set of "opportunity coefficients" for use with the Stouffer model (Equation 3) by the use of an iterative technique which considered only intervening opportunities and site attendance. He went on to derive a second set of indices by applying the psychological scaling technique of equal intervals to data pertaining to the rankings of site by a group of National Park Officials. This approach would yield a location independent measure of site attraction if the subjects ranking the various sites could disassociate themselves from considerations of accessibility. The indices derived from such a study would, however, represent estimates of the attraction of an area from an administrative rather than a user's point of view, and would not be correct unless the general public felt the same way about the parks as the Park Officials did.

Ellis and Van Doren (1966), Wolfe (1966), and Mitchell (1967) have established indices of attraction which are free of distance effects. Their approaches, however, have been based on the researcher's assignation of specific values to various site factors, and a more or less rigid statement of the way in which each factor contributes to the compage represented by the site attraction index. Ellis (1967, p. 8) states the equation by which he combines the influence of the various site factors as:
(9) $A_{d}=C_{d} S_{d} \frac{\left(W_{d}-.5 Q_{d}\right)}{1.5}$

$$
\text { where } \begin{aligned}
& C_{d}= \text { the relative capacity of } \\
& \text { the park } d, \text { chosen either } \\
& \text { as } .2, .6,1.0,2.0, \text { or } \\
& 3.0, \\
& S_{d}= \text { the estimated effect of } \\
& \text { any special factor, a value } \\
& \text { chosen as either } .75 \text { or } \\
& 1.25, \text { if present, } \\
& W_{d}= \text { the relative quality of } \\
& \text { water related resources of } \\
& \text { the park, chosen as either } \\
& .2, .6,1.0,1.5, \text { or } \\
& 2.0, \text { and }
\end{aligned}
$$

although he later suggests that the indices might be estimated more effectively by a factor analysis of quality ratings of the input variables felt to be of importance. The facilities index of Mitchell (1967), in his attempt to predict the number of visitors at urban playgrounds, assigns a series of nominal or ordinal numbers to various recreation facilities. The choice of "scores" assigned infers, for example, that nine and eighteen-hole golf courses are of equal importance, and that access to a swimming pool is as beneficial as access
to a beach of less than five acres. The form of his combining equation is linear, the weighting (B) coefficients being determined by leastsquares techniques. Schafer and Moeller (1971) discuss several other attempts to quantify attraction parameters.

A somewhat more theoretical approach to the estimation of attraction indices has been advanced by Cheung (1970). He suggests that the attractivity of a site may be defined as:

$$
(10) T_{j}=\sum_{e} a_{e} \sum_{m} r_{m} a_{m}
$$

$$
\text { where } \begin{aligned}
T_{j}= & \text { attractivity of site } j, \\
a_{e}= & \text { relative popularity of } \\
& \text { activity } e, \\
r_{m}= & \text { relative importance of } \\
& \text { facility m, and } \\
\mathbf{a}_{m}= & \text { rank numerical value of } \\
& \text { facility } m, \text { according to } \\
& i \text { its quantity or quality. }
\end{aligned}
$$

Cheung's measure $a_{e}$ was defined as proportional to the percentage of the subjects of a national recreation survey who participated in activity $e$, while $r_{m}$ was calculated to be a linear transformation of the rank correlation coefficient between attendance at all recreation sites in his system and the amount of each facility (e.g. the number of picnic tables) at those sites. The rank of the numerical value of the facility $\left(q_{m}\right)$ was assigned for each facility of each site on the basis of that site's position in a ranking of all sites based on the quality or amount of the particular facility under consideration.

Three major problems in Cheung's model may be identified.
First, the value $a_{e}$ is based on a national survey which may not reflect regional variations in participation levels in various recreation activities. More seriously, the participation proportions given,
even on a regional basis, would not be adjusted for regional disparities in the opportunity to partake in any given recreation activity. Second, the partial basing of $r_{m}$, the measure of the importance of facility $m$, on site attendance once more involves the use of figures which are at least partially determined by location of the site and alternative recreation sites. In Cheung's case, this objection is particularly strong because $84 \%$ of the variance in visitation is related to distance alone (Cheung, 1970, p. 15). Third, the technique employed in determining the values of $q_{m}$, although probably better than utilized by Mitchell, is questionable in that the transformation from the data to its numeric representation is defined very arbitrarily. In the case of picnic tables, for example, Cheung ranked all twelve sites in order of the number of tables observed, and then assigned values in such a way that the site with the most tables received a $q_{\text {table }}$ value of 12 , that with the second largest number 11 , etc. In spite of the shortcomings of Cheung's study, it is valuable as an initial approach to a theoretical, rather than empirical, estimation of site attraction indices. His use of a measure of the relative popularity of a recreation activity, although possibly inaccurate in this case, should prove to be of great assistance to fellow researchers.

Of the methods of determining indices of attraction which have been discussed, those which have involved considerations of attendance have been dismissed as being in violation of the statement that such indices must be free of the effects of location. The methods of Ellis and Mitchell satisfy this constraint by utilizing measures of site
characteristics alone. The problem with their approaches, however, is that even if the researcher identifies the major attraction inputs correctly, and is able to measure them accurately, he has no information upon which to base the form of the equation necessary to derive a single index of attraction. Additionally, the implicit assumption that the researcher and the recreators perceive the same set of site factors as being the determinants of site attraction is open to question.

Catton's "equal interval" indices, although based on the rankings of administrators rather than recreators, avoided these difficulties. He, unlike Rodgers and Cheung, made no attempt to relate the derived indices to site factors, a step which would enable him to calculate indices for sites for which he had no rankings.

Robertson and Ross (1969), in an investigation of the recreation potential of shorelines, proposed a model for establishing an index of suitability for various shoreline types. Their model, utilizing information concerning beach quality, view, and accessibility, indicated that site information could be combined in such a way as to provide a reasonable estimate of site character. They did not, however, relate their results to participation.

Recreation researchers have commonly assumed that a single interval attraction scale may be formed by mathematically combining the effects of several site components or attributes. Shepard (1964, p. 264 ff . appears to question this assumption in reporting that individuals appear to have great difficulty when attempting such combinations mentally (even when relatively simple combinatorial rules
are specified), seeming only to be able to form crude unidimensional scales. This implies that such scales may have little more than ordinal significance.
(B) Considerations of Distance

It is evident that the "geographic" distance which separates the individual from a site where a desired experience or good is offered is of little import except in the way in which it is perceived by the individual. Various transformatory schemes have been applied to geographic distance in order to make it correspond more closely to observed patterns of behaviour (Abler, et. al., 1971). Economists have commonly employed "economic" transformations, while sociologists have used "social" and "intervening opportunity" transformations. More recently, "psychological" transformations of geographic distance have been investigated by Gould (1967), Lee (1970), Lowrey (1970), and Lycan (1969).

These attempts to transform geographic distance all aim at the systematic explanation of the decline in patronization due to increased distance. The models most commonly used in estimating site patronage include a function of distance as an inhibiting factor. In the majority of cases, the function is expressed as:
(11) $f\left(D_{i j}\right)=D_{i j}{ }^{x}$

$$
\text { where } \begin{aligned}
D_{i j}= & \text { the geographic, economic, } \\
& \text { or horologic distance } \\
& \text { between origin } i \text { and site } \\
& j, \text { and } \\
x= & \text { a constant to be deter- } \\
& \text { mined empirically. }
\end{aligned}
$$

More recently, it has been noted that a more realistic function may be
found if the magnitude of $x$ is allowed to be a function of distance itself. Ross (1969), while studying vacation travel behaviour, solved the gravity model for the distance exponent for each of 58 city pairs. He noted that when the exponents were plotted against distance, the data separated into three distinct distance groups. Linear regression lines calculated for the three groupings showed high degrees of correlation and distinctive slopes and intercepts. Cheung (1970) and Rodgers (1966) have reported similar results.

Wolfe (1972), concerned with the fact that the gravity model overpredicts site visitation when distance is small and underpredicts it when distance is large, has recently advanced the inertia model. This model is formulated in such a way that the response to an increase in distance is a variable function of distance itself. His model (Equation 12) has yet to be tested thoroughly, but it would appear to be a new approach to the study of spatial interaction.
(12)

$$
\begin{aligned}
& v_{i j}=k \frac{P_{i}^{a} C_{j}^{b}}{D_{i j}{ }^{x}} D_{i j}\left[\frac{\operatorname{Ln}\left(\frac{D_{i j}}{m}\right)}{n}\right] \\
& \text { where } V_{i j}=\text { the number of } i n- \\
& \text { dividuals from origin } \\
& \text { i patronizing site } j \text {, } \\
& \mathbf{P}_{\mathbf{i}}=\begin{array}{l}
\text { the population of } \\
\text { origin } \mathbf{i},
\end{array} \\
& C_{j}=\text { the capacity of site } j \text {, } \\
& D_{i j}=\begin{array}{l}
\text { the distance from } i \\
\text { to } j \text {, and }
\end{array} \\
& \begin{aligned}
a, b, x, n, m= & \text { constants to be } \\
& \text { estimated. }
\end{aligned}
\end{aligned}
$$

It is interesting to note Wolfe's substitution of site capacity for the site attraction variable. Beaman (1972) has more recently suggested an impedance of distance function to express the probability of an
individual who has already travelled $x$ time distance units not travelling another unit. He suggests that the probability of this event is as shown in Equation 13. The extension of this reasoning shows that
(13) $\mathrm{g}(\mathrm{x})=\mathrm{ka} \mathrm{a}^{-\mathrm{bx} \quad \text { where } \mathrm{g}(\mathrm{x})=} \begin{aligned} & \mathrm{the} \text { probability of an } \\ & \text { individual who has al- } \\ & \text { ready travelled } x \text { time } \\ & \text { distance units stopping } \\ & \text { during another unit, } \\ & \text { a and } b= \text { constants to be esti- } \\ & \text { mated, and } \\ & k= \int_{0}^{\infty} 1 / \sum g(x),\end{aligned}$
the likelihood of a visitor from origin $i$ reaching site $j$ is therefore the product of the probabilities of his not stopping during any of the time distance units he must travel to reach site $j$. Denoting this likelihood as $g^{\prime}{ }_{i j}$, the number of visitors from origin $\mathfrak{i}$ to site $j$ can be seen to be
(14) $V_{i j}=g^{\prime}{ }_{i j} P_{i}$

$$
\text { where } \left.\begin{array}{rl}
V_{i j}= & \begin{array}{l}
\text { the number of individuals } \\
\\
\\
\text { sitem origin } i \text { patronizing }
\end{array} \\
g_{i j}^{\prime}= & \text { the likelihood of a } \\
& \text { visitor from origin } i \\
& \text { reaching site } j, \text { and }
\end{array}\right\}
$$

Note that Beaman includes no site attraction parameters at all.
It would seem that both Wolfe's and Beaman's models are more suited to the study of the way in which cities emit recreators than the way in which alternative sites attract patrons. Their major contributions are to the study of human response to distance, rather than the study of site visitation.
(C) Attraction-Distance Relationships

The form of the relationship between a function of distance and a measure of site attraction has generally been assumed to be such that distance inhibits interaction while attraction stimulates it. In the study of recreation flows, this assumption is generally true, but in certain circumstances it may be erroneous. Distance, for example, acts as an impediment when the benefits of a particular recreation trip are to be derived solely from activities carried out at the site. On trips classified as "driving for pleasure", on the other hand, the benefits derived are attributable to travel alone.

When considering attendance at a particular site, it must be kept in mind that although an "average" measure may be derived for any specific site, there are many factors which will influence it on a day-to-day basis. One of the most obvious of these is that of site attendance; the more people who patronize a given site, the less opportunity there is for others to find satisfactory recreation experience there. This is, of course, not true in all cases (see Cahn, 1968), but exceptions are generally uncommon (see Catton, 1966). Other causes of variation such as weather patterns and seasonal changes in foliage and user activities should not be overlooked.

In general, then, it may be said that a model having a reliable estimate of site attraction and a measure of intervening distance in an inverse relationship would be suitable for modelling recreation flows. Although variants of the gravity model will doubtless continue to be utilized because of their computational simplicity, it is clear from the foregoing discussion that even when the attraction and distance
parameters can be precisely specified it will still be subject to the constraints introduced by the spatial arrangements of the population and their alternative service sites. To date, the only approach to overcome this problem is the Space Preference technique. Recent use of this approach by Girt (1972) has supported the contention that a high level of individual choice prediction can be achieved in widely varying spatial situations.

## Résume

The above discussion of the literature clearly indicates a need for the development of a methodology for the establishing of measures which express the inherent attractiveness of service-offering sites. It is clear that such measures, hereafter referred to as indices of attraction, must be functions of the characteristics of the sites themselves, and be free of locational bias. It is reasonable to expect that indices of attraction which are attributable to site factors alone would be suitable for use with any of the above-mentioned participation prediction models, although the fact that the attraction scale is likely to have only ordinal properties will necessitate a different interpretation of their results.

It is also evident that the method of analysis designed to yield such indices must be such that it does not involve rigid assumptions about the effects of distance, and site characteristics but yet allows the consideration of the effects of alternative opportunities. The data necessary for determining attraction indices which satisfies these constraints consists of a set of observations of the form:

Individual $\mathfrak{i}$ residing at $X_{i} Y_{\mathfrak{i}}$ most often patronizes site $j$ located at $X_{j} Y_{j}$.

If reliable estimates of attraction indices are to be obtained from the analysis, it is imperative that data must be available on the individual's most of ten patronized site, rather than just the notation that an individual residing at $X_{i} Y_{i}$ patronized site $j$. A discussion of the sample subjects and data collection procedure utilized during the development and testing of the model will be deferred until the methodology has been made explicit. The second chapter of this study will present the methodology, while subsequent chapters will be concerned with the application of the technique developed.

## Chapter II -- THE ATTRACTION INDEX METHODOLOGY

In the previous chapter, it was established that a need exists for site attraction indices which are derived through the consideration of the spatial behaviour of individuals patronizing service sites, rather than through analytical techniques which fit the data to rigidly formulated models. This chapter of the thesis sets out a methodology by which such indices may be defined. Before beginning, however, it is fitting that three major assumptions be made explicit.

Main Assumptions of the Proposed Model
Assumption 1: The benefits of a trip made by any individual to any site offering the service being sought are attributable to the activity which takes place at the site, and are in no way related to the characteristics of the trip itself.

This assumption will have varying degrees of validity depending upon the type of service being sought. In the case of grocery shopping, it probably has high validity, while in the case of vacation camping it may be less true.

Assumption 2: All individuals given a choice between two alternative sites will rank these sites in the same order.

Two potential problems may be discussed here. First, if the stimuli are not completely discriminable (i.e. if one site is not obviously
better than the other), this assumption will be incorrect. In the majority of cases, however, it is expected that the degree of discrimination between alternative opportunities will be high, if not perfect. Second, the fact that individuals of different socio-economic groups may have characteristic responses to certain stimuli sets will introduce a noise factor to the model, although Ewing (1970) has found few systematic differences between socio-economic groupings.

Assumption 3: All individuals will have knowledge of all alternative opportunities situated closer to their residences than the site they prefer to patronize.

The assumption of complete knowledge is clearly naive in certain situations, but there are great operational problems which must be overcome before this assumption can be discarded. The degree to which it will bias the results of the study is indeterminate. It is believed that the level of bias will be a negative function of sample size and that the comparatively large numbers of individuals studied will minimize its effects. This is illustrated by the data presented in Table 2.1, where it is shown that even when the probability of an individual being aware of the existence of a specific site is relatively low, the number of subjects required to establish knowledge of the site at a reasonable degree of confidence is not overly large. The amount of bias introduced by individuals not satisfying this assumption should therefore be relatively small. The implications of this assumption will be discussed more fully after a short introduction to the basic logic of the proposed model.

TABLE 2.1 -- NUMBER OF SUBJECTS REQUIRED TO ESTABLISH SITE KNOWLEDGE

| Probability of a <br> site being known | .8 | .7 | .6 | .5 | .4 | .3 | .2 | .1 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Number of subjects <br> required to estab- <br> lish knowledge at. <br> the $95 \%$ level | 2 | 3 | 4 | 5 | 6 | 9 | 14 | 29 |

## The Numeric Method

The proposed model of human behaviour conceives of two com-ponents--attractiveness, $A_{j}$, a measure of the inherent attractiveness of site $j$, and distance, $D_{i j}$, a measure of the difficulty of travelling from the residence of visitor $i$ to site $j$. It is assumed that $D_{i j}$ is an impediment to travel, and that the effects of increasing distance are such that the degree of impedance always increases as distance increases--although the function relating impedance and distance need not necessarily be precisely defined. The model assumes only that it is strictly monotonic, i.e. a distance of $2 X$ is more difficult to travel than a distance of $X$. For every individual $i$, there exists a preference function $P$ which the individual attempts to maximize. No assumptions need be made about the exact nature of the function $P$. It may, for example, be. a simple addition of the two components $A_{j}$ and $D_{i j}$, or $i t$ may be of the form $P_{i j}=A_{j} D_{i j}$. The function may be graphically portrayed as an indifference surface on which the individual is indifferent between two alternative sites yielding equal values of $P$. It is assumed that an individual will select that alternative which is highest on the indifference surface, thus maximizing P. Under this assumption, working momentarily in one dimensional space, let us
consider an individual residing at point $\mathfrak{i}$ and two alternative sites located at $j$ and $k$ as shown in Figure 2.1. The individual's patronization of site $k$ leads us to the conclusion that, in his mind, the attractiveness of site $k$ is greater than that of site $j$--because he was willing to travel further, thus encountering more difficulty, to get there.

The extension of this reasoning to two dimensional space involves the assumption that the individual perceives a given distance to be of approximately the same magnitude regardless of the direction in which he must travel. Given this assumption, it may be stated that he implicitly judges the site he selects to be more attractive than any alternative site which is closer to his origin (i.e. any intervening opportunity) than the selected site (Fig. 2.2). No judgements can be made in regard to the relative attraction of sites which are beyond that which the individual selected.

The judgements which can be inferred in this fashion may be thought of as paired comparisons in which the individual has judged $A_{4}>A_{3}, A_{4}>A_{2}$, and $A_{4}>A_{1}$. No inferences can be made regarding the relative attractiveness of sites 1,2 , and 3 , except as they relate to site 4. These inferred inequalities form the only direct access to numerical values of $A$, although there is, of course, no means of measuring the difference between two A values. Beaman (1971) has suggested that this difference may be related to the extra "cost" the individual is prepared to pay to travel to his preferred site, but as the function relating extra cost to attractivity is not known, this approach will not be used here (although the effect of various functions will be

## FIGURE 2.1 -- ONE DIMENSIONAL SPACE

Residence Alternative Sites

## Distance

FIGURE 2.2 -- TWO DIMENSIONAL SPACE

$$
12
$$

30 4

The individual's residence is located at point 0 , while the points 1-4 represent alternative sites. If the individual chooses to viait site 4 it can be assumed that he judges that site to be more attractive than any site closer to his residence.
examined in Chapter IV).
The third main assumption, stated above, was concerned with the individual's knowledge of sites so situated as to be considered to be intervening opportunities. Referring to Figure 2.2, should the individual not be aware of the existence of site 3, the inferred judgement $A_{4}>A_{3}$ would be erroneous. Whether this judgement wili bias the results of the analysis or not depends upon the judgements between sites 3 and 4 which are inferred from the spatial choices of the other individuals in the sample. In all cases of judgements inferred between a known and an unknown site, the latter is biased downwards on the attraction scale. In view of the expectation that the likelihood of a site being known should be in very strong agreement with its attractiveness, this does not appear to be unreasonable.

Tabulating the Comparison Matrix C
If, for a number of subjects, the number of times that any site $i$ can be inferred to be more attractive than any other site $j$ is recorded, a site by site comparison matrix $C$ (Table 2.2) may be formed.

TABLE 2.2 -- COMPARISON MATRIX: EXAMPLE 1

| Site | 1 | 2 | 3 | 4 | 5 | 6 |
| :---: | :---: | :---: | :---: | :---: | :---: | ---: |
| 1 | - | 100 | 90 | 100 | 0 | 0 |
| 2 | 0 | - | 5 | 30 | 100 | 100 |
| 3 | 10 | 95 | - | 100 | 100 | 100 |
| 4 | 0 | 70 | 0 | - | 0 | 0 |
| 5 | 100 | 0 | 0 | 100 | - | 40 |
| 6 | 100 | 0 | 0 | 100 | 60 | - |

In this matrix, the ijth entry is the number of times a judgement $A_{1}>A_{j}$ could be inferred. In order to tabulate the matrix, one must take each sample individual in turn, calculate the distance from his residence to each of the alternative sites, and then, denoting the site he visited as $i$, increment $C_{i j}$ for every site $j$ which is closer to the individual's residence than site 1.

Calculation of the Proportion Matrix P*
From the comparison matrix $C$, the proportions matrix $P^{*}$ (Table 2.3) which shows the proportion of times that any site $i$ was judged to have a higher attraction than another site j may be calculated. An entry in the $P^{*}$ matrix is defined as:
(2.1) ${ }^{P *}{ }_{i j}=\frac{C_{i j}}{C_{i j}+C_{j i}}$

$$
\text { where } \left.\begin{array}{rl}
P_{i j}= & \text { the proportion of times } \\
\text { site } i \text { was chosen over } \\
\text { site } j,
\end{array}\right\}
$$

TABLE 2.3 -- PROPORTIONS MATRIX: EXAMPLE 1

| Site | 1 | 2 | 3 | 4 | 5 | 6 | Attraction | Confusion |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | - | 1.00 | .90 | 1.00 | .00 | .00 | .580 | .200 |
| 2 | .00 | - | .05 | 30 | 1.00 | 1.00 | .470 | .400 |
| 3 | .10 | .95 | - | 1.00 | 1.00 | 1.00 | .801 | .400 |
| 4 | .00 | .70 | .00 | - | .00 | .00 | .140 | .200 |
| 5 | 1.00 | .00 | .00 | 1.00 | - | .40 | .480 | .200 |
| 6 | 1.00 | .00 | .00 | 1.00 | .60 | - | .520 | .200 |

Because situations may arise in which site $\mathfrak{i}$ and site $j$ were never compared, the fact that the comparison is missing must be noted. Throughout this work, the value -1.00 will be used in such situations.

In an ideal situation, when all subjects comparing $A_{i}$ with $A_{j}$ have similar perceptions as to which is most favourable, $\mathrm{P}_{\mathrm{ij}}$ will take vaiues of 1 and 0 . However, in the real world, there will be some cases, especially when the sites being compared are very similar in attraction, when the issue will not be clear, and the value of $P_{i j}$ will be intermediate between 1 and 0.

Scaling the P* Matrix
Psychologists, in attempting to derive measurement scales (see Bock and Jones, 1968) from similar paired comparison matrices, have developed a number of approaches, many of which are based on the "Law of Comparative Judgement" proposed by Thurstone (1927). Thurstone's solutions require that the individual entries in a row $\mathbf{i}$ of the proportions matrix $P$ be considered as estimates of the true scale distance separating $A_{i}$ from $A_{j}$, for all $\mathbf{j} \neq \mathrm{i}$. In his Case $V$ solution, these estimates are transformed to normal $z$ equivalents (see Hays, 1967) by referring to the normal curve. Finally, the $Z$ values are averaged to yield the "true" scale value of $A_{i}$. A variety of other scaling techniques which treat the $P^{*}{ }_{i j}$ entries as non-metric measures of scale difference (see, for example, Kruskal, 1964) that is, having ordinal rather than interval properties, have been proposed. The fact that many of the sites being compared will have greatly different inherent attractions will result in a large proportion of the entries in the $P^{*}$
matrix having values of either 1 or 0 . Scale values for such completely discriminable stimuli cannot be found directly by using either the Thurstone or Kruskal methodologies. As an alternative approach which possesses neither of these problems, any value of $P_{i j}$ greater than one-half may be regarded as indicating a majority judgement preferring $i$, and any value less than one-half a majority preference for $j$. A scale which is as consistent as possible with these majority judgements may then be constructed--even though the scale derived can only be ordinal since no measures of interpoint distance are involved. To arrive at this ordinal scale, the rows of the $P$ matrix are summed and each divided by the number of valid entries in that row. (A valid entry is defined as one not equal to -1.00 .) The result is denoted as the index of attraction:
(2.2) $A_{i}=\sum_{j=1}^{N} e_{i j} P^{*}{ }_{i j} / \sum_{j=1}^{N} e_{i j}$

$$
\text { where } \begin{aligned}
A_{i}= & \text { the attraction index of } \\
& \text { site } \mathfrak{i}, \\
N= & \text { the total number of sites, } \\
P *{ }_{i j}= & \text { the proportion of times } \\
& \text { site } \mathfrak{i} \text { was chosen over } \\
& \text { site } j, \text { and } \\
e_{i j}= & 1 \text { if } P_{i j} \neq-1,0 \text { otherwise. }
\end{aligned}
$$

A measure which expresses the degree to which the judgements concerning any site were unanimous, the Index of Confusion ( $\mathrm{MU}_{\mathbf{i}}$ ) may be calculated at this time (Equation 2.3). It is defined as the proportion of valid entries in each row of the matrix $P^{*}$ which represent clear-cut decisions, and is calculated by dividing the number of entries in row $i$ which are either 1.00 or 0.00 by the number of valid entries in that row and subtracting the result from one. It is important to note that
the MU indices are entirely arbitrary and have, at best, only ordinal significance.
(2.3) $M U_{i}=1-\sum_{j=1}^{N} f_{i j} / \sum_{j=1}^{N} e_{i j}$

$$
\text { where } \begin{aligned}
M U_{i} & =\begin{array}{l}
\text { the confusion index of } \\
\text { site } i,
\end{array} \\
e_{i j} & =1 \text { if } P^{\star}{ }_{i j} \neq-1,0 \text { other- } \\
f_{i j} & =\begin{array}{l}
1 \text { if } P^{*}{ }_{i j}=1 \text { or } 0, \\
0 \text { otherwise. }
\end{array}
\end{aligned}
$$

## Testing the "Fit" of the Scale

The degree to which a scale expresses the data on which it is based can be utilized to assess its usefulness. In metric problems, goodness of fit is often expressed as the proportion of the variance in the original data which is explained by the scale defined. In Kruskal's non-metric methods, a comparative figure is defined as the "stress" of the solution, a measure of the discrepancy between the interpoint distances of the final scale and an arbitrary best fitting monotone transformation of the original dissimilarities (Kruskal, 1964). Because the entries in the $P^{*}$ matrix cannot be considered as measures of similarity or dissimilarity, neither of these approaches is suitable for assessing the fit of the scale defined from it. Instead, the transitivity measures developed by Kendall (1962) may be used to estimate the transitivity of the $P^{*}$ matrix, and an inspection of each of the inferred judgements upon which the $C$ matrix was based will allow statements regarding the overall "fit" of the scale to be made. Each of these procedures is discussed below.

Consider three sites--i, $j$, and $k$. If $A_{i}$ is greater than $A_{j}$, and $A_{j}$ is greater than $A_{k}$. then $A_{i}$ must clearly be greater than $A_{k}$.

Such a situation is termed a transitive or non-circular triad. The degree to which the $n(n-1)(n-2) / 6$ triads contained in a matrix of order $n$ are transitive provides a measure of the consistency of the judgements upon which the matrix is based. Three different types of transitivity--strong, moderate, and weak--have been suggested (Coombs et. al., 1970). A strongly transitive triad is defined as one in which, if $P{ }_{i j}>.5$ and $P{ }_{j k}>.5, P{ }_{i k} \geq \operatorname{Max}\left(P{ }_{i j}{ }_{i j} P^{\mathrm{F}}{ }_{j k}\right)$. Moderate transitivity holds if $P^{*}{ }_{i k} \geq \operatorname{Min}\left(P^{*}{ }_{i j}, P{ }_{j k}\right)$, and weak transitivity is satisfied if ${ }^{P}{ }_{i k}>.5$. It is clear that a strongly transitive triad also satisfies the moderate and weak conditions. In order to assess the transitivity of a triad under the strong and moderate interpretations, all three elements of the triad must be present. Under the weak interpretation, however, only the ijth and ikth elements are necessary. The possibility that the $P^{*}$ matrix will be incomplete thus suggests the use of the weak transitivity definition.

If the $P^{*}$ matrix is incomplete, as it often is, the standard formulae (see Kendall, 1962, p. 146) for determining the number of circular triads in the matrix cannot be calculated directly. Instead, all triads must be examined separately. Four different outcomes are possible when inspecting an incomplete matrix in this manner. These are:
(1) a triad may be non-circular,
(2) a triad may be circular or intransitive,
(3) a triad may be incomplete but of the form ${ }^{P}{ }_{i j}>.50$ and ${ }^{*}{ }_{i k}>.50$. If this is the case, the triad must be transitive regardless of the missing value $P^{*}{ }_{j k}$,
(4) a triad may be unknown because two elements are absent. Measures of the consistency of incomplete matrices cannot be computed analytically because the location of missing entries will partly determine the number of triads about which decisions can be made. Kendall's Coefficient of Consistency is defined as: (2.3) $K=1.0-\frac{d}{d_{\max }}$

$$
\begin{aligned}
\text { where } d= & \text { the number of circular } \\
& \text { triads observed, } \\
d_{\text {max }}= & \text { maximum possible number } \\
& \text { of circular triads. }
\end{aligned}
$$

Under the assumption that the maximum degree of inconsistency should be observed when judgements between any site $i$ and any other site $j$ are made at random, the following procedure has been adopted. Create a dummy $P^{*}$ matrix by replacing all the valid entries below the diagonal with a rectangularly distributed random number between (and including) 1.00 and 0.00 , and those above the diagonal with the complement. That is to say:

$$
\begin{aligned}
P_{i j}^{\prime}=X, P_{j i}^{\prime}=1.00-X \quad \text { where } P^{\prime}= & \text { the ijth entry in the } \\
& \text { dummy proportions matrix } P^{\prime}, \\
X= & \text { a uniformly distributed } \\
& \text { random number between } \\
& 1.00 \text { and } 0.000 .
\end{aligned}
$$

The number of intransitive triads in this dummy matrix is counted and stored. The operation is repeated several times, and a running sum and sum of squares of the number of circular triads is kept. After a number of iterations, the mean ( $M$ ) and standard deviation ( $S$ ) of the number of circular triads is calculated. The mean number of circular triads generated from the $P^{\prime}$ matrix is used as an estimate of the maximum number which could occur. It is easily seen that in the case of a small matrix this will be conservative. Following Kendall, $K$ is then
defined as:

$$
\text { (2.4) } K=1-d / M \quad \begin{aligned}
\text { where } d= & \text { the number of circular } \\
& \text { triads observed, and } \\
M= & \text { the average number of } \\
& \text { circular triads from the } \\
& \text { simulation. }
\end{aligned}
$$

The statistical significance of $K$ is in this case dependent upon both d and M. Kendall (1962) has shown that the chi square distribution, to which the distribution of $d$ tends as $n$ increases, may be employed for assessing the probability of finding as few as d circular triads for complete matrices of greater than 7th order. Chi square is defined as:
(2.5) $\left.x^{2}=8 / n-4\left[\frac{1}{4}(n(n-1)(n-2) / 6)-d+\frac{1}{2}\right)\right]+v$

$$
\text { where } \begin{aligned}
v & =n(n-1)(n-2) /(n-4)^{2} \\
& =\text { number of degrees of freedom. }
\end{aligned}
$$

Due to the fact that Kendall's definition of $\mathrm{X}^{2}$ is formulated in terms of complete matrices, we may expect that the $\mathrm{X}^{2}$ values derived for assessing the likelihood of given numbers of circular triads in incomplete matrices to be artificially high because there are fewer determinable triads. This overestimation will be most severe when the matrices are least complete; however, it is believed that it will have only a minor effect on the analyses to be conducted in this study. The determination of the magnitude of the overestimation being beyond the scope of this study, the $x^{2}$ values derived will be interpreted cautiously.

Kolmogorov-Smirnov statistics (Blalock, 1960) calculated for trial data sets revealed that the numbers of intransitive triads did
not deviate significantly from a normal distribution (Table 2.4). We may therefore calculate conservative estimates of the approximate upper and lower limits of $K$ for the value of $d$ observed. The necessary equations are:
$(2.6) \mathrm{K}_{1}=1-\frac{\mathrm{d}}{\mathrm{M}-3 \mathrm{~S}}$
and
(2.7) $K_{u}=1-\frac{d}{M+3 S}$
where $K_{u}$ and $K_{1}$ are the approximate upper and lower limits of $K$ at the $99 \%$ confidence level.

The coefficient of consistency $K$ provides a measure of the degree to which the matrix can be expressed as a unidimensional scale. As $K$ deviates from 1.0, the amount of disagreement between the scale and the matrix of proportions increases.

TABLE 2.4 -- CONSISTENCY PARAMETERS: EXAMPLE 1
Trial 1 Trial 2
Matrix order 43
$\begin{array}{ll}\text { Number of missing entries } & 278\end{array} 40$
Number of simulations 2030
Mean number of circular triads $\quad 2170.35 \quad 179.53$
$\begin{array}{lll}\text { Standard Deviation } 37.54 & 10.23\end{array}$
Maximum Deviation from the normal distribution when
7.5\%
7.0\%
plotted on probability paper
Kolmogorov-Smirnov $D_{\text {max }}$ at $\quad 30.4 \% \quad 24.8 \% ~$
$95 \%$ level

A second measure of fit may be defined as the proportion of the judgements $A_{i}>A_{j}$, inferred during tabulation of the $C$ matrix, which are in agreement with the ordinal attraction scale defined. This measure is computed by reinspecting the inferred comparisons and counting those for which the inference was consistent with the final scale. The ratio of "correct" inferences to the total number of inferences yields an index analogous to the coefficient of determination. This index has been designated as the Coefficient of Agreement, eta.

Visual Interpretation of the P* Matrix
Although the proportions matrix may be interpreted visually, it has been found helpful to reorder the rows and columns of the matrix in such a way as to place the highest scoring sites at the top and left of the matrix. Symbolic characters have then been assigned to represent different ranges of values in the permitted matrix (Table 2.5), missing comparisons being assigned a blank. Ideally, if the matrix were perfectly transitive, and all entries unanimous, all characters to the right and above the diagonal would be l's and all below and to the left O's. Such a situation would seldom arise. Several useful observations can be made from an inspection of the symbolic matrix.

| Site | 3 | 1 | 6 | 5 | 2 | 4 |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 3 |  | - | 1 | 1 | + | 1 | LEG |  |
| 1 | + |  | 0 | 0 | 1 | 1 | 0 | 0 |
| 6 | 0 | 1 |  | $=$ | 0 | 1 |  | .0-. 4 |
| 5 | 0 | 1 | $=$ |  | 0 | 1 |  | .4-.6 |
| 2 | - | 0 | 1 | 1 |  | - |  | .6-1 |
| 4 | 0 | 0 | 0 | 0 | + |  | 1. |  |

The inconsistent judgements can be identified as the 1's and +'s below the diagonal, or as 0 's and -'s above it, whilst the degree of confusion $M U_{j}$ for any site may be readily observed. Although the display of the $P^{*}$ matrix in this fashion adds no new information, it is useful in pinpointing sites which do not appear to fit into the general pattern of the system being studied.

## Résumé

A numerical method which defines an ordinal scale of site attraction indices has been described above. The data upon which the scale is based consists of a number of inferred judgements of the type $A_{i}>A_{j}$. A single judgement of this type is inferred each time that an individual is observed to patronize alternative site $\mathfrak{i}$ when alternative site $j$ lies closer to his residence, thus giving rise to the inference that site $\mathfrak{i}$ is more attractive than site $j$. A comparison matrix $C$ is tabulated, each cell $C_{i j}$ being incremented each time that an $A_{i}>A_{j}$ judgement can be inferred. After the inferred judgements of all sample subjects have been tabulated in this fashion, the proportions matrix $P^{*}$ is calculated, each entry $P^{*}{ }_{i j}$ being defined as the proportion of the individuals who compared sites $\mathbf{i}$ and $\mathbf{j}$ who judged site $\mathfrak{i}$ to be more attractive than site $j$. The scale value of the $i$ th site is then defined as the average value of the valid entries in the ith row of the matrix. Two measures of the extent to which the scale defined is in agreement with the inferred judgements have been presented: Kendall's Coefficient of Consistency $K$ expresses the degree to which the $P^{*}$ matrix can be explained with a unidimensional scale;
while the Coefficient of Agreement eta is equal to the proportion of the inferred judgements which are in agreement with the site ordering defined by the attraction scale.

This portion of the study has been concerned with the explication of the ordinal attraction index methodology. Subsequent sections will present the results of three initial applications of the technique developed.

## CHAPTER III -- MEASURES OF PICNIC SIte ATtraction

The previous chapters of this thesis have demonstrated the need for quantitative measures of inherent site attraction and suggested a method by which such measures may be derived. The task of this section is to present the results of the application of the technique to two data sets concerned with picnicking in the environs of two Ontario cities, London and Sarnia. Because the same data collection technique was employed in both cases, it will be described separately. The results will be presented individually below, while the analysis of a somewhat different data set will be the topic of the fourth chapter.

## Data Collection

Both sets of picnic data were collected by telephone surveys from systematic samples of households which were drawn randomly from the telephone directories of the respective cities. Research assistants were instructed to telephone the Nth telephone number of each page of the city listing and, after introducing themselves and the study, to ask the question "When you go picnicking for the day, where do you go most often?" If there was no answer, or if the phone was a commercial or business one, the assistants phoned the next residential number in the listing. The responses and street addresses of the
subjects contacted were recorded and later tabulated. Interviewing was conducted between $10 \mathrm{a} . \mathrm{m}$. and noon, and 1 p.m. and 3 p.m. Monday through Friday.

Following the completion of the data collection portion of the study, the home locations of the subjects were identified on large scale city maps with the aid of city directories (Polk, 1970 and Vernon, 1970). Locations which could not be fixed in this manner were determined by field searches. The digital coordinates of each subject were then determined and punched onto computer cards with the code number of the picnic site most often frequented. The set of alternative picnic sites for each city was determined from this data by considering all sites mentioned, subject to a distance constraint of 120 miles, to be alternatives to the site selected.

Several of the sample subjects named areas rather than specific sites as their most frequented alternative. Such responses can be included in the analysis if two conditions are met. These are: (1) all individuals naming an area can be assumed to prefer the same site (or group of sites) within the area, and (2) the scale value defined for the area is interpreted as an area score rather than a site score. In the case of the London data, the first condition was met because only one individual preferred each of the areas named as alternatives. The second condition was therefore not necessary--even though the exact site or sites preferred remained unidentified. The establishing of an area location was resolved by locating an approximate centroid for the area. However, in future studies, the subjects should be requested to name specific sites to minimize such problems.

The alternative sites were then located and digitized from smaller scale maps, their digital coordinates then being converted to the scale and orientation of the individual city maps.

There are undoubtedly a large number of potential alternative sites which were not identified by the individuals sampled. The omission of these unidentified alternatives will not bias the analysis because they were either unknown or invariably judged as being less attractive than any other sites to which they were compared. As none of them were ever observed to be judged more attractive than any other site, all would be assigned the attraction value 0.0 . This would not be interpreted to mean that all had the same attractiveness, but that a more discriminating scale could not be defined on the basis of the available data.

Two possible sources of bias introduced by the sampling technique are evident. First, the use of telephone interviews may well undersample low socio-economic groups because they are less likely to have telephones. Second, the hours of interviewing would result in the underrepresentation of families in which the wife worked, and would also exclude the majority of single householders. These shortcomings are compensated for, at least in part, by the speed and efficiency of the technique.

## The London Case

The London data set was collected during July, 1970. The majority of the 605 householders contacted named recreation areas near the city. Almost $40 \%$ (240) either did not go on picnics, did so at
private cottages, or could not name a most-frequented site. Thirtyseven sites, ranging from urban to National Parks, were identified as alternatives. Six other sites, mentioned as second choices, were also included in the alternative set. The areal distribution of the 605 respondents' residences is shown in Figure 3.1, while the 43 alternative opportunities are presented in Figure 3.2.

Following data preparation and verification, a $365 \times 43$ matrix containing the distance from each picnicking subject to each alternative site was calculated in the following manner. The equation of a straight line from the individual's residence to a site was determined, and the portion of that line which fell within the built-up area of the city was calculated. The distance assigned was the sum of the distance through the built-up area, weighted by a constant, and the distance through the rural area. The weighting factor was chosen to be 2.5, thus implying, for example, an average travel speed of 20 mph in the city and 50 mph outside it. The choice of the Pythagorean rather than the Manhattan metric is arbitrary.

The Comparison Matrix
The comparison matrix (C) resulting from the tabulation of the inferred judgements of the London subjects (Table 3.1) reveals a total of 3,991 individual judgemen ${ }^{c}$, an average of 10.9 per subject. In contrast to the paired comparison matrices of the psychologists, the sum of any $\mathbf{C}_{\mathbf{i j}}$ and the corresponding $\mathrm{C}_{\mathrm{ji}}$ is not expected to be constant throughout the matrix. This difference, caused because the inferred judgement $A_{i}>A_{j}$ is dependent upon $D_{j}>D_{i}$, results in a great variation
FIGURE 3.1

## AREAL DISTRIBUTION

OF SUBJECTS:

## LONDON


Each uncircled character to дəqunu әч7 sfuəsəadax sample subjects residing at that location. The characters have been
assigned as follows.
(N) $\operatorname{saqun} N$




## FIGURE 3.2 <br> OF SITES: LONDON





























in the number of comparisons made between any site $i$ and any other site $j$. In this case, the largest number (137) was made between site 8 and site 31, while there are 278 cases in which pairs of sites were never compared.

The Proportions Matrix
The proportions matrix P* (Table 3.2) calculated from this comparison matrix contains 1,528 valid entries, $94.6 \%$ of which are equal to 1 or 0 , thus supporting the belief that a large number of the entries would be unanimous. Additionally, $43 \%$ of the remainder deviate from these values by less than .25 . The symbolic representation of the reordered proportions matrix (Table 3.3) reveals that the individual entries which do not represent unanimous judgements (that is, those which are neither 0 nor 1) are clustered in the region of the diagonal. As these entries are based on judgements between sites ranking near each other on the attraction scale defined (Table 3.4), it is not surprising that they are not completely discriminable. Ewing (1971, p. 97) presents a similar matrix in which the same phenomenon may be observed.

The indices of confusion (Table 3.4), as well as providing measures of the difficulty the sample individuals had in ranking the sites, may provide a rough indication of the way in which the sites might be grouped in order to facilitate the study of the factors which contribute to the confusion. If the confusion indices are plotted against the rank of the attraction score derived for each site (Fig. 3.3), groups of "similar" sites may be defined. In this context,














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-
$\infty$







○安

-















## TABLE 3.3 -- SYMBOLIC MATRIX: LONDON


2015

41 19 28 16 24 06 29 10

2143122120201112124033320324113300340001233 0518987046910133822729419750475143335282660
11111111111111111111111111111111111111111 - 11111111111111111111111111111111111 0001111111111111111111111111111111111 000011111111111111111111111111111111111 00000 E 51111111111111111111111111111111 00000 \% 8 181111111111111111111111111111111 00000- $0^{2} 11811111111111111111111111111111$ $000000=11111111111111111111111111111111$ 00000000 11111111111111111111111111111 00000000 1111111111111111111111111111 0000000000 didildididililililililil 000000000000 1111111111111111111111111111111 000000000000 1:181111111111111111111111111 0000000000000 1181111111111111111111111111 00000000000000 151281/111111111111111111 000000000000000 E88411111111/11/111111 $0000000000000-000$ - 11111111111111111111 $0000000000000000=1=1111111111111111111$ $000000000000000=00$ 1011111111111111111111 $00000000000000=0 \%=10$ 11 11111111111111 $0000000000000=0010$ 11 11111111 11 000000000000000000010 wilimilidilil if 000000000000000000000 : 211811111111 is
 000000000000000000000 0\% wil1811111111 mild
 0000000000000000000000020 2811111 11 . 1Kili 00000000000000000000 N 0000 - iliti is 00000000000000000000000000000 $0000000000000000000000000000000-0$ 111 $1000000000000000000000000000000=00$ ह1 1 $000000000000000000000000000000-00$. 1 00000000000000000000000000000000000 施 0000000000800000
0000000000000000000000000000000000000
*A blank cell in the matrix indicates a
missing comparison.

FIGURE 3.3

SITE GROUPINGS FROM INDICES OF CONFUSION: LONDON

tABLE 3.4 -- ALTERNATIVE SITES: LONDON

| Index Number | Site Name | Attraction Value | Index of Confusion |
| :---: | :---: | :---: | :---: |
| 1 | Bayfield | . 733 | . 025 |
| 2 | Delhi | . 000 | . 000 |
| 3 | Dingmans | . 113 | . 027 |
| 4 | Fanshawe | . 158 | . 108 |
| 5 | Gibbons | . 041 | . 027 |
| 6 | Goderich | . 789 | . 000 |
| 7 | Grand Bend | . 529 | . 100 |
| 8 | Harris | . 000 | . 000 |
| 9 | Harrington | . 354 | . 135 |
| 10 | Hillsborough | . 700 | . 000 |
| 11 | Ipperwash | . 674 | . 025 |
| 12 | Kintore | . 000 | . 000 |
| 13 | Lake Huron Beaches | . 645 | . 025 |
| 14 | Lakeside | . 292 | . 135 |
| 15 | Leamington | . 975 | . 000 |
| 16 | Long Point | . 825 | . 100 |
| 17 | Pinafore | . 224 | . 081 |
| 18 | Pinery | . 574 | . 125 |
| 19 | Point Clarke | . 900 | . 000 |
| 20 | Point Pelee | 1.000 | . 000 |
| 21 | Port Bruce | . 392 | . 108 |
| 22 | Port Burwell | . 545 | . 154 |
| 23 | Port Franks | . 617 | . 075 |
| 24 | Port Huron | . 821 | . 050 |
| 25 | Port Stanley | . 306 | . 056 |
| 26 | Rock Glen | . 000 | . 000 |
| 27 | Rondeau | . 837 | . 100 |
| 28 | St. Clair | . 846 | . 051 |
| 29 | Canatara | . 772 | . 026 |
| 30 | Sauble Beach | . 000 | . 000 |
| 31 | Springbank | . 203 | . 158 |
| 32 | Stratford | . 453 | . 108 |
| 33 | Credit Forks | . 070 | . 081 |
| 34 | Tyrconnel | . 408 | . 079 |
| 35 | Waterworks | . 224 | . 058 |
| 36 | Wheatley | . 000 | . 000 |
| 37 | Wildwood | . 354 | . 135 |
| 38 | Kincardine | . 925 | . 000 |
| 39 | Highlands | . 434 | . 029 |
| 40 | Embro Pond | . 295 | . 086 |
| 41 | Inverhuron | . 974 | . 000 |
| 42 | Turnbulls | . 533 | . 079 |
| 43 | Colds tream | . 055 | . 081 |

similar means that the sites are close enough together in perceived attraction that the individuals cannot discriminate perfectly between them. These groupings cannot be extracted from the attraction index alone because of its ordinal nature. Two or more sites may belong to the same group because they are similar in nature, or because the site characteristics of the sites are combined in such a way as to yield perceived attractions of approximately the same magnitude.

The "Fit" of the London Attraction Scale
A complete proportions matrix of 43 rd order will contain 12,341 triads, but inspection of the incomplete London matrix reveals that 2,082 triads contain at least two missing elements and are thus indeterminate. The distribution of the four possible outcomes of the remaining 10,259 triads is shown in Table 3.5. The transitivity of the matrix, calculated according to the modified Kendall formula (Equation 2.3) was found to be

$$
K=1-7 / 2220=.9968
$$

The probability of $K$ being non-random, although obviously high in this case, is usually dependent solely upon the number of circular triads observed. However, as the fact that the maximum number of circular triads $\left(d_{\text {max }}\right)$ is an estimate when one deals with an incomplete matrix in this manner, the value of $K$ is also dependent upon variations in $\mathrm{d}_{\text {max }}$. The chi square statistic (Equation 2.5) for assessing the likelihood of finding as few as seven circular triads in a complete 43rd order matrix was calculated to be

$$
\begin{aligned}
x_{149}^{2} & =8 / 39\left[\frac{1}{4}(43)(42)(41) / 6\right]-6.5+(43)(42)(41) / 1521 \\
& =679.8
\end{aligned}
$$

The normal deviate corresponding to this value is

$$
Z=22.83
$$

clearly a highly significant result. Thus, it is shown that the probability of as few as seven intransitive triads being observed in a random matrix is small. The normal distribution of $d_{\max }$ allows the use of Equations 2.6 and 2.7 for determining approximate upper and lower confidence limits for $K$. At the $99 \%$ level these were found to be . 997 and . 944 respectively.

TABLE 3.5 -- CONSISTENICY PARAMETERS: LONDON

| Matrix order | 43 |
| :--- | ---: |
| Total number of triads | 12,341 |
| Number of transitive triads | 8,872 |
| Number of intransitive triads | 7 |
| Number of incomplete transitive triads | 1,380 |
| Number of indeterminate triads | 2,082 |
| Number of random simulations | 20 |
| Mean number of simulated <br> intransitive .triads <br> Standard deviation of numbers of <br> simulated intransitive triads | 2,220 |

In a perfectly transitive matrix (Table 3.6), $K$ takes the value of 1 and it is possible to derive an ordinal scale which is in perfect agreement with all the cells in the matrix. It is evident, however, that the consistency of the matrix says very little about the proportion of inferred judgements (upon which the comparison matrix is based) which are in agreement with the scale derived. If all cell

TABLE 3.6 -- PROPORTIONS MATRIX: EXAMPLE 2

|  |  |  |  |  |  |  | $\sum_{j=1}^{N} p_{i j}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Site | 1 | 2 | 3 | 4 | 5 | 6 | $\frac{N-1}{}$ |
| 1 | -- | .7 | .6 | .8 | .9 | .7 | .74 |
| 2 | .3 | -- | .7 | .9 | .8 | .6 | .66 |
| 3 | .4 | .3 | .- | .6 | .9 | .7 | .58 |
| 4 | .2 | .1 | .4 | -- | .7 | .6 | .40 |
| 5 | .1 | .2 | .1 | .3 | .- | .9 | .32 |
| 6 | .3 | .4 | .3 | .4 | .1 | -- | .30 |

Example calculation of the Measure of Agreement under the constraint that the number of individual judgements used to calculate each of the entries in the above matrix is equal to ten.

Comparison Matrix

| Site | 1 | 2 | 3 | 4 | 5 | 6 | $S^{*}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | - | 7 | 6 | 8 | 9 | 7 | 37 |
| 2 | 3 | - | 7 | 9 | 8 | 6 | 30 |
| 3 | 4 | 3 | - | 6 | 9 | 7 | 22 |
| 4 | 2 | 1 | 4 | - | 7 | 6 | 13 |
| 5 | 1 | 2 | 1 | 3 | - | 9 | 9 |
| 6 | 3 | 4 | 3 | 4 | 1 | - | 0 |
| $5 * *$ | 13 | 10 | 8 | 7 | 1 | 0 | - |

where $S^{*}=$ the sum of the above-diagonal (correct) judgements, and
$S^{* *}=$ the sum of the below-diagonal (incorrect) judgements.

Sum of S* = 111
Sum of S** $=39$
Eta $=111 /(111+39)=.74$
entries are based on the same number of paired comparisons, the proportion may be extracted from the matrix itself but if they are not, the calculation of the level of agreement becomes more complex. One way to resolve the problem is to reinspect the inferred paired comparisons, and to tabulate the number which are in agreement with the final attraction scale derived from the proportions matrix. In the case of the London data set, $97.52 \%$ of the inferred judgements agreed with the ordinal attraction scale.

## The London Attraction Scale

The London Attraction Scale and two less complicated measures of site attraction are presented in Table 3.7, along with the observed attendance at each site and the average distance from all individual's residences to that site. The three attraction measures are defined as follows:

Al.. the ordinal attraction scale derived from the proportions matrix,

A2.. the number of times a site was selected as a ratio of the number of times the site was inferred to have been considered, and

A3.. the atfendance at the site multiplied by the average distance to the site.

The first two of these measures appear to be similar until inspected quite closely. Al, the ordinal scale, is in reality the average probability of a site being designated as more attractive than any other site to which it has been compared; $A 2$, on the other hand, is the probability of a site being selected over all other sites. In many spatial situations, the two scales will place the sites in the
table 3.7 -- MEASURES OF SITE ATTRACTION: LONDON

| Site | Attendance |  | Average <br> Distance |  | Al | A2 |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: |$\quad$ A3

same order; however, in certain cases the results will be quite different. Figure 3.4 presents such a case, while Table 3.8 summarizes the data necessary for the calculation of A1 and A2 in this simple situation. It is evident that the procedure for calculating the A2 scale is independent of whether or not a comparison between any two sites could actually be inferred. As Ewing (1970, p. 21) points out, "this is precisely the kind of error that is made in models of consumer spatial preference using frequency of choice of alternatives as a basis." It is clearly unreasonable to infer comparisons between two stimuli, unless both can be assumed to be present. The fact that the A2 scale would provide the same ordering of attraction scores if all individuals surveyed were located at the same origin can, however, provide a measure of the spatial distribution of the sample. The more highly correlated the A1 and A2 attraction measures are, the more clustered the individuals in relation to the sites.

The third attraction measure, $A 3$, is analogous to the $A$ of the simple gravity model (Equation 11). The omission of the two constants $k$ and $x$ will not change the ordering of the result ( $x$ being assumed to be greater than 0 ). An inspection of the Spearman rank correlations between these measures (Table 3.9) reveals several interesting relationships. The negative relationship between attendance and distance is to be expected, but its low magnitude is surprising. The degree to which the attraction measures are related to attendance reveals the ordinal measure to be almost independent of visitation, thus supporting the hypothesis that the attraction of a site should not be dependent upon gross attendance.

## FIGURE 3.4

## hypothetical spatial arrangement of sites and individuals

$$
\begin{aligned}
& \text { A } \quad 9 \\
& 8 \quad 2
\end{aligned}
$$

C3*

## table 3.8 -- CALCULATION OF A1 AND A2: EXAMPLE 1

| Individual | Site Chosen | Decisio | ferred |
| :---: | :---: | :---: | :---: |
|  |  | A | B |
| 1 | B | * | 1 |
| 2 | B | 0 | 1 |
| 3 | A | 1 | 0 |
| 4 | B | * | 1 |
| 5 | A | 1 | 0 |
| 6 | B | * | 1 |
| 7 | B | * | 1 |

```
where 1 = chosen
    0 = rejected
    * = not considered
```

Comparison Matrix
A
B
A -
B 1
2

A - .66
A - .66
B . 33
Proportions Matrix
A B

## Attraction Scales

| Site | A1 | A2 |
| :---: | :---: | :---: |
| A | .66 | .66 |
| B | .33 | .71 |

TABLE 3.9 -- ATTRACTION CORRELATIONS: LONDON

|  | Attendance | Distance | A1 | A2 | A3 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Attendance | --- | -.387* | . 119 | .513* | .849* |
| Distance | -.387* | --- | .661* | .390* | . 032 |
| A1 | . 119 | .661* | --- | .811* | .490* |
| A2 | .513* | .390* | .811* | --- | .804* |
| A3 | .849* | . 032 | .490* | .804* | --- |

*significant at the $95 \%$ confidence level.

In view of the fact that the most distant sites in a system must be most attractive if they are to draw individuals to themselves, it seems reasonable to expect positive correlations between distance and attraction measures. The expectation is borne out by the data, most noticeably in the case of the ordinal attraction scale. It is evident, however, that given a data set in which all individuals resided at a single location, the correlation between distance and the ordinal attraction scale would be perfect. Discussion of this major shortcoming of the proposed model will be deferred until later in the chapter. The high correlation observed between the A1 and A2 measures of attraction indicates a relatively high degree of clustering of the individuals in relation to the sites they patronize. The discussion of the analysis of the London picnic data will be postponed until the results of the application of the methodology to a similar data set have been described.

## The Sarnia Case

The Sarnia data were collected during May, 1971. Fifty-seven percent of the 400 householders contacted named preferred picnic sites without apparent difficulty. In all, thirty alternatives were identified. Locations of the Sarnia residences and picnic sites are presented in Figures 3.5 and 3.6 respectively.

After data coding, punching, and verification, the $225 \times 30$ distance matrix was calculated under the same assumptions as that of the London case. The Sarnia comparison matrix (Table 3.10) contains 2,023 inferred comparisons, an average of 9.0 per individual. The proportions matrix calculated (Table 3.11) was $99.31 \%$ occupied. Again, the majority of the entries represented unanimous judgements (95.6\% were either 0 or 1 ), and the non-unanimous entries were located near the diagonal of the reordered matrix (Table 3.12). Site groupings based on the confusion indices (Table 3.13) are presented in Figure 3.7. Inspection of the proportions matrix revealed the 4,057 determinable triads to be distributed as shown in Table 3.14. The matrix was found to be highly transitive, $K$ taking the value .995, with upper and lower limits of .9955 and .9944 respectively. The normal deviate corresponding to the observed chi square value of 347.14 with 36 degrees of freedom was found to be 17.92, again a clearly significant result. Reinspection of the 2,023 inferred paired comparisons revealed $97.68 \%$ of them to be in agreement with the ordinal attraction scale derived from the proportions matrix.

The table of Spearman rank correlations (Table 3.15) between attendance, distance, and the three measures of attraction (Table 3.16)
FIGURE 3.5

## AREAL DISTRIBUTION

OF SUBJECTS:
SARNIA
ऽ aifs jo wofzejol -- (
Each uncircled character represents the number of sample subjects residing at that location. The characters have been assigned as follows.







~





















#       



 $\therefore 0^{\circ} 00^{\circ} 00009000000000000000000000$








 $00^{\circ} 0^{\circ} 00^{\circ} 00^{\circ}-00^{\circ} 00^{\circ} 00^{\circ} 00^{\circ} 00000000$ ㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇ - $0_{0}^{10} 00000080000000800808$


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 $00=000 \rightarrow 000=0000000,000000000000$











m 000000800080000000800808000000




- Oㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇ



TABLE 3.12 -- SYMbOLIC MATRIX: SARNIA

|  | $\begin{aligned} & 210210012102322112122100001201 \\ & 217493203740097631250491568885 \end{aligned}$ |  |  |
| :---: | :---: | :---: | :---: |
| 22 | 11111111111111111111111111 |  |  |
| 11 | 0 1111111111111/11111/111111 |  |  |
| 07 | 00111111111111111111111111111 |  |  |
| 24 | 00011111111111111111111111 |  |  |
| 19 | 0000 1111111111111111111111111 |  |  |
| 03 | 00000111111111111111111111111 |  |  |
| 02 | 000000 11111111111111111111111 |  |  |
| 10 | 000000-1111111111111111111111 |  |  |
| 23 | 00000000 111*1111111111111111 |  |  |
| 17 | 000000-0:-1111111111111111111 |  |  |
| 04 | 000000000\% 1581111111111111111 |  |  |
| 26 | 00000000000 11111111111111111 |  |  |
| 30 | 0000000000" 1111111111111111 |  |  |
| 29 | 00000000\%0-0\% $=1: 1111111111111$ |  |  |
| 27 | 0000000000000 =11111111111111 |  |  |
| 18 | 00000000000000 11111111111111 |  |  |
| 13 | 0000000000000000 111111111111 |  |  |
| 21 | 0000000000000000:111111111111 |  |  |
| 12 | 00000000000000000 181111111111 |  |  |
| 25 | 0000000000000000000 1111111111 |  |  |
| 20 | 000000000000000000-0 111111111 |  |  |
| 14 | 00000000000000000000 1111111 |  |  |
| 09 | 000000000000000000000 W111111 |  |  |
| 01 | 00000000000000000000-0 = 141811 | LEGEND | * |
| 05 | $0000000000000000000000=18188$ |  |  |
| 06 | $000000000000000000000000=1511$ | 0 - | 0 |
| 18 | 00000000000000000000000000 11 |  | .00.4 |
| 28 | $00000000000000000000000-0-\%$ 1 | $\cdots$ - | .40.6 |
| 08 | 0000000000000000000000000000 L | 6 - | -6-1. |
| 15 | 0000000000000000000000000 . | 1 - | 1. |

* A blank cell in the matrix indicates
a missing comparison.

TABLE 3.13 -- ALTERNATIVE SITES: SARNIA

| Index Number | Site Name | Attraction Value | Index of Confusion |
| :---: | :---: | :---: | :---: |
| 1 | Canatara | . 223 | . 172 |
| 2 | Pinery | . 784 | . 069 |
| 3 | Grand Bend | . 828 | . 000 |
| 4 | Ipperwash | . 669 | . 103 |
| 5 | Guthrie | . 148 | . 172 |
| 6 | Murphy Beach | . 145 | . 103 |
| 7 | Rock Glen | . 931 | . 000 |
| 8 | Bayview | . 029 | . 069 |
| 9 | Sun 0il | . 232 | . 036 |
| 10 | Forest | . 766 | . 034 |
| 11 | Goderich | . 966 | . 000 |
| 12 | Lakeport | . 368 | . 034 |
| 13 | Brights Grove | . 431 | . 034 |
| 14 | Rainbow Beach | . 250 | . 000 |
| 15 | Centennial | . 018 | . 071 |
| 16 | Campbell | . 500 | . 034 |
| 17 | Mitchell's Bay | . 677 | . 103 |
| 18 | Yacht Club Beach | . 093 | . 069 |
| 19 | Storybook Gardens | . 862 | . 000 |
| 20 | Huronview | . 310 | . 069 |
| 21 | Clay Creek | . 431 | . 034 |
| 22 | Stoke's Bay | 1.000 | . 000 |
| 23 | Ausable River | . 690 | . 069 |
| 24 | Kettle Point | . 897 | . 000 |
| 25 | Wildwood Beach | . 345 | . 000 |
| 26 | Metropolitan Beach | . 607 | . 000 |
| 27 | Cedar Bay | . 517 | . 069 |
| 28 | Kiwanis Park | . 066 | . 107 |
| 29 | Blue Point | . 570 | . 138 |
| 30 | Port Franks | . 607 | . 071 |

## FIGURE 3.7

## SITE GROUPINGS FROM INDICES OF CONFUSION: SARNIA



TABLE 3.14 -- CONSISTENCY PARAMETERS: SARNIA
Matrix order ..... 30
Total number of triads ..... 4,060
Number of transitive triads ..... 3,971
Number of intransitive triads ..... 5
Number of incomplete transitive triads ..... 81
Number of indeterminate triads ..... 3
Number of random simulations ..... 20
Mean number of simulated intransitive triads ..... 1,003
Standard deviation of numbers of simulated intransitive triads ..... 37.01
TABLE 3.15 -- ATTRACTION CORRELATIONS: SARNIA

|  | Attendance | . Distance | A1 | A2 | A3 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Attendance | --- | -. 130 | -. 051 | .554* | .429* |
| Distance | -. 130 | --- | .987* | .655* | .745* |
| A1 | -. 051 | .987* | --- | .712* | .809* |
| A2 | .554* | .655* | .712* | --- | .898* |
| A3 | .429* | .745* | .809* | .898* | --- |

[^0]table 3.16 -- MEASURES OF SITE ATTRACTION: SARNIA

| Site | Attendance | Average Distance | A1 | A2 | A3 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 22 | 1 | 931 | 1.000 | 1.000 | 931 |
| 11 | 2 | 406 | . 966 | . 667 | 812 |
| 7 | 8 | 363 | . 931 | . 727 | 2901 |
| 24 | 1 | 373 | . 897 | . 111 | 373 |
| 19 | 1 | 481 | . 862 | . 100 | 481 |
| 3 | 3 | 271 | . 828 | . 188 | 813 |
| 2 | 18 | 252 | . 784 | . 486 | 4541 |
| 10 | 2 | 262 | . 766 | . 080 | 524 |
| 23 | 1 | 230 | . 690 | . 026 | 230 |
| 17 | 1 | 222 | . 677 | . 020 | 222 |
| 4 | 27 | 217 | . 669 | . 403 | 5860 |
| 26 | 1 | 222 | . 607 | . 019 | 222 |
| 30 | 1 | 218 | . 607 | . 024 | 218 |
| 29 | 1 | 191 | . 570 | . 015 | 191 |
| 27 | 1 | 158 | . 517 | . 014 | 158 |
| 16 | 1 | 176 | . 500 | . 014 | 176 |
| 13 | 6 | 80 | . 431 | . 077 | 482 |
| 21 | 2 | 94 | . 431 | . 027 | 189 |
| 12 | 2 | 86 | . 368 | . 026 | 172 |
| 25 | 1 | 71 | . 345 | . 012 | 72 |
| 20 | 2 | 68 | . 310 | . 024 | 137 |
| 14 | 1 | 59 | . 250 | . 012 | 59 |
| 9 | 1 | 63 | . 232 | . 012 | 63 |
| 1 | 105 | 22 | . 223 | . 530 | 2357 |
| 5 | 5 | 31 | . 148 | . 037 | 157 |
| 6 | 4 | 30 | . 145 | . 039 | 120 |
| 18 | 1 | 24 | . 093 | . 007 | 24 |
| 28 | 1 | 25 | . 066 | . 007 | 25 |
| 8 | 2 | 16 | . 029 | . 010 | 33 |
| 15 | 2 | 16 | . 018 | . 010 | 32 |

reveals much the same pattern as that calculated for the London data set (Table 3.9). The strong relationship between Al and distance, combined with the moderately strong agreement between A1 and A2 indicates that the sample is considerably clustered.

## Discussion

The foregoing description of the application of the proposed methodology to the picnic data has revealed that highly consistent ordinal scales which are in strong agreement with the inferred judgements of picnickers may be defined from observations of consumer spatial behaviour. Several observations regarding the application of the technique to the Ontario data may be made at this time. The remainder of this chapter will consist of discussion of the above analyses, while the next chapter will be concerned with the analysis of a data set pertaining to day use patterns at selected National and Provincial parks in Saskatchewan.

The high degree of unanimity observed in the proportions matrices (Tables 3.2 and 3.11) is intuitively satisfying as it was expected that recreators would be able to discriminate perfectly between the majority of the alternative sites in a system. Inspection of the comparisons matrices (Tables 3.1 and 3.10), however, reveals that a large number of the entries in the proportions matrices were based on very small numbers of inferred comparisons. It is evident that if any $C_{i j}$ is equal to 0 when the corresponding $C_{j i}$ is not equal to 0 both resulting entries in the proportions matrix will be unanimous. If the sum of $\mathrm{C}_{\mathrm{ij}}$ and $\mathrm{C}_{\mathrm{ji}}$ is sufficiently large, the unanimity need not concern us unduly. If it is not, however, there is the
danger of a few inferred judgements disproportionately influencing the attraction scale. The addition of individuals 8 and 9 and sites $C$ and $D$ to Figure 3.4 illustrates this condition. The A1 and A2 attraction measures based on these supplemented data are presented in Table 3.17. It is evident that even though the magnitude of the attraction scores has changed considerably, the site ordering has remained constant. The addition of five new inferred comparisons (all of which can be correctly predicted because they are based on the ranking of $D$ as more attractive than $C$, which in turn was based on only one inferred comparison) raises the percentage of agreement from . 667 to .875 . Thus, it is shown that the inferred judgements of a few individuals who choose to patronize sites relatively far from their residences may have a very strong effect on both the scale derived and the level of agreement attained.

This possibility that the results of the analysis may be disproportionately dependent upon a few data observations is felt to be more a consequence of the spatial arrangement of the sample individuals in relation to the sites patronized than of the method itself. Returning momentarily to Figure 3.4, it is seen that individuals residing closer to site A than site B can never be inferred to have judged site A to be more attractive than any other site. In fact, their judgements are restricted to those in the set $B>A, C>A, C>B, D>A, D>B$, and $D>C$. Other judgement sets may similarly be defined for individuals at any location. It is also evident that in the spatial situation diagrammed in Figure 3.4, although the judgement $D>A$ allows one to infer $D>C$ and $D>B$, each of these judgements is independent. It is

TABLE 3.17 -- CALCULATION OF A1 AND A2: EXAMPLE 2

| Individual | Site Chosen | Decision Inferred |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | A | B | C | D |
| 1 | B | * | 1 | * | * |
| 2 | B | 0 | 1 | * | * |
| 3 | A | 1 | 0 | * | * |
| 4 | B | * | 1 | * | * |
| 5 | A | 1 | 0 | * | * |
| 6 | B | * | 1 | * | * |
| 7 | 8 | * | 1 | * | * |
| 8 | D | 0 | 0 | 0 | 1 |
| 9 | C | 0 | 0 | 1 | * |

$$
\text { where } \begin{aligned}
1 & =\text { chosen } \\
0 & =\text { rejected } \\
* & =\text { not considered }
\end{aligned}
$$

Comparison Matrix
Site A B C D
A-200
B 1 - 00
C $11 \ldots 0$
D 111 -

Attraction Scales

| Site | A1 | A2 |
| :---: | :---: | :---: |
| A | .22 | .40 |
| B | .11 | .56 |
| C | .67 | .50 |
| D | 1.00 | 1.00 |

also clear that no other site can be judged to be preferred to D.
The problem outlined in the previous paragraph becomes serious only when a large proportion of the individuals studied reside in close proximity. When they are interspersed among the alternative sites, the number of possible judgement sets increases greatly. The consideration of the preferences of individuals $1^{*}, 2^{*}$ and $3^{*}$ located as on Figure 3.4 changes the preference data to that presented in Table 3.18 . Inspection of the spatial arrangement of the individuals and sites reveals that all six possible paired comparisons could have been made, and in each case the outcome could have favoured either of the pair of sites. Although it is evident that the possibility to infer a paired comparison favouring either site of each possible pairing should be available in a good data set, the collection of such data would prove extremely difficult without prior knowledge of the alternative set perceived by the individuals to be sampled. A quantitative measure of the scale distortion caused by this spatial bias has not been developed because it was believed that an improved sampling method would do much to alleviate the problem. Threepoints should be noted however: before it can be stated that the paired comparison $A>B$ is impossible, it must be shown that no individual is located in such a position that the distance to $A$ is greater than that to $B, a$ situation which can occur only when the sample individuals are clustered and the alternative sites dispersed; the reliability of the ranking of sites which are not affected by spatial bias of this type is unaltered by the presence in the scale of sites suffering from spatial bias; and, finally, the ranking of spatially biased sites is
table 3.18 -- calculation of al and a2: example 3

| Individual | Site Chosen | Decision Inferred |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | A | B | c | D |
| 1 | B | * | 1 | * | * |
| 2 | B | 0 | 1 | * | * |
| 3 | A | 1 | 0 | * | * |
| 4 | 8 | * | 1 | * | * |
| 5 | A | 1 | 0 | * | * |
| 6 | 8 | * | 1 | * | * |
| 7 | B | * | 1 | * | * |
| 8 | D | 0 | 0 | 0 | 1 |
| 9 | C | 0 | 0 | 1 | * |
| 1* | B | * | 1 | 0 | 0 |
| 2* | A | 1 | 0 | 0 | 0 |
| 3* | B | * | 1 | 0 | 0 |
| where $\begin{aligned} & 1=c \\ & 0=r \\ & \star=n \end{aligned}$ | cted <br> considered |  |  |  |  |

Comparison Matrix
Site A B C D
$A-311$
B $1-22$
C $11-0$
$0111-$

Attraction Scales

| Site | A1 | A2 |
| :---: | :---: | :---: |
| A | .58 | .50 |
| B | .53 | .58 |
| C | .28 | .20 |
| D | .61 | .25 |

not necessarily incorrect--it is just less reliable.
The number of inferred comparisons upon which an entry in the proportions matrix is calculated presents more of a problem. Although this number will stabilize as the spatial locations of the subjects and alternative sites becomes more interspersed, the question of establishing confidence intervals for the values defined remains. Statistical techniques for defining such intervals have been developed (see Bock and Jones, 1967, p. 124 ff., for example), but they may be used only where discrimination is imperfect. They are thus inapplicable to matrices with large proportions of unanimous entries. Therefore, in this study, the problem is acknowledged but not treated, although it will be discussed further in the fourth chapter.

The site groupings defined by the consideration of the Indices of Confusion will be very useful in helping to determine the major inputs to inherent site attraction. Although this study is concerned with the definition of site attraction indices rather than the "explanation" of the indices defined, a short digression on the topic of the combinations of site variables leading to the attraction indices would not be inappropriate at this point.

The vast array of alternative sites at which individuals may picnic may be subdivided into many different classes, each having a characteristic set of site attributes. The individuals faced with the task of assessing the relative attraction of these various sites presumably make use of intuitive methods which combine various attributes of each site into a single measure (Shepard, 1964). Judging from the attraction scales defined, the methods utilized result in a
considerable amount of interclass mixing. From the London grouping (Figure 3.3), for example, it is seen that the third group contains sites noted for swimming, cottaging, amusement park activity, and Shakespearean plays. The set of site attributes which could be defined from a close inspection of the sites in this group is undoubtedly large, thus rendering the explanation of the attraction indices very difficult. The information that the differences in quantity and quality of the site attributes of similar sites contained in the same group are not sufficient to make their attraction indices completely discriminable should prove to be helpful for those attempting to predict the attraction of certain classes of sites. Although the Indices of Confusion are only ordinally defined, and the method of grouping is subjective, the groupings obtained appear reasonable, and should be of great assistance in the study of predicting attraction indices from site attributes.

Inspection of Tables 3.4 and 3.13 show that seven alternative sites may be considered to be common to both the London and Sarnia alternative sets. Assuming that both sample groups have similar notions of what constitutes site attractivity, the seven sites should be ordered identically by both groups. As can be seen from Table 3.19, the agreement between the orderings is poor. Spearman's Rank Correlation Coefficient was calculated to be -. 42 (Student's $t=1.03$, which is not significant at $95 \%$ ). The fact that the maximum deviations in ranking occur when the alternative is situated within one of the cities, and the minimum when the alternative is relatively distant from both cities is yet another indication of the spatial bias
inherent in these data sets. Similarly, low correlations are observed between these sites on both the A2 (-.52) and A3 (.24) attraction measures.
table 3.19 -- ALTERNATIVES COMMON TO BOTH LONDON AND SARNIA

| Site Name | Attraction Index |  |
| :--- | :--- | :--- |
| London | $\underline{\text { Sarnia }}$ |  |
| Pinery | $.574(5)$ | $.784(4)$ |
| Grand Bend | $.529(6)$ | $.828(3)$ |
| Ipperwash | $.674(3)$ | $.669(5)$ |
| Goderich | $.789(1)$ | $.966(1)$ |
| Springbank/Storybook | $.203(7)$ | $.862(2)$ |
| Port Franks | $.617(4)$ | $.607(6)$ |
| Canatara | $.772(2)$ | $.223(7)$ |

Of the three measures of attraction defined above for each data set, the Al scale is judged to be the best because of its lack of dependence on visitation and its avoidance of the problem of assuming comparisons between sites which cannot both be inferred to be compared. The other two measures have been presented for comparative reasons only, and will not be calculated for the data set which is to be analyzed in the next chapter.

## Résumé

The methodology applied above has been shown to reveal an ordinal attraction scale which is highly consistent with the inferred
judgements of the individuals upon which it was based. Certain shortcomings of the approach have been largely caused by spatial bias introduced by the relative locations of the sample subjects and their alternative sets. The fourth chapter of this thesis will consider a third data set which, although being aggregated origin-destination data collected at the recreation site rather than at the residence of the individual, involves a much more uniform spatial distribution.

## CHAPTER IV -- MEASURES OF DAY USE PARK ATTRACTION

The previous chapter of this thesis was concerned with the application of the attraction index methodology to the problem of estimating the inherent attraction of picnic sites in Southwestern Ontario. The task of this section is to present the results of its application to data concerning day use visitation at twelve Saskatchewan parks, and to discuss the stability of the scale defined under various assumptions regarding the relative contributions of varying numbers of visitors and the incremental effects of distance.

## Data Collection and Analysis

Data for this portion of the study was collected during 1969 under the direction of the National and Historic Parks Branch, Department of Indian Affairs and Northern Development, as part of the Canada Outdoor Recreation Demand Study (Kovacs, 1971 and Cheung, 1970). The data consisted of lists of the origins of randomly selected day users at each of the twelve parks (Fig, 4.1). Following a preliminary data tabulation which grouped the users according to their nearest community, the data were analyzed by the method utilized above. Due to inaccuracies in the data, it was necessary to assume an arbitrary distance constraint. This was taken to be 220 miles. Any one way trip greater than this limit was declared invalid and omitted from the data set. In this analysis, the origin-site distance was not

## FIGURE 4.1

: Salis any siograns so
noilngivisia tyayy
Site locations:
:suofzejot foefqns
f 27fs jo uofzejol -- (f)
 represents the number of Burptsax sfoofqns ordues at that location. The characters have been assigned as follows.

Character
-Nmホnon

weighted by an urban area component because of the difficulty of defining the built-up areas of many of the small communities included as origins. Additionally, over the relatively large range of distance being considered such a transformation would have only marginal effects.

The Saskatchewan comparison matrix (Table 4.1), containing the judgements inferred from the 3,254 individual trips observed (subject to a distance constraint of 220 miles) contains 2,203 inferred comparisons, an average of .677 per individual. The obvious difference between this ratio and those observed in the London and Sarnia data sets deserves comment at this point. In the picnicking examples, the data were collected at the individuals' residence rather than at the recreation sites themselves. The use of this approach yields a large amount of information concerning alternative recreation areas, thus allowing one to define a comparatively large alternative set for each individual. In the present situation, however, the alternative set is restricted to the subset of the parks which are closer to the individual's residence than that which he chose to patronize; thus the number of inferred comparisons is much smaller as a result of our inadequate information about alternative opportunities in the Saskatchewan spatial situation. It is obvious that knowledge of the complete set of alternative sites perceived by the sample subjects cannot be obtained from site based surveys, but must be gained through home-based data collection techniques. The site rankings obtained from a data set based on an incomplete alternative set may be considered valid only within the alternative set utilized. The addition

TABLE 4.1 -- COMPARISON MATRIX: SASKATCHEWAN (UNWEIGHTED)
$\begin{array}{lllllllllllll}\text { Site } & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12\end{array}$
$1 \quad 0.0 .1 . \quad 61.2 . \quad 2 . \quad 1 . \quad 1 . \quad 5 . \quad 8 . \quad 4 . \quad 14$.
2 7. 0. 4. 6. 6. 6. 2. 6. 8. 7. 3. 22.
3 13. 4. $0.14 . \quad 47.10 . \quad 3.10 .10 .14 . \quad 3.10$.
4 11. 0. 8. 0. 9. 10. 4. 6. 7. 19. 5. 8.
5 7. 0. 9. 14. 0. 2. 1. 9. 1. 7. 1. 6.
$6 \quad 5 . \quad 0 . \quad 5 . \quad 5 . \quad 8 . \quad 0 . \quad 2 . \quad 4 . \quad 4 . \quad 8 . \quad 2 . \quad 4$.
7 42. 3. 4. 10. 7. 13. 0. 1. 51. 44. 40. 43.
$8 \quad$ 20. 1. 2. 20. 4. 1. 1. 0. 2. 19. 0. 18.
$9 \quad 56 . \quad 2 . \quad 5 . \quad 53.50 . \quad 7 . \quad 1 . \quad 43 . \quad 0 . \quad 55 . \quad 0 . \quad 55$.
10 203. 3. 0. 227. 1. 1. 1. 2. 8. 0. 1. 9.
11 6. 6. 1. 4. 2. 5. 5. 0. 146. 99. 0. 5.
12 191. 1. 4. 30. 6. 7. 3. 7. 4. 33. 4. 0 .
of other alternative sites may result in partial reordering of the attraction scale, although it is believed that the majority of the original set would be placed in the same rank order.

The individual entries in the comparison matrix reveal that the number of visitors inferred to have compared any two sites varies from 0 in the case of sites 2 and 11 to 246 in the case of sites 4 and 10.

Further inspection shows that all of the entries shown in the proportions matrix (Table 4.2) were based on the responses of more than one individual. The entries deviate more from unanimity than those of the previous data sets, 52 of the 130 ( $40 \%$ ) deviating by more than .25 , whilst only $3 \%$ of the cells contain values of either 0 or 1 . The symbolic representation of the reordered matrix (Table 4.3) shows the unanimous entries to be largely those involving the second site, raising the question of the spatial arrangement of the subjects with respect to that site. Inspection of the data revealed that, although site 2 was the furthest site from many of the subjects, this was not always the case. Individuals residing close to site 2 had the opportunity (as defined by the maximum trip length permitted) of patronizing sites 1,10 , or 12 , thus inferring that site 2 was less attractive than the patronized site. Examination of the confusion indices (Table 4.5) indicates that the individuals sampled were unable to discriminate between the various sites at all well. The 434 determinable triads contained in the proportions matrix were distributed as shown in Table 4.4, while the coefficient of consistency $K$ was calculated to be .6792 , with upper and lower limits of .7615 and

TABLE 4.2 -- PROPORTIONS MATRIX: SASKATCHEWAN (UNWEIGHTED)

| Site | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 0.0 | 0.0 | 0.07 | 0.85 | 0.22 | 0.29 | 0.02 | 0.05 | 0.08 | 0.04 | 0.40 | 0.07 |
| 2 | 1.00 | 0.00 | 0.50 | 1.00 | 1.00 | 1.00 | 0.40 | 0.86 | 0.80 | 0.70 | 0.33 | 0.96 |
| 3 | 0.93 | 0.50 | 0.00 | 0.64 | 0.84 | 0.67 | 0.43 | 0.83 | 0.67 | 1.00 | 0.75 | 0.71 |
| 4 | 0.15 | 0.0 | 0.36 | 0.0 | 0.39 | 0.67 | 0.29 | 0.23 | 0.12 | 0.08 | 0.56 | 0.21 |
| 5 | 0.78 | 0.0 | 0.16 | 0.61 | 0.0 | 0.20 | 0.13 | 0.69 | 0.02 | 0.88 | 0.33 | 0.50 |
| 6 | 0.71 | 0.0 | 0.33 | 0.33 | 0.80 | 0.0 | 0.13 | 0.80 | 0.36 | 0.89 | 0.29 | 0.36 |
| 7 | 0.98 | 0.60 | 0.57 | 0.71 | 0.88 | 0.87 | 0.0 | 0.50 | 0.98 | 0.98 | 0.89 | 0.93 |
| 8 | 0.95 | 0.14 | 0.17 | 0.77 | 0.31 | 0.20 | 0.50 | 0.0 | 0.04 | 0.90 | $-1.00^{\star}$ | 0.72 |
| 9 | 0.92 | 0.20 | 0.33 | 0.88 | 0.98 | 0.64 | 0.02 | 0.96 | 0.0 | 0.87 | 0.00 | 0.93 |
| 10 | 0.96 | 0.30 | 0.0 | 0.92 | 0.13 | 0.11 | 0.02 | 0.10 | 0.13 | 0.00 | 0.01 | 0.21 |
| 11 | 0.60 | 0.67 | 0.25 | 0.44 | 0.67 | 0.71 | 0.11 | $-1.00^{*}$ | 1.00 | 0.99 | 0.00 | 0.56 |
| 12 | 0.93 | 0.04 | 0.29 | 0.79 | 0.50 | 0.64 | 0.07 | 0.28 | 0.07 | 0.79 | 0.44 | 0.00 |

*-1.00 indicates a missing value.

TABLE 4.3 -- SYMBOLIC PROPORTIONS MATRIX: SASKATCHEWAN (UNWEIGHTED)


LEGEND: 0 .. 0

$$
\begin{array}{ll}
-. . & .0-.4 \\
=. . & .4-.6 \\
+\ldots & .6-1 \\
1 . . & 1
\end{array}
$$

*A blank cell in the matrix indicates a missing comparison.

TABLE 4.4 -- CONSISTENCY PARAMETERS: SASKATCHEWAN (UNWEIGHTED)
Matrix Order ..... 12
Total number of triads ..... 220
Number of transitive triads ..... 193
Number of intransitive triads ..... 17
Number of incomplete transitive triads ..... 8
Number of indeterminate triads ..... 2
Number of random simulations ..... 20
Mean number of simulated intransitive triads ..... 53
Standard deviation of numbers of simulated intransitive triads ..... 6.09TABLE 4.5 -- ALTERNATIVE SITES: SASKATCHEWAN

| Index <br> Number |  | Attraction <br> Value | Index of <br> Confusion |
| :---: | :---: | :---: | :---: |
| 1 | . | .190 | .909 |
| 2 | . | .732 | .909 |
| 3 |  | .730 | .909 |
| 4 |  | .277 | .909 |
| 5 | .390 | .909 |  |
| 6 | .856 | .727 |  |
| 7 | .473 | .900 |  |
| 8 | .614 | .727 |  |
| 9 | .266 | .727 |  |
| 10 | .619 | .900 |  |
| 11 |  | .439 | .727 |

.5105 respectively. The normal deviate corresponding to the observed chi square value of 59.15 was 4.525 , once more a clearly significant result. Reinspection of the inferred paired comparisons contained in the raw data set revealed that $66.88 \%$ were in agreement with attraction scale defined (Table 4.5).

## Discussion of the Initial Results

The results of the Saskatchewan analysis, in particular the extremely low degree of unanimity observed in the proportions matrix and the relatively low fit of the scale defined, were somewhat discouraging, although in view of the nature of the data they were not completely unexpected. A great deal of the unanimity problem is undoubtedly due to the fact that the data (as a result of the survey technique employed) contains no information on the vast number of alternative recreation sites available to the subjects studied. Kovacs (1971, p. 15) notes that there are at least 74 other national and provincial parks in the study area, while there are doubtless many local, regional and private recreation areas which provide alternative destinations for day trips. If, as it appears from the indices of confusion, the twelve parks considered are all of similar attractivity relative to the complete spectrum of available alternatives, it is not surprising to find such a high degree of confusion in the proportions matrix. Additionally, there is no way of knowing from the data whether or not the individuals who visited any given park felt afterwards that the attraction of the site justified the expense of the trip to it. In other words, the assumption of a well-informed user implicit in most
models of recreational behaviour may not be valid. Ideally, the approach is best applied to situations in which it may be assumed that the individual has sufficient knowledge about all the alternative sites closer to his origin than the site he selects to make a rational decision as to which offers the greatest utility--presumably that which he patronizes most often. With the present data, we have none of this information, but know only that the individuals visited a specific site on at least one occasion.

## Notions of Consistency

In spite of the generally low fit of the attraction scale, the Saskatchewan data set is extremely valuable for the reason that it may be utilized for the purpose of testing the stability of the scale under various transformations of the input data, in reality a testing of the behavioural assumptions on which the model is based. An important methodological question may be posed at this time: should the final objective of this type of analysis be to maximize consistency between the scale defined and the data observed? If so, which notion of consistency should be utilized? Four of the many possible interpretations of consistency are discussed below.
(A) Maximum consistency is defined as that state which maximizes the number of cells in the proportions matrix ${ }^{P *}$ for which the values can be predicted correctly from knowledge of the attraction scale. This interpretation involves the assumption that each $\mathrm{P}^{*}{ }_{\mathbf{i} \mathbf{j}}$ has equal validity and weight, regardless of the number of inferred comparisons ( $\mathrm{C}_{\mathrm{ij}}+\mathrm{C}_{\mathrm{j} \mathbf{i}}$ ) it was computed from.
(B) Maximum consistency is reached when the number of inferred comparisons which are in agreement with the attraction scale defined is maximized. The assumption that each $P_{i j}$ should be weighted by $C_{i j}+$ $C_{j i}$ is necessary to this interpretation.
(C) Maximum consistency is realized when the impact of observed inconsistencies in the proportions matrix is minimized. This interpretation necessitates the weighting of such inconsistencies by a function of the seriousness of the inconsistency.
(D) Maximum consistency is attained by maximizing the reliability of the individual ${ }^{P *}{ }_{i j}$ from which the attraction scale is defined. This interpretation involves the assumption that if $P^{*}{ }_{i j}>P^{*}{ }_{i k}$ and $P^{*}{ }_{i k}>P^{*}{ }_{i m}$, then an observation $P^{*}{ }_{i j}>P^{*}{ }_{i m}$ gains validity, and should therefore be given more importance.

All of these interpretations suffer from the constraint that, although their objective functions may be defined, site orderings which maximize them cannot be reached by direct analytical methods but must be converged to by iteration, an approach which is impractical when the matrix is of even moderate size. The approach employed in this study, in contrast to those mentioned above, has no objective function but proceeds directly to a solution. It satisfies, to some extent, the requirements of interpretations $A$ and $B$ by assuming that all cells in the p* matrix have equal validity, but are weighted according to their magnitude.

Under conditions of perfect transitivity (when Kendall's $K$ is equal to 1.00 ), all interpretations would result in the same ordering,
but when dealing with spatial choices this condition is unlikely to occur. The presence of intransitivities in the $P^{*}$ matrix causes the coefficients eta and $K$ to decline, although not necessarily in direct relation to one another if the numbers of comparisons between any site $\mathbf{i}$ and any other site $\mathbf{j}\left(\mathrm{C}_{\mathrm{ij}}+\mathrm{C}_{\mathrm{j}}\right.$ ) are not constant throughout the matrix. Geographers, who in general deal with opportunity surfaces which are not isotropic, are likely to find the number of comparisons which can be inferred to have been made between sites to vary widely throughout the $P^{*}$ matrix. A brief illustration of the effect of varying these numbers is appropriate at this point. Consider, for example, the data presented in Table 2.1. If one increases the number of judgements between sites 1 and 3 by a factor of ten, retaining the same proportions in each cell, the data is as shown in Table 4.6(a). The $P^{*}$ matrix computed from these data and the attraction indices defined by the direct method would remain unchanged (see Table 2.2). Eta would decline from $62.33 \%$ to $42.71 \%$, although $K$ would be unchanged. The $C$ matrix, arranged according to the attraction scale, is presented in Table 4.6(b). It may readily be observed that the reordering of sites 1 and 3 (Table 4.6(c)) would raise the value of eta considerably (to 80.63\%). The question of whether or not such a reordering is justifiable depends solely upon the objectives of the researcher. If he is attempting to define an ordering unique to the particular data set with which he is working, he should inspect all possible site orderings and choose that which maximizes eta. If, on the other hand, he wishes to define a more general site ordering which would be less dependent upon vagaries in the data (and more likely to be defined in a replication

TABLE 4.6 -- COMPARISON MATRICES: EXAMPLE 2

| (a) |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Site | 1 | 2 | 3 | 4 | 5 | 6 |
| 1 | - | 100 | 900 | 100 |  |  |
| 2 | 0 | - | 5 | r 30 | 100 | 0 |
| 3 | 100 | 95 | - | 100 | 100 | 100 |
| 4 | 0 | 70 | 0 | - | 100 | 100 |
| 5 | 100 | 0 | 0 | 100 | - | 0 |
| 6 | 100 | 0 | 0 | 100 | $\overline{60}$ | 40 |
| (b) |  |  |  |  |  |  |
|  | 3 | 1 | 6 | 5 | 2 | 4 |
| 3 | 0 | 100 | 100 | 100 | 95 |  |
| 1 | 900 | 100 | 0 | 0 | 100 | 100 |
| 6 | 0 | 100 | - | 60 | 10 | 100 |
| 5 | 0 | 100 | 40 | - | 0 | 100 |
| 2 | 5 | 0 | 100 | 100 | - | 100 |
| 4 | 0 | 0 | 0 | 0 | 70 | - |
|  |  |  |  | eta $=42.71$ |  |  |
| (c) |  |  |  |  |  |  |
|  | 1 | 3 | 6 | 5 | 2 | 4 |
| 1 | 100 | 900 | 0 | 0 | 100 | 100 |
| 3 | 100 | - | 100 | 100 | 95 | 100 |
| 6 | 100 | 0 | 40 | 60 | 0 | 100 |
| 5 | 100 | 0 | 40 | - | 0 | 100 |
| 2 | 0 | 5 | 100 | 100 | - | 30 |
| 4 | 0 | 0 | 0 | 0 | 70 | - |

study), he should probably accept the site ordering defined by the direct method.

In this exploratory study, where the main emphas is is placed on the general technique of extracting an ordinal attraction scale, the latter objective is of greatest importance. The remainder of this section of the thesis will be concerned with the results of modifications introduced to test various behavioural assumptions and a discussion of the implications of their results.

Weighting the P* Matrix
In the case of a single individual (Fig. 2.1), the inferred judgement $A_{k}>A_{j}$ contributes an increment of one to the $k j t h$ cell of the comparison matrix. With grouped data, where $N$ individuals who were inferred to have judged $A_{k}>A_{j}$ can be considered to reside at location $i$, the increment would normally be $N$. The fact that the Saskatchewan data are grouped allows one to determine the influence that the number of visitors to a site exerts on the attraction scale derived. This can be achieved by making the $i$ th town's increment to the jkth cell of the comparison matrix a function of the number of individuals from $\mathbf{i}$ judging $A_{j}>A_{k}\left(f_{i j k}\right)$. The testing of different flow functions thus provides a measure of the attraction scale's dependence upon visitation rates.

A second modification was introduced to test the notion that (again referring to Fig. 2.1) the attraction of site $k$ is greater than the attraction of site $j$ by an amount which is a function of the extra distance an individual from $\mathbf{i}$ choosing site $k$ over site $j$ would have to travel to reach $k$. This extra distance is designated as $\mathrm{d}^{\boldsymbol{*}}{ }_{\mathbf{i j k}}$. This
amount can be assumed to act on the kjth cell of the comparison matrix in the same way as an increase in the number of judgements $A_{j}>A_{k}$ inferred from the spatial choices of individuals from town $\mathfrak{i}$. The testing of the effect of varying the distance function will again assess the stability of the attraction scale.

Additionally, following the reasoning of psychologists (see, for example, Coombs et. al., p. 131 ff .) on the subject of just noticeable differences ( $j n d$ 's), if the extra distance $d^{\star}{ }_{i j k}$ was of little significance (for example, less than one mile), the two sites $j$ and $k$ were assumed to be at the same distance and no judgement $A_{k}>A_{j}$ was inferred. The maximum "insignificant" d* was set arbitrarily.

These modifications were incorporated by calculating a single weight $W$ which was defined by the relation:
(4.1) $W_{i j k}=\Sigma f^{b}{ }_{i j k}{ }^{d \star^{a}}{ }_{i j k}$

$$
\begin{aligned}
& \text { for all } k \text { such that } A_{j}>A_{k} \text { and } \\
& d^{d}{ }_{i j k}> \text { jnd. } \\
& \text { where } W_{i j k}= \text { the increment to the } j k \text { th } \\
& \text { cell of the comparison } \\
& \text { matrix contributed by } \\
& \text { residents of origin } i, \text { and } \\
& b \text { and } a= \text { exponents defined arbi- } \\
& \text { trarily. }
\end{aligned}
$$

It can be seen that if the exponents are given values of 0.0 , the model will tabulate the flow data as single judgements, the $W_{i j k}$ being set equal to the number of $A_{j}>A_{k}$ judgements for which $d^{*}{ }_{i j k}>$ jnd, while if $a=1.0$ and $b=0.0$, the judgements will be weighted by the number of individuals from origin $\mathbf{i}$ patronizing site $k$, etc. During this portion of the analysis, the two exponents $a$ and $b$ were varied systematically, the best pair being defined as those which
maximized the proportion of inferred judgements agreeing with the attraction scale defined. Additionally, runs of the programme were made with different values of jnd, in order to assess the importance of this parameter.

Several conclusions are suggested by the results presented in Table 4.7. First, it is evident that, although the majority of the levels of agreement defined deviate only slightly from that of the original unweighted scale, values which are much higher have been defined for $a$ narrow range of $a$ and $b$. Second, varying the magnitude of the jnd (it was allowed to vary from 0 to 17.43 miles) appears to have had little effect on the measure of fit. The same may be said of the $d^{*}$ exponent (a). Lastly, it is clear that the level of agreement has a generally slow, although irregular, decline as the magnitude of $b$ increases. Each of these observations is discussed in detail below.

An inspection of the original data shows that each of the high levels of agreement is a result of a change in site ranking brought about by the particular $a$ and $b$ values used in the calculation of the weighting factors. As shown in Tables 4.7 and 4.8 , however, reorderings of the attraction scale which result in increases in the level of agreement do little to change the overall order of the attraction indices. A comparison of the site orderings defined from the original unweighted comparison matrix (Table 4.1) and that of the weighted matrix which reaches the highest level of agreement (Table 4.9) shows that, although the rank positions of six sites have changed, each has moved only one place in the ranking. of the three interchanges which do occur, only one (involving the inferred judgements of a single large community)

TABLE 4.7 -- LEVELS OF AGREEMENT FOR DIFFERENT WEIGHTING EXPONENTS

|  |  | Value of jnd* |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| a | $b$ | 0.00 | 0.25 | 0.50 | 1.00 |
| 0.0 | 0.0 | 67.64 | 67.64 | 67.64 | 67.74 |
| 0.5 | 0.0 | 67.64 | 67.64 | 67.64 | 67.64 |
| 1.0 | 0.0 | 67.64 | 67.64 | 67.64 | 67.64 |
| 1.5 | 0.0 | 67.64 | 67.64 | 67.64 | 67.64 |
| 2.0 | 0.0 | 67.64 | 67.64 | 67.64 | 67.64 |
| 0.0 | 0.5 | 76.49 | 76.49 | 67.64 | 67.64 |
| 0.5 | 0.5 | 67.64 | 67.64 | 61.01 | 61.01 |
| 1.0 | 0.5 | 61.01 | 61.01 | 61.01 | 61.01 |
| 1.5 | 0.5 | 61.01 | 61.01 | 61.01 | 61.01 |
| 2.0 | 0.5 | 61.01 | 61.01 | 61.01 | 61.01 |
| 0.0 | 1.0 | 69.86 | 69.81 | 60.83 | 60.83 |
| 0.5 | 1.0 | 69.81 | 69.81 | 60.96 | 60.96 |
| 1.0 | 1.0 | 60.96 | 60.86 | 60.96 | 60.96 |
| 1.5 | 1.0 | 60.96 | 60.96 | 60.96 | 60.96 |
| 2.0 | 1.0 | 60.96 | 60.96 | 60.96 | 60.96 |

*Values of jnd expressed in units of 17.43 mi .

TABLE 4.8 -- SITE RANKINGS FOR VARIOUS LEVELS OF AGREEMENT

| Level of Agreement (eta) |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 60.83 | 60.96 | 61.01 | 67.64 | 69.81 | 69.86 | 76.49 |

Rank

| 1 | 2 | 2 | 2 | 2 | 2 | 2 | 2 |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| 2 | 3 | 3 | 7 | 7 | 3 | 7 | 7 |
| 3 | 7 | 7 | 3 | 3 | 7 | 3 | 3 |
| 4 | 9 | 9 | 9 | 11 | 9 | 9 | 11 |
| 5 | 11 | 11 | 11 | 9 | 11 | 11 | 9 |
| 6 | 8 | 8 | 8 | 8 | 8 | 8 | 8 |
| 7 | 6 | 12 | 12 | 12 | 12 | 12 | 12 |
| 8 | 12 | 6 | 6 | 6 | 6 | 6 | 6 |
| 9 | 5 | 5 | 5 | 5 | 5 | 5 | 5 |
| 10 | 4 | 4 | 4 | 4 | 4 | 4 | 4 |
| 11 | 1 | 1 | 1 | 1 | 10 | 10 | 10 |
| 12 | 10 | 10 | 10 | 10 | 1 | 1 | 1 |

Lowest $R_{s}=R_{1,7}=1-\frac{48}{1716}=.972$
$T_{10}=13.07$ (significant at the $95 \%$ level)

COMPARISON MATRIX

|  | 1 | 2 | 3 | 4 | 5 | 0 | 7 | 8 | 9 | 10 | 11 | 12 |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| 1 | 0. | 0. | 6. | 15. | 12. | 12. | 6. | 6. | 13. | 21. | 11. | 10. |
| 2 | 50. | 0. | 35. | 46. | 46. | 46. | 22. | 46. | 51. | 50. | 23. | 55. |
| 3 | 07. | 26. | 0. | 71. | 87. | 53. | 17. | 40. | 54. | 71. | 17. | 54. |
| 4 | 39. | 0. | 12. | 0. | 13. | 32. | 19. | 5. | 31. | 51. | 24. | 34. |
| 5 | 30. | 0. | 10. | 35. | 0. | 9. | 8. | 24. | 8. | 30. | 8. | 27. |
| 6 | 25. | 0. | 25. | 26. | 29. | 0. | 8. | 23. | 13. | 35. | 6. | 22. |
| 7 | 92. | 16. | 29. | 65. | 46. | 66. | 0. | 10. | 113. | 101. | 71. | 96. |
| 8 | 41. | 6. | 11. | 41. | 20. | 9. | 9. | 0. | 15. | 39. | 0. | 35. |
| 9 | 114. | 4. | 35. | 108. | 101. | 42. | 4. | 68. | 0. | 111. | 0. | 112. |
| 10 | 56. | 12. | 0. | 50. | 3. | 3. | 6. | 6. | 14. | 0. | 6. | 20. |
| 11 | 36. | 24. | 8. | 28. | 16. | 21. | 21. | 0. | 83. | 47. | 0. | 28. |
| 12 | 89. | 3. | 23. | 67. | 39. | 45. | 22. | 20. | 25. | 76. | 25. | 0. |

## PROPORTIONS MATRIX

|  | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 1 | 0.0 | 0.0 | 0.08 | 0.28 | 0.29 | 0.32 | 0.06 | 0.13 | 0.10 | 0.27 | 0.23 | 0.10 |
| 2 | 1.00 | 0.0 | 0.57 | 1.00 | 1.00 | 1.00 | 0.58 | 0.88 | 0.93 | 0.81 | 0.49 | 0.95 |
| 3 | 0.92 | 0.43 | 0.0 | 0.86 | 0.90 | 0.68 | 0.37 | 0.78 | 0.61 | 1.00 | 0.68 | 0.70 |
| 4 | 0.72 | 0.0 | 0.14 | 0.0 | 0.27 | 0.55 | 0.23 | 0.111 | 0.22 | 0.57 | 0.46 | 0.34 |
| 5 | 0.71 | 0.0 | 0.10 | 0.73 | 0.0 | 0.24 | 0.15 | 0.55 | 0.07 | 0.91 | 0.33 | 0.41 |
| 6 | 0.68 | 0.0 | 0.32 | 0.45 | 0.76 | 0.0 | 0.11 | 0.72 | 0.24 | 0.92 | 0.28 | 0.33 |
| 7 | 0.94 | 0.42 | 0.63 | 0.77 | 0.85 | 0.89 | 0.8 | 0.53 | 0.97 | 0.94 | 0.77 | 0.81 |
| 8 | 0.87 | 0.12 | 0.22 | 0.89 | 0.45 | 0.28 | 0.47 | 0.0 | 0.18 | 0.87 .1 .00 | 0.55 |  |
| 9 | 0.90 | 0.07 | 0.39 | 0.78 | 0.93 | 0.76 | 0.03 | 0.82 | 0.0 | 0.89 | 0.0 | 0.82 |
| 10 | 0.73 | 0.19 | 0.0 | 0.50 | 0.09 | 0.08 | 0.06 | 0.13 | 0.11 | 0.0 | 0.11 | 0.21 |
| 11 | 0.77 | 0.51 | 0.32 | 0.54 | 0.67 | 0.72 | 0.23 .1 .00 | 1.00 | 0.89 | 0.0 | 0.53 |  |
| 12 | 0.90 | 0.05 | 0.30 | 0.66 | 0.59 | 0.67 | 0.19 | 0.45 | 0.18 | 0.79 | 0.47 | 0.0 |

CELL SAMPLE SIZE

|  |  |
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results in a significant number of comparisons which can be correctly predicted. The 11,9 interchange increases prediction by 146 , the 6,12 case by three, while the 7,2 swap actually decreases the number by one, indicating that an even better scale which placed site 7 before site 2 could be defined. Indeed, there is no assurance that the ranking which would maximize the level of agreement would ever be reached without examining each of the individual rankings which could be defined for this data set. One can, however, inspect the proportions matrix of any of the scales, either weighted or unweighted, and determine whether or not the inversion of any pair of sites which contain large intransitivities in their common cell below the diagonal would have a positive or negative influence on the level of agreement. This approach is very inefficient, however, since in general an exhaustive search requires the evaluation of $n$ ! rankings; in this case 12 ! rankings would be necessary.

The stability of the attraction scale under varying jnd's is certainly a function of the data set utilized. One would normally expect the number of comparisons infenred between any site $\mathbf{j}$ and any other site $k$ to decline as the value of the jnd was increased. Inspection of the data (see, for example, Tables 4.10 and 4.11 ) reveals that this is indeed the case, although the reductions are rarely very large. Their uniformity (a direct function of the spatial arrangement of the subjects and sites) is such that the proportions matrices, and hence the attraction scales derived, are almost identical. The sudden drops from the "anomalies" are in all cases the result of a large number of "correct" judgements made by the residents of a single large community

|  | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $1$ | 0. | 0. | 30. | 35 | 51. | 51. | 21. | $30 .$ | $44$ | $75$ | 42 |  |
| $2$ | $351^{\circ}$ | 0. | 265. | 345. | 345 | 345. | 133. | $345$ | $352$ | $351$ | $135$ | $353 .$ |
| 3 | 378. | 194. | 50. | 384. | 416 | 252. | 152. | 206. | $340$ | $384$ | $152$ | $351$ |
| 4 | 124. | 0 . | 50. | 0 | 50 | 96. | 80. |  | $11$ | 134. | 98 | $121$ |
| 5 | 106. | 0 . | 9. | 109. | 0 | 32. | 30. | 77. | 30. | $106 .$ | 30 | $\begin{aligned} & 102 \\ & 101 . \end{aligned}$ |
| 6 | 75. | 0. | 73. | 72. | 79. | 0. | 28. | $69 .$ | $34$ | $93 .$ | 28 | $710$ |
| 7 | 510. | 49. | 117 | 222 | 161 | 209. | 0. | $54 .$ | 554. | $536 .$ | 446 | 520 |
| 8 | 320. | 37. | 40 | 320. | 65. | 39. | 39. | $0$ | $76$ | $319 .$ |  |  |
| 9 | 1226 | 10. | 132. | 214 | 58 | 146. |  | $056 .$ |  | $217$ |  | $\begin{array}{r} 28 \\ 22 \end{array}$ |
| 10 | 162. | 36. | 0 . | 121. | 5 | 5. | 13. | 7. | 60. | 0 | 13. | 56 |
| 11 | 107. | 79. | 18. | 95. | 38. | 51. | 51. | 0. | 662 | 555. | 1 | 89 |
| 12 | 296. | 10. | 102 | 242. | 149. | 107. | 65. | 113. | 75. | 262. | 75. |  |

PROPORTIONS MATRIX

|  | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 0 | 9 | 10 | 11 | 12 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 0.0 | 0.0 | 0.07 | 0.22 | 0.32 | 0.40 | 0.04 | 0.09 | . 03 | 0.32 | 0.28 | 0.01 |
| 2 | 1.00 | 0.0 | 0.58 | 1.00 | 1.00 | 1.00 | 0.73 | 0.90 | 0.97 | 0.91 | 0.63 | 0.97 |
| 3 | 0.93 | 0.42 | 0.0 | 0.88 | 0.98 | 0.78 | 0.57 | 0.84 | 0.72 | 1.00 | 0.89 | 0.77 |
| 4 | 0.78 | 0.0 | 0.12 | 0.0 | 0.31 | 0.57 | 0.26 | 0.02 | 0.08 | 0.53 | 0.51 | 0.33 |
| 5 | 0.08 | 0.0 | 0.02 | 0.69 | 0.0 | 0.29 | 0.16 | 0.54 | 0.03 | 0.95 | C. 44 | 0.40 |
| 6 | 0.60 | 0.0 | 0.22 | 0.43 | 0.71 | 0.0 | 0.12 | 0.64 | 0.19 | 0.95 | 0.35 | 0.30 |
| 7 | 0.96 | 0.27 | 0.43 | 0.74 | 0.84 | 0.88 | 0.0 | 0.58 | 0.98 | 0.98 | 0.90 | 0.89 |
| 8 | 0.91 | 0.10 | 0.16 | 0.98 | 0.46 | 0.36 | 0.42 | 0.0 | 0.07 | 0.98 | 1.00 | 0.73 |
| 9 | 0.97 | 0.03 | 0.28 | 0.92 | 0.97 | 0.81 | 0.02 | 0.93 | 0.0 | 0.95 | 0.0 | 94 |
| 10 | - 0.68 | 0.09 | 0.0 | 0.47 | 0.05 | 0.05 | 0.02 | 0.02 | 0.05 | 0.0 | 0.02 | . 18 |
| 11 | 0.72 | 0.37 | 0.11 | 0.49 | 0.56 | 0.65 | 0.10 - | 1.00 | 1.00 | 0.98 | 0.0 | 54 |
| 12 | 0.99 | 0.03 | 0.23 | 0.67 | 0.60 | 0.70 | 0.11 | 0.27 | 0.06 | 0.82 | 0.46 | 0.0 |

CELL SAMPLE SIZE

|  | 1 | 2 | 3 | 4 | 5 | 0 | 7 | 8 | 9 | 10 | 11 | 12 |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
|  | 1 | 0 | 351 | 408 | 159 | 157 | 126 | 531 | 350 | 1270 | 237 | 149 |
| 2 | 351 | 0 | 459 | 345 | 345 | 345 | 182 | 382 | 362 | 387 | 214 | 363 |
| 3 | 408 | 459 | 0 | 434 | 425 | 325 | 209 | 248 | 472 | 384 | 170 | 453 |
| 4 | 159 | 345 | 434 | 0 | 159 | 168 | 302 | 326 | 1324 | 255 | 193 | 363 |
| 5 | 157 | 345 | 425 | 159 | 0 | 111 | 191 | 142 | 1188 | 111 | 68 | 250 |
| 6 | 126 | 345 | 325 | 168 | 111 | 0 | 237 | 108 | 180 | 98 | 79 | 238 |
| 7 | 531 | 182 | 269 | 302 | 191 | 237 | 0 | 93 | 564 | 549 | 497 | 585 |
| 8 | 350 | 382 | 248 | 320 | 142 | 108 | 93 | 0 | 1132 | 326 | 0 | 423 |
| 9 | 1270 | 362 | 472 | 1324 | 1188 | 180 | 564 | 1132 | 0 | 1277 | 662 | 1299 |
| 10 | 237 | 387 | 384 | 255 | 111 | 98 | 549 | 326 | 1277 | 0 | 568 | 318 |
| 11 | 149 | 214 | 170 | 193 | 68 | 79 | 497 | 0 | 662 | 568 | 0 | 164 |
| 12 | 300 | 363 | 453 | 363 | 250 | 238 | 585 | 423 | 1299 | 318 | 164 | 0 |


|  |  |  |  |  |  |  |  |  |  |  |  |  |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
|  | 1. | 2 | 3 | 4 | 5 | 0 | 7 | 8 | 9 | 10 | 11 | 12 |
| 1 | 0. | 0. | 30. | 30. | 51. | 51. | 21. | 30. | 40. | 70. | 38. | 1. |
| 2 | 349. | 0. | 265. | 344. | 344. | 344. | 133. | 344. | 350. | 349. | 135. | 349. |
| 3 | 369. | 191. | 0. | 374. | 379. | 243. | 150. | 203. | 332. | 374. | 150. | 344. |
| 4 | 118. | 0. | 50. | 0. | 50. | 91. | 78. | 4. | 106. | 123. | 96. | 117. |
| 5 | 105. | 0. | 6. | 105. | 0. | 32. | 30. | 75. | 30. | 105. | 30. | 101. |
| 6 | 72. | 0. | 72. | 69. | 76. | 0. | 26. | 68. | 31. | 87. | 26. | 68. |
| 7 | 477. | 47. | 116. | 216. | 159. | 202. | 0. | 54. | 518. | 502. | 415. | 487. |
| 8 | 318. | 37. | 40. | 318. | 64. | 39. | 39. | 0. | 76. | 318. | 0. | 310. |
| 9 | 1218. | 8. | 130.1208 .1152. | 142. | 10.1054. | 0.1211. | 0.1216. |  |  |  |  |  |
| 10 | 61. | 35. | 0. | 18. | 4. | 4. | 12. | 6. | 59. | 0. | 12. | 55. |
| 11 | 102. | 76. | 16. | 91. | 35. | 45. | 45. | 0. | 647. | 548. | 0. | 86. |
| 12 | 234. | 10. | 102. | 217. | 145. | 162. | 60. | 111. | 70. | 234. | 70. | 0. |

PROPORYIONS MATRIX

|  | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 1 | 0.0 | 0.0 | 0.08 | 0.20 | 0.33 | 0.41 | 0.04 | 0.09 | 0.03 | 0.53 | 0.27 | 0.00 |
| 2 | 1.00 | 0.0 | 0.58 | 1.00 | 1.00 | 1.00 | 0.74 | 0.90 | 0.98 | 0.91 | 0.64 | 0.97 |
| 3 | 0.92 | 0.42 | 0.0 | 0.88 | 0.98 | 0.77 | 0.56 | 0.84 | 0.72 | 1.00 | 0.90 | 0.77 |
| 4 | 0.80 | 0.0 | 0.12 | 0.0 | 0.32 | 0.57 | 0.27 | 0.01 | 0.08 | 0.87 | 0.51 | 0.35 |
| 5 | 0.67 | 0.0 | 0.02 | 0.68 | 0.0 | 0.30 | 0.16 | 0.54 | 0.03 | 0.96 | 0.46 | 0.41 |
| 6 | 0.59 | 0.0 | 0.23 | 0.43 | 0.70 | 0.0 | 0.11 | 0.64 | 0.18 | 0.96 | 0.37 | 0.30 |
| 7 | 0.96 | 0.26 | 0.44 | 0.73 | 0.84 | 0.89 | 0.0 | 0.58 | 0.98 | 0.98 | 0.90 | 0.89 |
| 8 | 0.91 | 0.10 | 0.16 | 0.99 | 0.46 | 0.36 | 0.42 | 0.0 | 0.07 | 0.98 .1 .00 | 0.74 |  |
| 9 | 0.97 | 0.02 | 0.28 | 0.92 | 0.97 | 0.82 | 0.02 | 0.93 | 0.0 | 0.95 | 0.0 | 0.95 |
| 10 | 0.47 | 0.09 | 0.0 | 0.13 | 0.04 | 0.04 | 0.02 | 0.02 | 0.05 | 0.0 | 0.02 | 0.19 |
| 11 | 0.73 | 0.36 | 0.10 | 0.49 | 0.54 | 0.63 | 0.10 .1 .00 | 1.00 | 0.98 | 0.0 | 0.55 |  |
| 12 | 1.00 | 0.03 | 0.23 | 0.65 | 0.59 | 0.70 | 0.11 | 0.26 | 0.05 | 0.81 | 0.45 | 0.0 |

CELL SAMPLE SIZE

|  | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| 1 | 0 | 349 | 399 | 148 | 156 | 123 | 498 | 348 | 1258 | 131 | 140 | 235 |
| 2 | 389 | 0 | 456 | 344 | 344 | 344 | 180 | 381 | 358 | 384 | 218 | 359 |
| 3 | 399 | 456 | 0 | 424 | 385 | 315 | 266 | 243 | 462 | 374 | 166 | 446 |
| 4 | 148 | 344 | 424 | 0 | 155 | 160 | 294 | 322 | 1314 | 141 | 187 | 334 |
| 5 | 156 | 344 | 385 | 155 | 0 | 108 | 189 | 139 | 1182 | 109 | 65 | 246 |
| 6 | 123 | 344 | 315 | 160 | 108 | 0 | 0 | 228 | 107 | 173 | 91 | 71 |
| 7 | 498 | 180 | 266 | 294 | 189 | 228 | 0 | 93 | 528 | 514 | 460 | 547 |
| 8 | 348 | 381 | 243 | 322 | 139 | 107 | 93 | 0 | 1130 | 324 | 0 | 421 |
| 9 | 1258 | 358 | 462 | 1314 | 1182 | 173 | 528 | 1130 | 0 | 1270 | 647 | 1286 |
| 10 | 131 | 384 | 374 | 141 | 109 | 91 | 514 | 324 | 1270 | 0 | 560 | 289 |
| 11 | 140 | 211 | 166 | 187 | 65 | 71 | 460 | 0 | 647 | 560 | 0 | 156 |
| 12 | 235 | 359 | 446 | 334 | 246 | 230 | 547 | 421 | 1286 | 289 | 156 | 0 |

being dropped from the analysis because the corresponding d* value is less than the new jnd. Under the ideal condition of an isotropic population surface such a condition would not occur, however, under real conditions it may often be noted.

The general decline in levels of agreement as the value of $b$ increases provides supporting evidence for the attraction scale's independence of the actual magnitude of the origin-site flows. As $b$ tends to 0 , the effects of large numbers of individuals choosing site $j$ over any other site $k$ tend to stabilize at a value of 1 , thus inferring that only one subject from each location $i$ has been sampled. As $b$ increases, on the other hand, increased importance is given to those flows which emanate from the larger cities, thus slanting the attraction scale to the viewpoint of the resident of the large urban area. The high rank correlations evident from the comparison of the site rankings corresponding to each of the measures of agreement defined indicate that the attraction scale is relatively invariant under flow transformations of this type.

The stability of the attraction scale under different transformations of $\mathrm{d}^{\star}$ is encouraging in that it infers that the extra information added to the analysis by considering the extra distance an individual travels to patronize a chosen site is not necessary for the determination of ordinal rankings.

## Discussion

The major conclusion to be drawn from the results of the testing of various modifications of the behavioural assumptions embedded
in the attraction index methodology is that, by and large, the modifications have had little effect on the rank ordering of the site attraction indices. The Spearman rank correlation coefficient between the ranking which gave the maximum degree of agreement with the inferred judgements of the individuals sampled and that which had the lowest "fit" was .972. Although a range of almost $16 \%$ was observed in the levels of agreement, it was shown that a few abnormally high values due to peculiarities in the data accounted for a great deal of this variation. Furthermore, it is evident that use of the weighting algorithm did little to change the site ranking defined.

The question of whether or not a researcher should attempt to transform the data in such a way as to attain the highest level of agreement remains unanswered. Although it has been shown above that such an attempt did not result in appreciable changes in the site rankings defined from the present data set, it is conceivable that for certain data sets the results may be very different. The possibility does exist, of course, that the analysis of an aggregated data set such as this one may in reality be the analysis of two dissimilar data sets which have been lumped together by the method of data collection. The separation of the Saskatchewan data into that concerning the individuals from large urban areas (however defined) and smaller towns and hamlets might well produce different orderings which would reveal the differences between the preferences of the two groups. This approach has not been followed here because of the exploratory nature of the study, but is certainly a factor which will bear investigation at a later date.

The extent to which the above analysis is data dependent is at
present unknown, a further data set for a replicatory study being unavailable at this time; however, it is suggested that such results would not differ greatly from those presented above. Cheung (1970), in his study of attendance at the twelve Saskatchewan parks treated in this study, used Equation 1.11 to define attraction indices from activity and facility information. The ranking of his site attraction indices, Table 4.12, is not significantly correlated with those defined from the analysis performed above. The shortcomings in his method of deriving attraction indices have been dealt with above.

## TABLE 4.12 -- CHEUNG ATTRACTION INDICES: SASKATCHEWAN

| Park | Attractivity |
| :---: | :---: |
| 1 | 96.12 |
| 2 | 45.26 |
| 3 | 126.40 |
| 4 | 112.05 |
| 5 | 76.56 |
| 6 | 61.46 |
| 7 | 88.75 |
| 8 | 113.11 |
| 9 | 96.10 |
| 10 | 59.01 |
| 11 | 104.28 |
| 12 | 26.60 |

Résumé
The initial analysis of the Saskatchewan data set indicated that $66.88 \%$ of the inferred judgements contained in the comparison matrix could be predicted correctly from the attraction scale defined, although a low degree of unanimity was observed in the proportions matrix, indicating that a large number of the sites were not completely discriminable. The nature of the original data, being collected at the alternative sites rather than at the individual's residences, was such that it obviously did not contain information regarding the vast number of sites which were not included in the CORDS survey. This fact does not invalidate the attraction scale defined, but merely contributes to the lack of unanimity of the proportions matrix on which it is based.

The testing of several behavioural assumptions embedded in the methodology revealed that, although site orderings which raised the level of agreement could be defined, these were very similar to those defined under. the original assumptions. This result should not be interpreted to mean that the different orderings defined are incorrect. Each is correct under its own behavioural assumptions. The added explanatory power of the best fitting scales defined from the weighted data, omitting the obvious anomalies caused by presumably unique data conditions, was generally little, and does not appear to justify the stronger behavioural assumptions upon which they are based.

The first four chapters of this work have demonstrated the need for an analytical method of measuring site attraction, outlined a method by which such measures may be defined from observations of
spatial behaviour patterns, and presented the results of applications of the method to different data sets. The final chapter will present an overall summary of the work, some conclusions which may be drawn from it, and suggestions as to the direction in which future research in this area should be directed.

## chapter v -- CONCLUSION

The presentation of analytic methodology for the determination of measures of inherent site attraction from observations of spatial behaviour patterns was the topic of the second chapter of this thesis. The third and fqurth chapters were concerned with the application of the method to the problem of deducing site attraction indices for different types of recreation sites in different spatial contexts. This final chapter will discuss the foregoing analyses, summarize the study, set forth and discuss the conclusions that have been reached, and make suggestions as to the directions in which further research on the measurement of attractivity and prediction of spatial choice might be profitably conducted.

## Further Discussion and Summary

The foregoing analyses have indicated that the determination of attraction scales which are highly consistent with observed spatial behaviour patterns is possible through the use of the proposed method0logy. Throughout the work, however, analyses have been plagued by sub-optimal spatial arrangements of the alternative sites and the residences of individuals who patronize these sites, as well as by the question of the reliability of the data. In order to demonstrate that a site ordering highly cornelated with the "correct" ordering can be recovered from data concerned with spatial behaviour, the following
experiment was conducted.
A set of twenty sites were located randomly in a hypothetical region one hundred units square. Each site $j$ was assigned an attraction value ( $A_{j}$ ). Five hundred individuals were then located randomly in this space (Fig. 5.1) each being considered to patronize the site which maximized the utility function

$$
u_{i j}=A_{j} / D_{i j}
$$

$$
\text { where } \begin{aligned}
U_{i j}= & \text { the utility of the recrea- } \\
& \text { tion experience of subject } \\
& \mathbf{i} \text { at site } j
\end{aligned}
$$

In this case, distance was calculated on the Manhattan metric. The comparison matrix tabulated from these data (Table 5.1) reveals an average of .63 inferred comparisons per subject, the largest number between any site $\mathbf{i}$ and any other site $\mathbf{j}$ being the 30 between sites 17 and 3, whilst 139 of the 190 possible comparisons could not be inferred to have been made. The proportions matrix (Table 5.2) contains 102 valid entries, all of which have values of either 0 or 1 . The symbolic portrayal of the reordered proportions matrix (Table 5.3) shows the missing entries to be concentrated mainly in the region of the lowest ranking sites. Inspection of the perfectly transitive proportions matrix revealed the 149 determinable triads to be distributed as shown in Table 5.4, while reinspection of the 316 inferred comparisons revealed none to be in disagreement with the attraction scale defined. The eleven attraction indices defined (A1) are presented with site attendance, the average distance from all individuals to each site,


$$
\begin{aligned}
& \text { moosoososonomosoóosma }
\end{aligned}
$$



## TABLE 5.3 -- SYMBOLIC MATRIX: SIMULATED



```
LEGEND *
\(\because \quad \because \quad 0\)
\(\because \quad 00.4\)
\(\because \quad . \quad .6=16\)
```

A blank cell in the matrix indicates a missing comparison.
the assigned attraction value, and attraction measure A3 (as defined in Chapter III) in Table 5.5, while the Spearman rank correlations between these measures are tabulated in Table 5.6.

TABLE 5.4 -- CONSISTENCY PARAMETERS: SIMULATED
Matrix order 20

Total number of triads 1140
Number of transitive triads 40
Number of intransitive triads 0

Number of incomplete transitive triads 159
Number of indeterminate triads 941
Number of random simulations 20
Mean number of simulated intransitive triads 10
Standard deviation of numbers of
simulated intransitive triads

The strong correlation between the assigned attraction scores and the ordinal attraction indices (Al) indicates that the original site ordering has been recovered quite well. Indices could not be calculated for nine sites which were so located that no individual who patronized them could be inferred to have rejected other sites in their favour. This problem could be overcome by increasing the number of randomly generated spatial behaviour patterns analyzed. It is clear that the site ordering defined is not the only one which would yield an eta of 1.00 for this data set. When all $n(n-1)$ entries in a perfectly transitive comparison matrix are occupied, only one ordering exists

TABLE 5.5 -- MEASURES OF SITE ATTRACTION: SIMULATED

| Number | Actual Attraction | Attendance | Average Distance | Al | A3 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 20 | 2000 | 22 | 71.78 | 1.000 | 1579 |
| 19 | 1900 | 66 | 52.73 | 1.000 | 3480 |
| 16 | 1600 | 64 | 61.76 | 0.900 | 3953 |
| 13 | 1300 | 51 | 67.70 | 0.750 | 3453 |
| 18 | 1800 | 28 | 71.90 | 0.750 | 2013 |
| 17 | 1700 | 81 | 62.01 | 0.667 | 5022 |
| 11 | 1100 | 67 | 58.89 | 0.625 | 3946 |
| 15 | 1500 | 33 | 72.76 | 0.600 | 2401 |
| 14 | 1400 | 18 | 73.08 | 0.500 | 1315 |
| 9 | 900 | 18 | 55.99 | 0.333 | 1008 |
| 12 | 1200 | 30 | 67.42 | 0.250 | 2023 |

TABLE 5.6 -- ATTRACTION CORRELATIONS: SIMULATED

|  | Attendance | Distance | Assigned <br> Attraction | A1 | A3 |
| :--- | :---: | :---: | :---: | :---: | :---: |
| Attendance | 1.000 | -.469 | .137 | .339 | .970 |
| Distance | -.469 | 1.000 | .182 | -.160 | -.391 |
| Assigned |  | .137 | .182 | 1.000 | .795 |
| Attraction | . .339 | -.160 | .795 | 1.000 | .365 |
| A1 | .970 | -.391 | .173 | .365 | 1.000 |
| A3 | . .970 |  |  |  |  |

which will achieve perfect agreement. When the matrix is incomplete, however, there may be many such orderings. The fact that the recovered scale does not agree perfectly with the attraction ranking is attributable to the missing entries in the $P^{*}$ matrix. The assigned ordering could be recovered by increasing the sample size until all possible comparisons had been made but this was not attempted in this study. The low correlation between Al and the distance variable tends to support the statement that the scale is free of distance effects.

Throughout the analyses discussed in Chapters III and IV of this study, it was emphasized that the spatial arrangement of the individuals and their alternative sites is critical to the deviation of accurate site rankings. The results achieved in the above simulation indicate that when the two are interspersed the rankings tend to agreement.

It is evident that spatial bias cannot exist when the behaviour of the sample individuals is in accord with the postulates upon which the model is based. When the postulated conditions are not met, however, the amount of spatial bias should be established. Although it is beyond the scope of this study to devise a coefficient to measure such bias, it is believed that one could be established, and its distribution identified through the use of Monte Carlo simulation techniques.

Rushton (1972) has suggested that the model can be made less sensitive to such spatial bias by rewriting Equation 2.1 as:


$$
\begin{aligned}
\text { where } N_{i j}= & \text { the number of individuals } \\
& \text { having the opportunity of } \\
& \text { preferring } \mathbf{i} \text { to } j \text {, and all } \\
& \text { other symbols are as in } \\
& \text { Equation 2.1. }
\end{aligned}
$$

Consider the following example where alternative sites $A$ and $B$ and origins 1 and 2 are situated in the manner shown in Figure 5.2a, and the values of the symbols $C$ and $N$ are:

$$
\begin{aligned}
& C_{A B}=100 \\
& C_{B A}=100 \\
& N_{1}=1000 \\
& N_{2}=1000
\end{aligned}
$$

Calculation of the P* value according to Equation 2.1 would yield the results
$P_{A B}^{*}=\frac{C_{A B}}{C_{A B}+C_{B A}}=\frac{100}{100+100}=.5$
The use of Rushton's modification would yield
$P_{A B}^{*}=\frac{C_{A B} / N_{2}}{C_{A B} / N_{2}+{ }_{C B} / N_{1}}=\frac{100 / 1000}{100 / 1000+100 / 1000}=.5$
Thus, in this situation buth approaches give the same result. However, if one changes the relative size of one of the origins maintaining the proportionality of the comparisons, the results will disagree. For example, if the assigned values are:

$$
\begin{aligned}
& C_{A B}=100 \\
& C_{B A}=10 \\
& N_{1}=100 \\
& N_{2}=1000
\end{aligned}
$$

under Rushton's suggestion
$P_{A B}^{*}=\frac{100 / 1000}{100 / 1000+10 / 100}=.5 \quad$ while Equation 2.1 would yield
$P_{A B}^{*}=\frac{100}{100+10}=.91$

As Rushton points out, the value of any $\mathrm{P}_{\mathrm{ij}}^{*}$ can therefore be manipulated by modifying the number of individuals sampled at certain locations, thus modifying the attraction scale.

This solution clearly goes a long way towards removing the effect of spatial bias by incorporating in each comparison the number of individuals in a position to make that judgement. In the London case, for example, many entries in the $\mathrm{P}^{*}$ matrix appear unanimous simply because it was impossible for any individual to make an opposing judgement. Such cases would clearly be disallowed under the Rushton proposal.

However, referring to Figure 5.2b, consider the effect of the position of point 2 relative to $B$. If the value of $C_{A B}$ remains constant as point 2 approaches $B$, the $P_{A B}^{*}$ given by Rushton's modification remains constant and so, therefore, does the eventual judgement about the relative attractions of $A$ and $B$. But the closer 2 lies to $B$, the greater A's attraction must be if it is to draw the same proportion of people. This contradiction appears to be a serious weakness of his suggestion. The unanimity of the $P^{*}$ matrix in the simulated case is a direct function of two.factors: each simulated individual attempted to maximize the same utility function; and each can be considered to have perceived the same attraction value for any alternative site $j$. In the real world, these conditions are not always met, causing individuals faced with identical alternative sets to make different decisions as to which are most preferable. The first of these factors need not concern us unduly as it is not necessary that all individuals being studied have the same utility function. The ordinal approach requires only that the function of distance utilized is monotonically increasing. The second
factor is more critical when studying real data. It is not reascnable for one to expect that all individuals would assign the same attraction value to any given alternative site. It

FIGURE 5 . 2 -- AREAL DISTRIBUTION OF SUBJECTS AND SITES: HYPTHETICAL
(a)

(b)

is more likely that an individual's estimate of attraction is based on the particular site facilities concerned with the recreation activity he is most interested in , rather than on the whole compage of site factors. Additionally, the estimate he assigned is bound to be based on conditions which existed at the site during past visits, rather than those existing at the time of estimation. These "noise" factors militate against achieving perfectly unanimous site attraction scores in the majority of real world situations, yet where the activity to be engaged in is well-defined (as would be the case in the study, for example, of swimming), their effects should be minimal.

## Conclusions

The purposes of this study have been twofold. The first was to present a methodology designed to isolate ordinal attraction indices for service-providing sites from data concerned with the spatial behaviour of individuals who patronize these sites. The second was to apply the proposed technique to data drawn from different geographic situations in order to test the usefulness of the model.

It was made evident from a brief review of the literature on the general topic of site attendance prediction that no generally applicable technique for determining measures of inherent site attraction had previously been defined, and that a methodology which achieved this goal would contribute substantially to the study of consumer spatial behaviour.

The general conclusion reached as a result of the foregoing analyses is that the methodology proposed is capable of extracting
ordinal scales which are in strong agreement with the true site orderings when the three main assumptions of the model are satisfied. Whilst the determination of the amount of bias introduced by data which does not completely fulfill these assumptions was not attempted, it is possible to say that in certain situations the bias may be quite serious. Such bias, being a function of the test data rather than any inherent weakness in the methodology presented, was not treated in the study but might well be investigated in future work.

The susceptibility of the methodology to bias caused by particular spatial arrangements of the origins of the sample individuals and their alternative opportunities is the cause of somewhat more concern. This problem, which was discussed in some detail in the third chapter, caused serious problems with the London and Sarnia data, but did not appear to be of consequence in either of the other data sets treated. Graphic illustrations of the effects of certain specific spatial arrangements were presented in the third chapter of the study. They show that sampling designs which are carefully constructed can remove this type of spatial bias completely. Such designs must ensure that the spatial arrangement of the subjects and sites is such that inferred judgements ( $A_{i}>A_{j}$ ) are possible for all values of $i$ and $j$. The ordinal attraction indices defined by the methodology presented above will be of considerable use in the prediction of spatial behaviour; however, it must be made explicit that they can be used only in formulations which recognize their ordinality. Their use in gravity models such as Equation 5.1 would be incorrect, although a similar model which defined site attraction as a monotonic function of
the ordinal scale would be suitable. The scale has considerable utility in its ordinal form, although Rushton's Preference Surface Model is the only well-known attendance prediction approach which is suitable for use with untransformed ordinal attraction indices. (5.1) $V_{i j}=\frac{K P_{i} A_{j}}{D^{X}{ }_{i j}}$

$$
\text { where } \begin{aligned}
& V_{i j}= \begin{array}{l}
\text { the number of visitors from } \\
\text { city to site } j,
\end{array} \\
& P_{i}= \text { the population of city } i, \\
& A_{j}= \text { the ordinal attraction index } \\
& \text { of site } j,
\end{aligned}
$$

In view of the large number of site factors which may influence the inherent attraction of a recreation site, it is unlikely that measures of attraction such as those derived above (or monotonic transformations of them) will lead directly to methods which would allow one to calculate similar measures by techniques which combine site factors. The scale, on the other hand, can provide a great deal of insight into the site variables of importance. The qualitative comparison of sites which were ranked in the first and fourth quartiles, for example, might well lead to the formulation of testable hypotheses concerned with the importance of specific site variables.

The index of confusio., defined for each site will be of assistance in the definition of groups of sites which lie close to one another on the true attraction scale. Should two or more of the sites in one of the defined groups be of similar nature (e.g. beach-oriented picnic areas), yet have widely varying amounts of particular site
attributes, insight of the importance of varying site factors (which are combined in such a way as to make the sites indiscriminable) may be gained. These indices also yield information regarding the relative spacing of the sites on the true attraction scale, and are thus of assistance in attempts to define transformations of the ordinal scale which might be used in attendance prediction trials.

The intransitivities identified in the proportions matrix $P^{*}$ may be utilized to identify sites about which the individuals sampled have difierent notions of attractiveness. Consider the following case. If the proportions matrix, the entry $P_{i j}^{*}$ is an intransitivity with the value of 1.00 . This means that on the ordinal scale defined $A_{j}>A_{i}$, but those individuals who patronized $\mathfrak{i}$ invariably judged $A_{i}>A_{j}$. The question, "How and why did these individuals' notions of the relative attractiveness of sites $i$ and $j$ differ from those of individuals who did not patronize site i?" may now be asked. There are, of course, many possible answers to such questions, but the irfiportant point is that not only has an atypical site compariscn been identified, but the individuals who consider it to be atypical have also been identified.

The foregoing study has raised several intriguing questions whic should be investictated in some depth. Six of these are discussed briefly below.

Sugcestions for Further Research
As the ordinal attraction indices defined by the foregoing method are by definition attributable to the characteristics of the
sites alone, one of the next phases in this study might well be the statistical explanation of these site attraction indices. This objective is hampered by the ordinal nature of the attraction indices themselves, and also by the fact that although many site variables (stimuli) may be measured on metric scales, it is not clear that human response to these is a simple function of the stimuli magnitudes. Ordinal techniques for multiple correlative approaches being presently undeveloped, this problem would be very difficult to solve, although the rewards of $i$ ts solution would be very high.

The establishment of a ratio index of inherent site attraction should be given a high priority in future research. Such a scale, more powerful and intuitively much more satisfying, would enable one to calculate trip utilities exactly, thus permitting spatial choices to be predicted on a probabilistic rather than a deterministic basis. Service trips emanating from origin $\mathbf{i}$ could be expected to be distributed in such a way that the probability of any alternative $j$ being patronized was a function of ratio of the utility of a trip to $j$ and the summed utilities of trips to all viable alternatives. The degree to which the expected origin-destination flows agreed with those observed might then be taken to be a measure of how well the utility function being tested modelled the composite utility function of the sample group being studied.

Assuming the utility function $U_{i j}=f\left(A_{j}, D_{i j}\right)$, and that each individual can solve the function for all viable alternative sites, there are bound to be many instances in which the $U$ values are very nearly equal. These are the values which would lie close to the same
contour on the individual's utility surface. It is reasonable to expect that if any $U_{i j}-U_{i k}$ is very small, the individual will not be able to consistently identify the largest of the two utilities as being the greatest. If, because of his inability to invariably choose the most utile alternative, the individual chooses to patronize a site for which the utility is slightly less, he cannot really be said to have chosen irrationally. The question is how to find the difference in magnitudes which the individual will perceive as really being different. The problem may be approached by following Weber's (see Bock and Jones, 1968) suggestion that a just noticeable difference (jnd) is a function of the change in stimulus with respect to the magnitude of the stimulus. One possible course of action would be to examine a well-structured data set and extract the value $e_{i j}=\frac{U_{i}-U_{j}}{\frac{\left(U_{i}+U_{j}\right)}{2}}$

$$
\text { where } \begin{aligned}
U_{i} & = \\
& \text { the utility of a trip to } \\
& U_{j}= \\
& \text { the utility of a trite and tronized site, }
\end{aligned}
$$

for all individuals who choose to patronize a less utile site. These values, if plotted in a cumulative frequency diagram, would allow one to define the value of the jnd by drawing a cutoff at the $50 \%$ point. Alternatively, they might be plotted as a histogram and the cutoff drawn subjectively. The jnd thus defined might then be used when assessing the fit of postulated utility functions by recording an "incorrect" choice as "correct" if the magnitude of the associated $\mathrm{e}_{\mathbf{i j}}$ was less than one jnd .

Students of recreation research have almost universally considered distance to be a factor inhibiting travel to recreation
alternatives. This is clearly a very naive assumption, although undoubtedly necessitated by the lack of suitable information upon which to base a method of defining the true effects of increasing distance. It is evident that for some types of recreation trips certain portions of the trip may be regarded as positive rather than negative stimuli (Keough, 1969). Each portion has a degree of positivity or negativity which is a function of the amount it deviates from the ideal distance the individual would most prefer to travel to engage in that type of recreation activity. If the distance at which the stimulus of additional travel changes from positive to negative can be identified, the effect of a certain distance may be redefined as being a function of the absolute value of this distance minus the value of the distance at which the stimulus changes sign. This concept may be shown graphically in Figure 5.2, where it can be seen that the problem very much resembles the " j scale" problems of Coombs (1964) and other psychologists. Although it is not clear that it could be solved by existing psychometric techniques, it would seem that an initial approach along these lines would prove beneficial to the study of human spatial movements.

The investigation of the distortions in attractivity scales which are caused by various different spatial arrangements of individual's origins and their alternative sites would be of great interest. In the third chapter of this study, it was suggested that the correlation between the average distance from all individuals to each site and the ordinal attraction index of that site could be interpreted as a measure of the degree to which the origins were clustered with

## FIGURE 5.3

## THE ATTRACTIVENESS OF TRAVEL


$D_{i j}^{*}=\left|D_{i j}-D_{i m}\right|=D_{j m}$
$D_{i k}^{*}=\left|D_{i k}-D_{i m}\right|=D_{k m}$
where $D_{i j}^{*}$ and $D_{i k}^{*}$ represent the "true" distance from the origin of individual $i$ to sites $j$ and $k$ respectively, and
$D_{1 m}$ the distance individual $i$ would most like to travel.
respect to the alternative sites. Although this condition would undoubtedly hold in the majority of cases, there is a finite probability that, even with the best possible arrangenent of origins and alternatives, the agreement would be perfect, causing the researcher to reject the results of the analysis. The development of a statistic which would more accurately reflect the degree to which the derived attraction indices were influenced by spatial bias inherent in the data would help to alleviate this problem.

The solution of problems such as these, if they can indeed be solved, will do much to allow us to fully comprehend patterns of the spatial interaction of individuals weighing the attraction of alternatives against measures of the costs of realizing those alternatives.

## REFERENCES

Abler, R., J. S. Adams, and P. Gould
1971: Spatial Organization. Prentice-Hall, Englewood Cliffs.
Beaman, J.
1971: Personal communication with the author.
Beaman, J.
1972: Distance and the 'Reaction' to Distances as a Function of Distance. CORD Technical Note 14, National and Historic Parks Branch, Department of Indian Affairs and Northern Development, Ottawa.

Blalock, H. M.
1960: Social Statistics. McGraw-Hill, Toronto.
Bock, R. D., and L. V. Jones.
1968: The Measurement and Prediction of Judgement and Choice. Holden-Day, San Francisco.

Cahn, R.
1968: Will Success Spoil the National Parks? Christian Science Monitor, May-August.

Carey, H. C.
1858: Principles of Social Science. Lippincott, Philadelphia.
Catton, W. R.
1966: From Animistic to Naturalistic Sociology. McGraw-Hill, Toronto.

Cesario, F. J.
1971: A New Model for Trip Distribution. A paper presented to the Regional Science Association at Ann Arbour, Nov. 14, 1971.

Cheung, $H$.
1970: A Day Use Park Visitation Model. Unpublished mimeograph, National and Historic Parks Branch, Department of Indian Affairs and Northern Development, Ottawa.

Coombs, C. H. 1964: A Theory of Data. John Wiley, New York.

Coombs, C. H., R. M. Dawes, and A. Tversky.
1970: Mathematical Psychology. Prentice-Hall, Englewood Cliffs.
Dodd, S. C.
1955: Dimensional Analysis in Social Physics, in B. Singh, ed., The Frontiers of Social Science. St. Martins, New York.

Ellis, J. B.
1967: A Systems Theory Model for Recreational Travel in Ontario: $\frac{\text { a Progress Report. }}{\text { Toronto. }}$ Ontario Department of Highways,

Ellis, J. B., and C. S. Van Doren.
1966: A Comparative Evaluation of Gravity and Systems Theory Models for Statewide Recreational Travel Flows, Journal of Regional Science, Vol. 6, 57-70.

Ewing, G. 0.
1970: An Analysis of Consumer Space Preferences Using the Method of Paired Comparisons. Ph.D. Thesis, McMaster University.

Girt, J. L.
1972: Models of Settlement Linkage in Newfoundland. Department of Geography, University of Guelph.

Gould, P. R.
1967: Structuring Information on Spatio-Temporal Preference, Journal of Regional Science, Vol. 7, 259-274.

Hays, W. L.
1967: Quantification in Psychology. Brooks-Cole, Belmont.
Kendall, M. G.
1962: Rank Correlation Methods. Hafner, New York.
Keough, B. M. .
1969: The Role of Travel in the Recreational Daytrip. M.A. Thesis, University of Western Ontario.

Kovacs, J.
1971: A Self-Administered Park Visitor Survey Technique. Technical Paper No. 1, National and Historic Parks Branch, Ottawa.

Kruskal, J. B.
1964: Multi-Dimensional Scaling by Optimizing Goodness of Fit to a Non-metric Hypothesis, Psychometrika, Vol. 29, 1-27.
Lee, T.
1970: Perceived Distance as a Function of Direction in the City, Environment and Behaviour, Vol. 2, 40-51.

Liebman, J. C., and N. Dee.
1970: A Statistical Study of Attendance at Urban Playgrounds, Journal of Leisure Research, Vol. 2, 145-159.

Lowrey, R. A.
1970: Distance Concepts of Urban Residents, Environment and Behaviour, Vol. 2, 52-73.

Luce, R. D.
1959: Individual Choice Behaviour. Wiley, New York.
Lycan, D. R.
1969: Interprovincial Migration in Canada: the Role of Spatial and Economic Factors, Canadian Geographer, Vol. 13, 237-254.

Mitchell, L. S.
1967: An Empirical Study of Urban Recreation Units. Ph.D. Thesis, Ohio State University.

Olsson, G.
1965: $\frac{\text { Distance and Human Interaction: a Bibliography and Review. }}{\text { Regional Science Institute, Pittsburgh. }}$
Polk, R. L., pub.
1970: Henderson's Sarnia, Ontario City Directory 1970, Winnipeg.
Robertson, R. W., and J. H. Ross.
1969: Toward a Recreation Feasibility Assessment Model. Geographical Studies, University of Victoria.

Rodgers, R. G. R.
1966: An Analysis of Some Elements of Demand for Ontario Provincial Parks. M.A. Thesis, University of Guelph.

Ross, J. H.
1969: Urban Vacation Hinterlands. M.A. Thesis, University of Victoria.

Rushton, G.
1969:
Analysis of Spatial Behaviour by Revealed Space Preference, American Association of Geographers, Annals, Vol. 59, 391400.

Rushton, G.
1967:
Analysis of Spatial Behaviour by Revealed Space Preference. Computer Institute for Social Science Research, Michigan State University.

Rushton, G.
1972: Personal communication.
Shafer, E. L., and G. H. Moeller.
1971: Predicting Quantitative and Qualitative Values of Recreation Participation, Recreation Symposium Proceedings. U.S.
Department of Agriculture, Upper Darby, Pa.

Shepard, R. N.
1964: On Subjectively Optimum Selection Among Multi-Attribute Alternatives, in M. W. Shelley and G. L. Bryan (eds.), Human Judgements and Optimality. Wiley, New York.

Stewart, J. Q.
1941: An Inverse Distance Variation for Certain Social Influences, Science, Vol. 93, 89-90.

Stouffer, S. A.
1940: Intervening Opportunities: A Theory Relating Mobility and Distance. American Sociological Review, Vol. 5, p. 867.

Thurstone, L. L.
1927: A Law of Comparative Judgement, Psychological Review, Vol. 34, 273-286.

Wennergren. E. B., and D. B. Neilsen.
1970: Probability Estimates of Recreation Demands, Journal of Leisure Research, Vol. 2, 112-123.

Vernon Directories Ltd., pub.
1970: London City Directory, 1970, Hamilton.
Wolfe, R. I.
1966: Parameters of Recreational Travel in Ontario: A Progress Report, Ontario Department of Highways, Toronto.
Wolfe, R. I.
1964: Perspective on Outdoor Recreation, Geographical Review, Vol. 54, 203-38.

Wolfe, R. I.
1972: The Inertia Model, Vol. 4, 73-76. Journal of Leisure Research.

I PURPOSE:
The program determines a set of ordinal attraction indices for a number of service sites, assesses the subjects agreement with the ranking derived, and appraises the degree of confusion involved in ordering the sites.

II METHOD:
Data regarding the location of the service sites and the residences of the consumers or visitors, along with information regarding the site they patronize, are analyzed in the following fashion.

Let us consider a consumer residing at location $A$, and having the option of patronizing sites at $B, C$, or $D$. If he chooses to patronize the site at $C$, when the distance from $A$ to $C$ is greater than the distance from $A$ to $B$, we may assume that he perceives the attraction of site $C$ to be greater than that of site $B$, because he is willing to travel farther to visit cite $C$. Such a condition gives us a paired comparison, $C>B$. If we inspect the spatial opportunities and choices of a group of subjects, we may tabulate a site-by-site matrix of these inferred comparisons. In this matrix, let us cail it $C$, an individual entry $C_{i j}$ will refer to the number of times site $i$ was judged more attractive than site $j$ in this manner. Note that site $j$ must be closer to the individual's residence than site $i$ before a judgement may be inferred. The $C$ matrix may then be used to determine an ordinal attractivity score for each site, the degree of confusion regarding the relative attractivity of each site, and the proportion of inferred comparisons which agree with the scale defined.

The dependence of the final scale on distance and attendance may be assessed by making the value of each increment to $C_{i j}$ a function of the extra
distance an individual has to travel to reach site $i$, and/or the number of individuals involved in this judgement. This is accomplished by setting the weighting factor ( $W_{i j k}$ ) to the value
$W_{i j k}=\sum f_{1_{j}}^{b}{ }_{j}{ }_{k}{ }_{1}^{a} j_{j}$ for all $K$ such that $A_{j}>A_{k}$ and $d^{*} \mathbf{i}_{j k}>j$ nd
where; $f=$ the number of people involved,
$\mathrm{d}^{*}=$ the extra distance,
$a$ and $b$ are preset exponents, and
$W_{i j k}=$ the increment to the $j k$ th cell of $C$ matrix contributed by residents of origin i.

Varying degrees of distance perception may be incorporated by setting a minimum value for $d^{*}$ (this is referred to as a jnd). If this jnd is not exceeded that particular increment is given a value of 1.

## III - PROGRAM OUTPUT

1) Comparison Matrix C:

Each entry $C_{i j}$ in this matrix is defined as the number of times an inferred judgement $i>j$ could be inferred from the data. (i.e. The number of times subjects were observed to have patronized site $i$ when site $j$ was closer to their residence.)
2) Proportions matrix P*

Each entry $P_{i}{ }_{j}$ is defined as

$$
P_{i j}^{*}=\frac{C_{i j}}{C_{i j}+C_{j i}}
$$

(should any pair of sites not have been compared the $\mathrm{P}_{\mathrm{i} j}$ is set to -1.00)
3) Symbolic $P^{*}$ matrix:

Each entry in this alphanumeric matrix has been assigned a character on the basis of the corresponding $P^{*}$ value. The matrix is permuted according to the attraction scale defined, comparisons between the most attractive sites being placed at the top left.
4) Ordinal attraction scores $A_{1}$

Each attraction score $A_{i}$ is defined as:

$$
\begin{aligned}
& A_{i}=\sum_{j=1}^{\eta} e_{i j} P_{i j}^{\star} /{ }_{j}^{\sum_{i=1}^{n}} e_{i j} \\
& \text { Where: } \quad A_{i}=\begin{array}{l}
\text { the attraction } \\
\text { index of site } i,
\end{array} \\
& \mathrm{~N}=\text { the total number } \\
& \text { of sites, and }
\end{aligned}
$$

5) Indices of Confusion - MU

Each confusion index $M_{i}$ is defined as:
$\left.M U_{i}=1-\sum_{j=1}^{n}\left(.5-\left|.5-e_{i j} P_{i j}^{*}\right|\right) / \sum_{j=1}^{n} e_{i j}\right)$

$$
\text { Where: } \begin{aligned}
M U_{i}= & \text { the confusion index } \\
& \text { of site } i \text { as measured } \\
& \text { on an interval scale. }
\end{aligned}
$$

6) The coefficient of agreement - eta

Eta is defined as:
 where eta $=$ the coefficient of agreement, and
$C_{i j}=\begin{gathered}\text { the } i j \text { th element of the } \\ \text { comparison matrix } C \text {. whic }\end{gathered}$ comparison matrix C. which has been permuted on the basis of the attraction scale defined.
7) The permuted comparison matrix $\mathrm{C}^{\prime}$

The matrix $C$ is reordered according to the attraction scale defined. In a perfect case, i.e. when eta takes the value 100, all entries will be above and to the right of the diagonal.

Eta and the $C^{\prime}$ matrix are presented for the derived ranking and for all subsequent assessed rankings. The other output is presented only once for each combination of weighting factors.

IV - PROGRAM LOGIC

1) Read parameter cards A-H, initialise.

Read parameter card I if necessary.
2) Read site data and store coordinates.
3) Read visitors data, one record at a time.

For each record;
i) calculate the distance to each alternative site $\mathbf{j}$ as

$$
D_{j}=\left(\left(Y_{j}-Y_{i}\right)^{M}+\left(X_{j}-X_{i}\right)^{M}\right)^{1 / M}
$$

if LDIST $\neq 1$, otherwise $\operatorname{read} D_{j}, j=1$, NP from unit LUNIT
ii) Denoting the patronized site as $n$, create a vector $N P$ in length, the ith value (IFLG.) being set to 0 if $D_{i}<D_{n}$, and 2 if $D_{i}>D_{n}$. Set $I F L G_{n}$ to 1 .
iii) For each of the NPOW (NROOT) weighting combinations specified calculate the weight $W_{i j k}$ (see section II).
iv) Write the IFLG vector to disk (Fortran unit 9), appending weighting factors and the subscript of the selected site.
4) Rewind unit 9 .
5) Tabulate comparison matrix $C$ :

For each record on unit 9 -
increment $C_{i j}$ by the weight of concern each time IFLG $_{j}=0$
Print. C matrix
Write $C$ matrix to disk unit 8
6) Calculate proportions matrix $P^{*}$ :

See section III (2)
Print $\mathrm{P}^{*}$ matrix.
7) Calculate Attraction scores:

See Section III (4)
8) Calculate Indices of confusion

See Section III (5)
Print permuted symbolic matrix, legend, Attraction scores and confusion indices.
9) Calculate eta by summing the upper right half of the reordered $C$ matrix, dividing this sum by the sum of the complete matrix, and expressing the result as a percentage.
See Section III (6)
10) Print permuted matrix for a visual check.
11) If more than one set of weighting factors is being used respeat steps 4 to 10 for each set.
12) If input rankings are to be assessed, repeat steps 9 and 10 for each ranking.

V - PROGRAM USE

1. Parameter cards

| Card | Cols. | Name | Purpose |
| :---: | :---: | :---: | :---: |
| A. | 1-2 | LIN | Input logical Fortran unit number for sites and visitors data sets. |
|  | 3-4 | LOUT | Output flag to control printout. |
|  |  |  | $\begin{aligned} \text { if Lout } & =99 \text { Individual choice lines are printed } \\ & =98 \text { Comparison matrix is printed } \\ & =97 \text { Proportions matrix is printed } \\ & =96=99,98, \text { and } 97 \end{aligned}$ |
|  |  | - | = $94=99$ and 97 |
|  |  |  | $=93=98$ and 97 |
|  |  |  | $=92 \text { none of these }$ $=\text { less than } 92=92$ |

5-7 NP Sets the number of sites in the system being studied. (Max = 100)

8-13 XMAX Set. maximum allowable distance a visitor may travel to any site. If locational coordinates are rectangular XMAX must be specified in coordinate units - if coordinates are entered as longitude and latitude XMAX should be expressed in miles.
B. 1-50 IFM Object time format for site data. Must read two real numbers.

| Card | Cols. | Name | Purpose |
| :---: | :---: | :---: | :---: |
| C. | 1-50 | IFM1 | Object time format for visitors data. Must read origin ID number, $X$ and $Y$ coordinates, and the flows to all sites. ( 1 integer, 2 real numbers, and NP integer fields) |
| D. | 1-50 | ITIT | Title for the job. |
| E. | 1-6 | XJND | Value of just noticeable difference. |
|  | 7-8 | ICHEK | Number of specified rankings to be assessed. |
|  | 9-10 | ITYP | Set to 1 if coordinates are in latitude and longitude. |
| F. | 1-5 | NPOW | Number of distance powers to be used. (Max $=15$ )* |
|  | 6-10 | POWER (1) | First distance power. |
|  | 11-15 | POWER(2) | Second distance power. |
|  | etcetera |  | etc. |
| G. | 1-5 | NROOT | Number of flow powers to be used. ( $\operatorname{Max}=15)^{*}$ |
|  | 6-10 | ROOT (1) | First flow power. |
|  | 11-15 | ROOT (2) | Second flow power. |
|  | etcetera |  | etc. |
| H. | 1-2 | MIN | Minkowski metric ... must be integer. (set to 01 for Manhattan distances, 02 for Pythagorean) |
|  | 3-4 | LDIST | Set to 1 if precalculated distance file is to be read in from logical unit LUNIT (see below). |
|  | 5-6 | LUNIT | Input logical Fortran unit number for distance file. |
| I. | 1-50 | IFM2 | Object time format for distance file. Must read NP real or exponential fields. |

* Although either NPOW or NROOT may have up to 15 values, NPOW (NROOT) must not exceed 15.

SITE DATA - NP records, to be read from unit LIN under format IFM.
This data set consists of one record for each site. Each record must contain the $X$ and $Y$ coordinates of the site. If geo-coords are used (ITYPE $=1$ on parameter card E) $X$ and $Y$ must be latitude and longitude respectively, expressed in decimal degrees.

VISITORS DATA - to be read from unit LIN under format IFM1.
This data set consists of one record for each origin. Each record must consist of the site identity number, the $X$ and $Y$ coordinates of the origin, and the number of individuals from that origin who patronised each site. The records must be in ID, $\mathrm{Y}, \mathrm{X}$, ( $\mathrm{N}(\mathrm{I}), \mathrm{I}=1, \mathrm{NP}$ ) form.

DISTANCE DATA - to be read from unit LUNIT under format IFM2.

- This data set consists of one record for each origin. Each record must consist of NP distances, the first being the distance from this origin to site 1 , the second to site 2, etc. Record form is $D(1), D(2), \ldots, D(N P)$.

RANKINGS DATA - ICHEK records, to be read from unit 5, under a 3013 format.

This data set consists of one record for each ranking to be assessed by the program.

VI - MODES OF OPERATION

1. Regular

The program defines attraction indices from observed flow patterns.
Either rectangular or geographic coordinate systems may be used.
Input Parameter Cards - A, B, C, D, E, F, G, H.
Input Data Sets - Sites, Visitors, Rankings (optional).
2. Distances Precalculated

As above, but precalculated distances are provided. Sites data may be blank records but must be provided. X and Y coordinates must be given for origins in Visitors data set even though they will not be used in calculation (specify dummies).
Input Parameter Cards - A, B, C,D, E, F, G, H, I.

Input Data Sets - Sites, Visitors, Distances, Rankings (optional). N.B. If LIN (card A.) = LUNIT (card H.) the Visitors and Distance records must be interleaved.

In addition to regular systems error messages, the program will alert the user to the following situations:

Message
TOO MANY PARKS
EXECUTION TERMINATED

NO LOCATION GIVEN FOR TOWN XXX

TOWN II GREATER THAN MAX.DIST.
TO PARK KK CASE REJECTED
DIST. - XXX.XXX NO. VISITORS - MM

## Meaning

NP greater than dimensioned area.
Redimension, or check parameter card 1

Coordinates missing for origin XXX Possible punching or formatting error.

Trip claimed was too long. Possible error in coordinates, or input format(s).

VIII - NOTE
In addition to the regular control cards, IBM360 users must make provision for temporary desk storage on units 8 and 9.






























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#### Abstract

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| 0065 | END |  |  |

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[^0]:    *significant at the $95 \%$ confidence level.

