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EVALUATION OF SEDIMENT TRANSPORT AND FRICTION FACTOR EQUATIONS USING MOBED

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Management Perspective

The calculation of flow depths, water levels and the rates of sediment transport in rivers require relationships which specify the flow resistance and sediment transport for given hydraulic conditions. Many such relationships have been introduced in the past and new equations are still being developed.

The mobile boundary flow model MOBED is formulated in such a way that it can use different combinations of friction factor and sediment transport equations. By conducting a set of laboratory experiments and comparing the results with simulations using MOBED, it was possible to test the validity of different combinations of these equations. The results show that certain combinations will give more realistic predictions than others. These results are useful for all who are interested in modelling of river flows and sediment transport.

Perspective de gestion

Le calcul des profondeurs de l'écoulement, du niveau de l'eau et des vitesses de transport des sédiments dans les rivières nécessite des équations qui établissent la résistance à l'écoulement et le transport des sédiments pour des conditions hydrauliques données. Un grand nombre de ces équations ont été intégrées dans des équations générales et de nouvelles sont encore en cours de préparation.

Le modèle d'écoulement limitrophe mobile MOBED est formulé de façon à pouvoir utiliser différentes combinaisons de facteurs de friction et d'équations de transport des sédiments. Après avoir fait une série d'expériences en laboratoire et comparé les résultats avec les simulations à l'aide du modèle MOBED, on a pu tester la validité des différentes combinaisons de ces équations. Les résultats indiquent que certaines combinaisons donneront des prévisions plus réalistes que d'autres. Ces résultats sont utiles à tous ceux qui s'intéressent à la modélisations des écoulements des rivières et du transport des sédiments.

Evaluation of Sediment Transport and Friction Factor Equations using MOBED

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Abstract

Three sediment transport rate equations and the same number of friction factor equations for mobile boundary channel flows were evaluated using a mobile boundary flow model called MOBED. The MOBED model was formulated in such a way that it is possible to use different friction factor and sediment transport relations easily. The model was used to simulate degradation downstream of a transition from fixed bed to movable bed in a laboratory flume using the nine possible combinations of sediment transport rate and friction factor equations. Comparing the predicted degradation and water surface elevation with the measured data, it was possible to draw conclusions regarding the suitability of different combinations of the selected equations.

Introduction

Mathematical models of river flows need equations of sediment transport and friction factor to compute the sediment transport rate and the slope of the energy-grade-line that appear in the set of governing equations. Therefore, the predictive ability of a river model depends not only on the accuracy of the numerical solution method adopted to solve the governing equations but also on the accuracy of the equations selected to calculate the sediment transport rate and friction factor. In this paper, an attempt is made to evaluae some of the available equations by adopting them in an unsteady flow model called MOBED [see Krishnappan (1981), (1983), (1985) and (1986)].

The equations selected for the evaluation are as follows:

<u>Sediment transport rate equations</u>
1) Ackers and White (1973)
2) Engelund and Hansen (1967)
3) Van Rijn (1984)

<u>Friction factor equations</u>
1) Kishi and Kuroki (1974)
2) Engelund (1966)
3) Brownlee (1983)

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Évaluation du transport des sédiments et équations du facteur de sélection à l'aide de MOBED

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Résumé

Trois équations de la vitesse de transport des sédiments et le même nombre d'équations du facteur de friction pour l'écoulement des canaux limitrophes mobiles ont été évaluées à l'aide du modèle d'écoulement limitrophe mobile MOBED. Le modèle MOBED a été formulé de façon à ce qu'il soit possible d'utiliser facilement différentes équations du facteur de friction et du transport de sédiment. Le modèle a été utilisé pour simuler la dégradation en aval d'une transition d'un lit fixe à un lit mobile dans une auge de laboratoire à l'aide des neuf combinaisons possibles de vitesses de transport des sédiments et d'équations du facteur de friction. En comparant la dégradation prévue et l'élévation de la surface de l'eau avec les données mesurées, on a pu tirer des conclusions relatives à la pertinence de différentes combinaisons d'équations choisies.

The evaluation was carried out by using MOBED with all nine possible combinations of the above equations to simulate flow conditions in a sediment transport flume in the laboratory. The salient features of MOBED and a brief description of the evaluation of selected equations are given below.

Salient Features of Model MOBED

The model MOBED is an unsteady, mobile boundary flow model capable of treating flows in a non-prismatic channel with irregular cross sections. The model solves the following set of governing equations:

$$\frac{\partial Q_s}{\partial t} + P \left(\frac{\partial z}{\partial t}\right) p + B\, C_{av} \left(\frac{\partial y}{\partial t}\right) + A \left(\frac{\partial C_{av}}{\partial t}\right) - q_s = 0 \qquad (1)$$

$$\frac{\partial Q}{\partial x} + B \left(\frac{\partial y}{\partial t}\right) - q = 0 \qquad (2)$$

$$\frac{\partial Q}{\partial t} + 2 \frac{Q}{A} \frac{\partial Q}{\partial x} + gA \left(\frac{\partial y}{\partial x}\right) - B \frac{Q^2}{A^2} \frac{\partial y}{\partial x} = gA (S_o - S_f) + \frac{Q^2}{A^2} A_x^{\eta} \qquad (3)$$

in which

- x = co-ordinate axis along the length of a stream
- z = vertical distance between a fixed datum and the mean bed level within a control volume
- y = vertical distance between the mean bed level and the mean water surface elevation within a control volume
- t = time
- g = acceleration due to gravity
- Q_s = volumetric sediment transport rate
- q_s = lateral inflow of sediment from overland flow, etc.
- Q = total flow rate
- q = lateral inflow of water from tributary, etc.
- P = wetted perimeter
- B = top-width of the stream
- A = flow cross-sectional area
- A_x^{η} = derivative of A with respect to x when the flow depth is held constant ($A_x^{\eta} = 0$ for prismatic channels)
- S_o = slope of the bed within a control volume
- S_f = slope of the energy grade line
- p = volume of sediment on the bed per unit volume of bed layer
- C_{av} = average volumetric sediment concentration within a control volume

There are six unknowns in the above set of governing equations. These are: Q, y, z, Q_s, C_{av} and S_f. Since there are only three equations, additional equations are required for closure. These additional equations come from the field of sediment transport and they are used to compute Q_s, C_{av} and S_f. The sediment transport rate equations give Q_s and C_{av} and the friction factor equation gives S_f. In MOBED, the sediment transport rate calculations are performed in a subroutine and hence it is possible to adopt different sediment transport rate equations in the model. Moreover, the slope of the energy grade line, S_f is expressed in a general form which makes it possible to use different friction factor equations in the model without having to restructure the model for each friction factor equation. Among all the currently available river models, MOBED is the only model that can accept different friction factor equations and hence it was chosen for the present evaluation.

The general form of S_f used in MOBED is shown below:

$$S_f = \text{const.} \left(\frac{R}{D_{65}}\right)^M \cdot \left(\frac{V^2}{gR}\right)^N \tag{4}$$

in which const, M and N are parameters that take different values for different friction factor equations. D_{65} is the size of sediment for which 65% by weight of sediment is finer. V is the average flow velocity. The values of the parameters, const, M, and N for different friction factor equations are summarized in a tabular form in Krishnappan (1985). The values for the selected equations are taken from that table and are listed below for easy reference.

Table 1. - Values of const, M and N for Selected Friction Factor Equations

Name of Equation	Type of Bed Form	const	M	N
Engelund	Dunes	$0.326\ (\gamma/\gamma_s)^{-4/7}$	-5/7	3/7
	Antidunes	0.022	-1/3	1
Kishi & Kuroki	Dune I	0.0052	1	3
	Dune II	0.013	0	1
	Transition I	$0.018\ (\gamma_s/\gamma)^{6/7}$	-3/7	1/7
	Flat bed	0.021	-1/3	1
	Antidunes	$0.0021\ (\gamma/\gamma_s)^2$	1/5	3
Brownlee	Lower Regime	$0.0205\ \sigma_g^{0.4131}$	-0.0753	-0.2813
	Upper Regime	$0.0125\ \sigma_g^{0.2786}$	-0.2813	1.0859

In the above table, γ stands for the specific weight of water; γ_s stands for the submerged specific weight of sediment and σ_g represents the geometric standard deviation of sediment size distribution.

It should be pointed out that the sediment transport rate and friction factor equations that are currently available are all developed for steady and uniform flow conditions and yet, these equations are commonly used in unsteady and non-uniform flow models because of lack of better knowledge.

Evaluation of Selected Equations

For the present evaluation of selected equations, the model MOBED was run with nine possible combinations of these equations to simulate the scour below a transition from a fixed to a movable bed in a sediment transport flume in the laboratory. The model predictions of depth of scour and water surface elevations were compared with those measured in the laboratory flume to draw conclusions regarding the suitability of different combinations of equations for use in river models.

The sediment transport flume used for this study is 22.8 m long and 2.0 m wide. The slope of the flume can be varied with the help of a motorized screw jack. The water supply for the flume is from a constant head tank. Water enters the flume through an inlet pipe, a diffuser and a head box fitted with baffles and flow straigteners. The downstream end of the flume terminates in a sediment trap which collects the sediment that is transported by the flow. Adjustable louvres at the downstream end facilitate the adjustment of flow depth in the flume. For the present study, a part of the flume length was fitted with a false bottom constructed of plywood and the remaining part was filled with a nearly uniform sand with median (D_{50}) size of 1.8 mm and the geometric standard deviation of 1.17. The D_{65} size is 2.0 mm. The surface of the false bottom was level with that of the sand and coated with a layer of sand to produce the same skin friction characteristics as that of a sand bottom.

A uniform flow with a flow rate of 0.180 m^3/s was established in the flume. The slope of the flume was set at 0.2 percent. The clear water entered the flume, flowed over the rigid bed and then over the sand bed. The flow had tractive force greater than the critical tractive force and hence the scour began to develop in the sand bed downstream of the transition point. The bed level and the water level were measured by traversing a bed profiler and a water level indicator along the length of the flume. The traces of bed level and water level mesured after ten minutes of the start of flow in the flume are shown in Fig. 1. It can be seen from this figure that a small amount of scour has already occurred at the transition point and the sand bed shows some signs of dune formation.

From the traces in Fig. 1, initial water and bed levels shown as solid lines were established and were used as initial conditions to model MOBED. The model was then run with different combinations of sediment transport rate and friction factor equations to pedict the bed and water surface elevations after 30 minutes from the initial traverse. The model predictions were then compared with measured bed and water surface elevations as shown in Figs. 2a, 2b and 2c.

In Fig. 2a, the model predictions with Kishi and Kuroki's friction factor equations and the three sediment transport equations are compared with the measured data. In Fig. 2b, the predictions using Brownlee's friction factor equation are shown and in Fig. 2c, the predictions using Engelund's friction factor equation are compared. From these figures, it can be seen that the model predictions deviate considerably from the measured data for certain combinations of

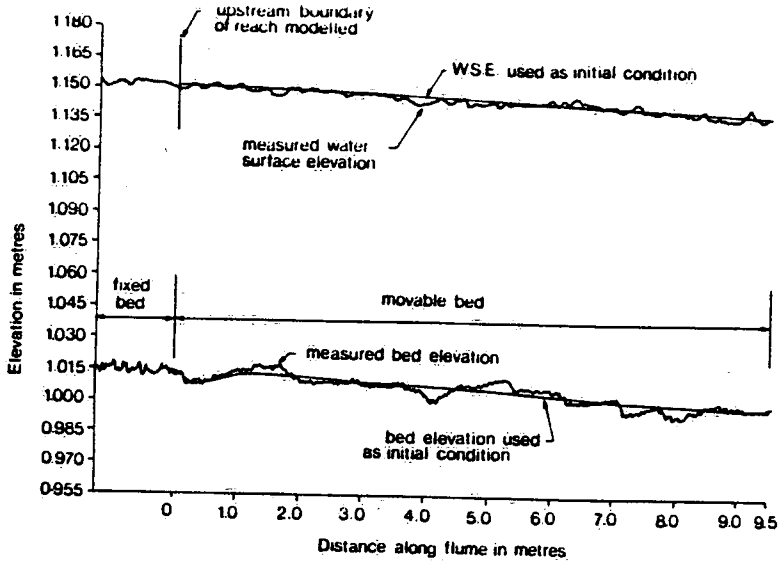

Fig.1 Initial bed and water surface elevations used for model simulation.

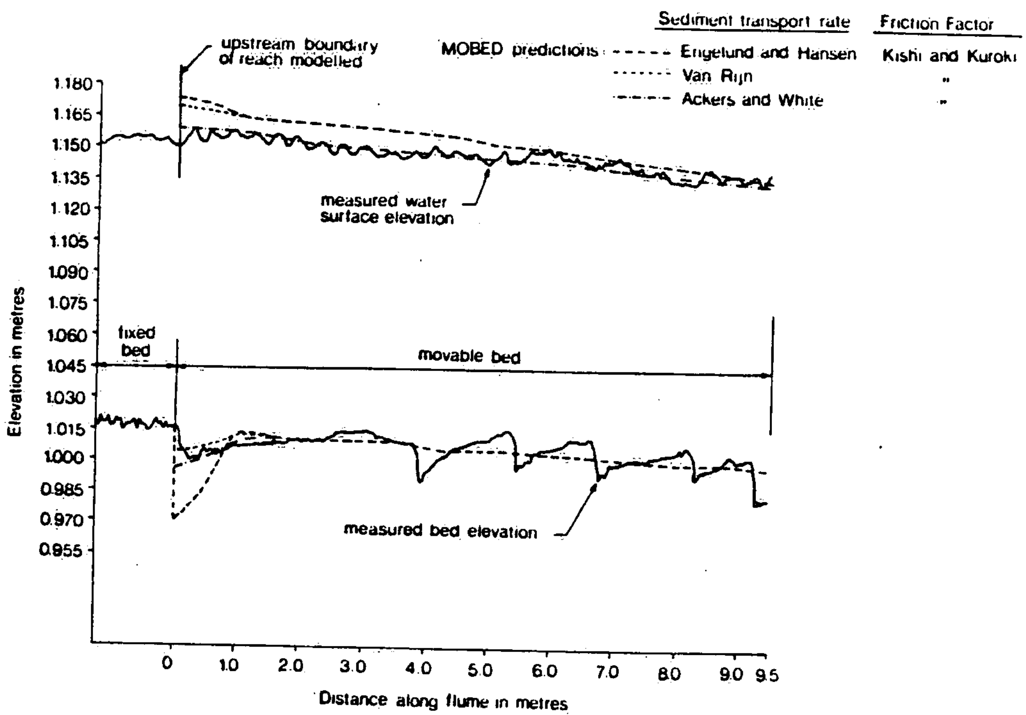

Fig 2a Comparison of model prediction and measurement.

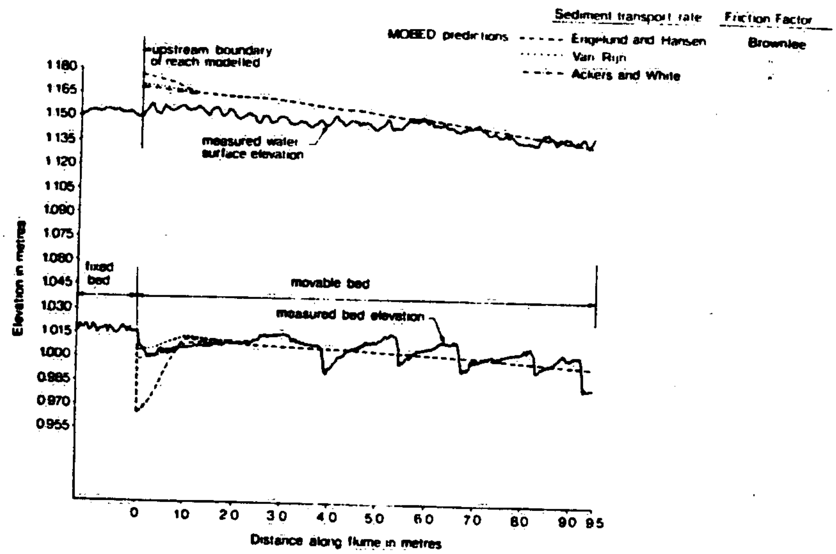

Fig 2b Comparison of model prediction and measurement

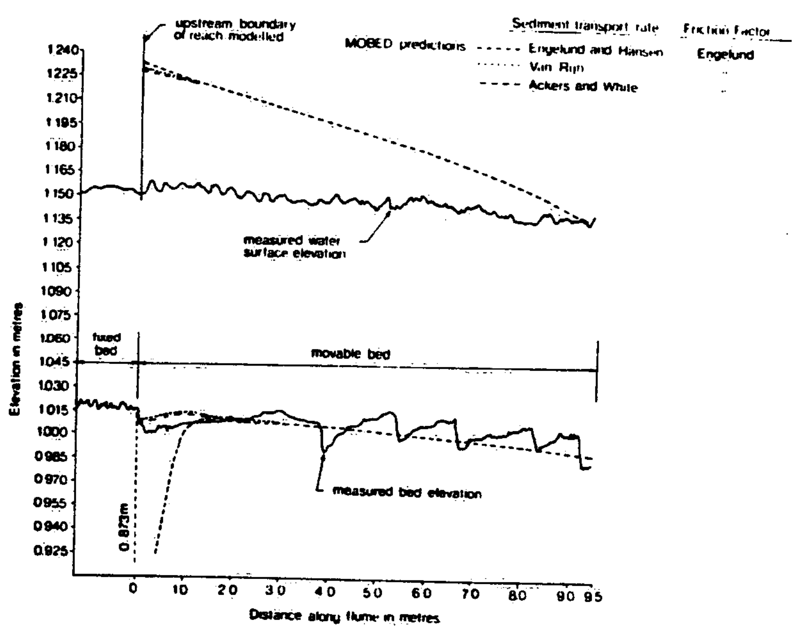

Fig 2C Comparison of model prediction and measurement equations. For example, the combination of Engelund's friction factor equation and Engelund and Hansen's sediment transport rate equation produces a large scour hole and a water surface elevation that deviates considerably from the measured data. The friction factor equations of Brownlee and Kishi and Kuroki produce water surface elevations that are reasonably closer to the measured water surface elevations. The sediment transport rate equations of Ackers and White and Van Rijn produce bed level patterns that are closer to the measured bed level. The best result was obtained with the combination of Kishi and Kuroki's friction factor equation and Ackers and White's sediment transport rate equation.

Summary and Conclusions

A number of existing friction factor and sediment transport rate equations were evaluated by adopting them in a river flow model. The results show that the combination of Kishi and Kuroki's friction factor equation and Ackers and White's sediment transport rate equation gives the best agreement with the measured data. The combination of Engelund's friction factor equation and Engelund and Hansen's sediment transport rate equation gives the least agreement. The other combinations formed by the equations of Brownlee and Kishi and Kuroki with Van Rijn and Ackers and White produce reasonable predictions.

Appendix

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