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MODEL SCALE FOR SAND BED CHANNELS

by

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SESSION II: SELECTION OF MODEL SCALE AND TESTING MATERIALS (I)

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Scope: For non-uniform and mobile boundary channel flows, Yalin (1971) had derived scale relationships of a distorted model by considering the following five basic criteria:

$$\lambda_{x_1} = 1; \lambda_{x_2} = 1; \lambda_s = n; \lambda_E = n; \lambda_{Fr} = 1 \qquad (1)$$

where λ stands for the ratio of model value of a property to the prototype value of the same property and the dimensionless numbers, x_1, x_2 and Fr are defined as follows:

$$x_1 = \frac{V_* D}{\nu} \text{ (Shear Reynolds Number)}$$

$$x_2 = \frac{\rho V_*^2}{\gamma_s D} \text{ (Mobility Number)} \qquad (2)$$

$$Fr = \frac{V}{\sqrt{gh}} \text{ (Froude Number)}$$

The meaning of symbols appearing in relations (1) and (2) above are given below:

S	=	slope of channel bed
E	=	slope of the energy-grade-line
n	=	distortion = λ_y/λ_x
V_*	=	shear velocity
D	=	grain size
ν	=	kinematic viscosity of fluid
ρ	=	density of fluid
γ_s	=	submerged specific weight of sediment
V	=	average flow velocity
h	=	average depth
g	=	acceleration due to gravity
x&y	=	horizontal and vertical dimensions respectively

The scale relationships of Yalin are

$$\lambda_D = (\lambda_y n)^{-1/2} ,$$

$$\lambda_{\gamma_s} = (\lambda_y n)^{3/2} ,$$

$$n^{1/2} \left(1 - \frac{\ln (\lambda_D/\lambda_y)}{\ln (11\ h'/D')}\right) = 1 \qquad (3)$$

and $\lambda_v = \lambda_y^{1/2}$

where h' and D' are prototype value of flow depth and grain size respectively.

In deriving the scale relationships, Yalin had employed the friction factor equation corresponding to flows over plain bed and noted that a model designed according to the above scale relationships would not reproduce the correct water surface profile if the prototype flow consists of bed undulations such as ripples and dunes. He further showed that if the prototype bed consists of ripples, then the model would appear to be rougher than what it should be and the flow depth in the model would be larger than the required depth. The model response would be just the opposite if the prototype bed forms are dunes.

In this paper, Yalin's method has been extended to include flows with bed forms. A friction factor equation which considers both the skin friction and the form drag of the bed forms is used to derive a new scale relationship that could be used in place of the third relationship in equation (3). The details of the derivation of this new scale relationship and its application to model flows with different bed form regimes are discussed in what follows.

Derivation of the New Scale Relationship:

A general expression for the slope of the energy grade line of mobile boundary flows with bed forms has been derived by Krishnappan (1985) as:

$$E = C \cdot \left(\frac{\gamma}{\gamma_s}\right)^L \cdot \left(\frac{R}{D}\right)^M \cdot \left(\frac{V^2}{gR}\right)^N \qquad (4)$$

where C, L, M and N are given by:

$$C = 6.82^{6/(1-4a_1)} \cdot C_1^{4/(1-4a_1)}$$

$$L = (4-4a_1)/(4a_1-1)$$

$$M = (3 - 4a_1 - 4b_1)/(4a_1 - 1) \qquad (5)$$

$$N = 3/(4a_1 - 1)$$

The quantities a_1, b_1 and c_1 appearing in (5) are paraemters of the following power relationship among the Mobility Number Y' formed using the shear stress pertaining to the stein roughness, the total Mobility Number Y and the relative hydraulic radius (R/D).

$$Y' = c_1 \, Y^{a_1} \left(\frac{R}{D}\right)^{b_1} \qquad (6)$$

Knowing the actual relationship among Y', Y and (R/D) in the above form, the values of a_1, b_1 and c_1 can be estabished for a particular type of bed form. Knowing a_1, b_1 and c_1, the slope of the energy grade line can be fully defined using equations (4) and (5).

Some of existing friction factor equations of mobile boundary flows were expressed in the form of equation (4) for different bed form configurations and the values of C, L, M and N are summarized in Table 1.

Table 1: Values of C, L, M and N for Different Friction Factor Equations.

Name of Formula	Type of Bed Form	C	L	M	N	$\left(\frac{3(L-M)}{(M-3L-2)}\right)$
Engelund (1966)	Dunes	0.326	-4/7	-5/7	3/7	-3/7
	Antidunes	0.022	0	-1/3	1	-3/7
Garde and Ranga Raju (1966)	Ripples and dunes	0.098	0	-1/3	1	-3/7
	Antidunes	0.028	0	-1/3	1	-3/7
Kishi & Kuroki (1974)	Dunes	.0052	2	1	3	-3/7
	Antidunes	.0021	2	1/5	3	-27/39

Using equation (4), the scale relationship for the slope of the energy grade line can be evaluated as:

$$\lambda_E = (\lambda_y/\lambda_{y_s})^L \cdot (\lambda_y/\lambda_D)^M \cdot (\lambda_{Fr})^{2N} \qquad (7)$$

With the basic requirements that $\lambda_E=n$, $\lambda_{Fr}=1$ and $\lambda_y=1$, and using the first two scale relationships of equation (3), equation (7) can be rearranged to get a relationship between distortion n and the vertical scale λ_y as:

$$n = \lambda_y^{3(L-M)/(M-3L-2)} \tag{8}$$

Discussion of Results:

It is interesting to note that the exponent of λ_y in equation (8) takes a value of (-3/7) for all three friction factor formulae listed in Table 1 in the case of dunes and for two out of three formulae in the case of antidunes (see the last column in Table 1). It is also surprising that three different equations with different exponents of the governing parameters produce the same scale relationship, namely,

$$n = \lambda_y^{-3/7} \tag{9}$$

for modelling flows with dune covered beds. All three equations are of empirical nature and are based on different approaches. For example, Engelund had expressed Y' as a function of Y alone without any consideration of (R/D) parameter whereas Kishi and Kuroki expressed Y' as a function of both Y and (R/D). Garde and Ranga Raju assumed a Manning type equation and determined the coefficient and exponents using different data sets from the ones used by Engelund and Kishi and Kuroki. Under these circumstances, it is a remarkable coincidence that all three equations give the same scaling law.

For the case of flows with antidunes, two out of three equations indicate that the modelling laws applicable to dune regime are also applicable to antidune regime. Only Kishi and Kuroki's equation gives two different scaling laws for dunes and antidunes. In the light of this result, the formulation of Kishi and Kuroki's equation for antidune regime should be re-examined. It is highly desirable, from a practical point of view, to have the same modelling laws apply to both types of bed forms.

The modelling law given by equation (9) is also practicable as it yields model distortions that are not too excessive. Table 2 gives the values of distortion n for a range of vertical scale of the model. Figure 1 shows a plot between n and λ_y.

Table 2: Values of Distortion n for Various Values of λ_y.

For λ_y =	1/100	1/50	1/30	1/25	1/20	1/15	1/10
n =	7.2	5.3	4.3	4.0	3.6	3.2	2.7

<u>Suggestion for Discussion</u>:

In discussing the scale relationships shown as equation (3), Yalin showed that a model that reproduces the energy loss due to skin friction correctly is incapable of yielding correct losses due to form

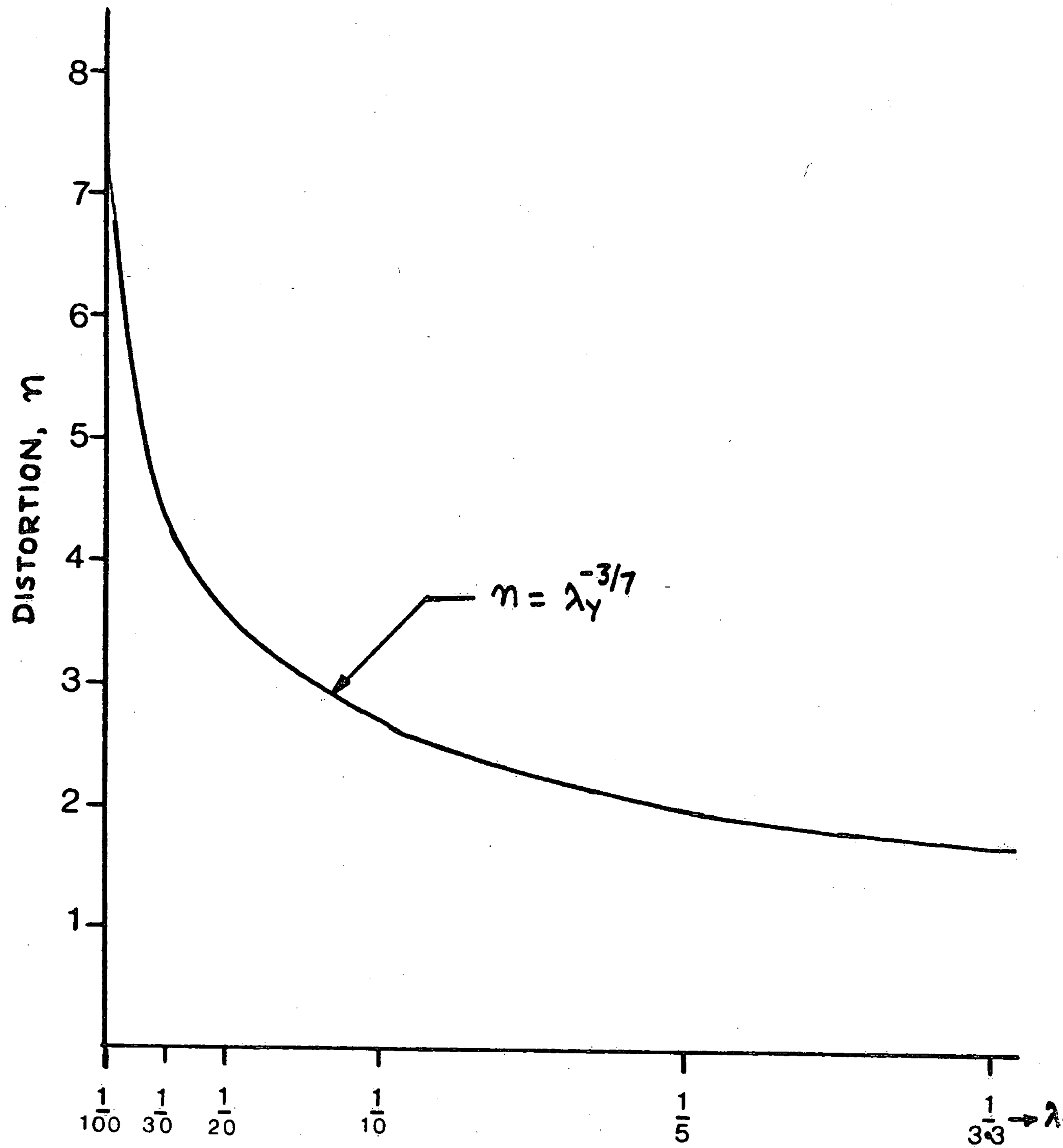

Fig.1 Relation between vertical scale and distortion for mobile boundary models

drag. The new model scale derived in this paper ensures correct total energy losses due to both skin friction and form drag whereas the individual components may not be correctly modelled. Importance of modelling individual components of energy losses correctly could be a topic for further discussion.

References

Krishnappan, B.G. Modelling of unsteady flows in alluvial streams, Journal of Hydraulic Engineering, ASCE, Vol. III, No. 2, 1985, pp 257-266.

Yalin, M.S. Theory of Hydraulic Models, Macmillan, 1971.