PERFORMANCE OF FLOAT ACTUATED
WATER LEVEL RECORDER SYSTEMS
by

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MANAGEMENT PERSPECTIVE

The Water Survey of Canada is responsible for measuring the discharge of over 3000 stations across the country. Accuracy of these measurements and of the records is vital. Furthermore as technology advances and new systems are proposed to make stage measurements, it is necessary to understand completely the accuracy of present systems. To these two ends, the Stevens "A" series float type recorder system was evaluated to determine its performance under a range of conditions. For typical installations errors can be kept to less than ±3 mm (0.01 ft).

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PERSPECTIVE-GESTION

La Division des relevés hydrologiques du Canada est chargée de la mesure du débit à plus de 3 000 stations dans le pays. La justesse de ces mesures et des relevés est une question vitale. En outre, comme les techniques et les systèmes de mesures des niveaux progressent, il est nécessaire de connaître parfaitement la justesse des systèmes actuels. À ces deux fins, on a évalué le limnigraphe Stevens de la série "A" afin de déterminer son rendement sous diverses conditions. Dans le cas des installations typiques, les erreurs de mesures peuvent rester inférieures à ± 3 mm (0,01 pi.).

SUMMARY

A theoretical analysis complemented by experimental measure-
ments was used to examine the behaviour of float activited water level
recorder systems when used with the Stevens A-71 recorder.   The
equations developed can be used to predict measurement errors for any
type of float type recording installation. The results show that for
typical Water Survey of Canada installations, errors in measuring
water levels can be kept to less than ±3 mm (0.01 ft.).

SOMMAIRE

On utilise une analyse théorique complétée de mesures expérimentales pour examiner le comportement de limnigraphes à flotteur lorsqu'on les utilise avec l'enregistreur Stevens A-71. Les équations établies peuvent être utlisées pour prévoir les erreurs de mesure de tout type d'installation de limnigraphes à flotteur. Les résultats montrent que pour les installations typiques de la Division des relevés hydrologiques du Canada, les erreurs de la mesure des niveaux d'eau peuvent rester inférieures à ± 3 mm (0,01 pi.).

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1.0   INTRODUCTION

The Water Survey of Canada operates about 3000 gauging stations across the country. The stage records obtained are converted to mean daily discharges by means of a stage-discharge curve developed at each gauging site. It is therefore important that all water level records be as accurate as possible.

The most basic instrument used by the Water Survey of Canada for the collection of continuous water level data is the Stevens "A" series float type recorder system. Stage records obtained with these instruments are usually reported as being correct to ±0.01 ft (±3 mm). Water Survey of Canada is reviewing the performance of these instruments for a wide range of operating conditions in order to determine if quoted data accuracies are realistic under all conditions. The results will provide information to develop standards and specifications for new transducer oriented measuring systems.

In this report, the performance of float actuated recording systems is examined with special emphasis on the Stevens A-71 water level recorder. The work was conducted at the hydraulics laboratory of the Research and Applications Branch of the National Water Research Institute, at the request of the Water Survey of Canada.

2.0   GENERAL EQUATIONS

2.1   Preamble

Automatic stage recorders give a continuous measurement of stage. These recorders are nearly always installed above a stilling well and are activated whenever feasible by a float-counterweight mechanism. A typical float actuated system is shown schematically in Figure 1. Movements of the float with changing stage are transferred through the pulley and gear mechanism to the pen carriage of the recorder which produces a trace on a chart moving at constant speed. As water levels in the stilling well change, there is a change in

length of the steel tape connecting float and counterweight, on either side of the recorder pulley. As a result, the submergence of the float will change. Additional change in the float submergence results if stages are high enough to submerge the counterweight and part of the supporting steel tape. Any mechanical resistance in the pulley bearings and mechanism of the chart recorder will also contribute to the submergence of the float. The general behaviour of the float system in Figure 1 is now examined through theoretical analysis.

2.1   Case 1. Counterpoise Above Water Surface

The forces acting on the float are shown in Figure 2 and the balance of these forces can be written as

$$B_f = W_p + W_{Tf} - P \tag{1}$$

in which B_f - buoyant force acting on the float, W_{Tf} = the weight of the steel tape or cable on the float side of the recorder pulley, W_f = the weight in air of the float and P = the tension in the tape or cable. The tension P at a given water level depends on whether the stage is rising or falling because the resisting force in the recorder mechanism, say, F_R acts in the opposite direction for these two cases. If one now considers that the recorder resistance might vary with water level (pen position on the recorder) and is also affected by changes in temperature, then the resisting force F_R may be more generally expressed by writing

$$F_R = F_R(\theta, y) \tag{2}$$

where θ = temperature in °C, y represents the water surface elevation. For a rising stage the force P can be expressed as

$$P_r = W_c + W_{Tc} - F_R(\theta, y) \tag{3}$$

where P_r = tension in the tape or cable on the rising stage, W_c = weight in air of the counterpoise and W_{Tc} = weight in air of the tape or cable on the counterpoise side of the recorder pulley. For a falling stage the tensile force P becomes

$$P_d = W_c + W_{Tc} + F_R(\theta, y) \tag{4}$$

where P_d = tensile force when water level is dropping. Examination of equation (3) and (4) shows that they differ only in the direction of the resisting force $F_R(\theta, y)$. Therefore, it will suffice to examine in detail only the conditions for rising stage and apply the results to the case of dropping stage.

Substituting equation (3) into equation (1) results in

$$B_f = W_f + W_{Tf} - W_{Tc} + F_R(\theta, y) \tag{5}$$

One may also write

$$B_f = \gamma_w A_f hy \tag{6}$$

$$W_f = \gamma_f A_f t \tag{6}$$

$$W_c = \gamma_c A_c l_c \tag{8}$$

In the above equations γ_w = specific weight of water, A_f = diametric area of the circular float, h_y = the depths of submergence of the float when the water level is at height y above the invert of the stilling well intake, γ_f = specific weight of the float, t = the cylinderical height of the float, γ_c = specific weight of the

counterpoise, A_c = cross-sectional area of the cylindrical counterpoise, l_c = the length of the counterpoise, γ_T = specific weight of the steel tape, A_T = cross-sectional area of the steel tape, L_f and L_c are shown in Figure 2.

Substituting equations (6) through (9) into equation (5), after some algebraic manipulation, results in

$$h_y = S_f t + S_T \frac{A_T}{A_f} (L_f - L_c) - S_c \frac{A_c}{A_f} l_c + F_R(\theta, y) \qquad (10)$$

in which S_f, S_T and S_c are the specific gravity of the float, tape and counterpoise respectively.

Referring to Figure 2 one now obtains

$$L_f = \frac{\pi d}{4} + L_R + L_w - y + h_y - t \qquad (11)$$

and

$$L_c = \frac{\pi d}{4} + L_R + y - l_c - (h_y - h_0) \qquad (12)$$

where d = diameter of the recorder pully, L_R is as shown in Figure 2 and h_0 is the depth of submergence of the float when the water surface is at the elevation of the intake invert. In equation (12), the length h_0 must be subtracted from h_y because as the water rises from y = 0, the float's submergence is decreased by the amount $(h_y - h_0)$ at the water surface height y. This change in submergence must be taken into account when determining the length L_c in equation (12). This quantity $(h_y - h_0)$ is not included in the equations presented by Stevens (1968). Now, substituting equations (11) and (12) into (10) gives

$$h_y = S_f t + S_T \frac{A_T}{A_f} [L_w - 2y + (l_c - t) + 2h_y - h_0] - S_c \frac{A_c}{A_f} l_c + \frac{F_R(\theta, y)}{\gamma_w A_f} \qquad (13)$$

Rearranging equation (13) gives

$$\frac{h_y}{t} = \frac{S_f + S_T\frac{A_T}{A_f}\left[L_w - \frac{2y}{t} + \left(\frac{l_c-1}{t}\right) - \frac{h_0}{t}\right] - S_c\frac{A_c}{A_f}\frac{l_c}{t} + \frac{F_R(\ ,y)}{_wA_f t}}{\left(1 - 2S_T\frac{A_T}{A_f}\right)} \quad (14)$$

Now, when $y = 0$, $h_y = h_0$. Using this boundary condition, one obtains from equation (14)

$$\frac{h_0}{t} = \frac{S_f + S_T\frac{A_T}{A_f}\left[\frac{L_w}{t} + \left(\frac{l_c-1}{t}\right)\right] - \frac{S_c A_c l_c}{A_f t} + \frac{F_R(\ ,y)}{_wA_f t}}{\left(1 - S_T\frac{A_T}{A_f}\right)} \quad (15)$$

Substituting equation (15) into (14), simplifying and after writing $z = h_y/t$, one obtains for the rising stage

$$Z_r = \frac{K}{\left(1 - S_T\frac{A_T}{A_f}\right)} - \frac{2S_T\frac{A_T}{A_f}\frac{y}{t}}{\left(1 - S_T\frac{A_T}{A_f}\right)} + \frac{F_R(\ ,y)}{_wA_f t\left(1 - S_T\frac{A_T}{A_f}\right)} \quad (16)$$

and for falling stage

$$Z_d = \frac{K}{\left(1 - S_T\frac{A_T}{A_f}\right)} - \frac{2S_T\frac{A_T}{A_f}\frac{y}{t}}{\left(1 - 2S_T\frac{A_T}{A_f}\right)} - \frac{F_R(\ ,y)}{_wA_f t\left(1 - S_T\frac{A_T}{A_f}\right)} \quad (17)$$

in which

$$K = S_f - S_c \frac{A_c l_c}{A_f t} + S_T \frac{A_T}{t} [\frac{L_w}{t} + (\frac{l_c}{t} - 1)]$$ (18)

Equations (16) and (17) are valid when the water level in the stilling well is low enough so that the counterpoise does not come in contact with the water surface ($y/L_w < 0.5$). The parameter K is a systems constant for a given gauge installation including material properties and geometric characteristics.

2.3   Case 2: Counterpoise Submerged

When the water level in the stilling well increases to values of $y/L_w > 0.5$, then at first the counterpoise becomes submerged and then with further rise in stage, the counterpoise and some length of the tape, say L_{TS}, become submerged. As a result of this submergence there is a buoyant force which affects the tension in the tape P. This is schematically shown in Figure 3. As a result, the tension P for a rising stage may be written as

$$P_r = W_c - \gamma_T A_T (L_f - L_c) - F_R(\theta, y) - B_r - B_c$$ (19)

where B_T and B_c are the buoyant forces acting on the tape and counterpoise respectively. The buoyant forces B_T and B_c may be written as

$$B_T = \gamma_w A_T L_{TS}$$ (20)

and

$$B_c = \gamma_w A_c l_c$$ (21)

From Figure 3 the length of submerged tape is

$$L_{TS} = 2y - L_w - h_y + h_0 - l_c \tag{22}$$

now substituting equations (20), (21), and (22) into equaton (19) the tension P_r becomes

$$P_r = \gamma_w(S_c - 1) A_c l_c - \gamma_w A_T[(S_T + 1)(L_w - 2y) + (S_T + 1)l_c +$$

$$(2S_T + 1)h_y - (S_T + 1)h_0 - S_r t] - F_R(\theta, y) \tag{23}$$

Substituting equation (23) into equation (1) results in

$$\frac{h_y}{t} = S_f - (S_c - 1) \frac{A_c l_c}{A_f t} + \frac{A_T}{A_f} [(S_T + 1)\frac{L_w}{t} + (S_T + 1)\frac{l_c}{t} - S_T]$$

$$+ \frac{A_T}{A_f} [(2S_T + 1) \frac{h_y}{t} - (S_T + 1)\frac{h_0}{t} - 2 (S_T + 1) \frac{y}{t}] + \frac{F_R(\theta, y)}{\gamma_w A_f t} \tag{24}$$

now when $y = 0$, $h_y = h_0$ and using this boundary conditions results in

$$\frac{h_0}{t} = \frac{S_f - (S_c - 1) \frac{A_c l_c}{A_f} + \frac{A_T}{A_f} [(S_T + 1)L_w + (S_T + 1)l_c - S_T t] + F_R(\theta, y)}{[1 - \frac{A_T}{A_f} S_T]} \tag{25}$$

Substituting equation (25) into (24), simplifying and setting $z = h_y/t$, one obtains for the rising stage

$$-Z_r = \frac{K_s}{\left(1 - \frac{A_T}{A_f} S_T\right)} - \frac{2(S_T + 1)\frac{A_T}{A_f}\frac{y}{t}}{\left[1 - (2S_T + 1)\frac{A_T}{A_f}\right]} + \frac{F_R(\theta, y)}{\gamma_w A_f t\left(1 - S_T\frac{A_T}{A_f}\right)} \quad (26)$$

and for a falling stage

$$Z_r = \frac{K_s}{\left(1 - S_T\frac{A_T}{A_f}\right)} - \frac{2(S_T + 1)\frac{A_T}{A_f}\frac{y}{t}}{\left[1 - (2S_T + 1)\frac{A_T}{A_f}\right] -} - \frac{F_R(\theta, y)}{\gamma_w A_f t\left(1 - S\frac{A_T}{T A_f}\right)} \quad (27)$$

in which

$$K_s = S_f - (S_c - 1)\frac{A_c l_c}{A_f t} + \frac{A_T}{A_f}\left[(S_T + 1)\frac{L_w}{t} + (S_T + 1)\frac{l_c}{t} - S_T\right] \quad (28)$$

Equations (26) and (27) are valid when the water level in the stilling well is such that the top of the counterpoise is just below the water surface. It is also clear from equations (16), (17), (26), and (27) that the relative submergence of the float Z depends on system parameters, depth of the water and direction of change in water level. These equations can now be used to determine the effect that this change in submergence has on the measurement of water levels in the stilling well, once the effect of the force $F_R(\theta, y)$ has been measured.

2.4   Measurement of Recorder Resistance

In equations (16), (17), (26) and (27), the parameters comprising the systems constant K are known for a given installation. In order to estimate the float submergence using the above equations it is necessary to conduct carefully controlled tests to determine the magnitude and variability of the force $F_R(\theta,y)$.

The required tests were conducted by setting the Stevens A-71 recorder over an acrylic stilling well having a diameter of 25 cm and a height of 150 cm. The stilling well was fitted with two needle valves. One valve was used to regulate inflow from a hose connected to a standard water tap in the domestic water supply line, while the second valve was used to regulate the outflow from the well. Using these valves it was possible to raise and lower water levels in the well at slow and constant rates. A tape, graduated in mm, was fastened to the outside of the stilling well for the purpose of reading water surface elevations.

In order to measure the force $F_R(\theta,y)$, the standard float was exchanged with a stainless steel rod having a diameter of 2.54 cm and a length of 20 cm. These dimensions were chosen because they would ensure larger changes in submergence of the float to give small changes in the buoyant force thus affording greater accuracy in determining $F_R(\theta,y)$. The steel float was fastened to one end of the standard steel tape used at hydrometric installations, and a variable weight counterpoise was attached to the other end. This counterpoise was simply a tin coffee can. Its weight could be varied by adding or removing sand. Once the tape was placed over the pully of the recorder, the system could be balanced so that part of the float was submerged by adding the appropriate amount of sand to the counterpoise. The test system is shown schematically in Figure 4.

After the recorder system was in static equilibrium, the water level was measured and water was slowly added to the stilling well. The recorder pulley was carefully observed and the water level

was allowed to rise until the recorder pulley was on the point of turning. At this point the inflow to the well was stopped, the new water elevation read and recorded. This procedure was repeated over a range of starting water levels to ensure that measurements occurred for different values of y in $F_R(\theta,y)$. A total of 125 measurements were made for rising water levels and descending water levels for each of two air temperatures $\theta = 27.2°C$ and $\theta = -19.5°C$.

The change in water level from the point of equilibrium to the onset of recorder motion, represents the change in submergence of the float. This information, together with the diameter of the float and the specific weight of water, yields the change in buoyant force required to overcome the frictional resistance of the recorder, $F_R(\theta,y)$. The computed mean values of $\bar{F}_R(\theta,y)$ and their standard deviations are given in Table 1 for rising water levels and in Table 2 for falling water levels. The means and standard deviations were used to compute the maximum resistance force at the 95% confidence level. These values are also given in Tables 1 and 2.

The data in Table 1 shows that, overall, there is little change in the resistance force as a result of changes in temperature, even though the temperature was changed from 27.2°C to -19.5°C. There is some variation in the resistance force over the five tests taken at each temperature indicating some effect of water level y. This variation may be considered to be small, therefore, for rising water levels, effects of temperature (θ) and water surface elevation (y) may not be important.

Examination of the resistance forces in Table 2 shows that on average they tend to be a little larger than observed in Table 1, indicating more resistance in the recorder when water levels are falling. The variability of the forces within each test group is also greater indicating a slight increase in the effect of y when water levels are dropping. Finally, the data in Table 2 show a small increase in resistance as a result of the change in temperature from

27.2°C to -19.5°C. However, considering the large range in temperature, the change in force may be considered to be small.

Over the 20 tests given in Tables 1 and 2, the resistance force varied from a low of 9.145×10^{-2} N to a high of 22.079×10^{-2} N. Analysis of the performance of the float-recorder systems through equations (16), (17), (26) and (27) will show to what extent the measured forces are significant.

3.0 ERRORS DUE TO CHANGES IN FLOAT SUBMERGENCE
3.1 Float Lag

The float lag is the difference in recorded water level when the same water level is observed at rising and at falling stage. When the water level is falling, the resistance in the recorder mechanism tends to "pull up" on the float, thereby causing it to ride higher in the water. On a rising stage the reverse is true with the effect of pushing the float down and increasing its submergence. If the recorder is set to read the true water level while the stage is rising, it will thereafter give the correct reading at the same stage each time the water surface passes through that point on a rising stage, assuming that everything else remains constant. However, on a falling stage, the recorder will give a water level reading which is too high by the amount of the float lag. The reverse is true if the change in direction is from a falling stage to a rising stage.

If one designates the float lag to be Δl_f, then one may write

$$\Delta l_f = \pm(Z_r - Z_d) \tag{29}$$

where Δl_f is positive when the change in direction is from rising to falling stage and negative when the change is in the oposite direction. After substituting for Z_r and Z_d from equations (16) and (17) the float lag becomes

$$\Delta l_f = \pm \left[\frac{F_R(\theta_r, y_r) + F_R(\theta_d, y_d)}{\gamma_w A_f \, t \, (1 - S_T \frac{A_T}{A_f})} \right] \qquad (30)$$

Now for a given water level at which the float lag Δl_f is observed, one has $y_r = y_d$ and equation (30) becomes

$$\Delta l_f = \pm \left[\frac{F_R(\theta_r, y) + F_R(\theta_d, y)}{\gamma_w A_f \, t \, (1 - S_T \frac{A_T}{A_f})} \right] \qquad (31)$$

For a cylindrical float $A_f = \pi D_f^2/4$ (D_f = float diameter, π = 3.14....) and the float lag becomes

$$\Delta l_f = \pm 4 \left[\frac{F_R(\theta_r, y) + F_R(\theta_d, y)}{\pi \gamma_w D_f^2 (1 - S_T \frac{A_T}{A_f})} \right] \qquad (32)$$

Equation (32) takes into account the fact that the resistance force may be different on the rising and falling stage because of a change in temperature. Note that Δl_f can be decreased by increasing the float diameter and by reducing the frictional resistance in the recorder drive. Obviously, because the diameter is represented in equation (32) by D_f^2, it should be more effective to increase the float diameter as long as it can be accommodated in the stilling well.

The float lag can be seen to be affected by γ_w which depends on fluid density, say ρ, and gravitational acceleration, say, g. The density of water changes negligibly over a range of temperatures from 4°C to 30°C (see Table 1). Therefore, the effects of ρ can be ignored for practical gauging procedures. Rouse (3) has shown that the gravitational acceleration on earth can be closely approximated by the empirical relationship

$$g = 9.801 - 0.0250 \cos 2\phi - 0.0000009h \tag{33}$$

in which ϕ = latitude in degrees, h = elevation in meters above sea level and g - acceleration due to gravity in m/s^2. Within Canada, stream gauges are most likely to be found in regions between 43° and 70° latitude and altitudes ranging from sea level to about 1000 m. For these limits, g varies from 9.735 to 9.826 which represents a change of about 0.93 percent. Therefore, for most applications changes in g with location of gauging sites do not need to be considered.

The float lag is not affected by submergence of the counterweight and tape, because the same conditions are always encountered on rising and falling stage and thus the effects of submergence will cancel out.

The data in Table 1 and 2 show that the recorder resistance is affected only slightly by changes in temperature and water level. The recorder resistance at falling stages was found to be greater than that measured at rising water levels. However, the difference relative to the absolute magnitudes may still be considered to be marginal. Therefore, one may write $F_R(\theta_r,y) \approx F_R(\theta_d,y) \approx F_R$ and equation (32) simplifies to

$$\Delta l_f = \frac{8 F_R}{\pi \gamma_u D_f^2 (1 - S_T \frac{A_T}{A_f})} \tag{34}$$

Equation (34) may be used to reveal the relative effects of changing frictional resistance and float diameter. Values of Δl_f were computed and plotted as Δl_f versus D_f with F_R as a parameter in Figure 5. The curves show how, for a given value of F_R, the float lag Δl_f can be reduced by increasing the float size. The curves also reveal the significance of the resistance force F_R in contributing to the float lag Δl_f. For example, when F_R = .05 N

(0.2 oz.), an increase in float size when $D_f \approx 17.5$ cm does not significantly reduce float lag. However, when $F_R = 1.2$ N (5 oz.) float size must be well in excess of 40.0 cm before the effect of increasing D_f on Δl_f becomes insignificant. The float diameters currently used by Water Survey of Canada with the Stevens A type recorder are 20.0 cm and 25.0 cm. When $D_f = 20.0$ cm, and $F_R = .05$ N. Figure 5 shows that $\Delta l_f \approx 0.4$ mm whereas when $D_f = 25.0$ cm $\Delta l_f = 0.25$ mm for the same value of F_R. If, however, the frictional resistance is increased to, say 0.4 N (1.4 oz) then when $D_f = 20.0$ cm, $\Delta l_f = 2.6$ mm and when $D_f = 25.0$ cm, $\Delta l_f = 1.7$ mm. Therefore, values of the frictional resistance should be kept below 0.4 N (1.4 oz) in order to keep errors to less than 3 mm. If errors are to be kept lower than that, then the effects of resistance F_R become more important for a given size of float. The float diameter required for a given recorder resistance to ensure that the float lag does not exceed a selected value is shown in Figure 6.

The maximum recorder resistance force given in Tables 1 and 2 is about 0.2 N at the 95% confidence level. It is clear from Figure 6 for float diameters of 20.0 cm or greater, the float lag will be less than 2 mm.

3.2   Line Shift

As the water level in the stilling well changes, the submergence of the float changes in accordance with equations (16), (17), (26) and (27). As a result, the recorded water level at a given stage will deviate from the true water level by an amount known as the line shift.

3.2.1   Case 1: Counterpoise unsubmerged

Considering a rising stage the submergence Z_r of the float is given by equation (16). If the water level changes from a given

stage y by an amount Δy then the change in submergence of the float may be expressed as

$$\Delta Z_r = \int_{Z_1}^{Z_2} dZ \tag{35}$$

After differentiating equation (16) one obtains

$$dZ_r = \frac{2S_T \frac{A_T}{A_f} \frac{d_y}{t}}{(1 - 2 S_T \frac{A_T}{A_f})} \tag{36}$$

Substituting equation (36) into (35) and integrating over the change in stage Δy, the change in float submergence becomes for rising stage

$$\Delta Z_r = - \frac{2S_T \frac{A_T}{A_f} \frac{\Delta y}{t}}{(1 - 2 S_T \frac{A_T}{A_f})} \tag{37}$$

In the case of falling stage one obtains

$$\Delta Z_d = \frac{2S_T \frac{A_T}{A_f} \frac{\Delta y}{t}}{(1 - 2 S_T \frac{A_T}{A_f})} \tag{38}$$

Equations (37) and (38) show that for a given stilling well installation the line shift error increases directly with the change

in stage. On a rising stage, line shift error is negative and on a falling stage the error is positive. The absolute value of ΔZ is plotted as a function of $S_T A_T / A_f$ with $\Delta y / t$ as a parameter in Figure 7. The curves show that the dimensionless line shift is very sensitive to changes in $S_T A_T / A_f$. This means that ΔZ can be significantly reduced by reducing $S_T A_T / A_f$. This can be done by reducing either S_T or A_T or both as well as by increasing the float area A_f. Typical values for installations operated by the Water Survey of Canada are $S_T = 7.8$, $A_T \approx 0.02208$ cm^2, $A_f = 314.1$ cm^2 and 490.8 cm^2 for diameters of 20 cm and 25 cm respectively. The corresponding values of $S_T A_T / S_f$ are ploted on the curve in Figure 7 to show the resultant value of ΔZ. The curves show that the reduction in ΔZ obtained by increasing the float size from a 20 cm to a 25 cm diameter is about 35 percent. A similar result could have been obtained by keeping a float with diameter of 20 cm and reducing $S_T A_T$ by a factor of 1.6. This might be achieved by reducing A_T (i.e., use wire instead of tape) and using a material with lower specific gravity than that of the presently used tape. Indeed such a change together with an increase in float diameter from 20 cm to 25 cm would result in a total reduction in $S_T A_T / A_f$ by a factor of about 2.6. From an operational perspective, however, it may not be practical to reduce $S_T A_T / A_f$ significantly by reducing S_T and A_T. A more convenient and effective method is to increase the float diameter. The change in submergence to be expected at a typical Water Survey of Canada installation can be determined from Figure 8 for different float sizes and changes in stage. The curves show that when a float with 20 cm diameter is used, a change in stage of 100 cm can occur before a line shift of 1 mm is exceeded. For a 25 cm float a line shift of 1 mm occurs for a change in stage of about 140 cm. It is also quite obvious from the curves in Figure 8, that except for large changes in stage, an increase in float diameter above 25 cm will not be very effective or useful. Finally, Figure 9 shows the variation of Δh_y with stage for float diameters

of 20 cm and 25 cm. The curves clearly show that the effect of float size becomes more significant as the change in stage Δy becomes large.

3.2.2   Case 2:   Counterpoise fully submerged

When the counterpoise is fully submerged the submergence of the float is defined by equation (26) and (27) for a rising and falling stage respectively. Applying the principles used in Case 1 to these equations, one obtains the line shift for rising stage as

$$\Delta Z_{rs} = \frac{2(S_T + 1) \dfrac{A_T}{A_f}}{[1 - (2 S_T + 1) \dfrac{A_T}{A_f}]} \cdot \frac{\Delta y}{t} \tag{39}$$

and for a falling stage is

$$\Delta Z_{ds} = - \frac{2(S_T + 1) \dfrac{A_T}{A_f}}{[1 - (2 S_T + 1) \dfrac{A_T}{A_f}]} \cdot \frac{\Delta y}{t} \tag{40}$$

The effect of the submergence of the counterpoise and the suspending tape can be determined by writing

$$\delta \Delta Z = \left| \Delta Z_r \right| - \left| \Delta Z_{rs} \right| \tag{41}$$

where $\delta \Delta Z$ = the difference in line shift at a given stage. Substituting equation (37) and (38) into equation (41) and after some algebraic manipulation one obtains

$$\delta \Delta Z = \frac{\Delta y}{t} \left[\frac{2 - 6K}{(1 - 2K)(1 - 2K - a)} \right] a \tag{42}$$

where $K = S_T A_T/A_f$ and $a = A_T/A_f$

In equation (42) for a typical recorder installation $a = 7.0 \times 10^{-5}$ whereas $S_T A_T/A_f \doteq 7.8$. Therefore one may consider that $K \gg a$ and equation (42) can be simplified to give

$$\delta\Delta Z = 2 \frac{\Delta y}{t} [\frac{1 - 3K}{(1 - 2 K)^2}] a \qquad (43)$$

Finally, recalling that $Z = h_y/t$, then one may write $\delta\Delta h_y/t = \delta\Delta Z$ and equation (43) becomes

$$\frac{\delta\Delta h_y}{\Delta y} = 2a [\frac{1 - 3K}{(1 - 2K)^2}] \qquad (44)$$

Considering that $a = 7.0 \times 10^{-5}$, then it is clear from equation (44) that $\delta\Delta h_y/\Delta y$ will be very small and thus the effect of submergence of counterpoise and steel tape on float submergence will be negligible.

4.0 CONCLUSIONS

Theoretical analysis was used to develop general mathematical models for float actuated water level recording systems for conditions of gradually rising and falling water levels. These models together with careful measurements of recorder resistance under controlled laboratory conditions were used to examine such a system when used with the Stevens A-71 water level recorder.

4.1 Laboratory measurements have shown that recorder resistance varies slightly with water level and air temperature and that the average resistance at falling stage is slightly greater than at rising stage. The maxiumum measured recorder resistance at the 95% confidence level was about .22 N.

4.2 Analysis has shown that the submergence of the counterpoise and steel tape has a negligible effect on water level measurement accuracy.

4.3 For the maximum measured recorder resistance of .22 N and float diameters of 20 cm or greater the float lag will be always less than about 1.5 mm. When the float diameter is increased to 25 cm, the float lag is about 1mm. The error introduced by the float lag is positive if the operating mode changes from that at rising stage to one at falling stage. The error is negative when the operating sequence is reversed.

4.4 Analysis has shown that the line shift varies significantly with the dimensionless parameter $S_T A_T / A_f$ (S_T = specific gravity of steel tape, A_T = cross-sectional area of steel tape, A_f = cross-sectional area of the float) and the change in water level Δy. The line shift can be significantly reduced by reducing $S_T A_T / A_f$. The most practical approach is to increase the float size A_f. For a given value of $S_T A_T / A_f$ the line shift increases as the change in water level increases. When a standard 20 cm float is used the line shift is 1 mm for a change in stage of about 1 metre. When the 25 cm float is used the permissable change in stage to maintain a line shift of 1 mm is increased to 1.4 metres. Clearly for large changes in stage, frequent gauge correction or adjustments to the gauge height records are required to keep the line shift error as small as possible. The error due to line shift is negative at rising stage and positive when water levels are dropping.

ACKNOWLEDGEMENTS

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Rouse, H. 1949. Engineering Hydraulics, John Wiley and Sons, New York, U.S.A.

Pulley to Drive Pen Carriage of Recorder

Chart Recorder

Steel Tape or Cable

Counter Poise
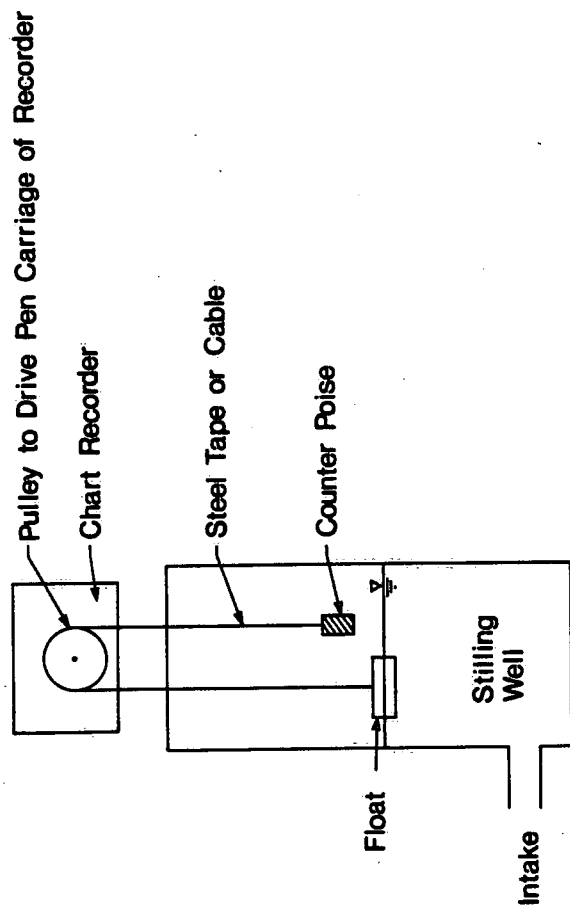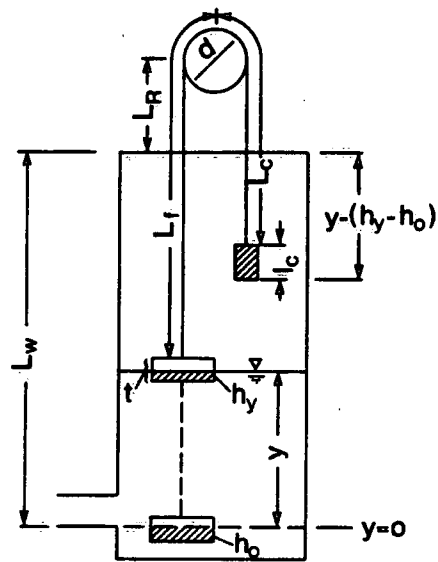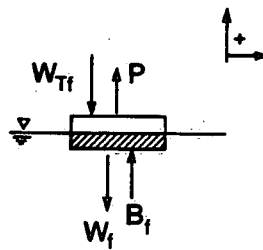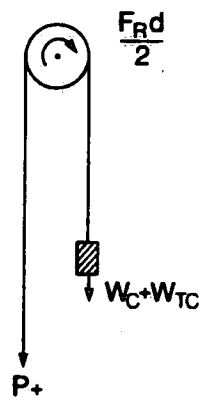
Float

Stilling Well

Intake

FIGURE 1 - SCHEMATIC OF FLOAT SYSTEM AND STILLING WELL

a) LENGTH VARIABLES

d) FORCES ON FLOAT   c) TENSILE FORCE IN TAPE

FIGURE 2 - DEFINITION OF LENGTH VARIABLES AND ACTING FORCES FOR CASE 1

a) LENGTH VARIABLES

d) FORCES ON FLOAT

c) TENSILE FORCE IN TAPE

FIGURE 3 - DEFINITION OF LENGTH VARIABLES AND ACTING FORCES FOR CASE 2

Recorder Pulley

Steel Tape in mm

Tin Can Containing Sand

Steel Rod Float

Acrylic Stilling Well

Outflow Needle Valve

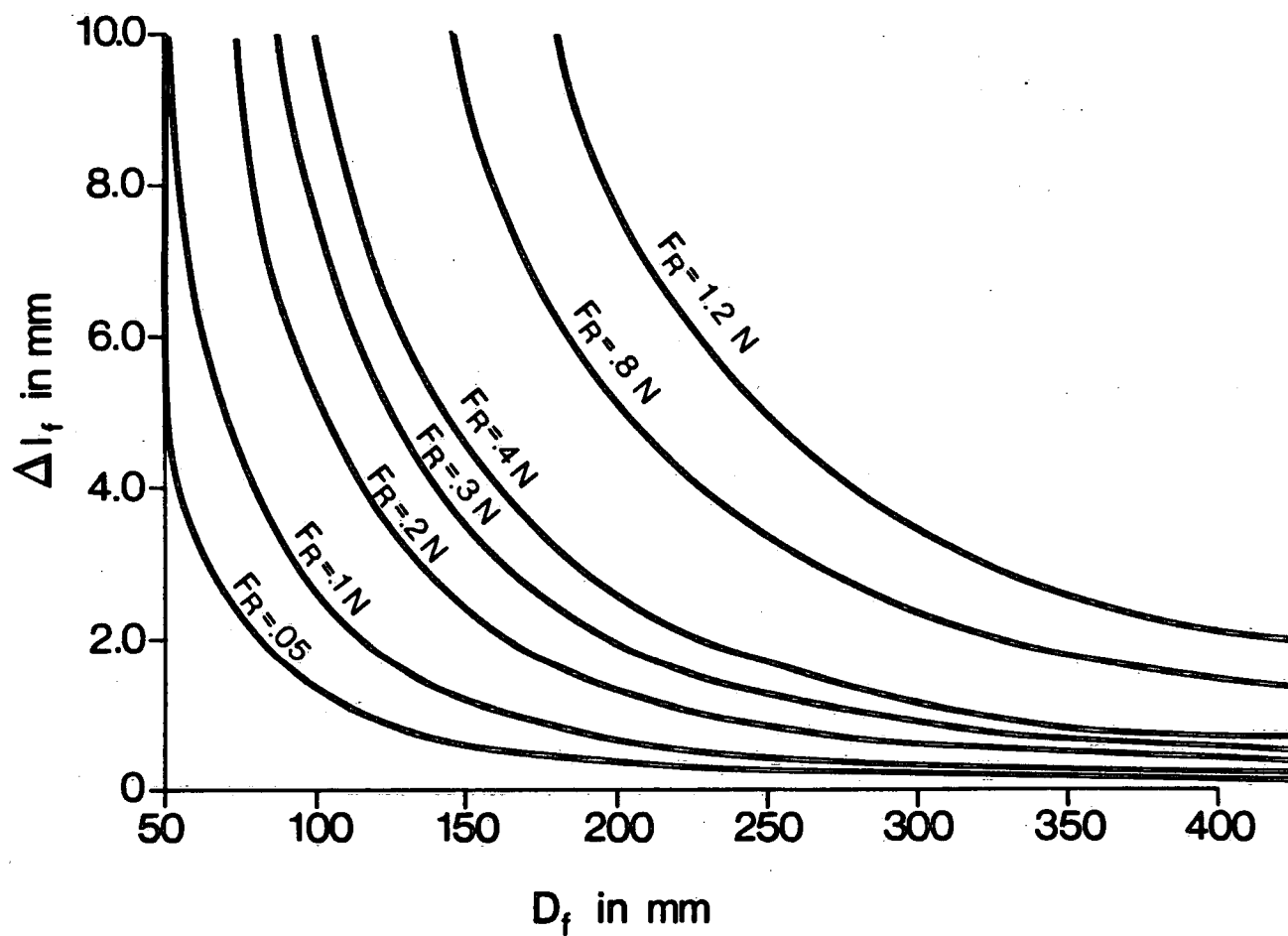Inflow Needle Valve
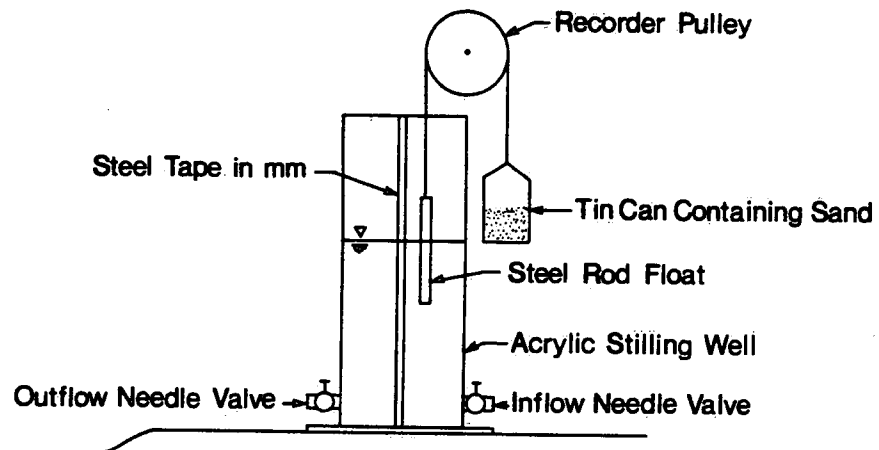
FIGURE 4 -SCHEMATIC OF TEST APPARATUS TO MEASURE
RECORDER RESISTANCE

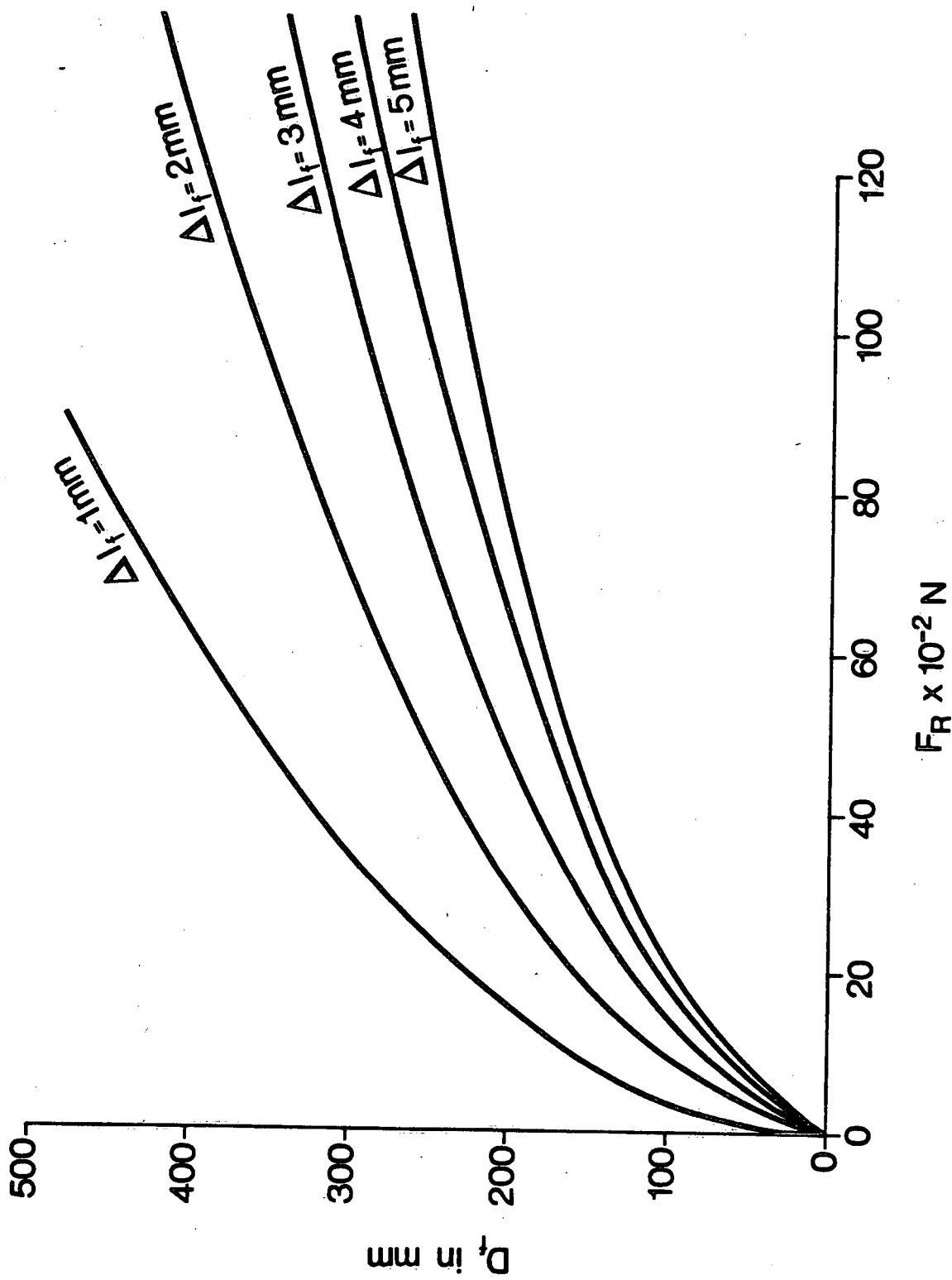FIGURE 5 -EFFECT OF D_f AND F_R ON FLOAT LAG

FIGURE 6 -FLOAT SIZE REQUIRED TO OBTAIN A FLOAT LAG OF 1mm FOR A GIVEN VALUE OF F_R
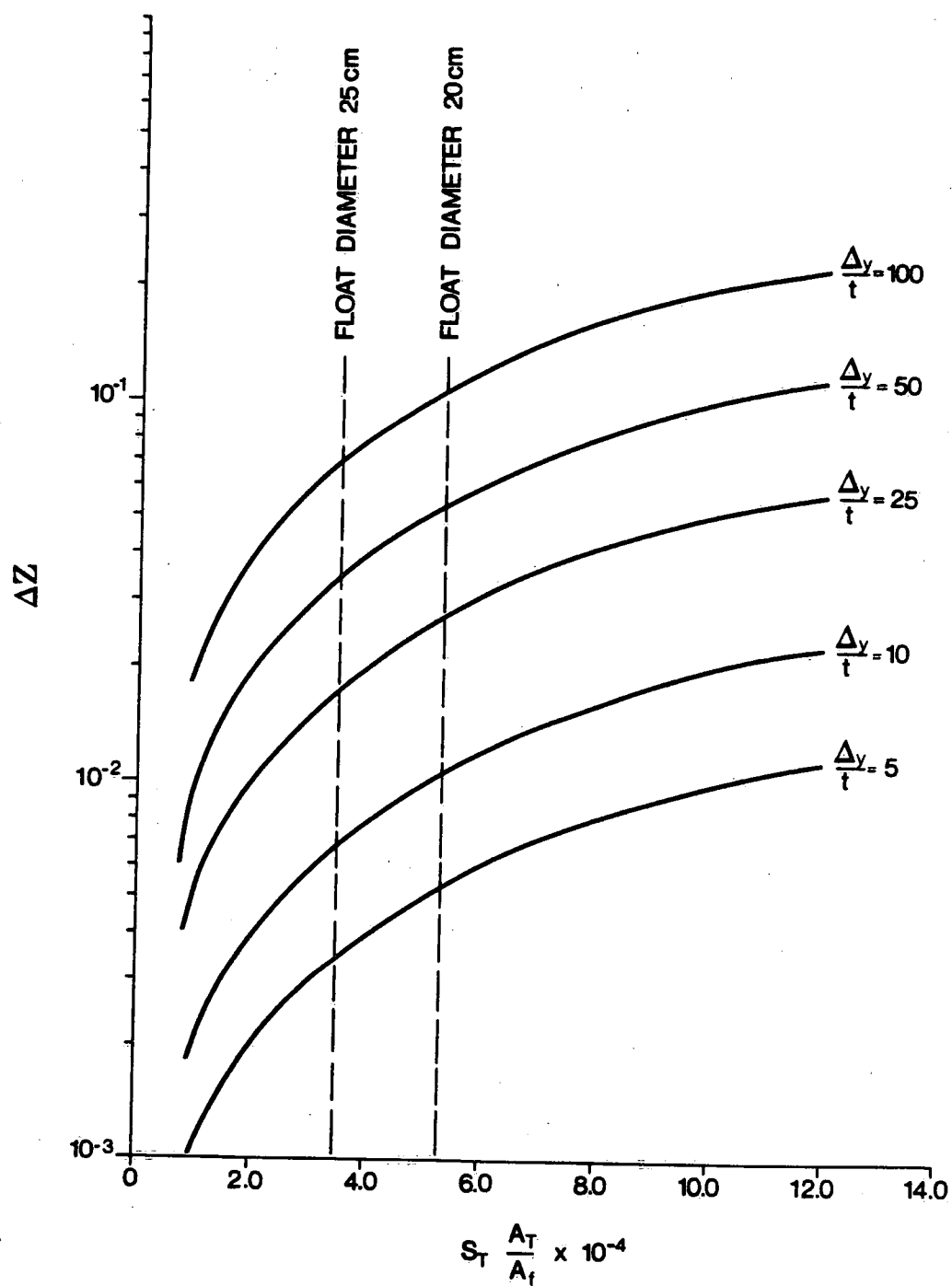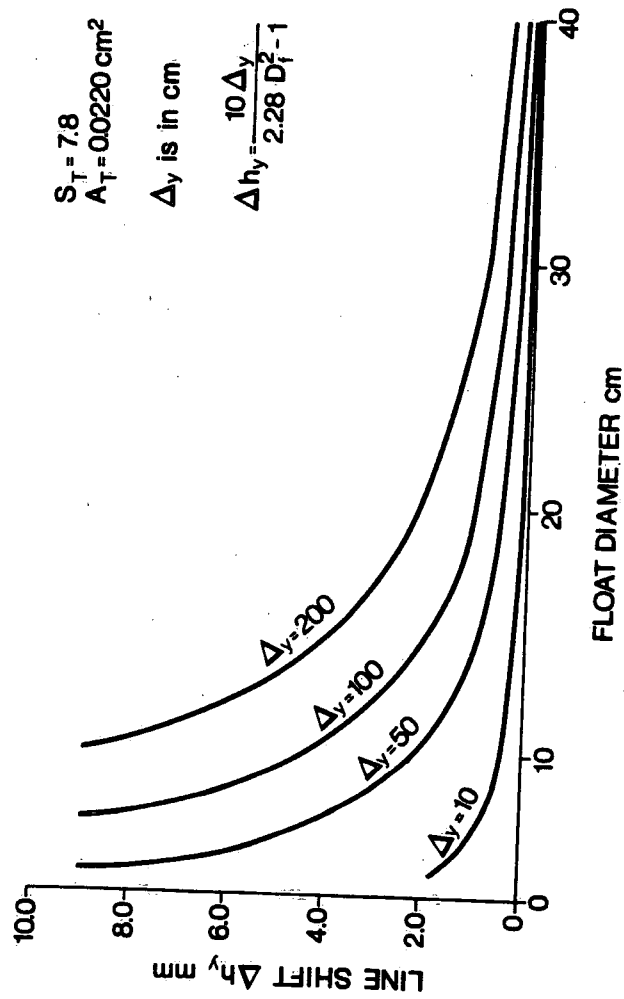
FIGURE 7 -VARIATION IN LINE SHIFT

FIGURE 8 - LINE SHIFT IN TERMS OF FLOAT DIAMETER AND CHANGE OF STAGE

TABLE 1

Recorder Resistance for Rising Water Levels

Test No.	n	Temperature °C	Mean Force N x10^{-2}	Std. Dev. N x10^{-2}	Max. Force at 95% Confidence N, x10^{-2}
1	25	27.2	13.304	10.162	17.577
2	"		10.381	5.190	12.561
3	"		8.810	5.688	11.199
4	"		10.978	4.597	12.907
5	"		9.765	3.540	11.252
6	25	-19.5	7.875	3.023	9.145
7	"		10.381	3.281	12.759
8	"		13.841	4.475	15.721
9	"		11.594	6.284	14.233
10	"		10.978	4.236	12.757

n = sample size   N = newtons

TABLE 2

Recorder Resistance for Falling Water Levels

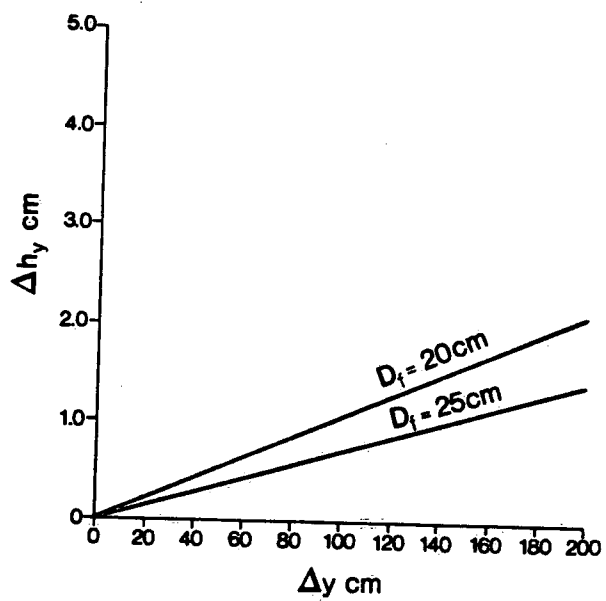| Test No.	n	Temperature °C	Mean Force N x10^{-2}	Stil. Dev. N x10^{-2}	Max. Force at 95% Confidence N, x10^{-2}
11	25	27.2	8.293	6.702	11.108
12	"		10.898	3.639	12.426
13	"		17.719	10.381	22.079
14	"		14.617	9.585	18.643
15	"		18.236	8.472	21.794
16	25	-19.5	13.841	7.975	17.191
17	"		11.336	3.898	12.973
18	"		12.449	6.026	14.980
19	"		13.324	6.503	16.055
20	"		10.878	3.918	12.524

n = sample size   N = newtons

FIGURE 9 -SENSITIVITY OF FLOAT SYSTEM FOR
A GIVEN FLOAT SIZE