

**NUMERICAL MODELLING OF SEEPAGE EROSION
IN SHORE BLUFFS CONSISTING OF
GLACIOLACUSTRINE SILTS**

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RÉSUMÉ

Nous avons étudié la stabilité des berges hautes soumises à l'érosion par infiltration en faisant une analyse de l'infiltration à l'aide d'un MCT. Quand les valeurs de k étaient inférieures à 1×10^{-4} cm/sec, nous avons obtenu des gradients hydrauliques très élevés dans la strate sensible de vase sableuse. La perméabilité anisotropique change l'orientation des vecteurs des gradients hydrauliques et augmente donc la stabilité des pentes des berges hautes. La stabilité des berges hautes soumises à l'érosion par l'infiltration est tributaire de la grandeur et de la direction des vecteurs des gradients hydrauliques de même que des propriétés géotechniques du sol.

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ABSTRACT

The stability of shore bluffs against seepage erosion was investigated by a FEM seepage analysis. Very high hydraulic gradients within the sensitive silt stratum were obtained for k values lower than $1 \times 10^{-4} \text{ cm s}^{-1}$. Anisotropic permeability changes the orientation of hydraulic gradient vectors and thus increases the stability of bluff slopes. Stability against seepage erosion depends on the magnitude and direction of hydraulic gradient vectors as well as on soil geotechnical properties.

INTRODUCTION

Shore bluffs are natural slopes characterized by marginal stability and rapid profile changes as a result of toe erosion due to waves. Slope stability computations using limit equilibrium methods are appropriate for cases where bluff recession occurs by sliding along planar or curved slip surfaces. The objective of this paper is to draw attention to a different mode of failure where bluffs recede by development of gullies. In certain areas along the lower Great Lakes gullies represent the major mode of bluff recession and they recede at considerably higher rates than the adjacent shore [1]. When combined with wave erosion, gully recession may reach 50 m per year or even more. This type of bluff retreat has been documented for numerous sites along Lakes Huron, Erie and Ontario. A tentative numerical model to assess quantitatively the stability of bluff slopes against this mode of failure is proposed and it is supported by field and laboratory data.

STUDY SITE

The study site is located approximately 4 km east of Port Burwell, Ontario, on the central north shore of Lake Erie. The regional geology and

stratigraphy along this stretch of bluff-type shore has been described elsewhere [2,3].

The three principal units occurring at the site are a stratum of clayey waterlain till overlain by a stratum of glaciolacustrine silt, which is in turn overlain by a top stratum of fine to medium sand. Annual topographic surveys carried out at the site over the period of five years [4] showed three principal modes of bluff retreat: (i) parallel retreat in which the crest and the toe recede at approximately equal rates; (ii) rotational slides characterized by a cyclic pattern comprising landsliding, formation of an offshore debris fan, erosion and steepening of the bluff slope and new landsliding; and (iii) gully growth at various stages of development (Fig. 1). The analysis described in this paper has been performed for location where bluffs recede by mode (i); however active gullies do exist in the immediate vicinity.

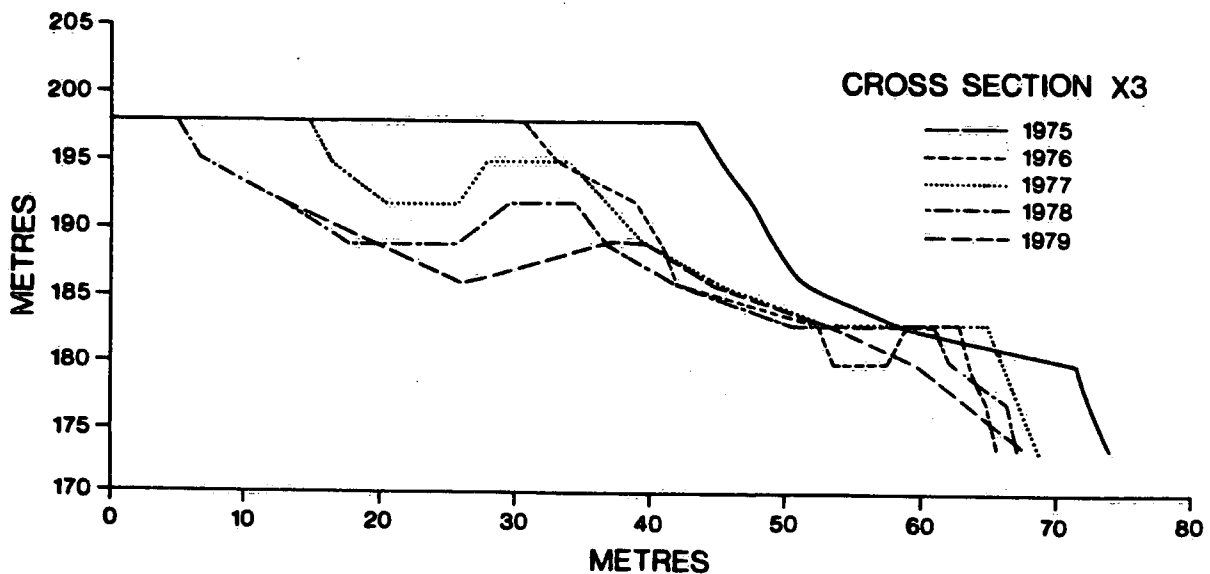


Figure 1. Example of gully development, Port Burwell study site.

While the first two modes of bluff retreat can be analyzed by limit equilibrium methods for planar and rotational slides, no computational method appears to be available to assess the stability of bluff slopes for the third mode. A theoretical model proposed by Hutchinson [5] is applicable only to seepage erosion occurring in a cohesionless layer confined by overlying and underlying aquicludes and cannot therefore be used.

FINITE ELEMENT SEEPAGE ANALYSIS

As the development of gullies is caused by groundwater, it appears logical to analyze the influence of seepage on the stability of the bluff slope. The finite element program PC-SEEP [6] is well suited to analyze the influence of heterogeneous and anisotropic stratigraphy present at the site.

A simplified stratigraphic cross-section (Fig. 2) is based on detailed results of borehole investigation, groundwater monitoring and laboratory

tests. Pore pressures were measured in the boreholes between the fall of 1975 and the spring of 1982. The hydraulic heads shown in Fig. 2 correspond to the situation in July 1976. The corresponding finite element mesh, consisting of 165 nodes and 278 elements, is shown in Fig. 3. Of special interest are four nodes (Nodes 41, 56, 72 and 89) that represent exit points for seepage within the silt stratum.

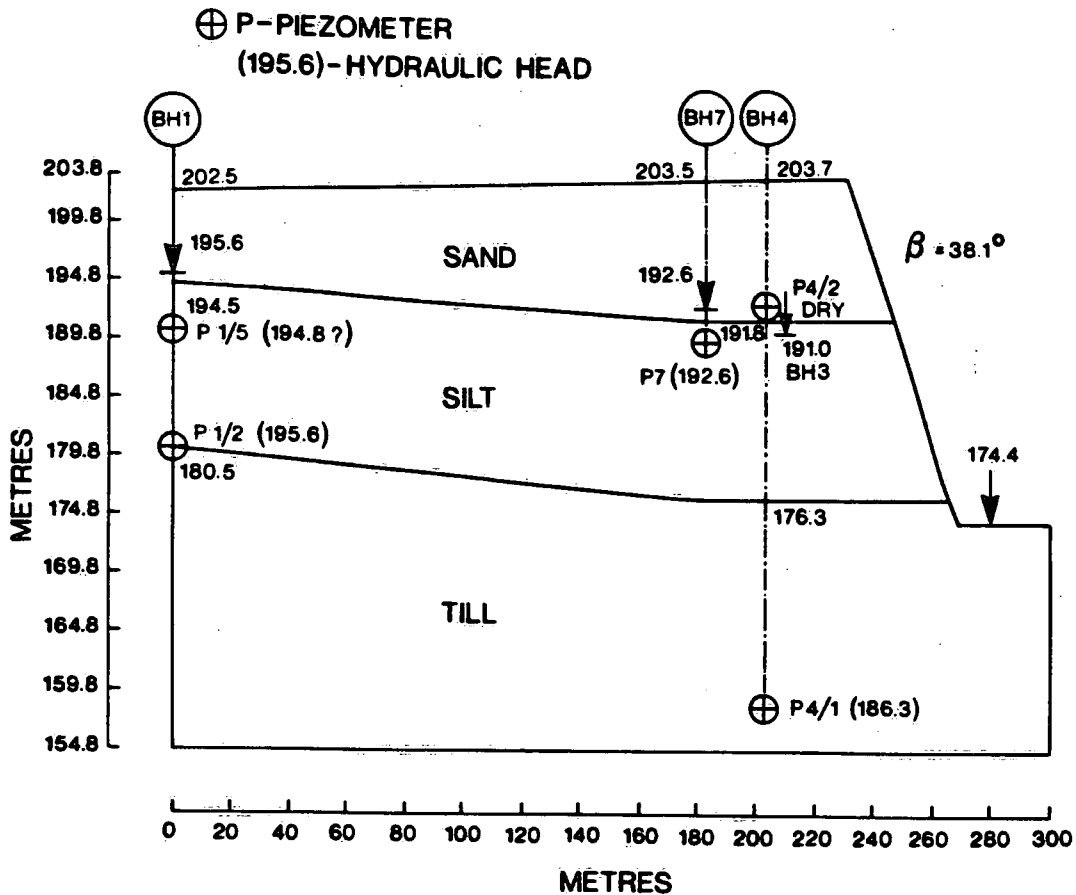


Figure 2. Simplified stratigraphic cross-section with measured hydraulic heads.

Effect of Isotropic Permeability

The coefficient of permeability, k , of silts is known to vary between about 5×10^{-3} and 5×10^{-7} cm s^{-1} [7]. Results of parametric study of the effect of k on the values of the hydraulic gradient vector $\{i\}$ (determined by its magnitude and its angle with horizontal, α , which is positive when $\{i\}$ is inclined downwards) are summarized in Table 1. Based on results of grain-size analysis, k values for sand and till are 5×10^{-3} cm s^{-1} and 1×10^{-8} cm s^{-1} , respectively.

The closest agreement with measured hydraulic head values was obtained for $k = 5 \times 10^{-5}$ cm s^{-1} (Fig. 4). The result, however, does not explain significantly lower heads measured in the deep piezometer in Borehole 4 (Fig. 2).

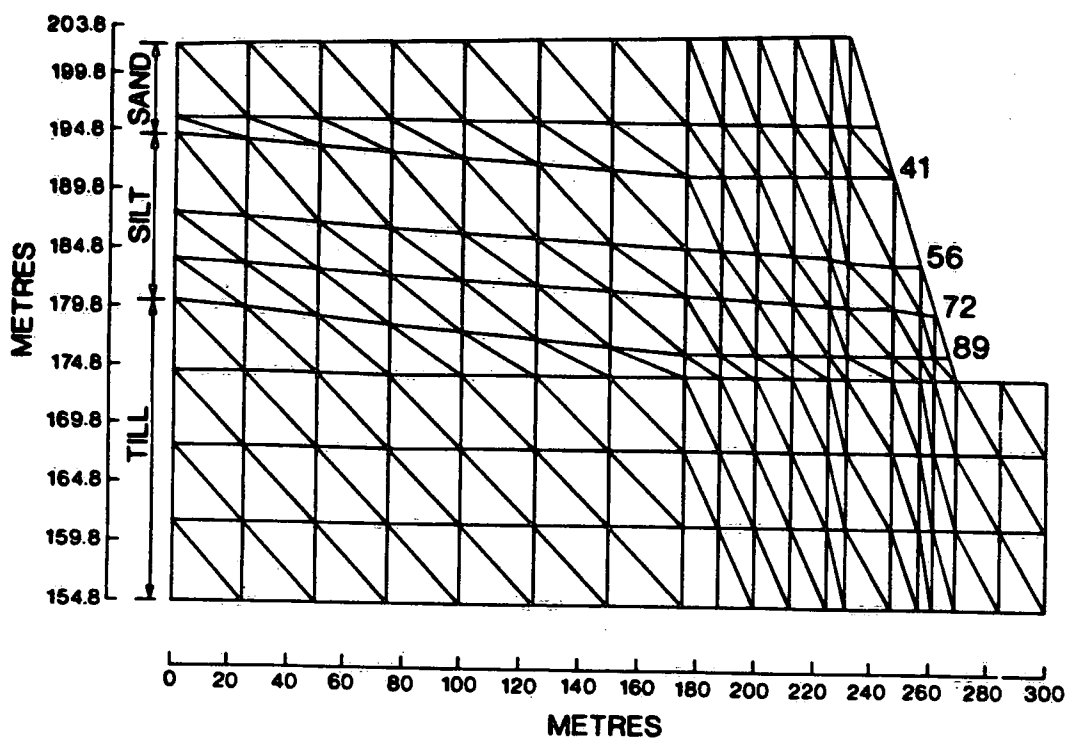


Figure 3. Finite element mesh showing four exit nodes at the surface of silt stratum.

TABLE 1
Magnitude $[i]$ and direction α of hydraulic gradient vector at four exit nodes for different k values (n.f. = no flow)

k (cm s ⁻¹)	Nodes							
	41		56		72		89	
	$[i]$	α	$[i]$	α	$[i]$	α	$[i]$	α
5×10^{-3}	n.f.		n.f.		n.f.		0.48	35.79
5×10^{-4}	n.f.		n.f.		n.f.		0.54	24.28
1×10^{-4}	n.f.		n.f.		n.f.		0.68	8.71
5×10^{-5}	n.f.		n.f.		0.49	10.69	0.72	3.85
5×10^{-6}	0.23	46.92	0.46	24.06	0.67	7.99	0.73	2.44
5×10^{-7}	0.23	50.58	0.62	18.84	0.71	4.54	0.73	2.61
			0.62	19.66	0.71	4.89		

Effect of Anisotropic Permeability

In view of the glaciolacustrine origin of the soil, as well as the occurrence of thin horizontal laminae of silty clay within the silt stratum, it is highly likely that the values of $[i]$ will be influenced by the ratio of the average horizontal and vertical permeability coefficients. The effect of anisotropy within the silt stratum was therefore investigated using the value of $k_h = 5 \times 10^{-4}$ cm s⁻¹ and the k_h/k_v ratio between 1 and 500. The results are summarized in Table 2.

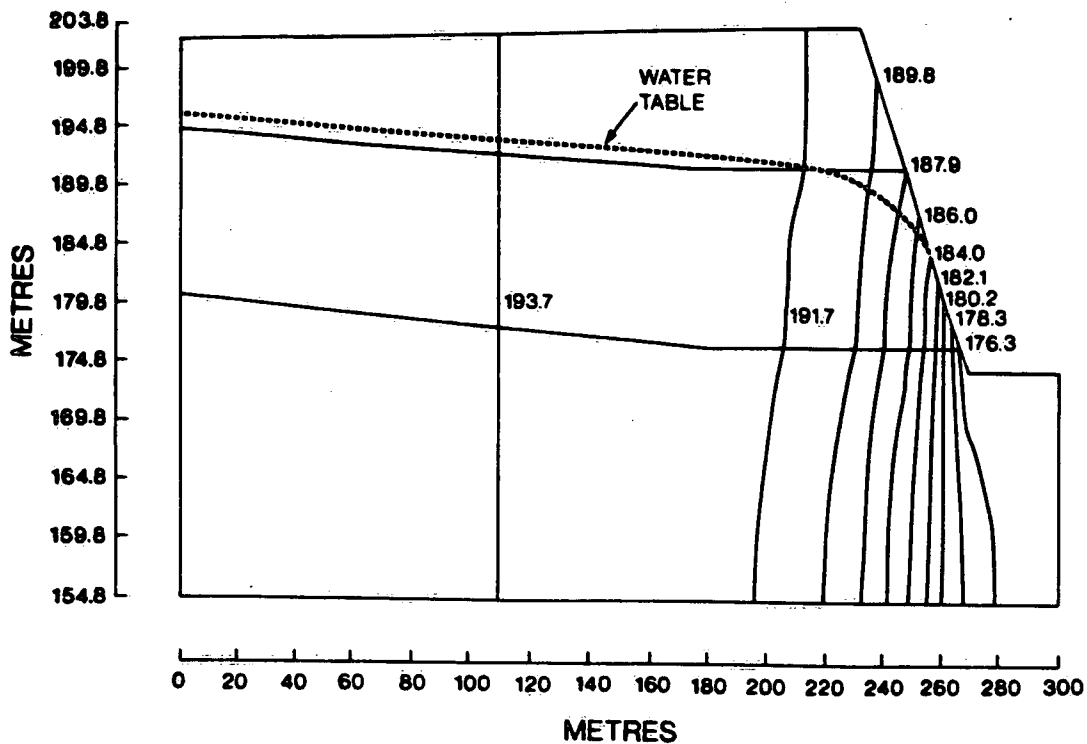


Figure 4. Contours of total hydraulic head in the bluff slope under conditions of isotropic permeability.

TABLE 2
Magnitude [i] and direction α of hydraulic gradient vector at four exit nodes for different anisotropic permeabilities

$k_h = 5 \times 10^{-4}$ (cm s⁻¹); n.f. = no flow

Ratio k_h/k_v	Nodes							
	41		56		72		89	
	[i]	α	[i]	α	[i]	α	[i]	α
1	n.f.		n.f.		n.f.		0.54	24.28
5	n.f.		n.f.		0.36	49.13	0.60	37.00
10	n.f.		n.f.		0.43	60.97	0.61	48.77
20	n.f.		n.f.		0.41	70.82	0.64	57.80
50	n.f.		0.68	84.82	0.73	75.81	0.70	67.27
100	n.f.		0.78	85.13	0.78	79.60	0.75	73.74
200	0.47	87.68	0.91	86.10	0.83	82.37	0.80	78.41
500	0.48	87.71	0.93	87.06	0.88	84.99	0.86	82.81

As can be seen from results presented in Table 2, increasing anisotropy increases both [i] and α values. The best agreement between measured and computed hydraulic heads, including the heads measured in the deep piezometer in Borehole 4 (Fig. 2), was obtained for $k_h/k_v = 50$. As the

direction of gradient vectors significantly deviates from the horizontal, the anisotropy clearly increases stability of the slope against seepage erosion.

Critical Hydraulic Gradient, i_{cr}

The influence of the hydraulic gradient vector $\{i\}$ on the potential for slope failure, assuming Darcian uniform seepage and isotropic permeability, has been studied by Kovacs [8] and Iverson and Major [9]. Following the approach outlined in [8], i_{cr} is determined by

$$i_{cr} = \frac{\gamma' (\tan \phi' \cos \beta - \sin \beta)}{\gamma_w [\cos (\beta - \alpha) + \tan \phi' \sin (\beta - \alpha)]} \quad (1)$$

where ϕ' is the effective friction angle, γ' is the submerged unit weight, γ_w is the unit weight of water and β is the slope angle. Following the derivation by Iverson and Major, i_{cr} is determined by

$$i_{cr} = \frac{\gamma' (\sin \phi' \cos \beta - \sin \beta \cos \phi')}{\gamma_w (\sin \beta \cos \lambda + \sin \lambda \cos \phi')} \quad (1a)$$

where λ is the angle of $\{i\}$ measured clockwise from the outward-directed normal to the slope surface. As $\sin \lambda = \cos (\beta - \alpha)$ and $\cos \lambda = \sin (\beta - \alpha)$, it can be easily shown that Equations 1 and 1a are identical. Using trigonometric identities [9], Equation 1a can be further reduced to

$$i_{cr} = \frac{\gamma' [-\sin (\beta - \phi')]}{\gamma_w \sin (\lambda + \phi')} \quad (2)$$

Equations 1 and 2 permit to investigate the influence of α on i_{cr} . Note that the results are in general influenced by soil properties as well as by the slope angle. It is further apparent that horizontal seepage through cohesionless soils is not necessarily the most adverse direction for slope stability, α_{cr} , which is obtained for the condition

$$\alpha_{cr} = \beta - \phi' \quad (3)$$

The results of direct shear tests for the silt stratum yielded zero effective cohesion, ϕ' (peak) = 42° and ϕ' (ultimate) = 37°. It is nevertheless of interest to investigate the effect of cohesion, c' , on i_{cr} because of variable percentage of clay content and slight plasticity determined on borehole samples of silt from the site. Although the derivation is not given in [9], it can be shown (Iverson, personal communication) that the effect of cohesion will increase the values of i_{cr} without changing the validity of Equation 3. In the case of cohesive soil, Equation 2 takes the form

$$i_{cr} = \frac{\gamma' [-\sin (\beta - \phi') + \frac{c' \cos \phi'}{\gamma_{cr} \gamma'}]}{\gamma_w \sin (\lambda + \phi')} \quad (4)$$

where y_{cr} is a critical soil depth at which failure occurs measured vertically downward from the slope surface. The relationship between i_{cr} and α for cohesive silt is more complex due to the combined effect of β and y_{cr} and, as expected, values of i_{cr} are in general higher than for cohesionless silt. In this case, the lowest value of i_{cr} was obtained for horizontal seepage and $\beta = \phi'$. The validity of Equation 4 should be empirically checked in view of the inherent and unlikely assumption of no volumetric change of soil in the proximity of the slope surface that is subjected to high hydraulic gradients.

Factors of Safety Against Seepage Erosion

The factors of safety F are defined as the ratio of the critical hydraulic gradient $[i]_{cr}$, determined from Equations 2 and 4, and the $[i]$ value obtained from the FEM analyses for the same values of α . The value of $\phi' = 42^\circ$ was used in the computations. The results for the three exit nodes are summarized in Table 3.

TABLE 3
Factors of Safety Against Seepage Erosion

	Nodes		
	56	72	89
(a) isotropic permeability, $k(\text{silt}) = 5 \times 10^{-5} \text{ cm s}^{-1}$, $c' = 0$ $F = [i_{cr}]/[i]$	0.15	0.09	0.09
(b) anisotropic permeability, $k_h = 5 \times 10^{-4} \text{ cm s}^{-1}$, $k_h/k_v = 50$, $c' = 0$ F	4.03	0.47	0.27
(c) isotropic permeability, $k(\text{silt}) = 5 \times 10^{-5} \text{ cm s}^{-1}$, $c' > 0$ F	1.55	1.05	0.89
(d) anisotropic permeability, $k_h = 5 \times 10^{-4} \text{ cm s}^{-1}$, $k_h/k_v = 50$, $c' > 0$ F	41.43	4.82	2.79

CONCLUSIONS

The computations presented in Table 3 suggest that both cohesion and soil anisotropic permeability are required to maintain F above unity at Node 89 located at the base of the silt stratum. The result is in agreement with field observations that no gully has formed at this location over the period of six years. The analysis has been carried out for $\beta = 38.1^\circ$ (Fig. 2). The actual overall slope angle measured ranged from 35.0° to 43.4° for six successive surveys.

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