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MEASURING HYDRODYNAMIC DISPERSION USING THE BOREHOLE DILUTION METHOD

by

Kent S. Novakowski

National Water Research Institute
867 Lakeshore Road
Burlington, Ontario, L7R 4A6
(416)336-4610

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ABSTRACT

The borehole dilution method is commonly employed to determine the local ground water velocity in sand and gravel aquifers. However, present analytical methods are limited because they do not account for hydrodynamic dispersion. In this paper, a new analytical model is developed by which measurements of hydrodynamic dispersion can be obtained along with determinations of ground water velocity from the results of borehole dilution experiments. The solution of the boundary value problem is obtained using the Laplace transform method and results are presented using a numerical inversion scheme. By defining a mixing factor, β, as the ratio of cross-sectional area available for ground water flow over the volume of the test interval, it is determined using the analytical model, that the size of β has an important effect on calculating the hydrodynamic dispersion coefficient and an accurate ground water velocity. For the case where β is large, unique values of the dispersion coefficient are obtained and the analytical model developed for this paper must be used in order to calculate accurate values of ground water velocity. Conversely, for the case where β is small, the effect of hydrodynamic dispersion is much less distinguishable and the traditional method of analysis can be employed to obtain relatively accurate values of velocity. The only means by which β can be adjusted under given test conditions, is by manipulating the volume of the test section. Thus, to enhance the effects of hydrodynamic dispersion in the results of borehole dilution experiments and obtain unique values of the dispersion coefficient, the volume of the test interval must be reduced.

RÉSUMÉ

La méthode de dilution en trou de sonde est couramment utilisée pour déterminer la vitesse locale d'écoulement des eaux souterraines dans les aquifères de sable et de gravier. Toutefois, les méthodes d'analyse actuelles sont limitées parce qu'elles ne tiennent pas compte de la dispersion hydrodynamique. Dans cet article, un nouveau modèle d'analyse est proposé qui permet de mesurer la dispersion hydrodynamique tout en déterminant la vitesse d'écoulement des eaux souterraines à partir des résultats d'expériences de dilution en trou de sonde. La solution du problème de la valeur limite est obtenue à l'aide de la méthode de transformation de Laplace et les résultats sont présentés à l'aide d'un modèle d'inversion numérique. En définissant un facteur de mélange, β, comme le rapport de l'aire de la section mouillée disponible pour l'écoulement des eaux souterraines en fonction du volume de l'intervalle d'essai, on détermine à l'aide du modèle d'analyse que la taille de β a une influence importante sur le calcul du coefficient de dispersion hydrodynamique et sur celui d'une vitesse précise d'écoulement des eaux souterraines. Lorsque la valeur de β est élevée, des valeurs uniques du coefficient de dispersion sont obtenues et le modèle d'analyse mis au point pour cette recherche doit être utilisé pour calculer les valeurs précises de la vitesse d'écoulement des eaux souterraines. Inversement, lorsque la valeur de β est faible, l'effet de la dispersion hydrodynamique est beaucoup moins notable et

la méthode classique d'analyse peut être utilisée pour obtenir des valeurs relativement exactes de la vitesse d'écoulement. Le seul moyen d'ajuster β en fonction des conditions d'essai données consiste à manipuler le volume de la section d'essai. Par conséquent, pour accroître les effets de la dispersion hydrodynamique dans les résultats des expériences de dilution en trou de sonde et pour obtenir des valeurs uniques pour le coefficient de dispersion, le volume de l'intervalle d'essais doit être réduit.

MANAGEMENT PERSPECTIVE

This paper describes a new analytical method for determining the characteristics of a geological material that cause the spreading and dilution of contamination as a result of ground water circulation. The field method that is used to provide results for interpretation using the analytical model, requires a small modification of a well established technique known as point dilution. The new analytical model and field method have a wide potential for application and may prove very valuable in ground water studies where a limited number of monitoring wells are available. The purpose of this technical note is only to present the conceptual model, the analytical method and potential limitations and applications with regard to the field method. Future work will focus on further development of the field method including a field assessment at a site where detailed information about the dispersive nature of the geological material, has already been obtained.

PESRPECTIVE DE GESTION

Cet article décrit une nouvelle méthode d'analyse permettrant de déterminer les caractéristiques d'un matériau géologique qui cause la dispersion et la dilution des contaminants par suite de la circulation des eaux souterraines. La méthode de terrain utilisée pour obtenir les résultats à interpréter à l'aide du modèle d'analyse nécessite une légère modification d'une méthode bien établie dite de dilution ponctuel. Le nouveau modèle d'analyse et la méthode de terrain offrent de nombreuses possibilités d'application et pourraient se révéler très précieux pour l'étude des eaux souterraines lorsqu'un nombre limité de puits de surveillance sont disponibles. Cette note technique n'a pour but que de présenter le modèle théorique, le modèle d'analyse et les limites et les applications potentielles de la méthode de terrain. D'autres travaux porteront sur les futurs développements de la méthode de terrain, notamment une évaluation dans un site pour lequel on dispose déjà d'informations sur les caractéristiques de dispersion du matériau géologique.

INTRODUCTION

Measurements of the hydrodynamic dispersion coefficient or the dispersivity of an aquifer material are commonly obtained using the results of advective tracer experiments. In most cases the experiments are conducted between two or more wells and require considerable instrumentation and a complicated field method. The quality of the results depend on the detail of the instrumentation, the nature of the advective flow field (Gelhar *et al*., 1985) and the effects of mixing in the injection and sampling instrumentation (Pickens and Grisak, 1981). In the case of tracer experiments conducted in a single well, where water labelled with tracer is pumped into and then out of the same well, the results are further complicated by macrodispersion and reversible dispersion processes (Gelhar *et al*., 1985). Consequently, in more recent studies of dispersion phenomenon, tracer methods using the natural flow field have been employed (Sudicky *et al*., 1983; Mackay *et al*., 1986; Killey and Moltyaner, 1988). In these experiments, measurements of dispersion are obtained based on transport between an injection well and multiple observation wells. However, in cases where the number of available monitoring wells is limited or where time is a factor, a single well test method would be of advantage. To date, there is no existing analytical or field method for determining dispersion coefficients from a single well under natural flow conditions.

The borehole dilution method (also known as point dilution) is a technique frequently used to measure horizontal ground water velocity in a single well (Halevy _et_ _al_., 1967; Drost _et_ _al_., 1968). The principle of the borehole dilution method is based on measurement of the decreasing concentration in a well as tracer is removed from the well by ground water flow in the horizontal direction. Vertical flow in the wellbore volume and effects due to diffusion, temperature, density and well construction can influence the results (Halevy _et_ _al_., 1967). Most of these problems are overcome by using a method in which the test section is isolated by inflatable packers, the tracer is injected into the test section instantaneously and then continuously and completely mixed through the duration of the experiment, and measurement of the tracer concentration is conducted in situ (Grisak _et_ _al_., 1977; Belanger, 1983). For a more thorough description of the borehole dilution method see Freeze and Cherry (1979, pg. 428).

The results of borehole dilution experiments are analysed using an expression derived from a simple ordinary differential equation which describes mass conservation in the borehole (Halevy _et_ _al_., 1967). In the nomenclature of this paper, the equation is given by:

$$\pi r_w^2 h_w \frac{dC}{dt} = - f\, 2 r_w h_w n\, \bar{v} C \qquad (1)$$

where r_w is the well radius, h_w is the mixing length or length of the test section, $\pi r^2_w h_w$ is the volume of the test section, n is the formation porosity, $\bar{v}$ is the average linear ground water velocity, C is the concentration of tracer in the wellbore and t is time. The term, $f2r_w h_w n \bar{v}$, represents the volumetric flux leaving the borehole. The factor, f, accounts for the disruption in the flow field due to the presence of the wellbore (Halevy *et al*., 1967, Palmer, 1988). The initial condition for (1) is given by:

$$C(0) = C_o \tag{2}$$

where C_o is the initial concentration of the tracer in the well. As is evident in equation (1), the boundary value problem, as stated, is missing a component of dispersive flux that may be important where advective transport is influenced by local-scale heterogeneity of the aquifer material. By using the solution to equation (1) to interpret the results of borehole dilution experiments conducted under conditions where the dispersive flux is large, some error in the calculation of velocity will occur.

The purpose of this note is to present and discuss a new analytical model in which a dispersive flux term is included in the boundary value problem to account for the influence of hydrodynamic dispersion in borehole dilution experiments. Using this model, a correct value of ground water velocity can be obtained from dilution experiments where dispersive flux is large. In addition, because a unique value of dispersivity can also be determined, the borehole dilution method can be used as a single well technique for determining dispersion coefficients under natural flow conditions.

DEVELOPMENT OF THE ANALYTICAL MODEL

Mass transport of a conservative solute under uniform and steady-state conditions of ground water flow is governed by:

$$\frac{\partial C}{\partial t} + \bar{v}\,\frac{\partial C}{\partial x} - D_L\,\frac{\partial^2 C}{\partial x^2} = 0 \qquad 0 \leq x \leq \infty \tag{3}$$

where C is the concentration of tracer in a semi-infinite medium. The coefficient of hydrodynamic dispersion, D_L, is equal to the dispersivity, α_L, times velocity plus a component of molecular diffusion. For a thorough discussion of D_L, see Bear (1979). The outer boundary condition for (3) is given as:

$$C\,(\infty,t) = 0 \tag{4}$$

and the initial condition is given as:

$$C\,(x,0) = 0 \tag{5}$$

for all x.

Borehole dilution is modeled using the inner boundary condition for (3) which, again using mass conservation, is given as:

$$\pi r^2_w h_w \frac{d C_w(t)}{dt} = - \xi [\bar{v} C (0,t) - D_L \frac{\partial C (0,t)}{\partial x}] \qquad (6)$$

where ξ is equal to $f2r_w h_w n$, $\pi r^2_w h_w$ is the volume of the isolated test section, and $C_w(t)$ is the concentration of tracer in the well. Equation (6) is identical to equation (1) except for the last term on the right hand side which represents dispersive flux. In addition, this boundary condition is valid for advectively dominated transport only, due to difficulties in accounting for diffusive transport in the x^- direction. Boundary conditions similar to this have been employed in analytical models used to simulate the results of advective tracer experiments (Hodgkinson and Lever, 1983; Moench, 1989). The initial condition for (6) is:

$$C_w(0) = C_o \qquad (7)$$

and continuity between the well and the formation is given by:

$$C_w(t) = C(0,t) \qquad (8)$$

The solution of equation (3) with respect to (4)-(8) is found in a straightforward manner using the Laplace transform method. The general solution to (3) in the Laplace domain is given by:

$$\bar{C}(x,p) = B \exp \{\psi x - \omega x\} \tag{9}$$

where B is a constant, p is the Laplace variable and ψ and ω are given by:

$$\psi = \frac{\bar{v}}{2D_L} \qquad \omega = \frac{1}{2} \sqrt{\left(\frac{4p}{D_L} + \frac{\bar{v}^2}{D_L 2}\right)} \tag{10}$$

Application of the Laplace transform to the inner boundary condition gives:

$$\frac{1}{\beta} [p \, \bar{C} \, (0,p) - C_o] = D_L \frac{\partial \bar{C}(0,p)}{\partial x} - \bar{v} \, \bar{C} \, (0, p) \tag{11}$$

where β is defined as the mixing factor (Palmer, 1988) and is given as:

$$\beta = \frac{\xi}{\pi r^2{}_w h_w} \tag{12}$$

After substitution of (9) into (11) and solving for B, the concentration of the tracer in the formation at distance x is obtained:

$$\frac{\bar{C} \, (x,p)}{C_o} = \frac{1}{[p + \bar{v}\beta - D_L \, (\psi - \omega)]} \exp \{\psi x - \omega x\} \tag{13}$$

and the solution for concentration in the well (the concentration at $x = 0$ by continuity) is given as:

$$\frac{\bar{C}\ (0,p)}{C_o} = \frac{1}{[p + \bar{v}\beta - D_L\ (\psi - \omega)]} \tag{14}$$

Because equation (14) is an irrational function of p, the complex inversion formula requires integration around a branch cut in the Bromwich contour. This will probably lead to an intractable definite integral in real space and thus analytical inversion is not attempted. A numerical inversion scheme (Talbot, 1981) is instead employed to obtain the function values in real space. Using a variety of extreme values for the coefficients in equation (14), the inversion scheme was found to be remarkably stable and will provide results for all practical values of β, $\bar{v}$ and α_L.

Verification of the solution equation (14) can partly be determined on the basis of comparison to the solution of equation (1). This is accomplished by setting D_L equal to zero (no dispersive component) so that equation (13) reduces to the expression:

$$\frac{\bar{C}\ (0,p)}{C_o} = \frac{1}{[p + v\beta]} \tag{15}$$

Analytically inverting (15) gives:

$$\frac{C(0,t)}{C_o} = \exp\{-\bar{v}\,\beta t\} \tag{16}$$

which is the solution for equation (1). Further verification of (14) would require the use of a numerical model or physical analog and is beyond the scope of this technical note.

DISCUSSION

<u>Influence of Hydrodynamic Dispersion</u>

Figure 1 shows two sets of dilution curves corresponding to β's of 0.5 m^{-1} and 50 m^{-1}. In both cases, dispersivity ranges from 0.001 to 1.0 m and velocity equals 10^{-6} ms^{-1}. The overall pattern of each of the family of curves with β equal to 0.5 m^{-1} and β equal to 50 m^{-1} are substantially different. Thus, the size of the mixing factor has an important effect on distinguishing the influence of hydrodynamic dispersion during borehole dilution experiments. Based on this example, it is apparent that β must be large when interested in interpreting borehole dilution experiments for the effects of dispersion. This is especially true considering that the range in dispersivity used for this example is extreme and for most aquifers, depending on scale, the dispersivity is more likely to be in the range 0.01 m to 0.1 m (Moltyaner and Killey, 1988).

In addition, a remarkable difference is evident in the slope and duration of the dilution curves for various values of α_L at large β. Thus, because borehole dilution experiments are traditionally analysed based on the slope of the dilution curve and equation (16), substantial error can be incurred due to the change in slope as a result of dispersive flux. For example, the relative error in the calculation of velocity using equation (16) and the value of t at C/C_0 equal to 0.5 for curves B and C at $\beta = 50\ m^{-1}$, in Figure 1, is 530% and 32%, respectively. Conversely, the relative error for curves B and C at $\beta = 0.5\ m^{-1}$, is virtually zero in both cases. Therefore, when using the traditional method to interpret the results of borehole dilution experiments, β must be small to prevent analytical error.

Practical Considerations

Borehole dilution experiments have commonly been performed in sand and gravel aquifers where velocities are high enough such that the experiments can be conducted in a reasonable time frame of 10-12 hours, for example, (Grisak et al., 1977; Belanger, 1983). Inspection of equation (12) indicates that the test interval volume is the only parameter by which β can be manipulated through experimental design. A typical β for an experiment conducted in a sand aquifer of porosity equal to 0.30, using modern borehole dilution equipment, is about 10-15 m^{-1} (Belanger, 1983). By reducing the test interval volume by 75% (using an inert filling material such as glass beads), the β will increase to about 40-60 m^{-1}. Because test duration is linearly proportional to β (see Equation (7), Grisak et al., 1977), the duration of the test is reduced by a factor of 4 and the determination of dispersive characteristics is better facilitated.

This is illustrated in Figure 2 where two sets of dilution curves for β equal to 10 and 60 m^{-1} are plotted for α_L of 0.01 and 0.1 m. The curves are calculated for a ground water velocity of 10^{-6} ms^{-1} and are shown plotted against logarithmic concentration and about 11 h of linear time. Using the traditional method of analysis, velocity is determined from the slope of a linear regression line fitted to the field data, and an expression for velocity found by rearranging equation (16). For the case where β equals 10 m^{-1}, fitting the regression line is quite easy for all values of α_L, although some small error will be incurred using equation (16). For the case where β equals 60 m^{-1}, the linear regression fit is progressively more difficult with larger α_L and the potential for error in the calculation of velocity increases. Conversely, however, the uniqueness of the fit when using the analytical model developed herein, is increased dramatically at the larger β and error in the velocity calculation can be eliminated. Furthermore, the model is very sensitive to changes in α_L at larger β and thus accurate values of α_L are obtained using the fitting process.

Unfortunately, two problems can potentially arise by reducing the volume of the test interval; 1) difficulties in attaining a complete mix in the test interval are increased (pers. comm., Belanger, 1989) and 2) the reduced volume will have an unknown influence on the flow field around the well thus modifying the factor, f. The latter effect should have little influence on the determination of dispersivity while the former will influence the results of both velocity and dispersivity calculations.

However, if the problems related to mixing in the interval of reduced volume can be overcome by experimental design and if new correction factors for the disruption in the flow field can be determined using analytical or numerical methods, the borehole dilution method may prove to be a powerful tool for assessing the dispersive characteristics of sand and gravel aquifers. For example, local dispersion coefficients can be easily obtained with this method in a stratified aquifer, by profiling open well screens using very short test intervals. Detail equivalent to that obtained by Killey and Moltyaner (1988) with respect to local dispersion coefficients is possible using appropriately spaced piezometers completed with a minimal well pack. Alternatively, macrodispersive processes can be investigated using the entire length of a well screen, providing a suitable mixing arrangement is employed.

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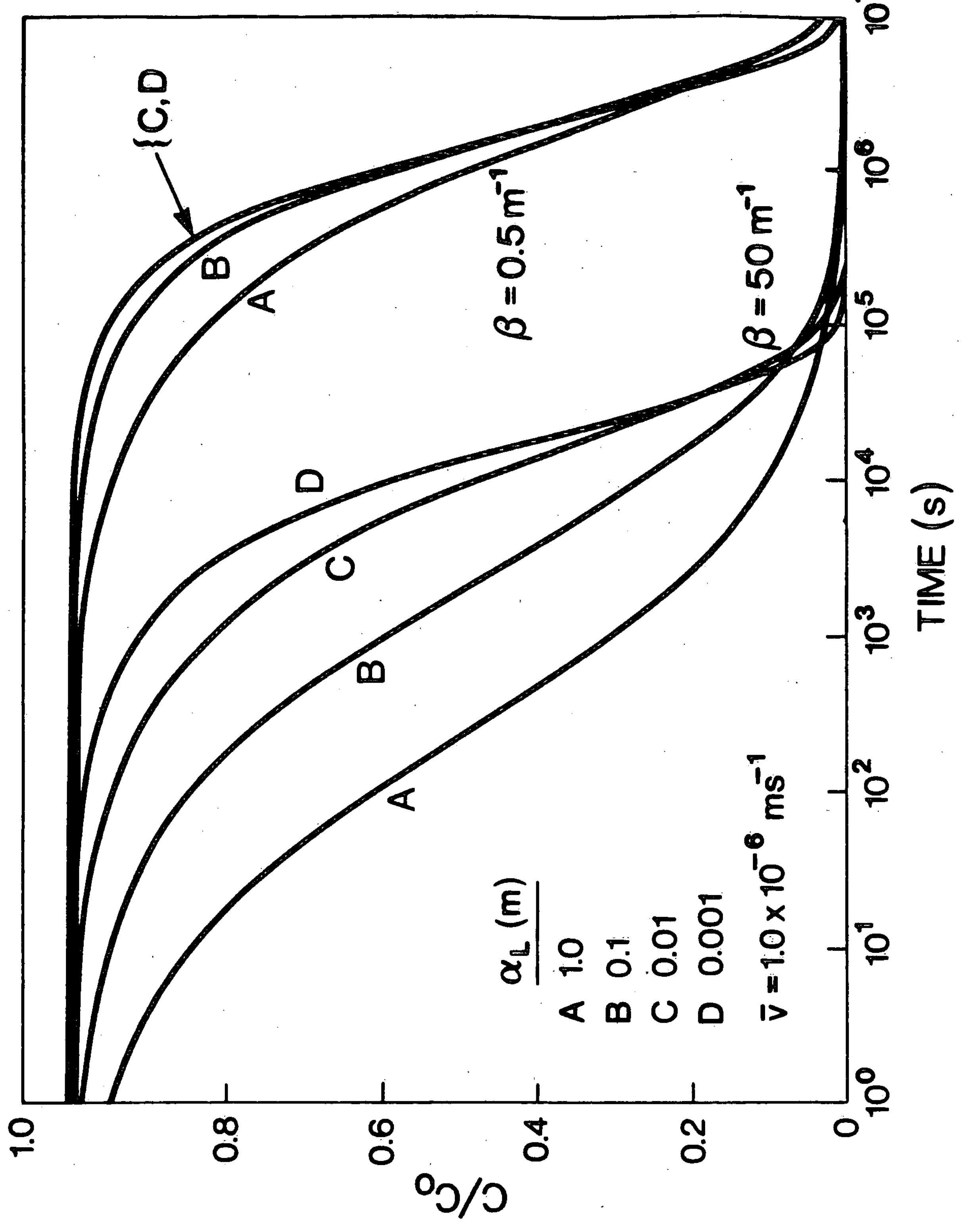

Figure 1. The influence of hydrodynamic dispersion on the results of a borehole dilution experiment.

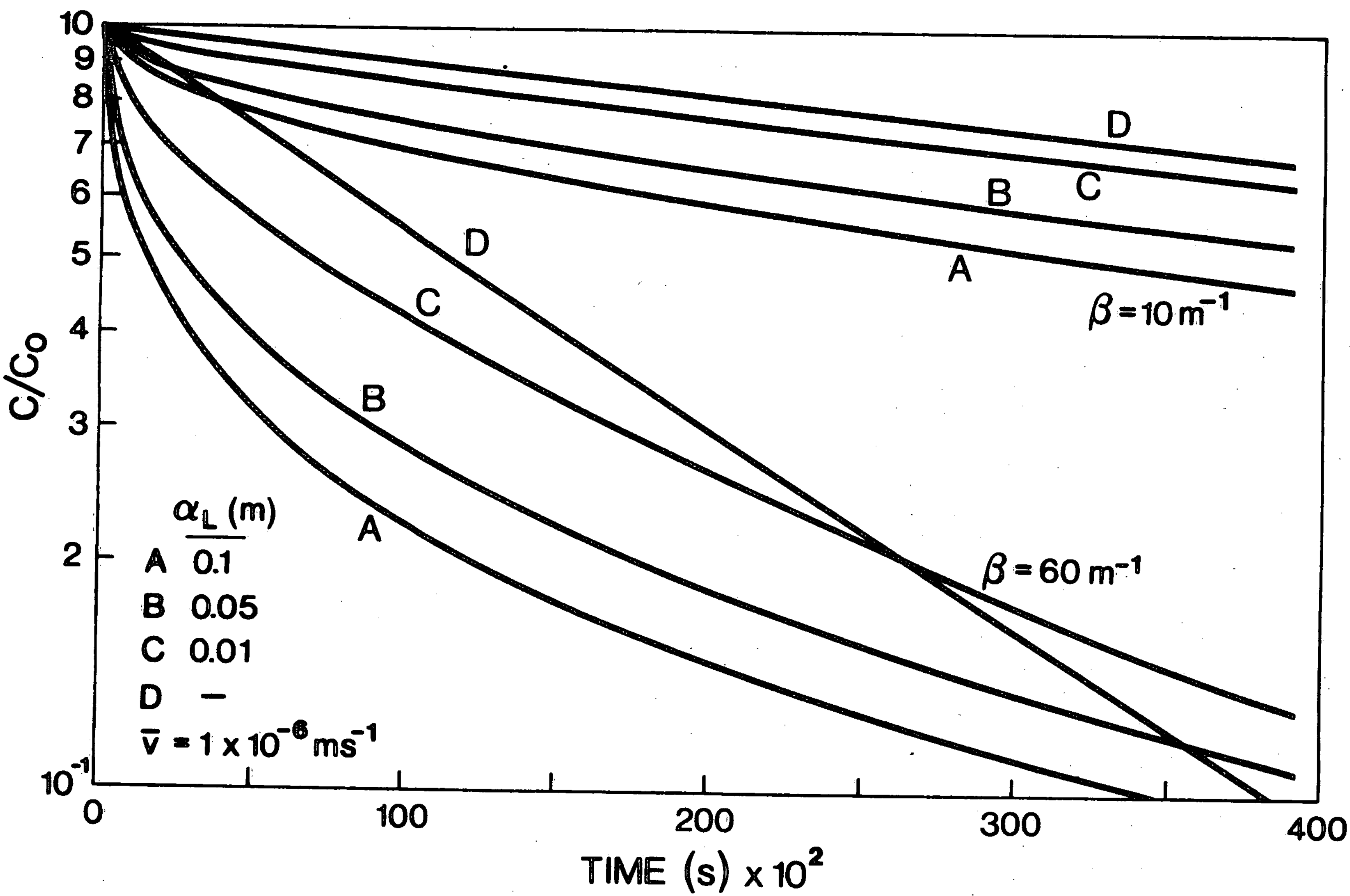

Figure 2. A comparison of the results of borehole dilution experiments conducted using a β of 10 and 60 m^{-1}.