ON THE RATE OF TRANSFER OF HEAT BETWEEN A LAKE AND AN ICE SHEET

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ABSTRACT

The flux of heat between water and ice is either ignored or set to a constant value in present sea ice models. More accurate treatment of this flux may be desirable in the vicinity of polynyas and ice margins, and for more accurate thermal simulation of lakes and reservoirs.

Observations of currents and temperatures under the ice in three large lakes of the headwater region of the Yukon River Basin permit the determination of the coefficient of sensible heat transfer between water and ice by various indirect methods. At a depth of 1 m and an estimated surface roughness of about 0.01 m, the coefficient of sensible heat transfer between water and ice was found to be $(0.8 \pm 0.3)\ 10^{-3}$. This value is within the limits of smooth and rough ice conditions found in laboratory investigations and appears to be somewhat less than that found in recent sea ice studies.

RÉSUMÉ

Dans les modèles actuels de glace de mer, le flux de la chaleur entre l'eau et la glace est soit ignoré, soit établi à une valeur constante. L'emploi d'une valeur plus exacte de ce flux est préférable pour les travaux effectués dans le voisinage des polynies et de la marge glaciaire, ainsi que pour obtenir une simulation plus précise de la température dans les lacs et les réservoirs.

L'observation des courants et des températures sous la glace dans trois grands lacs de la région des eaux nourricières du bassin du fleuve Yukon permet la détermination du coefficient de transfert thermique mesurable entre l'eau et la glace par diverses méthodes indirectes. À une profondeur de un mètre, et pour une rugosité de surface d'environ 0,01 m, le coefficient de transfert thermique mesurable entre l'eau et la glace est (0,8±0,3) 10^{-3}. Cette valeur est comprise dans les limites des régimes de glace vive et de glace rugueuse, que l'on a obtenues en laboratoire, et semble un peu inférieure à celle relevée lors d'études récentes de la glace de mer.

MANAGEMENT PERSPECTIVE

This study attempts to derive estimates of the sensible heat transfer between lake ice and water based on directly observed or easily modelled primary variables, namely the speed and temperature of the water one metre below the ice cover from field data collected from 1983 to 1985 in the headwater lakes of the Yukon River Basin. This quantity is required for studies on the thermal simulation of northern lakes and reservoirs. The estimation of the extent and location of open water areas in an ice sheet is important for the operation of intakes for hydroelectric power generation, for the evaluation of suitable overwintering habitat for waterfowl and the possible disruption of migratory routes for game.

PERSPECTIVE - GESTION

Dans cette étude, on tente d'estimer le transfert thermique mesurable entre la glace et l'eau des lacs en s'appuyant sur des variables primaires observées directement ou facilement modélisées, notamment le débit et la température de l'eau à un mètre sous la couverture de glace, d'après des données recueillies sur le terrain entre 1983 et 1985 dans les lacs nourriciers du bassin du fleuve Yukon. Cette valeur quantitative est nécessaire pour étudier la simulation thermique dans l'eau des lacs et des réservoirs septentrionaux. L'estimation de la superficie et une approximation de la situation géographique des zones d'eau libre dans la nappe glaciaire sont cruciales pour l'installation de prises d'eau à des fins de production d'énergie hydro-électrique, et pour déterminer quels habitats d'hivernage conviennent à la sauvagine ainsi que les probabilités de perturbation des routes migratoires empruntées par le gibier.

While the transfer of heat between water and ice is usually ignored or set to a constant value in the thermodynamic modelling of lake and ocean ice cover, recent studies related to through-flow in ice-covered lakes (cf. Patterson and Hamblin, 1988), have demonstrated the sensitivity of such water bodies to this exchange process. In this study we attempt to establish a bulk parameterization of the transfer of heat between water and ice suitable for numerical modelling purposes.

The heat transfer between rivers and their ice covers has been theoretically treated by Baines (1961) who quantified heat flux in terms of a Reynolds number appropriate to channel flow. A more useful viewpoint for lake and ocean modelling is the parameterization of the turbulent vertical heat flux in terms of a sensible heat transfer coefficient and the temperature and flow at some distance from the boundary. Gilpin _et al_. (1980) pioneered this approach in their laboratory study, and found a transfer coefficient that varied between (0.6 to 1.0) 10^{-3} depending on the surface roughness of the ice sheet. However, problems with laboratory methods are that the heat flux is referenced to a cross-sectionally averaged velocity, and that scales of turbulent motion representative of lake and ocean settings are not usually obtained in the laboratory.

We are not aware of field observations to determine thermal transfers between water and ice in lakes, despite the early interest in ice covered lakes by Scandinavian workers (cf. Liljequist, 1941). Melin (1947, 1948) drew attention to regulated waterflow and its effect on black ice thickness, particularly late in the season and showed convincingly the upwelling of warmer water near the outflow of Swedish lakes. More recently, Stigebrandt (1978) discussed the dynamics of through-flow in ice-covered lakes. However, none of the above authors have attempted to parameterize the heat transfer process.

A limited number of field studies on heat transfer through sea ice have been reported. Lewis and Perkin (1986) have summarized the results of Josberger and Meldrum (1985) and Bogorodskiy and Sukhorukov (1983) concerning the melting of sea ice in terms of the melting rate per degree Celcius versus under-ice current speed. In terms of the sensible heat transfer coefficient, the oceanic studies yield heat transfer coefficients ranging between 0.8 to 1.4 10^{-3}. Unfortunately, neither of these studies reported the physical characteristics of the under-ice thermal boundary layer. Lewis and Perkin (1986) conjecture that the above range of values represents the variation in the heat transfer coefficient from smooth to rough ice conditions.

In the following we discuss field measurements taken in Marsh and Tagish lakes in the Yukon River Basin during late winter (Figure 1). While these data were collected mainly as baseline information on winter circulation, they provide three means of estimating the coefficient of sensible heat transfer between water and ice sheets on fresh water. We then apply our parameterization of the heat transfer to the interpretation of ice thickness and outflow temperature distributions in Laberge and Tagish lakes.

The authors are grateful to M. Alford and E. Marles for their assistance in the field work. C.H. Mortimer, W. Campbell and E.L. Lewis are thanked for their suggestions of useful reference material.

THEORETICAL CONSIDERATIONS

According to the bulk aeordynamic formulation, the sensible heat flux, H, in a turbulent boundary layer is given by

$$H = C_s \rho \ C_p \ \Delta T U \tag{1}$$

where C_s is the sensible heat transfer coefficient, ρ is the density, C_p, the thermal heat capacity, ΔT is the difference over a height h, from the boundary, and U is the speed of flow at height h. Hence, to experimentally determine C_s, it is necessary to evaluate the sensible heat flux, while at the same time measuring the underice temperature gradient and flow speed at a fixed distance from the ice.

In standard atmospheric applications h is taken as 10 m and C_s has the values 1.5 10^{-3} for neutral conditions, 0.8 10^{-3} for very stable conditions and 1.6 10^{-3} for extremely unstable conditions (Fischer et al., 1979). From the requirements for the continuity of stress across the air-water boundary and that the boundary layer thickness be proportional to the friction velocity, the analogous height in the water should be scaled by $(\rho_a/\rho_w)-1/2$ where ρ_a and ρ_w are the densities of air and water, respectively. This leads to a height of order 1 m. Although this boundary layer height is adopted here, it is noteworthy that this reference depth has not been standardized. Josberger and Meldrum (1985) use 2.55 m as a reference depth for C_s whereas Langleben (1982) uses 1 m as the reference depth for the under-ice drag coefficient.

It is assumed that there is sufficient flow under the ice to maintain a turbulent boundary layer. In the case of lakes with little or no winter flow-through, Patterson and Hamblin (1988) proposed that a molecular heat transport term be added to equation (1).

In the present study the sensible heat transfer coefficient for the underside of lake ice is inferred from equation (1) using a number of intercomparable approaches employing the field data collected from Yukon lakes.

STUDY AREA AND METHODS

Field measurements were obtained from the outflow regions of three Yukon lakes (Tagish, Marsh, and Laberge) during late winter in 1985 and 1986 (Figure 1). Vertical profiles of water temperature were made using an internally recording CTD system (Applied Microsystems, Model 12); temperature accuracy is estimated to be 0.02°C. Ice thickness was measured at each hole using a simple ice staff accurate to 0.02 m.

Water currents were measured using two techniques. The first employed a Price Model 622 "winter pattern" current meter; the meter was deployed from a sounding reel mounted on a sled (see Alford and Carmack, 1987). The particular instrument used was chosen from a large inventory for its low threshold speed, estimated to be 0.02 m s^{-1} or less. Where measurements of lower speeds were required, radio-tracked under ice drogues were used. These drogues had a sail area of 10 m^2, and were considered to be reliable at speeds as low as 0.002 m s^{-2} (see Hamblin, 1989, for details).

EVALUATION OF C_s FROM FIELD DATA

Marsh Lake

A temperature section parallel to the outflow in Marsh Lake (Figure 2) shows that the temperature at 1 m depth, T_1, is 0.5°C and is essentially independent of horizontal position. Thus, at this depth, only the vertical heat flux need be considered. Since the vertical heat flux is composed of a diffusive term, $C_p \rho K_v \frac{\partial T}{\partial Z}$, and an advective term, $C_p \rho W T$, where Kv is the vertical eddy conductivity and W is the vertical velocity, it follows that from equating this flux to that of equation (1) that

$$C_s = \frac{(WT - Kv \frac{\partial T}{\partial z})}{U_1 T_1} \qquad (2)$$

The horizontal velocity at a depth of one metre, U_1, was observed to be 6 cm/s at station 4/11 near the outflow. A two-dimensional numerical model of the throughflow shows that flow lines are parallel to the isotherms under the steady winter conditions (Hamblin, 1989); hence, the vertical velocity approaching the outflow can be estimated from continuity considerations. Upstream of the outflow near station 16 a horizontal flow of 0.006 m s^{-1} was measured at a depth of 6 m; the slope of the corresponding isotherm (0.75°C) at this location and depth was 4.2×10^{-3}. A vertical velocity W of 2.4×10^{-5} m s^{-1} is thus indicated.

The vertical eddy conductivity was estimated by the dissipation method employing the overturning scales of motion as observed from a number of temperature profiles taken near the outflow. It is supposed that temperature inversions exemplified in Figure 3 represent shear induced events and are consequently amenable to the analysis described by Dillon (1982). The first step of the analysis is to reconstruct a undisturbed (monotonic) temperature profile using an objective method such as that described by Papadakis (1981), and here shown by the dashed line in Figure 3. The root mean square vertical displacement of each water parcel from its original position, L_T, is 0.9 m for this mixing event. In turn, this is related to the Ozmidov length scale (Dillon, 1982) and then to the turbulent kinetic energy dissipation rate when combined with the local stability frequency. Based on the assumption of a typical mixing efficiency for lakes of 5% (Fischer et al., 1979), we arrive at a vertical eddy conductivity of 1×10^{-4} m^2/s. The resulting value of C_s from (2) is 1.2×10^{-3}. This calculation of the vertical flux of heat suggests that the diffusive flux is twice that due to vertical advection. The principal shortcomings of this method arise from possible errors in the estimation of Kv and W.

A second method of determining the sensible heat transfer coefficient is based on observations of ice thickness at two times (1 March and 9 March) made concurrently with water temperatures and velocities at a depth of 1 m. Now, if the ice thickness is H_I, the thermal conductivity of ice is K_I, the density of the ice is ρ_I, and the latent heat of fusion is L, then the heat balance equation is

$$L\rho_I \frac{dH_I}{dt} = -\frac{K_I\ T_{SI}}{H_I} + C_s\ \rho_w\ C_p\ T_1\ U_1$$

which can be written in terms of C_s as

$$C_s = \left(L\rho_1 \frac{dH_I}{dt} + \frac{K_I\ T_{SI}}{H_I}\right)/\rho_w\ C_p\ T_1\ U_1 \qquad (3)$$

In this equation the surface temperature of the ice, T_{SI}, is assumed to be the average air temperature of −9.3°C during the interval between the two successive measurements as observed at the Whitehorse airport. By this assumption the effect of a thin (~5 cm) wind compacted layer of snow on the surface temperature of the ice is taken to be negligible. The distribution of ice thickness across the outflow of Marsh Lake (Figure 4) shows ablation of ice in the centre of the outflow where the velocities are highest and growth of ice at the extremities. Based on 19 observations of ice thickness and under-ice flow and temperatures, equation 3 yields an average value of C_s of 0.63 ± 0.25 10^{-3}. Further, in contrast to the studies of heat flux through sea ice (Joshburger and Meldrum, 1985; Bogorodskiy and Sukorukov, 1983), the effect of heat flux through the ice and snow cover may not be neglected as it accounts for at least one half of the upward total heat flux supplied by the water on the average.

This method, as in the previous one, is subject nonetheless to large uncertainties. The ice thickness is probably known only to within 0.02 m, the field temperatures were not measured on the same day as the currents, and current speeds measured far upstream are are close to the threshold of the current meter. Finally, the effect of snow cover and radiation on the surface temperature of the ice have been ignored. Nevertheless, the agreement between the two methods is encouraging.

Tagish Lake

The approach used here was to estimate C_s as the residual of the divergence in the horizontal heat flux, i.e., the difference in heat transport through sections normal to the outflow. Temperature profiles measured along three sections across the north end of Tagish Lake (Figures 5a, 6a and 7a) were used to compute the flow field based on the thermal wind equation (see Gill, 1987). Drogue measurements taken at 6 m depth supplied the reference level current for the thermal wind relation. The corresponding northward component of flow or outflow for each section (Figures 5b, 6b and 7b) yield cross-sectionally averaged transports of 136, 140 and 137 m^3/s, respectively, in good agreement with the nominal winter flow through rate of 140 m^3/s measured by the Water Survey of Canada (M.E. Alford, personal communication). The heat transport through section (14-12) is 7.85×10^5 kJ s^{-1}, through section B (2-7), is 7.36×10^5 kJ s^{-1} and through section C (13-11) is 7.63×10^7 kJ s^{-1}. A least squares best fit passing through the heat flux value of line A yields an average rate of heat loss to the ice of 3.72×10^4 kJ km^{-1}. With an average section width of 1.9 km, an average current speed computed from the thermal wind relation of 1 cm/s, and an observed average temperature of 0.8°C at a depth of one meter, the sensible heat transfer coefficient is found to be 0.58 $\pm$ 0.3 10^{-3}. Errors in this calculation arise from the close separation of the cross sections dictated by ice conditions, and thus the necessity to subtract large numbers from one another to establish a rate of heat flux to the ice.

DISCUSSION

Applications of the above parameterization can now be made to the intrepretation of other data collected during the study. For example, measurements of ice thickness, temperature, and current were taken along a section extending from the lake to the river outlet of Lake Laberge in March 1986 (Figures 8a and b). This section crossed an area of gradually thinning ice, eventually leading to an outlet polynya or area open water, and finally down river. From the measured flows and temperatures at 1 m below the ice the sensible heat flux has been calculated in Figure 8d employing equation (1) with $C_s = 0.8\ 10^{-3}$, $\rho = 1000$ Kg m^{-3} and $C_p = 4194$ $JKg^{-1}{}^{\circ}C^{-1}$. The current profiles on the underside of the ice for the three outermost stations yield a aerodynamic roughness of 0.02 m when fitted by a log-wall boundary layer. The computed profile of heat flux shows a strong relation to ice thickness. The heat flux at the ice edge approaches a maximum value of 240 Wm^{-2} close to that observed in Tagish Lake. A plot of temperature at 1 m below the ice or the water surface reveals in Figure 8c a distinct peak about midway across the ablating portion of the ice edge where due to the constricted depth upwelling does not supply heat at a rate sufficient to maintain the heat flux to the ice whereupon the outflow begins to cool.

A second application has been made to the ice thickness distribution on Tagish Lake. From the measured and inferred flows and temperatures at a depth of 1 m, the sensible heat transfer has been determined and plotted for Tagish lake in Figure 9a. In general, the associated ice thickness shown in Figure 9b is strongly correlated with the heat flux from water to the ice. Ice tends to be thinner on the western side of the lake where the upwelled warmer water is closer to the surface (Figures 5 to 7) and towards the outflow. The maximum heat flux measured in the vicinity of the permanent polynya on Lakes Tagish and Laberge is 230-240 Wm^{-2}. This lower bound on the heat flux required to maintain a permanent polynya may be reasonably compared to mean heat loss over the Dundas Island polynya in the Canadian Archipeligo of 329 Wm^{-2} (Der Hartog et al., 1983), 380 Wm^{-2} for the heat flux over the St. Lawrence Island polynya (Pease, 1985), or 340 Wm^{-2} over an open river in the Yukon basin (Alford and Carmack, 1987).

CONCLUSIONS

Observations of ice thickness, current and temperature under the ice in lakes of the headwater system of Yukon River Basin have been used to infer a coefficient of sensible heat transfer between water and ice of $(0.8 \pm 0.3)\ 10^{-3}$ at a depth of one meter below the ice. In turn, this value has been applied to the interpretation of outflow temperatures and ice thickness distributions. Only approximate comparisons between this value and those found from either the laboratory or the sea ice studies because the latter generally did not report the observation depth and roughness height. If a standard sea ice roughness of 10^{-3} m (Langleben, 1982) is assumed for the sea ice study of Josberger and Meldrum (1985), which may be compared to our value in Lake Laberge of 0.02 m, then their value of 0.8×10^{-3} at a depth of 2.55 m becomes 1.0×10^{-3} at one meter. This suggests that the heat transfer in lakes may be somewhat less than for oceanic ice sheets, but in both cases less than the neutral atmospheric boundary layer. It is recommended that in future investigations the depth and the roughness height be reported in the study or that a common reference depth for bulk transport formulae be adopted as in the case in the atmospheric boundary layer.

REFERENCES

Alford, M.E. and E.C. Carmack. 1987. Observations on the cover and streamflow in the Yukon River near Whitehorse during 1983-84. Inland Waters Directorate Scientific Series 152, Environment Canada, Ottawa.

Baines, W.D. 1961. On the transfer of heat from a river to an ice sheet. Transactions of Engineering Institute of Canada, 5(1): 27-32.

Bogorodoskiy, V.V. and K.K. Sukorukov. 1983. Physical conditions of bottom melting of the Arctic sea ice pack. Izvestiyza Atmospheric and Ocean Physics, 19(8): 667-669.

Den Hartog, G., S.D. Smith, R.J. Anderson, D.R. Topham and R.G. Perkin. 1983. An investigation of a polynya in the Canadian Archepelago, 3, Surface heat flux. J. Geophy. Res., 88: 2911-2916.

Dillon, F.M. 1982. Vertical overturns: a comparison of Thorpe and Ozmidov length scales. J. Geophys. Res., 87: 9601-9613.

Fischer, H.B., E.J. List, R.C.Y. Koh, J. Imberger and N.H. Brooks. 1979. Mixing in inland and coastal waters. Academic Press.

Gill, A.E. 1987. Atmosphere-Ocean Dynamics, Academic Press, New York, N.Y.

Gilpin, R.R., T. Hirata and K.C. Cheng. 1980. Wave formation and heat transfer at an ice-water interface in the presence of a turbulent flow. J. Fluid Mech., 99: 619-690.

Hamblin, P.F. 1989. Yukon River headwater lakes study 1983-1985: observations and analysis. Inland Waters Directorate, Scientific Series 175, Environment Canada, Ottawa.

Josberger, E.S. and D. Meldrum. 1985. Bottom ablation measurements and heat transfer coefficients from MIZEX-West, February 1983. MIZEX Bulletin VI MIZEX West. Wadhams ed. U.S. Army CRREL Special Report No. 85-6, 68-72.

Lewis, E.L. and R.G. Perkin. 1986. Ice pumps and their rates. J. Geophys. Res., 91: 11756-11762.

Langleben, M.P. 1982. Water drag coefficient of first-year sea ice. J. Geophy. Res., 87(1): 573-578.

Liljequist, Gosta. 1941. Winter tempertures and ice conditions of Lake Vetter with special regard to winter of 1939/40. Swedish Meteorological and Hydrographical Institute Communications Series No. 35, 29 pp.

Mellin, Ragnar. 1947. Investigations at the Meteorological and Hydrological Institute of Sweden concerning the ice conditions on lakes and rivers. Swedish Meteorological and Hydrological Institute Communications Series D, No. 1, 50 pp.

Mellin, Ragnar. 1948. Currents caused by water flowing through lakes. Extract of Proceedings of IUGG General Assembly, Oslo, Norway.

Papadakis, J.E. 1981. A simple method for monotonic approximation of functions. Pacific Marine Science Report 81-7, Institute of Ocean Sciences, Patricia Bay, B.C.

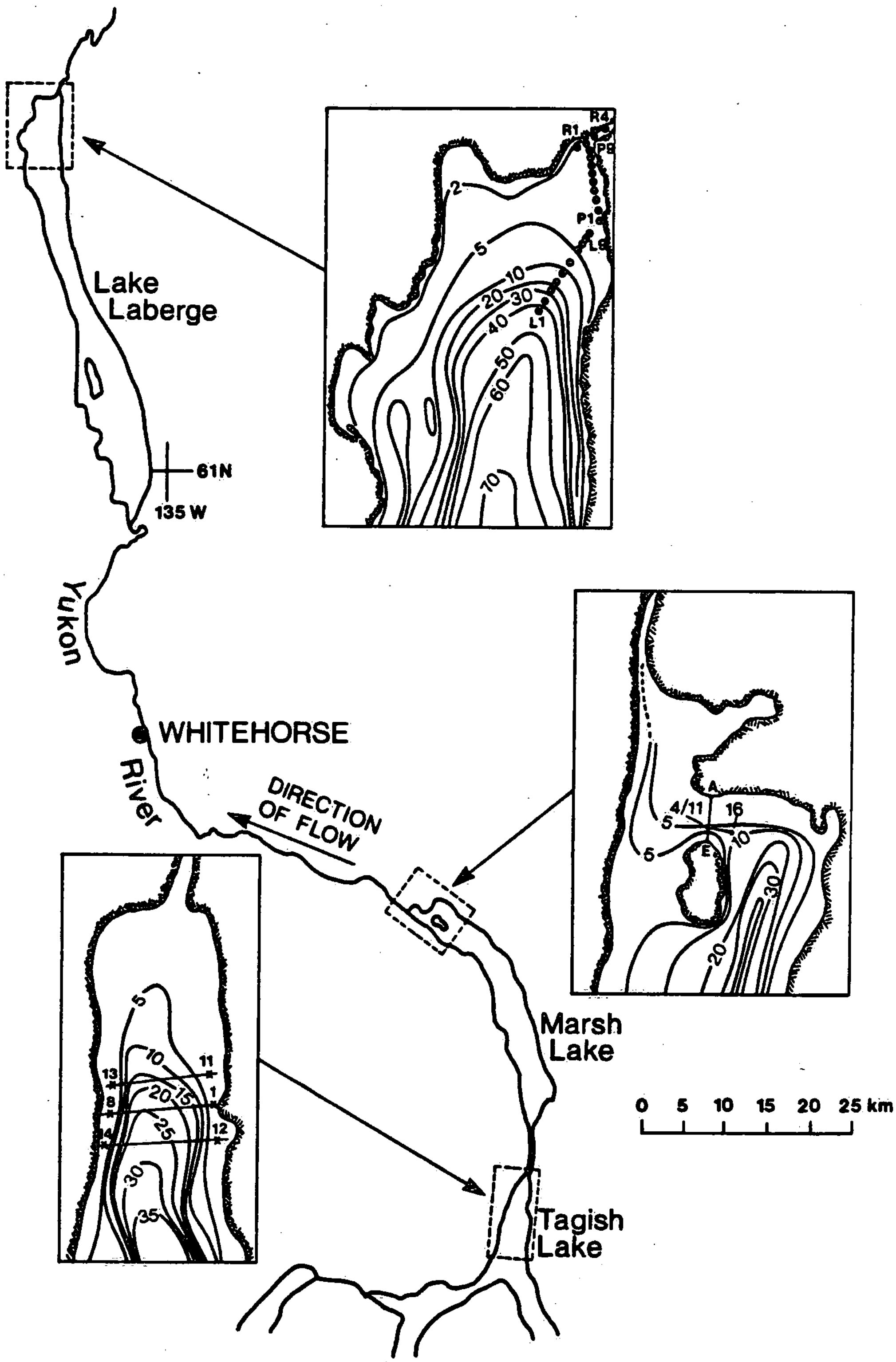
Patterson, J.C. and P.F. Hamblin. 1988. Thermal simulation of a lake with winter ice cover. Limnol. Oceanog., 33(3): 323-338.

Pease, C.H. 1985. Theory of wind-driven coastal polynyas. Mizex-Bulletin VI CRREL Special Report 82-6, Cold Region Research and Engineering Laboratory, Hanover, N.H., pp. 112-119.

Stigebrandt, A. 1978. Dynamics of an ice-covered lake with through-flow. Nordic Hydrology, 9: 219-244.

LIST OF FIGURES

Figure 9 Tagish Lake, March 1985, (a) heat flux distribution w/m^2 (b) ice thickness distribution (cm). The approximate margin of the ice edge is estimated and given by the dashed line.



Lake
Laberge
61N
135 W
Yukon
WHITEHORSE
River
DIRECTION
OF FLOW
Marsh
Lake
Tagish
Lake
0 5 10 15 20 25 km

Figure 1

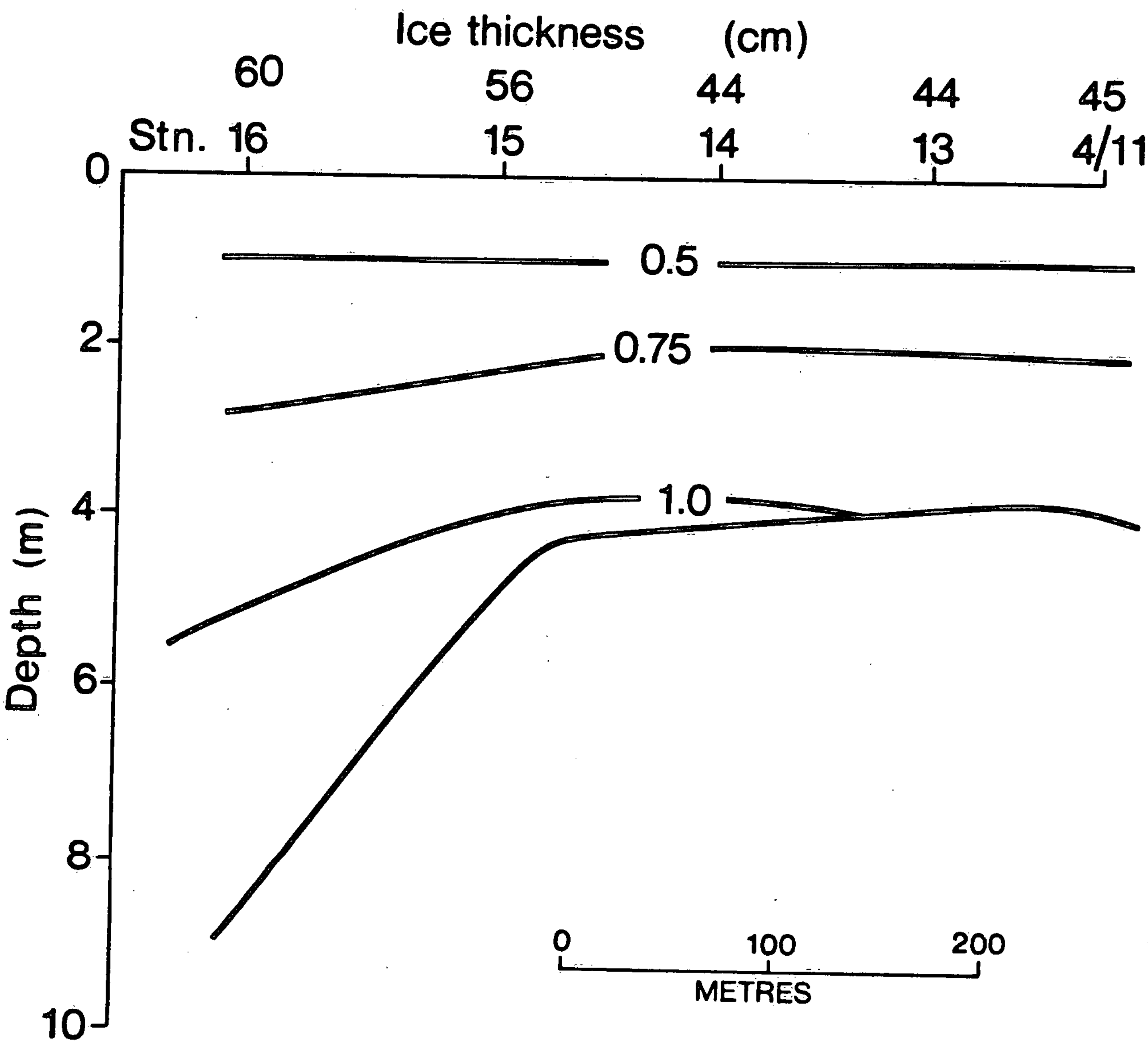

Ice thickness (cm)
60
56
44
44
45
Stn. 16
15
14
13
4/11
0
2
4
6
8
10
Depth (m)
0.5
0.75
1.0
0
100
200
METRES

Figure 2

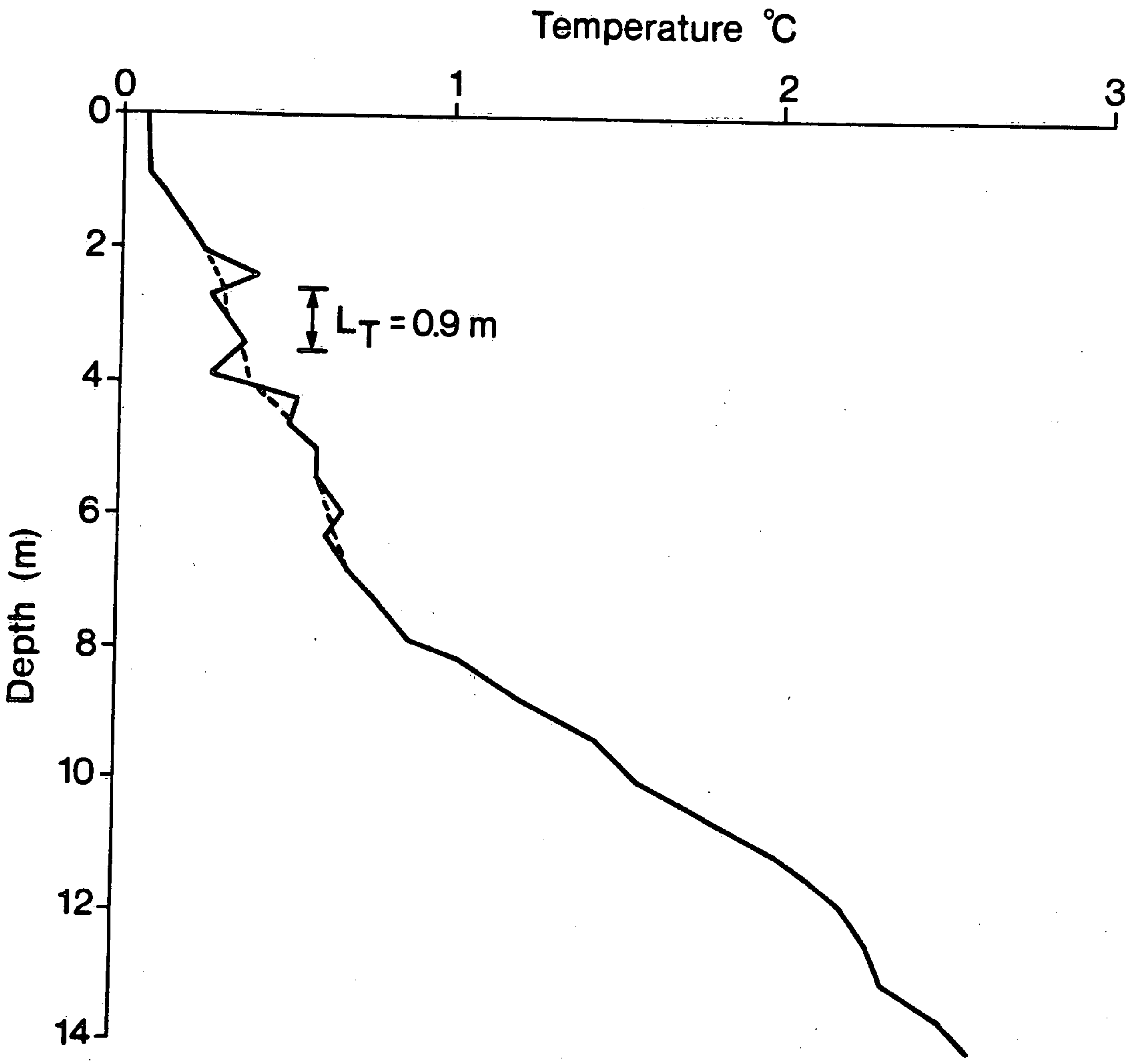


Figure 3

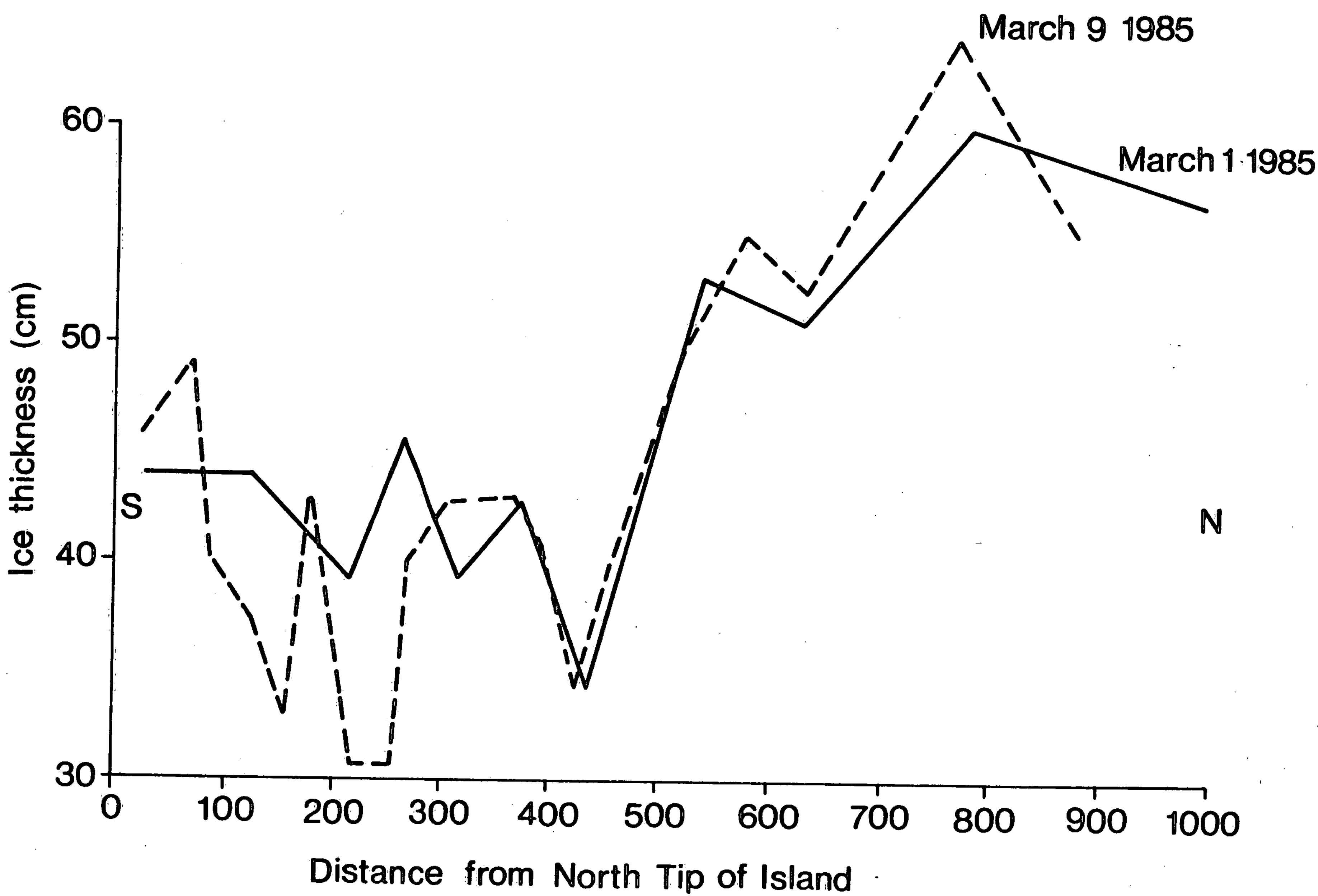
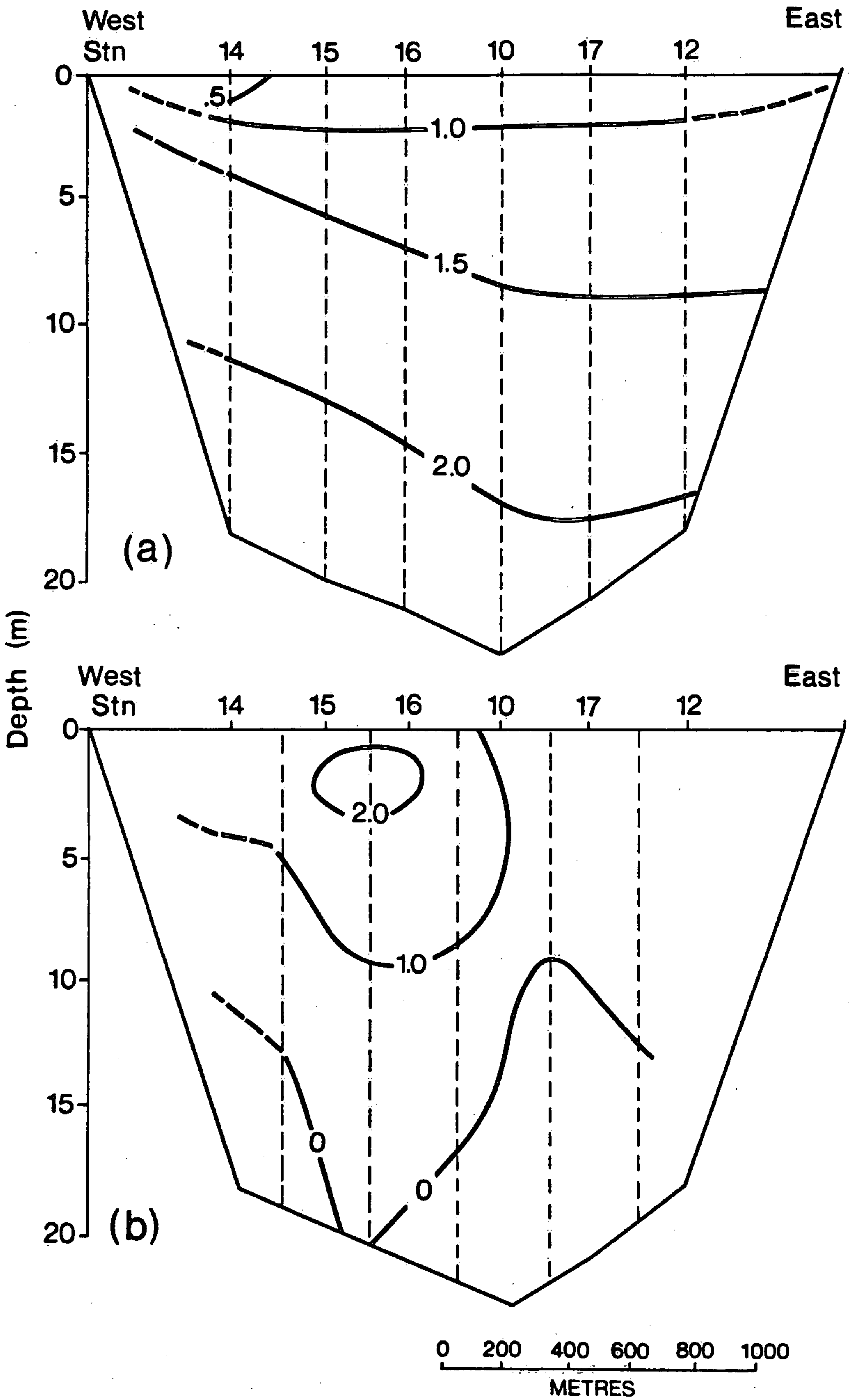
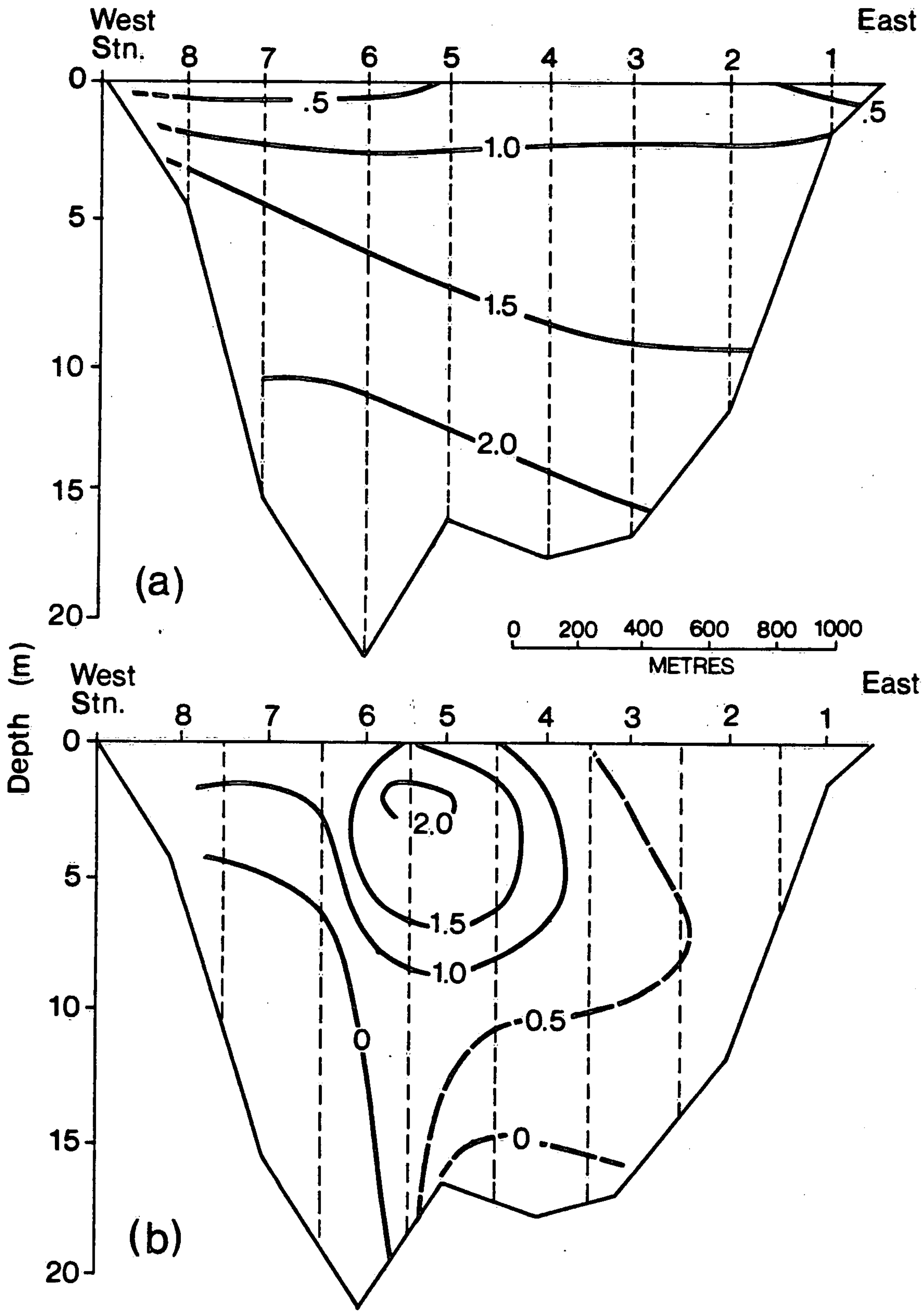

March 9 1985
March 1 1985
60
50
40
30
Ice thickness (cm)
S
N
0
100
200
300
400
500
600
700
800
900
1000
Distance from North Tip of Island

Figure 4



West
Stn
14
15
16
10
17
12
East
0
5
10
15
20
.5
1.0
1.5
2.0
(a)
Depth (m)
West
Stn
14
15
16
10
17
12
East
0
5
10
15
20
2.0
1.0
0
0
(b)
0 200 400 600 800 1000
METRES

Figure 5



West
Stn.
East
8
7
6
5
4
3
2
1
0
5
10
15
20
.5
.5
1.0
1.5
2.0
(a)
0
200
400
600
800
1000
METRES
Depth (m)
West
Stn.
East
8
7
6
5
4
3
2
1
0
5
10
15
20
2.0
1.5
1.0
0.5
0
0
(b)

Figure 6

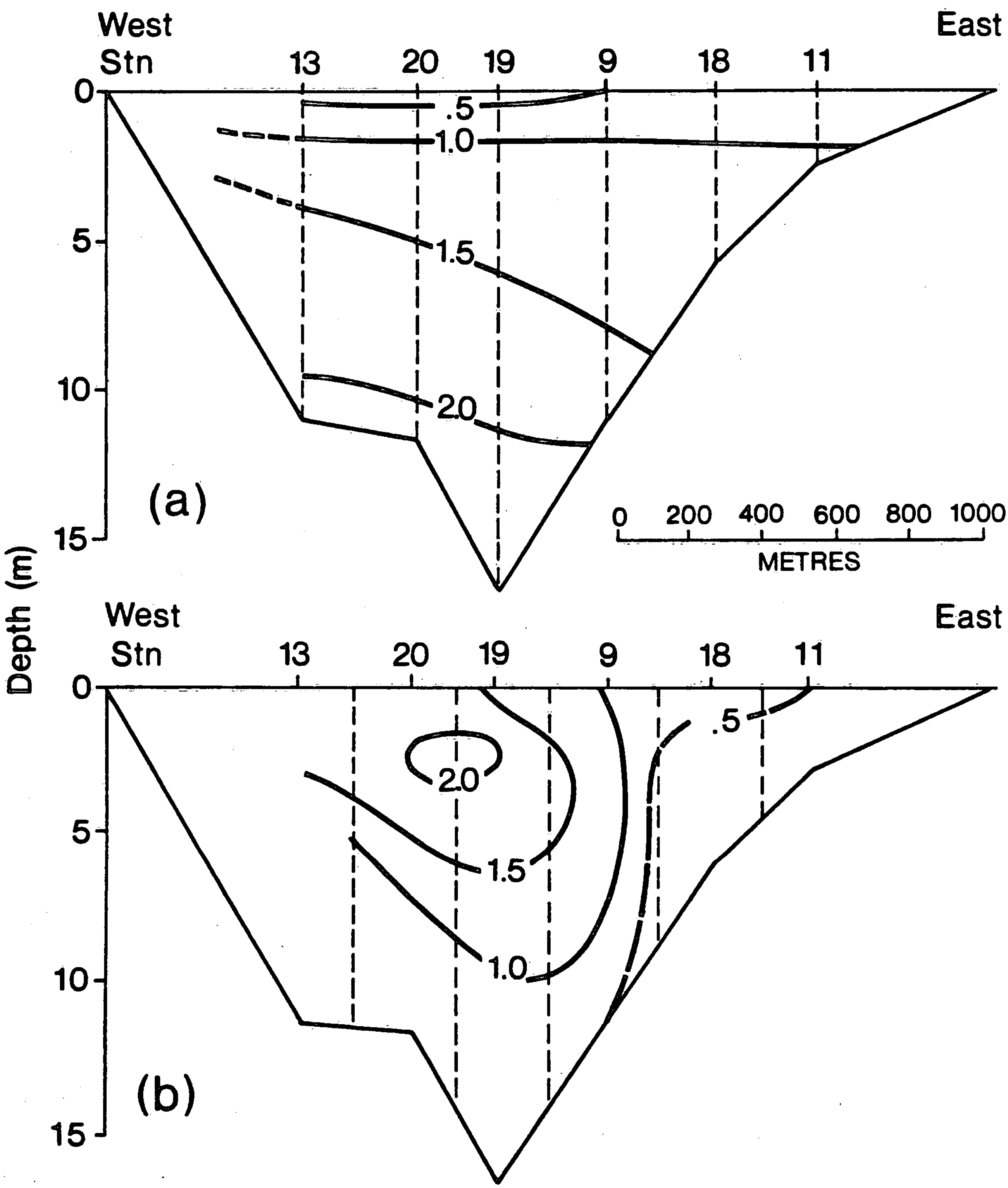

West
Stn
East
13
20
19
9
18
11
0
5
10
15
.5
1.0
1.5
2.0
(a)
0
200
400
600
800
1000
METRES
Depth (m)
West
Stn
East
13
20
19
9
18
11
0
5
10
15
2.0
1.5
1.0
.5
(b)

Figure 7

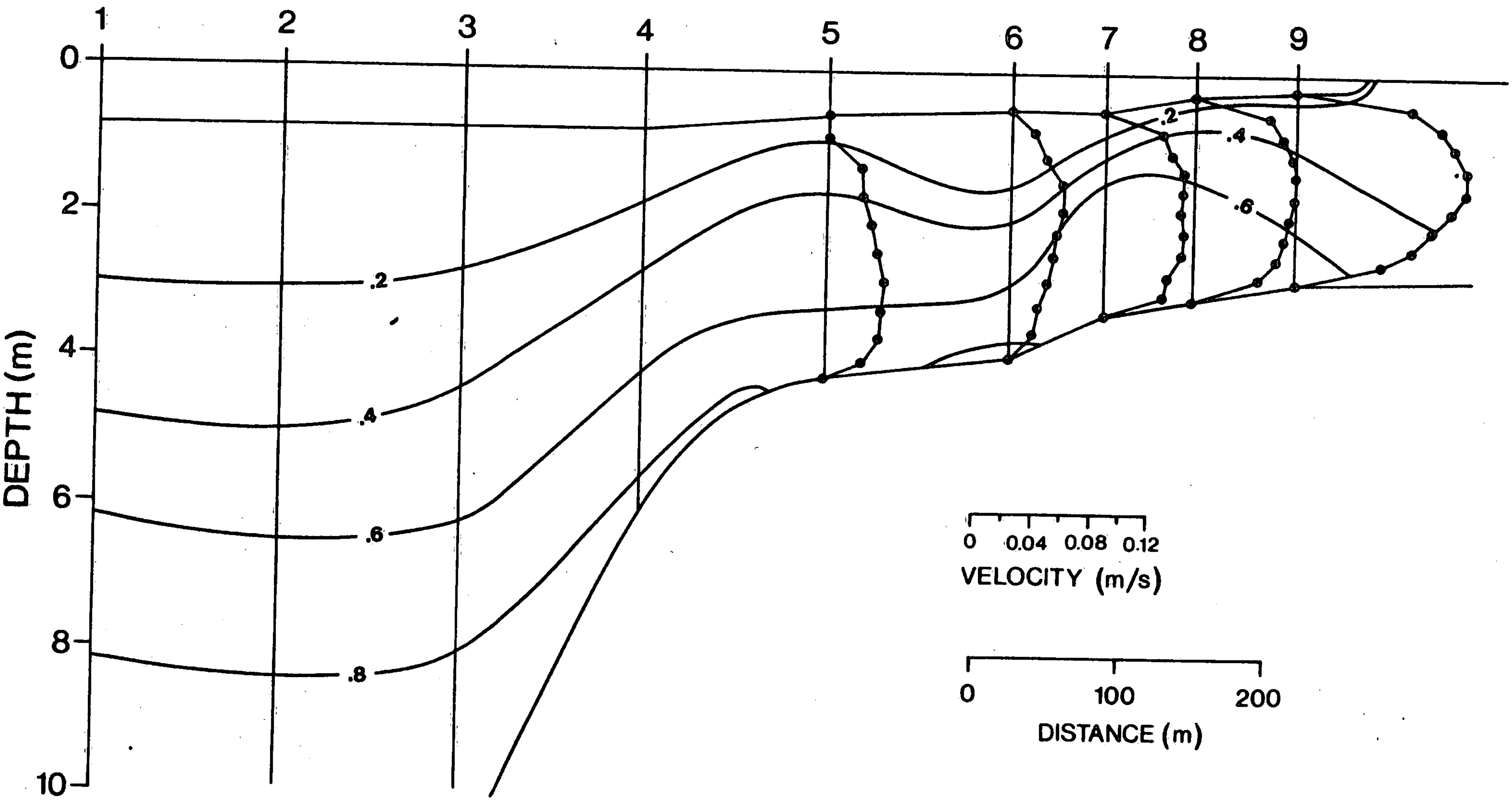

1
2
3
4
5
6
7
8
9
0
2
4
6
8
10
DEPTH (m)
.2
.4
.6
.8
.2
.4
.6
0
0.04
0.08
0.12
VELOCITY (m/s)
0
100
200
DISTANCE (m)

Figure 8a

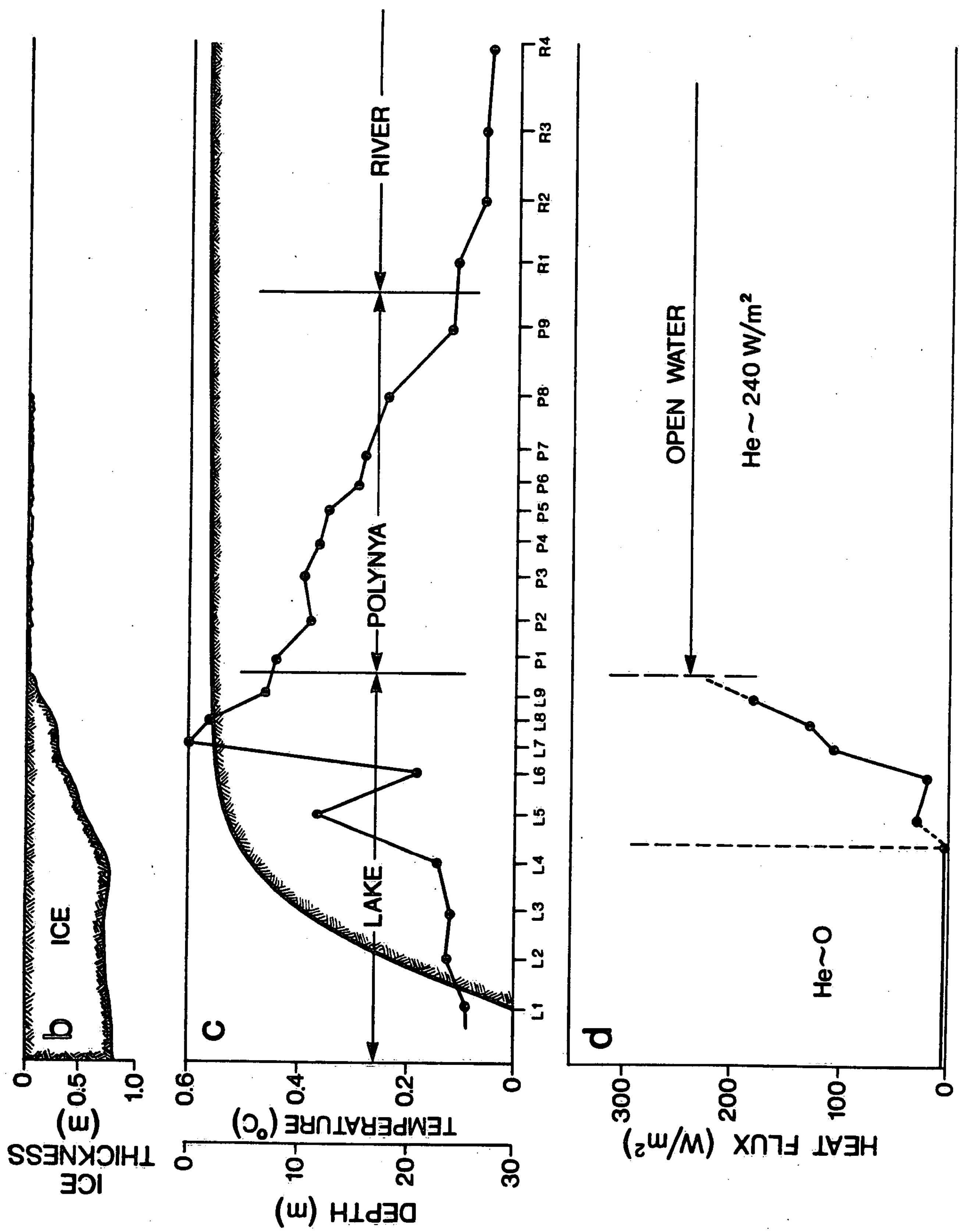


Figure 8b, 8c and 8d

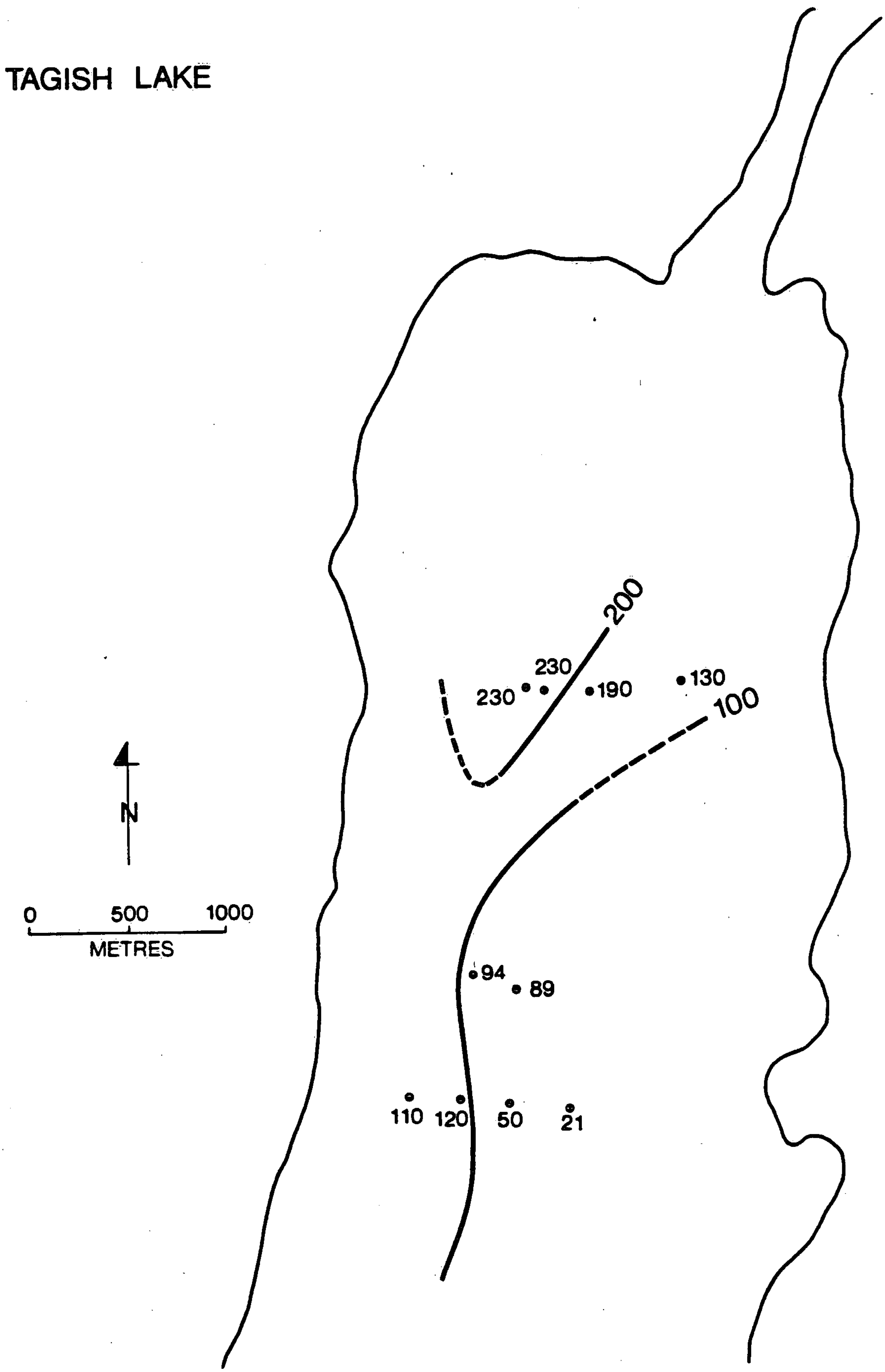

TAGISH LAKE
200
100
230
230
190
130
94
89
110
120
50
21
N
0
500
1000
METRES

Figure 9a

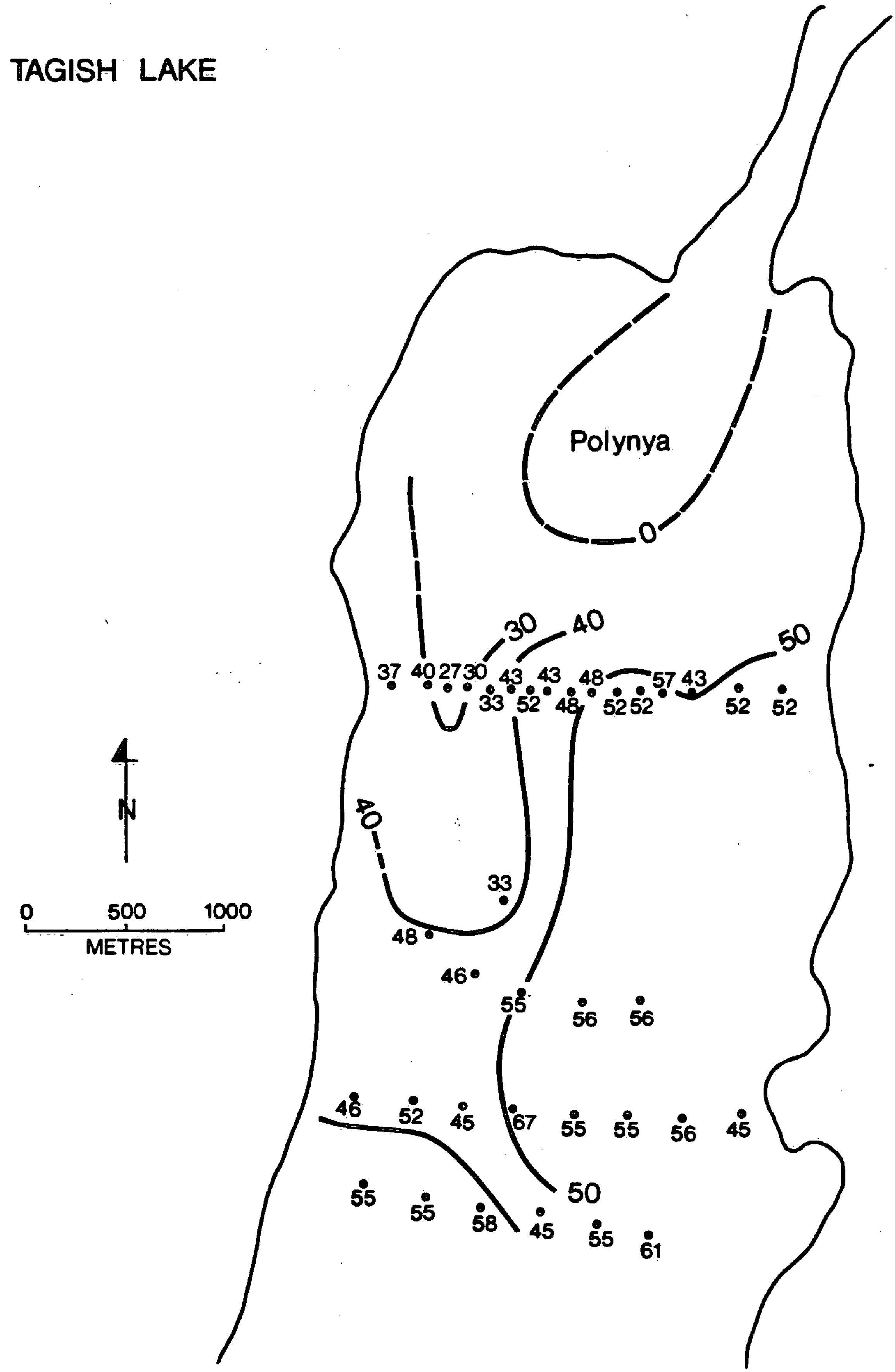

TAGISH LAKE
Polynya
0
30
40
50
37 40 27 30 43 43 48 57 43
33 52 48 52 52 52 52
40
33
48
46
55 56 56
46 52 45 67 55 55 56 45
50
55 55 58 45 55 61
N
0 500 1000
METRES

Figure 9b