MIXING BY WIND AND PENETRATIVE CONVECTION IN SMALL LAKES

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Executive Summary

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Lake mixing of two small lakes in the interior of British Columbia was studied to evaluate the relative importance of wind mixing and penetrative convection on lake stability. Penetrative convection is formed when temperature fluctuations lead to changes in the density of water. We confirmed that for two lakes penetrative convection was two to three times more important than wind mixing. An efficiency factor of 0.20 for penetrative convection energy was found for both sites. This constant will be useful in models to predict lake mixing.

This project was of immediate utility to the British Columbia Fish and Wildlife Branch. Our data set was intensive and demonstrated that factors other than storm driven upwelling of anoxic bottom water were producing fish kills in Green Lake.

Perspectives-gestion

On a étudié le mélange de lac sur deux petits lacs de l'intérieur de la Colombie-Britannique, pour y évaluer les importances relatives du mélange éolien et de la pénétration convective sur la stabilité du lac. La pénétration convective se manifeste quand les variations de température entraînent des changements de densité de l'eau. Nous avons pu confirmer que, pour deux lacs, la pénétration convective était deux à trois fois plus élevée que le mélange éolien. On a trouvé aux deux sites un facteur d'efficacité de 0.20 pour la pénétration convective. Cette constante sera utile dans les modèles utilisés pour prévoir le mélange de lac.

Le projet était d'utilité immédiate pour la direction des poissons et de la faune de Colombie-Britannique. Notre ensemble de données était probant et a montré que des facteurs autres que les remontées d'eau de fond anoxique sous l'effet des tempêtes entraînaient des mortalités de poissons dans le lac Green.

RÉSUMÉ

Des mesures physiques exhaustives ont été faites dans deux lacs adjacents (lac Mahoney et lac Green, en Colombie-Britannique) sur plusieurs mois d'été et d'automne. Le lac Mahoney présente une forte stratification de salinité (les matières totales dissoutes - MTD - près du fond atteignent 85,000 mg l^{-1}), résultant en une stagnation permanente et des gradients de température inhabituels aux basses couches, alors que le lac Green montre des caractéristiques dimictiques normales, avec des MTD ne dépassant pas 2500 mg l^{-1}.

Des enregistreurs automatiques déployés à chaque lac mesuraient la température de l'eau, celle de l'air, ainsi que la direction et la vitesse du vent. On a effectué chaque mois des mesures nouvelles des profils verticaux de salinité et de température. Au printemps qui a précédé la période de mesure, un fort écoulement de surface avait entraîné un apport d'eau douce très élevé dans les lacs.

Les deux lacs ont montré, entre juin et octobre, des pertes significatives d'énergie potentielle de stratification dues aux processus de transport éolien et de pénétration convective. Dans le lac Mahoney, ces pertes étaient en grande partie dues à la pénétration convective, et le mélange éolien n'y contribuait qu'à 28 pour cent. Dans le lac Green, le mélange éolien était presque deux fois plus élevé que dans le lac Mahoney, et le mélange par pénétration convective moyen de 34 pour cent plus élevé. On a trouvé qu'un facteur d'efficacité de 0.20 pour l'énergie de pénétration convective, plus élevé que les valeurs de 0.13 indiquées précédemment dans la documentation, correspondait bien à la perte mesurée d'énergie potentielle de stratification.

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SUMMARY

Comprehensive physical measurements were made at two adjacent lakes (Mahoney and Green Lake, British Columbia) for several months during the summer and autumn. Mahoney Lake is strongly salinity stratified (total dissolved solids [TDS] near the bottom are 85,000 mg l^{-1}), resulting in permanent stagnation and unusual temperature gradients in the lower layers, while Green Lake exhibits normal dimictic characteristics, with TDS no greater than 2500 mg l^{-1}.

Field data loggers deployed at each lake measured water temperature, air temperature and wind speed and direction. Monthly manual measurements were also made of vertical salinity and temperature profiles. During the spring preceding the period of measurement, high surface run-off contributed unusually large fresh water inputs to the lakes.

Both lakes exhibited significant losses of potential energy of stratification between June and October due to the processes of wind driven transport and penetrative convection. For Mahoney, the losses in potential energy of stratification were largely due to penetrative convection, with wind induced mixing contributing 28% of the total. Wind mixing for Green Lake was almost 100% larger than for Mahoney while average penetrative convective mixing was 34% larger. An efficiency factor of 0.20 for the penetrative convective energy, larger than values of about 0.13 previously reported in the literature, was found to fit the measured loss of potential energy of stratification well.

INTRODUCTION

Mahoney Lake is a meromictic lake in southern British Columbia (lat. 49°17'N, long. 119°35'W). Green Lake is only 1.5 km north-east of Mahoney Lake (see Fig.1), and is at approximately the same altitude. Green Lake is perched on the edge of the western scarp of the Okanagan Valley, with a drop of several hundred metres immediately to the north-east. This exposed location results in Green Lake having a windy climate, with strong afternoon winds during the summer and autumn months. Mahoney Lake is much more sheltered, with low and medium sized hills surrounding it. Both lakes receive a ice cover during the winter months, with Green Lake developing a thicker ice cover and showing a longer ice-on period than Mahoney Lake. Values for lake volumes are 1.36 and 1.14

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million m^3 and for lake surface areas are 0.198 and 0.126 million m^2 for Mahoney and Green Lakes respectively. Maximum lake depths are 18.3 and 20.1 m.

The unusual nature of the physical, chemical and biological properties of Mahoney Lake were first documented by Northcote and Halsey (1969), and data comparing Mahoney and Green Lakes were published later (Northcote and Hall 1983). Calculations of the energy required for completely mixing some meromictic lakes were made in earlier work by Hutchinson (1957).

During the spring and early summer of 1983, unusually large amounts of runoff caused substantial inputs of low salinity water to be introduced into both Mahoney and Green Lakes. The surface water layer that was introduced into Mahoney Lake was of lower salinity than the surface layer in preceeding or succeeding years (Northcote and Hall 1989). Inflows into Green Lake, also of higher volume than usual, became quickly mixed with the rest of the lake waters and did not form a distinct surface layer.

This analysis is an extension of recently published work on the mixing in Mahoney Lake (Ward et al. 1989).

MIXING IN LAKES

Two main processes drive the mixing in the upper layers of lakes during the summer and autumn, namely penetrative convection and wind mixing. Figure 2 shows the well mixed upper layer in the two lakes studied where a falling plume of cool water can erode the stability of the stratified layers in these lakes. An energy calculation may be carried out using the following equations (Imberger 1979).

Velocity u_f of falling plumes:

$$u_f = \left[\frac{\alpha\, g\, h\, \tilde{H}}{\rho_w C_p}\right]^{1/3} \qquad \text{................[1]}$$

in which
$\tilde{H}$ = rate of cooling at the water-air surface, W m^{-2}
h = depth of mixed upper layer, m
g = gravitational acceleration, m s^{-2}
C_p = specific heat of water, J kg^{-1} $°C^{-1}$ (= 4182 J kg^{-1} °C)
ρ_w = density of water, kg m^{-3} (1000 kg m^{-3})
α = thermal coefficient of expansion of water, $°C^{-1}$ (1.8×10^{-4} per °C)

The power per unit area available for stirring from cooling equals an efficiency Ck times the water density times the falling plume velocity cubed. Ck was found to be about 0.13 by Imberger (1979) in studies on Wellington Reservoir, Australia, and this value was found to be in reasonable agreement with work by Sherman et al. (1978).

The power per unit area available for mixing from wind induced motions is proportional to the cube of the shear velocity u^*. This is because the shear stresses at the surface (τ) are proportional to u^{*2}, and the mean drift speed is proportional to u^*, so that the associated power availability at the surface is proportional to u^{*3}. The total rate of working of the wind on the water is much larger than u^{*3}, and is more nearly equal to the product of the group velocity of the waves times the shear stress (τ), Turner (1973). However use of the shear velocity cubed as a reference for the energy used in mixing is a useful approach, and is extensively used in the literature. We will use the product $\rho_w u^{*3}$ herein for calculating an efficiency of wind energy utilization.

The shear velocity u^* is determined from a coefficient of drag Cd for the air-water interface. For a medium range of wind velocity (4 to 11 m s^{-1}), Cd equals 0.0012 for winds measured at 10 m above the ground surface (Large &Pond 1981). On this basis the shear velocity u^* is given by:

$$u^* = [\,0.0012\, u^2 \rho_a / \rho_w\,]^{1/2} \quad \text{.....................[2]}$$

in which ρ_a = density of air, kg m^{-3} (=1.225 kg m^{-3})
ρ_w = density of water, kg m^{-3} (=1.000 kg m^{-3})
u = wind velocity at 10 m, m s^{-1}

Using C^* as the efficiency of wind energy utilization, the wind power per unit area available for mixing is proportional to C^* times u^{*3}. Values for C^* are still being researched, and a wide range of published values has appeared. Wind stress experiments by Wu (1973) yielded a value of C^* of 0.23, and values reported by Imberger (1979) ranged from 0.03 to 0.23, when a remote station was used for wind data, depending on the amount of sheltering allowed for from surrounding hills. A value of 0.26 was used in studies on wind mixing in Babine Lake (Farmer & Carmack 1981). Values used for modelling with the US Army Corps of Engineers model CE-QUAL-RI are C^* = 0.12 and larger, dependent on the value of the Richardson number, Johnson & Ford (1981).

Assuming that the power available from penetrative convection and from wind energy are additive, then the time rate of change of potential energy of stratification per unit area is:

$$\text{Time rate of change of potential energy per unit area} = \underbrace{C_k \rho_w u_f^3}_{\text{convective current induced mixing}} + \underbrace{C^* \rho_w u^{*3}}_{\text{wind induced mixing}} \quad \text{..............[3]}$$

The potential energy of a horizontally layered lake is obtained by integrating the sum of the masses times the densities times the elevations of the individual layers. The potential energy is measured from a datum, usually the lowest point in the lake. Changes in potential energy of meromictic lakes are due to changes in temperature and redistributions of salinity in the water column.

The potential energy of stratification is the difference between the potential energy of the stratified lake, and the potential energy of a hypothetical totally mixed lake of the same average temperature and the same average salinity. For lakes that show negligible changes in water surface level during the period of analysis, the calculations may be carried out from the surface downwards (using the water surface as datum). The integrations do not need to go for the full depth, and may be terminated at a depth deemed to show no change during the measuring period. The potential energy of stratification, Ps, in joules, is given by:

$$Ps = \int_0^{z_m} (\rho - \bar{\rho})\, g\, z_m\, A\, dz \quad \text{............[4]}$$

in which z = the depth measured from the surface downwards, m
z_m = the depth of the level of no change (or bottom of the lake which is 8 m, 12

m for Mahoney & Green Lakes respectively)

$A(z)$ = is the area of layer at depth z, m^2

$\rho(z)$ = is density of layer at depth z, kg m^{-3}

$\tilde{\rho}$ = is mean density of layers above level of no change, kg m^{-3}

g = is gravitational acceleration, m s^{-2}.

The change in potential energy of stratification per unit area over a period of time Δt equals the power per unit area available for mixing from equation [3].

$$\Delta Ps/(A\Delta t) = \rho_w(Ck\, u_f^3 + C^* u^{*3}) \qquad \text{......................} [5]$$

FIELD MEASUREMENTS

Vertical temperature, conductivity and salinity measurements were made for both Mahoney and Green Lakes with a YSI Model 33 meter each month, at depth intervals of 0.5 to1.0 m. For both lakes data collection commenced in April 1983 and terminated in Oct. 1983 for Green Lake but continued through to March 1984 for Mahoney Lake. Details of measurements were published recently (Ward et al. 1989).

During the summer and autumn of 1983 data loggers were operated at both lakes. Data collection ran from 19th June to 6th Nov. for Mahoney Lake and from 15th June to 20th Oct. for Green Lake. Hourly measurements of wind velocity at 3 m above the water surface, water temperatures at depths of 0,2,6, and12 m, and air temperature were recorded by a Hymet package (Plassey recorder) positioned on a raft at mid-lake. The data were downloaded onto a floppy disk at the National Water Research Institute, Burlington, Ontario. Using a Fortran computer program, daily averages were computed, transferred to a Lotus 123 spreadsheet and then processes further appropriately. Water surface levels relative to a bench mark on the lake shore were also measured monthly.

Bathymetric information for both lakes was taken from contour maps of scale 1:3000 for Mahoney Lake and 1:3012 for Green Lake, with dept contours at intervals of 1 m. Areas were obtained by the use of a Placom KP-90 digital Planimeter.

RESULTS AND DISCUSSION

Isothermal depth-time plots were developed from these data and are shown in Fig.3. Several differences are apparent from the isotherms of the two lakes. First, Green Lake shows a well stratified profile throughout the summer and autumn months with warmer waters overlying cooler waters. However, Mahoney Lake shows the warmest region at the top of the thermocline and slightly cooler temperatures near the surface. A temperature inversion is also noted in the hypolimnion with the bottom layer 1 °C to 2 °C cooler than the 12 m layer. Secondly, during the summer months Green Lake temperatures vary from 10 °C to 22 °C in the surface waters whereas Mahoney Lake temperatures range from 16 °C to 28 °C; Mahoney being consistently higher. Thirdly, waters in the lowest levels of Mahoney Lake remain warmer during the winter months (between 8 °C and 16 °C) and in the lowest levels of Green Lake are at a stable 4 °C.

Daily average readings for the summer months obtained from the data logger for 0,2,6, and 12 m depths are plotted in Fig. 4. Since values from the 0 and 2 m levels were within the measuring accuracy they have been plotted together. Both lakes have surface temperatures that peak in August. Temperature fluxes in this layer are attributed to the synoptic meteorological weather patterns with periods of 4 to 5 days. At the 6 m level, temperatures also show significant peaks; however, Mahoney Lake's peak occurs during mid-October and Green Lake's during mid-September.

Using the monthly data, values of density were computed from the temperature and salinity readings. Field data were collected at approximately mid-month, and results from June 18, July 17, Aug. 11, Sept. 16, Oct. 16 and Nov. 18 were plotted by Ward et al. (1989). The depth to which mixing occurred was 8 m in Mahoney Lake and 12 m in Green Lake. Monthly values for potential energy of stratification for both lakes were computed from the density profiles using Equation 4 with 0.5 m depth layers. The values for rate of change of potential energy are plotted in Fig. 5. An overall decrease of 37 MJ was noted for Mahoney Lake during an 18 week period whereas Green Lake shows a 30 MJ decrease in potential energy of stratification for a 12 week period.

An analysis of the losses of potential energy of stratification was undertaken initially for Mahoney Lake using water temperature and wind data. First a heat budget for the whole lake above z_m= 8 m depth level was made, to enable losses from the lake (assumed to be all surface losses) per unit area to be obtained. Weekly average values of the heat loss from the surface were tabulated. Wind velocity values at 10 m above the lake surface were calculated from the measured values at 3 m, using two different velocity distributions, with the average ratio u_3/u_{10} = 0.82. Weekly average values of wind speed cubed, and shear velocity cubed were tabulated. Table 1 is an example of the type of tabulation used, except to save space monthly instead of weekly averages are shown.

The rate of change of potential energy of stratification per unit area was evaluated with equation 3, using mean areas (between $z = 0$ and $z_m = 8$ m or 12 m) of 110,000 m^2 and 78,000 m^2 for Mahoney and Green Lakes respectively. When this equation was used with Ck = 0.13 and C* = 0.23 for initial trial values, it was apparent that for Mahoney Lake, wind mixing was much less important than penetrative convective mixing for most months. Adjustment of coefficient values to ensure a good fit of the slope of the calculated lines with the measured data was carried out (Fig. 5). Achievement of a good fit would clearly be sensitive to changes in Ck, and insentive to changes in C*. For this reason, it was decided to stay with the previously published value for C* (C* = 0.23) and to determine the required value for Ck. With Ck = 0.20 and C* = 0.23, the fit between measured and calculated data was excellent. Fig. 5 presents the weekly values for the potential energy of stratification calculated using equation 3 with these coefficient values, starting with an initial value of 60 MJ on June 18th for Mahoney Lake. This value for Ck (= 0.20) is significantly larger than previously published values for Ck (= 0.13). The results showed that with Ck = 0.20 and C* = 0.23, the ratio of wind mixing to total mixing gave a 19-week avarage value of only 0.28 for Mahoney Lake, with a maximum weekly value of 0.68 and a minimum weekly value of 0.09.

Similar calculations were made for Green Lake using the same Ck and C* values used for Mahoney Lake. The depth at which autumnal mixing terminated an Green Lake was approximately 12 m, and hence potential energy of stratification values were calculated from $z = 0$ to $z_m = 12$ m. The results for the potential energy of stratification values from field measurements were more scattered for Green Lake than for Mahoney, and the fit is not as good (see Fig. 5). However, the general trend of the calculated values is in the correct order of magnitude, and shows rough agreement with the field values. Weekly values were used in the calculations for Fig. 5, and monthly average values are shown in Table 1. The ratio of wind mixing to total mixing gave a 17-week average value of 0.39 for Green Lake, with a maximum and minimum weekly values of 0.91 and 0.06, respectively. The long term average values of wind mixing for Green Lake was almost 100% larger than for Mahoney Lake, and the long term average value for penetrative convection mixing was 34% larger.

If the surface areas of the lakes instead of the mean area values were used for the calculations, then the value of Ck would be smaller than the determined value, and closer to previously published results.

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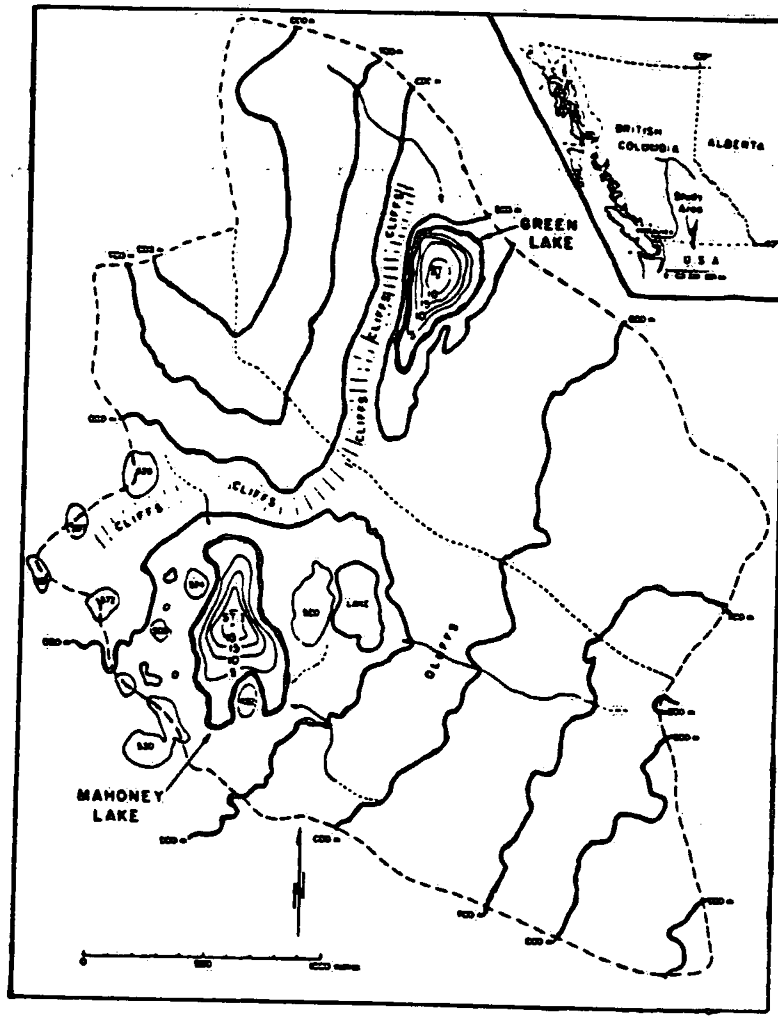
Table 1. Monthly Average Wind and Convective Cooling

PERIOD		AVERAGE VALUE FOR THE PERIOD								
		Wind Speed Cubed at 3 m	Wind Speed Cubed at 10 m	Shear Velocity Cubed[1]	Heat Loss by Cooling/ Unit Area	Velocity of Falling Plumes[2]	Velocity Cubed of Falling Plumes	Rate of Change of Potential Energy[3]		
From	To	($m^3 s^{-3}$)	($m^3 s^{-3}$)	($m^3 s^{-3} \times 10^{-6}$)	($W s^{-1}$)	($m s^{-1}$)	($m^3 s^{-3} \times 10^{-6}$)	Wind ($J m^{-2} d^{-1}$)	Convective ($J m^{-2} d^{-1}$)	Total ($J m^{-2} d^{-1}$)
MAHONEY LAKE										
June 18	July 17	11.86	21.5	0.0383	24.0	0.0043	0.0810	0.76	1.40	2.16
July 18	Aug. 10	7.95	14.4	0.0257	20.9	0.0041	0.0705	0.51	1.22	1.73
Aug 11	Sept. 17	10.80	19.6	0.0349	54.5	0.0057	0.1839	0.69	3.18	3.97
Sept. 18	Oct. 17	9.30	16.9	0.0301	46.9	0.0054	0.1583	0.60	2.74	3.34
Mean Values		9.98	18.1	0.0323	36.6	0.0049	0.1234	0.64	2.14	2.77
GREEN LAKE										
June 18	July 17	20.35	36.9	0.0658	35.6	0.0049	0.1202	1.31	2.08	3.38
July 18	Aug. 10	15.48	28.1	0.0500	12.6	0.0035	0.0426	0.99	0.74	1.73
Aug 11	Sept. 17	20.92	37.9	0.0676	60.0	0.0059	0.2026	1.34	3.50	4.85
Sept. 18	Oct. 17	22.71	41.2	0.0734	94.6	0.0068	0.3194	1.46	5.52	6.98
Mean Values		19.87	36.0	0.0642	50.7	0.0053	0.1712	1.28	2.96	4.23

$C^* = 0.23$, $Ck = 0.20$, Wind Speed Ratio 0.82 (u_3/u_{10})

[1]Calculated from equation 2, [2]Calculated from equation 1, [3]Calculated from equation 3.

Figure 1. Location map of the study area showing generalized geological and morphological features.



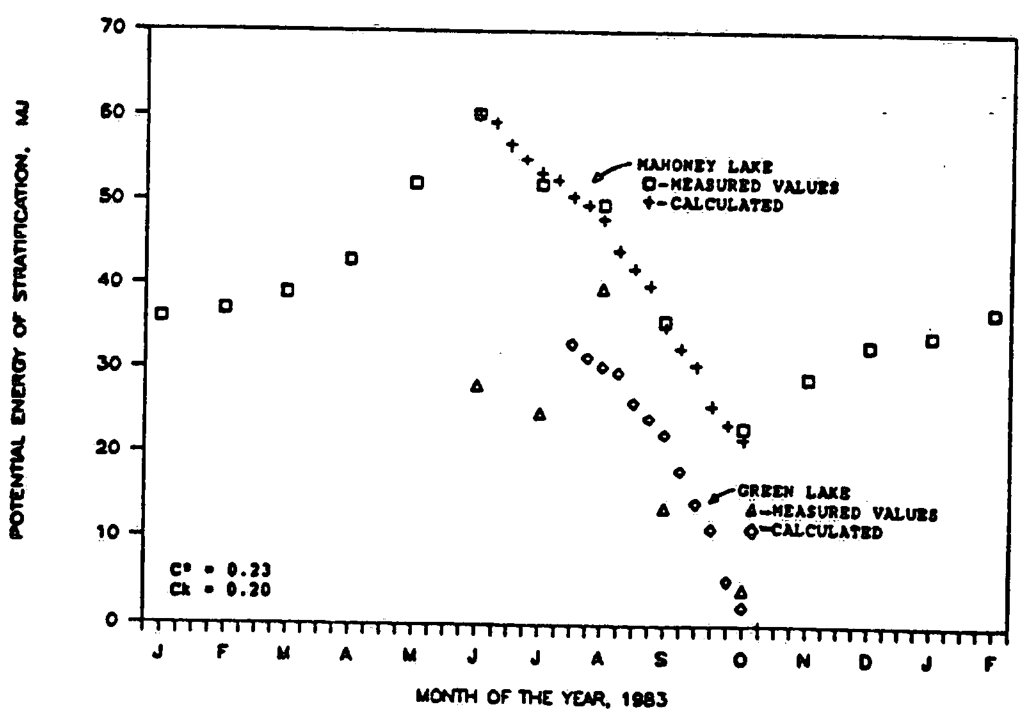


Figure 5. Monthly Values of Potential Energy of Stratification for Mahoney and Green Lakes.

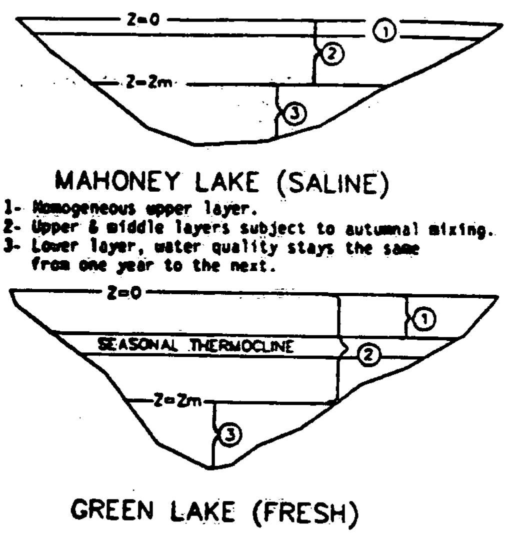


Figure 2. Layer Properties for Mahoney and Green Lakes.

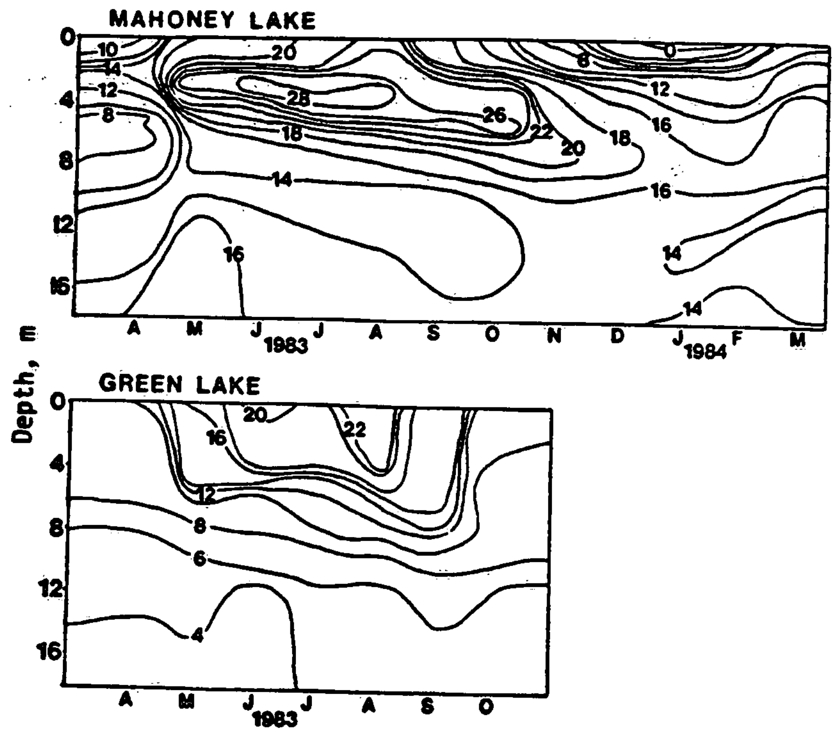


Figure 3. Isothermal depth-time plots.

Figure 4. Daily average temperature readings at depths of 0-2m, 6m, and 12m for Mahoney and Green Lakes.