AN ALGORITHM TO PLOT HASSE DIAGRAMS ON MICROCOMPUTERS AND CALCOMP PLOTTERS

Efraim Halfon[1], Jo-Ann Hodson[2] and Karon Miles[2]

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[1] Lakes Research Branch

[2] Computer Programming Section
Research Support Division

National Water Research Institute
Canada Centre for Inland Waters
Burlington, Ontario, Canada L7R 4A6

MANAGEMENT PERSPECTIVE

A computer algorithm has been developed to plot Hasse diagrams. Hasse diagrams are often used in lattice and graph theory. At NWRI Hasse diagrams are used to display the ranking of the hazard of toxic contaminants in the environment and to rank contaminanted sites, for example along the shores of Lake Ontario and Lake St. Clair. In a Hasse diagram, each circle represents a chemical and each line shows whether experiments used to test the hazard of different chemicals produce contradictory results. When Hasse diagrams are used to display large lattices the problem is that tens of circles and hundreds of straight and curved lines might have to be plotted. If a Hasse diagram is plotted by hand, a possibility exists that a line might be drawn between the wrong circles or possibly even left out entirely. This algorithm allows the plotting of any size Hasse diagrams on microcomputer screens and Calcomp plotters. These figures are then ready for publication. This algorithm a) permits the display of results within minutes on a computer screen and within hours on the plotter, b) is useful to scientists at NWRI and worldwide, c) prepares publication ready figures, and d) it saves several man-hours for each Hasse diagram to the drafting department.

PERSPECTIVE-GESTION

Un algorithme informatique permettant de tracer des diagrammes de Hasse a été créé. Les diagrammes de Hasse sont souvent utilisés dans la théorie des graphes et des treillis. À l'INRE, les diagrammes de Hasse sont utilisés pour représenter les niveaux de risque associés aux substances toxiques dans l'environnement et pour faire le classement des sites contaminés, par exemple le long des rives du lac Ontario et du lac Ste-Claire. Dans le diagramme de Hasse, chaque cercle représente une substance chimique et chaque ligne indique si les expériences effectuées pour évaluer les dangers associés aux différentes substances donnent des résultats contradictoires. Quand on veut représenter des treillis de grande taille par un diagramme de Hasse, des dizaines de cercles et des centaines de lignes droites ou courbes doivent être tracés. Certaines erreurs peuvent être commises quand on trace un diagramme de Hasse à la main : il est possible qu'on fasse des erreurs en traçant les lignes ou qu'on oublie de tracer certaines lignes. Le programme d'ordinateur permet de tracer des diagrammes de Hasse de toutes tailles sur des écrans de micro-ordinateurs ou à l'aide de traceurs Calcomp. Les figures ainsi obtenues sont alors prêtes à être publiées. Cet algorithme a) permet de représenter les résultats en quelques minutes sur un écran de micro-ordinateur et en quelques heures sur papier à l'aide d'un traceur, b) rend service à des scientifiques à travers le monde, c) permet d'obtenir des figures publiables, et d) pour chaque diagramme de Hassse, il permet d'économiser plusieurs heures-personnes dans les ateliers de graphisme.

ABSTRACT

A computer algorithm has been developed to plot Hasse diagrams. Hasse diagrams are often used in lattice and graph theory. Hasse diagrams have also been used to display results of ranking exercises, where each level of the diagram represents a ranking level and where each line represents the logical connections between levels. These diagrams have also been used to assess the best structure of simulation models. Hasse diagrams are a useful tool to display lattices and hierarchies in an easy to understand fashion. Hasse diagrams can have a very complex structure with circles on different levels and straight or curved lines connecting some circles on different levels. When Hasse diagrams are used to display large lattices tens or hundreds of circles and lines might have to be plotted. If a Hasse diagram is plotted by hand, a possibility exists that a line might be drawn between the wrong circles or possibly even left out entirely. The computer program allows the plotting of any size Hasse diagrams on microcomptuter screens and Calcomp plotters. These figures are then ready for publication. This algorithm a) permits the display of results within minutes on microcomputer screens and within hours on the plotter, b) is useful to scientists worldwide, c) prepares publication ready figures, and d) it saves several man-hours for each Hasse diagram to drafting departments.

RÉSUMÉ

Un algorithme informatique permettant de tracer des diagrammes de Hasse a été créé. Les diagrammes de Hasse sont souvent utilisés dans la théorie des graphes et des treillis. Ces diagrammes ont aussi déjà été utilisés pour représenter les résultats d'opérations de classement. Dans ces figures, chaque niveau du diagramme correspond à un rang et chaque ligne indique les liaisons logiques entre les niveaux. On a aussi utilisé ces diagrammes pour déterminer la meilleure structure dans les modèles de simulation. Les diagrammes de Hasse sont utiles pour représenter de façon compréhensible des treillis et des hiérarchies. Avec leurs cercles représentant différents niveaux et leurs lignes droites ou courbes reliant certains de ces cercles, les diagrammes de Hasse peuvent avoir une structure très complexe. Lorsqu'on se sert des diagrammes de Hasse pour représenter des treillis de grandes dimensions, des dizaines ou des centaines de cercles et de lignes peuvent être nécessaires. Certaines erreurs peuvent être commises quand on trace un diagramme de Hasse à la main : il est possible qu'on fasse des erreurs en traçant les lignes ou qu'on oublie de tracer certaines lignes. Le programme d'ordinateur permet de tracer des diagrammes de Hasse de toutes

tailles sur des écrans de micro-ordinateurs ou à l'aide de traceurs Calcomp. Les figures ainsi obtenues sont alors prêtes à être publiées. Cet algorithme a) permet de représenter les résultats en quelques minutes sur un écran de micro-ordinateur et en quelques heures sur papier à l'aide d'un traceur, b) rend service à des scientifiques à travers le monde, c) permet d'obtenir des figures publiables, et d) pour chaque diagramme de Hassse, il permet d'économiser plusieurs heures-personnes dans les ateliers de graphisme.

INTRODUCTION

Hasse diagrams are a useful tool to display lattices and hierarchies in an easy to understand fashion. The formal mathematical and logical development of Hasse diagrams can be found in Preparate and Yeh (1973) and in Harary (1969). Reggiani and Marchetti have used Hasse diagrams to display results of ranking exercises (Reggiani and Marchetti, 1975), where each level of the diagram represents a simulation model and where each line represents the logical connections between levels. Halfon (1978) and Halfon (1983a,b) have also used these diagrams to assess the best structure of simulation models. Halfon and Reggiani (1986) applied Hasse diagrams to rank the environmental hazard of 39 chemicals and Halfon and Brueggemann (1988) ranked the hazard of eight chemicals spilled in 1986 in the Rhine River at the Sandoz plant in Basel, Switzerland.

When Hasse diagrams are used to display large lattices the problem is that a large number of circles and lines might have to be plotted. If a Hasse diagram is plotted by hand, a possibility exists that a line might be drawn between the wrong circles or possibly even left out entirely. The computer program developed at our institute allows the plotting of any size Hasse diagrams on microcomputer screens. The algorithm can be modified for plotting on Calcomp plotters. These figures are then ready for publication.

The main problem in plotting a Hasse diagram is that a straight line connecting two circles might pass over a third circle located at an intermediate level. For example in Fig. 1 the line connecting circles 1 and 17 passes over circles 3 and 27, while the line connecting 27 and 9 paases over circle 5. The logical solution is to draw a circular line to avoid passing over the circle(s) in between. This solution so easy to explain is very difficult to program, because of the many variables involved in the process. Figure 2 shows a solution to this problem as computed by the algorithm described below.

HASSE DIAGRAMS

A Hasse diagram is composed of two parts, a series of circles located at discrete levels and lines connecting them. Any number of circles might be present on a given level, from one to a large number. Any two circles not on the same level might be connected by a line. Circles, located at two adjacent levels, can usually be connected by a straight line. If however two circles are located at levels separated by one or more levels where circles are present, then a straight line might pass over one or more circles located in between.

The solution of the problem can be found by use of analytical geometry using equations for straight lines, circles, perpendicular lines and distances. The first step in drawing a Hasse diagram is to identify the square area available for the graph. These numbers are used as inputs. The second step is to set the radius, r, of each circle, C, for each level yk. The coordinates of the centre of the circles are xc,yc. At each level yk there might be one or more circles. The number of circles at each level and the connections between circles are given as inputs.

COMPUTATION OF LINE EQUATIONS

Assume that two circles, C1 and C2 are located at levels $y_{level}=1$ and

$y_{level}=3$ and that these two circles are connected by a line, L1. The center of C1 is located at x1,y1 and the center of C2 is located at x2,y2. The general equation for a straight line is

$$y = mx + q \quad (1)$$

Case 1: two circles connected by a straight line

The slope of the line, L1, between x1,y1 and x2,y2 is

$$m = (y1-y2)/(x1-x2) \quad (2)$$

and the intercept is

$$q = y1 - mx1 \quad (3)$$

To connect C1 and C2 the following steps must be performed:

1) Find the levels, ym of x1,y1 and x2,y2.

2) Determine the number of yk levels between y1 and y2.

3) Compute the location where L1 crosses each yk level, this point is given by the coordinates

$$y_k = mx - mx2 + y2 \quad (4)$$

$$x_k = (y + mx2 - y2)/m \quad (5)$$

4) If the point x_k,y_k is not located within an existing circle Ck (+/- a tolerance factor) on the yk level, draw the straight line, L1, and exit. This step implies a checking of the distance of L1 from any circles Ck located on the y_k level. The distance, $d_{L1,Ck}$, between L1 and a circle Ck on level yk must be larger than the radius, r, of Ck plus a tolerance level, thus

$$d_{L1,Ck} > radius + tolerance \quad (6)$$

5) If a straight line crosses a circle, Ck, then the algorithm must draw a curved line between x1,y1 and x2,y2 so that the distance dL1,Ck respects the

constraint of Eq. 6.

<u>Case 2: two circles connected by a curved line</u>

6) Calculate the distance D of L1 between x1,y1 and x2,y2, where

$$D = SQRT\ [(x2-x1)^2 + (y2-y1)^2] \qquad (7)$$

7) Find the midpoint, x3y3, of L1, where

$$x3 = (x1+x2)/2 \qquad (8)$$

$$y3 = (y1+y2)/2 \qquad (9)$$

8) The equation of the line, L2, through x3,y3 and perpendicular to L1 is

$$y = -x/m + (y3 + x3/m). \qquad (10)$$

9) The equation of a circle, Cm, with the centre at xm,ym and radius rm is

$$(x-xm)^2 + (y-ym)^2 = rm^2. \qquad (11)$$

10) To connect x1,y1 and x2,y2 with a curved line, an arc, both points must be located on the circumference of a circle, Cm. The center of Cm must be located at a point x4,y4, on a line, L2 (Eq. 10), perpendicular to L1 connecting x1,y1 and x2,y2. The curvature of Cm should be as flat as possible to connect x1,y1 and x2,y2 with an arc, L3, of minimum length. As a first hypothesis the algorithm assumes that the circle Cm has a radius rm = 5D, where D (Eq. 7) is the distance between x1,y1 and x2,y2. The equation of Cm is therefore

$$(x-x4)^2 + (y-y4)^2 = (5D)^2. \qquad (12)$$

11) The coordinates, x4,y4, of the center of Cm must be located on L2 perpendicular of the line L1, thus Eq. 12 now becomes

$$(x-x4)^2 + [y-(-x4/m + y3 + x3/m)]^2 = (5D)^2. \quad (13)$$

Since x1,y1 is a point on the circle, then

$$(x1-x4)^2 + [y1-(-x4/m + y3 + x3/m)]^2 = (5D)^2. (14)$$

To solve for x4, Eq. 14 must be reorganized into a canonical form:

$$x1^2 - 2x1x4 + x4^2 + (x4/m + a)^2 = 25D^2 \quad (15)$$

where a = y1 - y3 - x3/m. Then

$$x1^2 - 2x1x4 + x4^2 + x4^2/m^2 + 2ax4/m + a^2 = 25D^2. \quad (16)$$

Since the canonical form of a quadratic equation is

$$ax^2 + bx + c = 0 \quad (17)$$

Eq. 16 can be rearranged as

$$(1+1/m^2)\ x4^2 + (-2x1 + 2a/m)\ x4 + (x1^2 + a^2 - 25D^2) = 0. \quad (18)$$

The two roots of Eq. 18 are called x4A and x4B. For each of these two solutions the position of y4A and y4B can be calculated as follows:

$$y4A = -x4A/m + y3 + x3/m \quad (19)$$

$$y4B = -x4B/m + y3 + x3/m \quad (20)$$

since both x4A,y4A and x4B,y4B are located on L2 perpendicular to L1. We now have two equations (Eq. 21 and 22) for two circles, Ca and Cb, with radius 5D passing through x1y1 and x2y2, one with its centre at x4A,y4A and the other at x4B,y4B:

$$(x-x4A)^2 + (y-y4A)^2 = (5D)^2 \quad [Ca] \quad (21)$$

$$(x-x4B)^2 + (y-y4B)^2 = (5D)^2 \quad [Cb]. \quad (22)$$

Since one or more yk levels might be present between x1,y1 and x2,y2 the following five steps are repeated:

i) Find each yk level between x1,y1 and x2,y2

ii) for each yk level the algorithm computes xk by substituting yk for y in Ca (Eq. 21), then

$$xka = x4A +/- SQRT [(5D)^2 - (yk-y4A)^2] \quad (23)$$

where the + sign is used when x4a < x3 and the - sign when x4a > x3. Equation 23 thus computes the xka value on Ca (Eq. 21) at the yk level. The same procedure is used to compute xkb by substituting yk for y in Cb (Eq. 22).

For each circle on the yk level, check whether xkA or xkB falls within the circle or its tolerance in the x direction.

iii) For the vertical line through the centre (xcen) of each circle on the yk level, calculate ykA and ykB, as

$$ykA = y4A +/- SQRT [(5D)^2 - (xcen-x4A)^2], \quad (24)$$

then check whether ykA or ykB fall within the circle or its tolerance in the y direction.

iv) If either circle (arc) can find an clear path from x1,y1 to x2,y2, the arc is drawn. Then continue to the next pair of circles to be connected.

v) If neither circle equation can produce an undisturbed arc between x1,y1 and x2,y2, reduce the radius in the circle equations, i.e. from 5D to 4.8D to 4.6D and repeat from step 9, above, until a path is found. Note that rm = .5D is the smallest radius allowed, since the centre of the Cm would be midway between x1,y1 and x2,y2.

vi) If no arc is found, then the radius, r, of all Hasse diagram circles is reduced and the whole procedure starts over from the computation of the slope and intercept of each line L1 (Eq. 2 and 3). This procedure always has a solution because eventually all circles are small enough that a path between x1,y1 and x2,y2 is found.

RESULTS

Figures 1 and 2 show the output of the program on a Calcomp plotter. Figure 1 shows lines intersecting circles 3, 27 and 5. Figure 2 shows a properly drawn Hasse diagram. Note the difficulty in fitting a line between circle 1 on level 1 and circle 17 on level 6. The solution to this problem was obtained by drawing a curved line and by reducing the radius of all circles. For interpretation of the meaning of Hasse diagrams and various applications see Reggiani and Marchetti (1975), Halfon and Reggiani (1978, 1986), Halfon and Brueggemann (1989) and Halfon (1983a,b; 1989).

DISCUSSION

Hasse diagrams are a useful graphic tool commonly used in algebra to display lattices (e.g., a genealogical tree is a special case of a Hasse diagram). This usefulness might be reduced if a large Hasse diagram with tens of circles and possibly hundreds of lines must be plotted by hand. The possibility always exists that errors might occur. In this paper we presented an algorithm that automates the drawing of Hasse diagrams in a publication ready form using Calcomp plotters. This algorithm can also be extended to plotting Hasse diagrams on a computer monitor. Eventually, this algorithm will become part of an expert system that will be used to rank the hazard of toxic contaminants in the environment, to rank the hazard of different location on earth contaminated by toxic pollutants and to assess which experiments, criteria or attributes must be measured for an optimal ranking. This work implied the development of large data bases and the ranking of tens or hundred of pollutants. The ranking method is based on partial ordering, a vectorial approach that recognizes that when many criteria are used for ranking, contradictions about the ranking according to each criteria exist. Hasse diagrams can display both the ranking and the contradictions among criteria so that an analysis of the original data base is easily accomplished.

REFERENCES

Halfon, E. 1983a. Is there a best model structure ? I. Ecological Modelling, 20: 135-152.

Halfon, E. 1983b. Is there a best model structure ? II. Ecological Modelling, 20:153-163.

Halfon, E. 1989. A comparison of an index function and a vectorial approach method to rank waste disposal sites. Environ. Sci. Technol. (in press).

Halfon, E. and Brueggemann, R. 1989. Environmental hazard of eight chemicals present in the Rhine River. Wat. Sci. Tech., 21:815-820.

Halfon, E. and Reggiani, M.G. 1978. Adequacy of ecosystem models. Ecological Modelling, 4: 41-50.

Halfon, E. and Reggiani, M.G. 1986. On ranking chemicals for environmental hazard. Environ. Sci. Technol., 20:1173-1179.

Harary, F. 1969. Graph Theory. Addison-Wesley, Reading, Ma.

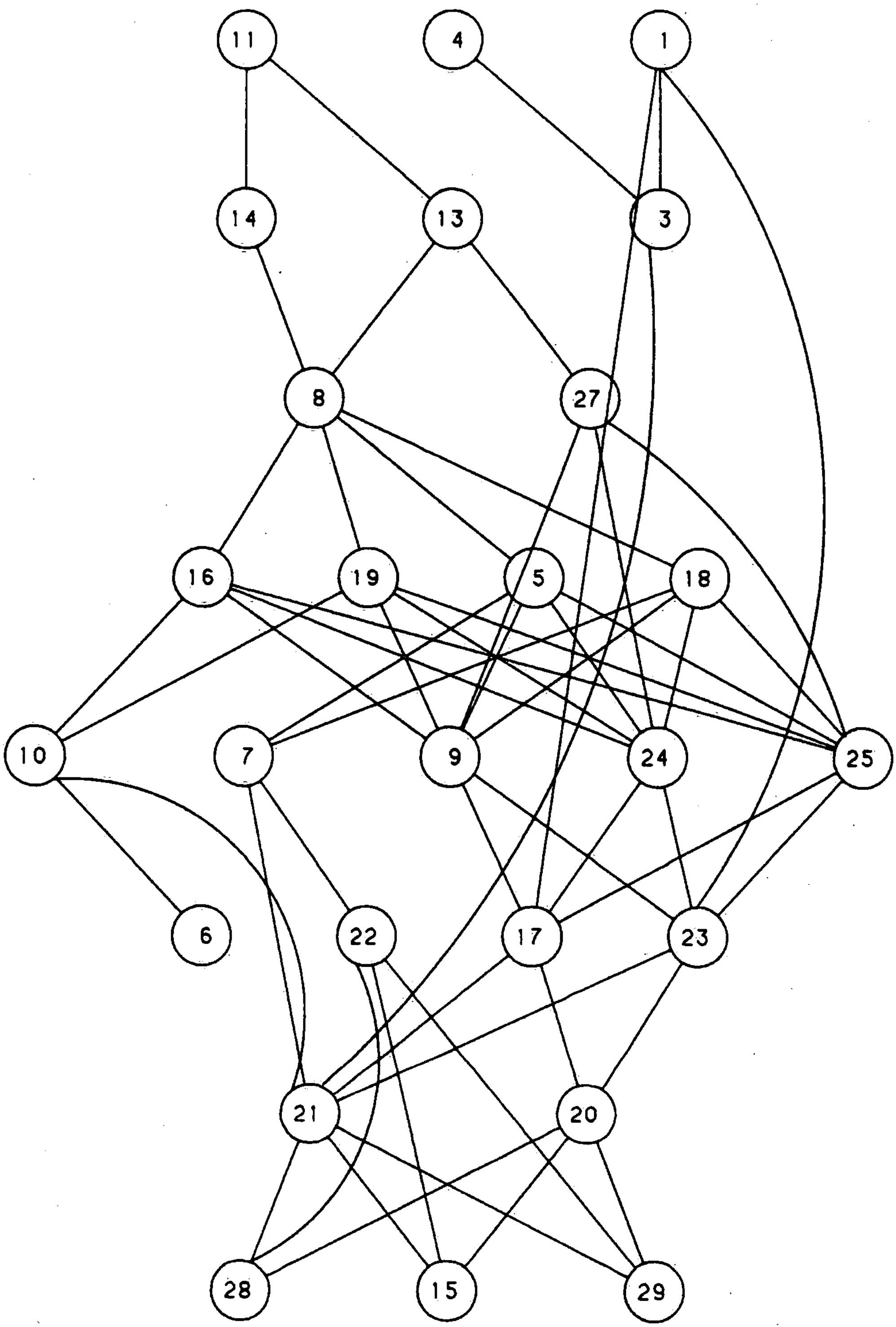
Preparate, F.P. and Yeh, R.T. 1973. Introduction to Discrete Structures, Addison-Wesley, Reading, Ma.

Reggiani, M.G. and Marchetti, F.E. 1975. Trans. Syst. Man Cybernetics, SMC-5, 322-330.

FIGURE LEGENDS

Figure 1: A Hasse diagram. Note that the line that connects circles 1 and 17 passes over circles 3 and 27, while the line connecting 27 and 9 paases over circle 5. The logical solution is to draw a circular line to avoid passing over the circle(s) in between. Figure 2 shows a solution to this problem as computed by the algorithm.

Figure 2: Same Hasse diagram as in figure 1. The line that connects circles 1 and 17 has been modified into a cricle to avoid passing over circles 3 and 27. A similar solution was found for the line connecting 27 and 9.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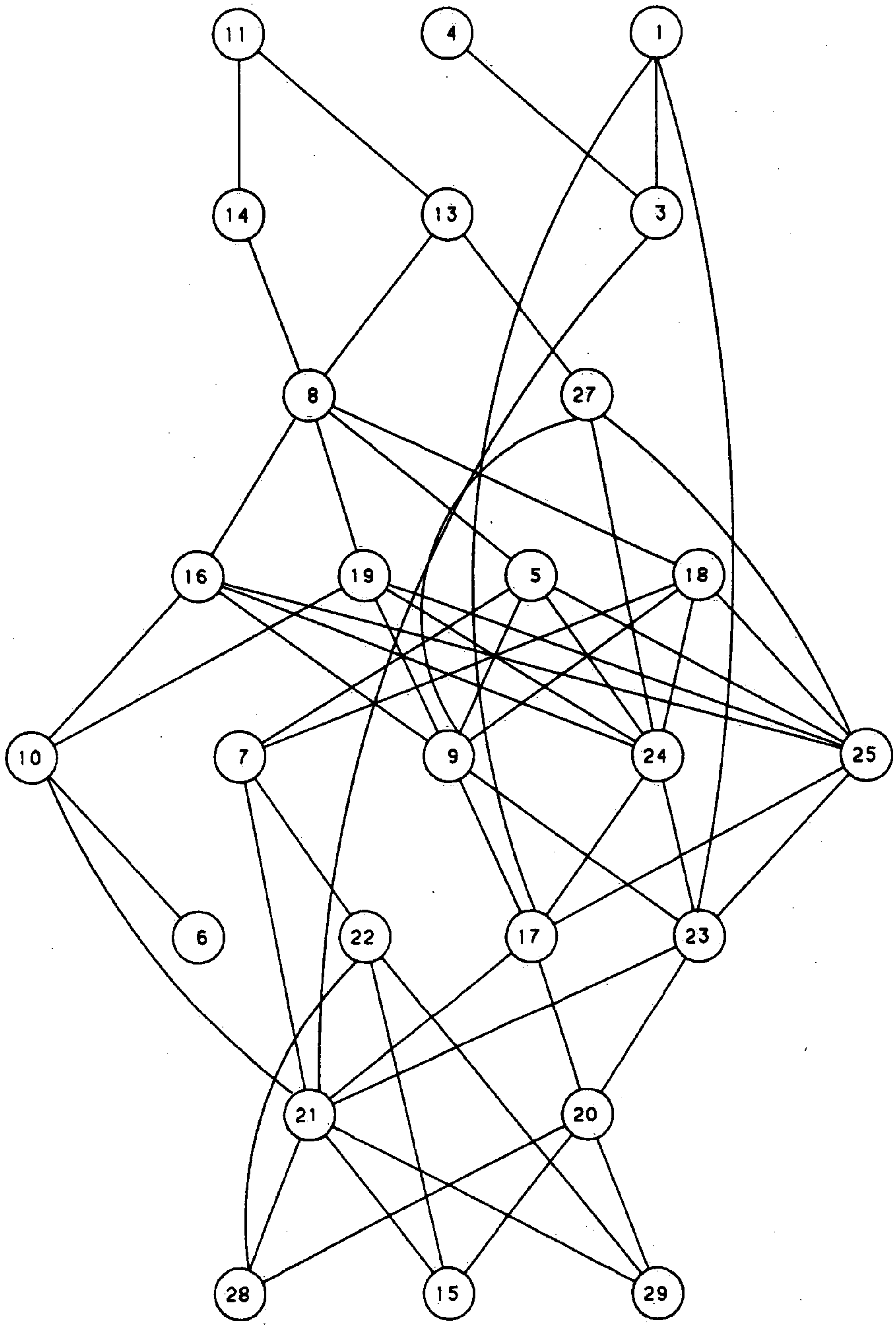
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