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**MODELLING BIOFILM CONSUMPTION  
IN OPEN-CHANNEL FLOW**

by

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## ABSTRACT

An analytical model of the loss of substrate in an open channel flow through the consumption by a bottom biofilm is presented. This idealized model considers the flux of substrate by diffusion through the viscous sublayer into the biofilm, with diffusion and reaction within the biofilm. Solutions for zero-order as well as first-order kinetics are presented and it is shown that the rate of change of the concentration in the main flow is directly related to the kinetics in the biofilm. Available field data were analysed in an attempt to investigate the dependence of the dimensionless loss rate coefficient on the shear Reynolds number of the flow.

## RÉSUMÉ

On présente un modèle analytique de la perte de substrat dans le courant d'un chenal, consommé par un biofilm de fond. Ce modèle idéalisé examine le flux de substrat par diffusion à travers la sous-couche visqueuse jusque dans le biofilm, en fonction de la diffusion et des réactions à l'intérieur du biofilm. Des résultats, obtenus pour une réaction cinétique d'ordre zéro ainsi que du premier ordre, sont présentés, et l'on montre que la rapidité de modification de la concentration dans le courant principal est directement liée aux réactions cinétiques dans le biofilm. On tente par l'analyse des données relevées sur le terrain d'étudier le phénomène de l'assujettissement du coefficient sans unité de vitesse de perte au nombre de Reynolds relatif au cisaillement de l'écoulement.

## MANAGEMENT PERSPECTIVE

One of the ways in which contaminants are removed from rivers and streams is through consumption by the slime layer on the bottom, a layer of bacteria and algae commonly referred to as a biofilm. In shallow, gravel bed streams, biofilm consumption may be the main mechanism for the removal of trace organics and other contaminants which do not undergo decay.

Very little has been done on the modelling of biofilm consumption in open-channel flows. This report presents an analytical model of the process. The model can be used in conjunction with experiments to investigate the process of biofilm consumption and how it can be affected by the streamflow. The results will be useful for understanding the fate and pathways of contaminants in river systems.

## PERSPECTIVE - GESTION

L'une des possibilités d'élimination des contaminants dans les cours d'eau est leur consommation par des bactéries et des algues dans une couche de vase de fond, appelée communément biofilm. Dans les cours d'eau peu profonds à lit de gravier, la disparition dans le biofilm pourrait constituer le principal mécanisme d'élimination des substances organiques à l'état de traces et d'autres contaminants qui ne se décomposent pas.

La modélisation de l'absorption par le biofilm dans les courants de chenaux est encore très rudimentaire. Ce rapport présente un modèle analytique du processus qui peut être utilisé en conjonction avec des expériences visant à étudier le processus de consommation par le biofilm et à voir comment le phénomène peut être modifié par l'écoulement des eaux. Les résultats aideront à déterminer la destination et les voies de cheminement des contaminants dans les réseaux fluviaux.

## INTRODUCTION

Stream beds are often covered by a layer of slime which is actually a film of bacteria and algae attached to rocks and cobbles. Microbial activity within the biofilm can remove the dissolved chemicals from the water above the film and can contribute significantly to the purification process in shallow flowing streams. Boyle and Scott (1984) showed that benthic films played a dominant role in the oxygen balance in the River Culm in England, and Srinanthakumar and Amirtharaja (1983) showed that attached biofilms were much more effective in removing organic carbon than the suspended biomass in a swift, shallow mountain stream. Gantzer et al. (1988) showed that biofilms can determine the rate at which trace organics can be removed from aquatic systems.

Many models have been proposed for the diffusion and consumption of substrate in biofilms. (The term substrate refers to the nutrients and chemicals which undergo reaction in a biofilm.) However, these were developed mainly for waste treatment processes in biological reactors such as trickling filters. Relatively little attention has been paid to the modelling of biofilm consumption in stream flows, where the bulk concentration in the flow and the concentration at the liquid-biofilm interface are both changing in the downstream direction. There is as yet no general agreement on the kinetics of the biofilm. Vaughan and Holder (1984), Holder and Vaughan (1987) and LaMotta (1976) found zero order reactions in their experiments using glucose and benzoate as substrate. Gantzer et al. (1988) concluded

that the consumption of trace organics in their biofilm experiments followed first order kinetics, as did Srinanthakumar and Amirtharaja (1983) for the uptake of organic carbon. Kornegay and Andrews (1968) and Rittmann and McCarty (1980) concluded that the process followed Monod kinetics.

This report presents a model of biofilm consumption of substrate in an open channel flow and demonstrates how the change in concentration in the flow with downstream distance is related to the kinetics of consumption in the biofilm.

#### MODEL DEVELOPMENT

The concept of an idealized biofilm model is shown in Figure 1. A uniform flow in the x-direction, with depth  $h$  and mean velocity  $U$ , flows over a biofilm of thickness  $L_f$  and density  $X_f$ . The y-axis starts at the liquid-biofilm interface and is positive downwards. Concentrations in the main flow are considered to be uniform throughout the depth down to a thin viscous sublayer just above the film. The substrate is transported by molecular diffusion through the sublayer down to the biofilm surface. Within the biofilm, diffusion and consumption of the substrate occur simultaneously. Advective transport in the film is considered to be negligible and the concentration gradient in the x-direction is much smaller than the gradient in the y-direction. The model considers the substrate mass balance in the bulk flow and in the biofilm, with the transport through the viscous sublayer acting as the link between the two.

For a steady state biofilm, the substrate mass balance within the biofilm can be written as

$$D_f \frac{\partial^2 C_f}{\partial y^2} = R_f \quad (1)$$

where  $C_f$  is the substrate concentration within the biofilm;  $D_f$  is the diffusion coefficient and  $R_f$  is the rate of consumption of substrate. Solutions to Eq. 1 will be different depending upon the kinetics chosen for the rate of consumption and upon the boundary conditions. If the diffusional resistance in the biofilm is small and the concentration at the interface is large enough, the substrate can penetrate to the bottom of the film. Otherwise, the substrate may be depleted before the bottom of the film is reached, giving rise to a different boundary condition. Three different cases will be considered here.

#### Case 1 - Zero Order Kinetics-full Penetration

Eq. 1 can be written as

$$D_f \frac{\partial^2 C_f}{\partial y^2} = r_f X_f \quad (2)$$

where  $r_f$  is the zero order rate constant. The boundary conditions are



$$C_f = C_s(x) \quad y = 0 \quad (3)$$

and

$$\frac{\partial C_f}{\partial y} = 0 \quad y = L_f \quad (4)$$

The concentration at the liquid-biofilm interface,  $C_s$ , will not be constant but will decrease in the downstream direction.

For this set of boundary conditions the solution is

$$C_f = C_s(x) - \frac{r_f X_f}{D_f} \left( L_f y - \frac{y^2}{2} \right) \quad (5)$$

In the very thin viscous sublayer over the biofilm, it will be assumed that the concentration decreases linearly. The diffusion of substrate through the viscous sublayer is equal to the diffusion into the biofilm. Therefore

$$\frac{D_m}{L_s} (C_w - C_s) = -D_f \frac{\partial C_f}{\partial y} \Big|_{y=0} \quad (6)$$

in which  $C_w$  is the concentration in the main flow,  $D_m$  is the molecular diffusivity in the viscous sublayer, and  $L_s$  is the sublayer thickness.

Eq. 6 gives

$$\frac{D_m}{L_s} (C_w - C_s) = r_f X_f L_f \quad (7)$$

This relates the concentration in the water flow above the biofilm to the interface concentration  $C_s$  and thus to  $C_f$ .

In the flow above the biofilm, it will be assumed that the loss to the biofilm is the only reason for the substrate concentration to decrease, i.e., there are no other sources or sinks. The change in the mass flux of substrate in the downstream direction is thus equal to the flux into the biofilm. Therefore, the mass balance equation can be written as:

$$\frac{d}{dx} (Uh C_w) = -r_f X_f L_f \quad (8)$$

This gives

$$C_w = C_0 - \frac{r_f X_f L_f}{Uh} x \quad (9)$$

in which  $C_0$  is the concentration at  $x = 0$ .

Using Eqs. (9) and (7),

$$C_s = C_o - \frac{r_f X_f L_f}{Uh} x - \frac{r_f X_f L_f}{D_m} L_s \quad (10)$$

Therefore, if the reaction in the biofilm is zero order and the substrate can penetrate to the bottom of the film, the model shows that the substrate concentration in the water above the film will also follow a zero order process and decrease linearly with distance downstream. The rate of decrease will depend on the film density and thickness, the rate constant, the flow velocity and flow depth. Typical concentration profiles through the water and the film are depicted in Figure 2(a). The concentration variation through the biofilm remains unchanged as one moves downstream. The profile at  $x_2$  has the same shape as that at  $x_1$  except that the value at the interface is smaller. Because the flux is constant, the drop in concentration through the various sublayer is also constant. This picture will continue until the substrate is depleted before the bottom of the biofilm is reached.

#### Case 2 - Zero Order Kinetics - Incomplete Penetration

At some point downstream, the substrate concentration in the water will decrease to such a value that the substrate will be depleted before the bottom of the biofilm is reached. This will also

happen if the diffusional resistance is large enough. When this happens, the depth of penetration or the effective biofilm thickness will decrease in the downstream direction. For this case, the boundary condition at the interface,  $y=0$ , remains unchanged but the bottom boundary condition becomes

$$\frac{\partial C_f}{\partial y} = 0 \qquad y = \delta(x) \qquad (11)$$

The depth of penetration,  $\delta(x)$ , will decrease in the downstream direction.

With these boundary conditions the solution of Eq. (2) is

$$C_f = C_s(x) - \frac{r_f X_f}{D_f} \left( \delta y - \frac{y^2}{2} \right) \qquad (12)$$

Making use of the fact that the concentration becomes zero at  $y=\delta$ , one obtains from Eq. (12)

$$\delta(x) = \left[ \frac{2 D_f}{r_f X_f} C_s(x) \right]^{1/2} \qquad (13)$$

It can be seen that the depth of penetration decreases as the half power of the interface concentration. Thus the effective thickness of the biofilm becomes smaller as one moves downstream.

Applying Eq. (6) which equates the flux across the viscous sublayer to the diffusion flux into the film, one obtains

$$\frac{D_m}{L_s} (C_w - C_s) = [2D_f r_f X_f C_s]^{1/2} \quad (14)$$

The mass balance equation for the bulk flow is

$$\frac{d}{dx} (U h C_w) = - [2 D_f r_f X_f C_s]^{1/2} \quad (15)$$

Closed form solutions for  $C_w$  and  $C_s$  cannot be obtained and equations (14) and (15) have to be solved numerically. Typical profiles are shown in Figure 2(b).

For the case of full penetration, the bulk flow concentration decreases linearly with distance downstream, as given by Eq. (9). With incomplete penetration, the decrease of  $C_w$  with  $x$  will no longer be linear and will become progressively smaller as  $x$  increases.

### Case 3 - First Order Kinetics

The mass balance equation for a biofilm with first order kinetics is

$$D_f \frac{\partial^2 C_f}{\partial y^2} = k_f X_f C_f \quad (16)$$

where  $k_f$  is the first order rate constant. The boundary conditions are given by Eqs. (3) and (4), the same as for case 1. The solution for  $C_f$  is

$$C_f = C_s(x) \frac{\cosh [\alpha(L_f - y)]}{\cosh [\alpha L_f]} \quad (17)$$

in which

$$\alpha = \left[ \frac{k_f X_f}{D_f} \right]^{1/2} \quad (18)$$

It can be seen that when  $y = L_f$

$$C_f = \frac{C_s(x)}{\cosh (\alpha L_f)} \quad (19)$$

The concentration at the bottom of the biofilm is thus always non-zero for finite values of the film thickness. Therefore, with first order kinetics for substrate consumption, the substrate can always penetrate to the bottom of the biofilm.

Applying Eq. (6) which equates the flux through the viscous sublayer to the flux into the biofilm, one gets

$$\frac{D_m}{L_s} (C_w - C_s) = C_s \alpha D_f \tanh (\alpha L_f) \quad (20)$$

The mass balance in the water column gives

$$\frac{d}{dx} (U h C_w) = - C_s D_f \tanh (\alpha L_f) \quad (21)$$

From Eqs. (20) and (21), one can write

$$U \frac{dC_w}{ds} = - k_w C_w \quad (22)$$

in which

$$k_w = \frac{1}{h} \left[ \frac{\frac{D_m}{L_s} \beta}{\frac{D_m}{L_s} + \beta} \right] \quad (23)$$

and

$$\beta = D_f \alpha \tanh (\alpha L_f) \quad (24)$$

Solving Eq. (22) gives

$$C_w = C_0 \exp(-k_w x/U) \quad (25)$$

in which  $C_0$  is the substrate concentration at  $x = 0$ . Therefore, if the consumption within the biofilm follows first order kinetics, the decrease in concentration in the bulk flow also follows a first order process, with the concentration decaying exponentially with downstream distance. The decay rate constant, given by Eq. (23), depends on the film properties, the sublayer properties, the flow velocity, and the flow depth. Typical concentration profiles are shown in Fig. 2(c).

It is interesting to note that the decay rate can be controlled either by the diffusion resistance in the viscous sublayer or by the process within the biofilm. The thickness of the viscous sublayer is governed by the bottom shear stress. If the turbulence in the flow is small, the sublayer thickness will be large.  $D_m/L_s$ , which represents a velocity of diffusion through the sublayer, will be small. If the biofilm consumption rate happens to be large, the term  $\beta$  can be much larger than  $D_m/L_s$ . Eq. (23) then reduces to

$$k_w = \frac{D_m}{hL_s} \quad \frac{D_m}{L_s} \ll \beta \quad (26)$$



In the other limit, when the diffusion velocity is large, Eq. (23) becomes

$$k_w = \frac{D_f \alpha \tanh(\alpha L_f)}{h} \quad \frac{D_m}{L_s} \gg \beta \quad (27)$$

Therefore, the rate of decrease of concentration in the bulk flow can be governed by the turbulence of the flow or by the properties of the film depending on the relative magnitudes of the diffusion resistance. This is in contrast to the case with zero order kinetics in which the rate of decrease of the mass of substrate depends only on the film properties, as seen from Eq. (8).

#### ANALYSIS OF AVAILABLE DATA

In order to verify the models presented in the previous section, data from open channel flow experiments are required. With measurements of the change in the bulk flow concentration, one can determine whether the biofilm consumption follows a zero order or first order process. For complete model verification, independent measurements of all the model parameters are required. These include the biofilm density and thickness, the diffusion coefficients and properties of the flow including the viscous sublayer thickness. Unfortunately, such experimental data do not seem to be available.

The only available data from open channel flows in which biofilm consumption is the sole or primary reason for the loss of substrate appear to be those obtained by Carey et al. (1984). They measured the change in concentration of 2,4-dichlorophenol (2,4-DCP) and 3,4-dichlorophenol (3,4-DCP) along the Canagagigue Creek, a small stream in Southern Ontario. Because the loss of chlorophenols could not be attributed to volatilization or to adsorption on suspended solids and settling, consumption by the biofilm which lined the bottom of the stream was identified as the only likely cause for the decrease in concentration along the stream. Eight experiments were conducted. In each case it was found that the concentrations decreased exponentially with distance downstream, indicating first order kinetics. The loss rate constant varied in value between  $0.072 \text{ h}^{-1}$  and  $0.347 \text{ h}^{-1}$  for 2,4-DCP. Values for 3,4-DCP were similar. If the values for all the biofilm and viscous sublayer parameters were available, Eq. (23) can be used to calculate the rate coefficients for all the different runs and these can be compared with the experimental values. However, none of these values are available and so a direct comparison cannot be made.

Even though a comparison of model prediction with measurements cannot be made, one can still make use of Eq. (23) to investigate the possible variations of the loss coefficient through the use of dimensional analysis. The expression for  $k_w$  indicates that  $k_w$  depends on the biofilm properties, the diffusion coefficients, the thickness of the sublayer and the flow depth. It is known that the thickness of

the viscous sublayer depends on the bottom shear stress and the viscosity of the fluid. Therefore, one can write

$$k_w = f(h, u_*, \nu, D_m, \text{film properties}) \quad (28)$$

in which  $u_*$  is the shear velocity and  $\nu$  is the kinematic viscosity of water. For a given substrate and a constant set of film properties, Eq. (28) can be reduced to

$$k_w = f(h, u_*, \nu) \quad (29)$$

Using dimensional analysis, one gets

$$k_w h / u_* = f(u_* h / \nu) \quad (30)$$

This indicates that, for a given substrate and a given biofilm, the dimensionless loss coefficient depends only on the shear Reynolds numbers of the flow. The functional dependence has to be obtained through experiments, although it will be reasonable to expect that the dimensionless loss coefficient should increase with the Reynolds number in some way. The reason is that, as the Reynolds number increases, the thickness of the viscous sublayer should be reduced, thus reducing the resistance to diffusion of substrate into the biofilm.

The dimensionless numbers were calculated from the data from Carey et al. (1984) and then used to investigate the behaviour of  $k_w$  based on Eq. (30). The values of the various parameters are given in Table 1. Unfortunately, the range of flow encountered was not large, and so the variation in Reynolds number was quite small. Figure 3 shows a plot of  $k_{wh}/u_*$  versus  $u_*h/v$ . With the narrow range of Reynolds numbers and with the scatter inherent in field data, the results are not very conclusive. There is some indication that  $k_{wh}/u_*$  increases with  $u_*h/v$ . However, this cannot be ascertained until more data over a wider range of Reynolds numbers are available.

#### SUMMARY

An analytical model of the loss of substrate from open channel flows through the process of biofilm consumption has been developed. Steady state biofilm and a single, rate-limiting substrate are assumed. Zero and first order kinetics are considered. It has been shown that the rate of loss of substrate in the downstream direction is directly related to the kinetics in the biofilm. Thus it is possible to investigate the biofilm kinetics using measurements in the bulk flow.

Using the model equations and dimensional analysis, it is shown that the dimensionless form of the first order loss rate coefficient depends on the shear Reynolds number of the flow. An attempt is made

to investigate the dependence using a set of field data. However, the results are inconclusive because the flow range is too small.

In order to fully test the model presented, controlled laboratory experiments in channel flows will be required.

#### ACKNOWLEDGEMENTS

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**FIGURES**

**Figure 1. Idealized Biofilm Model**

**Figure 2. Changes in concentration profile in the downstream direction**

**Figure 3. Dimensionless loss rate coefficient versus shear Reynolds number for data from Canagagigue Creek**

Figure 1

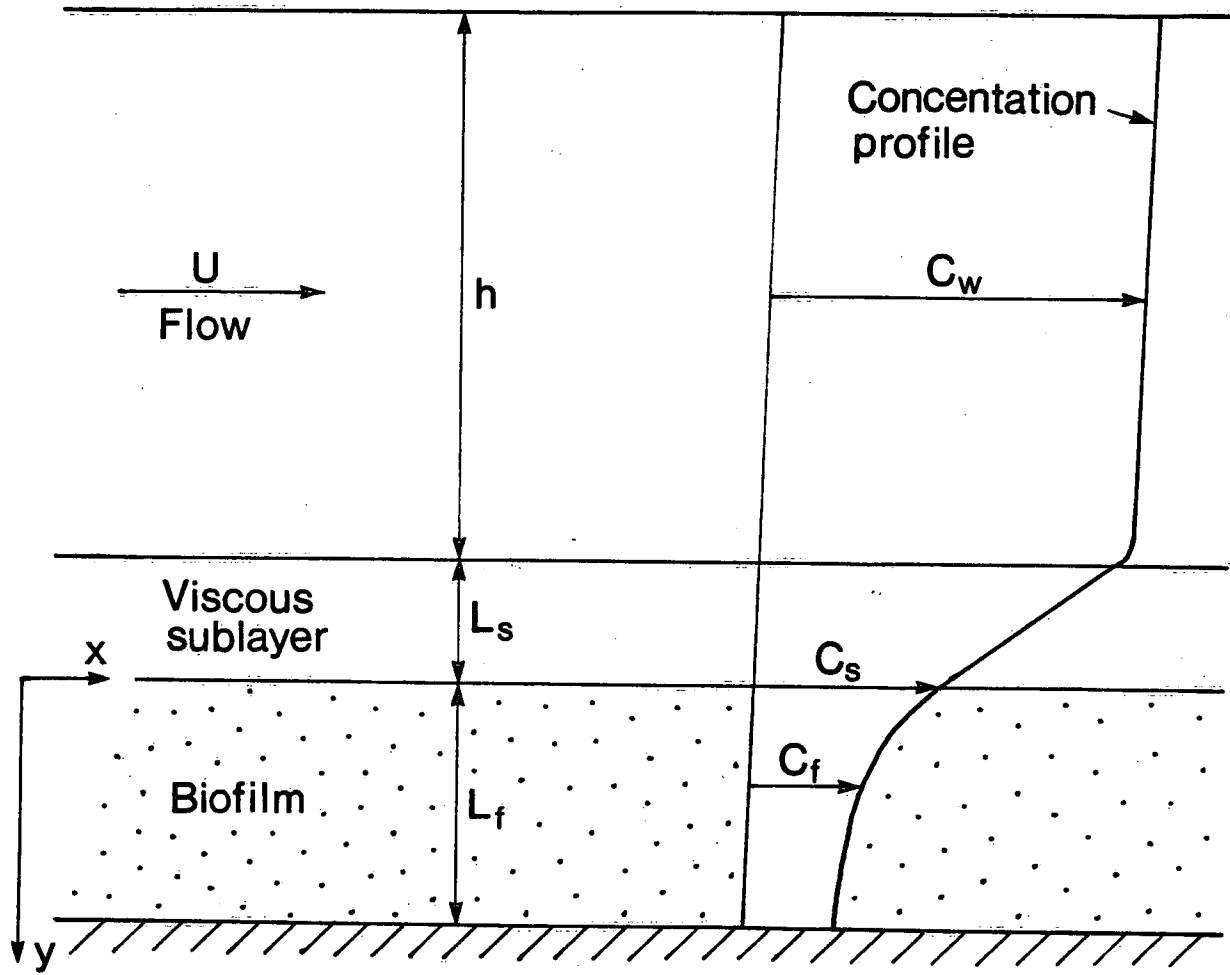
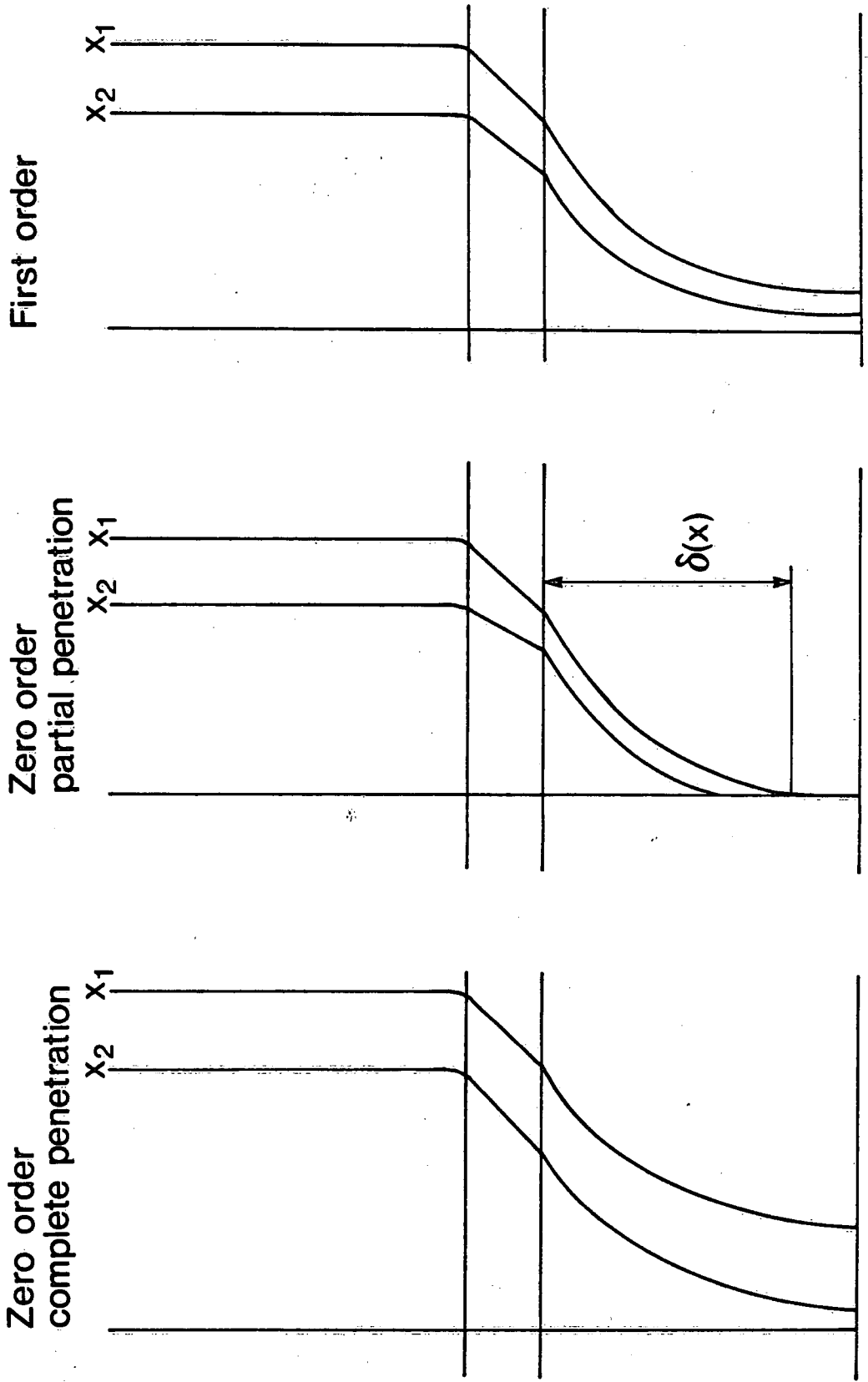




Figure 2



(c)

(b)

(a)

Figure 3

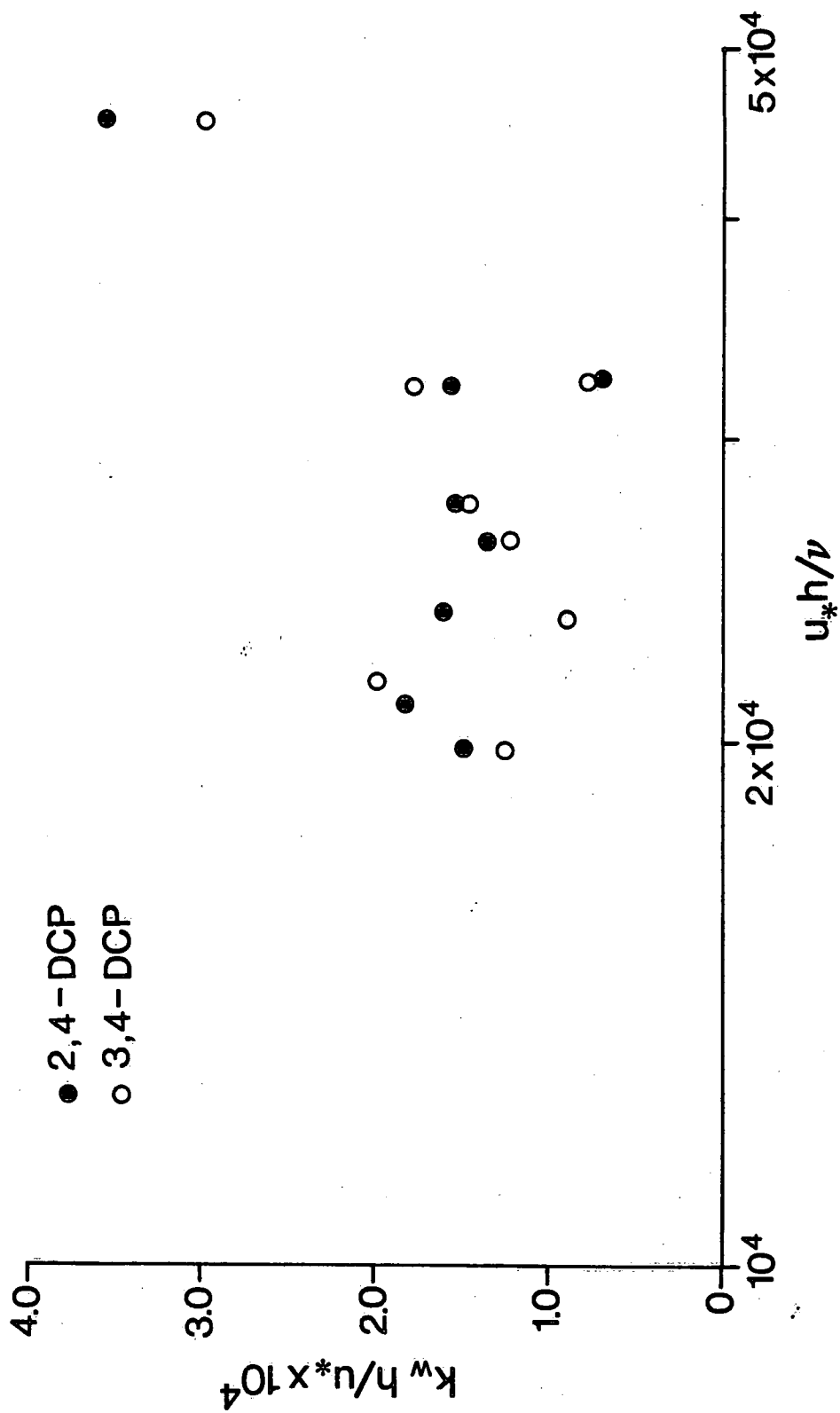


Table 1. Dimensionless Loss Rate Coefficients and Shear Reynolds Number Calculated from Data of Carey *et al.* (1984)

| Date (1981) | h(cm) | $u_*$ (cm/s) | $v$ (cm <sup>2</sup> /s) | $\frac{u_* h}{\nu}$  | 2,4-DCP                  |                       | 3,4-DCP                  |                       |
|-------------|-------|--------------|--------------------------|----------------------|--------------------------|-----------------------|--------------------------|-----------------------|
|             |       |              |                          |                      | $k_w$ (s <sup>-1</sup> ) | $\frac{k h}{u_*}$     | $k_w$ (s <sup>-1</sup> ) | $\frac{k h}{u_*}$     |
| April 7     | 30.0  | 9.55         | 1.20x10 <sup>-2</sup>    | 2.39x10 <sup>4</sup> | 5.17x10 <sup>-5</sup>    | 1.62x10 <sup>-4</sup> | 2.92x10 <sup>-5</sup>    | 0.92x10 <sup>-4</sup> |
| May 5       | 32.0  | 9.86         | 1.15x10 <sup>-2</sup>    | 2.75x10 <sup>4</sup> | 4.78x10 <sup>-5</sup>    | 1.55x10 <sup>-4</sup> | 4.58x10 <sup>-5</sup>    | 1.49x10 <sup>-4</sup> |
| June 2      | 27.5  | 9.14         | 0.96x10 <sup>-2</sup>    | 2.62x10 <sup>4</sup> | 4.55x10 <sup>-5</sup>    | 1.37x10 <sup>-4</sup> | 4.19x10 <sup>-5</sup>    | 1.26x10 <sup>-4</sup> |
| July 7      | 21.0  | 7.99         | 0.84x10 <sup>-2</sup>    | 2.00x10 <sup>4</sup> | 5.72x10 <sup>-5</sup>    | 1.50x10 <sup>-4</sup> | 4.86x10 <sup>-5</sup>    | 1.28x10 <sup>-4</sup> |
| Aug. 4      | 25.0  | 8.72         | 1.00x10 <sup>-2</sup>    | 2.18x10 <sup>4</sup> | 6.39x10 <sup>-5</sup>    | 1.83x10 <sup>-4</sup> | 7.03x10 <sup>-5</sup>    | 2.00x10 <sup>-4</sup> |
| Sept. 1     | 41.0  | 11.17        | 1.00x10 <sup>-2</sup>    | 4.58x10 <sup>4</sup> | 9.64x10 <sup>-5</sup>    | 3.54x10 <sup>-4</sup> | 8.17x10 <sup>-5</sup>    | 3.00x10 <sup>-4</sup> |
| Sept. 29    | 37.5  | 10.68        | 1.22x10 <sup>-2</sup>    | 3.28x10 <sup>4</sup> | 2.00x10 <sup>-5</sup>    | 0.70x10 <sup>-4</sup> | 2.17x10 <sup>-5</sup>    | 0.76x10 <sup>-4</sup> |
| Dec. 8      | 46.2  | 11.85        | 1.70x10 <sup>-2</sup>    | 3.22x10 <sup>4</sup> | 4.03x10 <sup>-5</sup>    | 1.57x10 <sup>-4</sup> | 4.56x10 <sup>-5</sup>    | 1.77x10 <sup>-4</sup> |