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WAVE DIRECTIONAL SPECTRA IN MIXED SEAS

by

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MANAGEMENT PERSPECTIVE

The method of analysis of directional wave data presented herein will allow the type of precise examination of data needed to make advances in the understanding of the directional distribution of surface wave energy. This method is now being applied to the data collected during the PERD funded "Wave-Turbulence Interaction" experiment, and will lead to advances in modelling of mixing processes beneath waves and in wave forecasting.

Dr. J. Lawrence
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Perspective-gestion

La méthode d'analyse des données directionnelles de vagues présentées dans ce document permettra d'effectuer le type d'examen précis des données nécessaire pour progresser dans la compréhension de la distribution directionnelle de l'énergie des vagues de surface. On l'applique présentement aux données recueillies lors de l'expérience "Interaction vague-turbulence", financée par le GRDE, et elle fera avancer la modélisation des processus de mélange sous les vagues, de même que les prévisions de vagues.

RÉSUMÉ

Des expériences sur l'interaction air-eau ont été faites pendant trois ans (1985-1987) à la plate-forme d'observation des vagues de l'Institut national de recherche sur les eaux (INRE) sur le lac Ontario, dans le cadre du projet "Déferlement de vagues en eau profonde et interaction vague-turbulence". Une partie de ces expériences visait à étudier les spectres directionnels des vagues dans différentes conditions environnementales, par ex., fetch, direction du vent (gradient de fetch) et vitesse du vent. Le capteur de direction de la vague consistait en six sondes de capacité, disposées aux pointes et au centre d'un pentagone. La principale méthode d'analyse est la méthode du maximum de vraisemblance (MMV) en utilisant directement les données de hauteur. Par comparaison avec les capteurs traditionnels de vagues sur bouée flottante, les données des sondes servent à calculer les pentes et les courbures locales, et à reconstruire les distributions directionnelles qu'on observerait avec des bouées en trèfle ou des bouées de tangage-roulis-pilonnement. On compare la discrimination directionnelle de ces trois méthodes en utilisant les données de test avec des vagues planes venant de diverses directions. Pour les trois méthodes, on compare la dispersion quadratique moyenne pour un cas où la mer du vent et la contre-houle se mêlent.

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ABSTRACT

Air-water interaction experiments were undertaken during a three year period (1985-1987) at the National Water Research Institute's (NWRI) Waves Tower on Lake Ontario for the "Deep Water Wave Breaking and Wave-Turbulence Interaction" project. A part of this experiment was to investigate wave directional spectra under different environmental conditions, i.e., fetch, wind direction (fetch gradient) and wind speed. The wave direction sensor was an array of six capacitance gauges arranged at the apices and centre of a pentagon. The principal method of analysis is the Maximum Likelihood Method (MLM) using the elevation data directly. For comparison with traditional floating buoy wave sensors, the gauge data is used to compute local slopes and curvatures and to construct the directional distributions that would be observed by pitch-roll-heave and cloverleaf buoys. The directional discrimination of these three methods is compared using test data with plane waves from various directions. The root-mean square spread for a mixed wind-sea and counter-swell case is compared for the three methods.

1. INTRODUCTION

Much of the information on directional spectra of wind waves and swell has been gathered from moored buoys. Most wave directional buoys measure pitch, roll and heave (eg. Hasselmann et al. 1980), but a few attempts to observe higher derivatives of the surface slope have been attempted (eg. Mitsuyasu et al. 1975). The resulting directional resolution of these methods is not very much better than the expected rms spread of wind seas and may be totally inadequate for distinguishing more than one swell component in a frequency bin or delineating the skewed distribution that may occur in rapidly turning winds or the appearance of double peaks (Phillips, 1957) in response to resonant wind forcing.

The Maximum Likelihood Method (MLM) has much greater resolving power (Jefferys, 1981 & 1986) but the resulting resolution (window) is dependent on the data (Pawka, 1983), and the observed directional distributions will be somewhat distorted. An efficient correction procedure to the MLM has been derived (Krogstad et al., 1988), but we have not implemented this in the results presented below.

The data were collected from a fixed tower in Lake Ontario, (Tsanis & Donelan, 1987). The National Water Research Institute's Waves Tower is in 12 m of water, 1.1 km off the beach at the west end of Lake Ontario. In the Tower's location the shoreline is straight and the bottom slope is gentle. The annual water level variation is less than 0.5 m. The location of the tower makes possible fetches from 1.1 km for west winds up to 300 km for east winds. Finally, every year Lake Ontario sees a number of wind episodes with speeds in excess of 15 m/s.

Capacitance type wave staffs are used as sensing elements to provide information for the water surface elevation changes, eg. Oakley et al. (1977). The wave staffs are teflon-coated wires 6 m in length with an overall diameter of 4.8 mm. Six wave staffs are arranged in a pentagon with one at the center (the distance between the wave staff at the center and the others is 0.25 m) and are held taut with rubber shock cords (490 N tension). The array is symmetrical resulting in smaller azimuthal differences in directional sensitivity and its size is appropriate for the wave periods of interest, i.e., 0.6 to 8.0 seconds. The wave staffs had a very stable linear calibration during the experimental period and were cleaned almost every week from any impurities and algae that had accumulated on them.

The wave staff data, after proper filtration and amplification, are digitized on the tower and transmitted via a system of modems, a microcomputer and an underwater cable to the data acquisition system in another microcomputer in a trailer on shore. The data are digitized at 20 Hz and the length of the different episodes are at least 80 minutes to ensure adequate wave statistics.

In the following sections we outline the methodology of the three analysis methods (section 2), explore the response of each method to mixed seas (section 3) and compare the observed spreading distribution for a case in which a strongly forced wind sea propagates against decaying swell (section 4).

2. OUTLINE OF METHODS

The three directional spectral methods used in the present work are briefly described.

(a) **Pitch-Roll Buoy**, eg. Longuet-Higgins (1964). The directional spectra can be determined by the information yielded by the motion of a buoy. The vertical displacement and angles of pitching and rolling of the buoy, can provide information on the water surface elevation $\zeta(x,y,t)$ and its slopes $\zeta_x(x,y,t)$ and $\zeta_y(x,y,t)$ in x and y directions, respectively. In our study the elevation and the slopes are measured by using the information from all six wave staffs. The elevation of the wave staff at the center of the pentagon $\zeta_3(0,0,t)$ is used and the rest of the wave staffs are used for evaluating the slopes as follows

$$\zeta_x(0,0,t) = \frac{\partial\zeta}{\partial x}(0,0,t) = \frac{1}{2}\left[\frac{\zeta_1-\zeta_3}{r(1,1)} + \frac{2\zeta_3-(\zeta_5+\zeta_6)}{r(1,5)}\right] \tag{1}$$

$$\zeta_y(0,0,t) = \frac{\partial\zeta}{\partial y}(0,0,t) = \frac{1}{2}\left[\frac{\zeta_2-\zeta_4}{2r(2,2)} + \frac{\zeta_5-\zeta_6}{2r(2,5)}\right] \tag{2}$$

where r(i,j) are the coordinates of the wave staffs (i=1,2 for the x and y coordinate, respectively, and j=1,6 for the wave staffs). Figure 1 shows the wave staff array, the numbering of the wave staffs and the coordinate system orientation.

(b) **Cloverleaf Buoy**, eg. Cartwright and Smith (1964) & Mitsuyasu et al. (1975). The directional spectra can be determined by the information yielded by the motion of a cloverleaf buoy. The cloverleaf buoy measures the vertical acceleration ζ_{tt}, the slopes ζ_x, ζ_y, and curvatures ζ_{xx}, ζ_{yy}, ζ_{xy} of the wave surface $\zeta(x,y,t)$. In our study, a second order polynomial

$$\zeta(x,y,t) = a_1(t) + a_2(t)\,x + a_3(t)\,y + a_4(t)x^2 + a_5(t)\,y^2 + a_6(t)\,xy \tag{3}$$

is used to describe the water surface. Using Eq. (3) for the six wave staffs yields a system of six linear equations with six unknown coefficients $a_i(t)$, i=1,6. The system of equations is solved and the coefficients $a_i(t)$, i=1,6 are evaluated. The coefficients represent the elevation, slopes and curvatures of the water surface at the point (x,y)=(0,0), i.e.,

$$a_1(t) = \zeta\,(0,0,t) \tag{4α}$$

$$a_2(t) = \zeta_x(0,0,t) = \frac{\partial\zeta}{\partial x}(0,0,t) \tag{4b}$$

$$a_3(t) = \zeta_y(0,0,t) = \frac{\partial\zeta}{\partial y}(0,0,t) \tag{4c}$$

$$2a_4(t) = \zeta_{xx}(0,0,t) = \frac{\partial^2\zeta}{\partial x^2}(0,0,t) \tag{4d}$$

$$2a_5(t) = \zeta_{yy}(0,0,t) = \frac{\partial^2\zeta}{\partial y^2}(0,0,t) \tag{4e}$$

$$a_6(t) = \zeta_{xy}(0,0,t) = \frac{\partial^2\zeta}{\partial x\partial y}(0,0,t) \tag{4f}$$

(c) **Maximum Likelihood Method (MLM)**, eg. Capon (1969) & Jefferys et al. (1981). According to the Maximum Likelihood Method the energy incident from direction θ is evaluated by minimizing the influence from all the other components. The minimization uses standard Lagrangian theory and leads to an estimate of the energy in the plane wave

$$E(\omega_k,\theta) = [\,\chi^{*T}(\omega_k,\theta)\,C^{-1}(\omega_k)\,\chi(\omega_k,\theta)\,]^{-1} \tag{5}$$

where k is the frequency index and $C^{-1}(\omega_k)$ is the inverse of the cross spectral density matrix. In terms of the energy spectrum $E(\omega_k,\theta)$ from directions θ_i in the frequency band near ω_k, the cross spectal density matrix is given by:

$$C(\omega_k) = \sum_{i=1}^{N}\chi(\omega_k,\theta_i)\,\chi^{*T}(\omega_k,\theta_i)\,E(\omega_k,\theta_i) \tag{6}$$

where $\chi(\omega_k, \theta_i)$ is the complex phase lag between the jth sensor and the origin for a wave of frequency ω_k approaching from direction θ_i.

The cross spectra between the elevations, slopes and or curvatures needed for the computation of the directional spectra using the above techniques, make use of the following values:
Number of wave staffs = 6
Number of directions = 72
Sampling rate = 20 Hz
Spectral analysis = FFT method
Number of samples in data block/FFT = 4096
Number of blocks = 24
Frequency interval 8 X 20 / 4096 = 0.039 Hz
(averaging every 8 intervals is performed for spectral smoothing).

3. DIRECTIONAL DISCRIMINATION IN MIXED SEAS

Fig. 2 shows the normalized (to peak) angular distribution of wave energy $E(\theta)$ for plane waves of 0.10 m amplitude at three frequencies, 0.154, 0.583 and 1.013 Hz from different angles, 60°, 300° and 180°, respectively. The numbers, in the upper right corner in these plots indicate the frequency and in the bottom left corner the frequency index. The dashed, dash-dotted and solid lines represent the directional estimates based on the pitch-roll buoy, cloverleaf buoy and the MLM techniques, respectively. A perspective plot of the previous case, for all angles (0° - 360°) and frequencies (0.037 - 1.40 Hz) based on the above techniques (on the same scale) is shown in Fig. 3. Close inspection of the spreading of the wave energy reveals the following: (a) the directional spread calculated by means of the pitch-roll and the cloverleaf buoy equivalent techniques are independent of frequency and (b) the directional spectra evaluated via the MLM technique are frequency dependent. At low frequencies the spreading is wider than at high frequencies. This is a direct consequence of the nonlinearity of the MLM technique.

The angular distribution of wave energy based on the pitch-roll buoy technique is wider than the corresponding one based on the cloverleaf buoy technique, and both are wider than the corresponding one based on the MLM. A test on the angular discrimination of the above techniques for plane waves at frequency 0.662 Hz, is shown in Fig. 4. Fig. 4a shows the angular distribution of wave energy for three plane waves at 50°, 60° and 70°, Fig. 4b for plane waves at 30° and 90°, Fig. 4c for plane waves at 20° and 100° and Fig. 4d for plane waves at 130° and 350°. The plane waves are 10° apart in Fig. 4a, 60° apart in Fig. 4b, 80° apart in Fig. 4c and 140° apart in Fig. 4d. The pitch-roll buoy technique begins to resolve the plane waves when they are 140° apart while the cloverleaf buoy technique begins to resolve plane waves when they are 80° apart. The MLM has the greatest resolving power as indicated in Fig. 4a, where it is able to resolve waves even when they are only 10° apart.

4. COMPARISON OF MEASURED SPECTRAL WIDTHS

During the extensive observational period (October to December in three successive years) we captured a few occasions of multiple seas. Lake Ontario is about 300 km long

and so it is generally subject to the influence of only one meteorological system at a time. However when a strong easterly (long fetch) blow was followed by a rapid shift to the prevailing westerlies, we observed a growing offshore wind sea propagating counter to the residual swell from the east. The most interesting of these events is depicted in 3-D (Fig. 5) and 2-D (Fig. 6) representations of the pitch-roll, cloverleaf and MLM calculations. In addition to these Fig. 6 includes a $\mathrm{sech}^2\beta(\theta-\theta_p)$ fit to the directional estimates, where θ_p is the angle at the peak of the distribution and β is a spreading parameter. The swell with relatively narrow spread is shown at 60° and the wind sea (somewhat broader) is centered at 215°. Figs. 5 and 6a show the directional spreads from the MLM, pitch-roll and cloverleaf equivalent calculations of the swell. The cloverleaf fails completely for these long waves probably because the quantization error (2.5 mm) in surface elevation means that curvature errors of 200% or more occur randomly for these long waves. On the other hand similar slope errors are only about 10% so the pitch-roll equivalent calculation does not fail as badly, but it is still more than twice as wide as the MLM.

The spreading at the peak of the wind sea is shown in Fig. 6b. Here all methods appear to work and the cloverleaf and MLM are nearly in agreement, although the cloverleaf is still somewhat wider. The pitch-roll buoy is considerably wider.

An overall look at the root-mean square spread for the swell and the wind-sea case of Fig. 5, is given in Fig. 7. The mean square spread is given by the definite integral $\int_{-\pi}^{+\pi}(\theta-\theta_c)^2E(\omega_k,\theta)d\theta$ normalized by the area under the spectrum $\int_{-\pi}^{+\pi}E(\omega_k,\theta)d\theta$ where θ_c is the angle at the centroid of the distribution. For the wind-sea above the peak the cloverleaf and MLM are in close agreement. On the other hand the pitch-roll spreads are appreciably wider and only reach agreement with the other methods for the rather wide distribution beyond twice the peak frequency.

5. ACKNOWLEDGEMENTS

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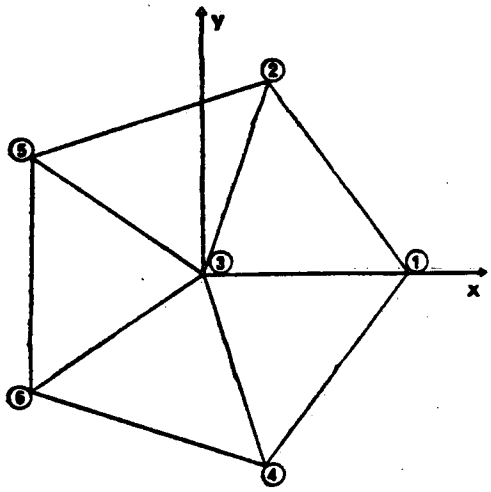


Fig. 1 The wave staff array

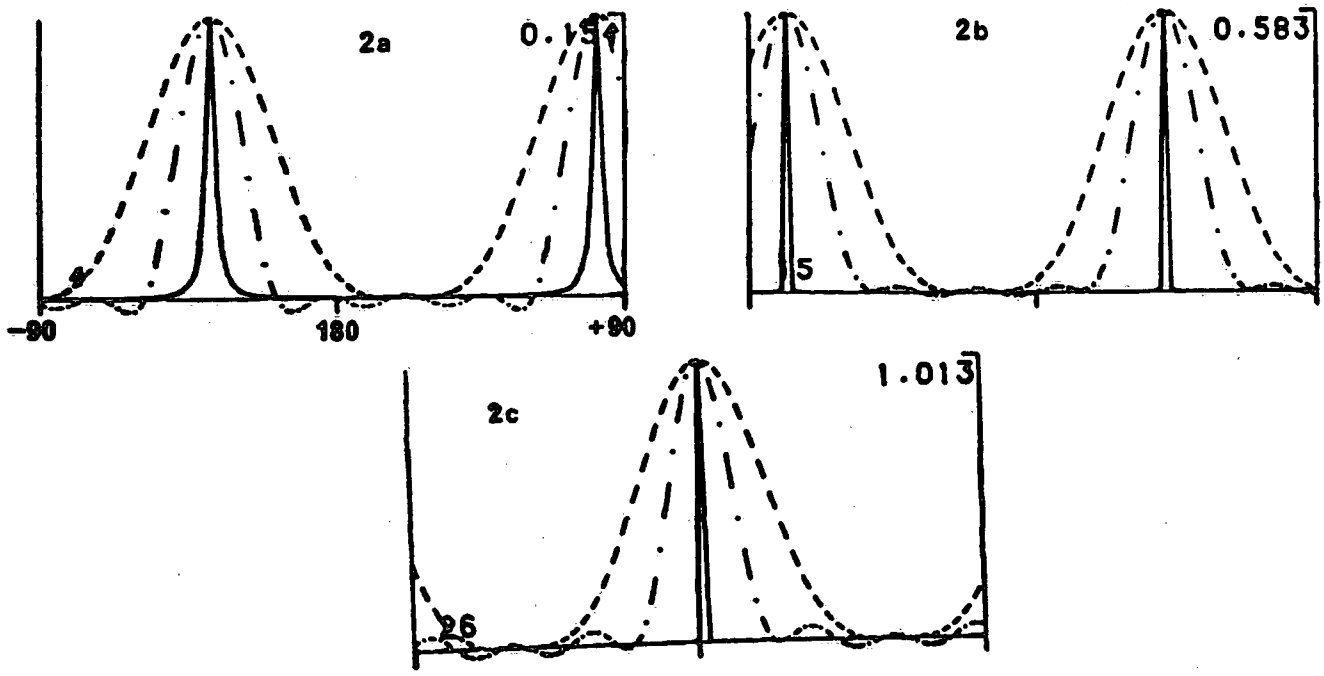
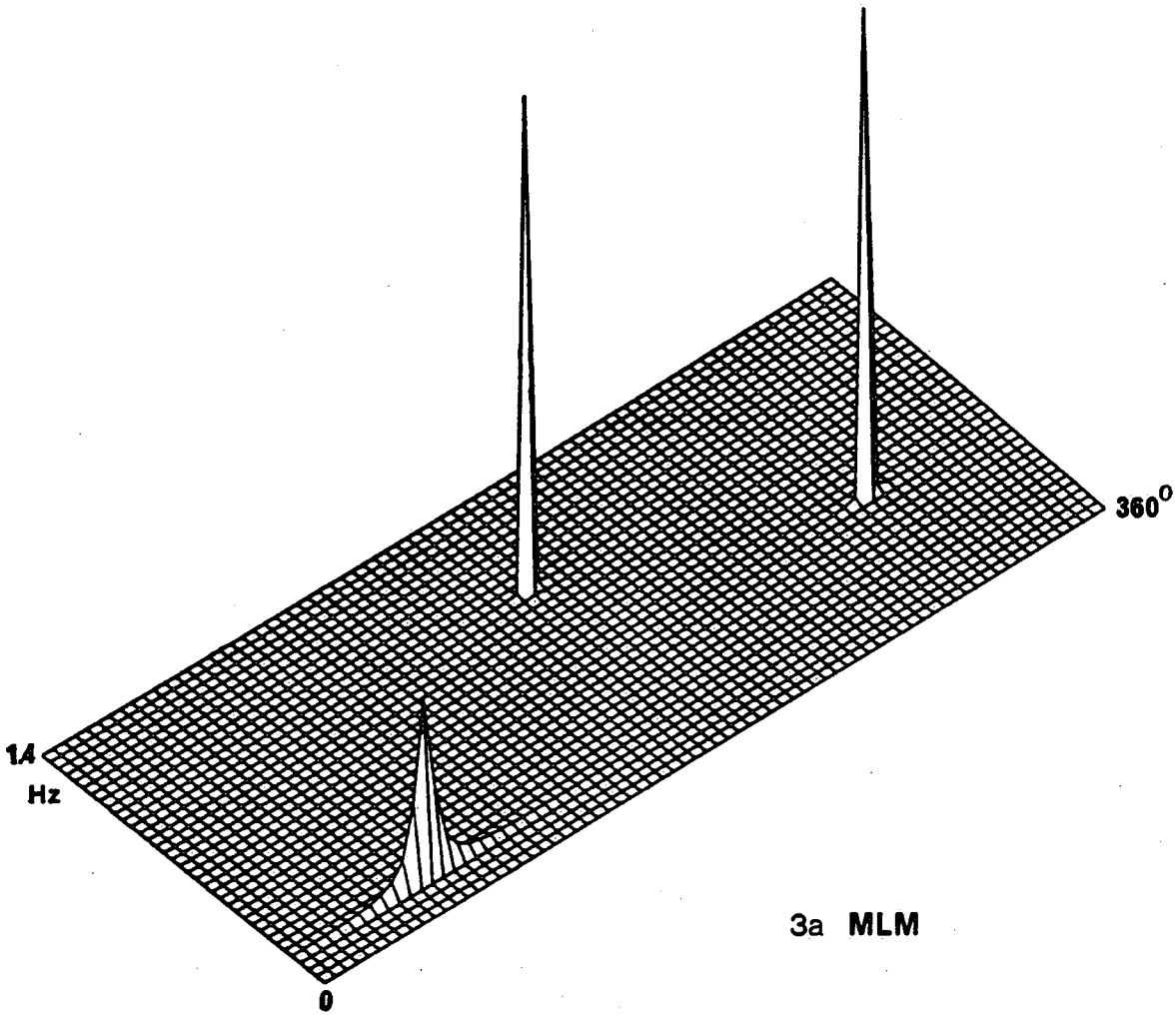
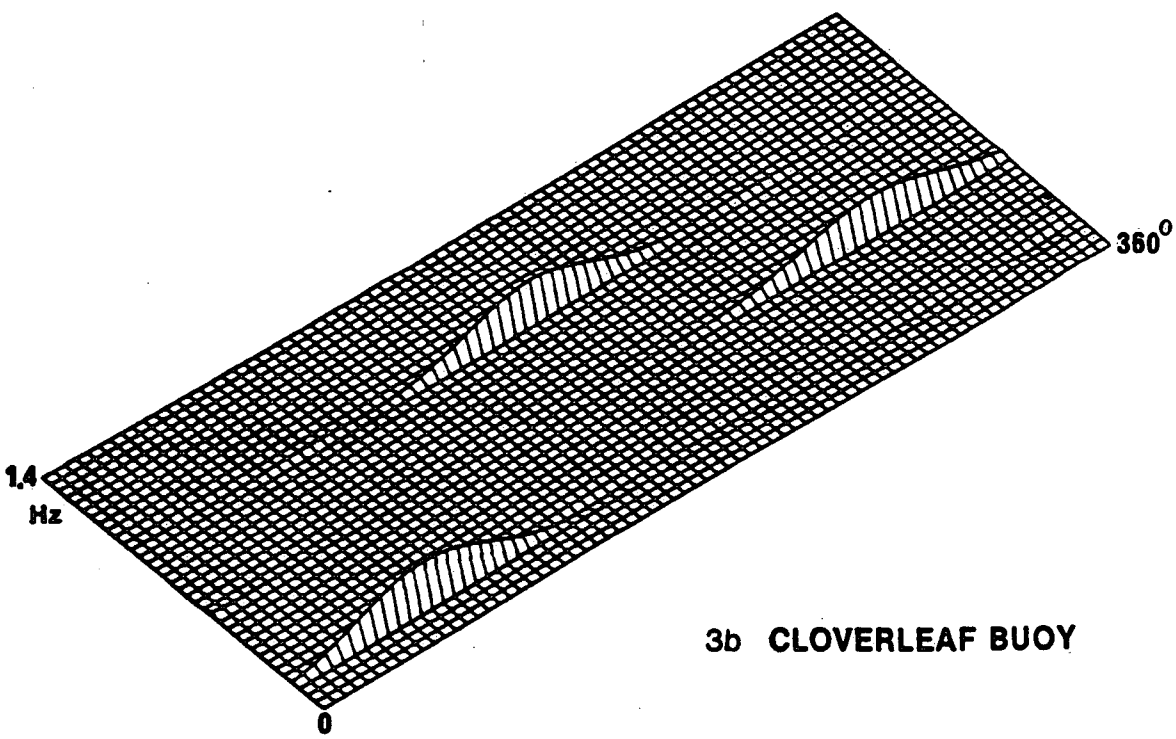


Fig. 2 2-D presentation of plane waves from different angles at different frequencies.
--------- Pitch-roll buoy; ---------- Cloverleaf buoy; ——— MLM



3a MLM



3b CLOVERLEAF BUOY

Fig. 3 3-D presentation of plane waves from different angles at different frequencies.

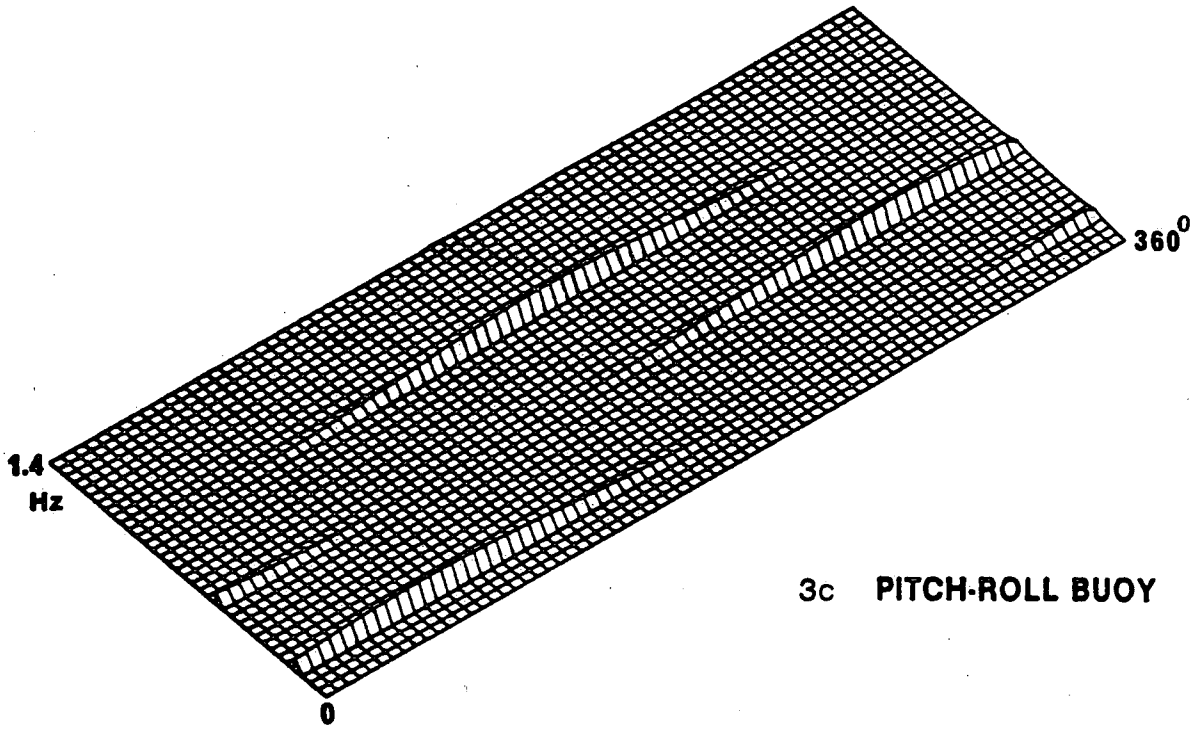


Fig. 3 3-D presentation of plane waves from different angles at different frequencies.

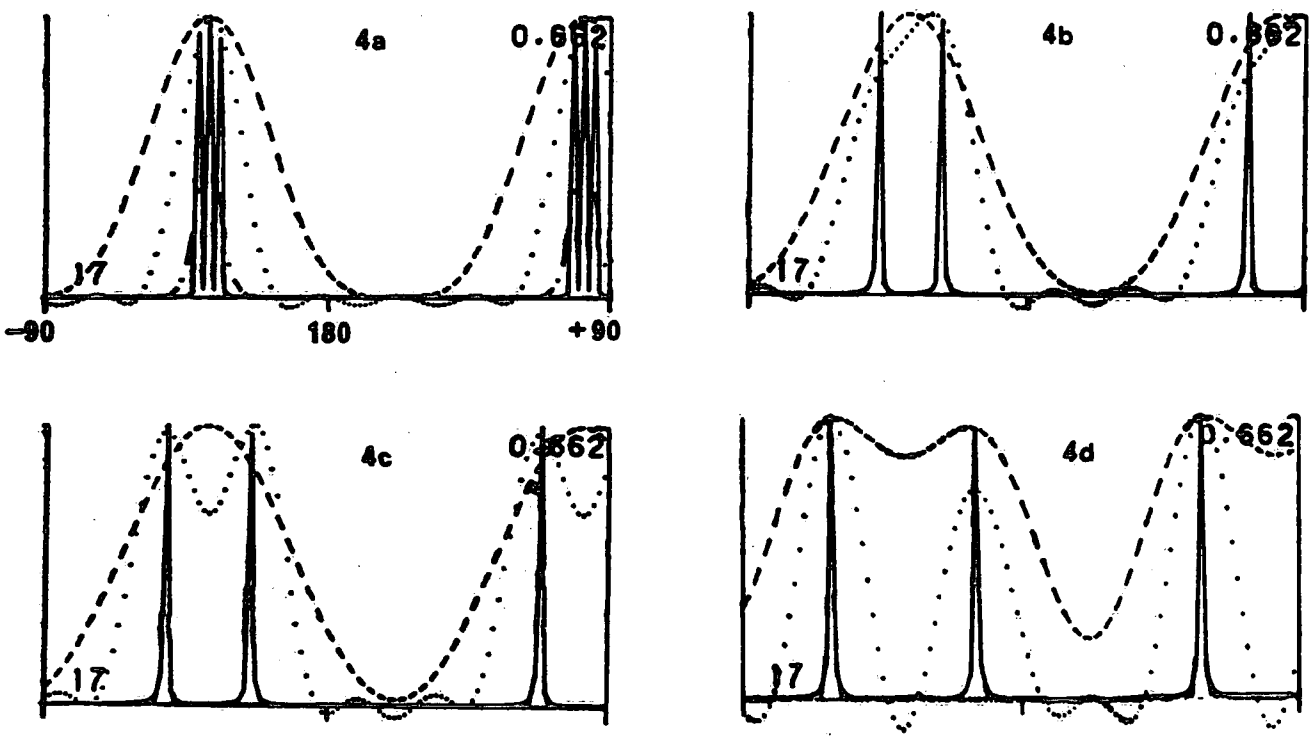


Fig. 4 Test of angular discrimination of different techniques.
---- Pitch-roll buoy; ..... Cloverleaf buoy; —— MLM

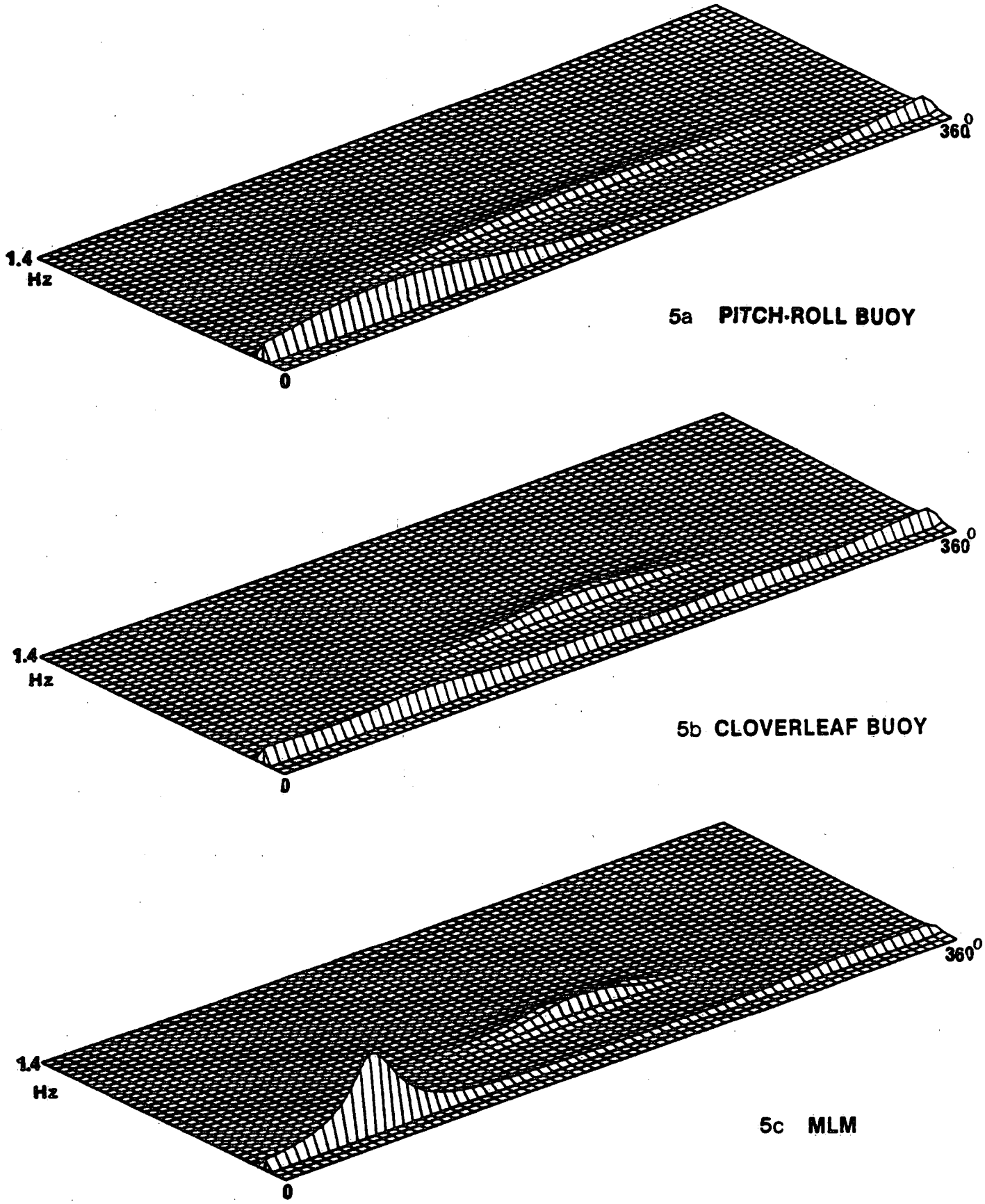


Fig. 5 3-D presentation of wave directional spectra in an offshore wind with an onshore swell.

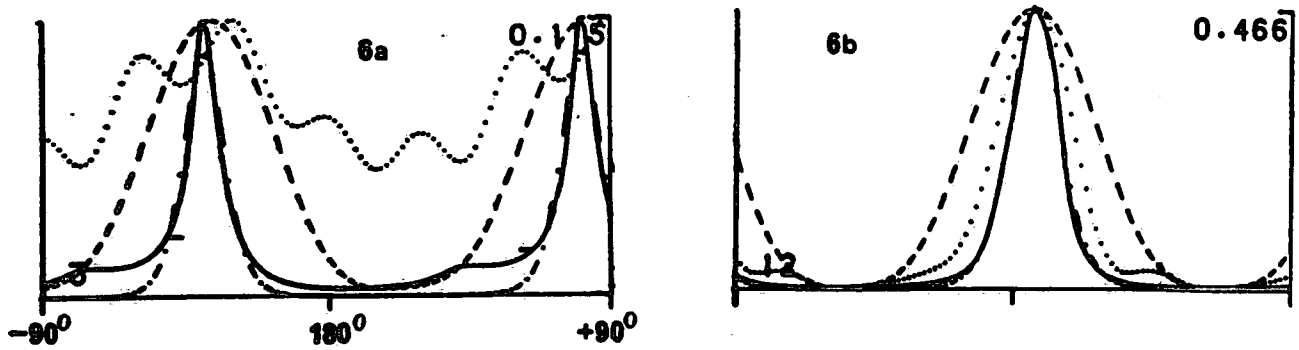


Fig. 6 2-D presentation of wave directional spectra in an offshore wind with an onshore swell. (----- Pitch-roll buoy; ..... Cloverleaf buoy; —— MLM)

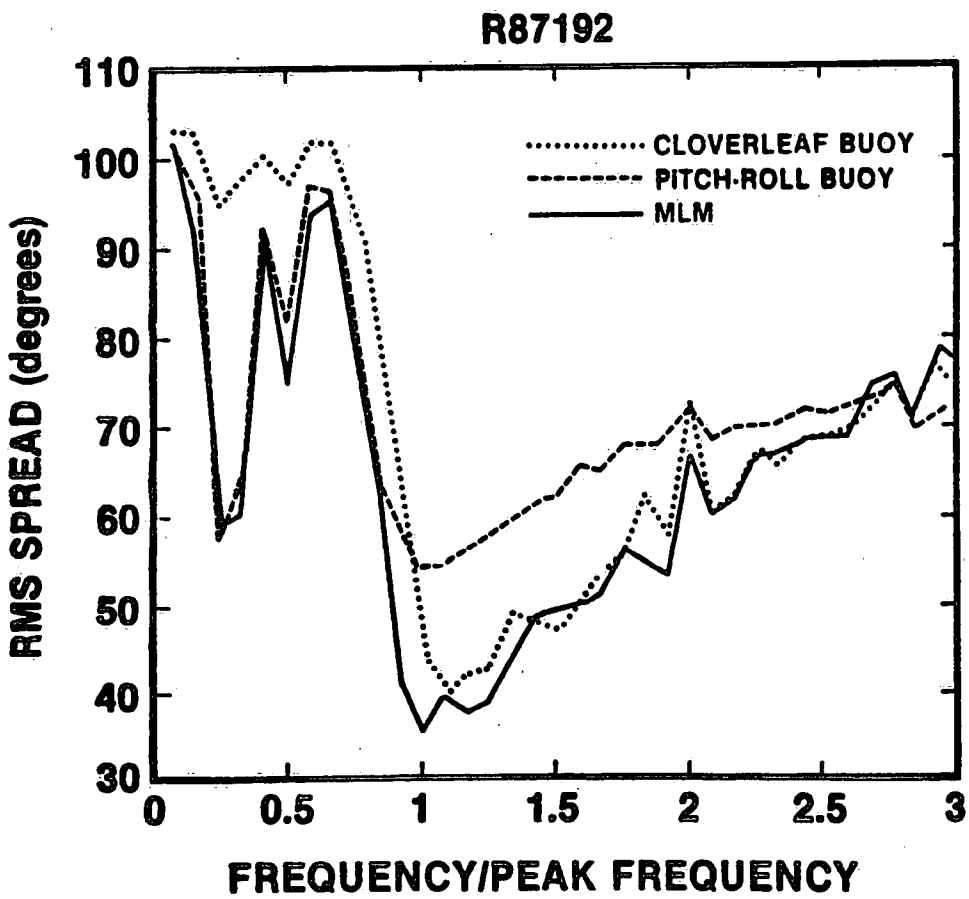


Fig. 7 Frequency dependance of RMS spread of the mixed sea of Fig. 5.