NWRI CONTRIBUTION 90-114

COMPARISON OF MODEL AND PROTOTYPE VALUES OF MANNING'S "n" FOR ARTIFICIAL ROUGHNESS ELEMENTS

by

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January 1990

MANAGEMENT PERSPECTIVE

Urban drainage channels are protected against erosion by lining their sides and bed with a matrix of inter-locking concrete blocks. Designers of such channels need to know the hydraulic roughness of these protection matrices in order to determine sufficient freeboard for all flow conditions. The roughness parameter preferred by many engineers is Manning's "n". Values of Manning's "n" for such erosion control products are usually determined in a hydraulic flume. Such tests should be conducted at full scale because significant errors in values of Manning' "n" may result from model tests.

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PERSPECTIVE GESTION

Les canaux de drainage urbains sont protégés contre l'érosion en garnissant les côtés et le fond d'une matrice de blocs de béton emboîtés. Les concepteurs de ces canaux doivent connaître la rugosité hydraulique de ces matrices de protection afin de déterminer une revanche suffisante pour toutes les conditions d'écoulement. Le paramètre de rugosité préférés par de nombreux ingénieurs est le "n" de Manning. Les valeur de n pour de tels produits de lutte contre l'érosion sont habituellement établies dans un canal hydraulique. De tels essais devraient être menés à l'échelle réelle car des essais sur modèles pourraient produire des erreurs importantes dans les valeurs de n.

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SUMMARY

Full scale and model tests were conducted on inter-locking concrete blocks which are placed in a matrix as erosion protection in urban drainage channels. It was determined that Manning's roughness coefficient "n" from scaled up model data is about 15% lower than values obtained from prototype tests. The results suggest that great care must be used when using values of Manning's "n" obtained from model tests.

RESUME

Des essais à l'échelle réelle et sur modèles ont été menés sur des blocs de béton qui s'emboîtent pour former une matrice de protection contre l'érosion dans les canaux de drainage urbains. Il a été établi que le coefficient de rugosité "n" de Manning découlant des données de modèles ramenés à l'échelle est environ 15 % inférieur aux valeurs obtenues dans les eaais sur prototypes. Les résultats révèlent qu'il faut être très prudent dans l'utilisation des valeurs de n provenant d'essais sur modèles.

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1.0 INTRODUCTION

The capacity of a uniformly flowing open channel is determined by obtaining appropriate values of the crossectional area and the hydraulic radius of the flow. Many engineers prefer to use Manning's equation for this purpose. For a given channel, the design discharge, the slope and the Manning's roughness coefficient, usually denoted as "n", are given. The slope of the channel is primarily governed by topography and the value of "n" depends on the type of material used to line the channel. Urban drainage channels are often protected against erosion by lining their sides and bottom with a matrix of inter - locking, concrete blocks. Manning's "n" for these matrices is usually determined from tests conducted in a tilting flume.

Recently, tests of a particular erosion control product were conducted to determine, in addition to Manning's "n", the flow velocity at which the protection matrix begins to fail. In order to obtain sufficiently high velocities for the latter tests, it was necessary to model the prototype matrix. Model blocks were fabricated, taking care that all physical properties were preserved in the proper proportions. In contrast to the maximum velocity determinations, the tests for Manning's "n" were conducted on the full size blocks because of the uncertainty due to possible scale effects. The data from the model were used to determine the model values of Manning's "n" and these are compared with the prototype values obtained.

The work was conducted as part of the cost recovery program of the Hydraulics Laboratory at the National Water Research Institute in Burlington, Ontario, Canada.

2.0 MODELLING CONSIDERATIONS

2.1 Dimensional Analysis

For a free surface, uniform flow in a channel, having a rectangular cross-section and the bed lined with a matrix of inter-locking concrete blocks, the mean velocity can be expressed in the following general functional form:

$$V = f(R, U_*, \rho, \mu, g, a, b, c, k, p, \theta, r) \qquad (1)$$

where: V = the mean velocity of the flow in in the cross-section, R = the hydraulic radius, U_* = the shear velocity, ρ = the density of the fluid, μ = the dynamic viscosity of the fluid, g = the acceleration due to gravity, a, b, c, θ, r and p are geometric parameters of each concrete block as shown in Figure 1, k = the equivalent sand grain roughness of the surface of the individual blocks, and f denotes a function. Taking g, R and ρ as the repeating variables, dimensional considerations yield

$$\frac{V}{U_*} = f_1 \left[\frac{V}{\sqrt{gR}}, \frac{a}{R}, \frac{b}{R}, \frac{c}{R}, \frac{r}{R}, \frac{p}{R}, \frac{k}{R}, \theta, \frac{g^{1/2} R^{3/2}}{\nu} \right] \qquad (2)$$

where f_1 denotes another function and ν = the kinematic viscosity. As long as the flow is in the rough turbulent regime, the viscous effects should not be important. In addition, if the slope of the channel is less than about 2%, then the effect of Froude number should have a negligible effect on V/U_*. Therefore the Froude number and the parameter $g^{½}R^{3/2}/\nu$ can be removed from equation (2) which, after some further rearranging, becomes

$$\frac{V}{U_*} = f_2 \left[\frac{a}{b}, \frac{c}{b}, \frac{r}{b}, \frac{p}{b}, \theta, \frac{k}{b}, \frac{R}{b} \right] \tag{3}$$

where f_2 denotes another function. In the model construction it is quite easy to preserve the quantities a/b, r/b, θ, and p/b. Therefore, these parameters are not of present concern and can also be removed from further consideration. As a result equation (3) is now reduced to the form

$$\frac{V}{U_*} = f_3 \left[\frac{c}{b}, \frac{k}{b}, \frac{R}{b} \right] \tag{4}$$

where f_3 denotes another function.

The Manning's equation is given as

$$V = \alpha \frac{R^{2/3} S^{½}}{n} \tag{5}$$

where S is the water surface slope for a uniform flow, n = Manning's roughness coefficient and α = a coefficient. The value of α = 1.0 when the S.I. system of units is used and α = 1.486 when the British system of units is used. Equation (5) can be rearranged in the following form:

$$\frac{V}{U_*} = \alpha \frac{R^{1/6}}{n\sqrt{g}} \qquad (6)$$

Finally, upon combining equations (4) and (5) one may write

$$\frac{n\sqrt{g}}{R^{1/6}} = f_4 \left[\frac{c}{b}, \frac{k}{b}, \frac{R}{b} \right] \qquad (7)$$

Equation (7) states that the dimensionless form of Manning's "n"is a function of the two dimensionless parameters which account for the width of separation between adjacent blocks, the relative roughness due to surface texture of the block material and a parameter which accounts for the depth of the flow.

The difficulty with modelling the concrete prototype blocks to obtain Manning's "n" is due to the uncertainty in achieving true geometric similitude in the two ratios c/b and k/b. Tests were conducted to determine the significance of such scale effects.

2.2 Scaling Relationships

The required ratios to scale up the results from the model to the prototype can be obtained by writing

$$\left[\frac{\lambda_n \ \lambda_g^{1/2}}{\lambda_R^{1/6}} \right] = 1 \qquad (8)$$

and

$$\frac{\lambda_c}{\lambda_b} = \frac{\lambda_k}{\lambda_b} = \frac{\lambda_R}{\lambda_b} = 1 \qquad (9)$$

where λ denotes the model/prototype ratio of the subscripted variables. Considering that the ratios of length dimensions are the same (ie: the model is undistorted), then $\lambda_c = \lambda_k = \lambda_R = \lambda_b = \lambda_L$ where λ_L simply denotes the scale ratio. The scale ratio for the model blocks is 1:10. As a result the scaling ratio for Manning's "n" may now be written as

$$\lambda_n = \lambda_L^{1/6} = 1:1.468 \qquad (10)$$

Equation (8) was used to scale up the model values of n to the prototype values for comparison with values of n obtained from tests conducted on full sized blocks.

3.0 EXPERIMENTAL SET-UP AND PROCEDURE

3.1 Description of the Flume

The experiments were conducted in a glass walled flume, rectangular in cross-section, having the following dimensions:

Width	1.0 m
Length	21.5 m
Height of Walls	0.75 m

Water is supplied to the flume from a large constant head tank which is fed by three axial flow pumps with a combined output of 0.8 m^3/s. From the constant head tank the water passes through a 16 in (0.4 m) diameter pipe which is terminated by a diffuser in the head box of the flume. Baffles and screens placed in the head box further ensure a satisfactory velocity distribution over the cross-section of the flow at the entrance to the rectangular flume channel. The depth of the flow can be controlled by a vertical leaf gate at the end of the flume. The flume can be tilted by means of a system of motor driven screw jacks to a maximum slope of about 5%. The maximum available discharge in the flume is 0.4 m^3/s.

3.2 Measurement of Hydraulic Variables

Discharge measurements were made using a large calibrated volumetric tank. The overflow from the constant head tank was diverted into the volumetric tank where it was measured to an accuracy of about 5%. Once the overflow was measured, the discharge in the flume could simply be determined from the relationship

$$Q_f = Q_p - Q_o \qquad (11)$$

where Q_f = the discharge in the flume in m^3/s, Q_p = the discharge from the pumps entering the constant head tank in m^3/s and Q_o = the overflow in m^3/s.

Measurements of flow depth were made using three point gauges which could be read to an accuracy of 0.1 mm.

The slope of the flume was measured using a calibrated mechanical counter attached to the jack-gear mechanism. This gave an accuracy in the slope measurement of about 0.05%.

3.3 Placement of the Prototype Blocks

Placement of the blocks was complicated by the fact that they have an effective width of 0.43 m and therefore, the flume bed, having a width of 1.0 m, is equivalent to 2.33 blocks. In addition, the perimeter of each block consists of two pairs of "tabs" and "recesses" (Figure 1) which are mated with those of adjacent blocks to create the inter-lock. In order to cover the flume bed completely, while at the same time maintaining the surface characteristics of the matrix, the following procedure was used. To place the first row of blocks along the glass wall of the flume, it was necessary to cut the protruding "tabs". Once these "tabs" were removed the first row of blocks could be placed against the glass wall of the flume. However, because of the Inter - lock "recesses", cavities remained between the edge of the blocks and the flume wall. These cavities were filled with the "tabs" cut off earlier, resulting in a smooth, continuous fit. Once the first row was placed it was a simple matter to place the second row by inter-locking it with the first row. The remaining flume width was then filled by cutting each block to fit. The block matrix was placed over the full length of the flume bed as shown in Figure 2.

3.4 Placement of the Model Blocks

The model blocks were placed for the purpose of determining the maximum velocity that the blocks could be subjected to without being lifted out of their inter-locking positions. The flume was prepared by fastening a false bottom consisting of 3/4 plywood, to the bottom of the flume extending over the full length and width. This newly formed flume bed was then prepared by fastening a "geo-textile" filter cloth to the plywood, beginning at a point ten metres from the head of the flume and extending downstream for a length of nine metres.

The model blocks were then placed in their normal inter-locking configuration to cover the "geo-textile" completely. In the 10 metre reach upstream of the leading edge of the block matrix, a layer of plywood of the same thickness as the filter cloth and block matrix combined, was fastened to the false bottom. This ensured a smooth vertical bed transition from the head box into the test reach. All plywood was treated with wood preservatives to prevent swelling during the tests. Similar to the placement of the prototype blocks, care was taken to ensure that a smooth fit was achieved along the glass walls of the flume. The completed matrix is shown in Figure 3.

3.5 Test Procedure

Three point gauges were placed along the centre line of the flume at locations chosen to minimize the effects of the upstream and downstream transition zone while maintaining the maximum length of working section. The point gauges were adjusted to a common datum and the average bed elevation was noted.

For each test the desired flow depth was set on the point gauges based on the average of the measured bed elevations and the flume was tilted to the desired slope. The discharge and tailgate were then adjusted to obtain uniform flow (ie: water surface just touching the point gauges at all three locations). When uniform flow was established, the depth and slope were noted and the discharge was measured. The same procedure was used for both model and prototype measurements.

Tests were conducted over a wide range of discharges up to the maximum attainable while flow depth was kept constant. The data for the tests are given in Table 1 and Table 2 for the model and prototype respectively.

4.0 DATA ANALYSIS

4.1 Computation of Manning's "n" for Block Matrices

The value of n computed from equation (5) is an effective roughness for the entire flow boundary. In the tests conducted, the roughness was not the same along the entire wetted perimeter because the sides of the flume are made of glass whose roughness coefficient should be significantly less than that of the blocks being tested. Therefore, in order to obtain the value of n for the block matrices the effect of the glass wall has to be taken into account. A method given in Chow (1959) was used.

In this method, the wetted perimeter is divided into two parts: the bottom, which is composed of the block matrix and the glass sides. The Manning's "n" for the total wetted perimeter can thus be related to the values of n for the block matrix and the glass by the equation

$$n = \frac{\left[P_g n_g{}^2 + P_b n_b{}^2\right]^{1/2}}{P^{1/2}} \qquad (12)$$

where P = the wetted perimeter for the entire flow = (B + 2h) metres for a flow depth of h metres and a flume width of B metres, P_g = the wetted perimeter of the glass side walls = 2h metres, P_b = the wetted perimeter of the matrix = B metres, n_b = roughness coefficient of the block matrix and n_g = the roughness coefficient of the glass walls. The roughness coefficient of the glass walls was assigned the value of .010 which is a value recommended by Chow (1959). This value is not expected to be very much in error. It can be shown that even if n_g is in error by 10%, the error in n will only be about 2%. Values of n_b

for the model were scaled up to prototype values by multiplying the values obtained from the model tests by the scaling factor $\lambda_L = 1.468$. The computed values of the roughness coefficients for model and prototype are given in Table 3.

4.2 Model and Prototype Results

The mean and standard deviation for the values of the roughness coefficients in Table 3 were computed and these are given in Table 4. The coefficient of variation "C" given as "100% X (standard deviation/mean)" for the model results is about 10.5% and for the prototype results is about 4.3% indicating that the model tests provided less consistent data. The reason for this is that the model tests were conducted at steeper slopes and thus faster flows. This makes the measurement of flow depth less certain, however, the model results are sufficiently precise for present purposes.

The results in Table 3 show that the scaled up model value under estimates the prototype value by about 15%. The form of equation (5) shows that this would result in an error of 15% when the scaled up value of n is used with the Manning's equation. The reason for this difference must be attributed to the fact that the value of the parameters c/b and k/b in equation (7), could not be kept the same in the model and the prototype. The effect of R/b is insignificant because it can be shown that the value of Manning's "n" is not affected by the depth of flow in the range of the experimental conditions used. This can be demonstrated as follows: The dimensionless form of Manning's roughness coefficient can be expressed as

$$\frac{n\sqrt{g}}{k_s^{1/6}} = \frac{\left(\frac{R}{k_s}\right)^{1/6}}{\sqrt{8}\left[a + 2\log\left(\frac{4R}{k_s}\right)\right]} \qquad (13)$$

where k_s = the equivalent sand grain roughness of the total roughness of the block matrix and a is a constant. The value of the constant a = 1.14 (Vennard, 1966). The equivalent grain size k_s, is dependent only on the effect of the slots, joints and surface texture of the material of which the blocks are made and therefore is constant for a given condition. Equation (13) is plotted as $n/k_s^{1/6}$ vs. R/k_s in Figure 4. The curve in Figure 4 clearly shows that for values of $R/k < 200$, values of n are not significantly affected by changes in depth or equivalently, changes in R. Values of k for the prototype block matrix are of the order of 0.02 m and therefore values of R/k for the prototype and model tests were less than 100. As a result, the difference in the prototype and model values of Manning's roughness coefficient should not be due to any disproportionate differences in the value of R used in the model and prototype. Therefore, the parameter R/b can be removed from equation (7) to give

$$\frac{n\sqrt{g}}{R^{1/6}} = f_4 \left[\frac{c}{b}, \frac{k}{b}\right] \qquad (14)$$

where f_4 denotes another function. This means that the difference in the scaled up values and the prototype values of Manning's n, must be due to differences in the fit of the joints, accounted for by c and the surface texture of the block material, accounted for by k. These two variable combine to provide a lower total resistance to the flow which is reflected by a lower value of n.

The difference of 15% in the scaled up and the prototype values of Manning's n does not appear to be too excessive at first glance. However, it must be kept in mind that if Manning's equation is used to compute the channel slope, the value of n must be squared. Under these circumstances, the error of 15% in the value of n will result in an error of 23% in the channel slope. Such errors are excessive and situations leading to such errors must be avoided.

Therefore, values of Manning's n obtained with scale models should be treated with caution and whenever possible, values based on full scale data should be used.

5.0 CONCLUSIONS

5.1 The value of Manning's "n" for a rectangular channel is virtually independent of the depth of flow for values of R/k_s < 200. For values of R/k_s from 200 to about 1000 values of n can be expected to increase by about 12%.

5.2 The scaled up values of n were lower than the prototype values by about 15%. This difference can result in errors of about 25% when Manning's equation is used to determine channel slope.

5.3 Great care must be exercised when selecting values of n based on model tests. Whenever possible, values of n should be based on full scale information.

ACKNOWLEDGEMENT

This report is based on information obtained during tests conducted for OAKS Precast Industries. The writer is grateful for their permission to use the information required for this report.

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TABLE 1

EXPERIMENTAL MODEL DATA

Test No.	Depth m	Discharge m^3/s	Velocity m/s	Slope %
M1	0.15	0.155	1.03	0.25
M2	0.15	0.245	1.63	0.50
M3	0.15	0.279	1.86	0.75
M4	0.15	0.288	1.92	1.00
M5	0.15	0.344	2.29	1.25
M6	0.15	0.383	2.56	1.50
M7	0.10	0.085	0.85	0.25
M8	0.10	0.147	1.47	0.50

TABLE 2

EXPERIMENTAL PROTOTYPE DATA

Test No.	Depth m	Discharge m^3/s	Velocity m/s	Slope %
P1	0.30	0.261	0.87	0.20
P2	0.30	0.222	0.74	0.15
P3	0.30	0.278	0.93	0.25
P4	0.30	0.205	0.68	0.125
P5	0.30	0.243	0.81	0.175
P6	0.30	0.285	0.95	0.225
P7	0.30	0.209	0.70	0.138
P8	0.30	0.237	0.79	0.163
P9	0.30	0.271	0.90	0.188
P10	0.30	0.272	0.91	0.190
P11	0.30	0.213	0.71	0.125
P12	0.30	0.234	0.78	0.150

TABLE 3

VALUES OF MANNING'S n

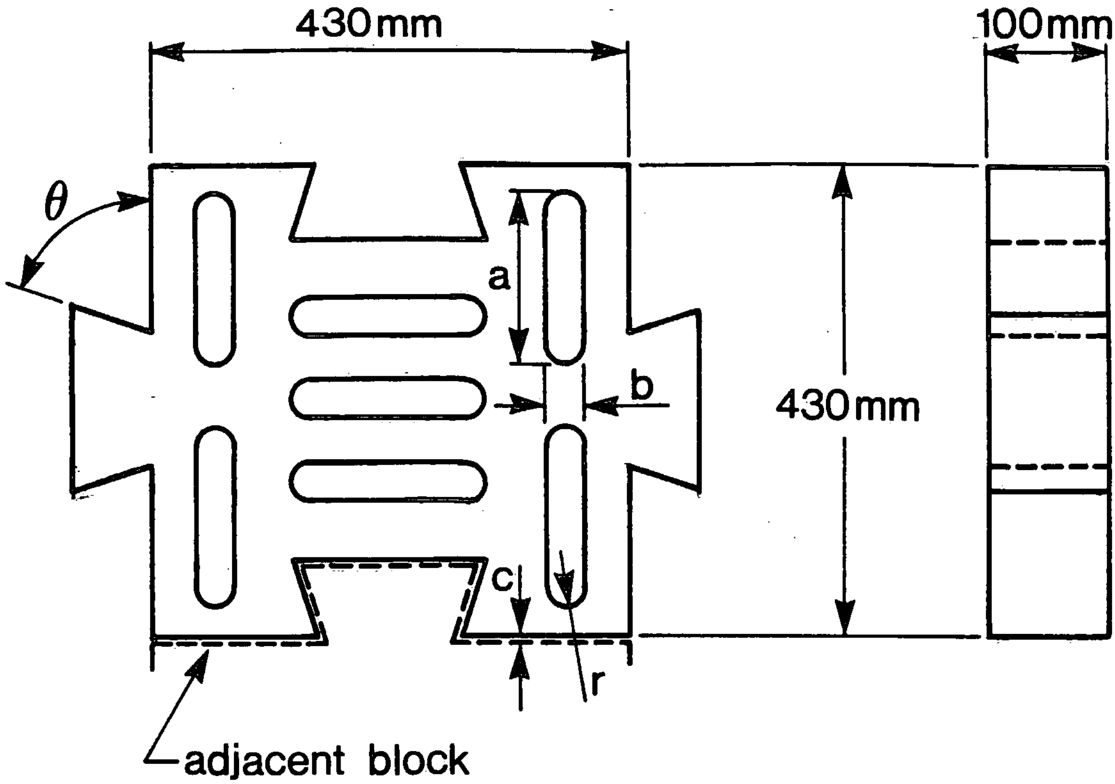
Test No.	n_{bm}	n_{bs}	n_{bP}
M1	0.0119	0.0175	
M2	0.0103	0.0151	
M3	0.0113	0.0166	
M4	0.0129	0.0189	
M5	0.0120	0.0176	
M6	0.0118	0.0173	
M7	0.0115	0.0169	
M8	0.0090	0.0132	
P1			0.0199
P2			0.0203
P3			0.0210
P4			0.0201
P5			0.0199
P6			0.0192
P7			0.0207
P8			0.0197
P9			0.0183
P10			0.0184
P11			0.0192
P12			0.0190

n_{bm} = Manning's "n" for the model tests
n_{bs} = Scaled up values of
n_{bP} = Manning's "n" for the prototype tests
M denotes model tests
P denotes prototype tests

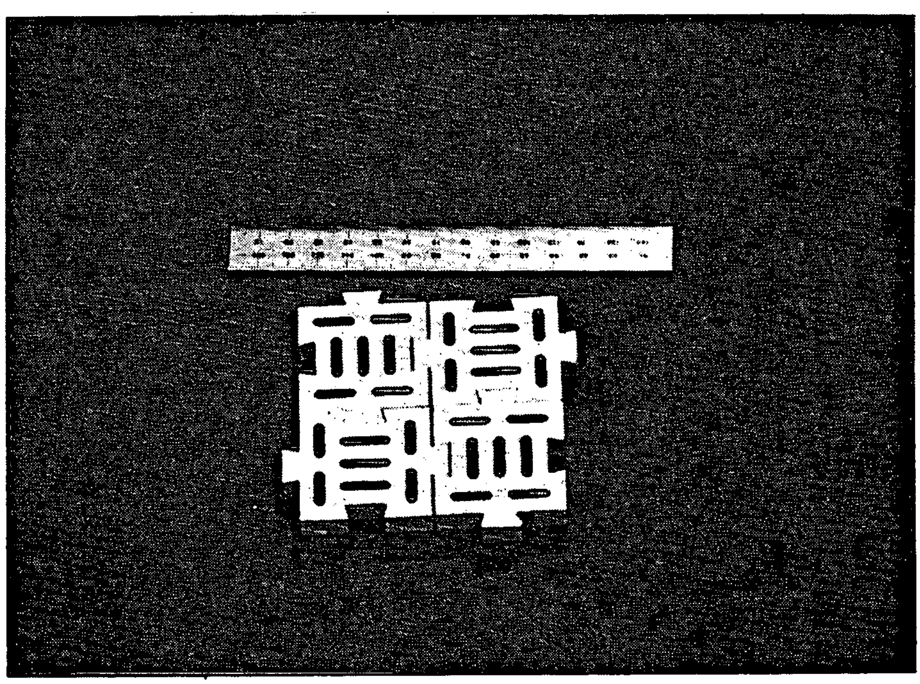
TABLE 4

MEAN VALUES OF MANNING'S n

Type	Mean	Std. Deviation	C %
Scaled Up from Model	0.0167	0.00175	10.5
Prototype	0.0196	0.00085	4.3



a) Typical block dimensions



b) Interlocking configuration of blocks

Figure 1 Properties of concrete blocks.

Figure 2 Matrix of prototype blocks in flume.

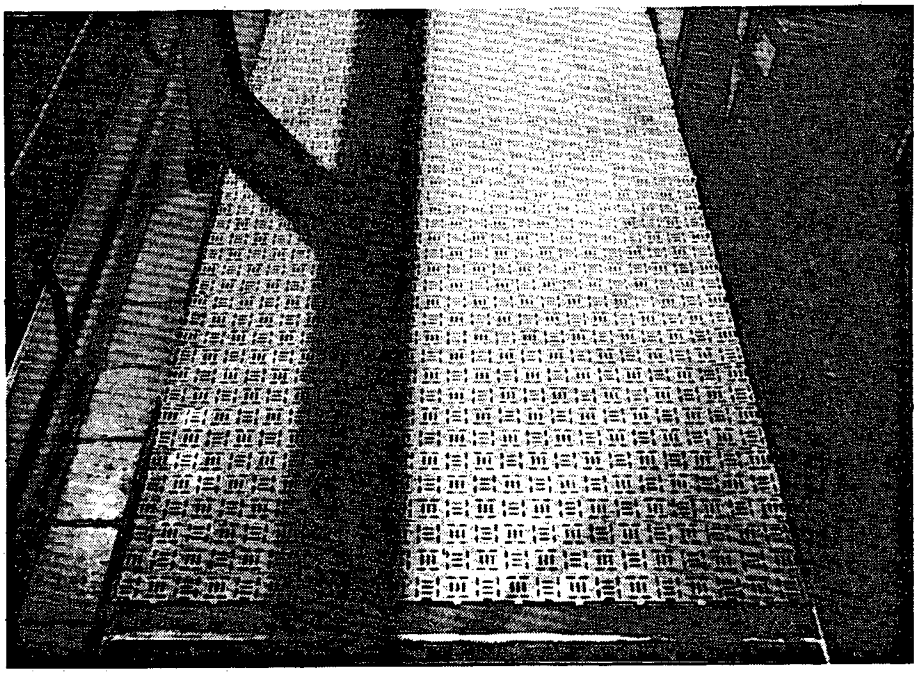


Figure 3 Matrix of model blocks in flume.

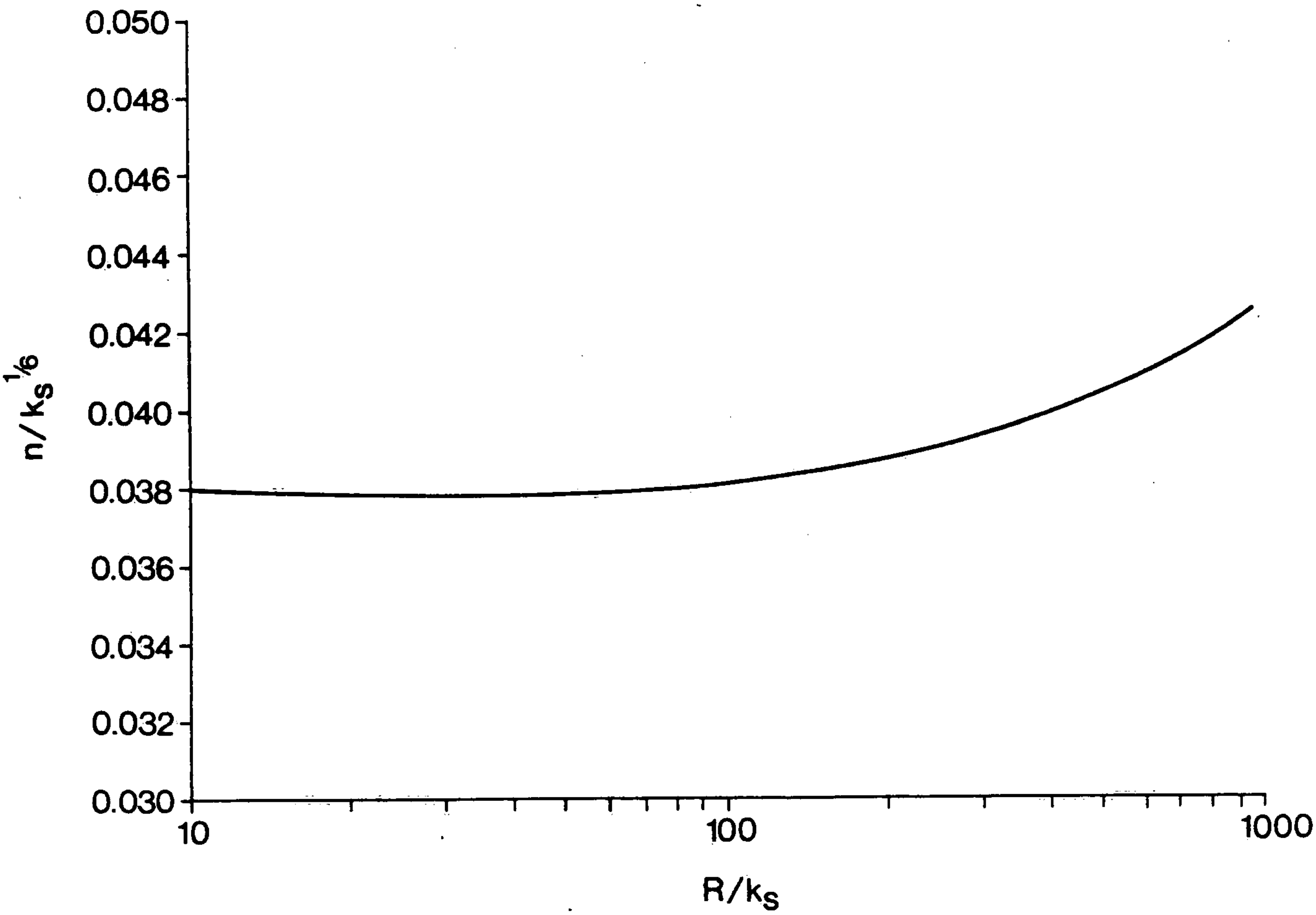


Figure 4 Effect of flow depth on Manning's "n"