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PHYSICAL AND MATHEMATICAL MODEL COMPARISONS FOR WINDERMERE BASIN

by

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MANAGEMENT PERSPECTIVE

This paper shows that increased confidence in hydraulic modelling can be obtained in some cases by a combination of physical and mathematical modelling. Such practices are particularly productive where space limitations dictate the use of distorted physical models. In such cases, the physical model is built large enough to ensure that the effects of Reynolds number are sufficiently small and the mathematical model is calibrated with the physical model. The mathematical model can then be used to predict the behaviour of the prototype. This approach was shown to be feasible for a simple prototype configurations.

PERSPECTIVE - GESTION

Ce rapport montre qu'il est possible d'obtenir, dans certains cas, une plus grande confiance dans la modélisation hydraulique par une combinaison des modèles physiques et mathématiques. Ces méthodes sont particulièrement efficaces lorsque les limites d'espace imposent l'emploi de modèles physiques imparfaits. Dans de tels cas, le modèle physique est construit avec des dimensions assez grandes pour assurer que les effets du nombre de Reynolds sont suffisamment faibles et le modèle mathématique est étalonné avec le modèle physique. Le modèle mathématique peut alors servir à prévoir le comportement du prototype. Cette méthode s'est révélée applicable aux configurations d'un prototype simple.

SUMMARY

A two dimensional depth-averaged model is used to simulate the hydraulic circulation in a distorted model of Windermere basin. The results from this model are compared with the experimental results. In the distorted model, the bed friction influence is underestimated relative to the convective influence resulting in overprediction of the recirculation zones depending on the degree of distortion.

RÉSUMÉ

Un modèle bi-dimensionnel à profondeur moyennée est utilisé pour simuler la circulation hydraulique dans un modèle imparfait du bassin de Windermere. Les résultats de ce modèle sont comparés aux résultats expérimentaux. Dans le modèle imparfait, l'influence du frottement du lit est sous-estimée par rapport à l'influence convective, ce qui donne lieu à une surestimation des grandeurs des zones de recirculation selon le degré de déformation.

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Abstract

A two-dimensional depth-averaged model is used to simulate the hydraulic circulation in a distorted model of Windermere basin. The results from this model are compared with the experimental results. In the distorted model the bed friction influence is underestimated with respect to the convective influence resulting in overprediction of the size of the recirculation regions.

Introduction

The Windermere basin lies at the south east corner of Hamilton Harbour, Ontario, Canada. The inflow consists primarily of runoff from the Red Hill Creek watershed and outflow from the Hamilton sewage plant. Mean daily flows are about 3.5 m^3/s and the 100 year average storm flow is about 64 m^3/s. The flows from the basin pass into Hamilton Harbour through a constriction spanned by a small railway bridge. A cleanup of the Windermere Basin is to be undertaken, accompanied by dredging a portion of the basin. The dredged material, containing toxic substances, will be stored in a confined area behind impermeable berms, within the perimeter of the present basin, effectively creating a new but smaller basin. A physical model study was conducted to study the flow patterns of the proposed basin design, and a mathematical model was used for verification.

Physical Basin Model

The physical model was designed as a fixed bed model. Available floor space and accuracy of measuring water depths, dictated a horizontal scale ratio of 1:60 and a vertical scale ratio of 1:15 resulting in a model with a 4:1 distortion. As a result the model was restricted to simulating prototype flows greater than about 17 m^3/s in order to ensure that viscous scale effects were minimized.

The model was constructed inside a water tight enclosure consisting of concrete blocks built on the laboratory floor. Standard procedures, using plywood templates, sand and mortar, were used to construct the model bed.

The surface was then spray-painted with a light blue latex paint to provide a good background for overhead photographs and video tape recordings. A 1m x 1m black grid was painted on the model bed to facilitate the determination of flow velocities and rates of change of flow patterns. The berms were built of wood framing covered with sheet metal which was then covered with coarse sand or gravel to simulate the design rip rap. The railway bridge at the basin outlet was built from plywood, with particular attention being paid to the proportioning of the piers and abutments to ensure proper development of local flow patterns.

The depth in the model was 175 mm with a central trap of 350 mm. According to Froudian similitude, the flow rate was 11.4 l/s representing a prototype flow of 40 m^3/s. The discharge was measured to an accuracy of about 2% with a 90^o V notch weir in a headbox upstream of the model basin. All water levels were measured using stilling wells fitted with Mitutoyo point gauges, having a resolution of 0.05 mm. The gauges were set to a common reference level, equivalent to the prototype project datum of 74.0 m chart datum. Flow patterns were visualized by using Potassium Permangenate. In addition to the dye, 22 weighted ping-pong balls were released at the entrance of the basin at various times as an additional aid to visualize flow paths and to calculate the surface velocities in the model basin. The model is given in Figure 1, where the vectors drawn indicate the magnitude in cm/s and direction of the velocities in different locations in the basin.

Mathematical Basin Model

A two-dimensional horizontal (large width to depth ratio) flow model is used to simulate the hydraulic circulation in the Windermere Basin Model. The equations of motion in the x and y directions and mass continuity are simplified under the assumptions (a) the water is incompressible and homogeneous, (b) Coriolis forces are negligible for the size of the Model Basin (c) the flow is quasi-hydrostatic. The resulting equations are given as

$$\frac{\partial U}{\partial t} + U\frac{\partial U}{\partial x} + V\frac{\partial U}{\partial y} = - g\frac{\partial \zeta}{\partial x} - \frac{C_b}{H} U\sqrt{U^2+V^2} \quad (1)$$

$$\frac{\partial V}{\partial t} + U\frac{\partial V}{\partial x} + V\frac{\partial V}{\partial y} = - g\frac{\partial \zeta}{\partial y} - \frac{C_b}{H} V\sqrt{U^2+V^2} \quad (2)$$

$$\frac{\partial}{\partial x}(UH) + \frac{\partial}{\partial y}(VH) + \frac{\partial \zeta}{\partial t} = q \quad (3)$$

where U and V are the depth-averaged velocities, g is the acceleration due to gravity, ζ is the free surface elevation relative to the still water level h, H is the total water depth, q(x,y,t) ($[q]=L^3/L^2/T$) is the specific discharge of a source or a sink and C_b is a dimensionless bed friction coefficient defined as

$$C_b = \frac{n^2 g}{H^{1/3}} \quad (4)$$

where n is the Manning's coefficient. A smoothing factor t_h in the form

$$t_h = 1 - 4\frac{\nu_h \Delta t}{(\Delta x)^2} \tag{5}$$

is used in the calculation of the time derivative of the velocities U and V in a horizontal rectangular grid with mesh size Δx (Krestenitis, 1988) where Δt is the time step of the calculation and ν_h is the numerical horizontal eddy viscosity. The system of equations (1) to (3) are solved numerically using an explicit finite difference scheme (Koutitas, 1988).

Using an inflow $Q = 0.0114\ m^3/s$ and a Manning's coefficient n=0.011 in the distorted model of Windermere basin, equations (1) to (5) are solved for a space step of Δx=0.20 m, a time step of Δt=0.04 sec and a value of smoothing factor t_h=0.995 (ν_h=0.00125 m^2/s). The resulting hydraulic circulation of the above simulation is shown in Fig. 2. The magnitude and the direction of the velocity vectors and the size of the recirculation zones are similar to those determined experimentally.

Discussion

Froudian models have been used for some years to study the dispersion of waste by Lagrangian techniques. Laboratory space limitations dictate the use of vertically distorted physical models that alter the characteristics of turbulent mixing and advection of pollutants. This results in model behavior which varies from the prototype behavior depending on the distortion used. For example failure to maintain the jet characteristics, i.e., Reynolds number ($R_j=U_jh/\nu$, U_j is the jet velocity), while retaining the same jet Froude number ($Fr_j=U_j/\sqrt{g\,h}$) leads to a different hydrodynamic pattern in the basin. The resulting velocity pattern in the basin based on the 1:1 undistorted model given in Fig. 3 is different than the one of Fig. 2. The advection of the distorted model is larger, resulting in smaller spreading of the inflow jet, which is consistent with the results of Roberts and Street, 1982. Table 1 shows the different parameters for the three numerical model applications and their corresponding Peclet numbers ($Pe_j=U_j\Delta x/\nu_h$). By equating the horizontal turbulent Peclet number in the 1:1 model and the prototype the mathematical model was able to predict the circulation in the prototype basin. Comparison between the velocity patterns in the prototype basin (shown in Fig. 4) and the 1:1 model reveal good agreement. The validity of this assumption will be tested by field studies in the prototype after construction of the Windermere basin is complete.

Conclusions

A distorted physical model was used to study the circulation and flow patterns of the Windermere basin and compared with results from a two-dimensional horizontal flow model. The mathematical model was successfully used to predict the hydrodynamic pattern of a distorted model of the Windermere basin. The distorted model overpredicts the size of the recirculation regions by reducing the spreading of the inflow jet due to imbalance of bed friction influence versus convective influence.

References

Koutitas, C. 1988. Mathematical Models in Coastal Engineering, Pentech Press, London.

Krestenitis, Y.N., 1988. Numerical Study of the Wind-induced circulation and examination of the open-sea boundary conditions. Case study of Thermaikos Gulf. Tech. Chron. - A, Greece, Vol. 8, No. 4.

B.R. Roberts and R.L. Street. 1982. Mixing in Surface Jets Under Distorted Conditions, Proceedings of the Conference Applying Research to Hydraulic Practice, ASCE, pp. 206-215.

TABLE 1

	4:1	**1:1**	**Prototype**
Inflow Q (m^3/s)	0.011400	0.001425	39.7368
Inlet Depth h (m)	0.1750	0.04375	2.6250
Inlet Velocity U_j (m/s)	0.0814	0.0407	0.3154
Manning's n	0.0115	0.0045	0.0090
Length Scale L (m)	10	10	600
Time Scale $\tau = L/U_j$ (sec)	123	246	1902
Space Step Δx (m)	0.20	0.20	12.00
Time Step Δt (sec)	0.04	0.04	0.3097
Smoothing Factor t_h	0.995	0.995	0.995
Eddy Viscosity ν_h (m^2/s)	0.00125	0.00125	0.5812
Inlet Froude Number Fr_j	6.21×10^{-2}	6.21×10^{-2}	6.21×10^{-2}
Inlet Reynolds Number R_j	1.415×10^4	1.768×10^3	8.222×10^5
Inlet Peclet Number Pe_j	13.024	6.512	6.512

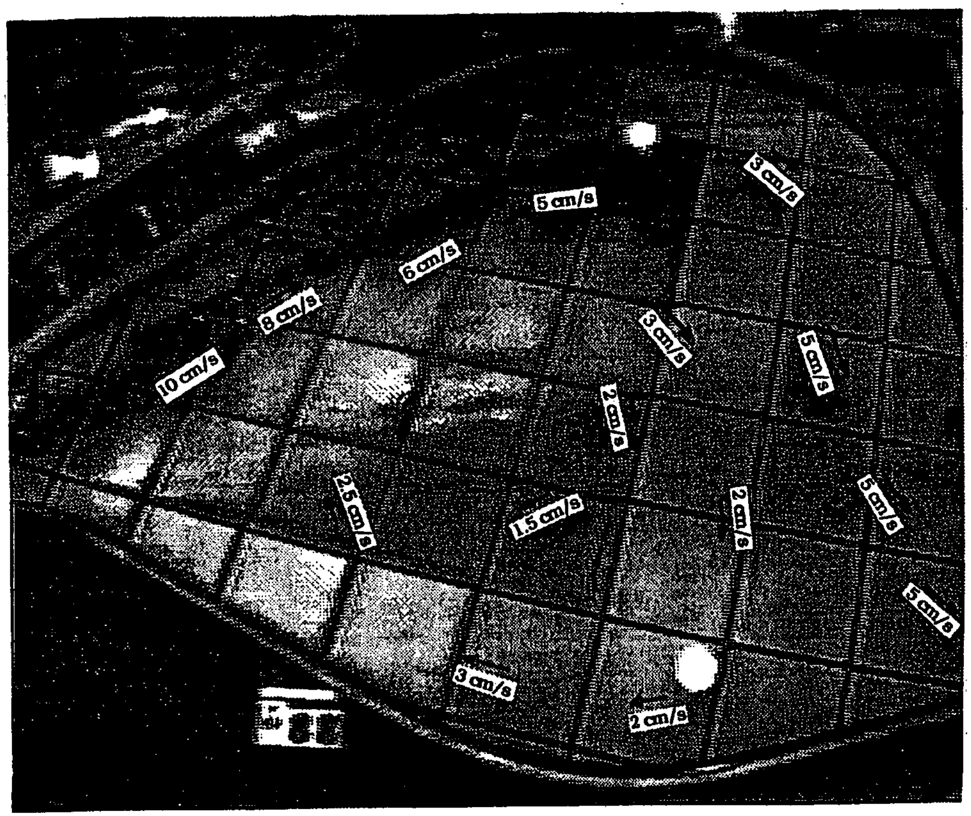
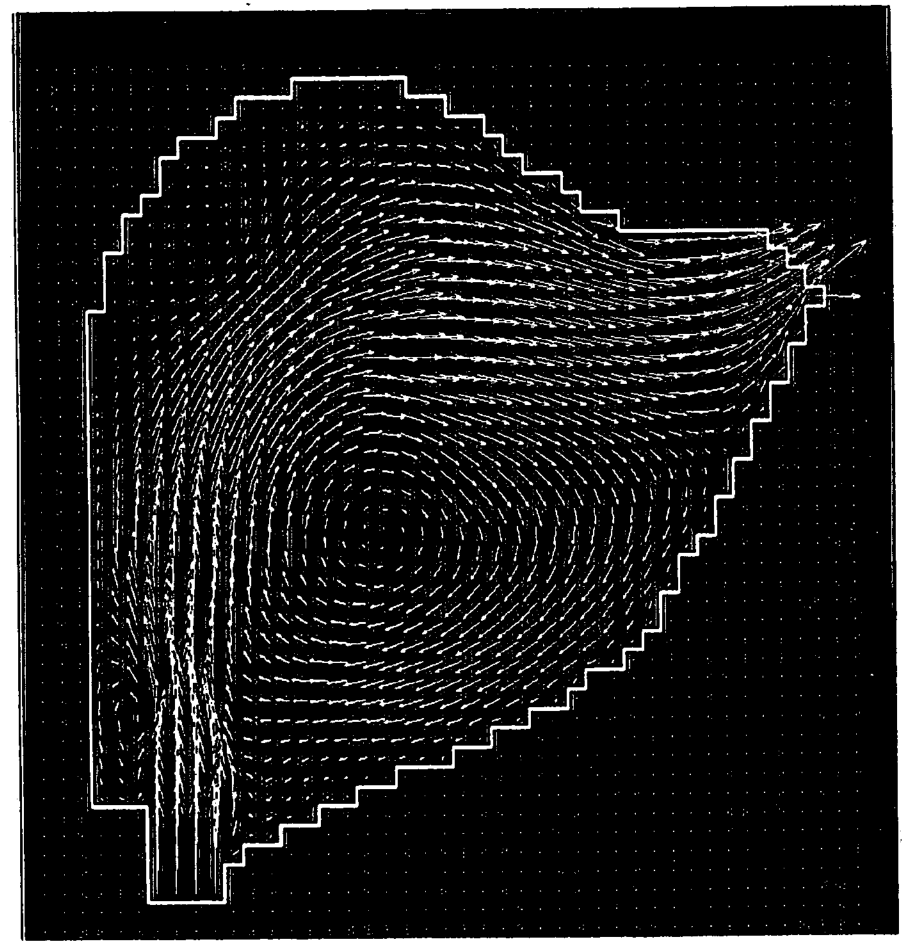
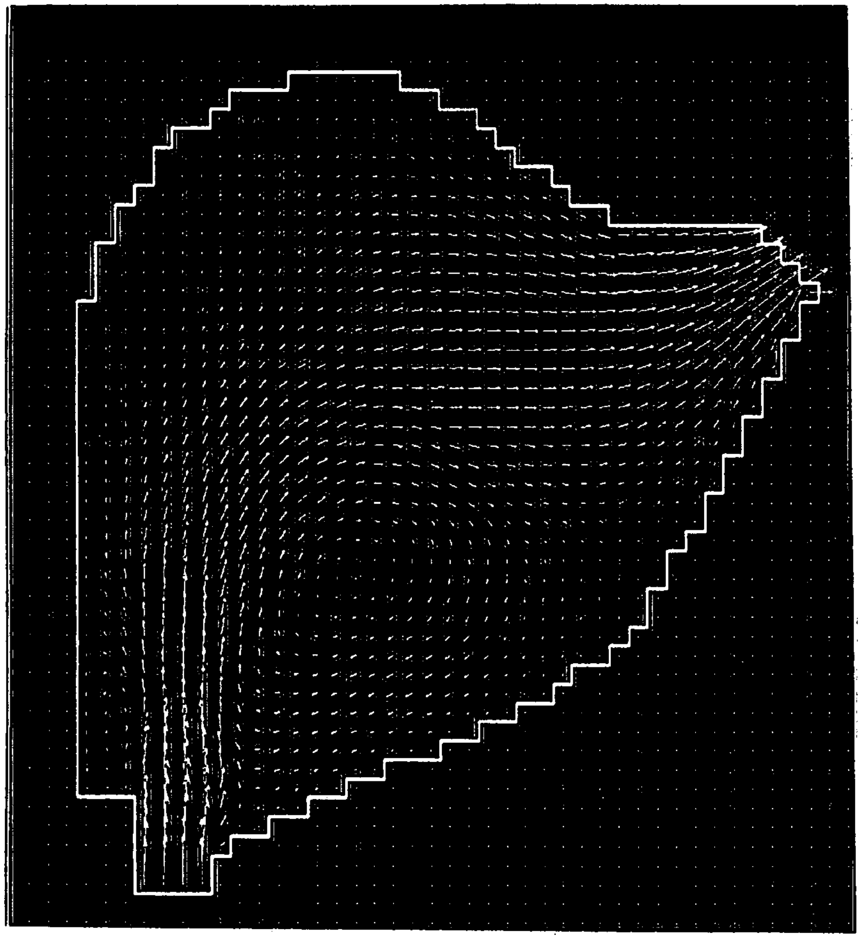


Figure 1



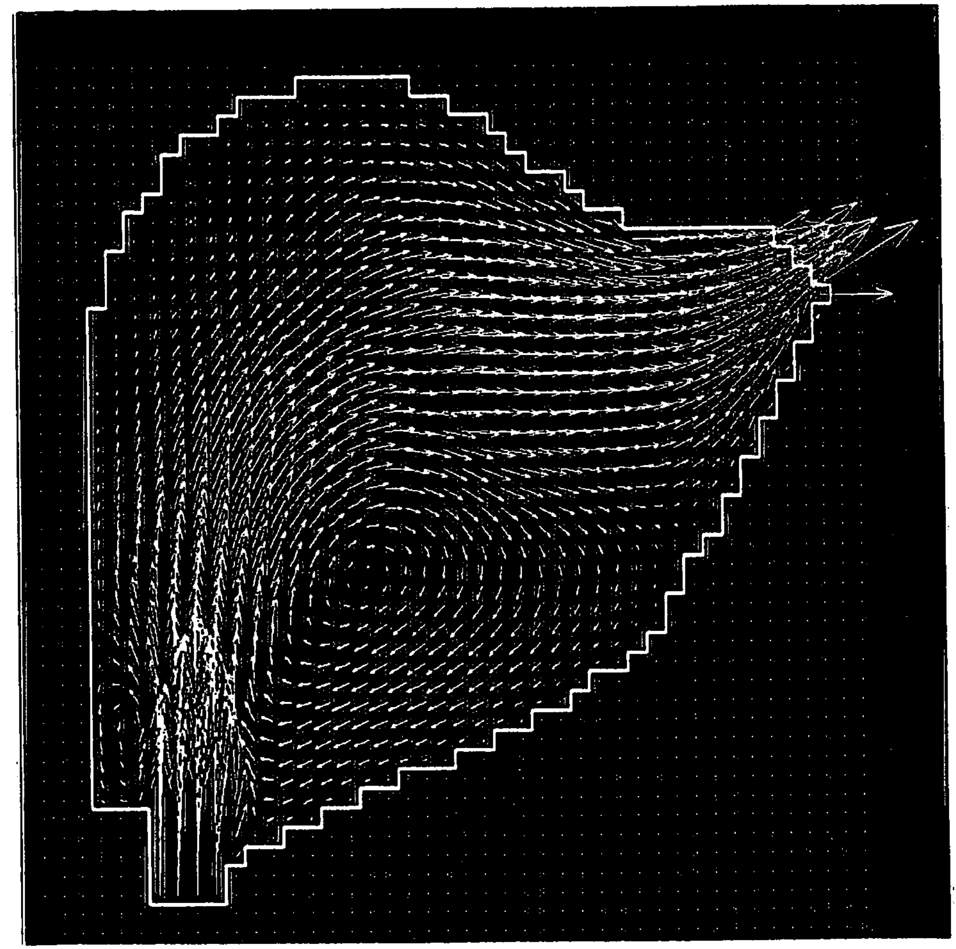
Vector equal to one grid distance represents a velocity of 1.6 cm/s

Figure 2



Vector equal to one grid distance represents a velocity of 1.6 cm/s

Figure 3



Vector equal to one grid distance represents a velocity of 4.5 cm/s

Figure 4