

NWRI Cont. 91-20
C1

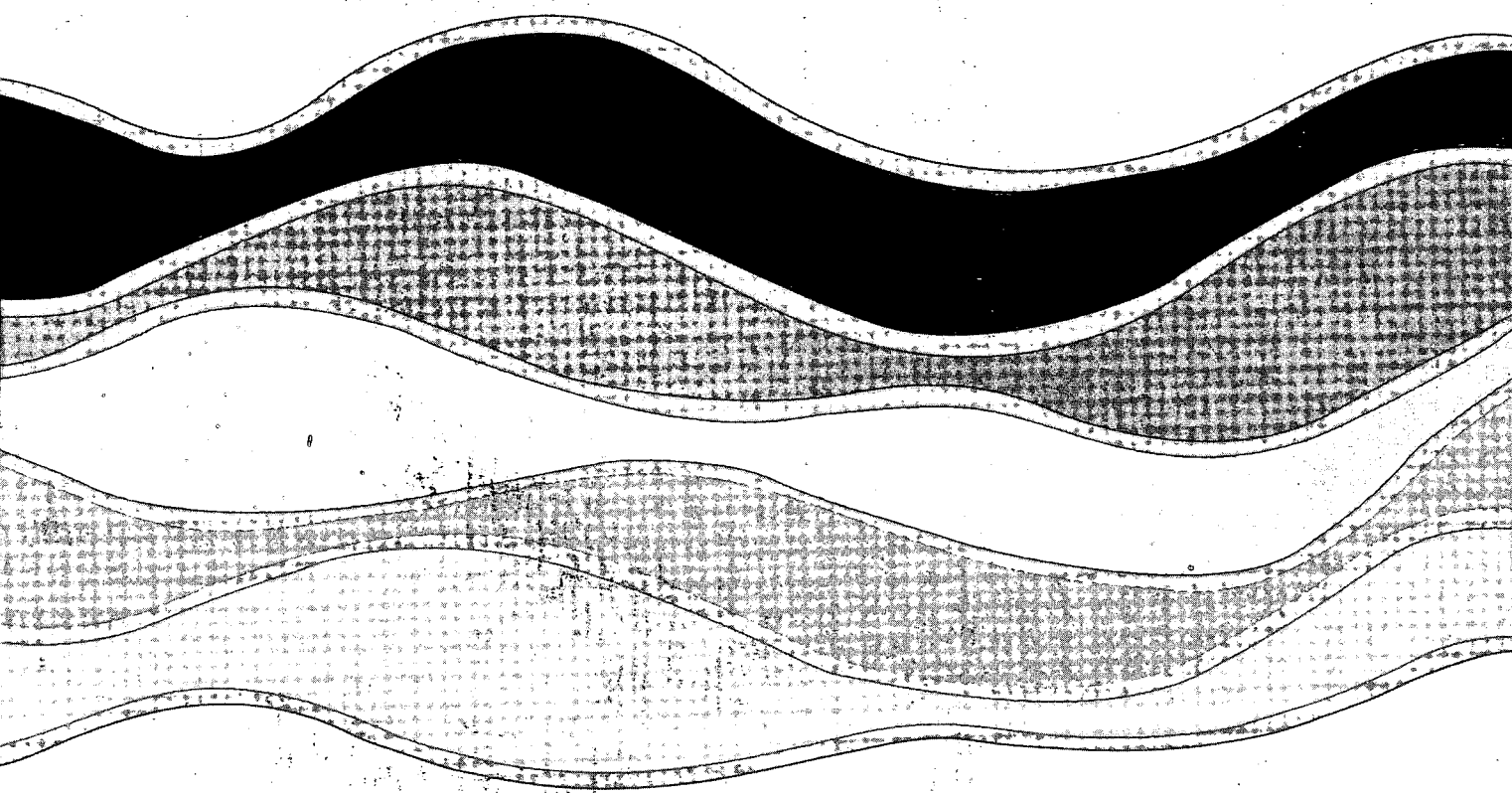
CCIW

MAR 31 1992

LIBRARY

NATIONAL
WATER
RESEARCH
INSTITUTE

INSTITUT
NATIONAL
de RECHERCHE
sur les
EAUX



**A SIMPLE MODEL OF THE
FRASER RIVER SALT WEDGE**

P. F. Hamblin

NWRI Contribution No. 91-20

TD
226
N87
No. 91-
20
c. 1

**A SIMPLE MODEL OF THE
FRASER RIVER SALT WEDGE**

P.F. Hamblin

**Lakes Research Branch
National Water Research Institute
867 Lakeshore Road, P.O. Box 5050
Burlington, Ontario L7R 4A6**

NWRI Contribution No. 91-20

MANAGEMENT PERSPECTIVE

In response to the need for better understanding of the transport of contaminants and their relation to salt water in the Fraser River Estuary, a modelling program of the salt wedge was undertaken. It is shown that while the model hindcasts the salt intrusions reasonably well over a wide variety of river discharge and tidal conditions, further improvement of the model simulations requires more accurate input of river flow. It is recommended that the present model be incorporated into a two-dimensional tidal model of the Fraser River Estuary.

PERSPECTIVES DE LA DIRECTION

Afin de mieux comprendre le transport des contaminants et leur rapport avec l'eau salée dans l'estuaire du Fraser, un programme de modélisation du coin salé a été entrepris. Le modèle permet une prévision a posteriori raisonnablement bonne des intrusions salées pour divers débits de cours d'eau et de conditions tidales; toutefois, pour améliorer les simulations du modèle, il faut des données plus précises sur l'écoulement du cours d'eau. Il est recommandé d'incorporer le modèle actuel dans un modèle tidal bidimensionnel de l'estuaire du Fraser.

ABSTRACT

A simple mathematical model of the Fraser River salinity intrusion is presented which is designed to be suitable for microcomputer applications. One of the novel features of the model is that it allows for landward migration of the boundary between fresh and sea water on the flood and thus is relevant to low river discharge conditions ($Q < 3000 \text{ m}^3/\text{s}$) as well as higher flows. The model is applied to the main arm of the Fraser River Estuary by assuming a trapezoidal cross-section with an average breadth of 350 m, surface width three times the bottom width and a bottom drag coefficient of 1.0×10^{-3} and to the north arm by assuming a rectangular cross-section. Fifteen sets of observations of the intrusion of the salt wedge and its maximum excursion in the main arm reported by Ages (1979) and Geyer (1985) are compared to modelled results based on measured river discharges at Hope and semi-diurnal tides at Pt. Atkinson over river flow conditions ranging from freshet to low flow. Although the agreement is reasonable it could likely be improved by including forcing from diurnal tidal constituents.

RÉSUMÉ

Les auteurs présentent un modèle mathématique simple d'une avancée d'eau salée conçu pour des applications sur micro-ordinateur. Une des nouvelles caractéristiques du modèle est qu'elle permet le déplacement vers la terre de la limite entre l'eau douce et l'eau salée en période de crue et il est donc bien adapté à des conditions de cours d'eau à faible débit ($Q < 3000 \text{ m}^3/\text{s}$) et à des débits plus élevés. Le modèle est appliqué au bras principal de l'estuaire du Fraser en supposant une coupe trapézoïdale avec une largeur moyenne de 350 m, une largeur de la surface égale à trois fois la largeur du fond et un coefficient de traînée du fond de $1,0 \times 10^{-3}$, et au bras nord en supposant une coupe rectangulaire. Quinze ensembles d'observations de l'intrusion du coin salé et de son excursion maximale dans le bras principal signalés par Ages (1979) et Geyer (1985) sont comparés aux résultats obtenus par modélisation et fondés sur le débit du cours d'eau à Hope et de marées semi-diurnes à Pointe Atkinson pour des conditions de débit de cru à débit lent. Même si l'accord est suffisant, il pourrait vraisemblablement être amélioré en incluant le forçage d'éléments de marée diurne.

INTRODUCTION

Along with the Great Lakes and St. Lawrence River, the Government of Canada has identified the Fraser River as a priority for clean-up action. Besides numerous observational studies of the dynamics of the Fraser River Estuary (Geyer and Farmer, 1989), a number of mathematical modelling studies have been undertaken. A one-dimensional tidal model has been developed and applied to the Fraser River Estuary by Ages and Woollard (1976). A further application of this model to contaminant transport has been reported by Lam et al. (1988) and by Ages and Woollard (1988). It has been pointed out by those studying contaminants in the estuary that flow simulations could be improved by taking into account the effects of the intrusion of salt into the Fraser River Estuary. Moreover, certain chemical reactions are influenced by the ionic strength of water so that knowledge of the extent of saline water is required. Furthermore, Kostaschuk et al. (1989) have pointed out the connection between the salt wedge and suspended sediment transport, a well-known factor in contaminant transport. For these reasons, the development of a mathematical model of the salt wedge was undertaken. The modelling strategy adopted was to develop a routine that could be readily combined with a two-dimensional finite element tidal model (Hamblin et al. 1990) which is presently being applied to the Fraser River, and is sufficiently simple so that it can be run routinely on microcomputers. As well, a realistic representation of the controlling processes provides the potential for wider application in contrast to the purely empirical approach of Kostaschuk and Atwood (1990).

A finite difference and time variable model of the Fraser River salt wedge has been described by Hodgins et al. (1977). Their approach required flow and salinity specifications at the seaward boundary and thus, is somewhat limited in its applicability. Geyer (1985) has formulated a finite difference time-

stepping model which assumed critical flow conditions at the mouth of the estuary.

Although obviating the need for flow and salinity specifications, that approach did not apply to low river flow conditions when sea water enters the estuary over the entire water column. Modelling of the Fraser River plume in the Strait of Georgia has been studied by Royer and Emery (1985) and by Stronach et al. (1988).

A novel feature of the present model is that it allows for landward migration of the incoming seawater and thus should be relevant to low flow conditions as well. Geyer (1985) showed that depth irregularities in the main arm cause only minor changes to the speed of propagation of the tidal wave. This suggests that constant depth can be reasonably assumed for the salt wedge problem in the Fraser River, thus permitting analytical methods to be used rather than numerical.

In the data reports on saline intrusion on the Fraser River to be employed subsequently in model validation, tidal elevations near the mouth of the estuary and river discharges are given. Ward (1976) has demonstrated that river flow is required to estimate the salt wedge excursion. To provide river flows at various stages of the tide, it is first necessary to review the relevant aspects of tidal propagation in estuaries. Next, the theory of internal gravity wave propagation is developed and, finally, applications of the theory to a number of test cases are presented and discussed.

SALT WEDGE DYNAMICS

(1) Damped Tidal Wave Propagation

Ippen (1966) considered a tidal long wave of frequency, ω , amplitude, a_0 and wave number, k , entering an estuary. The elevation of the tidal wave, η , is given at the distance, x , landward of the estuarine entrance by

$$\eta = a_0 e^{-\mu x} \cos (\omega t - kx)$$

and the associated tidal flow, u_T , by

$$u_T = a_o \frac{\omega}{h} e^{-\mu x} \frac{\cos(\omega t - kx + \theta)}{\sqrt{\mu^2 + k^2}} \quad . \quad (1)$$

The phase lag, θ , of the current behind the tidal elevation is related to the bottom drag coefficient, C_D , the depth, h , and the root mean square current, U_{RMS} , by $\tan 2\theta = C_D U_{RMS}/(\omega h)$. According to Geyer (1985) the observed phase lag from harmonic analyses of Fraser Estuary data is 27° which implies a bottom drag coefficient of 1.6×10^{-3} . Although the wave number is not known for the purposes of this application it is based on an estimated wave length of 150 km. Since the damping factor, μ , is given by $\mu = k \tan \theta$ (Ippen 1966) the e-folding distance of the wave decay is 300 km. Thus, over the intrusion length of 30 km in the Fraser River, the decay of current may be safely neglected. Equation (1) has been used to estimate the tidal flow which, in turn, has been added to the flow associated with the river discharge to compute the background flow that interacts with the salt wedge intrusion.

(2) Internal Gravity Wave Propagation

The classical internal hydraulics theory of Benjamin (1968) is based on the four conservation statements. With reference to Figure 1 for an intrusion of density $\rho + \Delta \rho$ at rest in a steady flow of u and layer areas, A_1 and A_2 , the continuity equation is

$$u(A_1 + A_2) = u_1 A_1 \quad .$$

Geyer (1985) has shown that to a good approximation for a trapezoidal river channel the momentum force equation may be written as

$$u^2(A_1 + A_2) + (A_1 + A_2) \frac{P}{\rho} = u_1^2 A_1 + \Delta \frac{P}{\rho} g A_2 h_2 \quad .$$

In the momentum equation the usual assumption of hydraulic theory is made, that the flow is rapidly varied so that friction may be neglected. It may be noted that this equation is exact for the case of a rectangular channel studied by Benjamin (1968). The conservation of energy along the stagnation streamline dividing the intrusion from the river flow formulated by Benjamin has been modified by Geyer (1985) to include the head loss due to bottom friction over the length of the intrusion, L , according to:

$$1/2 u^2 + \frac{P}{\rho} = g \frac{\Delta \rho}{\rho} h_2 + C_D |u + u_T + u_R| (u + u_T + u_R) L / h_2 \quad .$$

Benjamin (1968) also employed conservation of energy along the surface streamline,

$$\frac{P}{\rho} + \frac{u^2}{2} = \frac{u_1^2}{2} \quad .$$

The appropriateness of the assumption of steady flow for the intrusion problem is examined by Geyer (1985) who found that time dependent terms could be reasonably neglected.

The above four algebraic equations may be solved directly by standard methods. In the case of no friction ($C_D = 0$) which occurs in an arrested wedge, it is easily shown that,

$$A_1 = A_2$$

$$u^2 = \frac{\Delta \rho}{\rho} g \frac{h_2}{2}$$

which is identical to Benjamin's result for a rectangular channel, $h_1 = h_2$. However, in the case of a trapezoidal channel of a top width of three times the bottom breadth, the lower layer thickness is

$$(\sqrt{5} - 1)/2$$

or 62% of the total depth. Thus, the sloping sides of the river channel have the effect of increasing the propagation speed of the gravity wave beyond the vertically walled case.

When the wedge is either advancing or retreating and bottom friction is not negligible, a closed solution may be found in the case of propagation into still water ($u_T = u_R = 0$)

$$\frac{A_2}{A_1} = \frac{1 + \sqrt{1 + 16 C_D L / h_2}}{2} \quad \text{and}$$

$$u^2 = \frac{\Delta \rho}{\rho} g h_2 \frac{A_1}{A_1 + A_2} \left[\frac{2A_1 + A_2}{2A_2 + A_1 + 2A_1 C_D L / h_2} \right]$$

Friction has the effect of deepening the lower layer, and at the same time, decreasing the phase speed of the intrusion. Sensitivities of the propagation speed and of the upper layer depth to friction and to intrusion length are shown in Figure 2 for intrusion into still water and in Figure 3 for an adverse flow in which the intrusion is gradually pushed seawards. In this case, the internal gravity wave speed is higher than that of the arrested wave since the bottom stress acts in the opposite direction. As will be seen shortly, this

enhancement of the wave speed has important consequences on the mixing between the layers.

(3) MIXING BETWEEN THE SALT AND FRESH WATER

In a layered model, the effect of mixing between the two layers is usually accounted for by an entrainment coefficient and velocity. For example, Fischer et al. (1979) introduced an entrainment coefficient which is a function of the bottom drag coefficient and bed slope to represent the mixing of an underflowing river inflow in a lake or reservoir. Similarly, Grubert (1990) has suggested an entrainment velocity, W_e for salt wedge estuaries of the form,

$$W_e = 2.4 C_D^{1.5} \frac{(u_1 - u_2)^3}{\frac{\Delta \rho}{\rho} g R}$$

The density difference between layers is $\Delta \rho$, the acceleration of gravity, g , and the hydraulic radius, R is given by:

$$R = \frac{b_o h_2 + b_o h_2^2/h_o}{b_o + 2\sqrt{h_2^2 + b_o^2}} = \frac{2}{3} h_2$$

for a trapezoidal cross-section of bottom width, b_o . u_2 is the bottom layer flow speed but for the purposes of application of these formula ($u_1 - u_2$) is taken as twice the wave speed u (see Figure 1).

The decrease in salinity of the intruding bottom layer, ΔS in the time interval, Δt , is related to the entrainment velocity and layer salinity, S , by

$$\Delta S = \Delta t W_e S bi / A_2 \quad .$$

Substitution for the interfacial width, b , yields

$$\Delta S = \Delta t W_e S \frac{(h_o + 2h_2)}{(h_o h_2 + h_2^2)}$$

where h_o is the total depth.

Another parameter often used to quantify turbulent mixing between stratified layers in an estuarine shear flow is the Richardson Number, Ward (1976),

$$Ri = \frac{\Delta \rho}{\rho} \frac{g R}{(u_1 - u_2)^2} \quad .$$

The position of the salt wedge is found by integration of the characteristic equation, $\frac{dx}{dt} = u$, where the flow velocity, u , is given by the

previous expressions or calculated numerically, and the origin is taken as the mouth of the estuary at the beginning of the tidal cycle. If the flood velocity exceeds the internal wave speed, then the excursion arising from the difference in flow speeds is added to the position of the salt wedge.

As an example to illustrate the method and possible mixing between the riverine and marine layers, Figure 4 shows the intrusion distance over a semi-diurnal tidal cycle in the main arm of the Fraser River obtained by computing the gravity wave speed at 15-min intervals over the tidal cycle and calculating the excursions of the wave. Similarly, the 15-min excursions associated with the entrainment velocity, W_e , are cumulated to show that the bottom layer grows rapidly at the initial and final stages of the intrusion but more slowly at

intermediate times when the shear between the layers is least. Eventually, the lower layer entrains almost all of the upper layer. By the end of the flood, it is likely that the two-layer assumption breaks down. The Richardson number over the intrusion also varies considerably. At the beginning of the ebb tide when the arrested wedge starts to retreat, the shear between the layers is

sufficient that the Richardson number drops below the critical value of 0.25 as hypothesized by Ward (1976). At this point, mixing between the layers would likely homogenize the water column. A similar disappearance of the two-layer structure has been observed by Geyer (1985) and by Carey (J.H. Carey, pers. com.). Also shown on Figure 4 is the salinity of the head of the intrusion which starts from 30 ppt, but is rapidly diluted by the river water. In the computation of the internal wave speed, the decrease of the buoyancy force driving the intrusion is taken into account.

Another factor leading to a decrease of the wave speed is the retarding effect of the interfacial friction between the two layers. For layer depths as thin as those in the Fraser River Estuary, this effect is overwhelmed by bottom drag and is therefore neglected in the model formulation.

APPLICATION

The area of study (Figure 5) covers the main and north arms of the Fraser River. The maximum penetration of the salt wedge is to the Oak St. bridge in the north arm and as far as the western third of Annacis Island in the main arm (Ages and Woollard, 1976).

(1) Main Arm

Figure 6 shows a river cross-section redrawn from Ages (1979) and an idealized symmetrical trapezoid taken to represent the geometry of the main arm. For the purposes of modelling, the bottom width is taken as 350 m, the top width as 1,050 m and the depth as 10 m plus the contribution from, Q , the

river flow, $3(Q - 720)/5,600\text{m}$. The entrance to the main arm was taken for modelling purposes at Sand Heads (Figure 5).

For a detailed comparison of the model hindcasts of the intrusion of the Fraser River salt wedge, data on the river discharge, the tidal range of the semi-diurnal tide at Point Atkinson and the intrusion times and distances were estimated from five plots of Ages (1979) and from three figures of Geyer (1985). The river flows at Hope were multiplied by 0.8 to represent the discharge in the main arm. The bottom drag coefficient was set to 1×10^{-3} for simulations presented in Figures 7, 8 and 9. In the simulations to follow the decrease in salinity due to mixing was ignored. Comparison of observed and modelled salt wedge excursions for high flow conditions in Figure 7 show reasonable agreement over distances ranging to 8 km. There is somewhat poorer agreement between the 10 km excursions associated with the moderate flow conditions of Figure 8. In the eight cases, the discharge is sufficiently high that the sea water is prevented from entering the river over the entire water column.

An additional seven cases where only the maximum excursion is given Geyer (1985) include two events during low flow conditions. Figure 9 is a scatter diagram of the modelled maximum intrusion distances versus the observed distances for all 15 cases. The 45° line in Figure 9 is a reference line and does not represent a best-fit curve. It is evident that the two low flow cases are underestimated despite the fact that the model predicted that sea water entered the estuary over the entire water column. It is possible that in low flow cases, it is necessary to consider prior intrusions since the starting point for the salt intrusion may not be the river mouth as in the higher flow cases.

(2) North Arm

The model was also applied to the north arm by assuming that the entrance to the arm is located at the outer edge of Sturgeon Bank. The channel was taken to be rectangular, of depth ranging between 7.5 and 10.5m and an average breadth of 300m (Ages, pers. com.). The discharge was specified as 15% of the value of Hope. Simulation of the intrusion distances varied between the limits indicated in Figure 5. No detailed data are available

for comparison as in the main arm, but it is likely that the simulations slightly underestimate the maximum excursion. The modelled intrusions of January and March, 1978 do not quite reach the Oak St. Bridge which is considered to be the upstream limit of the salt wedge (Ages and Woollard, 1976).

(3) Diurnal Tidal Effects

Finally, an examination of the sensitivity of the modelled intrusions to the phase of the diurnal tide was undertaken for the August 4, 1976 event. Hodgins et al. (1977) stated that the duration of the flood period is dependent on the phasing of the diurnal tide. Under the assumption that the amplitude of the diurnal tide is 90% of the semi-diurnal (Geyer, 1985) and the theory of damped tidal waves presented earlier, it is apparent in Figure 10 that intrusion distances are highly sensitive to the phasing of the diurnal tide varying from 9.5 to 16 km for a single event. Since the phase of the diurnal tide is not given in the data reports used in this study, this could account for the discrepancies between modelled and observed results. Little progress can be made on refining the salt wedge model unless more accurate tidal and river flows are available than can be inferred from tidal wave theory and the water levels at Port Atkinson.

CONCLUSIONS

A simple modelling approach for the intrusion of sea water in a shallow estuary has been formulated based on the steady, two-layer hydraulic equations of internal gravity wave propagation. Application to 15 observations taken from the literature demonstrated that the model is valid over a wide range of flow conditions on the basis of reasonable values for model coefficients. In particular, the model should be further tested at low flow conditions.

The next stage of development should be the use in the salt wedge model of flow velocities from a Fraser River tidal model, the incorporation of salt wedge influences on the tidal model and their further testing on field data from both the main and north arms. Besides simulations of the intrusion distance, it would be useful to compare modelled flows directly with current meter and profile data.

ACKNOWLEDGEMENTS

The author would like to thank A. Ages, W. R. Geyer and R. A. Kostaschuk for supplying useful reference material.

FIGURE CAPTIONS

- (1) Definition of model variables in a salinity intrusion. Tr is the tidal range.
- (2) Non-dimensional phase speed (solid line) and non-dimensional lower layer thickness (dashed line) in the case of intrusion into still water.
- (3) Non-dimensional phase speed (solid line) and non-dimensional lower layer thickness (dashed line) in the case of intrusion into adverse flow of non-dimensional speed -0.6 .
- (4) Intrusion distance over time for event August 4, 1976 (Ages, 1979). Dashed line is the thickness of the mixed layer (m), the numbers to the left of intrusion curve are the Richardson numbers and the numbers to the right are the salinity ($^0/_{\infty}$) of the head of the bottom layer.
- (5) Fraser River Estuary, north and main arms. High and low flow limits to simulated salt intrusions are shown.
- (6) Fraser River cross-section at 2 km upstream of Sand Heads (Ages, 1979).
- (7) Comparison of modelled (small dots) and observed intrusion distances (larger dots) for high flow conditions; 1976 data from Ages (1979); other cases from Geyer (1985).
- (8) Same as Figure 7 but for moderate flow cases.
- (9) Observed and modelled maximum excursions from data of Ages (1979) and Geyer (1985).
- (10) Sensitivity of the salt intrusion to various phase lags between the semi-diurnal and diurnal tides for the case of August 4, 1976. The water level at the mouth of the estuary is given for each case.

REFERENCES

- Ages, A. 1979. The salinity intrusion in the Fraser River: salinity, temperature and current observations, 1976, 1977. Pacific Marine Sciences Report, 79 - 14, Institute of Ocean Sciences, Sidney, B.C.
- Ages, A. and A. Woollard. 1988. Tracking a pollutant in the Lower Fraser River: a computer simulation. Wat. Pol. Res., J. Can., Vol. 23: pp 122 - 140.
- Ages, A. and A. Woollard. 1976. The tides in the Fraser Estuary. Pacific Marine Science Report, 76-5, Institute of Ocean Sciences, Sidney, B.C.
- Benjamin, T.B. 1968. Gravity currents and related phenomena. J. Fluid Mech., 27: 513 - 539.
- Fischer, H.B., List, E.J., Koh, R.C.Y., Imberger, J. and Brooks, N.H. 1979. Mixing in inland and coastal waters. Academic Press.
- Geyer, W.R. and Farmer, D.M. 1989. Tide-induced variation of the dynamics of a salt wedge estuary. J. Phys. Ocng. 19: pp 1060 - 1072.
- Geyer, W.R. 1985. The time-dependent dynamics of a salt wedge. Spec. Rep. 101, College of Ocean and Fishing Sciences, University of Washing, Seattle, Wash.
- Grubert, J.P. 1990. Interfacial mixing in estuaries and fiords. J. Hydr. Engrg. Vol. 116, No. 2: pp 176 - 187.
- Hamblin, P.F., Hunt, A.H. and Spigel, R.H. 1990. Circulation Model and Observations in Lyttelton Harbour. Manuscript submitted for publication.

- Hodgins, D.O., Osborn, T.R. and Quick, M.C. 1977. Numerical model of stratified estuary flows. J. Waterway Port Coastal and Ocean Div., A.S.C.E. 103, WWI: 25 - 42.
- Ippen, A.J. 1966. Estuary and coastline hydrodynamics, McGraw-Hill, p. 744.
- Kostaschuk, R.A. and Atwood, L.A. 1990. River discharge and tidal controls on salt wedge position and implications for channel shoaling: Fraser River, British Columbia. Can. J. Civil Eng. 17: pp 452- 459
- Kostaschuk, R.A., Luternauer, J.L. and Church, M.A. 1989. Suspended sediment hysteresis in a salt wedge estuary: Fraser River, Canada. Marine Geology. 87: pp 273 - 285.
- Lam, D.C.L., McCrimmon, R.C. Carey, J. H. and Murthy, C.R. 1988. Modelling the transport and pathways of tetrachlorophenol in the Fraser River. Wat. Pol. Res., J. Can., Vol. 23: pp 141 - 159.
- Royer, L. and Emery, W.J. 1985. Computer simulations of the Fraser River plume. J. Marine Research 43: pp 289-306.
- Stronach, J.A., Crean, P.B. and Murty, T.S. 1988. Mathematical modelling of the Fraser River plume. Wat. Poll. Res., J. Can., Vol. 23: pp 179 -212
- Ward, P.R.B. 1976. Seasonal salinity changes in the Fraser River Estuary. Can.J. Civil Eng., 3: pp 342-348

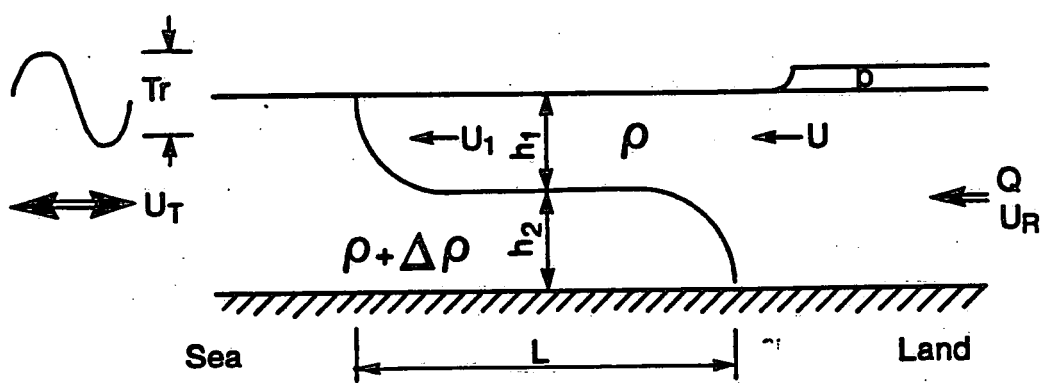


Figure 1

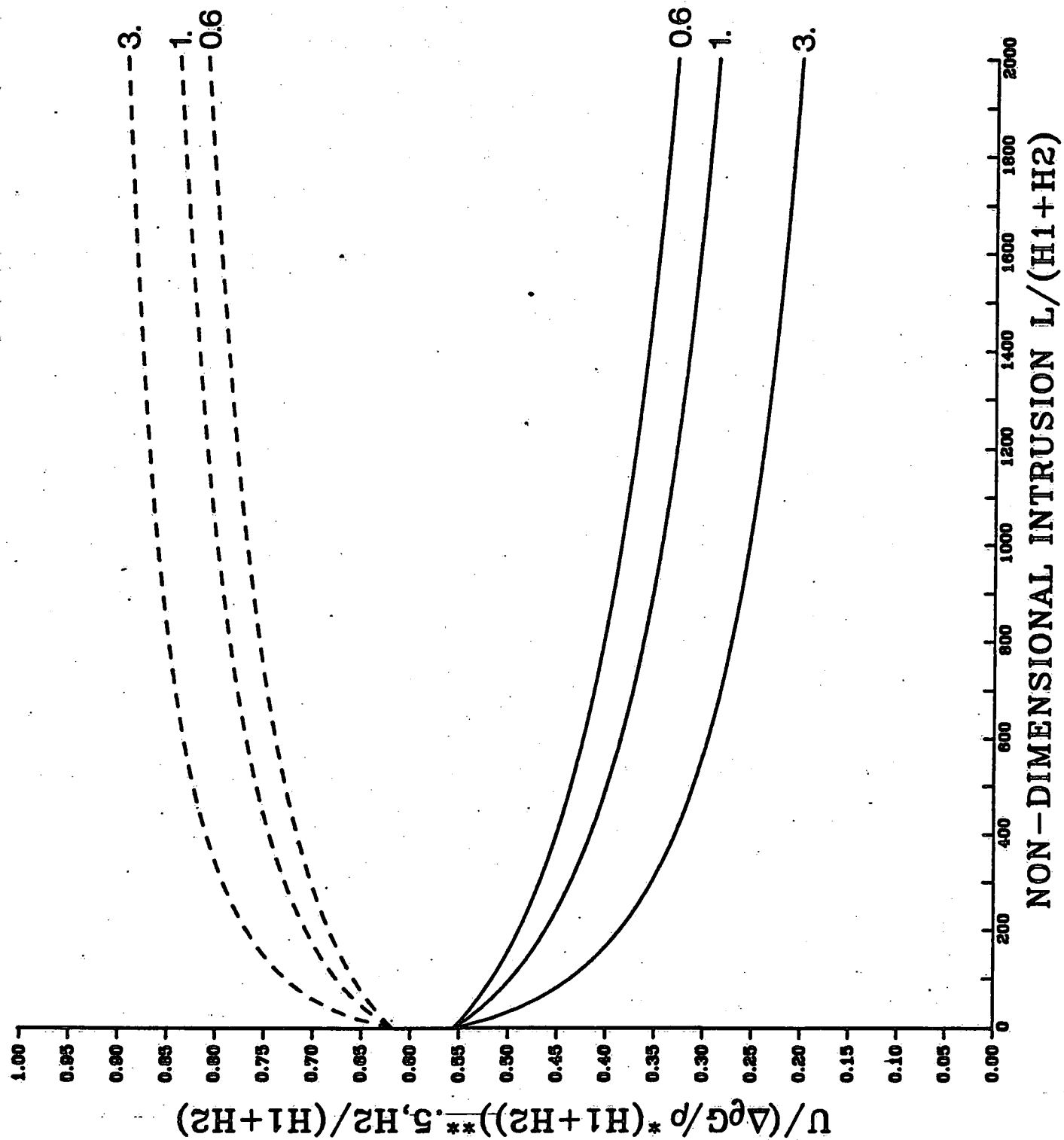


Figure 2

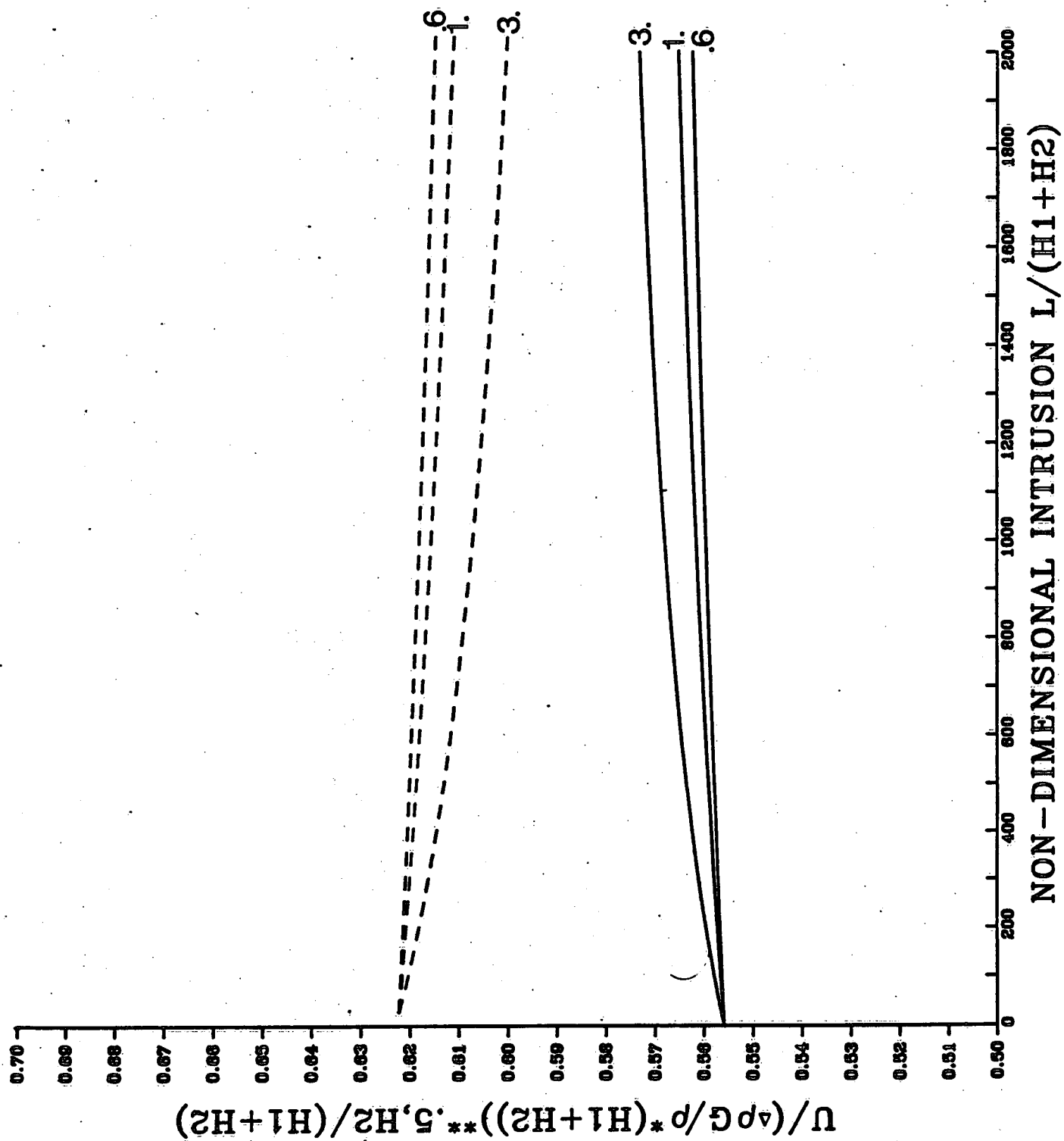


Figure 3

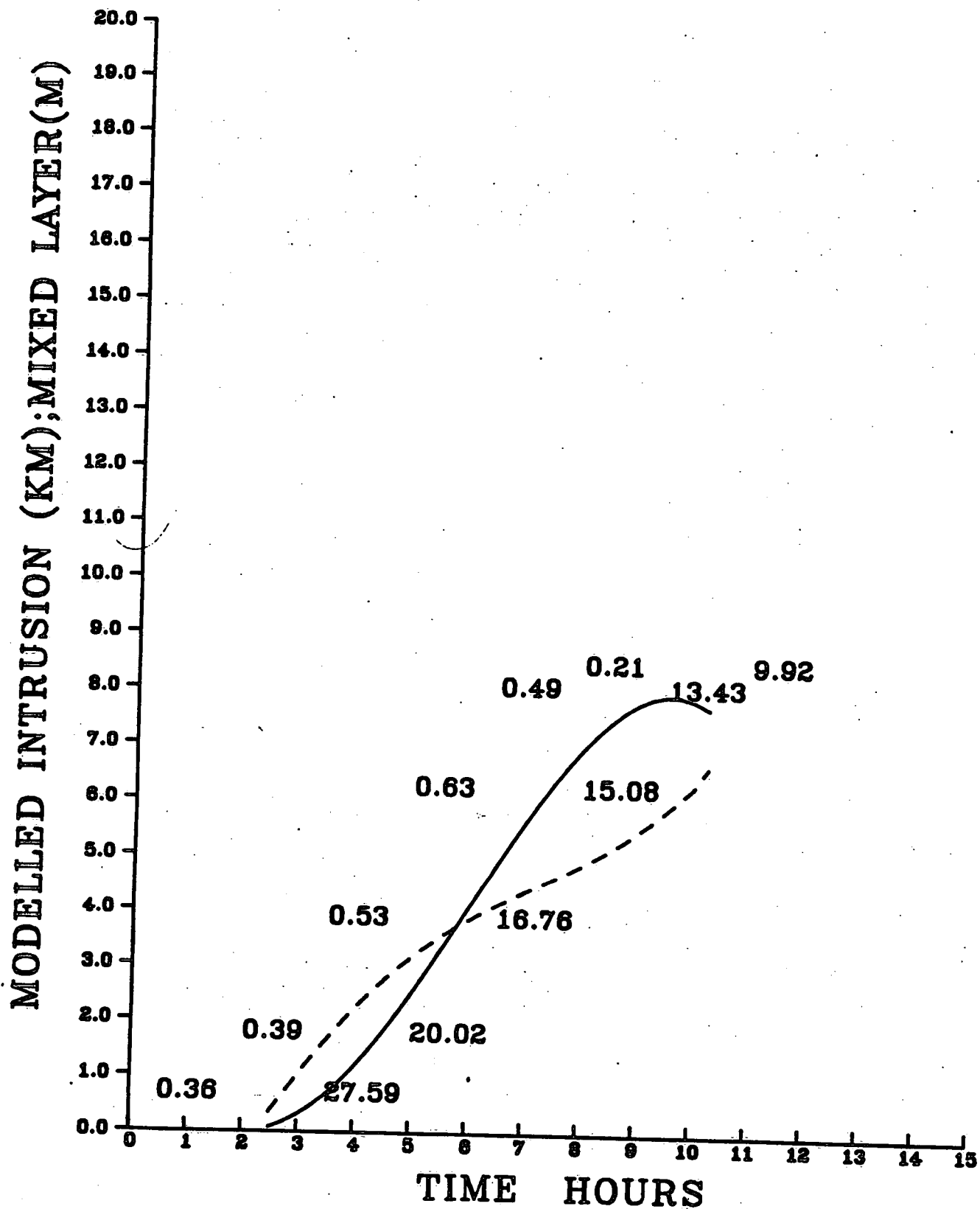


Figure 4

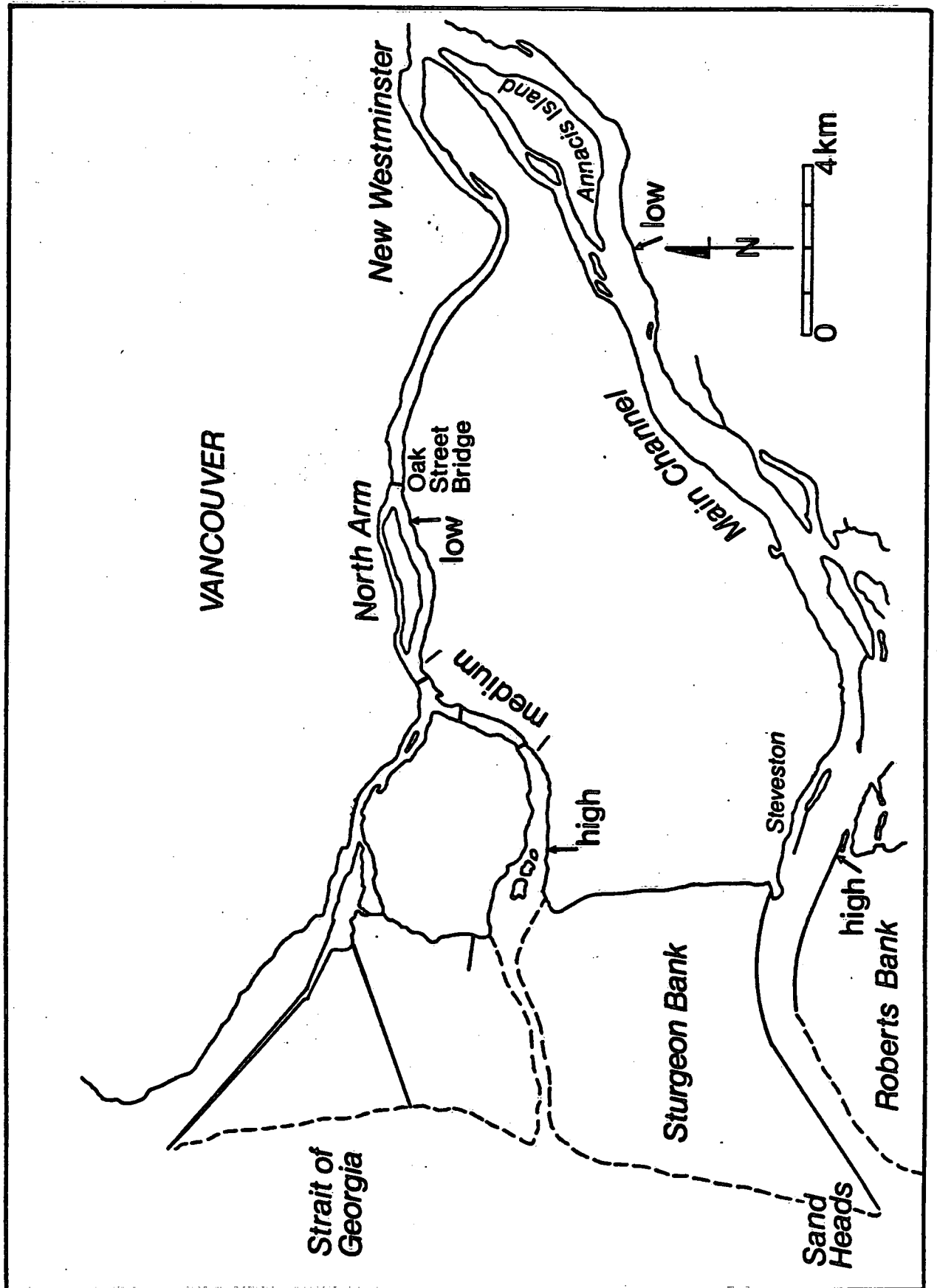


Figure 5

CROSS SECTION AT 1.5 km FROM SANDHEADS

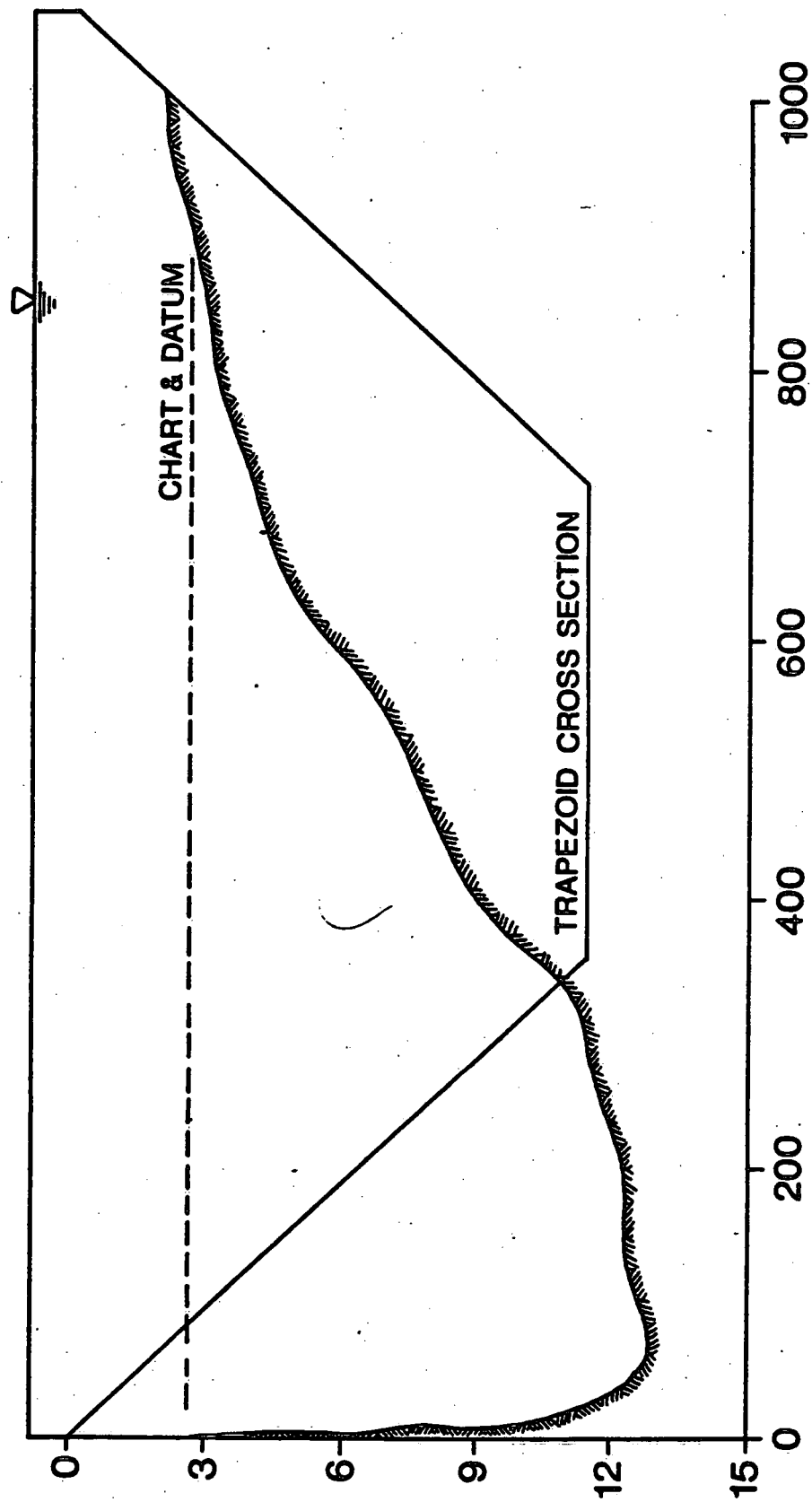


Figure 6

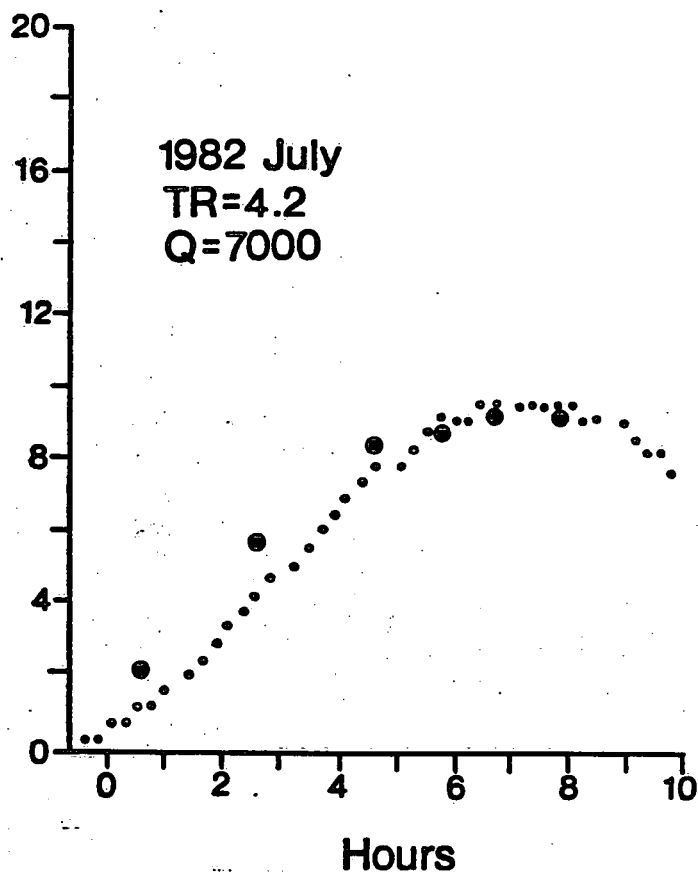
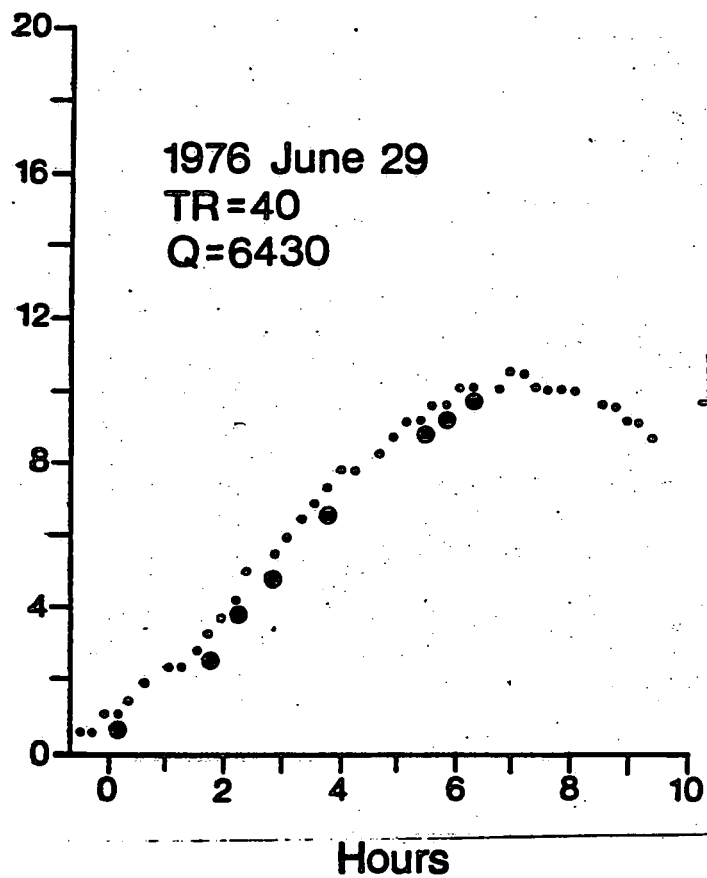
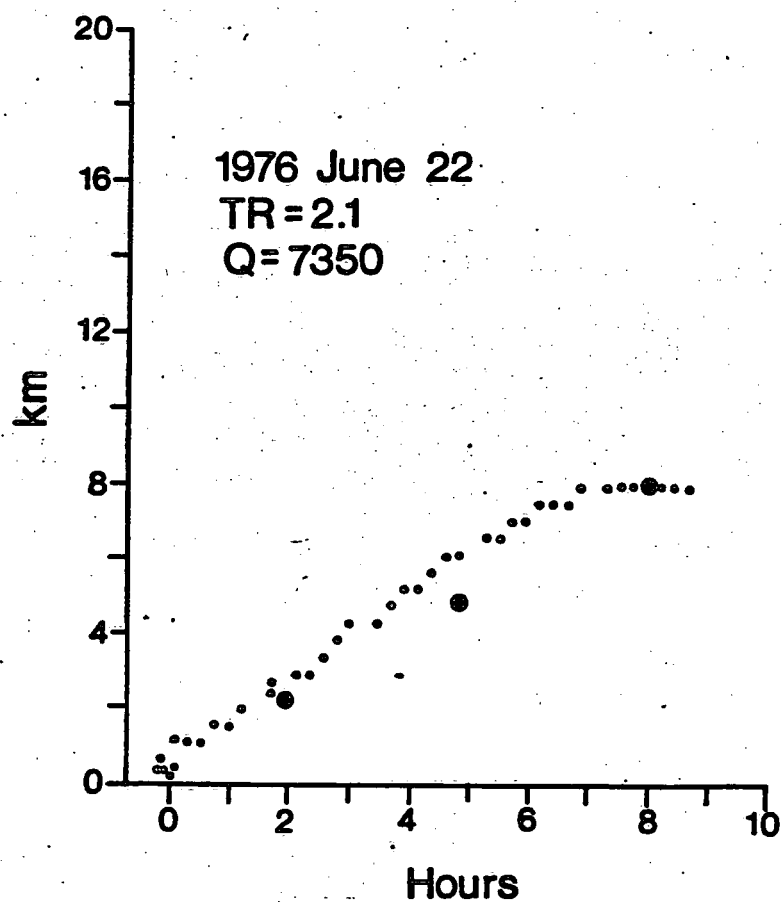


Figure 7

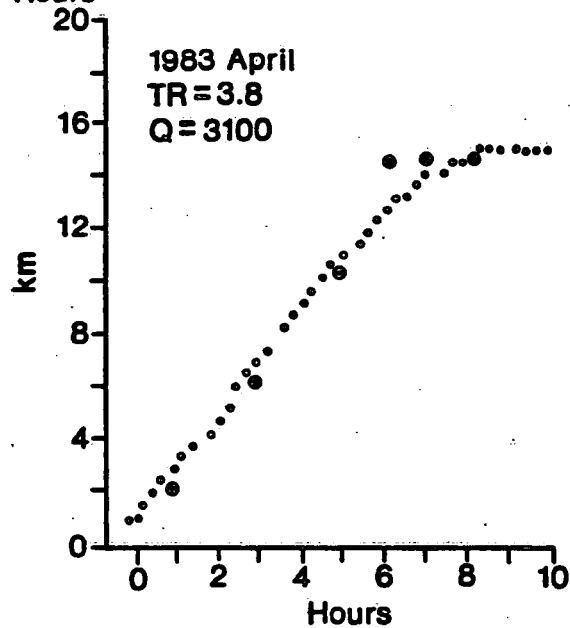
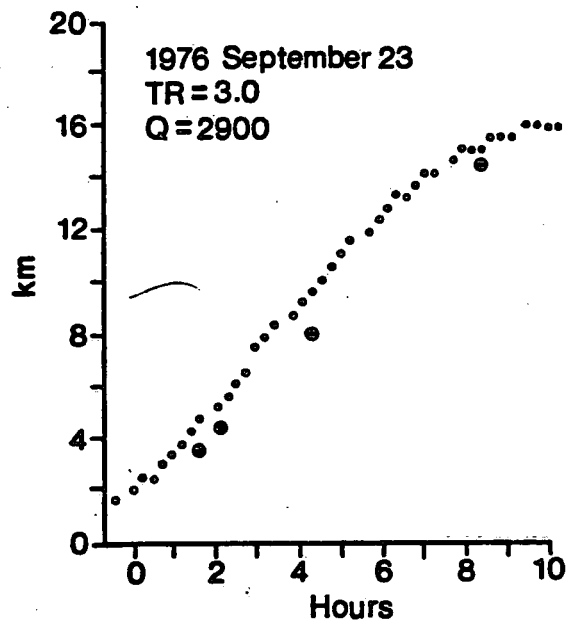
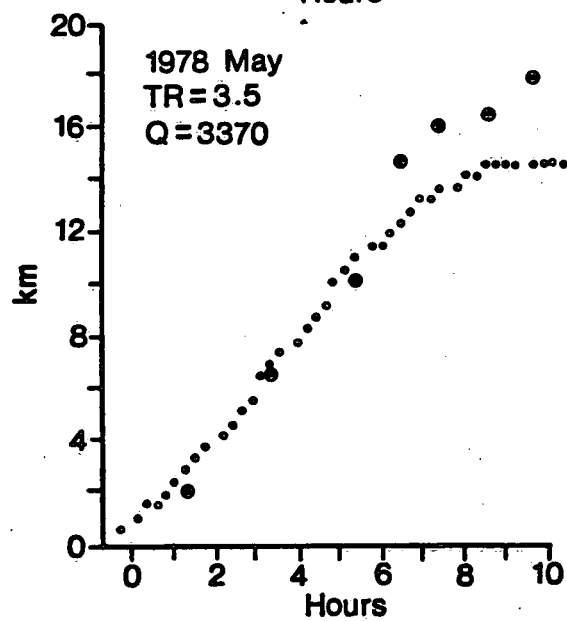
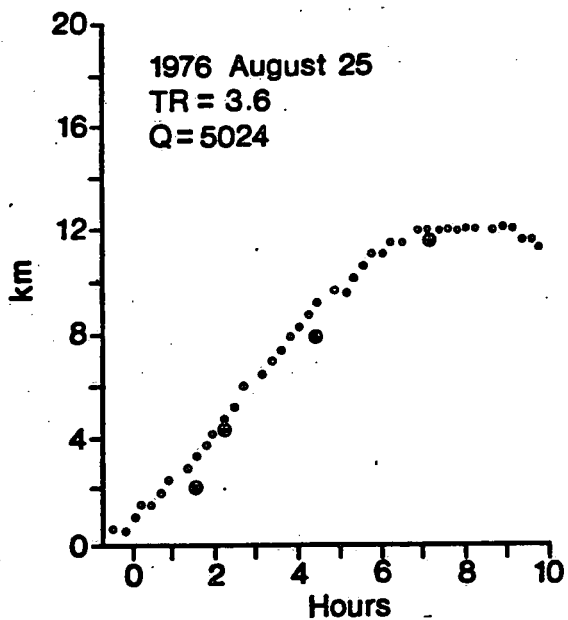
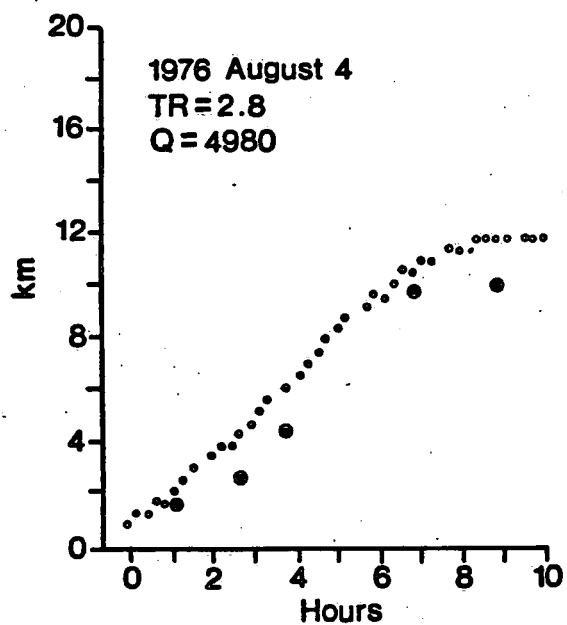


Figure 8

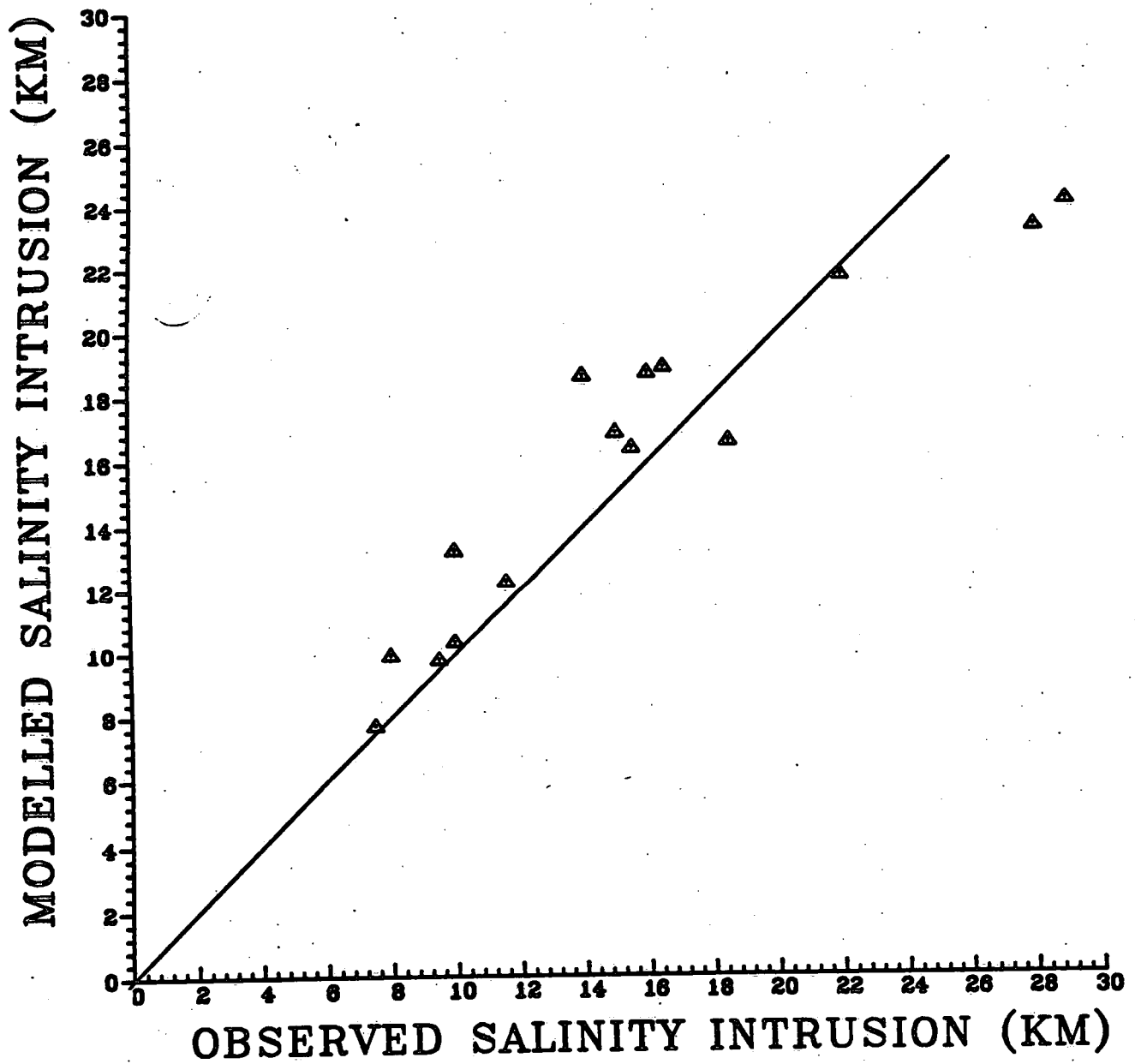


Figure 9

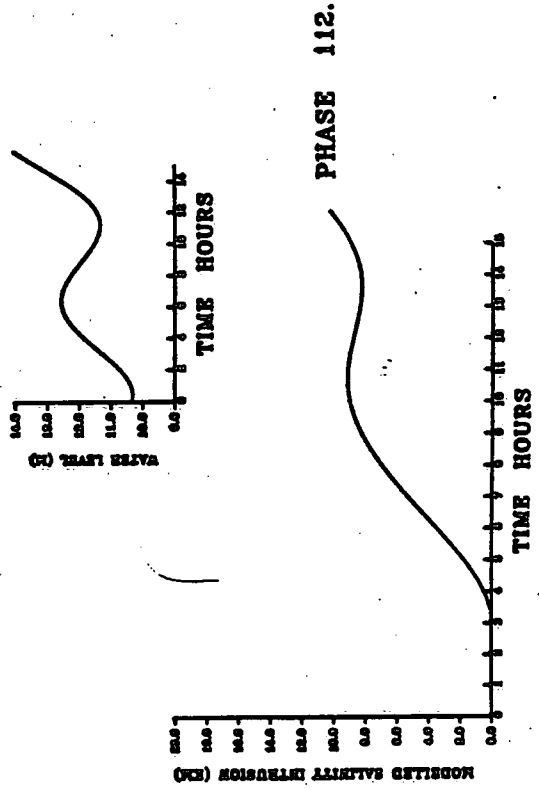
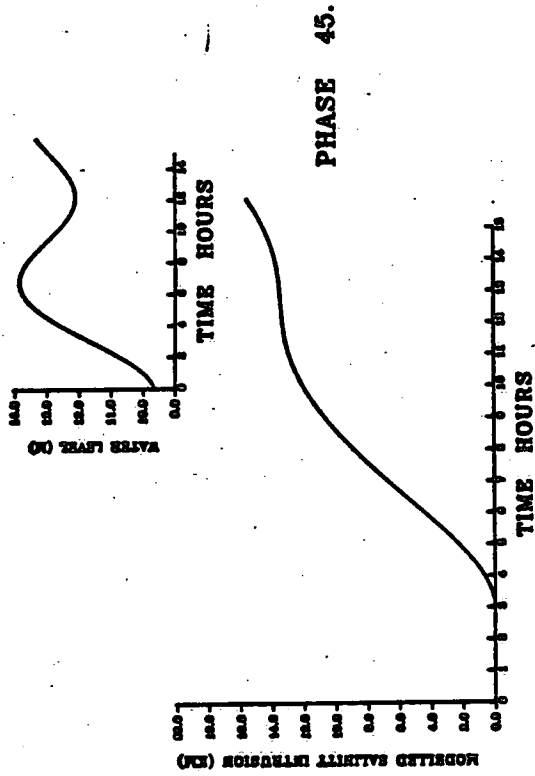
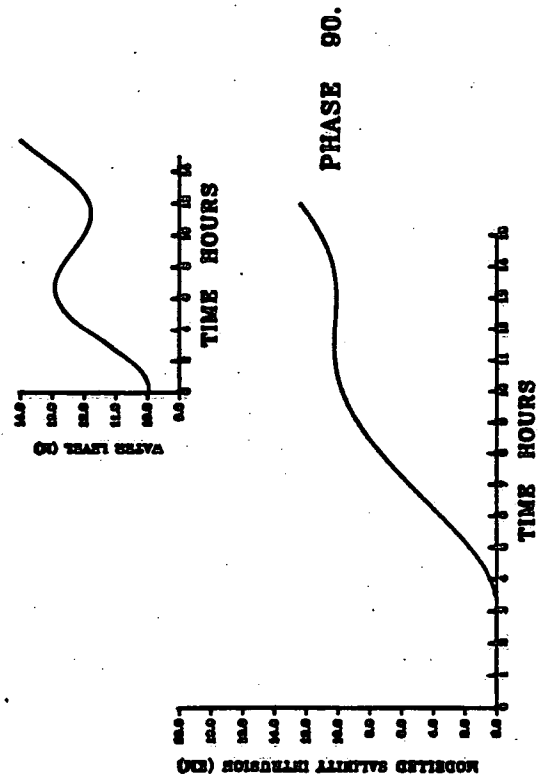
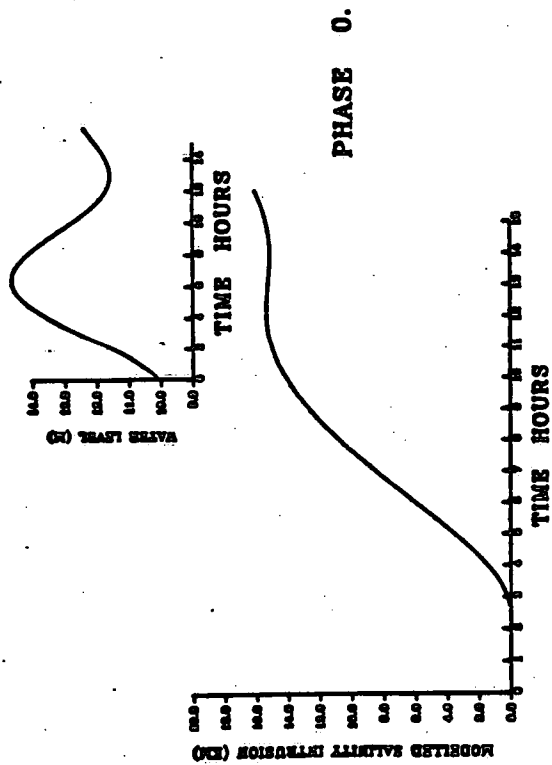


Figure 10



3 9055 1017 0380 8



NATIONAL WATER RESEARCH INSTITUTE
P.O. BOX 5050, BURLINGTON, ONTARIO L7R 4A6



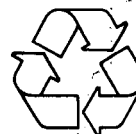
Environnement
Canada

Environnement
Canada

Canada

INSTITUT NATIONAL DE RECHERCHE SUR LES EAUX
C.P. 5050, BURLINGTON (ONTARIO) L7R 4A6

Think Recycling!



Pensez à recycler!