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MODELLING OF COHESIVE SEDIMENT TRANSPORT

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NWRI Contribution No. 91-79

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MANAGEMENT PERSPECTIVE

One of the weak links in existing water quality models is the cohesive sediment transport component.

Cohesive sediments play an important role in the transport of a number of persistent and toxic contaminants. For a proper prediction of transport and ultimate fate of sediment-bound contaminants, a cohesive sediment transport model is an essential pre-requisite.

In this paper, a modelling framework is outlined for predicting the transport of cohesive sediments in natural rivers.

SOMMAIRE À L'INTENTION DE LA DIRECTION

Une des faiblesses des modèles existants de qualité de l'eau est l'élément transport des sédiments cohérents.

Les sédiments cohérents jouent un rôle important dans le transport d'un grand nombre de contaminants persistants et toxiques. Pour obtenir une bonne prévision du transport et du devenir final des contaminants fixés à des sédiments, un modèle de transport des sédiments cohérents est une condition préalable essentielle.

Dans le présent document, un plan de modélisation est indiqué en vue de prévoir la transport de sédiments cohérents dans les cours d'eau naturels.

ABSTRACT

A modelling framework for predicting the transport of cohesive sediments in natural rivers is presented in this paper. It consists of two components: a transport component and a flocculation component. The transport component adapts the advection - diffusion equation expressed in natural curvilinear co-ordinate system to treat the transport and mixing of fine sediment entering a steady flow as a continuous source. The flocculation component is based on a coagulation equation and includes four different collision mechanisms. The application of the model and verification of essential components of the model are presented.

RÉSUMÉ

Le présent document présente un plan de modélisation pour prévoir le transport des sédiments cohérents dans les cours d'eau naturels. Il comporte deux éléments : un élément transport et un élément floculation. L'élément transport adapte l'équation advection-diffusion, exprimée dans un système canonique de coordonnées curvilignes, pour traiter le transport et le mélange des sédiments de faible granulométrie pénétrant dans un écoulement constant comme une source continue. L'élément floculation repose sur une équation de coagulation et comprend quatre mécanismes de collision différents. On indique dans le présent document l'application du modèle et la vérification des éléments essentiels du modèle.

INTRODUCTION

Cohesive sediments in the size classes of silt and clay (<64 μm) are known to adsorb and transport a large number of contaminants that are classified as priority pollutants by the US-EPA (Chapman et al. 1982). For a proper design of environmental monitoring, prediction and management of toxic contaminants bound to sediments, modelling of the transport of cohesive sediments becomes a pre-requisite. A number of models of cohesive sediment transport are already in existence. Some of the notable ones are (i) SERATRA and FETRA by Onishi & Wise (1979) and Onishi and Thompson (1984); (ii) University of California model by Ziegler and Lick (1986); (iii) finite element hydrodynamic and cohesive sediment transport modelling system by Hayter (1987); (iv) TABS-2 by U.S. Army Corps of Engineers (Thomas & McAnally, (1985); and (v) WASP-4 by U.S. Environmental Protection Agency (U.S. EPA 1988).

In most of these models of cohesive sediment transport, the flocculation process of cohesive sediment is not explicitly modelled and often the cohesive sediment particles are assumed to behave as individual particles rather than as agglomeration of particles i.e. flocs. Since the transport characteristics of sediment flocs differ considerably from those of the constituent primary particles (see Partheniades, (1962, 1965, 1986); Krone, (1962, 1963, 1972, 1978); Mehta, (1986, 1988); Lick, (1982); Kranck, (1980, 1986)), the predictive capabilities of the above models can at best be only approximate and could introduce errors which could then cascade into the prediction of the sediment bound contaminant. Therefore, there is a definite need for explicit treatment of flocculation process in modelling the transport of cohesive sediments.

Attempts have already been made to include the flocculation process explicitly in sediment transport models by Valioulis and List (1984 a, b). In their work, Valioulis and List modelled the sedimentation process in a settling basin and considered the flocculation process in terms of a system of first order ordinary differential equation expressing the conservation of sediment mass in different size fractions during the development of particle size distributions (coagulation equation). They considered three collision mechanisms, namely, differential settling, turbulent shear and Brownian motion.

In this paper, an alternate method is proposed to calculate the size distribution of the flocculating sediment as it is transported in a steady flow in a natural river.

PRESENT METHOD

In the development of the present method, the motion of sediment is considered in two stages, namely, a transport stage and a flocculation stage. These two stages are assumed to occur alternatively over a fixed time interval. The transport stage of the model is based on a mixing model (RIVMIX) development by Krishnappan and Lau (1985). The flocculation stage is based on a simulation model proposed by Krishnappan (1990). The salient features of the present method are described below.

a) Transport stage:

A depth-averaged advective-dispersion equation expressed in a natural curvilinear co-ordinate system (see Fig 1) introduced by Yotsukura and Sayre (1976) is used to analyze the sediment motion during this stage. The form of the equation solved is shown below:

$$(1) \qquad \frac{\partial C_k}{\partial x} = \frac{\partial}{\partial \eta}\left(\frac{M_x U h^2 E_z}{Q^2}\ \frac{\partial C_k}{\partial \eta}\right) + \frac{M_x \lambda_1}{U} C_k + \frac{M_x}{U}\lambda_2$$

where C_k	is depth-averaged volumetric concentration of the kth size fraction.
x	is distance measured along the longitudinal co-ordinate axis (Fig. 1)
U	is the depth-averaged velocity component in the x direction.
h	is the flow depth.
Q	is the volumetric flow rate
E_z	is the transverse dispersion coefficient
M_x	is the metric coefficient of the co-ordinate system (Fig. 1).
η	is the normalized cumulative discharge defined

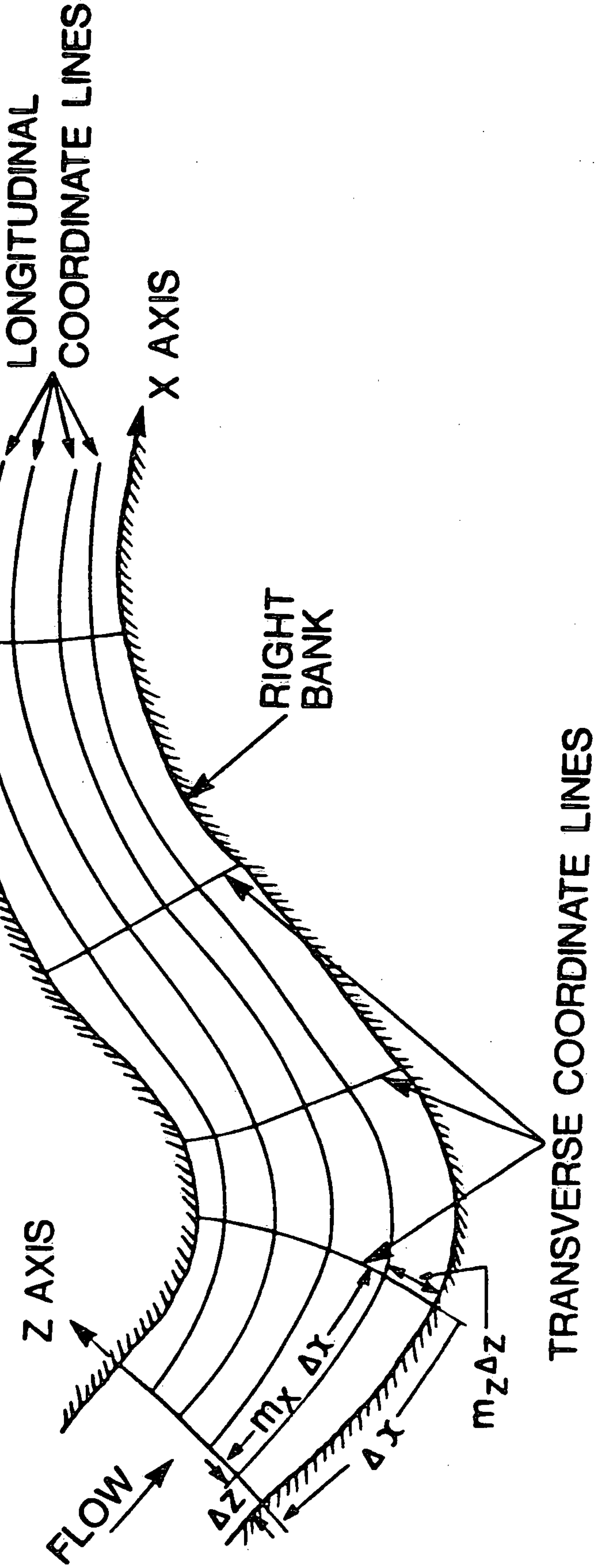


Fig.1 CURVILINEAR COORDINATE SYSTEM

(2) $$\eta = \frac{1}{Q}\int_{o}^{z} M_z \, \mu h dz$$

	z	is the transverse distance co-ordinate
	λ_1	is the rate coefficient for reactions etc.
and	λ_2	is the rate of source or sink.

Equation (1) expresses the mass balance of a sediment fraction as it undergoes advection and turbulent diffusion and dispersion in a steady flow in a natural stream when the sediment enters as a continuous source. It also includes terms reflecting material loss or gain due to reactions that are of first order in character and sources and/or sinks in the flowfield.

The boundary conditions used are as follows:

i) Upstream boundary: A known transverse distribution of sediment concentration at the first cross-section is used as the upstream boundary condition. This can be obtained either by direct measurement or from some other calculations.

ii) At the side banks: The sediment flux across these boundaries are zero and hence the concentration gradients across those boundaries are taken as zero.

The sediment deposition at the stream bed and the possible re-entrainment of the deposited sediment are taken into account using the source-sink term λ_2.

There is a great deal of uncertainty in quantifying the rates of erosion and deposition of cohesive sediment at the stream bed because of the large number of controlling parameters that govern these processes. In addition to the bed shearstress and the physical characteristics of sediment such as size and specific weight which are the main characteristic parameters for the coarse grained non-cohesive sediments, physico chemical and biological properties of the sediment-water mixture and the time history of erosion and deposition cycles also become important governing parameters for the near bed processes of cohesive sediment transport. Attempts to quantify the net rate of sediment flux across the sediment-water interface are not highly successful and further research is needed to advance the knowledge in this area.

For the purpose of present model, the net amount of sediment deposited is expressed as:

$$(3) \qquad \lambda_{2k} = - W_{sk} (1\text{-}S)\, C_{bk}$$

where W_{sk} is the settling velocity of a sediment fraction, C_{bk} is the nearbed concentration of the sediment fraction and S is a scouring parameter. S is allowed to vary to test the sensitivity of the erosion and deposition process. When S = 0, implies that all the settling sediment deposit where as S = 1, implies the rate of settling is equal to the rate of erosion. A value of s greater than 1 implies the rate of erosion exceeds the rate of deposition.

Equation (1) is solved by re-writting it in the form of a general advection diffusion equation and using a finite difference scheme proposed by Stone and Brian (1963). The details of the solution procedure can be found in Krishnappan and Lau (1985).

b) Flocculation stage

The flocculation stage of the sediment transport was treated using a coagulation equation which expresses that rate of change of number of sediment particles per unit fluid volume in a particular size class, i, as:

$$(4) \qquad \frac{\partial N(i,t)}{\partial t} = - N(i,t) \sum_{j=1}^{\infty} K(i,j)\, N(j,t) + \frac{1}{2} \sum_{j=1}^{\infty} K(i-j,j)\, N(i-j,t)\, N(j,t).$$

where N(i, t) and N(j, t) are number concentrations of size classes i and j respectively at time t and K(i,j) is the collision frequency function which is a measure of the probability that a particle of size class i collides with a particle of size j in unit time. The first term on the right hand side of equation (4) gives the reduction in the number of particles in size class, i, due to the flocculation of particles of class i with all other size classes. The second term gives the generation of the new particles in class i by the flocculation of pairs of particles of smaller size classes. In this process, the volume of the sediment particles is conserved. In otherwords, if r_i and r_j are the radio of particles of classes i and j, then the radius r of the newly formed particle is

(5) $$r_i^3 + r_j^3 = r^3$$

The collision frequency function, k(i, j) takes different functional forms depending on the collision mechanism. The various collision mechanisms for which the collision frequency functions have already been derived are (1) Brownian Motion, (2) laminar or turbulent fluid shear, (3) particle inertia in turbulent flows and (4) differential settling of sediment mixtures. In the present model, all the above four mechanisms are considered and an effective collision function, K_{ef}, is calculated following Heubsch (1967) as:

(6) $$K_{ef} = K_b + (K^2_{sh} + K^2_I + K^2_{ds})^{1/2}$$

where K_b, K_{sh}, K_I and K_{ds} are given by the following relations:

(7) $$K_b\,(r_i, r_j) = \frac{2}{3}\,\frac{KT}{\mu}\,\frac{(r_i + r_j)^2}{r_i r_j} \quad \textit{(Brownian Motion)}$$

(8) $$K_{sh}\,(r_i, r_j) = \frac{4}{3}\,(\frac{e}{v})^{1/2}\,(r_i + r_j)^3 \quad \textit{(Fluid Shear)}$$

(9) $$K_I\,(r_i, r_j) = 1.21\,\frac{\rho_s}{\rho_f}\,(\frac{e^3}{v^5})^{1/4}\,(r_i + r_j)^2\,|\,r_i^2 - r_j^2\,| \quad \textit{(Inertial)}$$

(10) $$K_{ds}\,(r_i, r_j) = \frac{2}{9}\,\frac{\pi g}{v}\,(\frac{\rho_s - \rho_f}{\rho_f})\,(r_i + r_j)^2\,|\,r_i^2 - r_j^2\,| \quad \textit{(Differential settling)}$$

In the above relation, k is Boltzmann constant, T is the temperature in degree Kelvin, μ is viscosity of fluid, ν is kinematic viscosity, ε is turbulent energy dissipation per unit mass, ρ_s and ρ_f are densities of sediment and fluid respectively and g is acceleration due to gravity.

The solution of (4) in continuous radius space is tedious because of the wide range of particle sizes and hence a discrete equation in logarithmic radius space development by Yue and Deepak (1979) was adopted. Accordingly, the continuous radius space was divided into a set of discrete ranges as:

(11) $$r_i = r_1 \, 2^{(i-1)/3} \qquad i = 2, 3 \,........M$$

In such a scheme, each range can be considered as a bin containing particles of certain size range. r_1 is the geometric mean radius of particles in the first bin and M is the total number of bins. The above scheme also implies that the mean volume of particle in bin i is twice that of the preceding bin (particles are assumed to be spherical). Each bin contains particles with volumes ranging from (v_i - 0.5 Δv_i) to (v_i + 0.5 Δv_i), where $v_i = (4/3)\pi r_i^3$ and $\Delta v_i = 2^i(v_1/3)$. Fig. 2 depicts the division of particle size ranges.

When particles of bin i flocculate with particles in bin j (j<i), the newly formed particles will fit into bins i and i+1. The proportion of particles fitting in bins i and i+1 are calculated by considering the conservation of mass before and after flocculation. For example, if N_{ij} is the number of newly formed particles and N_x and N_y are the numbers going into bins i and i+1, then the conservation of mass gives:

(11a) $$\rho_s N_{ij} V_i + \rho_s N_{ij} V_j = \rho_s N_x V_i + \rho_s N_y V_{i+1}$$

(12) $$\text{i.e.} \qquad V_i + V_j = V_i \frac{N_x}{N_{ij}} + V_{i+1} \frac{N_y}{N_{ij}}$$

or

(13) $$V_i + V_j = V_i f_{ij} + V_{i+1} (1 - f_{ij})$$

where f_{ij} is the fraction going to bin i and (1 - f_{ij}) is the fraction going to bin i+1. A schematic sketch of such particle allocation is shown in Fig. 3.

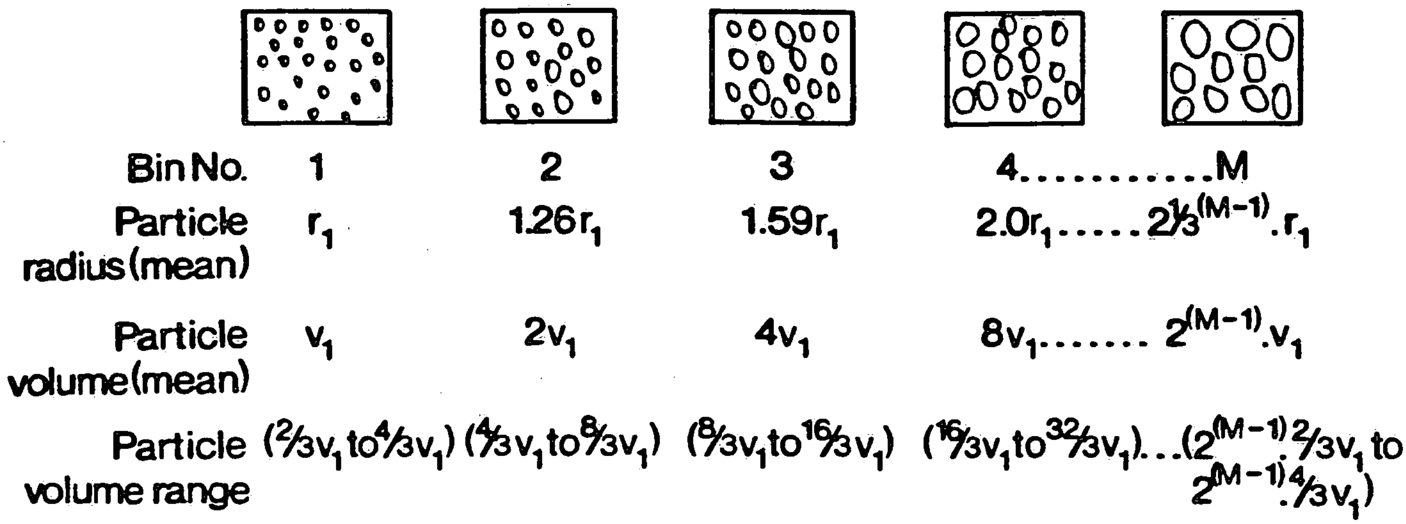
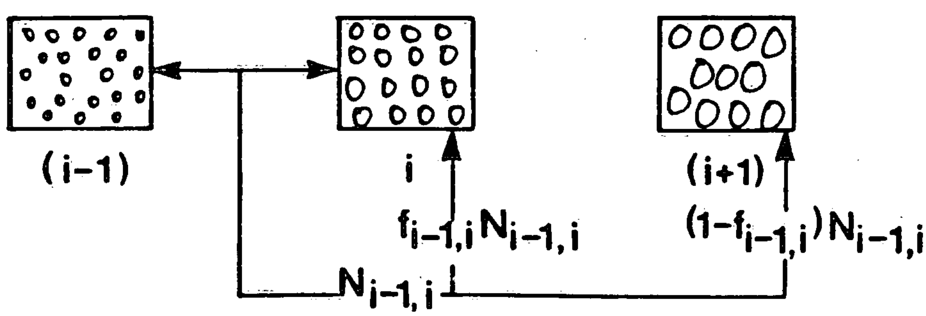


Fig. 2 . Discretization of particle size ranges



$N_{i-1,i}$ – No. of flocculated particles
$f_{i-1,i}$ – Particle allocation function

Fig. 3 . Schematization of allocation of flocculated particles

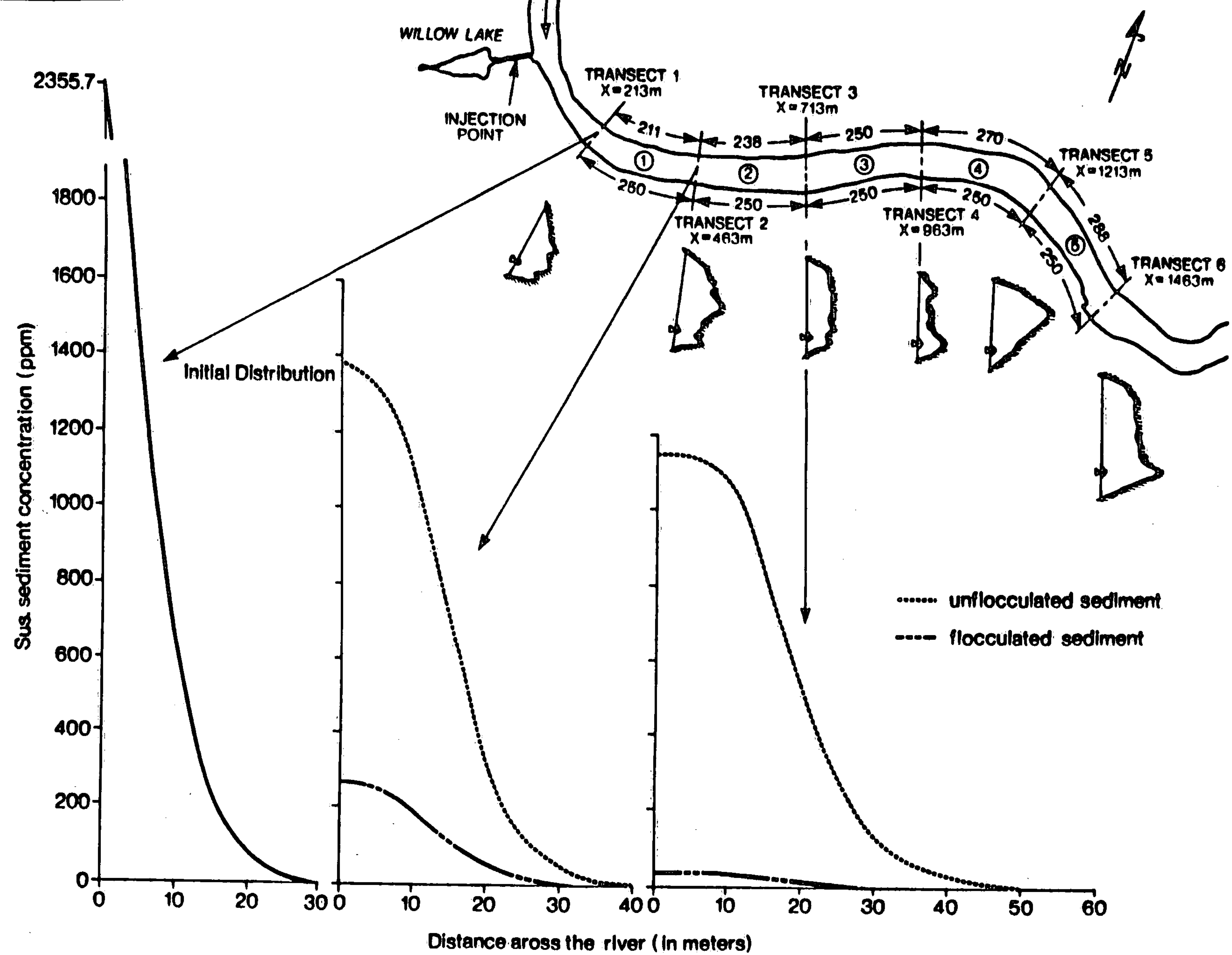


Figure 4 Distribution of suspended sediment across the river at different transects

From equation (13), the allocation function f_{ij} becomes:

$$f_{ij} = (V_i + V_j - V_{i+1})/(V_i - V_{i+1})$$

With the particle allocation scheme proposed above, equation (4) becomes:

$$(14) \quad \frac{\Delta Ni}{\Delta t} = - \sum_{j \neq i} K_{ef}(i,j) N_i N_j + \sum_{j<i} f_{ij} K_{ef}(i,j) N_i N_j + \sum_{j<i-1} (1 - f_{i-1}, j) K_{ef}(i-1,j) N_{i-1} N_j$$

Applying the above equation to each of the bins, the change in particle size distribution can be computed over a time interval of Δt.

PROCEDURE FOR MODEL APPLICATION

The steps involved in applying the model for a practical problem are illustrated by an example. The model was applied to a stretch of the Grand River near Kitchener, Ontario, Canada. The plan view and the cross-sectional shapes at a number of transects are shown in Fig. 4. As can be seen from this figure, this stretch of the river includes bends, changing river widths and irregular cross-sectional shapes. A tributary (Willow Creek) enters the river at the upstream reach. The complete hydraulic characteristics of the river reach can be found in Krishnappan and Lau (1982). A hypothetical sediment with an initial size distribution as shown in Fig. 5 is assumed to enter the river reach from the tributary Willow Creek. This sediment was then divided into 16 size classes according to Eq. (11) (r_1 = 1.0 μm and M = 16). The lateral variation of the sediment concentration at transect 1 for each size class (the upstream boundary for the calculation domain) was calculated using an analytical solution of a simplified form of advection - diffusion equation, treating the discharge from the tributary as a continuous point source. Using this distribution as the upstream boundary condition, the sediment distribution at

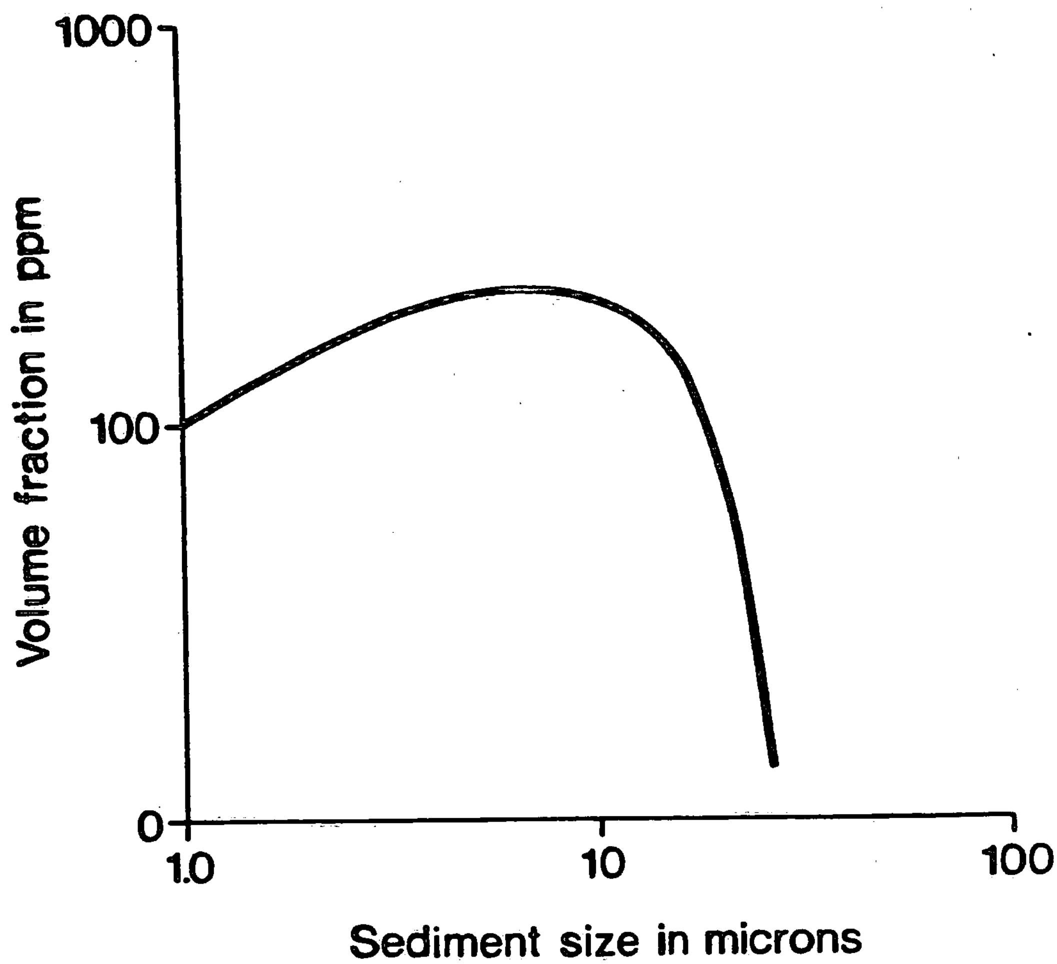


Figure 5 Size distribution of sediment used in the predictions.

transect #2 was calculated by solving the advection diffusion equation (1) for each size fraction. When solving equation (1), the river width was divided into 20 segments and the concentration of each fraction of sediment was calculated for each segment. Knowing the concentration of all sediment fractions in each segment, the flocculation model was then applied and a new size distribution was calculated for that segment. The time duration Δt of flocculation was calculated as the transit time for the parcel of water to traverse the distance between the transects. This process was repeated for all segments. This, then completes the calculation for transect #2. Similar calculations were repeated for all the transects in the calculation domain.

The calculated transverse distribution of suspended sediment concentration for some of the transects are shown in Fig. 4. In performing these calculations, two distinct cases were considered. In case one, shown in dotted line, the sediment was considered to be transported in unflocculated form (flocculation stage was suppressed) and in case two, the sediment was allowed to flocculate. In both cases, the scouring parameter S is assumed to be zero i.e. only sediment deposition is present without any resuspension. From Fig. 4, it is evident that the flocculation component affects the suspended sediment distributions drastically. Whereas unflocculated sediment is largely transported through the reach, flocculated material is virtually deposited over a distance of 713 m (see Fig. 6 which gives the suspended sediment flux as a function of distance along the river reach).

MODEL VERIFICATION

A direct verification of the model is not possible at this present time because of the lack of experimental data on flocculation in flowing medium. However, there is experimental data on flocculation of settling sediment in still water. Kranck (1980) measured settling of sediment mixtures in still water under unflocculated and flocculated state. To make use of the data of Kranck to test the validity of 'the flocculation component, the model was simplified for the still water case, and a direct comparision of model predictions with the experimental data of Kranck was made. The results of this verification are summarized in Krishnappan (1990). Some of the results of the

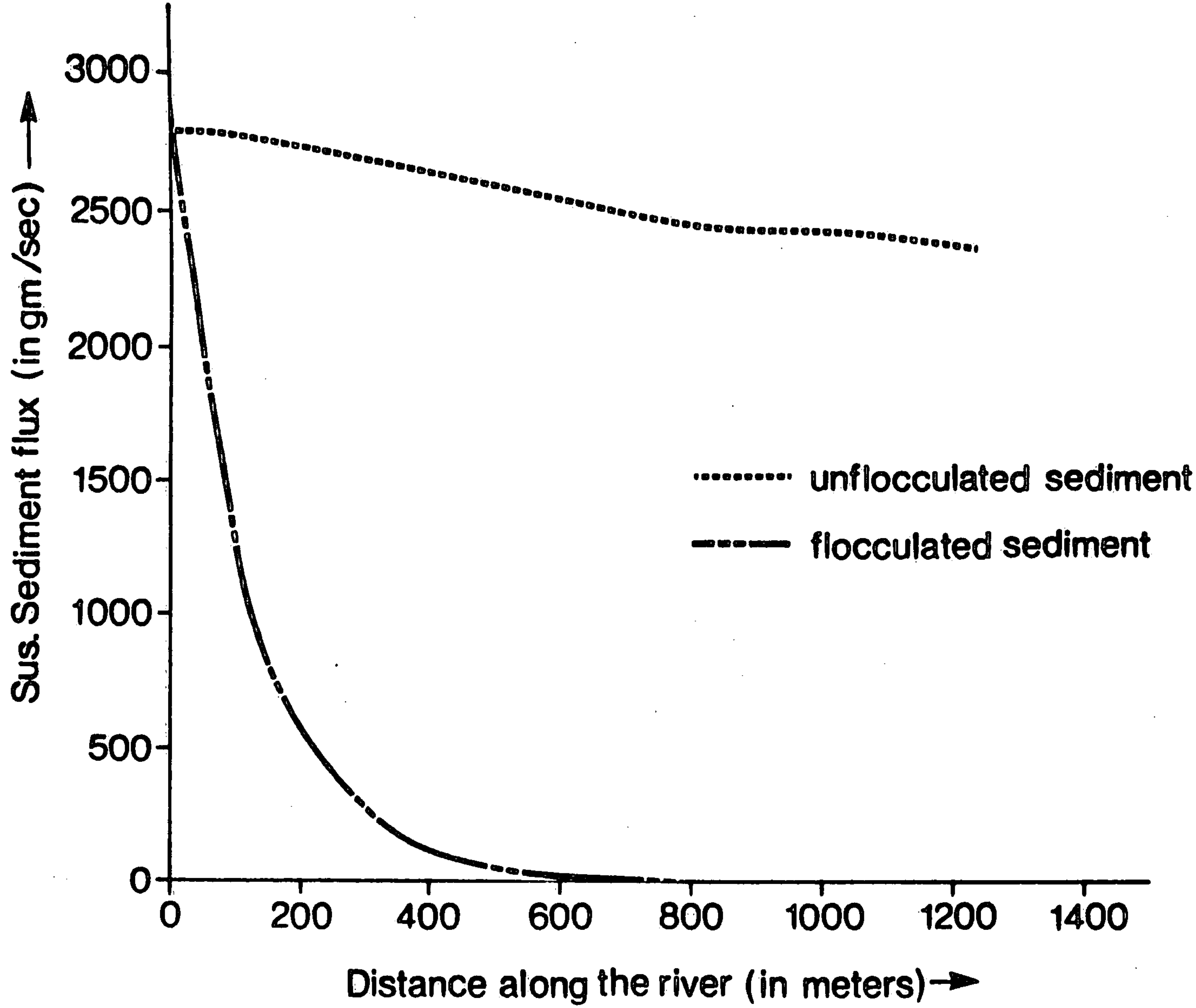


Figure 6 Suspended sediment flux variation along the river

verification are shown here in Figs. 7 and 8. Fig. 7 is for single grain settling and Fig. 8 is for floc settling. In these figures, the total sediment concentration in a settling column is plotted as a function of time. It can be seen from these two figures that there is a drastic difference between these two modes of settling and the model predictions for both cases, agree reasonably will with the measured data. Predictions of size distributions were also made and compared with measured data. These comparisons are not shown here due to page limit. The reference of Krishnappan (1990) can be consulted for these details.

SUMMARY AND CONCLUSIONS

A modelling framework for predicting the transport of cohesive sediments in natural rivers is presented in this paper. It consists two components: a transport component and a flocculation component. The transport components employs the advection - diffusion equation in natural co-ordinate system to treat the transport and mixing of sediment entering a steady flow in a stream as a continuous steady source. The flocculation component is based on a coagulation equation including four different collision mechanisms, namely, Brownian motion, fluid shear, particle inertia and differential sedimentation. The steps involved in the application of the model are examined by applying the model for a hypothetical sediment entering a stretch of a river reach in southern Ontario, Canada. The essential features of the model were verified using existing laboratory measurements and the model predictions agree seasonably well with measured data.

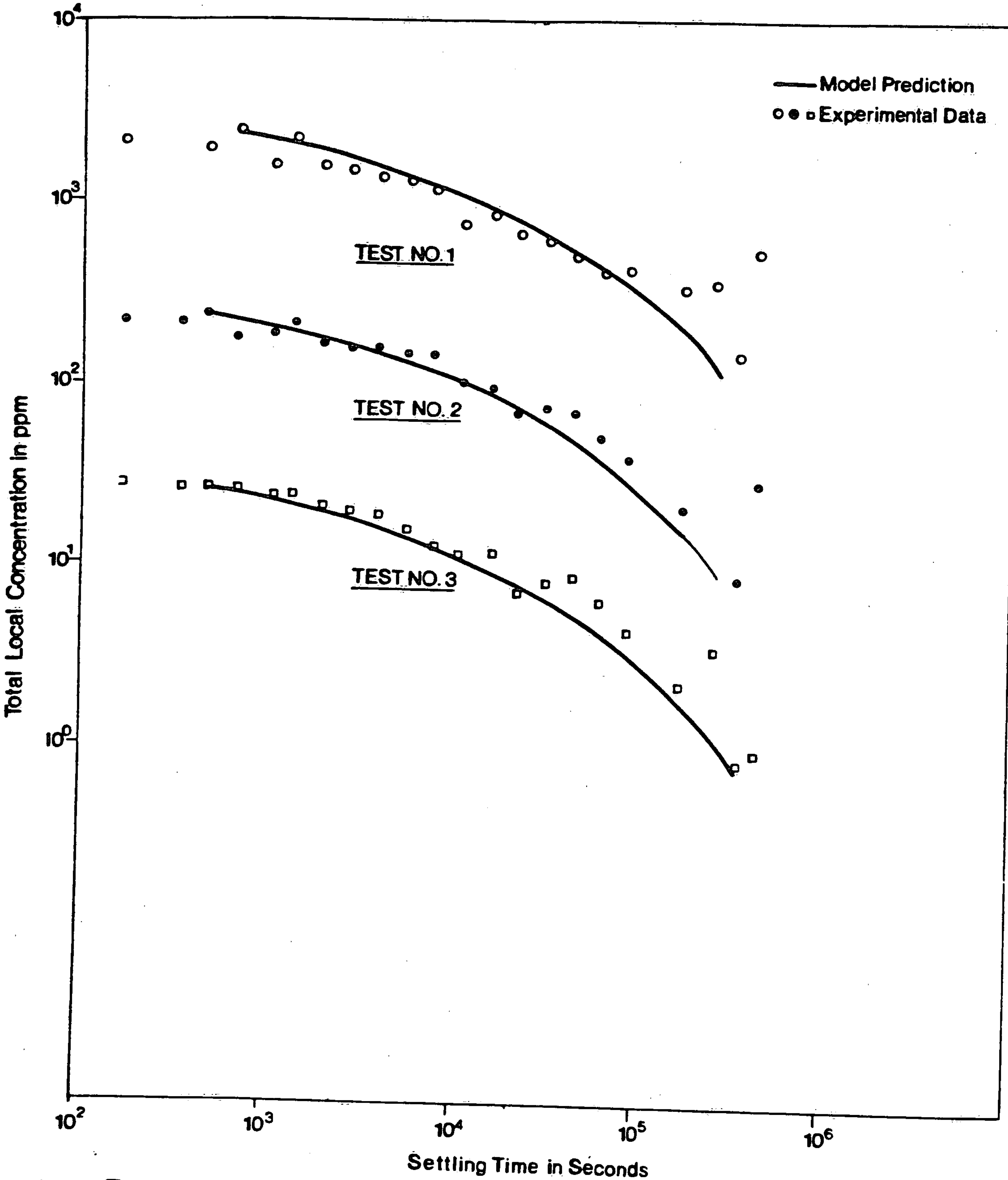


Figure. 7 CHANGE IN TOTAL SEDIMENT CONCENTRATION AS A FUNCTION OF TIME (SINGLE GRAIN SETTLING)

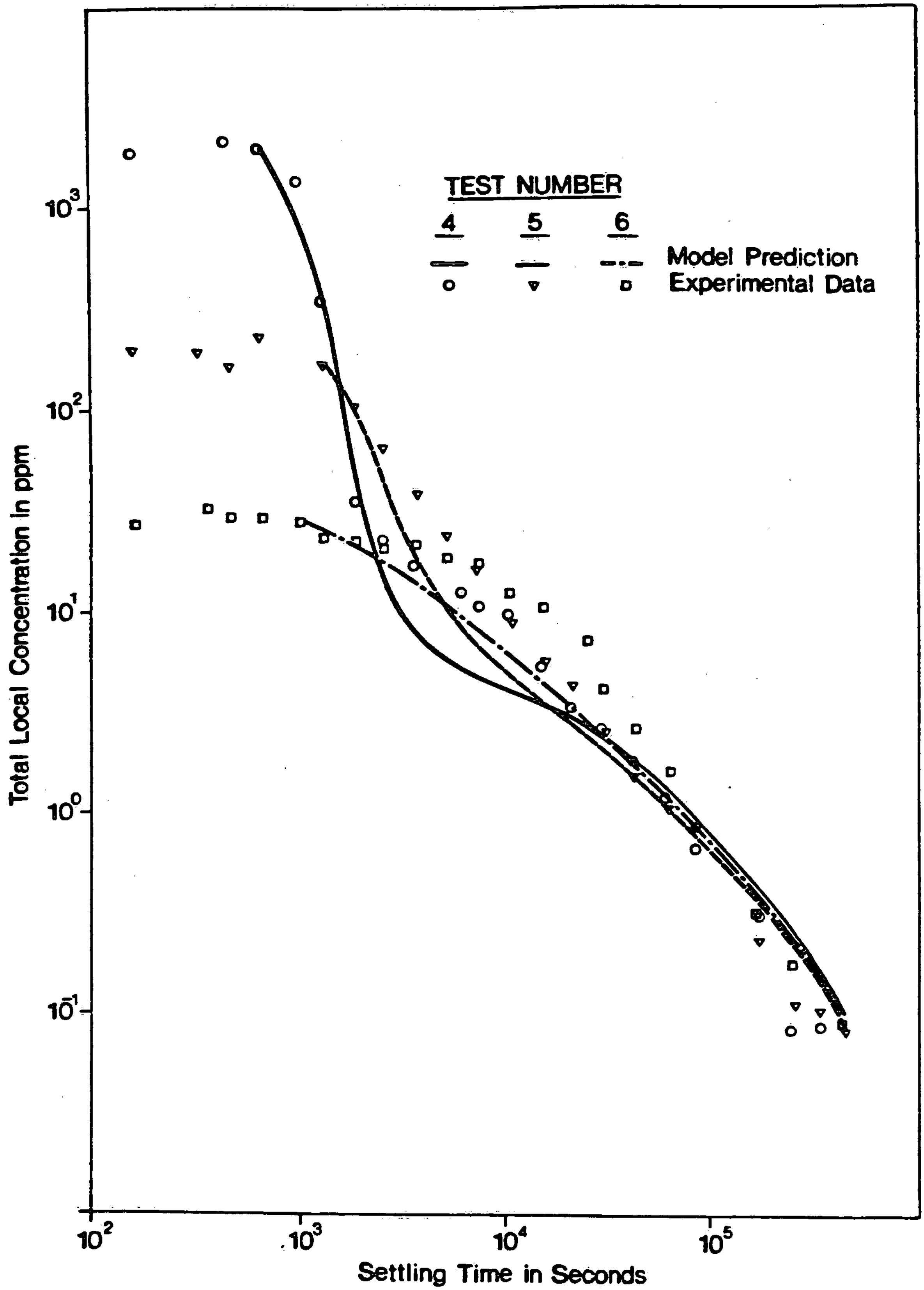


Fig. 8 Change in Total Concentration as a Function of Time (Floc Settling)

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