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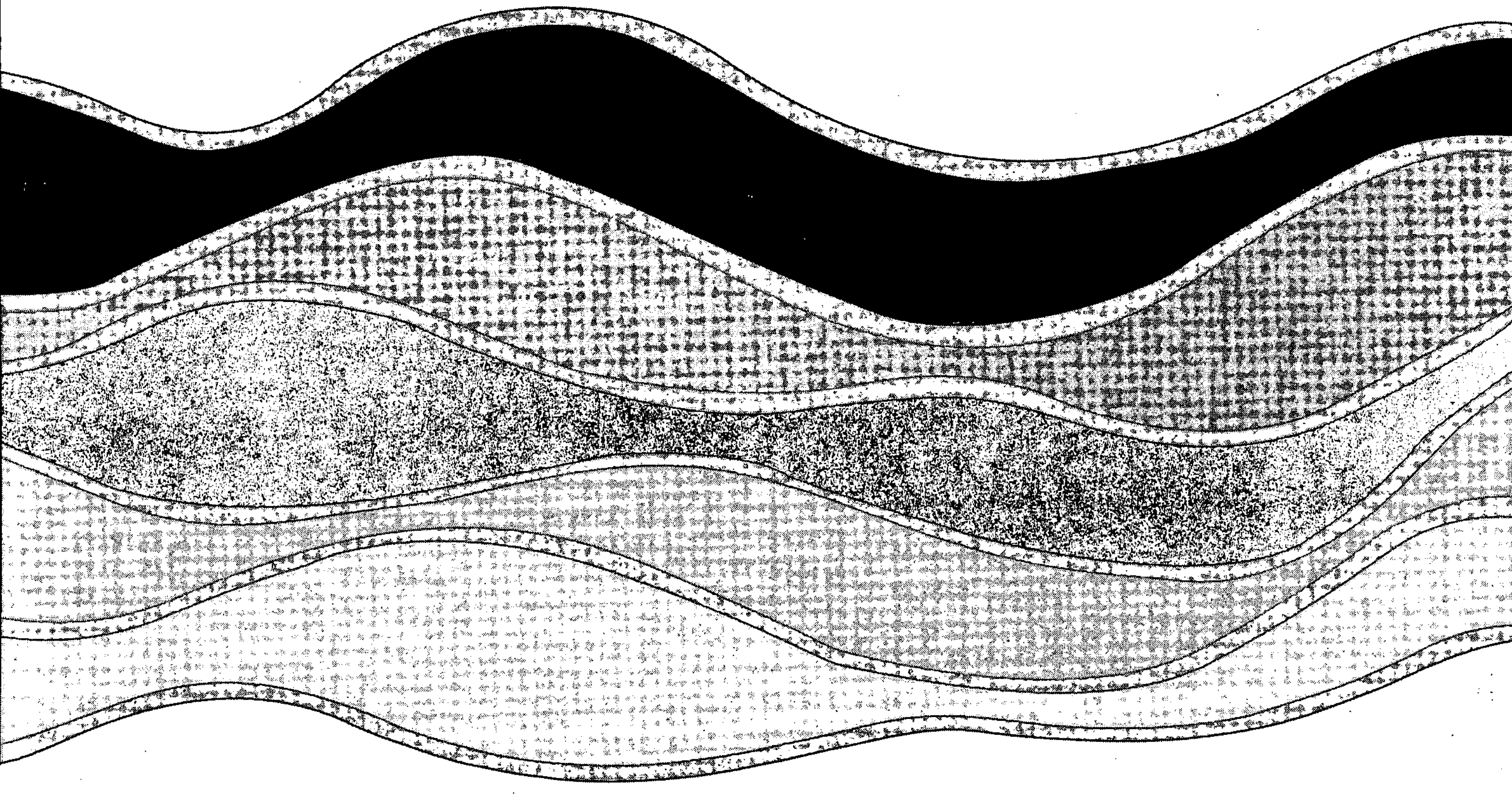

DETERMINING FORMATION PROPERTIES IN THE PRESENCE OF FINITE THICKNESS SKIN AND PARTIAL PENETRATION

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NWRI Contribution No. 91-81

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MANAGEMENT PERSPECTIVE

One of the most important aspects regarding the siting and design of nuclear and hazardous waste facilities, has to to with estimates of groundwater travel time. Estimates of travel time are currently calculated using average measurements of hydraulic gradient and a measurement of hydraulic conductivity. Most hydrogeologists use standard techniques (such as the slug test) to determine estimates of hydraulic conductivity in the low-permeability material that acts as an hydraulic barrier at waste facilities. Unfortunately, it has recently been shown that these methods are widely inaccurate under most conditions, particularily when employed in clay environments (such as are found at most hazardous waste sites in Southern Ontario). Thus, as a means of improving the field methods and increasing the accuracy of the estimate of hydraulic conductivity, a new analytical model is developed which more realistically describes the physical testing configuration for a particular type of hydraulic test known as the constant-head test.

It was found, using parametric analysis, that the constant-head test offers several advantages over the standard techniques. In particular, it was determined that this method is very sensitive to the case where a zone of material adjacent to the measurement well is of greater permeability than the formation. In addition, this method is found to be an efficient alternative to the very-long-term slug tests which are sometimes employed to determine hydraulic conductivity in low-permeability clays and landfill clay liners.

SOMMAIRE À L'INTENTION DE LA DIRECTION

L'un des aspects les plus importants au sujet de l'emplacement et de la conception des installations d'entreposage des déchets radioactifs et des déchets dangereux concerne les estimations du temps de parcours des eaux souterraines. Les estimations de ce temps sont habituellement calculées à l'aide de mesures moyennes du gradient hydraulique et d'une mesure du coefficient de perméabilité. La plupart des hydrogéologistes utilisent des techniques standards (comme l'essai de puits) afin d'établir les estimations du coefficient de perméabilité du matériau peu perméable qui sert de barrière hydraulique dans les installations d'entreposage des déchets. Malheureusement, il a été démontré dernièrement que ces méthodes sont généralement imprécises dans la plupart des conditions, notamment lorsqu'elles sont appliquées dans des milieux argileux (comme ceux de la plupart des sites d'entreposage des déchets dangereux du sud de l'Ontario). Donc, pour améliorer les méthodes pratiques et rendre l'estimation du coefficient de perméabilité plus précise, on a mis au point un nouveau modèle analytique qui décrit de façon plus réaliste la configuration des essais physiques pour un type donné d'essai hydraulique connu sous le nom d'essai du perméamètre à charge constante.

À l'aide de l'analyse paramétrique, on a trouvé que l'essai du perméamètre à charge constante est beaucoup plus avantageuse que les techniques habituelles. Notamment, il a été établi que cette méthode est très sensible lorsqu'une zone de matériau adjacente au puits de mesure est plus perméable que la formation rocheuse. De plus, cette méthode est une solution efficace aux essais de puits à très long terme qui sont parfois utilisés pour calculer le coefficient de perméabilité des argiles peu perméables et des revêtements d'argile des sites d'enfouissement.

ABSTRACT

Hydraulic tests conducted using a source condition of constant head are frequently employed to determine the hydraulic properties of low-permeability media. In this paper, an analytical model is developed for analysing the results of a constant head test conducted under conditions where the influence of finite-thickness skin and partial penetration are present at the source well. The analytical model is derived by application of the Laplace transform method with respect to time and the finite Fourier cosine transform with respect to the vertical coordinate. Verification of the solution is undertaken by comparison to other solutions with slightly different boundary conditions. The solution is numerically evaluated by analytical inversion of the Fourier transform and numerical inversion of the Laplace transform. The solution is employed to produce type curves of dimensionless flow rate versus dimensionless time so as to investigate the influence of finite-thickness skin and partial penetration on the results of constant head tests. In addition, an investigation is conducted to determine the utility of type curve analysis for uniquely determining formation properties under these conditions. Results show that the presence of a skin zone of finite-thickness having permeability less than the formation produces an inflection point in the type curves at which point the dimensionless flow rate tends asymptotically towards a steady value. For the case where the skin and formation permeabilities are similar, these type curves can be employed to uniquely define both the skin and formation properties. For larger constrasts in permeability, the type curves become non-unique. Where the skin zone is very small, the shape of the type curves mimic the curve for a uniform, fully confined medium, and thus only the properties of the skin are measured using type curve analysis. Conversely, the type curves for the case where the skin zone is of greater permeability than the formation show a unique and interpretable shape for each skin thickness and ratio of formation/skin permeability over most practical values of hydraulic diffusivity. In addition, it was found that the effect of partial penetration can be pronounced in wells completed with small screen lengths. In many cases, however, the influence of partial penetration can be avoided by proper test design.

RÉSUMÉ

Des essais hydrauliques effectués avec une source à charge constante sont souvent employés afin d'établir les propriétés hydrauliques de milieux peu perméables. Dans le présent document, un modèle analytique est mis au point pour analyser les résultats d'un essai de perméamètre à charge constante où l'influence d'une épaisseur déterminée du revêtement et la pénétration partielle sont présents dans le puits d'alimentation. Le modèle analytique est dérivé par application de la méthode de la transformation de Laplace en fonction du temps et de la transformation de Fourier à nombre fini de coefficients (cosinus) en fonction des coordonnées verticales. La vérification de la solution est effectuée par comparaison d'autres solutions avec des conditions limites légèrement différentes. La solution est évaluée numériquement par inversion de la transformation de Fourier et par inversion numérique de la transformation de Laplace. La solution est employée pour produire des courbes types de débit adimensionnel par rapport au temps adimensionnel de manière à étudier l'influence de l'épaisseur à valeur finie du revêtement et de la pénétration partielle sur les résultats des essais du perméamètre à charge constante. De plus, une étude est menée afin de déterminer l'utilité de l'analyse de courbes types dans le seul but d'établir les propriétés de la formation rocheuse dans ces conditions. Les résultats montrent que la présence d'une zone de revêtement à épaisseur finie moins perméable que la formation rocheuse produit un point d'inflexion dans les courbes types où le débit adimensionnel tend de façon asymptotique vers une valeur constante. Dans le cas où les perméabilités du revêtement et de la formation rocheuse sont semblables, ces courbes types peuvent être utilisées pour définir d'une manière unique les propriétés du revêtement et de la formation rocheuse. Dans les cas de perméabilité à contraste plus marqué, les courbes types perdent leur unicité. Lorsque la zone de revêtement est très faible, la forme des courbes types

imite la courbe d'un milieu uniforme, entièrement confiné, et donc, seules les propriétés du revêtement sont mesurées au moyen de l'analyse des courbes types. Inversement, lorsque le revêtement est plus perméable que la formation rocheuse, les courbes types ont une forme unique et interprétable pour chaque épaisseur de revêtement et rapport entre la perméabilité de la formation rocheuse et le revêtement pour presque toutes les valeurs de diffusivité. De plus, on a constaté que l'effet d'une pénétration partielle peut être marqué dans les puits crépinés sur de faibles longueurs. Dans de nombreux cas, cependant, on peut éviter l'influence d'une pénétration partielle grâce à un plan d'essai approprié.

INTRODUCTION

There are three possible approaches to conducting a field test in a single well to measure the hydraulic conductivity and specific storage of an aquifer or aquitard. The two most commonly employed methods are 1) the pumping test where the flowrate of abstracted or injected water is maintained at a constant value and hydraulic head in the well varies with time and 2) the slug test where both the flowrate and hydraulic head in the well vary with time. The third, less frequently employed, method is conducted using a constant hydraulic head in the well (hereafter refereed to as the constant head test). Interpretation of the results of constant head tests are carried out using measurements of flowrate versus time. Although the constant head test is not employed widely in hydrogeological studies, this method is often used in the investigation of low-permeability materials, in particular, fractured or porous sedimentary and crystalline rocks (Doe and Remer, 1980; Doe *et al.*, 1987) and marine clays and clay tills (Wilkinson, 1968; Olson and Daniel, 1981; Tavenas *et al.*, 1990). The are principally two reasons for this: 1) it is often difficult to establish a steady flow rate in low-permeability media for a constant flow rate test and 2) unpractically long test durations are required to obtain enough information to analyse a slug test conducted in low-permeability formations.

When investigating sedimentary or crystalline rock, the constant head test is usually conducted by injecting water into a test section isolated by pneumatic packers. Water is injected into the test section by application of a steady pressure to a system of tanks at ground surface. The application of steady pressure is usually confirmed by downhole measurement using a pressure transducer (Doe and Remer, 1980; Price *et al.*, 1982; Bliss and Rushton, 1984; Doe *et al.*, 1987). Flowrate measurements are typically obtained using relatively rudimentary methods such as by measurement of the change in water level in the injection tanks at various times during the test. The test is completed when the flowrate becomes approximately steady. The results are interpreted using the analytical solution for steady state flow conditions in a confined, radial flow field otherwise known as the Theim equation (eg. Doe and Remer, 1980).

Constant head tests conducted in clay formations are performed in a similar manner. In this case, water is injected into augered boreholes or self-boring permeameters using a constant-head mariotte bottle located at ground surface (Tavenas *et al.*, 1990). Flowrate is reliably determined by sequential measurement of water level in the mariotte bottle during the course of the test. The injection pressure is determined based on the elevation of the mariotte bottle above the static water level in the well. The results are interpreted by plotting flowrate with respect to time, t, and extrapolating to $t \rightarrow \infty$ to estimate a flowrate at steady state (Tavenas *et al.*, 1990). In cases where the flow is not fully confined (partial penetration), a common condition for unconsolidated formations, the interpretation is modified using shape factors which have been derived to account for the patterns of flow around the test section (Hvorslev, 1951; Moye, 1967). More recently, Dagan (1978) derived a numerical scheme for the interpretation of constant-head tests at steady state conditions. The scheme is based on the distribution of sources of unknown strength along the borehole axis to account for the effects of unconfined flow and is valid only for the case where the length of well screen or test section is greater than five times the well radius. Braester and Thunvik (1984) investigated the application of Dagan's solution using a finite element model which included both the effects of partial penetration and a zone of reduced permeability adjacent to the well (see following discussion regarding skin effects) and found that the formulae developed by Dagan, Hvorslev and Moye all yield similar estimates of hydraulic conductivity. In addition, Braester and Thunvik found that by using the steady-state approximation, only the properties of the zone of reduced permeability are determined and not the true properties of the formation.

A more rigorous interpretation of the results of constant-head tests, for the fully confined case and a homogeneous medium, can be obtained by matching the change in flowrate versus time to the type curve developed by Jacob and Lohman (1952). In this case, a unique determination of both hydraulic conductivity and specific storage can be obtained provided accurate measurements of flowrate are determined. With the advent of accurate flowrate transducers having the capability to measure a broad range in flow, tests conducted in consolidated media can now be routinely interpreted in this manner. As aforementioned, accurate flowrate measurements for the case of tests conducted in clay materials can be obtained without the use of electronic means of measurement.

Unfortunately, the use of the Jacob and Lohman type curve is limited to those field situations where the length of the screen or measuring interval is large relative to the diameter of the well and where the effects of three-dimensional flow around the ends of the measuring interval are minimal. Type curves have not been developed for cases where the effects of the flow patterns around the well influence the transient flowrate measurements. Analytical and numerical studies have been undertaken in which transient flowrates were incorporated for the purpose of investigating the shape factor at steady state conditions (Randolph and Booker, 1982; Tavenas *et al.*, 1990).

The results of hydraulic tests conducted in both consolidated and unconsolidated media are sometimes influenced by the presence of a zone of reduced or enhanced permeability adjacent to the borehole. This is referred to as borehole skin in reservoir engineering terminology and can arise due to the invasion of drilling fluids, changes in the stress field adjacent to the well, changes in the geochemistry of the formation adjacent to the well or simply due to heterogeneity in the hydraulic conductivity field. Doe *et al.* (1987) provides a very thorough description of the physical manifestation of skin effects for both porous and fractured media.

Skin effects can be accounted for in the results of hydraulic tests by either assuming a skin of infinitesimal thickness whereby hydraulic head in the source well is proportional to the hydraulic head in the formation according to a head loss factor or by employing a radially-composite model where two governing equations are used, one for the finite-thickness skin around the well and one for the formation. Novakowski (1990) provides a more thorough description of the differences between these two approaches and suggests that, for the sake of wide applicability and more realistic depiction of the physical system, the use of a composite model is more desirable for hydrogeological studies.

Several analytical models have been developed to account for a radially-composite medium with a fully-penetrating well for both constant-rate pumping tests (Karasaki, 1986; Olarewaju and Lee, 1987; Butler, 1988; Novakowski, 1989) and slug tests (Moench and Hsieh, 1985). In both cases, it was determined that the effect of the skin is to shift the type curves in the time domain without influencing the shape of the type curve except for the case where the skin zone becomes very large in the radial direction (Moench and Hsieh, 1985; Novakowski, 1989). Consequently, it is very difficult to measure anything other than the properties of the skin in the presence of a skin of reduced permeability relative to the formation. Where the skin is of enhanced permeability, the type curves for different skin/formation ratios overlie each other and are thus non-unique except at very early time in the case of constant-rate pumping tests (Novakowski, 1989) and very late time in the case of slug tests (Moench and Hsieh, 1985).

Similarly, Uraiet (1980), used a finite-difference model having both inner and outer boundaries of constant head to investigate the effects of infinitesimally-thin skin. It was found that, except for early time where the effect of a finite-thickness skin is predominant, the type curves for flow rate are shifted vertically along the flow rate axis with virtually no difference in shape. Thus, as with the case for constant rate and slug tests, only the properties of the skin region are determined from later time data where the skin is of reduced permeability relative to the formation. Uraiet did not investigate the early-time flow rate data for means of distinguishing the formation properties, focusing more on the effect of the outer boundary at late time.

Olarewaja and Lee (1987) and Olarewaja *et al.* (1991) developed an analytical model for a fully-confined composite medium where the well test is conducted using a constant-head. Type curves of dimensionless flow rate and cumulative flow rate (the integral of dimensionless flow rate with respect to time) versus dimensionless time are presented for the case where the formation has an outer boundary and the permeability of the skin zone is ten times that of the formation. This condition is common in hydrocarbon reservoirs where wells are often stimulated to increase production by acidification or hydraulic fracturing. The type curves are developed for several ratios of the radius of the inner region to the radius of the formation region and show significant departure from the uniform medium case (skin and formation properties equivalent) where the skin thickness is large relative to the thickness of the formation (i.e. approaching one-half the extent of the formation). Because these type curves reflect physical conditions not typical to hydrogeology, especially with regard to the boundedness of the formation, use of these curves to determine formation properties for hydrogeological problems is not practical.

The effect of partial penetration on the shape of type curves for constant rate tests and slug tests has long been recognized (Hantush, 1964; Earlougher, 1977). In the petroleum industry, the influence of partial penetration is sometimes referred to as pseudoskin due to the similar manner in which the effects of skin and partial penetration are manifest in well test data. Although several studies have been conducted to investigate the effect of partial penetration alone, very few have been conducted to investigate the combined effects of skin and partial penetration. Jones and Watts (1971) developed a semi-empirical equation for a skin factor (infinitesimally-thin skin) on the basis of a numerical modelling study, which related the two effects. However, no known studies have been conducted to investigate the combined effect on the shape of type curves for any of the three hydraulic testing types, although some models are available which could be employed to do so (eg. Dougherty and Babu [1984] for a slug test conducted with an infinitesimally-thin skin).

The purpose of this paper is to develop an analytical model for constant-head tests which accounts for the presence of a skin zone of finite-thickness and for the effect of partial penetration. The model is employed to develop type curves to investigate the influence of skin and partial penetration on the transient flowrate from the test well. In addition, means are sought by which to distinguish the properties of the formation from the results of constant-head tests influenced by these effects. The investigation is conducted within the context of field conditions typical for tests conducted in both consolidated and unconsolidated media.

MATHEMATICAL DEVELOPMENT

Development of the following analytical model is conducted using the Laplace transform and finite Fourier cosine transform methods. Due to the complexity of the final solution, analytical inversion will be undertaken for the Fourier transform only. The Laplace transform will be inverted numerically using the Talbot (1979) algorithm. An example of the forward and inverse transform pair for the finite Fourier cosine transform can be found in Churchill (1972).

To provide a basis for the development of the more complicated model, it is instructional to derive the fundamental solution, in the Laplace domain, equivalent to that derived by Jacob and Lohman (1952). The governing equation for radial flow in a semi-infinite and fully confined domain is given as:

$$\frac{\partial^2 h}{\partial r^2}+\frac{1}{r}\frac{\partial h}{\partial r}=\frac{S_s}{K_r}\frac{\partial h}{\partial t} \qquad r\geq r_w \tag{1}$$

where t is time, r_w is the well radius, S_s is specific storage, K_r is hydraulic conductivity in the radial direction, h is induced hydraulic head and r is radial distance from the axis of the well. The initial condition for equation (1) is

$$h(r,0)=0 \qquad r_w<r<\infty \tag{2}$$

and the outer boundary condition is given by:

$$h(\infty,t)=0 \tag{3}$$

The inner boundary condition is

$$h(r_w,t)=H_0 \tag{4}$$

where H_0 is hydraulic head in the well. Equation (1) is translated into a dimensionless form using the following dimensionless variables:
where the subscript D denotes a dimensionless parameter.

The solution to equation (1) is obtained using the Laplace transform method with respect to time. In the Laplace domain the particular solution is
where p is the Laplace variable, K_0 is the modified Bessel function of the first kind, order

$$
\begin{aligned}
r_D &= \frac{r}{r_w} \\
h_D &= \frac{h}{H_0} \\
t_D &= \frac{t K_r}{r_w^2 S_s}
\end{aligned}
\tag{5}
$$

$$
\bar{h}_D(r_D, p) = \frac{K_0(\sqrt{p}\, r_D)}{p\, K_0(\sqrt{p})}
\tag{6}
$$

zero and the overbar indicates the Laplace transformed parameter.

To find flow rate, Darcy's law is employed at the well face. A well of finite diameter is used rather than erroneously assuming an infinitesimally-thin line source. The difference between the two approaches can be substantive at early time for the constant-rate test (Barker, 1991) and it is assumed that the same is true here. Thus, the equation for the flow rate from the well becomes:

$$
-Q_w(t) = 2\pi K_r L r_w \left.\frac{\partial h}{\partial r}\right|_{r=r_w}
\tag{7}
$$

where L is the thickness of the aquifer and Q_w is the flowrate. Equation (7) can also be expressed in dimensionless form using

$$
-Q_{wD}(t_D) = \left.\frac{\partial h_D}{\partial r_D}\right|_{r_D=1}
\tag{8}
$$

where Q_{wD} is the dimensionless flow rate given by

$$
Q_{wD}(t_D) = \frac{Q_w(t)}{2\pi K_r L H_0}
\tag{9}
$$

The solution to equation (8) is obtained by taking the gradient of (6) at r_D=1 which gives

$$Q_{wD}(p)=\frac{K_1(\sqrt{p})}{\sqrt{p}\,K_0(\sqrt{p})} \tag{10}$$

where K_1 is the modified Bessel function of the first kind, order one. Jacob and Lohman analytically inverted equation (10) in dimensional form.

Analytical Model for Finite-Thickness Skin and Partial Penetration

To account for the effects of partial penetration, a term is added to the governing equation for flow in the vertical direction. The effect of finite-thickness skin is accounted for as aforementioned, by employing two governing equations, one for each region of the media. Figure 1 shows schematically the boundary value problem. Note that the skin region is assumed to extend to the lower boundary in z. Clearly this is not representative of the ideal field case in unconsolidated formations, however, it is estimated that this assumption has minimal effect on the flow rate calculated in the well. Also, in many field situations it is likely that the skin region does extend for some distance below the screened portion of the well completion due to changes in stress and compaction of the clay material. In the case where the tests are conducted in consolidated material using packers, the skin region is likely to extend the entire length of the formation.

The upper boundary condition as depicted in Figure 1 is also poorly representative for some field cases in unconsolidated material where the upper boundary is unconfined. In such cases, an upper boundary condition of specified head is often used to simulate the unconfined conditions (Dagan, 1978; Tavenas *et al.*, 1990). For practical purposes, in the following development it is assumed that the presence of an unconfined upper boundary has no influence on the results of a hydraulic test conducted in clay material, particularly at early time. Thus, to simulate an unconfined condition using the following solution, the ratio of screen or test section length over the thickness of the formation is minimized (ie. to 0.01 or less).

In the following derivation, general and particular solutions are developed for both the skin zone and the formation. It should be noted however that it is the solution for the skin zone only that is employed in the derivation of the solution for flow from the well.

Thus, the solution for hydraulic head in the formation is presented for clarity only and is not germane to the purpose of the investigation.

The governing equations for radial flow are given as follows:

$$K_{r1}\frac{\partial^2 h_1}{\partial r^2}+\frac{K_{r1}}{r}\frac{\partial h_1}{\partial r}+K_{z1}\frac{\partial^2 h_1}{\partial z^2}=S_{s1}\frac{\partial h_1}{\partial t} \qquad r_w\leq r\leq r_s \tag{11}$$

$$K_{r2}\frac{\partial^2 h_2}{\partial r^2}+\frac{K_{r2}}{r}\frac{\partial h_2}{\partial r}+K_{z2}\frac{\partial^2 h_2}{\partial z^2}=S_{s2}\frac{\partial h_2}{\partial t} \qquad r\geq r_s \tag{12}$$

where the subscript 1 denotes the skin zone, subscript 2 denotes the formation, K_r is the hydraulic conductivity in the radial direction, K_z is the hydraulic conductivity in the vertical direction and r_s is the radial thickness of the skin zone. Specific storage is assumed to be isotropic and unique to each region. The initial condition for equations (11) and (12) is

$$h_1(r,z,0)=h_2(r,z,0)=0 \tag{13}$$

and the outer boundary condition for the formation is given by:

$$h_2(\infty,z,t)=0 \tag{14}$$

Continuity between the skin zone and the formation is defined by:

$$h_1(r_s,z,t)=h_2(r_s,z,t) \tag{15}$$

$$K_{r1}\frac{\partial h_1(r_s,z,t)}{\partial r}=K_{r2}\frac{\partial h_2(r_s,z,t)}{\partial r} \tag{16}$$

The inner and outer boundary conditions in the z-direction are, respectively:

$$\frac{\partial h_1(r,0,t)}{\partial z}=\frac{\partial h_2(r,0,t)}{\partial z}=0 \tag{17}$$

$$\frac{\partial h_1(r,L,t)}{\partial z}-\frac{\partial h_2(r,L,t)}{\partial z}=0 \tag{18}$$

Initially, the boundary value problem will be solved for a point source of constant strength, H_0, from which the final solution for a line source will be obtained by integration. Thus, the inner boundary condition in the radial direction is given as:

$$h_1(r_w,z,t)=H_0\delta(z-z') \tag{19}$$

where z' is the location of the point source in the z-direction and δ is the Dirac delta function.

As a means of facilitating the solution of this boundary value problem (BVP), the following dimensionless variables are employed:

$$\begin{gathered} t_D=\frac{K_{r2}t}{S_{s2}r_w^2} \qquad r_{Ds}=\frac{r_s}{r_w} \qquad r_D=\frac{r}{r_w} \\ L_D=\frac{L}{r_w} \qquad h_{D1}=\frac{h_1}{H_0} \qquad h_{D2}=\frac{h_2}{H_0} \\ \alpha_1=\frac{K_{z1}}{K_{r1}} \qquad \alpha_2=\frac{K_{z2}}{K_{r2}} \\ \gamma=\frac{K_{r2}}{K_{r1}} \qquad \xi=\frac{S_{s1}}{S_{s2}} \end{gathered} \tag{20}$$

where, again, the subscript D denotes a dimensionless variable.

To solve the boundary value problem, the Laplace and finite Fourier cosine transform are successively applied to the governing equations and boundary conditions. The Laplace transform is applied with respect to time and the finite Fourier transform is applied with respect to the z-coordinate. In dimensionless form, the subsidiary equations are given as follows:

$$\frac{d^2\bar{h}_{D1}}{dr_D^2}+\frac{1}{r_D}\frac{d\bar{h}_{D1}}{dr_D}-q_1^2\bar{h}_{D1}=0 \tag{21}$$

$$\frac{d^2\tilde{h}_{D2}}{dr_D^2}+\frac{1}{r_D}\frac{d\tilde{h}_{D2}}{dr_D}-q_2^2\tilde{h}_{D2}=0 \tag{22}$$

where the tilde indicates the double transformed parameter and

$$\begin{aligned} q_1^2&=\alpha_1\omega^2+\gamma\xi p \\ q_2^2&=\alpha_2\omega^2+p \end{aligned} \tag{23}$$

where p is the Laplace transform variable as before and

$$\omega=\frac{n\pi}{L_D} \tag{24}$$

where n is the Fourier transform variable. The transformed continuity conditions between the skin zone and formation are given by:

$$\tilde{h}_{D1}(r_{Ds},n,p)=\tilde{h}_{D2}(r_{Ds},n,p) \tag{25}$$

$$\frac{d\tilde{h}_{D1}}{dr_D}(r_{Ds},n,p)=\gamma\frac{d\tilde{h}_{D2}}{dr_D}(r_{Ds},n,p) \tag{26}$$

and the inner and outer boundary conditions in the radial direction, respectively, are given by:

$$\tilde{h}_{D1}(1,n,p)=\frac{\cos(\omega z_D')}{p} \tag{27}$$

$$\tilde{h}_{D2}(\infty,n,p)=0 \tag{28}$$

where equation (27) is found using the sifting property of the Dirac function.

The general solution to equations (21) and (22) is found in a straight forward manner and after application of the outer boundary condition, equation (28), the solution for h_{D1} and h_{D2}, respectively, are given by:

$$\bar{h}_{D1}(r_D,n,p)=AI_0(q_1r_D)+BK_0(q_1r_D) \tag{29}$$

$$\bar{h}_{D2}(r_D,n,p)=DK_0(q_2r_D) \tag{30}$$

where A,B and D are constants to be determined from the boundary conditions and I_0 is the modified Bessel function of the second kind, order zero.

The particular solution is found by substitution of the general solution into the boundary conditions, equations (25)-(27). This leads to a system of three equations with three unknowns which is solved using Cramer's rule. Hence, the particular solution in the Laplace and Fourier domains for dimensionless hydraulic head in the skin zone is:

$$\bar{h}_{D1}(r_D,n,p)=\frac{\cos(\omega z_D')}{p}\frac{[I_0(q_1r_D)\beta_1-K_0(q_1r_D)\beta_2]}{[I_0(q_1)\beta_1-K_0(q_1)\beta_2]} \qquad 1\leq r_D\leq r_{Ds} \tag{31}$$

and for the formation:

$$\bar{h}_{D2}(r_D,n,p)=-\frac{\cos(\omega z_D')}{p}\frac{K_0(q_2r_D)}{[I_0(q_1)\beta_1-K_0(q_1)\beta_2]} \qquad r_D\geq r_{Ds} \tag{32}$$

where

$$\beta_1=\gamma q_2K_0(q_1r_{Ds})K_1(q_2r_{Ds})-q_1K_0(q_2r_{Ds})K_1(q_1r_{Ds}) \tag{33}$$

$$\beta_2=\gamma q_2I_0(q_1r_{Ds})K_1(q_2r_{Ds})+q_1I_1(q_1r_{Ds})K_0(q_2r_{Ds}) \tag{34}$$

and I_1 is the modified Bessel function of the second kind, order one.

To employ the particular solution, the transforms must be inverted and the solution for a point source integrated from b_1 to b_2 (Figure 1) to provide the solution for a well screen or test section of finite length. Hence, recalling that analytical inversion is undertaken for the Fourier transform only, and upon completion of the integration, the solution for the hydraulic head in the skin zone, in the Laplace domain, is:

$$\bar{h}_{D1}(r_D,z_D,p)=\frac{[I_0(q_3r_D)\beta_3-K_0(q_3r_D)\beta_4]}{L_D p\psi}(B_2-B_1)$$
$$+\frac{2}{\Pi}\sum_{n=1}^{\infty}\frac{\cos(\omega z_D)}{p}\frac{[I_0(q_1r_D)\beta_1-K_0(q_1r_D)\beta_2]}{\zeta}\cdot\frac{1}{n}[\sin(\omega B_2)-\sin(\omega B_1)] \tag{35}$$

and for the hydraulic head in the formation:

$$\bar{h}_{D2}(r_D,z_D,p)=-\frac{K_0(q_4r_D)}{L_D p r_{Ds}\psi}(B_2-B_1)$$
$$-\frac{2}{\Pi}\sum_{n=1}^{\infty}\frac{\cos(\omega z_D)}{p}\frac{K_0(q_2r_D)}{\zeta}\cdot\frac{1}{n}[\sin(\omega B_2)-\sin(\omega B_1)] \tag{36}$$

where

$$B_1=\frac{b_1}{r_w} \qquad B_2=\frac{b_2}{r_w}$$
$$q_3^2=\gamma\xi p \qquad q_4^2=p$$
$$\zeta=I_0(q_1)\beta_1-K_0(q_1)\beta_2 \tag{37}$$
$$\psi=I_0(q_3)\beta_3-K_0(q_3)\beta_4$$

and

$$\beta_3=\gamma q_4K_0(q_3r_{Ds})K_1(q_4r_{Ds})-q_3K_0(q_4r_{Ds})K_1(q_3r_{Ds}) \tag{38}$$

$$\beta_4=\gamma q_4I_0(q_3r_{Ds})K_1(q_4r_{Ds})+q_3I_1(q_3r_{Ds})K_0(q_4r_{Ds}) \tag{39}$$

To determine the flow rate from the well, the solution for the skin zone is employed along with an expression for Darcy's law similar to that given by equation (7). In dimensionless form, the equation for dimensionless flow at the well, Q_{Dp}, is given by:

$$-Q_{Dp}(t_D)=\int_{B_1}^{B_2}\left.\frac{\partial h_{D1}}{\partial r_D}\right|_{r_D=1} dz_D \tag{40}$$

To equate Q_{Dp} to Q_{wD} both sides of equation (40) are divided by (B_2-B_1). Thus, the dimensionless flowrate is given by:

$$Q_{Dp}(t_D)=\frac{Q_w(t)}{2\pi(b_2-b_1)K_{r1}H_0} \tag{41}$$

Unfortunately, it is necessary to use the permeability of the skin, K_{r1}, rather than that of the formation in the dimensionless flow rate due to the way in which Darcy's Law is formulated in this case.

The solution to equation (40) is obtained by application of the forward Laplace transform in conjunction with the gradient of equation (35) with respect to radial distance. Upon conducting the integration in equation (40), the solution is found in the Laplace domain:

$$\begin{aligned} Q_{Dp}(p)= & -\frac{(\gamma\xi)^{1/2}}{L_D p^{1/2}}\frac{[I_1(q_3)\beta_3+K_1(q_3)\beta_4]}{\psi}(B_2-B_1) \\ & -\frac{2L_D}{\Pi^2}\sum_{n=1}^{\infty}\frac{q_1}{p}\frac{[I_1(q_1)\beta_1+K_1(q_1)\beta_2]}{\zeta} \\ & \cdot\frac{1}{n^2(B_2-B_1)}[\sin(\omega B_2)-\sin(\omega B_1)]^2 \end{aligned} \tag{42}$$

Note that the solution has two parts, the first term on the right hand side (RHS) being the solution for fully confined radial flow and the second term (the summation) accounting for the effect of partial penetration.

Verification and Special Cases

As a means of verifying the solutions, equations (35),(36) and (42) are algebraically reduced to less complicated problems and then compared to solutions independently derived from the new BVP. Three alternative boundary value problems are used for verification. These include; 1) fully confined flow with finite-thickness skin, 2) partial penetration in a homogenous medium, and 3) fully confined flow in a homogeneous and isotropic medium. In the latter case, verification is achieved by direct comparison to a published solution (Jacob and Lohman, 1952).

Manipulation of equations (35) and (36) for the case for fully confined flow with finite-thickness skin is achieved by setting b_1 to zero and b_2 to the thickness of the aquifer, L. Thus, equations (35) and (36) are algebraically reduced to, respectively:

$$\bar{h}_{D1}(r_D,p)=\frac{[I_0(q_3r_D)\beta_3-K_0(q_3r_D)\beta_4]}{p\psi} \tag{43}$$

$$\bar{h}_{D2}(r_D,p)=-\frac{K_0(q_4r_D)}{pr_{Ds}\psi} \tag{44}$$

for hydraulic head in the skin zone and formation. The solution for flow rate is found in the same manner and is given by:

$$Q_{Ds}(p)=-\frac{(\gamma\xi)^{1/2}}{p^{1/2}}\frac{[I_1(q_3)\beta_3+K_1(q_3)\beta_4]}{\psi} \tag{45}$$

where Q_{Ds} is defined by equation (9) with K_r replaced by K_{r1}.

Equations (43)-(45) are also obtained by direct solution of equations (11)-(16) (after elimination of the term in the z-direction in [11] and [12]) where equation (4) is employed as the inner boundary condition in the r-coordinate. Equation (45) is obtained using equation (43) and equation (8). Comparison of equations (43)-(45) to the solution of Olarewaju and Lee (1987) can not be conducted directly because of the difference between the outer boundary condition employed in each case.

To obtain the solution for the case of partial penetration in a homogenous medium, γ, ξ and r_{Ds} are set equal to one and either equation (35) or (36) is reduced to:
where q_5 is equal to $(\omega^2+p)^{1/2}$ and the Wronskian $I_0(x)K_1(x)+I_1(x)K_0(x)=1/x$ is employed. Similarly, the solution for flow rate is given by:
where Q_{Dpp} is defined by equation (41) except that K_{r1} is replaced by K_r. Again, equations

$$\bar{h}_{D2}(r_D,z_D,p)=\frac{K_0(q_4 r_D)}{L_D p K_0(q_4)}(B_2-B_1)$$
$$+\frac{2}{\Pi}\sum_{n=1}^{\infty}\frac{\cos(\omega z_D)}{p}\frac{K_0(q_5 r_D)}{K_0(q_5)}$$
$$\cdot\frac{1}{n}[\sin(\omega B_2)-\sin(\omega B_1)] \tag{46}$$

$$Q_{Dpp}(p)=\frac{K_1(q_4)}{L_D q_4 K_0(q_4)}(B_2-B_1)+\frac{2L_D}{\Pi^2}\sum_{n=1}^{\infty}\frac{q_5 K_1(q_5)}{p K_0(q_5)}$$
$$\cdot\frac{1}{n^2(B_2-B_1)}[\sin(\omega B_2)-\sin(\omega B_1)]^2 \tag{47}$$

(46) and (47) are also obtained by direct solution of the BVP. In this case, equation (11) is used as the governing equation (only one governing equation required) and boundary conditions (13), (14), (17), (18) and (19) are employed. Equation (47) is found by substitution of equation (46) into equation (40).

Finally, to obtain the solution for a fully confined, homogeneous and isotropic medium equivalent to equations (6) and (10) for hydraulic head and flow rate, respectively, any one of equations (35), (36), (42)-(47) can be employed. For example, by substituting into equation (35), γ and ξ equal to one and b_2 equal to L with b_1 equal to zero, equation (6) is obtained. Alternatively, by setting b_1=0 and b_2=L, equation (47) reduces to :

$$Q_{Dpp}(p)=\frac{K_1(\sqrt{p})}{\sqrt{p}K_0(\sqrt{p})} \tag{48}$$

which is identical to equation (10). As aforementioned, equations (6) and (10) are verified against the published solution of Jacob and Lohman (1952).

Numerical Evaluation

To develop type curves for use in the following discussion, equation (42) is numerically evaluated in three steps. First, the Talbot (1979) algorithm is employed to invert the first term on the RHS and to calculate the terms necessary for the summation.

Secondly, the series is calculated using a total of 300-500 terms and finally, the first term and the sum are added to provide the value for flow rate from the well at any given point in time. In addition, as a means of improving computational efficiency, the less complex solutions given by equations (10), (45) and (47) are employed in place of (42), where warranted. All calculations are conducted using double precision format.

The Talbot algorithm was selected for numerical inversion of the Laplace transform due to the robust behavior this method has been shown to exhibit in inverting analytical problems of a similar nature (Novakowski, 1989; Karasaki,1990). As a means of evaluating the Talbot algorithm for the problem developed herein, a comparison was made to the DeHoog *et al.*(1982) algorithm which is also known to be robust (Novakowski, 1991). The results showed that the Talbot algorithm is stable over a wider range of values than the Dehoog method, for this particular problem.

Because of the large computational demands required to numerically invert the Laplace transform in conjunction with series evaluation, the epsilon-partial sums algorithm (EPAL) developed by Wynn (1956) was tested as a means by which to improve efficiency. Unfortunately, because of the alternating and slowly converging nature of the series for particular values of L, b_1 and b_2, the EPAL algorithm will not yield consistently reliable results. Consequently, the series is calculated directly to a specified number of terms. By monitoring the convergence of the series, it was found that accuracy of three significant figures is obtained using approximately 400 terms.

RESULTS AND DISCUSSION

Although constant head tests are primarily employed for investigating low-permeability materials, the possible range of hydraulic diffusivity that can be measured using this method is large when considering results from both consolidated and unconsolidated media. For example, typical values of hydraulic diffusivity for glacial clay range from 10^{-8} to as high as 10^{-3} and values for sedimentary and crystalline rock range from less than 10^{-4} to 10^{2}. This gives a total range of approximately 10 orders of magnitude. Therefore, the following discussion is divided into two sections, one for the range of hydraulic diffusivities and test conditions typical of consolidated media and one for those typical of unconsolidated media. Because the duration of constant head tests usually doesn't exceed three orders of magnitude of time (10^{1} s to 10^{4} s) only eight orders of magnitude in dimensionless time is required to define a type curve for each range of diffusivity.

Figures 2a and 2b show type curves, Q_{wD} versus t_D, for each range, assuming confined flow in a homogeneous medium (i.e. Jacob and Lohman solution). Note that the type curve is relatively linear at early time in Figure 2a and late time in Figure 2b. Consequently, recalling that hydraulic diffusivity is directly proportional to t_D, unique type curve fits are difficult to obtain in low-permeability clay and high-permeability rock using this type curve. Unique type-curve fits are probably limited to the range of t_D between 10^{-1} and 10^{4}. Furthermore, in comparison to the type curve for a constant-rate hydraulic test (Theis curve), the overall degree of curvature is considerably less. Hence, additional reason as to the popularity of the constant-rate test versus constant-head test for investigating fully confined permeable media.

It should also be noted that it is possible to manipulate the initial or final value of t_D by means of test design. By scrutiny of the coefficients in the definition of t_D, equation (5) or (20), it is apparent that well diameter can have substantial influence on the value of t_D. Thus, in particular field cases where an estimate of hydraulic diffusivity is available, it may be possible to select the diameter of the borehole or augeice such that the initial or final value of t_D can be favourably adjusted into a portion of the type curve that exhibits greater curvature, thus increasing the likelihood of obtaining a unique type-curve fit. For example, minimizing the borehole diameter for tests conducted in moderate- to highly-permeable rock will decrease the initial value of t_D into the mid- to low-range on Figure 2b, thus improving the possibility of a good fit to the type curve. This method also applies to the following discussion where type curves of different shape are presented.

All discussion related to the effects of partial penetration are based on the assumption that the well screen or test section is centered between the inner and outer boundaries in z. This is done for simplicity in interpretation and is not a prerequisite for use of the analytical model.

Constant Head Tests Conducted in Consolidated Media

In many sedimentary and crystalline rocks, the direction and rate of groundwater flow is governed by secondary porosity which usually occurs in the form of fractures. In many cases, particularly in the vicinity of ground surface, horizontal fractures predominate. In older sedimentary rock and in most crystalline rock, the rock matrix often has minimal permeability. Thus, during a field hydraulic test, it is quite common for the majority of radial flow to be in the horizontal fractures only and thus fully confined conditions prevail. In the case where a skin zone of finite-thickness is present, equation (45) can be employed to interpret the results of a constant-head test. In the following, the storage coefficients of the skin zone and formation are assumed to be equal.

Figure 3 shows the type curves for the case where a skin zone of r_{Ds} equal to ten has a range of permeability relative to the formation from 1 to 1000 (γ= 1, 5, 10, 100, 1000). In dimensional terms, an r_{Ds} of ten is equivalent to a skin zone 0.5 m in thickness, around a well 0.1 m in diameter. This is approximately what might be expected for a typical mud-invaded well drilled in sedimentary rock. In all cases, the type curves for a skin zone of reduced permeability lie above the curve for a uniform medium and, particularly for lower permeability skin, approach an asymptotic value for large dimensionless times. The shape of these curves are diagnostic, particularly for cases where the permeability of the skin zone is only slightly less than that of the formation and where the constant-head test is conducted over the range of t_D in which the inflection point is evident. However, if the test is conducted in the range outside of that containing the inflection point, for example in the higher-permeability range, misinterpretation of the value of hydraulic diffusivity will occur if the type curve for a uniform medium is employed. In addition, the distinction between the type curves for larger values of γ (i.e. 100 to 1000) is very subtle and non-unique results may be obtained from type-curve analysis under these conditions. Interestingly, if a particular test conducted in higher permeability media is interpreted in the absence of flow rate measurements using the steady-state approximation and a standard estimate of the radius of influence, the value obtained for hydraulic conductivity will more closely reflect that of the formation than that of the skin zone for a skin thickness of the order discussed above.

Type curves can also be generated for the case where the permeability of the skin zone is greater than the permeability of the formation. Figure 4 shows type curves for an r_{Ds} of 10, with a range of γ from 1.0 (the uniform medium case) to 0.01. Contrary to similar conditions for constant-rate tests (Novakowski,1989), in which the type curves are indistinguishable, the type curves shown in Figure 4 are distinctively shaped for each value of γ, but only at smaller values of t_D. Therefore, if good flow rate data is available for a constant head test conducted in low-permeability rock with a skin zone having enhanced permeability, the data could be uniquely interpreted for the properties of both the skin zone and the formation using equation (45). Conversely, if the test is conducted in rock of higher permeability, interpretation of the data for the hydraulic conductivity of the formation can be conducted using the type curve for a uniform media, without significant error or knowledge of the permeability of the skin zone. Values of the storage coefficient determined from this match, however, may be significantly in error.

The effect of the thickness of the skin zone is shown in Figure 5 where type curves are plotted for dimensionless radial thicknesses of 1.1, 2, 5, 10, and 25. The permeability of the skin for this case is an order of magnitude lower than the formation (γ=10). The type curve for r_{Ds} equal to 1.1 is almost identical in shape to the curve for a uniform medium (no skin) except the location is shifted vertically upward by one order of magnitude of Q_{Ds}. Therefore, the presence of a very thin skin can not be uniquely determined using the results of a constant-head test in this environment. Furthermore, interpretation of constant-head test results using the type curve for a uniform medium will lead to an estimate of the permeability of the skin and not the formation. Fortunately, and contrary to the case for a constant-rate test (Novakowski, 1989), with thicker skin zones the presence of the skin zone is uniquely identifiable for the lower range of formation permeabilties although some problems of non-uniqueness probably exist for larger γ. At early time (lower permeability), the decline in flow rate is greater (for r_{D_s} 2-25) and tends asymptotically toward the curve for a uniform medium. Thus, interpretation of the curve prior to the inflection point using the type curve for a uniform medium will lead to an underestimate of the permeability of the formation. Conversely, if the inflection point is present in the flow rate data, equation (45) can be used to uniquely determine the permeability of the formation for small γ.

For the case where the consolidated media is relatively unfractured or where the permeability of the fractures is similar to the permeability of the matrix, the effects of partial penetration may also influence the results of the constant head tests. This situations often occurs in Mesozoic and Cenozoic sedimentary and igneous rock. Figure 6 and 7 show the effect of partial penetration, in the absence of skin, on type curves developed for various lengths of test section in an aquifer of a dimensionless thickness of ten where ϕ=1 is equivalent to the fully confined case. Note the similarity in shape of the uppermost type curves relative to those for finite-thickness skin. However, for the case where the presence of the confining layer has little influence on the response (the uppermost curve in Figure 6 and the upper three curves in Figure 7), the type curves are virtually non-unique over most values of t_D. The upper curves in Figure 7 are typical of the flowrate data that might be observed in small diameter wells with short (ie. cm in length) test sections completed in thick formations. In this case, analysis of the constant head test must be conducted using an estimate of the flow rate at steady conditions and an appropriate shape factor. No information regarding the storativity of the formation is obtained. Figure 7 also shows the effect of test section length on the shape of the type curves. For small L_D (i.e. small test section length where ϕ is fixed at 0.01), the type curve is singularity flat and departs from the solution for a uniform medium at values of

t_D below the range presented here. As the length of test section is increased (larger L_D), the effect of partial penetration is increasingly diminished. For extreme cases such as the curve for L_D=20000 shown in Figure 7, the shape of the type curve mimics the curve for a uniform medium over most of the range of t_D presented here. Unfortunately, this family of type curves is unique to the value of ϕ and thus is applicable only in the case where neither the upper nor lower boundaries influence the test response. Further, where appropriate field conditions warrant the use of these curves, unique type curve fits can only be obtained for tests conducted in wells completed with longer test sections (B_2-$B_1 \geq 10$). In dimensional terms, a screen or test section length of 1.0 m in a typical N-sized borehole (76 mm diameter) would yield a value of B_2-B_1 equal to 26. Therefore, for most small diameter boreholes completed in consolidated material, analysis of constant head tests influenced by the presence of partial penetration can be conducted using the type curve for a uniform medium. In the case where the permeability of the medium is large, the well diameter is large or the test section is small, type curve analysis is intractable although for particular situations, interpretation of the test results can be undertaken using the steady flow rate and an appropriate shape factor.

Figure 8 shows the effect of finite-thickness skin on the shape of the type curves for tests conducted in partially penetrating wells. The ratio of screen or test section length to formation thickness, ϕ, is equal to 0.01 and r_{Ds} is equal to 10 for all cases. In comparison of the curves in Figure 8 for a skin of reduced permeability relative to the formation, to those shown in Figure 3, it is apparent that the effect of partial penetration is relatively minimal in terms of the shape of the curves. Therefore, where conditions of partial penetration exist in conjunction with skin of reduced permeability, analysis of flow rate data for interpretation of the properties of the formation can be undertaken by type curve analysis subject to the same conditions as those given for the fully confined case. Conversely, however, the curves for skin of enhanced permeability (γ=0.1 and 0.01) as presented in Figure 8, show that unique type curve match over most of this range of t_D is not possible for tests conducted in the presence of partial penetration. It is important to note that the presence of a skin zone of reduced permeability acts to enhance the curvature of the type curves for the partial penetration case, thus helping to increase the likelihood of obtaining a unique type curve fit. In the presence of partial penetration, the effect of the thickness of the skin zone is the same as for the fully confined case as shown in Figure 5.

Constant Head Tests Conducted in Unconsolidated Media

Hydraulic tests conducted in low-permeability clay material are widely known to be influenced by the effects of partial penetration (Tavenas et al., 1990) and a skin zone of reduced permeability (D'Astous *et al.*, 1989; Feenstra *et al.*, 1991). Several specialized devices and methods such as the self-boring permeameter have been developed to counteract these effects. Some methods, such as those based on large diameter auger holes (Boast and Kirkham, 1971) in conjunction with removal of the skin zone (D'Astous *et al.*, 1989), have been shown to provide reasonably accurate measurements of formation permeability. Unfortunately, conventional hydraulic tests conducted in large diameter boreholes often require weeks to conduct and are probably influenced by the presence of a residual skin effect even when the majority of the skin zone is removed. Therefore, in the following discussion, the possible use of the constant-head test method for investigating the hydraulic properties of clays in these conditions is explored.

Figure 9 shows the type curves for dimensionless flow rate from a partially penetrating well in the absence of a skin zone. The curves are constructed for a dimensionless aquitard thickness of 200 and for ϕ ranging from 0.1 to 0.005. In dimensional terms, these conditions are equivalent to a the case where a well of 0.5 m diameter is completed in an aquitard about 100 m in thickness. Screen lengths range from 0.5 m to 10 m. In the region where t_D is less than 10^{-1} all of the type curves fall on the curve for a fully confined medium. Therefore, interpretation of constant head tests conducted in auger holes of large diameter completed in very low-permeability clay can not be undertaken using type curve analysis due to the uniform slope of the type curve in this region. However, it is possible to analyse such a test using the steady approximation provided it is recognized that the traditional plot of flow rate versus the inverse root of time (Tavenas *et al.*, 1990) is not likely to be linear for this range of t_D. If a smaller diameter probe is employed, the initial value of t_D will be higher, more in the range of the family of curves shown in Figure 9. However, the equivalent value of L_D will also be much larger and the effect of partial penetration can probably be ignored particularly if long screen lengths are employed.

In low-permeability clay materials, the thickness of the skin zone is often very small and occurs as a smeared clay layer formed as the auger flights or probe cutters are inserted or removed. Typical skin thicknesses have been determined to range from a few mm to several cm for larger diameter auger holes (D'Astous *et al.*, 1989), although the stress effects associated with the presence of the hole are likely to extend the skin to somewhat greater distances. Figure 10 shows the type curves for a constant head test conducted in a small-diameter probe (L_D of 5000) where the results are influenced by a finite-thickness skin of dimensionless thickness ranging from 1.5 to 25. Curves are shown for the case where the permeability of the skin is one order of magnitude greater than the formation and for the case where it is two orders less. For the lower range of t_D, the curves for finite skin mimic the slope of the curve for a uniform medium but are shifted vertically in the y-axis. Therefore, irrespective of skin thickness, the measurement of permeability using the constant head test method in a large diameter auger hole, will lead to an accurate estimate of the value for the skin but not the formation. The same will be true for an analysis conducted using the steady approximation. However, where a smaller diameter probe is employed, there is more likelihood of obtaining a type curve fit which provides estimates of both skin and formation permeability (provided the uniqueness problems aforementioned are not encountered).

CONCLUSIONS

An analytical model was developed to provide type curves for the analysis of field hydraulic tests conducted using a constant hydraulic head. The model was developed to account for the influence of a skin zone of finite-thickness and the effects of a partially penetrating well completion. Two governing equations are employed, one for the skin zone and one for the formation. A constant first-type boundary is employed for the source condition and flow rate from or into the well is determined using Darcy's Law evaluated over the length of the well screen. The boundary value problem is solved by application of the Laplace transform with respect to time and the finite Fourier cosine transform with respect to the vertical coordinate, z. The particular solution is found in the Laplace and Fourier domains and the Fourier transform is analytically inverted. This results in an infinite series where the series coefficients must be numerically inverted from the Laplace domain. Verification of the solution was undertaken by algebraically reducing the problem to less complex solutions. The new solutions were then independently derived from the new BVP or verified against an existing solution.

The presence of a skin zone of finite thickness surrounding the test well substantially influences the shape of the transient flow rate data for tests conducted in both consolidated and unconsolidated media. Type curves developed for a range in hydraulic diffusivity typical for consolidated media show that the curves tend asymptotically to a steady value much more rapidly than in the absence of a finite skin for the case where the permeability of the skin is less than the formation. Because non-unique type curve fits may be obtained where the permeability of the skin is much less than the formation, type curve analysis is only recommended where the skin and formation permeabilities are relatively similar. The steady approximation for flow rate can be employed to determine the permeability of the formation in the case where the flow rate has reached the asymptotic steady condition. Where the thickness of the skin zone is very small, as is often found in wells completed in low-permeability clay material, the type curves mimic the shape of the curve for a uniform medium over all but the largest values of hydraulic diffusivity. Consequently, using either type curve analysis or the steady approximation to analyse test results will lead to an estimate of the permeability of the skin and not the formation. Conversely, the type curves for the case where the permeability of the skin is greater than the permeability of the formation will provide unique type curve fits for both the skin and formation permeability over many values of hydraulic diffusivity.

The effect of partial penetration on the results constant head tests can be substantial if the test well is completed with a short screen or test section length. The shape of type curves influenced by partial penetration is very similar to those influenced by a skin zone of reduced permeability. Consequently, some non-uniqueness is possible in data obtained from wells in which the influences of both finite skin and partial penetration are pronounced. Again, where the flow rate data has reached the asymptotic value, analysis can be conducted using the steady approximation and application of an appropriate shape factor. In addition, it is usually possible to design the well completion such that the effects of partial penetration are minimized. In most cases, analysis of the flow rate data can be undertaken using the type curve for a uniform, fully confined medium.

In general terms, the constant head test method offers a number of advantages relative to constant-rate and slug tests. In particular, the flow rate during a constant head test is more sensitive to the thickness of a skin zone of reduced permeability than the transient hydraulic head during a constant-rate test. Thus, better estimates of the skin thickness and permeability of the skin and formation can be obtained using the constant head test method. Both methods are subject to non-uniqueness for some conditions. Further, the constant head method is much superior for the case where the skin zone is of greater permeability than the formation. Unique fits are obtained over a substantial range in hydraulic diffusivity using the results from a constant head test, whereas the results from a constant-rate test are entirely non-unique. The influence of very thin skin zones common to clay materials remains a difficult problem, and none of the three hydraulic testing methods can be employed to determine anything but the properties of the skin zone.

NOTATION

A	constant determined from boundary conditions.
b_1	lower z coordinate of well screen, L.
b_2	upper z coordinate of well screen, L.
B	constant determined from boundary conditions.
B_1	b_1/r_w.
B_2	b_2/r_w.
D	constant determined from boundary conditions.
h	hydraulic head, L.
h_D	dimensionless hydraulic head, h/H_0.
H_0	hydraulic head in the well, L.
K_r	radial hydraulic conductivity, L/T.
K_z	vertical hydraulic conductivity, L/T.
L	aquifer or aquitard thickness, L.
L_D	dimensionless aquifer/aquitard thickness, L/r_w.
n	Fourier variable.
p	Laplace variable.
q_1	$(\alpha_1\omega^2+\gamma\xi p)^{1/2}$
q_2	$(\alpha_1\omega^2+p)^{1/2}$
q_3	$(\gamma\xi p)^{1/2}$
q_4	$p^{1/2}$
q_5	$(\omega^2+p)^{1/2}$
Q_w	volumetric flow rate, L^3/T.
Q_{Dp}	flow rate, $Q_w(t)/2\pi(b_2-b_1)K_{r1}H_0$.
Q_{Ds}	flow rate, $Q_w(t)/2\pi LK_{r1}H_0$.
Q_{wD}	flow rate, $Q_w(t)/2\pi LK_rH_0$.
Q_{Dpp}	flow rate, $Q_w(t)/2\pi(b_2-b_1)K_rH_0$.
r	radial distance, L.

r_D dimensionless radial distance, r/r_w.

r_w well radius, L.

S_s specific storage, 1/L.

t time, T.

t_D dimensionless time, uniform medium, tK_r/r_wS_s.

t_D dimensionless time, finite skin, tK_{r2}/r_wS_{s2}.

z vertical distance, L.

z' location of point source, L.

z_D dimensionless vertical distance, z/r_w.

z_D dimensionless location of point source, z'/r_w.

α K_z/K_r.

β_1 $\gamma q_2K_0(q_1r_{Ds})K_1(q_2r_{Ds})-q_1K_0(q_2r_{Ds})K_1(q_1r_{Ds})$.

β_2 $\gamma q_2I_0(q_1r_{Ds})K_1(q_2r_{Ds})+q_1I_1(q_1r_{Ds})K_0(q_2r_{Ds})$.

β_3 $\gamma q_4K_0(q_3r_{Ds})K_1(q_4r_{Ds})-q_3K_0(q_4r_{Ds})K_1(q_3r_{Ds})$.

β_4 $\gamma q_4I_0(q_3r_{Ds})K_1(q_4r_{Ds})+q_3I_1(q_3r_{Ds})K_0(q_4r_{Ds})$.

γ K_{r2}/K_{r1}.

δ dirac function.

ζ $I_0(q_1)\beta_1-K_0(q_1)\beta_2$.

ξ S_{s1}/S_{s2}.

ϕ $(B_2-B_1)/L_D$.

ψ $I_0(q_3)\beta_3-K_0(q_3)\beta_4$.

ω $n\pi/L_D$.

Subscripts 1 and 2 denote skin zone and formation respectively and K_0,K_1,I_0 and I_1 are modified Bessel functions.

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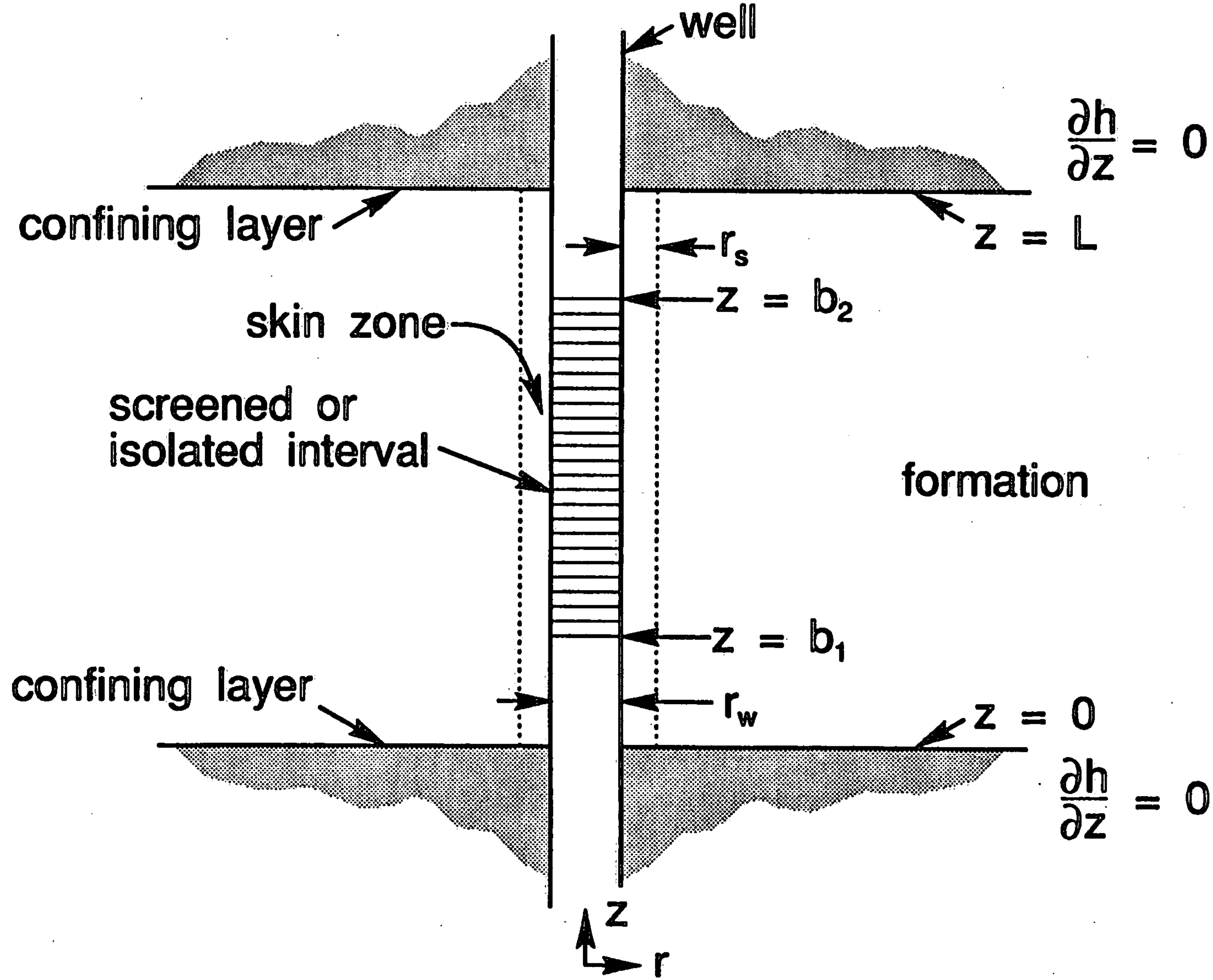
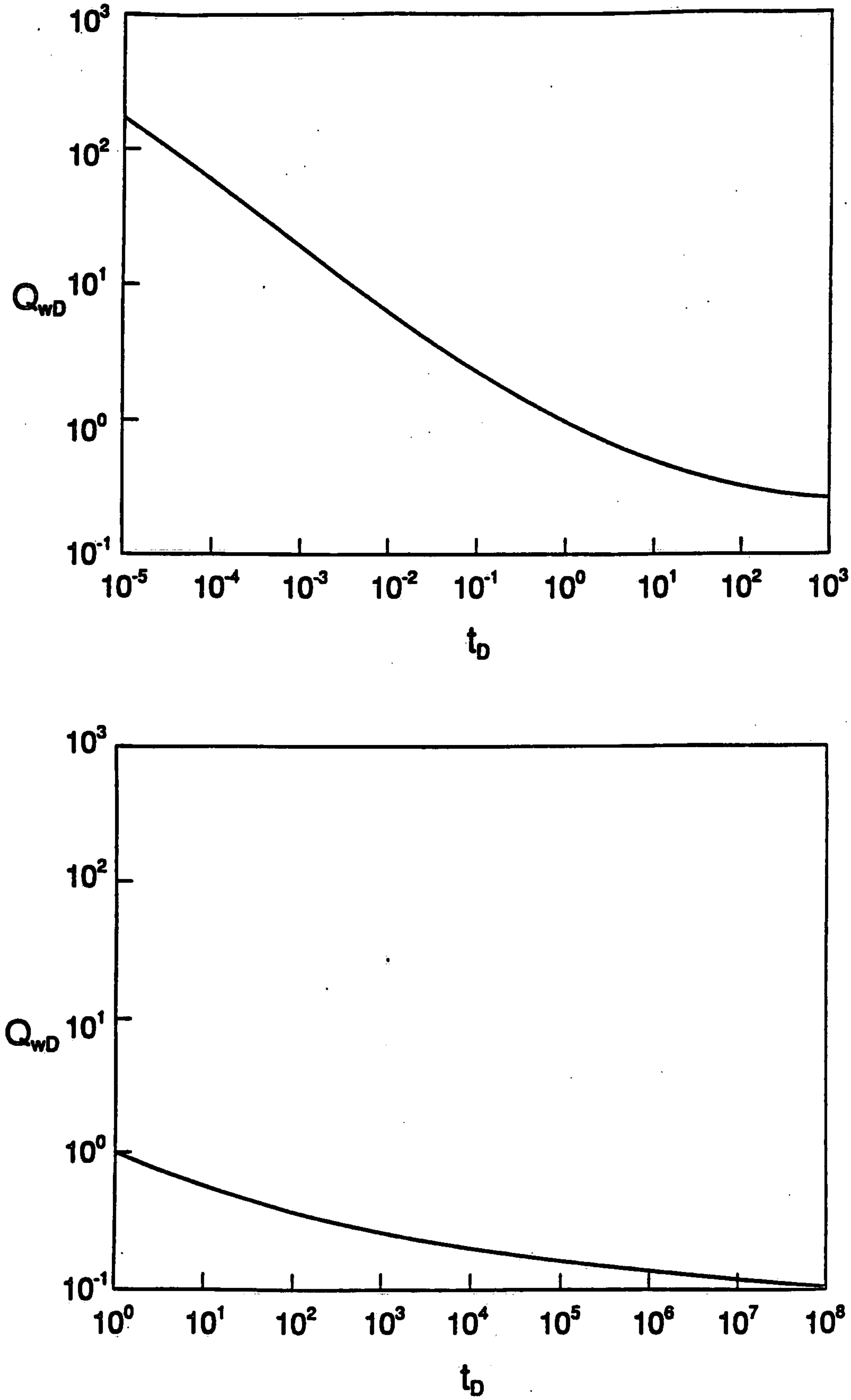


Figure 1. Schematic diagram of the boundary value problem.



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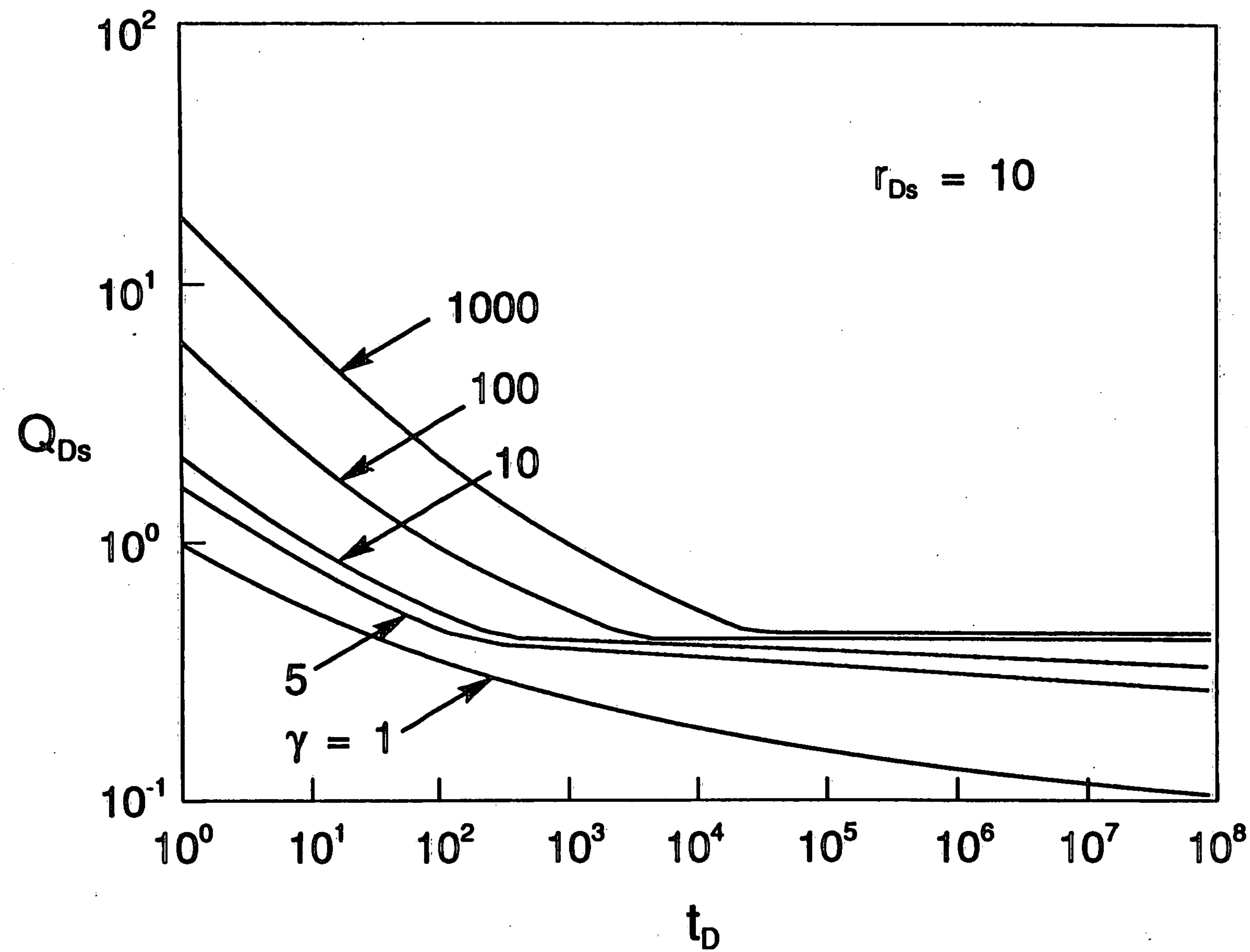

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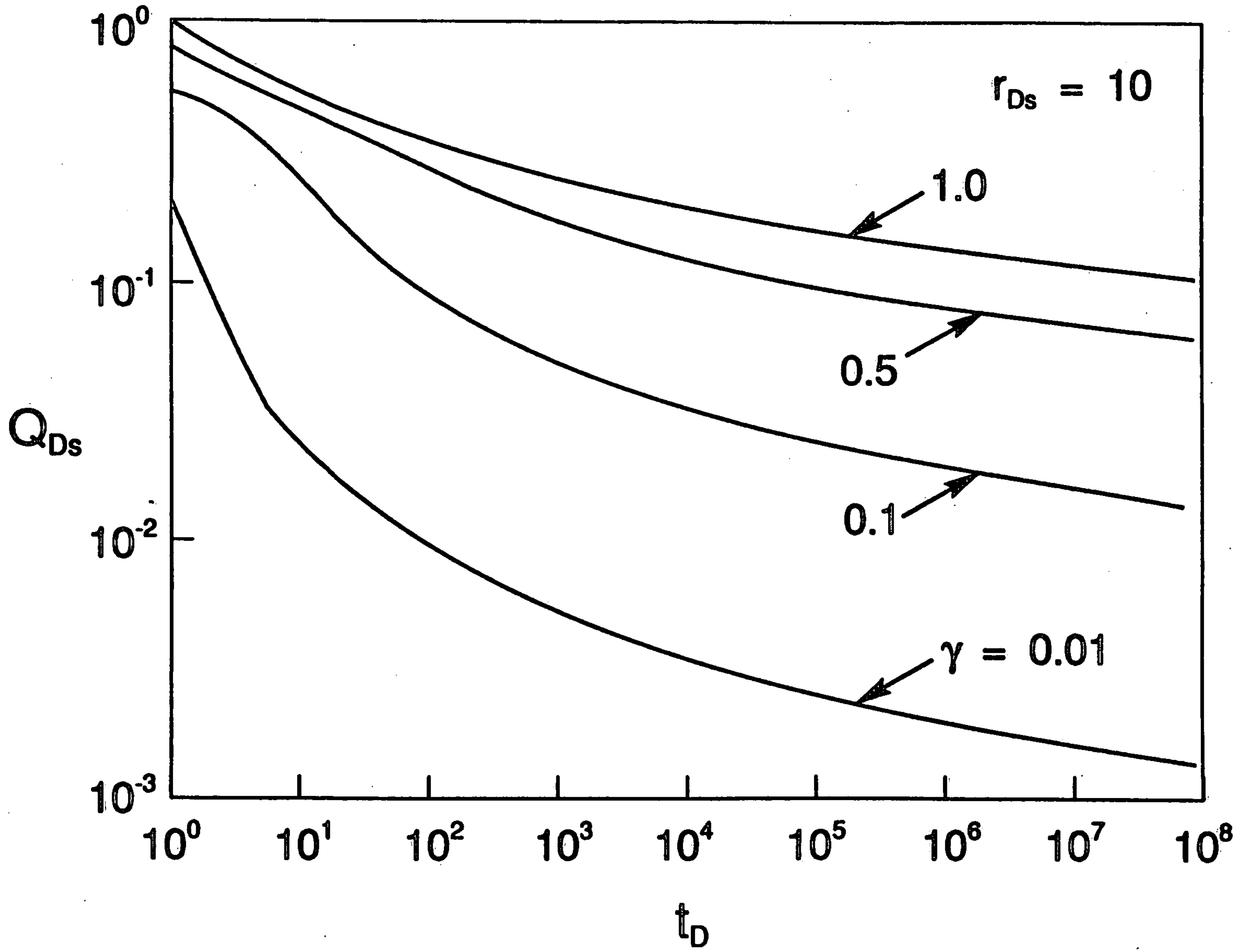

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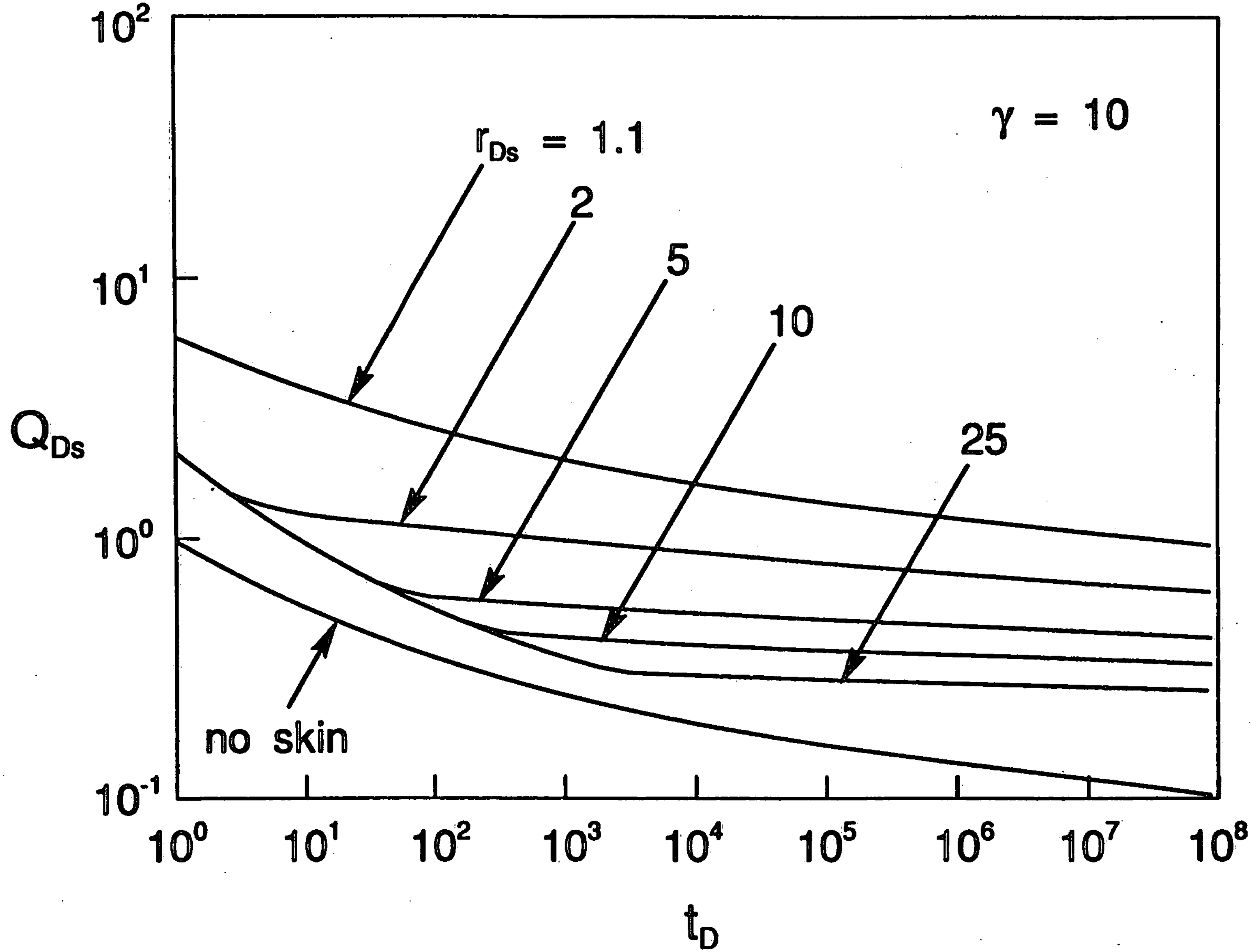

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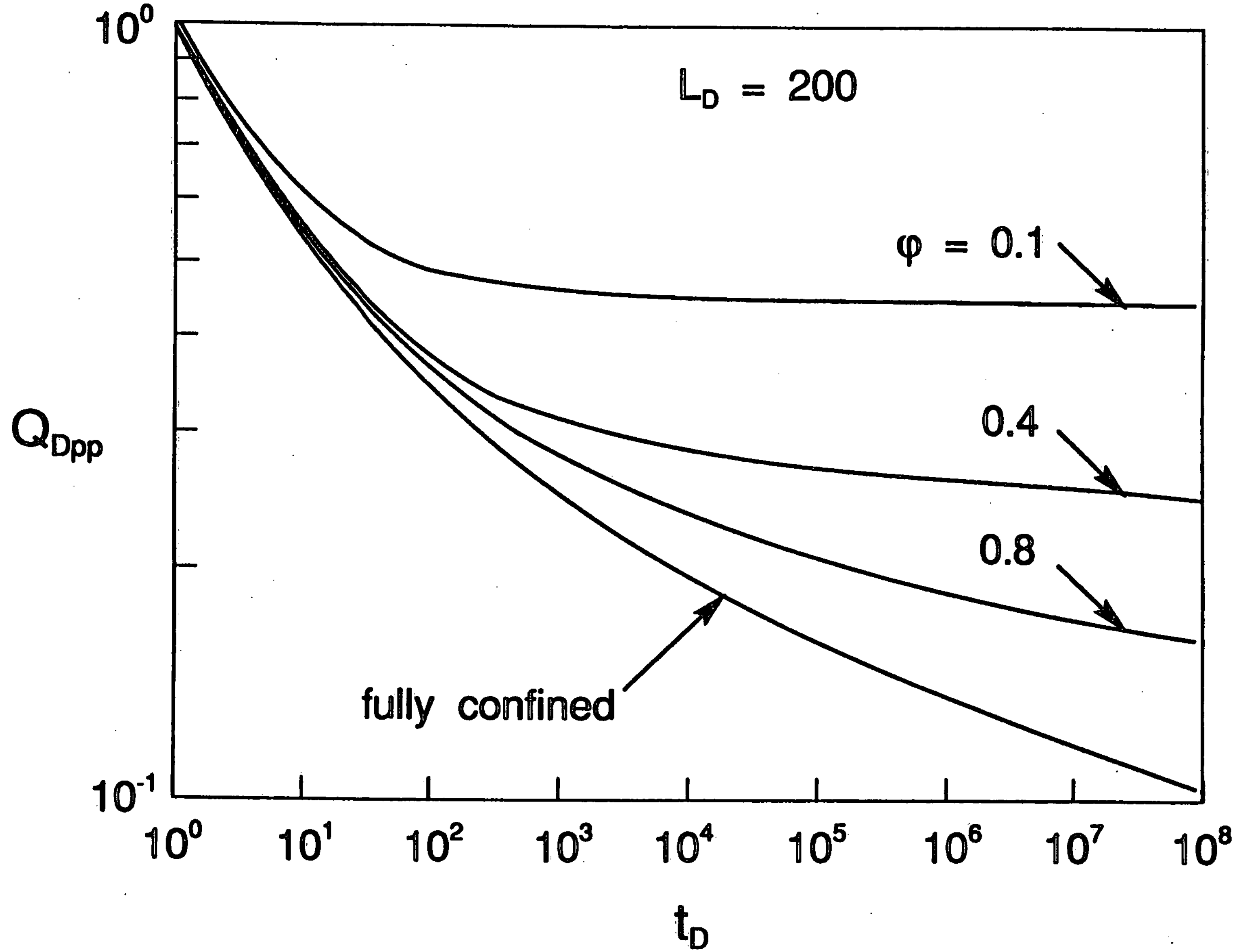

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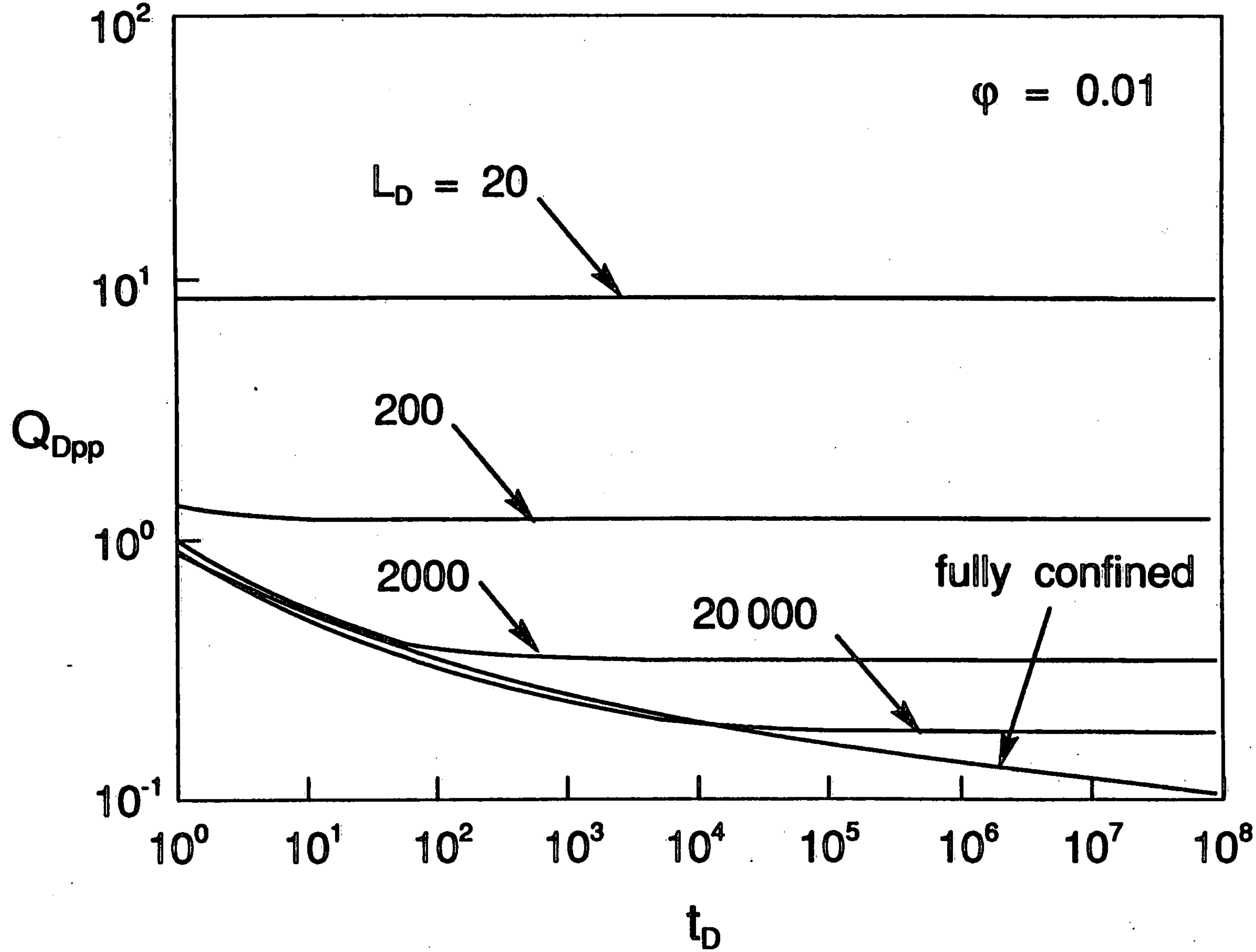

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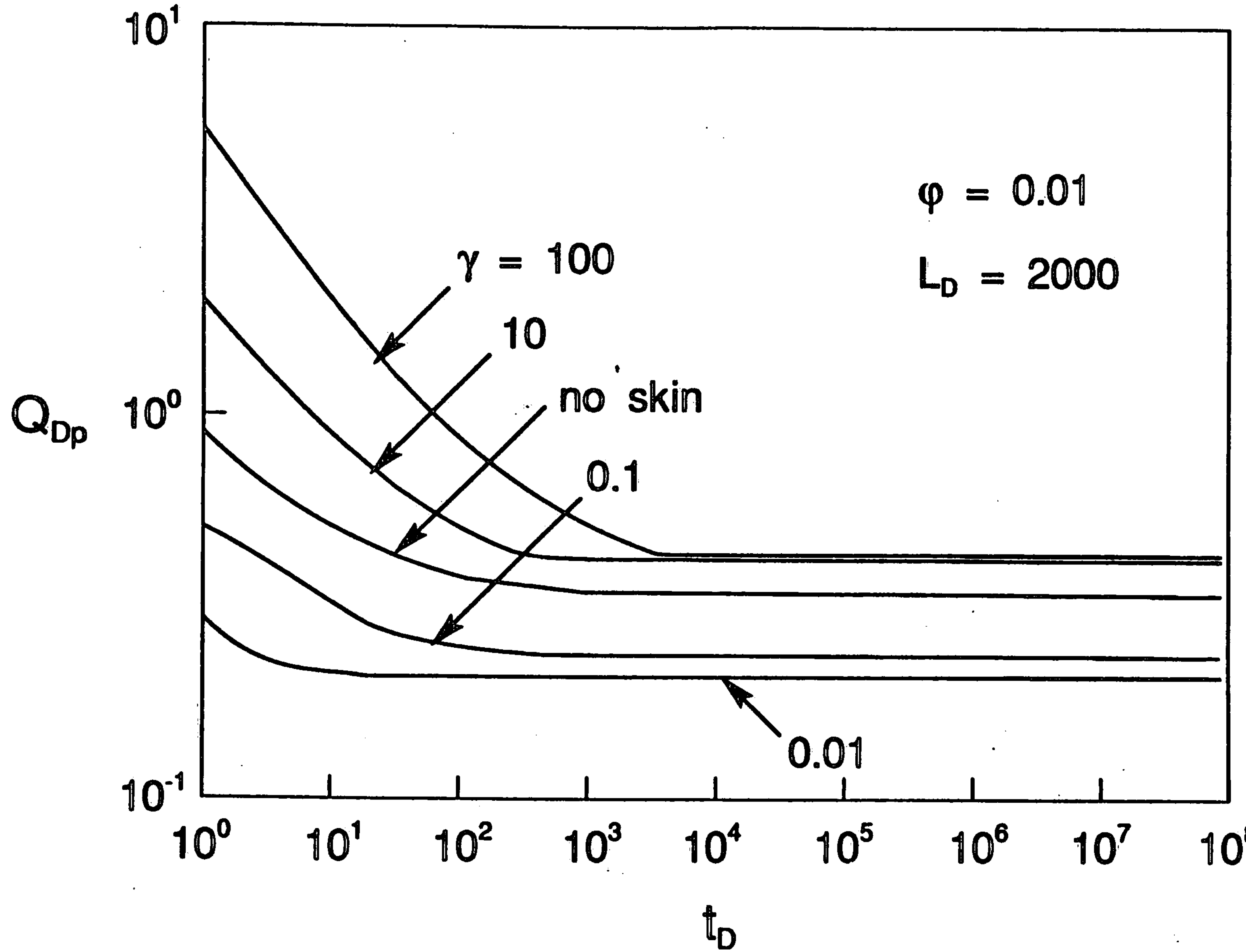

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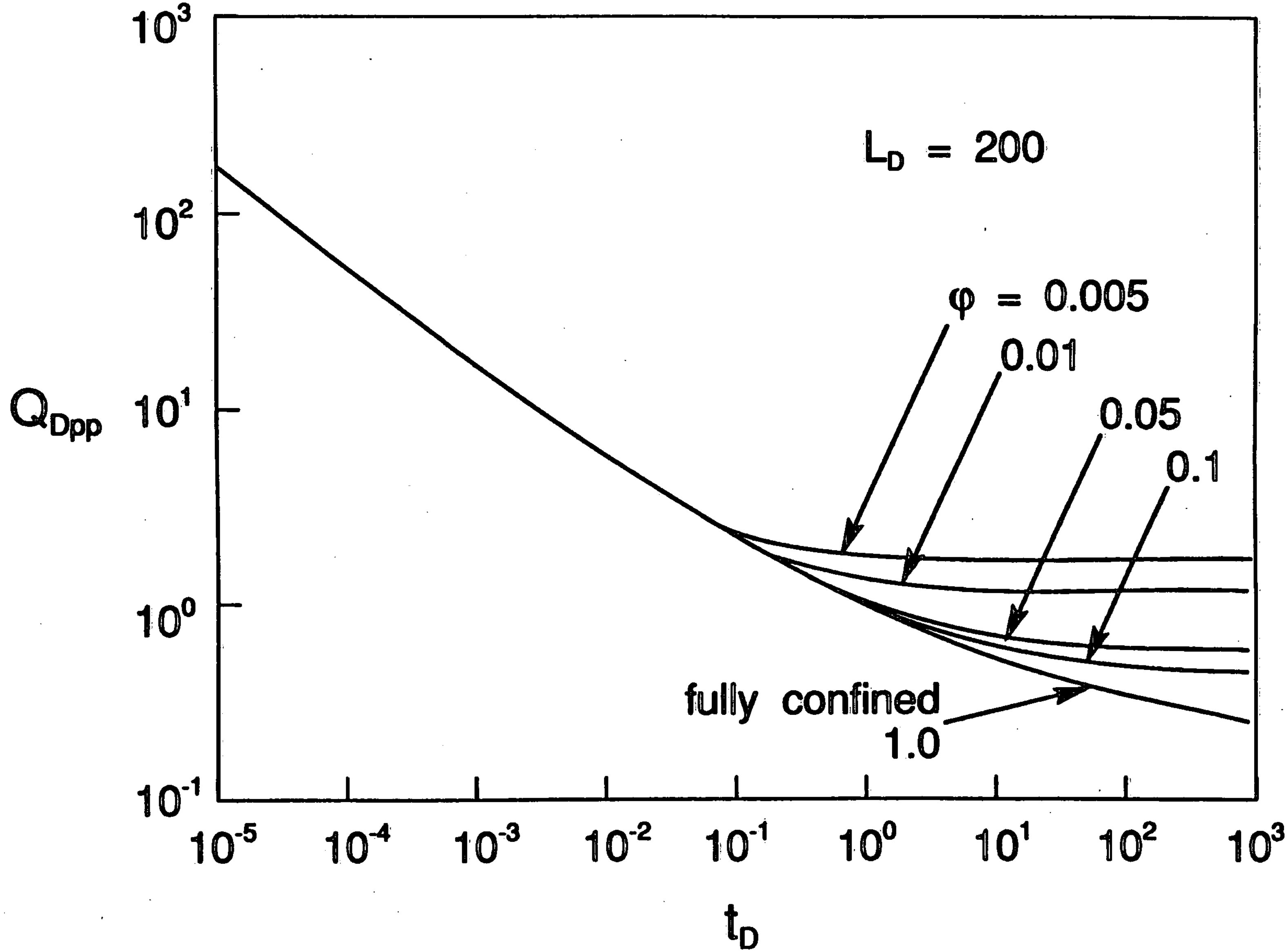

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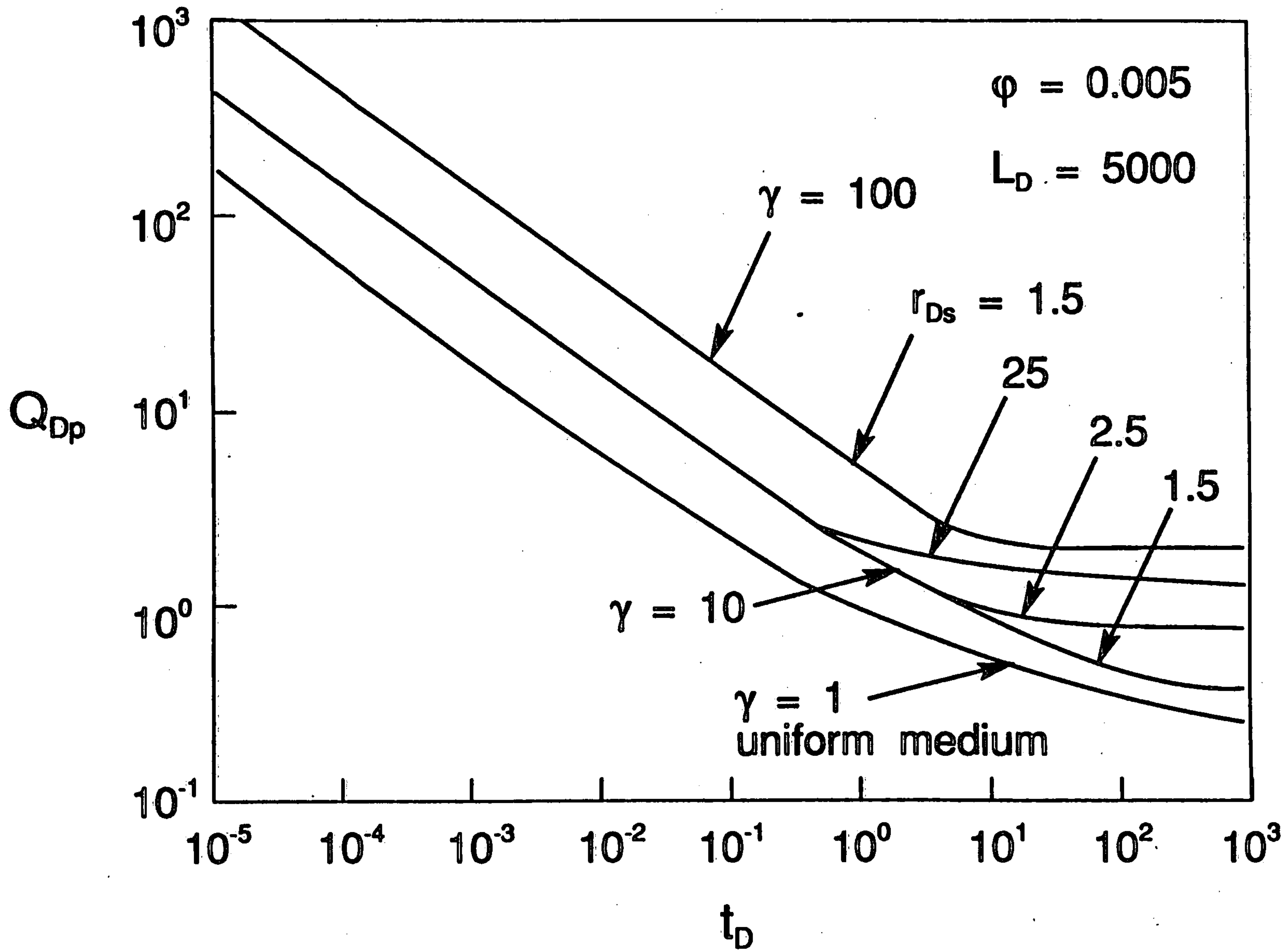

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