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UNCERTAINTY IN CURRENT METER CALIBRATION

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MANAGEMENT PERSPECTIVE

Increased awareness of river pollution and the importance of water quality monitoring has made it necessary to improve the accuracy of discharge measurements. One of the factors contributing to the error in a flow velocity measurement is the uncertainty in the current meter calibration itself. This uncertainty must be determined experimentally. In this report, repeated calibrations of five Price winter current meters, obtained in the towing tank of the Hydraulics Laboratory (HL) at the National Water Research Institute (NWRI), are examined to determine the uncertainty in a new form of calibration equation at the 95% confidence level. The results provide important information for the development of data quality control standards and development of an updated calibration strategy by the Surveys and Information Systems Branch (SISB) for measurement of flow in rivers with solid ice cover.

SOMMAIRE À L'INTENTION DE LA DIRECTION

En raison d'une sensibilisation accrue à la pollution des cours d'eau et à l'importance du contrôle de la qualité de l'eau, il a été nécessaire d'améliorer la précision des mesures du débit. L'un des facteurs responsables de l'erreur au niveau de la mesure de la vitesse du débit est l'incertitude de l'étalonnage même du courantomètre. Cette incertitude doit être déterminée expérimentalement. Dans le cadre de ce rapport, les étalonnages répétés de cinq courantomètres d'hiver de marque Price, effectués dans le bassin à chariot mobile du Laboratoire d'hydraulique (LH) de l'Institut national de recherche sur les eaux (INRE), sont examinés pour déterminer l'incertitude sous une nouvelle forme d'équation d'étalonnage à un seuil de confiance de 95%. Les résultats fournissent des informations importantes pour l'établissement de normes visant le contrôle de la qualité des données et l'élaboration par la Direction des relevés et systèmes d'information d'une stratégie d'étalonnage modifiée pour mesurer le débit dans les cours d'eau entièrement recouverts de glance.

ABSTRACT

Five Price winter current meters were calibrated separately, each ten times, for a total of fifty calibrations. A new form of calibration equation fitted to the data by least squares methods gave excellent results. Analysis showed that, even at velocities of 10 cm/s, the uncertainty due to the calibration equation was 2% or less at the 95% confidence level for the five meters tested.

RÉSUMÉ

Cinq courantomètres d'hiver de marque Price ont été étalonnés séparément, à raison de dix fois chacun, pour un total de 50 étalonnages. Une nouvelle forme d'équation d'étalonnage ajustée à la courbe des données par des méthodes des moindres carrés a donné d'excellents résultats. L'analyse a démontré que, même à des vitesses de 10 cm/s, l'incertitude due à l'équation d'étalonnage était de 2% ou moins à un seuil de confiance de 95% pour les cinq courantomètres testés.

TABLE OF CONTENTS

UNCERTAINTY IN CURRENT METER CALIBRATION

by

P.Engel[1] and *K.Wiebe*[2]

INTRODUCTION

Increased awareness of river pollution and the importance of water quality monitoring has made it necessary to improve the accuracy of discharge measurements. The determination of river discharge requires the measurement of the flow velocity. The velocity is measured by placing a meter into the flow and recording the rate of rotation of the rotor, usually in revolutions per second. The relationship between the linear velocity of the flow and the revolutions per second is determined by calibrating the meter in a towing tank. The current meter calibrations are normally expressed by some form of equation from which calibration tables are prepared for use in the field. One of the factors contributing to the error in a flow velocity measurement is the uncertainty in the current meter calibration itself (Smoot and Carter 1968).

In this paper, repeated calibrations of five rod suspended Price current meters, conducted in the towing tank of the Hydraulics Laboratory (HL) at the National Water Research Institute (NWRI), are examined to determine the calibration uncertainty.

CALIBRATION EQUATION

In developing a new calibration equation for rod suspended Price meters, it was shown by Engel (1989), that for a frictionless current meter, the dimensionless rotor response could be expressed as

$$\frac{ND}{V} = \frac{1}{\pi}\left[\frac{K-1}{K+1}\right] \tag{1}$$

where N=the rate of rotation of the rotor, D = the effective diameter of the rotor, V = the average flow velocity or towing speed, $K = \frac{C_{D1}}{C_{D2}}$, C_{D1} = the drag coefficient of the conical elements on the stoss-side and C_{D2} = the drag coefficient of the conical elements on the lee-side. The value of K must be determined experimentally.

Equation (1) reflects the typical response characteristics of the Price current meter in a two dimensional flow field if there is no frictional resistance in the bearings and other contact surfaces. ND/V is dependent only on the value of K which reflects primarily the shape and orientation of the conical elements of the rotor. The sensitivity of the meter is dependent on both D and K.

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For a given meter the value of K and D are constant and a practical calibration equation is normally expressed in a form of V as a function of N. Therefore, equation (1) may be rearranged to give

$$V = \frac{D}{\pi}\left[\frac{K+1}{K-1}\right]N = AN \tag{2}$$

where A = the meter constant. Equation (2) is linear, with slope A and passes through the origin of a $V vs. N$ plot. Such a behaviour would be ideal for a current meter. It is known, however, that calibration curves are nonlinear, particularly in the region of lower velocities. This effect can best be illustrated with the plot of $ND/V vs. V$ in Figure 1. The average curve fitted to the data shows that the meter response is very nonlinear for velocities less than about 30 cm/s. For velocities greater than 30 cm/s the values of ND/V are approximately constant, indicating that the rotor response in this range tends to be linear. A single, continuous calibration equation, which combines the linear and the frictional components of the rotor response, was developed by Engel (1989) and is given as

$$V = AN + Be^{-kN} \tag{3}$$

where A, B and k are coefficients to be determined by calibration in a towing tank.

Typical examples of the goodness of the fit of equation (3) can be seen in Figures 2 in which curves of equation (3) are superimposed on the plotted data for two of the five meters tested. The data are plotted as $\frac{N}{V}$ vs. V. The ratio $\frac{N}{V}$ was used because of its high sensitivity to changes in V. It represents the rotation of the meter rotor for each meter of distance along the towing tank and can therefore be considered to be a form of meter rotor efficiency. The curves fit the data extremely well over the full range of velocities tested. This accuracy cannot be obtained with linear calibration equations presently used for calibrations of Price meters.

The Effects of A, B and k

The coefficient A, as shown in equation (2), can be expressed as

$$A = \frac{D}{\pi}\left[\frac{K+1}{K-1}\right] \tag{4}$$

which shows that it depends on the shape and orientation of the conical rotor cups and the size of the rotor. For a given meter type, the rotor geometry is the same with minor differences due to normal fabrication variances. Therefore, one should expect very little variation in A from one meter to another.

The coefficient B represents the threshold velocity of the meter. Theoretically, the threshold velocity is the maximum towing velocity for which the rotor will remain stationary. In other words, it is the flow velocity at which the rotor is on the verge of the beginning of rotation. Using dimensional analysis, it was shown by Engel (1989) that B can be expressed as

$$B = \frac{T_o}{\rho D^{\frac{7}{2}} g^{\frac{1}{2}}} \tag{5}$$

where T_o = the resistance in the meter at the point of beginning of rotation which occurs at the threshold velocity, ρ = density of the fluid and g = the acceleration due to gravity. One can expect that the threshold velocity increases as T_o increases. Clearly, for best performance, T_o should be kept as small as possible. In the case of the Price meter, the dependence of the threshold velocity on T_o has significant implications. The Price meter has "cat-whisker" electrical contact brushes which form part of the pulse signal circuit. The overall resistance torque T_o is dependent on how snugly these contact brushes are set. It is therefore important that these adjustments and settings made, at the time of meter calibration, are maintained during use in the field. Equation (5) also shows that the threshold velocity is inversely proportional to the rotor diameter. Therefore, for a given static resistance T_o, the threshold velocity can be significantly decreased by increasing the rotor diameter. Finally, the effect of fluid density on B can be seen in Figure 3 in which data for the average calibrations of three Price meters in both air and water are plotted as V vs. N. The curves clearly show that the threshold velocity, when the fluid is air, is much larger than when the fluid is water. Fortunately, changes in density of the water, as a result of temperature changes are small and therefore, the density of the water does not affect the response of the meter rotor significantly for standard calibrations. It is also interesting to note in Figure 3 that both curves merge into a single curve indicating that the meter behaves similarly in both air and water in the range where the factors contributing to the nonlinearity become insignificantly small.

The exponent kN in equation (3) is dimensionless and therefore, k has the units of s/rev. Physically, k is a decay constant, the magnitude of which dictates the rate at which the non-linear component of equation (3) approaches the linear component. The rate of change in the non-linear component reflects the rate of change of the resistance in the meter. Since the threshold velocity is directly proportional to T_o, then k should be directly related to B. It was shown by Engel (1989) that k decreases as B decreases. Physically, one would expect that $k \to 0$ as $B \to 0$, implying that when $k = 0$ the meter operates as an ideal frictionless meter.

UNCERTAINTY EQUATION

It can been shown that for equation (3) the error in the computed velocity may be expressed as

$$\delta V = \left\{ \left(\frac{\partial V}{\partial A}\delta A\right)^2 + \left(\frac{\partial V}{\partial N}\delta N\right)^2 + \left(\frac{\partial V}{\partial B}\delta B\right)^2 + \left(\frac{\partial V}{\partial k}\delta k\right)^2 \right\}^{\frac{1}{2}} \tag{6}$$

Equation (6) states that the error δV in V is the square root of the sum of the squares of the errors due to an error in A, N, B and k. The error in N can be considered to be very small because the measurement of time and rotor revolutions are very precise. Therefore the quantity $(\frac{\partial V}{\partial N}\delta N)$ can be eliminated from equation (6) and the error in V becomes

$$\delta V = \left\{ \left(\frac{\partial V}{\partial A}\delta A\right)^2 + \left(\frac{\partial V}{\partial B}\delta B\right)^2 + \left(\frac{\partial V}{\partial k}\delta k\right)^2 \right\}^{\frac{1}{2}} \tag{7}$$

The partial derivatives are obtained by differentiating equation (3); substituting into equation (7) and rearranging to give the relative error in the velocity in terms of the relative errors in A, B and k as

$$\frac{\delta V}{V} = \left\{ \frac{1}{(1+\beta)^2} \left[\beta^2 \left(\frac{\delta A}{A}\right)^2 + \left(\frac{\delta B}{B}\right)^2 + k^2 N^2 \left(\frac{\delta k}{k}\right)^2 \right] \right\}^{\frac{1}{2}} \tag{8}$$

in which $\beta = \frac{AN}{Be^{-kN}}$. The relative error ratios $(\frac{\delta V}{V})$, $(\frac{\delta A}{A})$, $(\frac{\delta B}{B})$ and $(\frac{\delta k}{k})$ can be expressed as ratios of the standard deviation to the corresponding mean and as such become coefficients of variation (Herschy, 1978). The coefficient of variation is a basic measure of the relative uncertainty in the mean value of the variable it represents. Normally, the uncertainty is expressed at the 95% confidence level which, in the case of the velocity V, has been expressed by Engel (1991) as

$$E_V = \pm \frac{100 t_{0.975} C_V}{\sqrt{n-1}} \tag{9}$$

where E_V = the relative uncertainty in the computed velocity in percent at the 95% confidence level, $t_{0.975}$ = the confidence coefficient at the 95% confidence level from Student's t distribution for $(n-1)$ degrees of freedom (Spiegel, 1961) and C_V is the coefficient of variation for the velocity V. The same reasoning applies to A, B and k for which the relative uncertainties can be given as E_A, E_B and E_k. Replacing the relative error ratios in equation (8) with the percent uncertainties, the relative uncertainty in the calibration becomes

$$E_V = \left\{ \frac{1}{(1+\beta)^2} \left[\beta^2 E_A^2 + E_B^2 + k^2 N^2 E_k^2 \right] \right\}^{\frac{1}{2}} \tag{10}$$

Uncertainties in hydrometry are generally expressed as percentages. This practice has been recommended by the International Standards Organization (ISO 5168) and experience in the field has proved this approach to be convenient both in statistical analysis of the data and in the use to which data is put (Herschy 1978). Equation (10) is used to examine the effectiveness of the meter calibrations obtained when equation (3) is fitted to the data by least squares methods.

RESULTS

Values of A, B and k

Mean values of A and the uncertainty E_A at the 95% confidence level for each of the five meters tested were computed and these are given in Table 1. Examination of the data shows that indeed the variation in A from meter to meter is trivial. The uncertainty E_A is more pronounced, but its magnitude is very small and therefore does not have a great impact on the effect of A for the Price meters.

Examination of Table 1 shows that average values of B vary considerably from meter to meter. Although the magnitude of B is small, its relative effect at low velocities is significant. The uncertainty in determining B is much higher than that for A. Values of E_B are between about 12% and 25% for the five meters tested. This uncertainty is largely due to towing tank environment. At very low towing velocities, residual velocities in the tank due to the disturbance from previous meter tows (Kamphuis 1971) and density currents can cause significant variability in the meter response. This effect can be reduced by increasing the waiting times between tows, but cannot be completely eliminated.

The uncertainties E_k in Table 1 are considerably larger than E_B varying between 38% and 79% for the five meters tested. This reflects the high sensitivity of k in equation (3). Considering the dependence of k on B, the high uncertainty must be largely a magnification of the uncertainty in determining B.

Effect of Uncertainty in A, B and k

Examination of equation (10) shows that the uncertainty in the velocity E_V varies with the flow velocity reflected by the rate of rotation of the meter rotor N. Values of E_V were computed for different values of N over the range of towing velocities used for the tests using the values of E_A, E_B and E_k. Results for the two meters in Figure 2 are plotted as E_V vs. N in Figures 4. Smooth curves were drawn through the plotted points to facilitate the analysis. The curves clearly show that the greatest uncertainty in the calibration occurs at low values of N. Although uncertainties in B and k are very large, the effect of this variability is small because β in equation (10) is large and increases as N increases. Therefore, as N increases, E_V decreases, initially rapidly, with the rate of change decreasing as N increases. Even at the low values of N (i.e. low velocities), the uncertainties E_V are quite low. For $N = 0.15$, representing a velocity of about 0.10 m/s, the largest value of E_V is about 2.0%. Such low values indicate that the calibrations in the towing tank are very consistent and that equation (3) is a good representation of the response characteristics of the Price meters when they are mounted on a rod suspension.

CONCLUSIONS

Using theoretical analysis, an equation for the uncertainty in the calibration equation for rod suspended Price current meters was obtained.

Calibrations of five rod suspended Price current meters, each calibrated ten times, were successfully conducted in a towing tank to provide the necessary data for the determination of calibration uncertainty.

Analysis of the data showed that the uncertainty of A, at the 95% confidence level, was very low and that values of A varied only slightly from meter to meter. This is primarily due to the high precision in the fabrication of the rotor elements of the meters. The uncertainty E_A is mainly due to normally expected experimental error.

Analysis of the data showed that the uncertainty of B, at the 95% confidence level, was quite high and much higher than that of A. The reason for this is mainly due to residual currents in the towing tank as a result of disturbances created by towing the current meters. This uncertainty can be reduced by increasing the time interval between successive meter tows.

Analysis of the data showed that the uncertainty of k, at the 95% confidence level, was considerably higher than that of B. The primary reason for this is that k is dependent on B, with k decreasing as B decreases. Therefore, the factors that affect B can be expected to affect k with the uncertainties being amplified.

The effect of uncertainties in the coefficients A, B and k is greatest at the lowest velocities and decreases as velocities increase. This shows that the greatest errors in meter calibrations are likely to occur at the low velocities. The results confirm that equation (3) provides the most accurate calibration for rod suspended Price winter current meters. Considerations should be given to adopting this equation to replace the linear equation format presently used.

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APPENDIX I. REFERENCES

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APPENDIX II. NOTATION

The following symbols are used in this paper

A = calibration coefficient;
B = calibration coefficient;
C_{D1} = drag coefficient on stoss-side of conical elements;
C_{D2} = drag coefficient on lee-side of conical elements;
C_V = coefficient of variation;
D = effective diameter of meter rotor;
e = the base for natural logarithms;
E = percent uncertainty of designated variable;
$K = \frac{C_{D1}}{C_{D2}}$;
k = calibration coefficient;
N = rate of rotation of meter rotor;
$\Pi = 3.14...$;
$t_{0.975}$ = 95% confidence coefficient;

V = flow or towing velocity;
$\beta = \frac{AN}{Be^{-kN}}$;
δ = operator denoting error in designated variable;
∂ = operator denoting partial derivative of designated variable;

TABLE 1 Means and Uncertainties at 95% level for A, B and k

Meter	$\overline{A}$ $[m/rev]$	E_A [%]	$\overline{B}$ $[m/s]$	E_B [%]	$\overline{k}$ $[s/rev]$	E_k [%]
6-226	0.6783	0.1006	0.012172	16.562	3.721	36.202
6-273	0.6788	0.0821	0.009295	13.616	3.375	43.847
6-322	0.6791	0.0366	0.007401	22.851	2.497	79.415
6-449	0.6817	0.0906	0.006832	11.732	1.583	37.695
6-487	0.6829	0.0805	0.004795	24.760	1.298	65.397

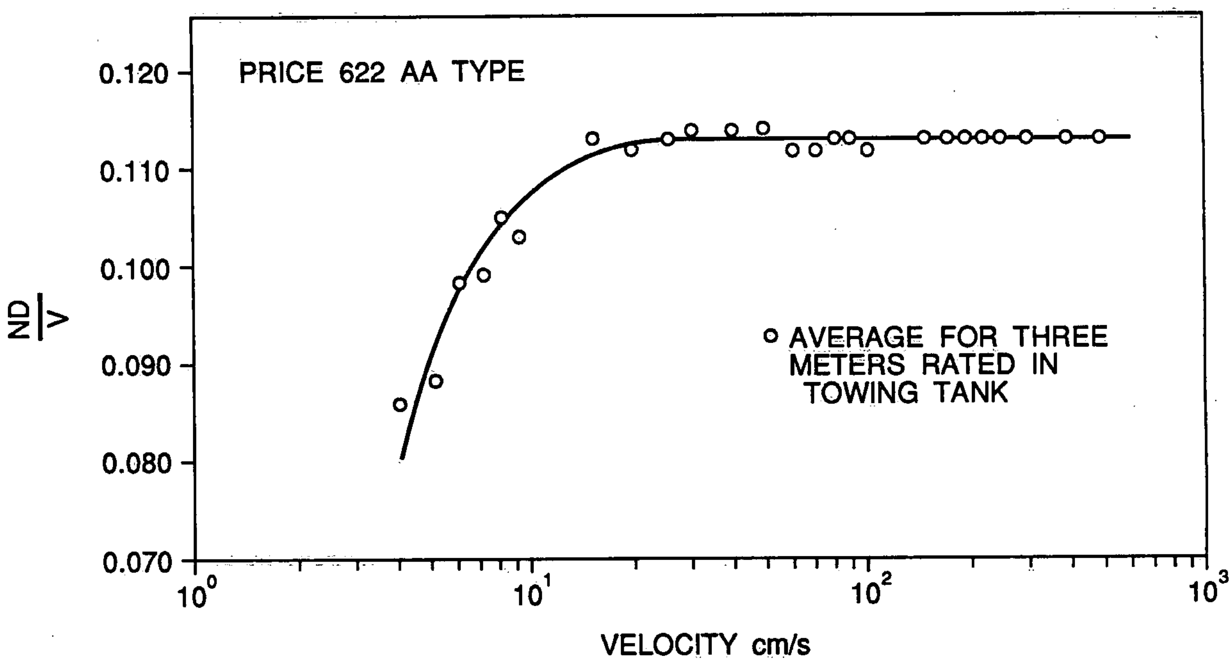
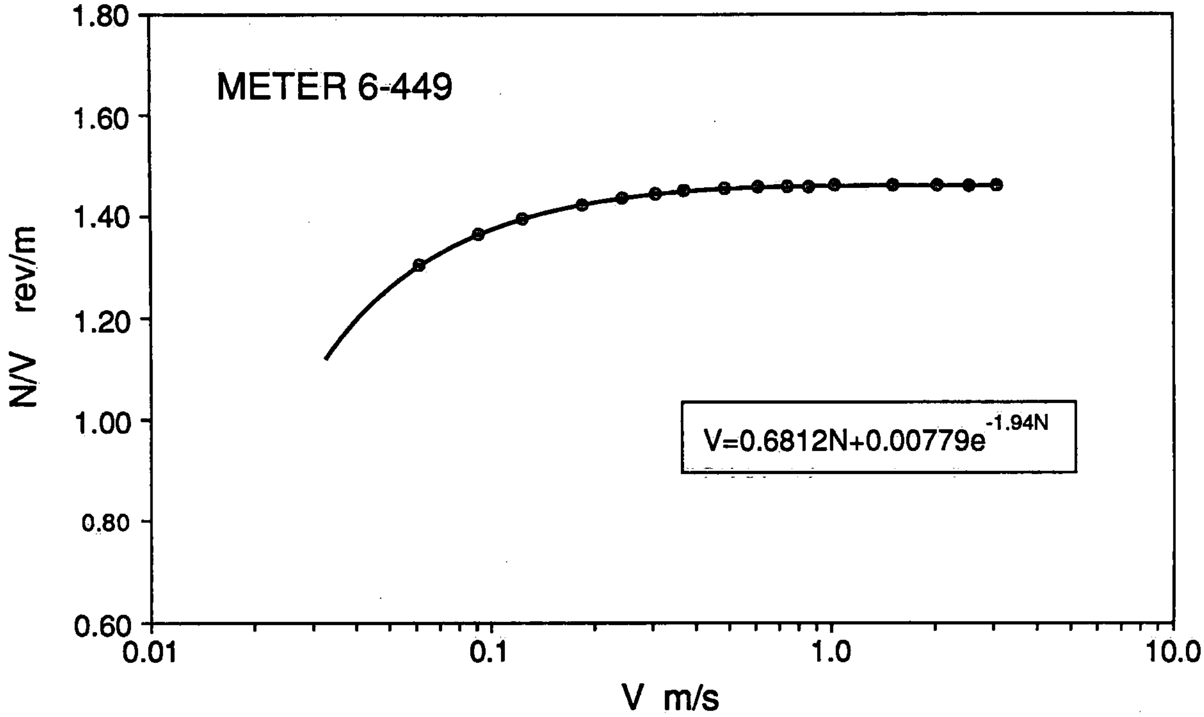


FIGURE 1 TYPICAL PRICE METER PERFORMANCE CURVE



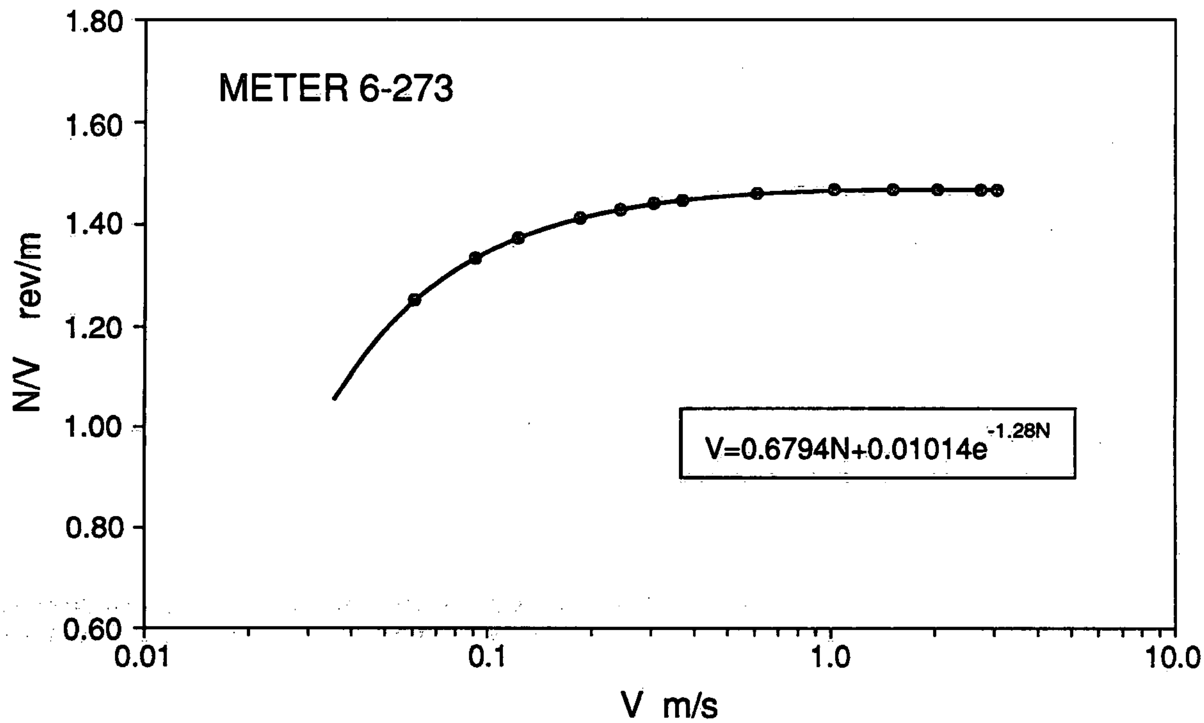


FIGURE 2. TYPICAL FIT OF CALIBRATION EQUATION

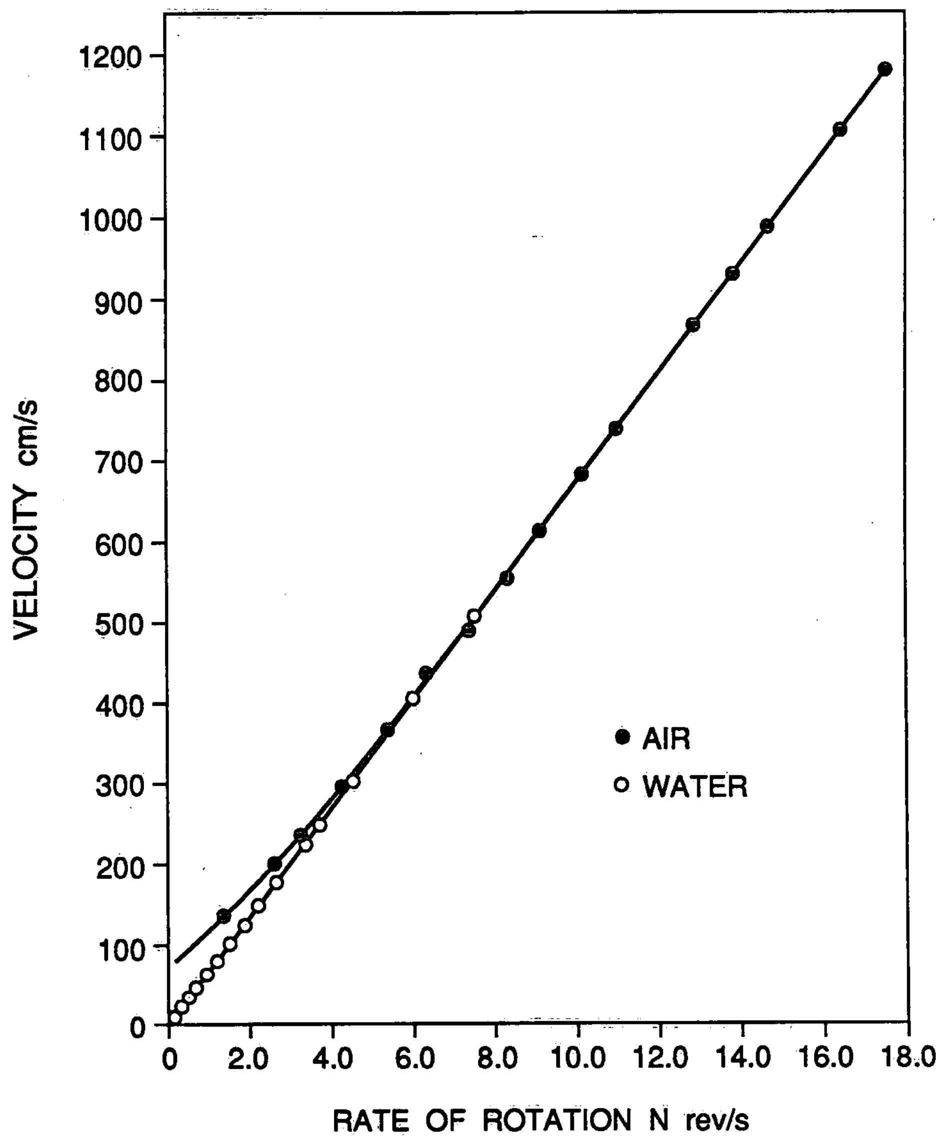


FIGURE 3. CALIBRATION CURVES FOR PRICE METER IN AIR AND WATER

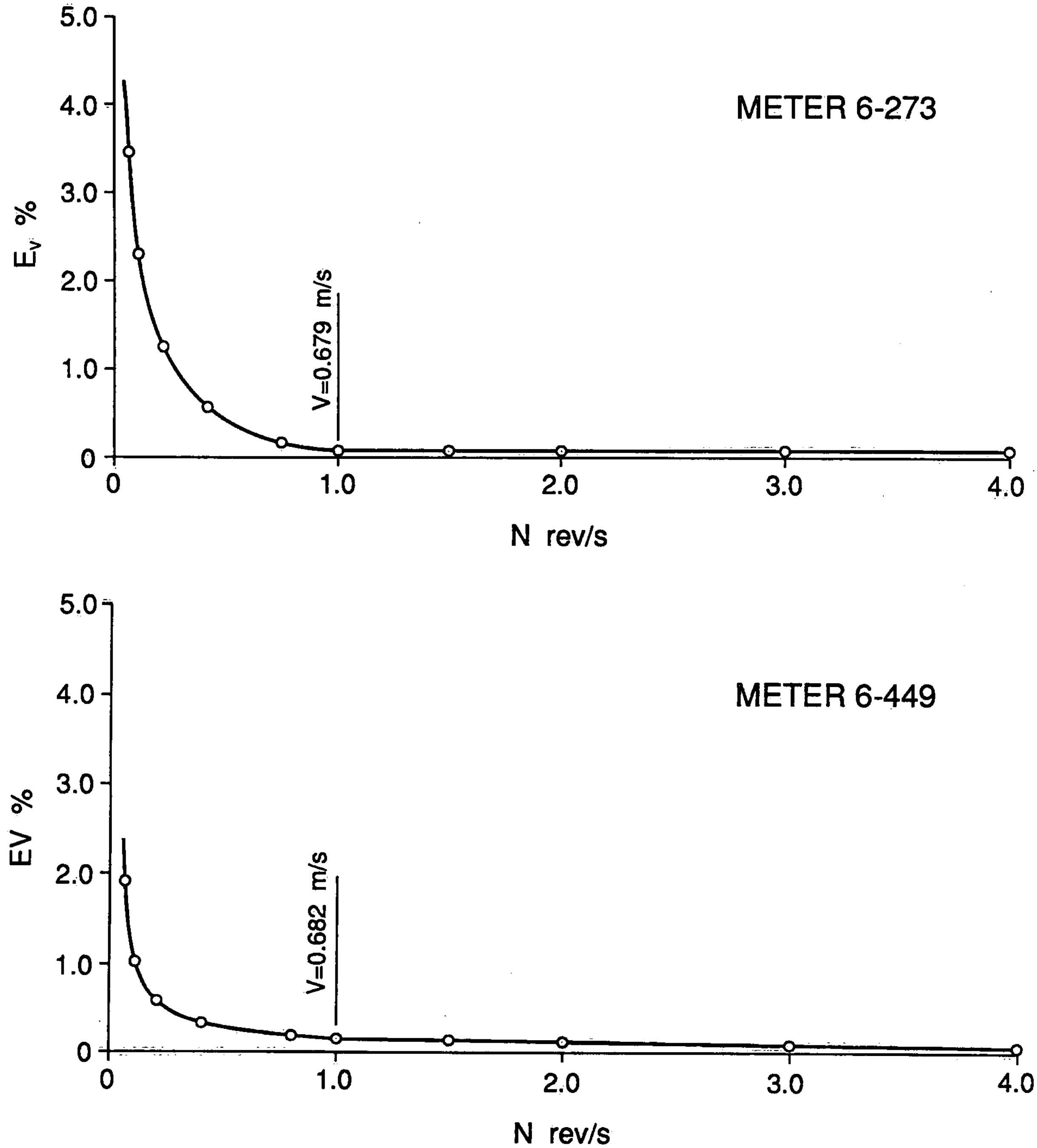


FIGURE 4. TYPICAL CALIBRATION UNCERTAINTIES

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