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ON THE PARTITIONING OF TOTAL SHEAR STRESS FOR UNIFORM FLOW OVER SIMULATED SAND WAVES

P. Engel and B.G. Krishnappan

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MANAGEMENT PERSPECTIVE

The total shear stress exerted by the flow on a bed of triangular elements simulating sand waves is due to the sand-grain roughness and the roughness due to the shape of the sand waves. Sediment transport is due to the sand-grain shear stress only and therefore, it is important to have a reliable method of determining this component of the total shear stress. In this report, several methods of separating the shear stresses are examined using available data from carefully conducted laboratory experiments. The results provide basic information for mathematical modelling of sediment transport processes in addressing the sediment issues of the Fraser River Action Plan (FRAP).

SOMMAIRE À L'INTENTION DE LA DIRECTION

La force de cisaillement totale exercée par l'écoulement sur un lit composé d'éléments triangulaires simulant des dunes sous-marines est liée à la rugosité des grains de sable, et à la rugosité due à la forme des dunes sous-marines. La transport des sédiments s'effectue grâce à la force de cisaillement imputable à la rugosité des grains de sable. Il est donc important de disposer d'une méthode fiable permettant de déterminer la part de cette force dans la force de cisaillement totale. On examine ici plusieurs méthodes permettant de distinguer les diverses forces de cisaillement au moyen des données disponibles émanant d'expériences en laboratoire effectuées avec beaucoup de précautions. Les résultats fournissent une base d'information utile à la modélisation mathématique des processus de transport des sédiments qui pourra servir aux études sédimentologiques dans le cadre du Plan d'action du Fraser.

ABSTRACT

Using theoretical and dimensional analysis together with available experimental data, several methods of partitioning the total shear stress exerted by open channel flow over a bed composed of triangular elements have been examined. It has been shown that the concept of isolated roughness flow is valid. Analysis indicates that partitioning is most readily accomplished by using the slope-separation method when sand wave steepness is less than 0.07. When the steepness is greater than 0.07 the slope separation method can be used with an adjustment factor. The effective shear stress due to the form roughness of bedforms can be computed using principles of energy losses due to the sudden flow expansion immediately downstream of the crest. Further tests over a wider range of flow conditions are required to confirm present results for general application.

RÉSUMÉ

Au moyen d'analyses théoriques et dimensionnelles, ainsi que des données expérimentales disponibles, plusieurs méthodes de distinction des diverses composantes de la force de cisaillement totale exercée par l'écoulement libre sur un lit composé d'éléments triangulaires ont été examinées. On a montré que le concept de l'écoulement rugueux isolé est valide. L'analyse indique que la détermination des composantes est facilitée par l'utilisation de la méthode de séparation des pentes dans les cas où la pente des dunes sous-marines est inférieure à 0,07. Lorsque la pente est supérieure à cette valeur, la méthode de séparation des pentes peut être utilisée avec un facteur correcteur. La force de cisaillement réelle due à la rugosité des formes et du fond du lit peut être calculée au moyen des principes de perte d'énergie causée par une expansion brusque de l'écoulement immédiatement en aval de la crête. Des essais supplémentaires réalisés pour une gamme plus importante de conditions d'écoulement et de géométrie des formes du lit sont nécessaires pour confirmer les résultats et permettre une application généralisée de la méthode.

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ON THE PARTITIONING OF TOTAL SHEAR STRESS FOR UNIFORM FLOW OVER SAND WAVES

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INTRODUCTION

The total shear stress exerted by the flow on a bed of sand waves is due to the sand-grain roughness usually denoted as τ' only and the form roughness denoted as τ''. Sediment transport is due to the shear stress τ' and therefore it is important to have a practical method of determining this component of the total shear stress. There are two basic methods presently in use for separating the sand-grain shear stress from the form shear stress normally referred to as the Einstein and Barbarossa method (Einstein and Barbarossa, 1952) and the Taylor and Brooks method (Taylor and Brooks, 1962). The first method, hereafter denoted as the depth-separation method, is based on dividing the hydraulic radius (depth for a two dimensional flow) into sand-grain roughness and form roughness components. The second method, hereafter denoted as the slope-separation method, seeks to achieve a similar objective by dividing the total friction slope into sand-grain roughness and form roughness components. A third method based on the principle of energy losses due to sudden flow expansion at the crest of bedforms proposed by Yalin (1964), seeks to determine the effective shear stress τ'' due to the form roughness.

In this report, the three partitioning methods are examined using available data from carefully conducted laboratory experiments with triangular elements to simulate sand waves. An extensive laboratory investigation by Vittal (1972) focused on the independent determination of τ' from measurements obtained with a Preston tube and τ'' by integrating the measured pressure distributions on the upstream and leeside of the triangular bed forms. These data, together with data from other tests with triangular bed forms from Engel (1981), are used to examine the validity of the partitioning concept and methods of determining the two shear stress components. The results provide basic information for mathematical modelling of sediment transport processes in addressing the sediment issues of the Fraser River Action Plan (FRAP).

PRELIMINARY CONSIDERATIONS

Addition of Partitioned Stresses

Morris (1954) found, from tests in pipes, that individual roughness elements will act as isolated bodies if they are a critical minimum distance apart. The elements must be so spaced that the wake zone and vortex generating zone at each element are completely developed and dissipated before the next element is reached. This condition is called isolated roughness flow. The common practice of considering the bed shear stress to be the sum of the sand-grain shear stress τ' acting on the bed surface and an "apparent" shear stress τ'' resulting from the form roughness assumes the validity of the principle of isolated roughness flow. This can be expressed as

$$\tau = \tau' + \tau'' \tag{1}$$

where τ = the total shear stress at the bed. The application of this principle to flows over sand waves was first proposed by Einstein and Barbarossa (1952) and adopted by Laursen (1958), Taylor and Brooks (1962), Yalin (1964), Engelund (1966) and Vanoni and Hwang (1967). Independent measurements of τ' and τ'' from Vittal (1972) were added together and plotted versus the total bed shear stress computed as $\tau = \rho g R S$ (R = hydraulic radius, S = the water surface slope of a uniform flow, ρ = the density of the fluid) in Figure 1 for several values of bed form steepness $\frac{\Delta}{\Lambda}$ (Δ = height of the bedforms from trough to crest and Λ = the length of the bedforms from crest to crest). All data points plot satisfactorily along the 45° line for all values of $\frac{\Delta}{\Lambda}$ from 0.05 to 0.20 for smooth and sand coated bed forms. This confirms the validity of equation (1) for smooth and rough surfaces.

Variation of the Sand-Grain Shear Stress

When the bed is composed of fixed triangular bedforms, the average sand-grain shear stress τ' for a two-dimensional turbulent flow can be expressed by the functional relationship

$$\tau' = f(\Delta, \Lambda, D_{50}, h, u_*, \rho, \nu, g) \tag{2}$$

where D_{50} = the median diameter of the material covering the bedform surfaces, h = the depth of uniform flow, u_* = the shear velocity for the total flow, ν = the kinematic viscosity of the water, g = the acceleration due to gravity and f denotes a function. Noting that $\tau = \rho u_*{}^2$, equation (2) can be written in dimensionless form as

$$\frac{\tau'}{\tau} = f_1\left(\frac{\Delta}{\Lambda}, \frac{D_{50}}{\Delta}, \frac{\Delta}{h}, \frac{u_*}{\sqrt{gh}}, \frac{g^{\frac{1}{2}} D_{50}^{\frac{3}{2}}}{\nu}\right) \tag{3}$$

where f_1 denotes a function. Data from Vittal (1972) were plotted as average values of $\frac{\tau'}{\tau}$ versus $\frac{\Delta}{\Lambda}$ with 95% confidence limits in Figure 2. The data are for one sand size and one bedform height and therefore, the plot is for only one value of $\frac{D_{50}}{\Delta}$ and $\frac{g^{\frac{1}{2}} D_{50}^{\frac{3}{2}}}{\nu}$. A smooth curve was fitted to the data and extended to the plane bed condition ($\frac{\Delta}{\Lambda} = 0$) for which the shear stress ratio must be equal to 1.0. The plot shows that the effect of $\frac{\Delta}{h}$ and $\frac{u_*}{\sqrt{gh}}$ is very small as shown by the narrow confidence limits and therefore, these variables may be removed from equation (3) resulting in

$$\frac{\tau'}{\tau} = f_2\left(\frac{\Delta}{\Lambda}, \frac{D_{50}}{\Delta}, \frac{g^{\frac{1}{2}} D_{50}^{\frac{3}{2}}}{\nu}\right) \tag{4}$$

where f_2 denotes another function. The curve in Figure 2 clearly shows the decrease of $\frac{\tau'}{\tau}$ as the bedform steepness $\frac{\Delta}{\Lambda}$ increases, showing that the relative importance of τ' decreases as the dune steepness increases. This is in agreement with results obtained by Engel and Lau (1980).

EXAMINATION OF PARTITIONING METHODS

Theoretical Analysis

The partitioning of the total bottom shear stress is accomplished by computing τ' on the assumption that methods valid for plane beds with sand-grain roughness are applicable to similar surfaces which are not flat. To understand the difference between the values of τ' obtained with the depth-separation method and the slope-separation method, one may simply consider the case of a two dimensional flow over a rough boundary using the Manning equation. When depth-separation is used, τ' can be expressed as

$$\tau'_h = \frac{\rho g n_s^2 U^2}{h'^{\frac{1}{3}}} \tag{5}$$

where n_s = Manning roughness coefficient for a plain sand-grain surface, U = the mean flow velocity in the cross-section and h' = the partitioned depth of the two dimensional flow due to the sand-grain roughness and the subscript h denotes depth-separation. Similarly, the total bed shear stress can be written as

$$\tau = \frac{\rho g n^2 U^2}{h^{\frac{1}{3}}} \tag{6}$$

where n = the Manning roughness coefficient for the total bed roughness. Combining equations (5) and (6) and noting that $\frac{\tau'}{\tau} = \frac{h'}{h}$, one obtains

$$\frac{\tau'_h}{\tau} = \left(\frac{n_s}{n}\right)^{\frac{3}{2}} \tag{7}$$

When slope-separation is used, the Manning equation is written as

$$\tau'_s = \frac{\rho g n_s^2 U^2}{h^{\frac{1}{3}}} \tag{8}$$

where the subscript s denotes the slope separation. Once again the stress ratio can be obtained by combining equations (8) and (6) to give

$$\frac{\tau'_s}{\tau} = \left(\frac{n_s}{n}\right)^2 \tag{9}$$

Finally, combining equations (7) and (9) results in

$$\frac{\tau'_s}{\tau'_h} = \left(\frac{n_s}{n}\right)^{\frac{1}{2}} \tag{10}$$

Examination of equation (10) shows that τ'_s will always be smaller than τ'_h by a factor of $(\frac{n_s}{n})^{\frac{1}{2}}$. In the presence of sand waves, n_s depends only on the sand size whereas n depends on the sand size and sand wave geometry. Therefore, the ratio $\frac{n_s}{n}$ can be expressed as

$$\frac{n_s}{n} = \frac{n_s}{n_s + n_\Delta} \tag{11}$$

which, after some rearranging, can be written as

$$\frac{n_s}{n} = \frac{1}{1 + \frac{n_\Delta}{n_s}} \tag{12}$$

in which n_Δ = the Manning roughness coefficient due to the form drag of the bedforms. For a plane bed, $n_\Delta = 0$ and $\frac{n_s}{n} = 1$. In this case $\tau'_h = \tau'_s$. In the presence of sand waves, for a given sand size,

$\frac{n_\Delta}{n_s} > 1$ because the form roughness is greater than the sand-grain roughness and $\frac{n_s}{n}$ decreases as the size of sand waves increases. As a result, in accordance with equations (7) and (9), $\frac{\tau_h'}{\tau}$ and $\frac{\tau_s'}{\tau}$ both decrease but the latter is always smaller than the first with the difference increasing as n_Δ increases (sand waves become larger).

From the above discussion, it is evident that the depth-separation and slope-separation methods give different results. However, in order to determine which method gives better results, it is necessary to determine h' and S' from the available data, compute the corresponding stresses and compare them with the measured values. To determine h' and S', different friction factor relationships can be used. In the present study, the approach of Engelund (1966), which is based on the logarithmic velocity profile, is employed.

Depth-Separation

In this method, τ_h' is evaluated from:

$$\tau_h' = \rho g R' S \tag{13}$$

and

$$\tau_h'' = \rho g R'' S \tag{14}$$

where R' = the hydraulic radius corresponding to the sand-grain roughness, R'' = the hydraulic radius corresponding to the form roughness and S = the energy slope. The values of R' and R'' are such that their sum is equal to R (R = the hydraulic radius due to the total flow boundary). Values of R' are determined by trial and error using the relationship

$$\frac{U}{\sqrt{gR'S}} = 2.5 \ln\left(b\frac{R'}{k_s}\right) \tag{15}$$

where b = a coefficient and k_s = the equivalent sand-grain roughness of a plane bed. The coefficient b is a function of the sand-grain roughness Reynolds number $Re_* = \frac{\sqrt{gR'S}k_s}{\nu}$ and can be expressed as

$$b = e^{\kappa B_s - 1} \tag{16}$$

in which

$$B_s = 8.5 + [2.5\ln(Re_*) - 3]e^{-0.127[\ln(Re_*)]^2} \tag{17}$$

and κ = the Von Karman constant having a value of 0.4. Equation (17) was developed by Yalin (1992) and is given as the solid curve in Figure 3.

To examine the depth-separation method, values of R' were computed from equation (15) and these in turn were used to compute values of τ_h' from equation (13). Values of τ_h' were plotted versus the measured values from Vittal (1972) in Figure 4 for $\frac{\Delta}{\Lambda}$ = 0.05, 0.067, 0.098 and 0.20 and two different water surface slopes. The plots clearly show that the depth-separation method over-estimates values of τ' and that this discrepancy increases as $\frac{\Delta}{\Lambda}$ increases. This trend is not affected by changes in the water surface slope.

Slope-Separation

In this method, τ'_s and τ''_s are evaluated from

$$\tau'_s = \rho g R S' \tag{18}$$

and

$$\tau''_s = \rho g R S'' \tag{19}$$

in which S' and S'' are the energy slopes corresponding to the sand-grain and form roughness respectively. The values of S' and S'' are such that their sum is equal to S. Values of S' are determined by trial and error from the relationship

$$\frac{U}{\sqrt{gRS'}} = 2.5 \ln \left(b \frac{R}{k_s} \right) \tag{20}$$

together with equations (16) and (17) in which, for this case, $Re_* = \frac{\sqrt{gRS'}k_s}{\nu}$.

To examine the slope-separation method, values of S' were computed from equation (20) and these in turn were used to compute values of τ'_s from equation (18). Values of τ'_s were plotted versus the measured values from Vittal (1972) in Figure 5 for $\frac{\Delta}{\Lambda}$ = 0.05, 0.067, 0.098 and 0.20 and two different water surface slopes. The plots show that the computed shear stresses agree quite well with the measured values for bedform steepness of 0.05 and 0.067 but over-estimate the stresses when $\frac{\Delta}{\Lambda} = 0.098$ with the discrepancy increasing as the steepness increases to 0.20. Comparison of Figures 4 and 5 shows that the slope-separation method gives much better results than the depth-separation method. For $\frac{\Delta}{\Lambda} < .07$, the slope-separation method gives good agreement with measured values whereas, for the same conditions, the depth-separation method significantly over estimates the sand-grain shear stress. For values of $\frac{\Delta}{\Lambda} > 0.07$ both methods over-estimate the shear stress but the discrepancy is much greater with the depth-separation method. This suggests that the slope-separation method may be physically more sound since it associates the energy losses with corresponding fractions of the total energy slope (Yalin, 1977). Therefore, only the slope-separation method is given further considerations in this report.

Adjustment Coefficient C_Δ

The reason for the difference between the computed and measured sand-grain shear stress may be due to the combined effect of the accelerating flow and the inappropriate application of the log-law in flows with undulating bed. Carefully conducted flume experiments over an inclined plate of finite length by Cordosa et al (1991), have shown that the shear stress decreased to less than 50% of the uniform flow value in the downstream direction. This condition can be expected to increase as $\frac{\Delta}{\Lambda}$ increases.

Frequently, attempts have been made to account for most of the discrepancies in the computed sand-grain shear stress by applying an adjustment factor based on the change in sand-grain roughness area as sand wave steepness changes. Arisz and Davar (1991) assumed that the sand-grain friction on a plane surface, parallel to the flow, is equal to the sand-grain friction on an identical inclined surface after correcting for the difference in surface area. However, the validity of this adjustment was not demonstrated.

The length of the horizontal projection of the sand-grain surface depends on the steepness of the bedforms and the length of flow separation on the lee side as shown in Figure 6. With reference to Figure 6, the ratio of sand-grain roughness length to bedform length can be written as

$$\frac{\ell_s}{\Lambda} = 1 - \frac{\ell_w}{\Lambda} \tag{21}$$

where ℓ_s = the projection of the length of sand-grain roughness surface on the plane bed and ℓ_w = the horizontal length of the flow separation zone. All quantities in equation (21) are known except ℓ_w. The length of the flow separation ℓ_w has been determined by Engel (1981) from experiments conducted with triangular bedforms. Results showed that $\frac{\ell_w}{\Lambda}$ can be expressed as

$$\frac{\ell_w}{\Lambda} = f_w\left(\frac{\Delta}{\Lambda}\right) \tag{22}$$

where f_w denotes a function. The results from Engel (1981) are plotted as $\frac{\ell_w}{\Lambda}$ versus $\frac{\Delta}{\Lambda}$ with $\frac{D_{50}}{\Delta}$ as a parameter in Figure 7 together with data from Vittal (1972). A smooth curve was drawn through the plotted points to facilitate the analysis. The curve shows the dominant effect of the bedform steepness. The effect of $\frac{D_{50}}{\Delta}$ is small.

The ratio $\frac{\ell_s}{\Lambda}$ may be considered to be an adjustment coefficient, denoted as C_Δ, to account for the effect of the reduction in sand-grain roughness area on the stoss-side of the bedforms. One may then express this coefficient as

$$C_\Delta = 1 - \frac{\ell_w}{\Lambda} \tag{23}$$

Values of C_Δ were computed and plotted in Figure 8 as a function of $\frac{\Delta}{\Lambda}$. The adjustment coefficient decreases as $\frac{\Delta}{\Lambda}$ increases with the rate of change decreasing.

Values of τ_s', obtained with equations (18) and (20), were computed as $C_\Delta \tau_s'$ and plotted as $C_\Delta \frac{\tau_s'}{\tau}$ versus $\frac{\Delta}{\Lambda}$ in Figure 9. The curve for the measured values from Vittal (1972), given in Figure 2, was superimposed on the plot for comparison. It can be seen that the adjustment provided with C_Δ is effective when the bedform steepness is greater than about 0.07. This can be further shown by superimposing plots of $C_\Delta \tau_s'$ as a function of τ, on Figure 5 for $\frac{\Delta}{\Lambda}$ = 0.098 and 0.20. The agreement is quite good, indicating that the use of C_Δ provides satisfactory results for engineering purposes. It can also be seen from Figure 5 that agreement between τ' and τ is satisfactory for $\frac{\Delta}{\Lambda}$ = 0.05 and 0.067 and therefore, an adjustment coefficient is not required when $\frac{\Delta}{\Lambda} \leq 0.07$.

Form Roughness Separation

In accordance with the isolated flow principle, the sand-grain shear stress can be determined as the difference

$$\tau_f' = \tau - \tau_f'' \tag{24}$$

where τ_f' = the sand-grain shear stress and τ_f'' = the effective shear stress due to form roughness determined with the form roughness method. The effective shear stress τ_f'' can be determined directly by using the principle of energy loss due to a sudden expansion (Yalin, 1964). The energy loss may be written as

$$h_\Lambda = S_f''\Lambda = K\frac{(U_c - U_t)^2}{2g} \tag{25}$$

where h_Λ = the head loss over the bedform length Λ, S''_f = the energy slope due to the form roughness, U_c = the average velocity in the cross-section over the crest of the two dimensional bedforms, U_t = the average velocity in the cross-section over the lowest point in the trough of the two dimensional bedforms and K = the expansion coefficient. The continuity equation for the flow over the bedforms can be written as

$$Uh = U_c\left(h - \frac{\Delta}{2}\right) = U_t\left(h + \frac{\Delta}{2}\right) \tag{26}$$

Combining equations (25) and (26) and simplifying one obtains

$$S''_f = \frac{K}{\Lambda}\left(\frac{\Delta}{h}\right)^2 \frac{U^2}{2g} \tag{27}$$

Finally, it can be shown that

$$\tau''_f = \rho g h S''_f = \frac{1}{2} C_D \rho U^2 \frac{\Delta}{\Lambda} \tag{28}$$

and therefore, combining equations (27) and (28), one can express K as

$$K = C_D \frac{h}{\Lambda}\frac{\Lambda}{\Delta} \tag{29}$$

where C_D = the drag coefficient for the bedforms. Values of C_D and K were computed for data from Vittal (1972) and K is plotted as a function of $\frac{h}{\Lambda}$ with $\frac{\Delta}{\Lambda}$ as a parameter in Figure 10. The curves show that K increases as $\frac{h}{\Lambda}$ increases and the effect of bedform steepness decreases as $\frac{\Delta}{\Lambda}$ decreases. The range of K for the data is from 0.80 to 1.7 in comparison to the value of 1.0 for sudden expansion of pipes. In the case of a sudden flow expansion in a pipe, the approach conditions up to the section of expansion are uniform. As a result, the drop of pressure due to separation is only considered in obtaining the form drag due to the pipe expansion. However, in the case of triangular bedforms, in addition to the low pressure on the lee-side of the bedforms due to the sudden expansion at the crest, there exist high pressures on the upstream face of the bedforms, giving rise to larger drag forces. As a result, values of K can be larger for sand waves than for pipe expansions (Vittal, 1989).

Using equation (27), values of τ''_f were computed using the relationship $\tau''_f = \rho g h S''_f$. These values were plotted as τ''_f versus τ'' in Figure 11 for smooth bedforms with $\frac{\Delta}{\Lambda}$ as a parameter. The smooth bedform data were used because the curves for K in Figure 10 were determined with the data for sand coated bedforms and the effect of surface roughness is small. The plots show that the agreement between computed and measured values of the form shear stress is virtually independent of the bedform steepness. When $\frac{\Delta}{\Lambda} = 0.05$, the agreement is excellent. In the case of $\frac{\Delta}{\Lambda} = 0.067$ and $\frac{\Delta}{\Lambda} = 0.098$, τ''_f over-estimates the measured shear stress by a virtually negligible constant value. When $\frac{\Delta}{\Lambda} = 0.20$, the scatter in the data has increased but the plotted points are evenly distributed about the equal yield line. These results suggest that the form roughness separation provides a reliable means of partitioning the bed shear stresses for values of $\frac{\Delta}{\Lambda} \geq 0.05$. There are no data for $\frac{\Delta}{\Lambda} < 0.05$. Clearly, when $\Delta = 0$, the sand bed has become a plane bed and $\tau''_f = 0$. It follows, that there must be some minimum value of $\frac{\Delta}{\Lambda}$ (minimum Δ) at which the uncertainty in determining K is too large to make the determination of τ''_f reliable. Examination of Figure 7 suggests that lengths of flow separations in the lee of the bedforms can be clearly defined and measured for values of $\frac{\Delta}{\Lambda}$ as low as 0.02. Further test are required to determine the minimum practical value of $\frac{\Delta}{\Lambda}$ for which the form roughness separation method can be used.

CONCLUSIONS

Examination of available information on the partitioning of the total shear stress for a uniform flow over triangular roughness elements has led to the following conclusions.

The concept of isolated roughness flow was confirmed to be valid. This means that the total bed shear stress is equal to the sum of the sand-grain shear stress and the form roughness shear stress.

The ratio of sand-grain shear stress to total shear stress decreases as the bedform steepness increases. This means that the sand-grain friction becomes less important as the form roughness of the bed increases.

The sand-grain shear stress obtained with the depth-separation method is larger than that obtained with the slope-separation method. The difference increases as the steepness of the bedforms increases. For a plane bed, both methods give the same results.

Review of existing data indicates that the slope-separation method gives satisfactory results in the range $0 \leq \frac{\Delta}{\Lambda} \leq 0.07$. When $\frac{\Delta}{\Lambda} > 0.07$, a correction coefficient must be applied. The correction coefficient which accounts for the reduction in active sand-grain surface area appears to give reasonable results for engineering purposes.

Available data indicate that the effective shear stress due to the form roughness of bedforms can be computed using principles of energy losses due to the sudden flow expansion immediately downstream of the bedform crests. The results obtained with data from Vittal (1972) provide sufficient accuracy for most practical purposes. More tests are required to obtain data over a wider range of flow conditions.

APPENDIX I. REFERENCES

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APPENDIX II. NOTATION

The following symbols are used in this report:

b = a coefficient;
B_s = a function of the Reynolds Number $\frac{u_* k_s}{\nu}$;
C_D = drag coefficient for bedforms;
C_Δ = sand-grain shear stress adjustment coefficient;
D_{50} = median diameter of sand grains;
f = a function;
g = acceleration due to gravity;
h = average depth of flow;
h_Δ = head loss over a bedform;
k_s = equivalent sand-grain roughness;
K = energy loss coefficient for sudden flow expansions;
ℓ_s = length of bedform surface exposed to the flow;
ℓ_w = length of flow separation for triangular bedforms;
n = Manning's roughness coefficient;

n_s = Manning's roughness coefficient for sand-grain roughness;
n_Δ = Manning's roughness coefficient for form roughness;
R = hydraulic radius;
Re_* = flow Reynolds number $\frac{u_* k_s}{\nu}$;
R' = sand-grain roughness component of hydraulic radius;
R'' = form roughness component of hydraulic radius;
S = water surface slope of a uniform flow;
S' = sand-grain roughness component of water surface slope;
S'' = form roughness component of water surface slope;
S''_f = form roughness component of water surface slope from energy principles;
U = average velocity of flow;
u_* = shear velocity;
U_c = average flow velocity at the crest of bedforms;
U_t = average flow velocity above lowest point in trough of bedforms;

Δ = height of bedforms trough to crest;
$\delta\tau$ = the difference between computed and measured values of form roughness shear stress;
κ = Von Karman's universal constant;
Λ = length of bedforms crest to crest;
ν = kinematic viscosity of the fluid;
ρ = density of the fluid;
τ = shear stress exerted by the flow on the bed;
τ' = shear stress component due to sand-grain roughness;
τ'' = shear stress component due to form roughness;
τ'_h = sand-grain shear stress computed with depth-separatiom method;
τ'_s = sand-grain shear stress computed with slope-separation method;
τ'_f = shear stress component computed as $\tau'_f = \tau - \tau''_f$;
τ''_f = shear stress due to form roughness computed from energy principles.

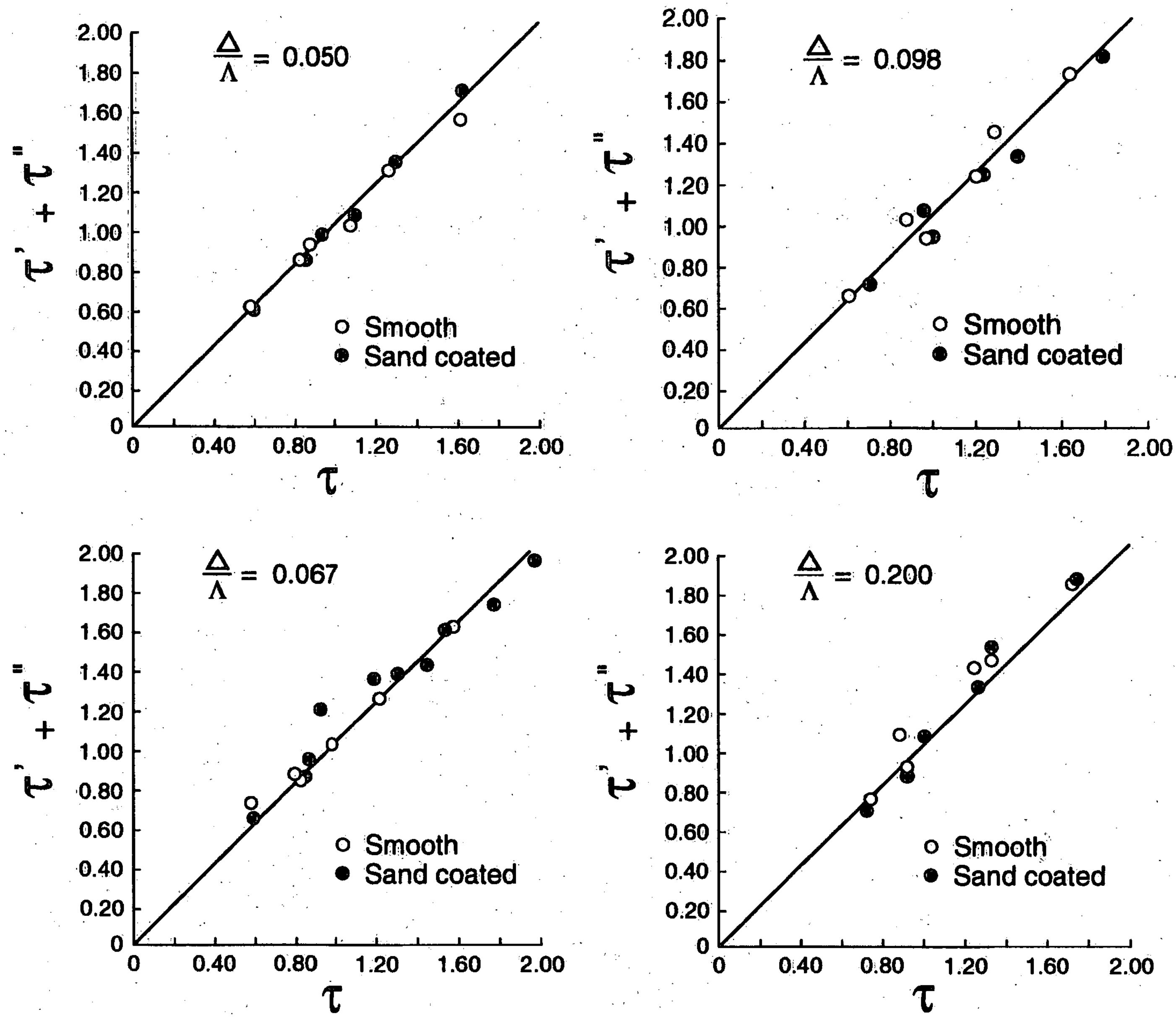

Figure 1. Comparison of added stress components with total shear stress

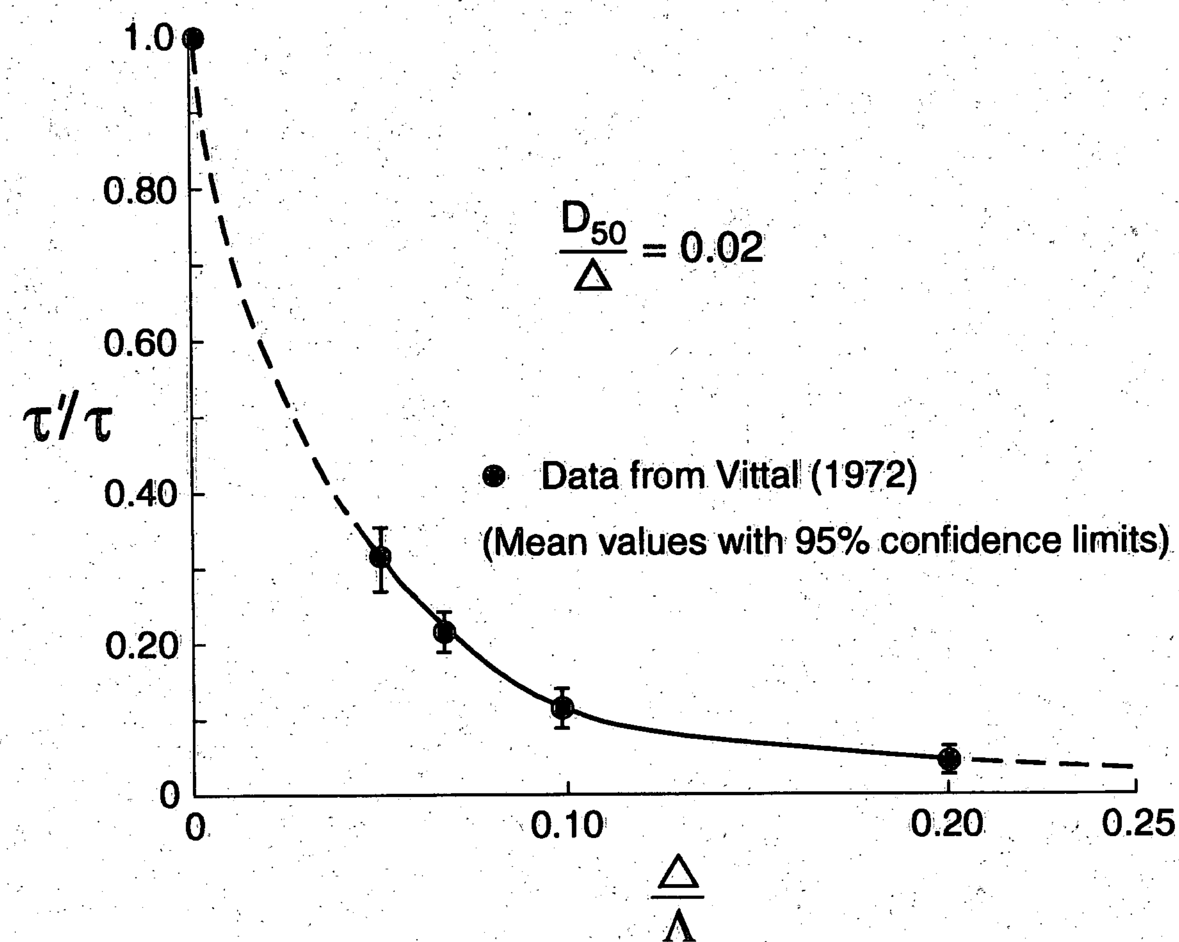

Figure 2. Variation of skin shear stress ratio for D_{50}/Δ

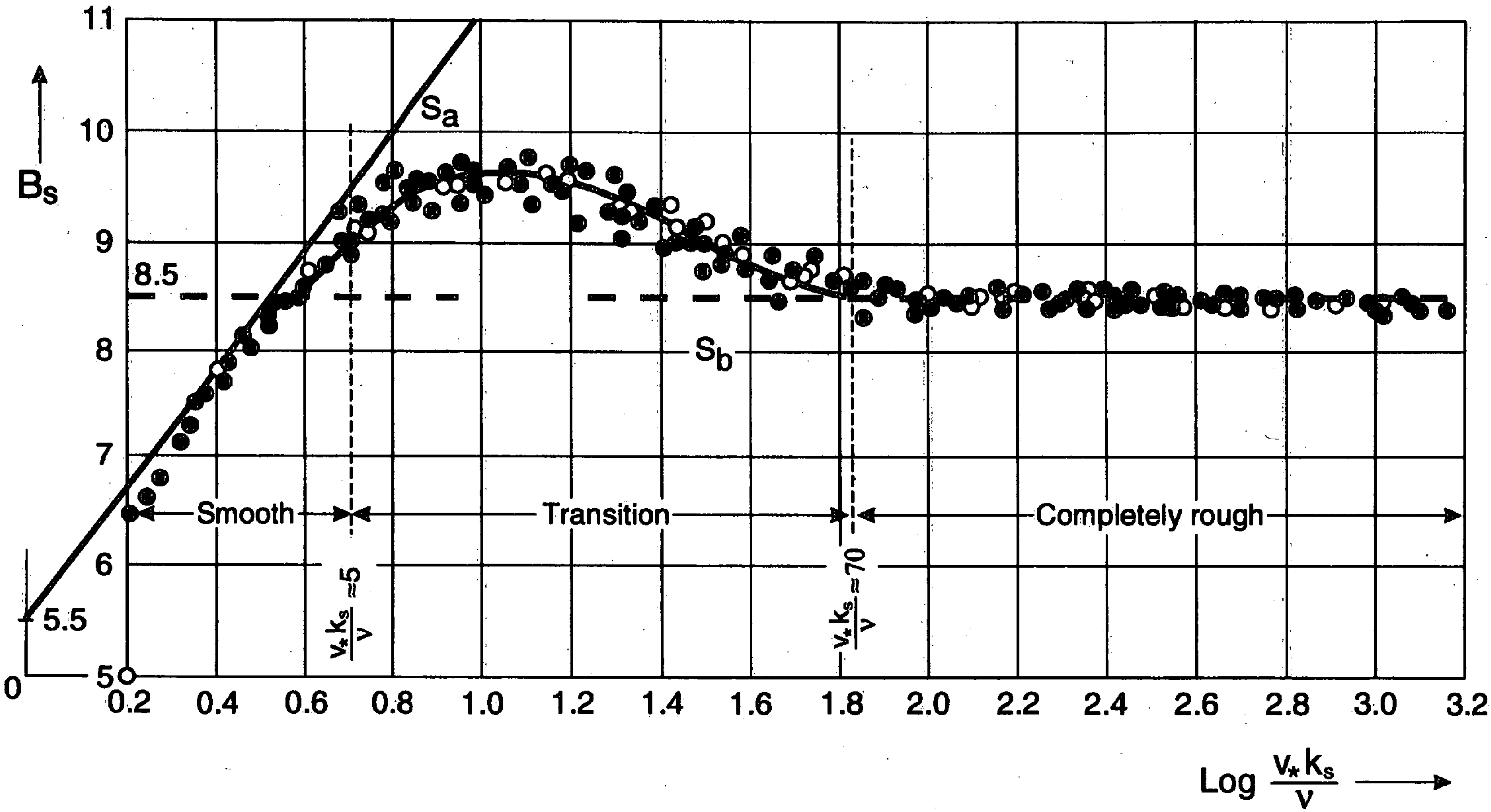

Figure 3. Variation of B_s (from Yalin, 1977)

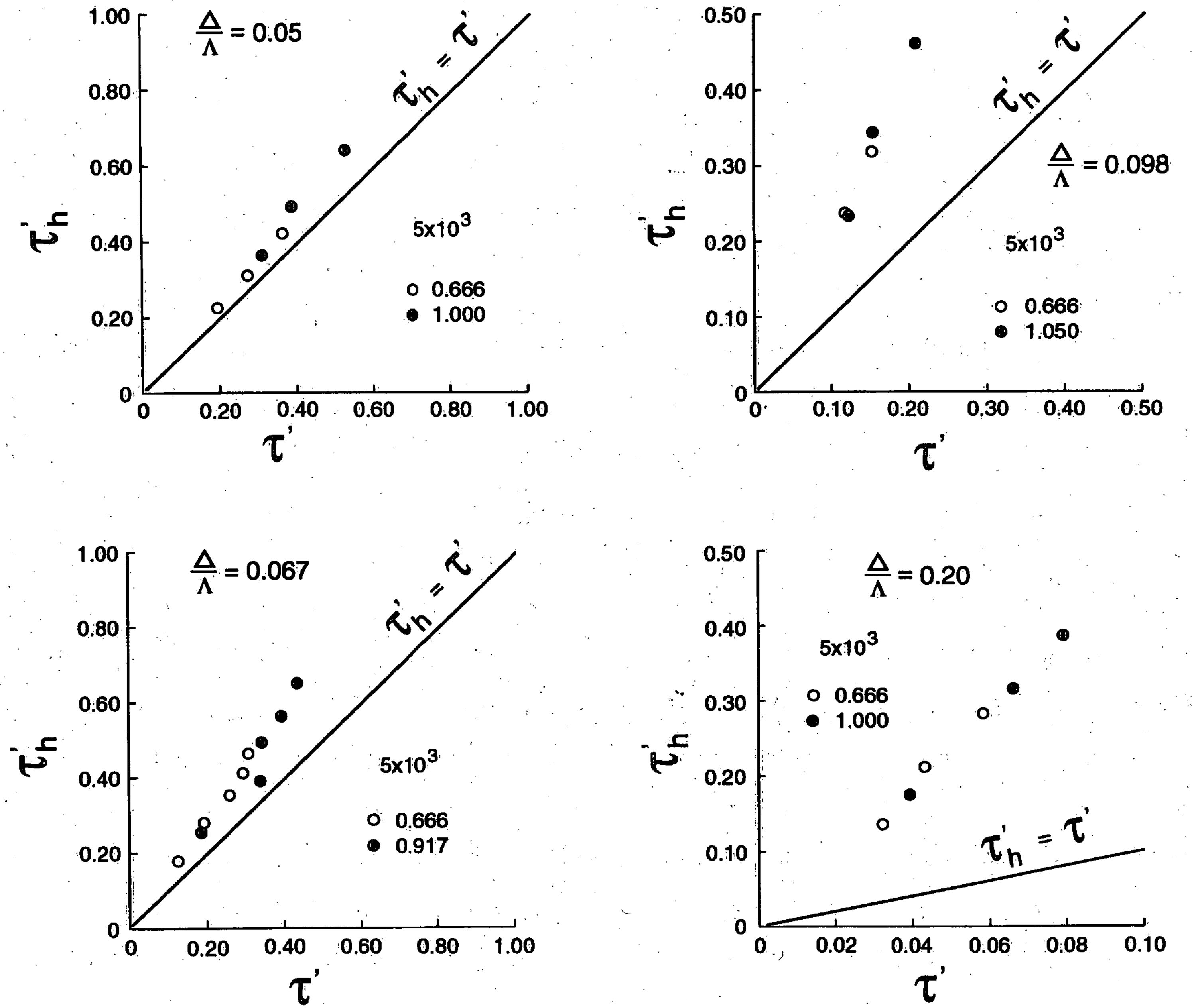

Figure 4. Comparison of depth-separation method with measured shear stress

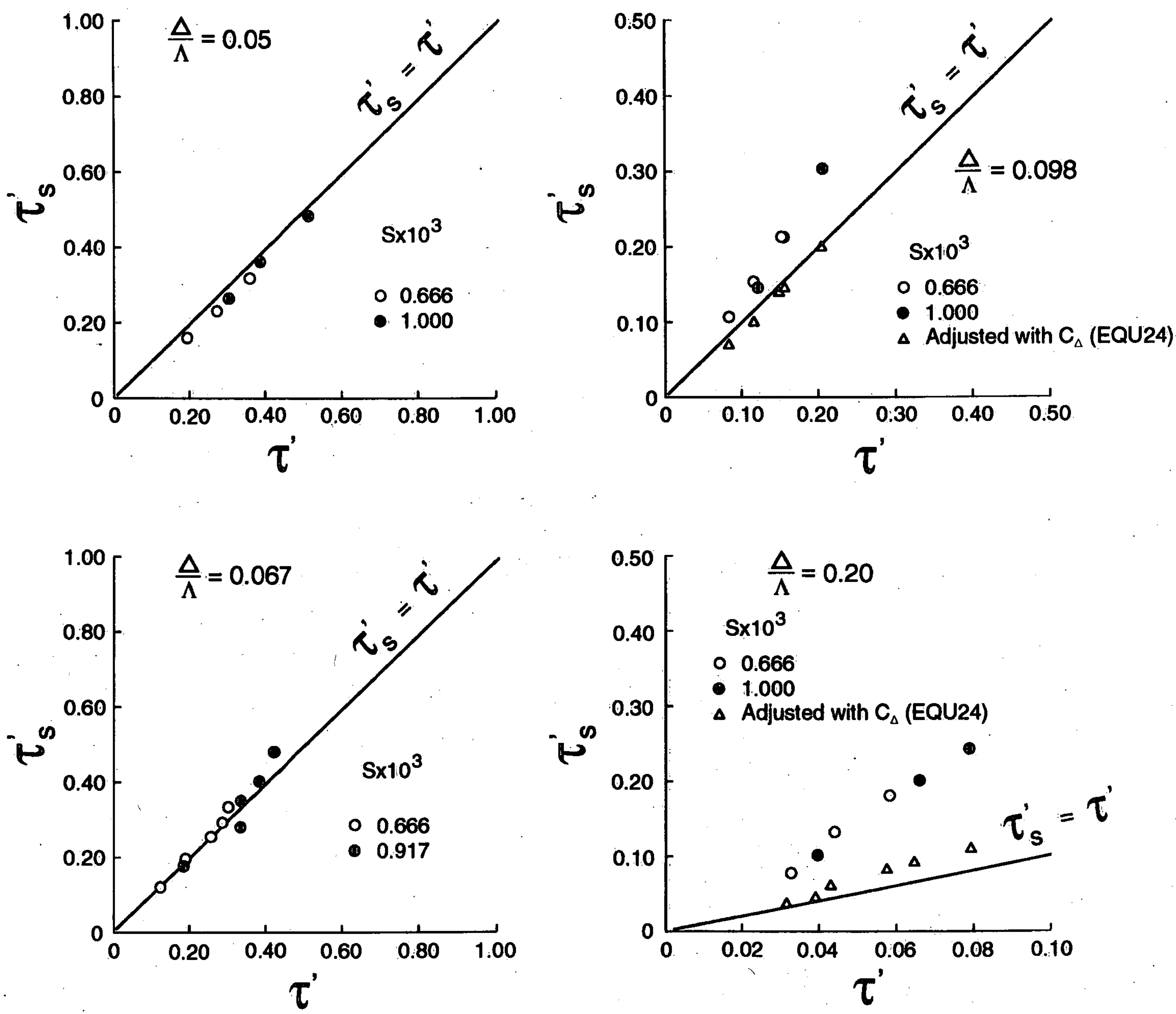

Figure 5. Comparison of slope-separation method with measured shear stress

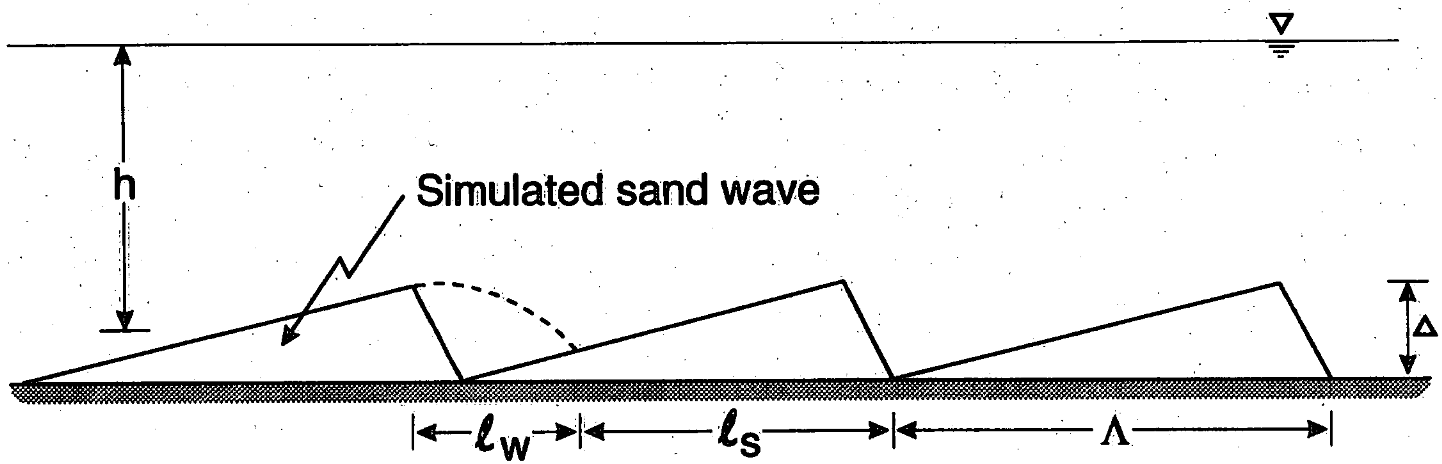

Figure 6. Schematic definition of ℓ_w and ℓ_s

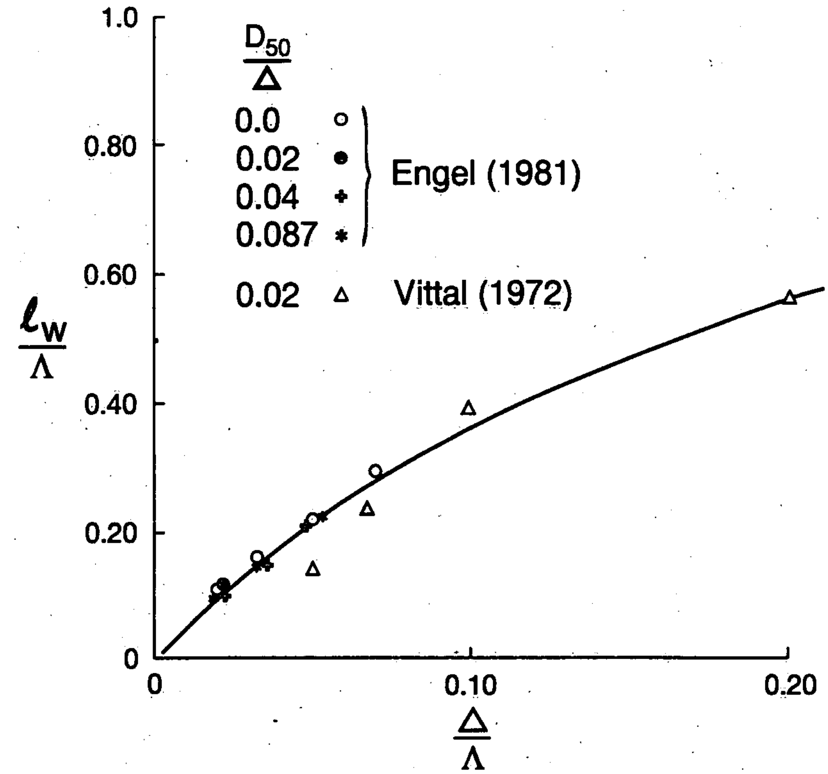

Figure 7.Variation of dimensionless length of flow separation

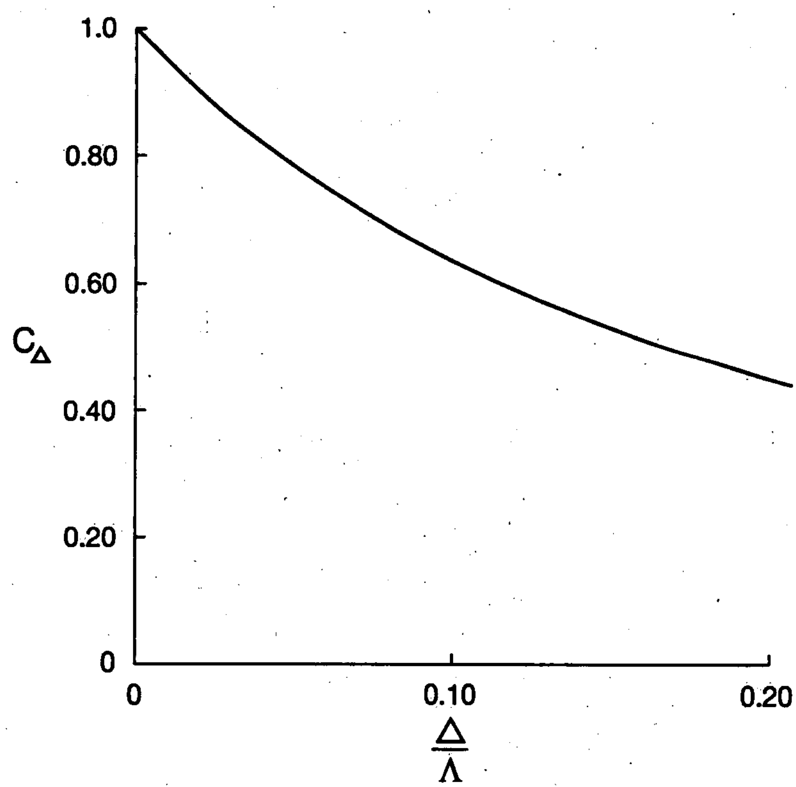

Figure 8. Shear stress adjustment coefficient

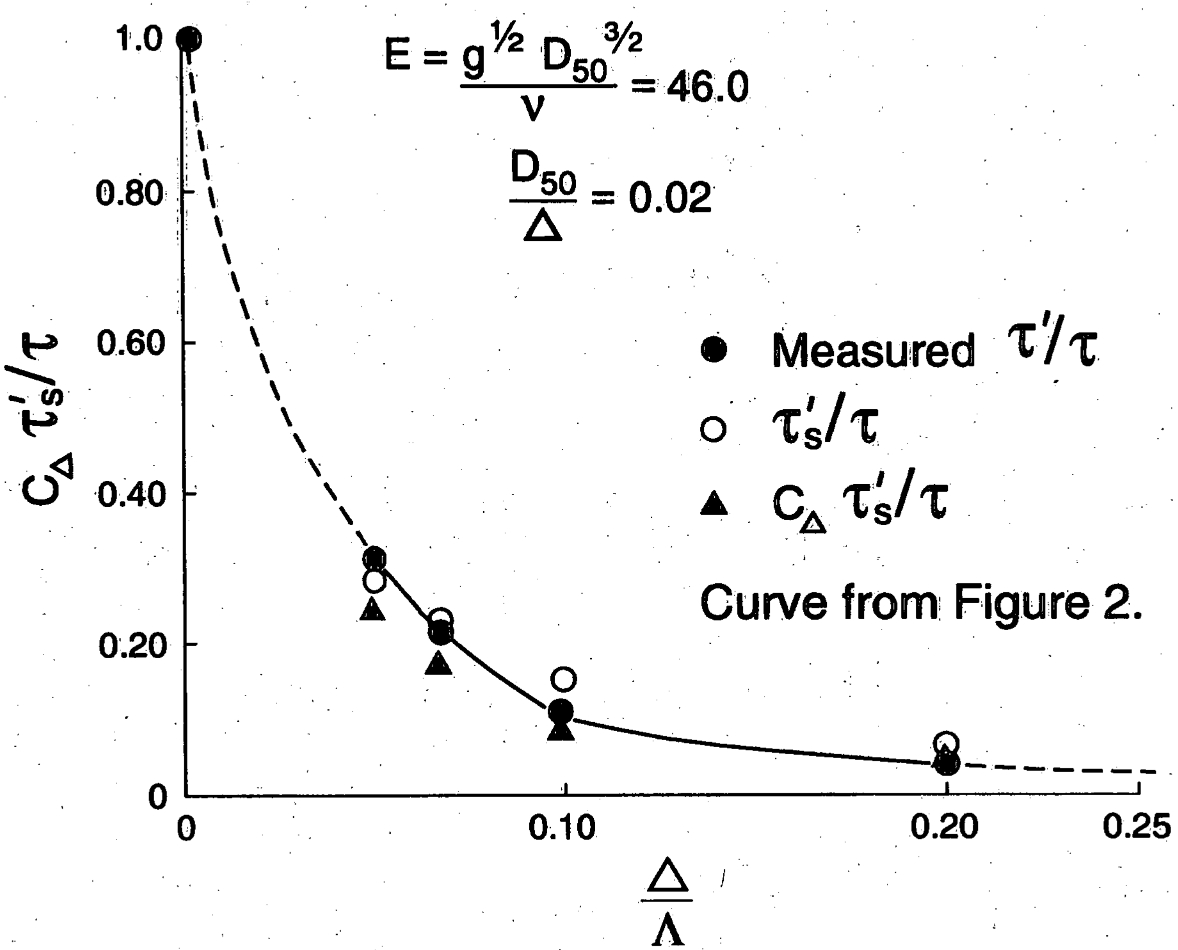

Figure 9. Adjusted shear stress ratio for Slope- separation method

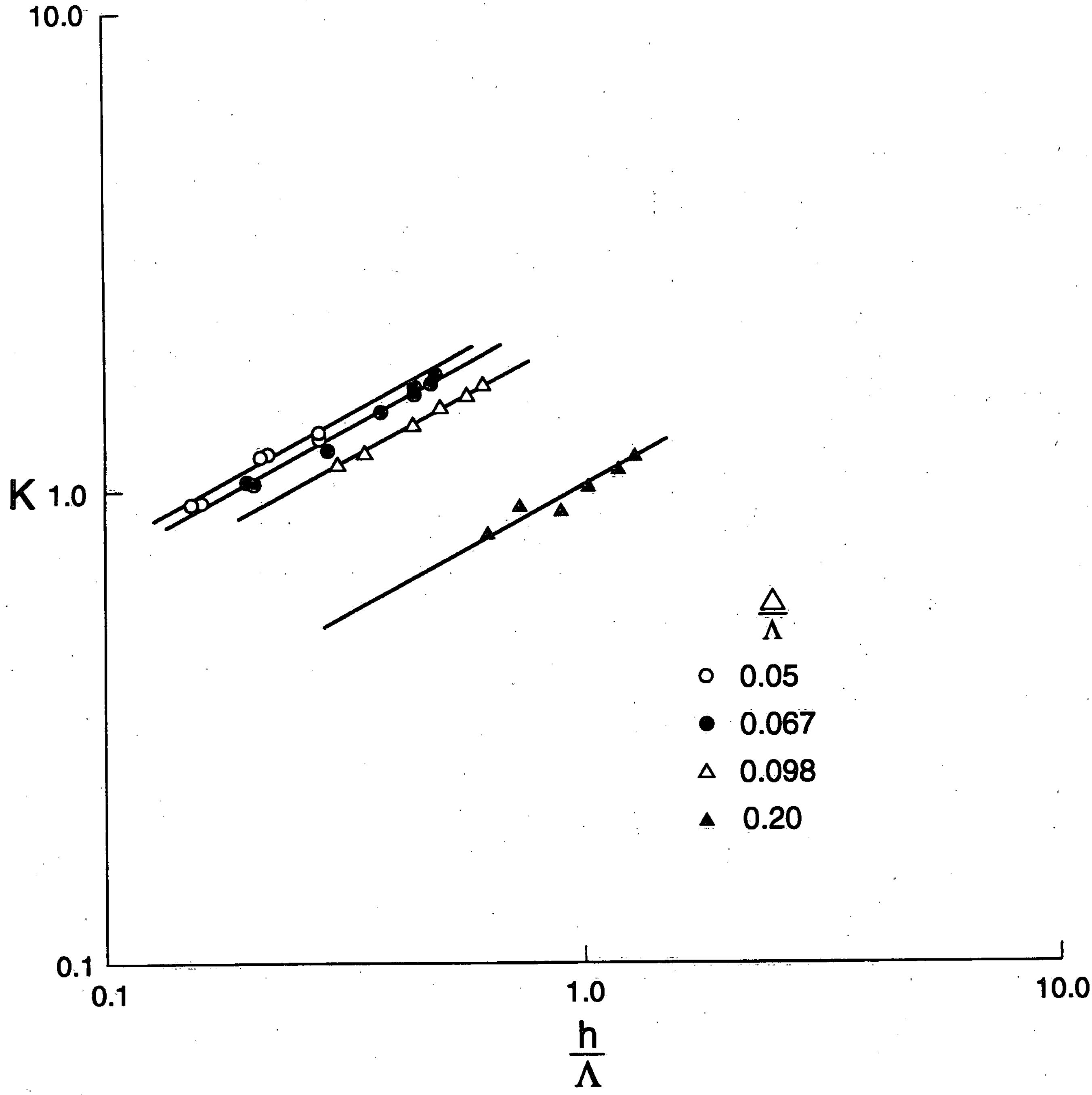

Figure 10. Flow expansion coefficient

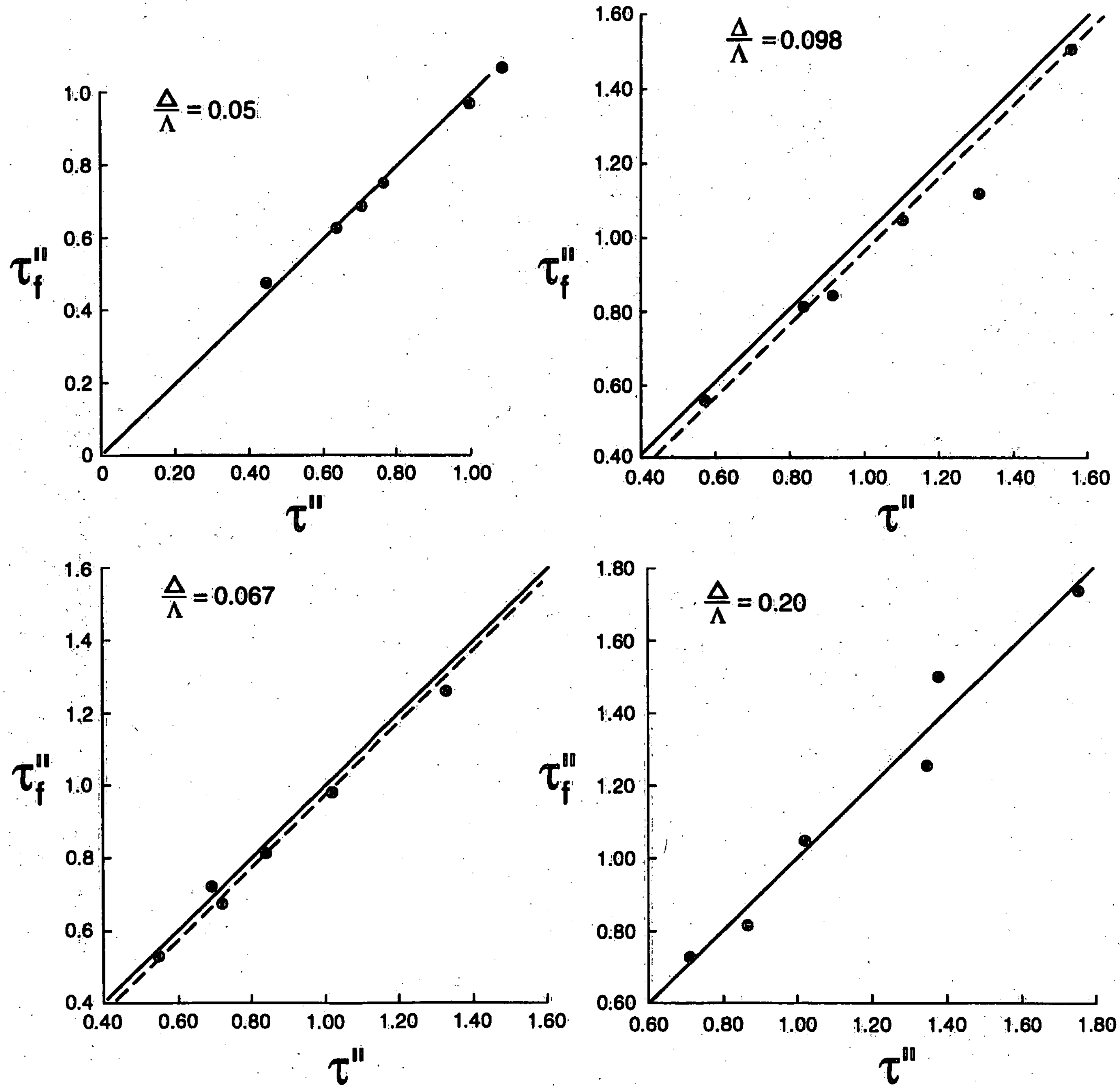

Figure 11. Comparison of form roughness shear stress separation method with measured shear stress

NATIONAL WATER RESEARCH INSTITUTE
P.O. BOX 5050, BURLINGTON, ONTARIO L7R 4A6

Environment Canada Environnement Canada

Canada

INSTITUT NATIONAL DE RECHERCHE SUR LES EAUX
C.P. 5050, BURLINGTON (ONTARIO) L7R 4A6