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CONSIDERATIONS FOR THE CALIBRATION OF ROD SUSPENDED PRICE CURRENT METERS

P. Engel

**NWRI CONTRIBUTION NO. 94-88** 

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# CONSIDERATIONS FOR THE CALIBRATION OF ROD SUSPENDED PRICE CURRENT METERS

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### MANAGEMENT PERSPECTIVE

Increased awareness of river pollution and the importance of water quality monitoring has made it necessary to reexamine the accuracy of discharge measurements. One of the factors contributing to the error in flow velocity measurements is the uncertainty in the current meter calibration itself. Present practice is to calibrate each current meter individually. An alternative approach is to develop an average calibration equation, known as a group calibration, based on a large number of current meters of the same type. A group calibration will be a considerable improvement if its uncertainty is not significantly different from that of calibrations of individual meters. In this report, individual and group calibrations are examined to provide basic information for the review of present calibration methods. The work is done in support of the Water Survey of Canada, Integrated Monitoring Branch, Atmospheric Environment Service.

# SOMMAIRE À L'INTENTION DE LA DIRECTION

Notre sensibilité accrue à la pollution des cours d'eau et l'importance de la surveillance de la qualité de l'eau ont rendu nécessaire un réexamen de la précision des mesures des débits. L'incertitude relative à l'étalonnage des courantomètres est l'un des facteurs qui contribuent à l'erreur relative aux mesures d'écoulement de l'eau. À l'heure actuelle, la pratique est d'étalonner chaque courantomètre séparément. Une autre possibilité serait de développer une équation d'étalonnage par calcul d'une moyenne de groupe établie à partir d'un grand nombre d'appareils du même type. Ce serait là une amélioration notable si l'incertitude associée à cette équation ne différait pas significativement de celle de l'étalonnage séparé des courantomètres. Dans ce rapport, nous comparons les étalonnages de courantomètres pris un à un et des étalonnages par calcul d'une moyenne de groupe pour recueillir les données de base sur lesquelles appuyer l'examen des méthodes actuelles d'étalonnage. Ce travail est fait pour le compte de la Division des relevés hydrologiques du Canada, Direction de la surveillance intégrée, Service de l'environnement atmosphérique.

### ABSTRACT

Thirty nine rod suspended Price current meters were calibrated individually. A new calibration equation fitted to each set of data by least squares methods gave excellent results. It was shown that for the meters tested, the uncertainty of group calibrations was substantially greater than the uncertainty of calibrating individual meters. Group calibration uncertainty was attributed to manufacturing variances in the fabrication of meter rotors. The largest errors occurred when a calibration for a particular meter was used with another meter. Recommendations for further tests at low velocities have been made.

# **RÉSUMÉ**

Trente-neuf courantomètres Price suspendus à des tiges ont été étalonnés séparément. Une nouvelle équation d'étalonnage ajustée à chaque ensemble de données par des méthodes des moindres carrés a procuré d'excellents résultats. On a pu établir que, pour les courantomètres testés, l'incertitude de l'échantillonnage par calcul d'une moyenne de groupe est substantiellement supérieure à celle de l'étalonnage séparé des courantomètres. L'incertitude associée à la moyenne de groupe a été attribuée à des variations dans la fabrication des rotors des courantomètres. La plus forte erreur a été observée lorsque l'étalonnage d'un courantomètre donné servait pour un autre. Il est recommandé de procéder à de nouveaux essais à bas régime de fonctionnement.

# TABLE OF CONTENTS

	Page
MANAGEMENT PERSPECTIVE	i i
SOMMAIRE À L'INTENTION DE LA DIRECTION	ii
ABSTRACT	iii
RÉSUMÉ	iv
INTRODUCTION	1
FORM OF THE CALIBRATION EQUATION	1
Physical Properties of $A$ , $B$ and $k$	3
UNCERTAINTY OF CURRENT METER CALIBRATIONS	4
Uncertainty Equation	4
Individual Meter Calibration	5
Group Calibration	5
Effect of Variability in $E_{A_b}$ , $E_{B_b}$ and $E_{k_b}$	6
Implications of Changing $E_{A_b}$ and $E_{k_b}$	6
Mean Values of Calibration Coefficients	7
Effect of Interchanging Meter Calibrations	7
CONCLUSIONS	8
ACKNOWLEDGEMENTS	8
APPENDIX I. REFERENCES	9
TABLES	10
FIGURES	

# CONSIDERATIONS FOR THE CALIBRATION OF ROD SUSPENDED PRICE CURRENT METERS

by  $P. Engel^1$ 

#### INTRODUCTION

The determination of river discharge requires the measurement of flow velocity. The velocity is measured by placing a meter into the flow and recording the rate of rotation of the rotor, usually in revolutions per second. The relationship between the linear velocity of the flow and the revolutions per second is determined by calibrating the meter in a towing tank. The current meter calibrations are normally expressed by some form of equation from which calibration certificates are prepared. One of the factors contributing to the error in a flow velocity measurement is due to the uncertainty in the current meter calibration (Smoot and Carter, 1968).

The Water Survey of Canada is the largest user of Price current meters in Canada. Presently, it is standard procedure to calibrate all meters on a regular basis. Each one of these calibrations has associated with it an uncertainty which is greatest in the low velocity range (Engel and Wiebe, 1993). Low velocity uncertainties are largely due to the towing tank environment because of residual velocities in the tank (Kamphuis, 1971; Engel, 1993). These disturbances can be reduced by increasing the waiting times between successive calibration runs, however, this requires large increases in the total calibration time for each meter. Present methods of current meter calibrations are already very time consuming and further increases in calibration time are not desirable. It is therefore important to examine calibration accuracies while at the same time focusing on reducing calibration time. One possible alternative is to adopt an average equation known as a group calibration (Charlton, 1978). Such a procedure is useful if it can be shown that the uncertainty of the group calibration is not significantly greater than that for calibrations of individual meters.

In this report, basic considerations of calibration accuracy are examined to provide information for the review of present calibration methods. Individual calibrations of thirty nine rod suspended Price current meters were obtained in the towing tank of the Hydraulics Laboratory at the National Water Research Institute. The results, together with data from individual calibrations of five meters, each repeated 10 times (Engel and Wiebe, 1993), are used to examine the uncertainties obtained with individual meter and group calibrations. The work was done in support of the Water Survey of Canada, Integrated Monitoring Branch, Atmospheric Environment Service.

# FORM OF THE CALIBRATION EQUATION

In developing a new calibration equation for rod suspended Price meters, it was shown by Engel (1989), that for a frictionless current meter, the dimensionless rotor response could be expressed as

$$\frac{ND}{V} = \pi \left[ \frac{K-1}{K+1} \right] \tag{1}$$

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where N=the rate of rotation of the rotor, D = the effective diameter of the rotor, V = the average flow velocity or towing speed,  $K = \frac{C_{D1}}{C_{D2}}$ ,  $C_{D1}$  = the drag coefficient of the conical elements on the stoss-side and  $C_{D2}$  = the drag coefficient of the conical elements on the lee-side. The value of K must be determined experimentally.

Equation (1) reflects the typical response characteristics of the Price current meter in a two dimensional flow field if there is no frictional resistance in the bearings and other contact surfaces. ND/V is dependent only on the value of K which reflects primarily the shape and orientation of the conical elements of the rotor. The sensitivity of the meter is dependent on both D and K. For a given meter the value of K and D are constant and a practical calibration equation is normally expressed in a form of V as a function of N. Therefore, equation (1) may be rearranged to give

$$V = \frac{D}{\pi} \left[ \frac{K+1}{K-1} \right] N = AN \tag{2}$$

where A = the meter constant. Equation (2) is linear, with slope A and passes through the origin of a Vvs.N plot. Such a behaviour would be ideal for a current meter. It is known, however, that calibration curves are nonlinear, particularly in the region of lower velocities. A single, continuous calibration equation, which combines the linear and the frictional components of the rotor response, was developed by Engel (1989) and is given as

$$V = AN + Be^{-kN} \tag{3}$$

where A, B and k are coefficients to be determined by calibration in a towing tank. Values of these coefficients for the 39 calibrations conducted for this report are given in Table 1. Variations in A are 2% and less whereas values of B and k vary considerably from meter to meter. The variability in A is largely due to manufacturing variances and may be improved by tightening fabrication tolerances. The variability in B and k is due to the presence of residual currents in the towing tank and differences in the friction of the rotor bearings and electrical contacts in the meter head. It may be possible to improve calibration performance by changing towing procedures at the lower velocities and using frictionless technology such as optic fibers to eliminate the electrical contacts.

Typical examples of the goodness of the fit of equation (3) can be seen in Figures 1 to 10 in which curves of equation (3) are superimposed on the plotted data for ten of the thirty nine meters tested. The data are plotted as  $\frac{N}{V}$  vs. V. The ratio  $\frac{N}{V}$  was used because of its high sensitivity to changes in V. It represents the steady state rotation of the meter rotor for each metre of distance travelled in the towing tank and can therefore be considered to be a form of meter rotor efficiency. The curves fit the data quite well over the full range of velocities tested. Superimposed on the plots are the curves obtained from linear calibrations presently used by Water Survey of Canada. Agreement is quite good for velocities greater than 0.30 m/s. As velocities decrease below 0.30 m/s, the difference between the curves increases with the WSC curves giving larger values of  $\frac{N}{V}$ . Clearly, linear calibration curves should not be used if low flow accuracy is important. Therefore, only equation (3) is used as a basis for further analysis on calibration uncertainties in this report.

# Physical Properties of A, B and k

The coefficient A, as shown in equation (2), is given as

$$A = \frac{D}{\pi} \left[ \frac{K+1}{K-1} \right] \tag{4}$$

which shows that it depends on the shape and orientation of the conical rotor cups and the size of the rotor. For a given meter type, the rotor geometry is the same with minor differences due to normal fabrication variances. Therefore, variation in A from one meter to another should depend in part on quality control in the fabrication process.

The coefficient B represents the threshold velocity of the meter for a particular value of K. Theoretically, the threshold velocity is the maximum towing velocity for which the rotor will remain stationary. In other words, it is the flow velocity at which the rotor is on the verge of the beginning of rotation. Using dimensional analysis, it was shown by Engel (1989) that B can be expressed as

$$B = \frac{bT_o}{\rho D^{\frac{7}{2}}g^{\frac{1}{2}}} \tag{5}$$

where b = a coefficient,  $T_o = the$  resistance in the meter at the point of beginning of rotation which occurs at the threshold velocity,  $\rho = density$  of the fluid and g = the acceleration due to gravity. One can expect that the threshold velocity increases as  $T_o$  increases. Clearly, for best performance,  $T_o$  should be kept as small as possible. Equation (5) also shows that the threshold velocity is inversely proportional to the rotor diameter. Therefore, for a given static resistance  $T_o$ , the threshold velocity can be significantly decreased by increasing the rotor diameter. Finally, the effect of fluid density on B can be seen in Figure 11 from (Engel, 1976) in which data for the average calibrations of three Price meters in both air and water are plotted as Vvs.N. The curves clearly show that the threshold velocity, when the fluid is air, is much larger than when the fluid is water. Fortunately, changes in density of the water, as a result of temperature changes, are small compared to the change in density of air and therefore, normal changes in temperature of the water should not affect the response of the meter rotor significantly. The threshold velocity must also be dependent on the geometry of the drive elements of the rotor and therefore, the coefficient b must be a function of K which is included implicitly in the calibration of the meter.

The exponent kN in equation (3) is dimensionless and therefore, k has the units of s/rev. Physically, k is a decay constant, the magnitude of which dictates the rate at which the non-linear component of equation (3) approaches the linear component. The rate of change in the non-linear component reflects the change in dynamic friction and slippage in the coupling between the water and the rotor elements and eddies formed in the vicinity of the rotor. Since the threshold velocity is directly proportional to  $T_o$ , then k should be directly related to B. It was shown by Engel (1989) that k decreases as B decreases. Physically, one would expect that  $k \to 0$  as  $B \to 0$ , implying that when k = 0 the meter operates as an ideal frictionless meter.

# UNCERTAINTY OF CURRENT METER CALIBRATIONS

# **Uncertainty Equation**

It can be shown that, in accordance with the format of equation (3), the error in the computed velocity may be expressed as

$$\delta V = \left\{ \left[ \frac{\partial V}{\partial A} \delta A \right]^2 + \left[ \frac{\partial V}{\partial N} \delta N \right]^2 + \left[ \frac{\partial V}{\partial B} \delta B \right]^2 + \left[ \frac{\partial V}{\partial k} \delta k \right]^2 \right\}^{\frac{1}{2}}$$
 (6)

Equation (6) states that the error in velocity,  $\delta V$ , is the square root of the sum of the squares of the errors in A, N, B and k. The partial derivatives are obtained by differentiating equation (3), substituting into equation (6) and rearranging to give the relative error in the velocity in terms of the relative errors in A, B, k and N as

$$\frac{\delta V}{V} = \left\{ \frac{1}{(1+\beta)^2} \left[ \beta^2 \left( \frac{\delta A}{A} \right)^2 + \left( \frac{\delta B}{B} \right)^2 + k^2 N^2 \left( \frac{\delta k}{k} \right)^2 + (\beta - kN)^2 \left( \frac{\delta N}{N} \right)^2 \right] \right\}^{\frac{1}{2}}$$
(7)

in which  $\beta = \frac{AN}{Be^{-kN}}$ . The relative error ratios  $(\frac{\delta V}{V})$ ,  $(\frac{\delta A}{A})$ ,  $(\frac{\delta B}{B})$ ,  $(\frac{\delta k}{k})$  and  $(\frac{\delta N}{N})$  can be expressed as ratios of the standard deviation to the corresponding mean and as such become coefficients of variation (Herschy, 1978). The coefficient of variation is a basic measure of the uncertainty in the value of the variable it represents. Uncertainties of A, B, k and N in equation (3) can be computed by using small sampling theory.

The true value of a variable is the mean value of a very large sample. Such large samples are not feasible and true values are inferred based on limited sample sizes. For example, the true value of A is then said to lie between confidence limits defined by the relationship

$$A = \overline{A} \pm t_{0.975} S_A \tag{8}$$

where  $\overline{A}$  = the mean value of A from a limited sample,  $t_{0.975}$  = the confidence coefficient at the 95% confidence level from Student's t distribution for (n-1) degrees of freedom (Spiegel, 1961),  $S_A$  = the standard deviation of A about the sample mean  $\overline{A}$  and n = the number of values of A composing the limited sample. Equation (8) can be made dimensionless by dividing both sides by  $\overline{A}$ . In addition, by denoting the coefficient of variation as  $C_A$ , then  $C_A = \frac{S_A}{A}$  and one obtains

$$\frac{A}{A} = 1 \pm t_{0.975} C_A \tag{9}$$

The quantity  $t_{0.975}C_A$  in equation (9) represents the relative uncertainty in determining the true value of A obtained from n different observations of A and may be expressed as

$$E_A = 100t_{0.975}C_A \tag{10}$$

where  $E_A$  = the relative uncertainty of A in percent at the 95% confidence level. Similarly, the uncertainties for B, k and N can be computed as  $E_B$ ,  $E_k$  and  $E_N$ . Replacing the relative errors in equation (7) with the corresponding uncertainties, one obtains

$$E_V = \left\{ \frac{1}{(1+\beta)^2} \left[ \beta^2 E_A^2 + E_B^2 + k^2 N^2 E_k^2 + (\beta - kN)^2 E_N^2 \right] \right\}^{\frac{1}{2}}$$
 (11)

in which  $E_V$  = the uncertainty of the computed velocity at the 95% confidence level. For all practical values of N,  $\beta \gg kN$  and therefore equation (11) can be further simplified to give

$$E_V = \left\{ \frac{1}{(1+\beta)^2} \left[ \beta^2 (E_A^2 + E_N^2) + E_B^2 + k^2 N^2 E_k^2 \right] \right\}^{\frac{1}{2}}$$
 (12)

It has been found that the uncertainty in the velocity of the towing carriage is about 0.05% at the 95% confidence level. This translates into a value of  $E_N \approx .075\%$ . The effect  $E_N$  is only significant when values of  $E_A$  are very small (i.e. of the order of  $E_N$ ). It is known, however, that  $E_A \gg E_N$  and therefore, for engineering purposes, the uncertainty  $E_N$  can be omitted from equation (12) resulting in

$$E_V = \left\{ \frac{1}{(1+\beta)^2} \left[ \beta^2 E_A^2 + E_B^2 + k^2 N^2 E_k^2 \right] \right\}^{\frac{1}{2}}$$
 (13)

Equation (13) is used to examine the uncertainties obtained with individual meter calibrations and group calibrations.

# **Individual Meter Calibration**

Mean values of the calibration coefficients given as  $\overline{A}_s$ ,  $\overline{B}_s$  and  $\overline{k}_s$  and the corresponding uncertainties  $E_{A_s}$ ,  $E_{B_s}$  and  $E_{k_s}$  for each of five meters, calibrated 10 times, are given in Table 2. Uncertainties in the computed velocities given as  $E_{V_s}$  were computed for different values of the rate of meter rotor rotation N for each of five meters tested. The results are plotted in Figure 12 as  $E_{V_s}$  versus V. The curves clearly show that repeatability of a given calibration is very good and better than  $\pm 0.3\%$  for towing velocities greater than 1.0 m/s. For velocities less than that, the uncertainty increases, with the rate of change increasing, reaching values greater than  $\pm 5\%$  at velocities less than 0.1 m/s. Considering that geometric properties of each meter are constant throughout the tests, the uncertainties must be attributed to experimental error, conditions in the towing tank and the difficulty of maintaining constant pressure with the electrical contact wire (i.e. cat whisker) in the data acquisition mechanism of the meter. The uncertainties of individual calibrations represent the standard against which all other calibration strategies should be compared.

## **Group Calibration**

The mean values for the coefficients of the 39 calibrations in Table 1, expressed as  $\overline{A}_b$ ,  $\overline{B}_b$  and  $\overline{k}_b$  and the corresponding uncertainties  $E_{A_b}$ ,  $E_{B_b}$  and  $E_{k_b}$ , are given in Table 3. Using these mean values, the group calibration equation for rod suspended Price meters is given by

$$V_b = 0.68037N + 0.00927e^{-3.5564N} (14)$$

Uncertainties in the computed velocity given as  $E_{V_b}$  were computed for given values of the rate of meter rotor rotation N by substituting the uncertainties from Table 3 in equation (13). The results are plotted in Figure 13 as  $E_{V_b}$  versus V superimposed on the uncertainties for individual meter calibrations from Figure 12. It can be seen at once that  $E_{V_b}$  is greater than  $E_{V_a}$ . For velocities greater than 0.7 m/s,  $E_{V_b}$  is constant at about  $\pm 1.2\%$ . This represents an increase over individual

calibration uncertainty by a factor of about 4 which is entirely due the uncertainty in A. As velocities decrease from 0.7 m/s, the uncertainty increases with the rate of change increasing. The difference between individual and group calibrations decreases, until  $V \approx 0.2$  m/s. For velocities less than that, group calibration uncertainty remains greater than that for individual calibrations, due to the difference in  $E_A$ , but their rates of increase is about the same.

# Effect of Variability in $E_{A_b}$ , $E_{B_b}$ and $E_{k_b}$

The importance of the uncertainty in  $A_b$  can be shown by examining the sensitivity of  $E_{V_b}$  to changes in  $E_{A_b}$ . Keeping  $E_{B_b}$  and  $E_{k_b}$  constant, at the values given in Table 3,  $E_{A_b}$  was varied from 0.25% to 1.25% and values of  $E_{V_b}$  computed with equation (13) over the range of towing velocities used in the tests. The results are plotted in Figure 14 as  $E_{V_b}$  versus V with  $E_{A_b}$  as a parameter. The curves clearly show the effect of variability in  $A_b$ . The effect begins to make itself felt at velocities of about 0.25 m/s and increases as towing velocities increase, until when  $V \ge 1$  m/s, the effect remains constant. For a given velocity,  $E_{V_b}$  increases as the uncertainty in  $A_b$  increases.

The effect of  $B_b$  is felt at low velocities and this can be shown by varying the uncertainty  $E_{B_b}$  while keeping the uncertainties  $E_{A_b}$  and  $E_{k_b}$  constant at the values given in Table 3. Values of  $E_{B_b}$  were varied from 25% to 150% and the corresponding values of  $E_{V_b}$  computed over the range of test velocities. The results are plotted as  $E_{V_b}$  versus V with  $E_{B_b}$  as a parameter in Figure 15. The curves clearly confirm that the effect of  $B_b$  is restricted to velocities less than 0.5 m/s with the effect increasing as velocities decrease. At a given value of V,  $E_{V_b}$  increases as  $E_{B_b}$  increases, however, the increase is only marginal even at velocities as low as 0.1 m/s. This suggests that efforts to improve the uncertainty of  $E_{B_b}$  are not justified.

The effect of  $k_b$  is also felt at low velocities and this can be shown by varying the uncertainty  $E_{k_b}$  while keeping the uncertainties  $E_{B_b}$  and  $E_{A_b}$  constant at the values given in Table 3. Values of  $E_{k_b}$  were varied from 25% to 200% and the corresponding values of  $E_{V_b}$  computed over the range of test velocities. The results are plotted as  $E_{V_b}$  versus V with  $E_{k_b}$  as a parameter in Figure 16. The curves show that the effect of  $k_b$  is felt for velocities up to 0.75 m/s with the effect increasing as velocities decrease. At a given value of V,  $E_{V_b}$  increases as  $E_{k_b}$  increases, with the rate of change increasing as  $E_{k_b}$  increases.

# Implications of Changing $E_{A_b}$ and $E_{k_b}$

It was shown that the uncertainty in the computed velocity for individual meter calibrations was about  $\pm 0.3\%$  for  $V \geq 0.5$  m/s. The curves in Figure 14 show that in order to achieve this accuracy for group calibrations, it is necessary to reduce the uncertainty of  $A_b$  from the present value of about  $\pm 1.2\%$  to  $\pm 0.3\%$ . It remains to be determined how much improvement in the fabrication tolerances is required to achieve this reduction in  $E_{A_b}$ . If sufficient improvements can be made economically, group calibrations will be possible.

The value of  $E_{k_b}$  for the present group calibration is 188.08%. Examination of Figure 16 shows that the calibration uncertainty at velocities less than 0.5 m/s can be improved by reducing  $E_{k_b}$  from 188% to 100%. In order to achieve this, calibration procedures must be changed to reduce the effect of residual currents in the towing tank. One way of reducing the residual currents is by increasing the waiting times between successive meter tows for individual calibrations, thereby allowing the currents to decay. However, this will provide only limited improvement because large increases in waiting times will result in excessive total time to calibrate each meter. Instead further tests should be conducted to explore the possibility of low velocity group calibrations. Since such

calibrations are one time efforts, increased waiting times can be used together with longer towing lengths to reduce the effects of residual currents. The resulting mean values of  $B_b$  and  $k_b$  can then be used as characteristic values for low velocity segments of all individual calibrations of rod suspended Price current meters. The effect of the electrical contacts cannot be improved with the present system; instead non-contact methods using fiber optics or magnetic switches should be used.

# Mean Values of Calibration Coefficients

The uncertainty in the true value of the mean of the coefficient A, say  $\mu_A$ , is given by

$$E_{\mu_A} = \frac{100t_{0.975}C_A}{\sqrt{n-1}} \tag{15}$$

where  $E_{\mu_A}$  is the uncertainty in  $\mu_A$  at the 95% confidence level and  $C_A = \frac{S_A}{A}$  = the coefficient of variation of A in which  $S_A$  = the standard deviation of A. Similarly, the uncertainties for  $\mu_B$  and  $\mu_k$  can be given by  $E_{\mu_B}$  and  $E_{\mu_k}$ . The uncertainty in the mean values of the coefficients for individual calibrations given as  $E_{\mu_{A_a}}$ ,  $E_{\mu_{B_a}}$  and  $E_{\mu_k}$  are compared with the corresponding uncertainties for group calibrations given as  $E_{\mu_{A_b}}$ ,  $E_{\mu_{B_b}}$  and  $E_{\mu_{k_b}}$ .

The mean values of the calibration coefficients for the individual meter calibrations, together with the 95% chance that they are the true values, obtained from Engel and Wiebe (1993), for five current meters, each calibrated ten times, are given in Table 4. Examination of the data show that values of  $\overline{A}_s$  vary very little from one meter to another. For each of the five calibrations shown, the 95% uncertainty  $E_{\mu_A}$  is 0.1% or less which can be attributed to experimental error. In contrast to this, values of  $\overline{B}_s$  vary considerably with  $E_{\mu_B}$  ranging from 12% to 25% for the five meters tested. This uncertainty in  $\mu_B$  reflects the towing tank environment. The greatest uncertainty for each individual calibration is in the value of  $\mu_k$ , varying from about 36% to 79% for the five meters tested. Theoretically, k accounts for the dynamic friction in the meter, but the uncertainty  $E_{\mu_k}$  is also a further confirmation of the variability in the low velocities in the towing tank due to residual currents in spite of the fact that the towing carriage speed can be determined very accurately.

The mean values of the coefficients for the group calibration, together with the 95% chance that they are the true values, are given in Table 5. The value of  $\overline{A}_b$  is very similar to that obtained for individual meter calibrations but the uncertainty  $E_{\mu_{A_b}}$ , although quite small in the absolute sense, is about twice as large. This increase in uncertainty reflects the difference in the meter characteristics which are mostly due to fabrication variances in the rotor geometry. Values of  $E_{\mu_{B_b}}$  and  $E_{\mu_{B_b}}$  are very similar and values of  $E_{\mu_{k_b}}$  are somewhat lower than those of  $E_{\mu_{k_b}}$ . This is further confirmation that the differences between individual and group calibrations are mainly due to the variability in rotor geometry from one meter to the next.

### Effect of Interchanging Meter Calibrations

Given the uncertainty of group calibrations relative to individual meter calibrations, one can expect that greater errors will be incurred if one adopts for one meter the calibration intended for another. To examine the errors that can result from such interchanges, 25 of the 39 calibrations given in Table 1 were used. For 21 values of N, differences in the velocities between pairs of calibration equations were computed and expressed in percent. Each equation was paired with the other 24 for a total of 600 pairings. The results are plotted as  $\frac{\Delta V}{V}$  versus V in Figure 17.

The plot shows that  $\frac{\Delta V}{V}$  slightly exceeds  $\pm 2\%$  for velocities greater than 0.5 m/s. As velocities decrease below 0.5 m/s, the relative velocity difference increases with the rate of change increasing rapidly as V decreases, reaching values near  $\pm 10\%$  when  $V \approx 0.10$  m/s. The error, as a result of interchanging meters, is about twice the uncertainty at the 95% confidence level obtained with the group calibration equation. This is due to the fact that the group calibration equation represents the centroid of the individual calibrations of all the meters in the group. Clearly, interchanging of current meter calibrations should be avoided.

#### CONCLUSIONS

Using theoretical analysis and extensive tests in a towing tank, the following results were obtained:

The best calibration results with rod suspended Price meters, presently used by the Water Survey of Canada, are obtained by calibrating each meter individually. For velocities greater than 0.7 m/s, the uncertainty in the computed velocity is about  $\pm 0.3\%$  at the 95% confidence level. For group calibrations the uncertainty increases to  $\pm 1.2\%$ .

For velocities less than 0.5 m/s, the difference in the calibration accuracy of individual meter and group calibrations is less significant. This is largely due to the presence of residual currents in the towing tank. Tests should be conducted to determine if the effects of residual velocities can be significantly reduced. If repeatability of calibrations at the low velocities cannot be improved, there is no advantage in refining calibration procedures for the low velocity range.

Differences in meter calibrations, reflected by the calibration coefficient A, are mostly due to small fabrication variances in rotor geometry. In order to obtain uncertainties for group calibrations, equal to those of individual meter calibrations, the uncertainty in A for group calibrations must be reduced by a factor of about four. Efforts should be made to determine if such improvements can be obtained economically by stricter quality control in meter fabrication.

The effect of the uncertainty due to the calibration coefficient B is restricted to very low velocities. Efforts to improve the uncertainty  $E_B$  are not justified.

The effect of the uncertainty due to the calibration coefficient k is restricted to velocities less than 0.75 m/s. Calibration uncertainty can be sufficiently improved by reducing the uncertainty in k by a factor of two. Tests should be conducted in the towing tank to see if such improvement can be achieved by changes in calibration procedures.

Care should be taken to avoid adopting for one meter an individual calibration intended for another meter of the same type. This can lead to errors of up to  $\pm 2\%$  for velocities greater than 0.5 m/s and larger errors for low velocities.

#### ACKNOWLEDGEMENT

The calibrations were conducted by B. Near and C. Bil. Their support is greatly appreciated.

### APPENDIX I. REFERENCES

Charlton, F.G, 1978: Current Meters. In Hydrometry, Edited by R.W. Herschy, John Wiley and Sons, Toronto.

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Engel, P., 1976: A Universal Calibration Equation for Price Meters and Similar Instruments. Environment Canada, Inland Waters Directorate, Canada Centre for Inland Waters, Scientific Series No. 65.

Engel, P., 1989: A New Calibration Equation for Vertical Axis Current Meters. NWRI Contribution 89-131, National Water Research Institute, Canada Centre for Inland Waters, Burlington, Ontario.

Engel, P., 1993 Stilling Times for the Calibration of Price Current Meters: A Review. NWRI Contribution 93-52, National Water Research Institute, Canada Centre for Inland Waters, Burlington, Ontario.

Engel, P. and K. Wiebe, 1993: Uncertainty in Current Meter Calibration. NWRI Contribution 93-57, National Water Research Institute, Canada Centre for Inland Waters, Burlington, Ontario.

Herschy, R.W., 1978: Accuracy. In Hydrometry, Edited by R.W. Herschy, John Wiley and Sons, Toronto.

Smoot, G.F. and R.W. Carter, 1968: Are Individual Current Meter Ratings Necessary. Journal of the Hydraulics Division, ASCE, vol. 94, No. Hy2.

Spiegel, M.S., 1961: Theory and Problems of Statistics. Schaum Outline, Schaum Publishing Company, New York, New York, U.S.A.

TABLE 1 VALUES OF k, A AND B FOR 39 METERS TESTED

No. 6-002 6-042 6-050 6-051 6-058 6-067 6-074 6-080 6-134	[s/rev]  4.151 1.367 0.000 6.350 2.800 7.533 3.730 0.000	0.6888 0.6877 0.6744 0.6846 0.6747 0.6787 0.6837	[m/s] 0.00741 0.00535 0.00478 0.01057 0.00896 0.01381 0.00749
6-042 6-050 6-051 6-058 6-067 6-074 6-080 6-134	1.367 0.000 6.350 2.800 7.533 3.730	0.6877 0.6744 0.6846 0.6747 0.6787	0.00535 0.00478 0.01057 0.00896 0.01381
6-042 6-050 6-051 6-058 6-067 6-074 6-080 6-134	1.367 0.000 6.350 2.800 7.533 3.730	0.6877 0.6744 0.6846 0.6747 0.6787	0.00535 0.00478 0.01057 0.00896 0.01381
6-050 6-051 6-058 6-067 6-074 6-080 6-134	0.000 6.350 2.800 7.533 3.730	0.6744 0.6846 0.6747 0.6787	0.00478 0.01057 0.00896 0.01381
6-051 6-058 6-067 6-074 6-080 6-134	6.350 2.800 7.533 3.730	0.6846 0.6747 0.6787	0.01057 0.00896 0.01381
6-058 6-067 6-074 6-080 6-134	2.800 7.533 3.730	0.6747 0.6787	0.00896 0.01381
6-067 6-074 6-080 6-134	7.533 3.730	0.6787	0.01381
6-074 6-080 6-134	3.730		
6-080 6-134		0,6837	0.00740
6-134	0.000		
		0.6746	0.00941
	7.605	0.6800	0.01249
			0.00488
			0.00674
			0.01504
6-231	3.245	0.6822	0.00766
6-247	5.290	0.6792	0.01152
6-248	10.72	0.6822	0.02118
6-253	2.699	0.6820	0.00782
6-262	2.970	0.6791	0.01431
6-280	1.590	0.6784	0.01309
6-294	1.850	0.6790	0.01100
6-309	1.145		0.00477
			0.01457
			0.02447
			0.01039
			0.00725
	· · · · · ·		0.00908
			0.00592
			0.01496
			0.00739
· · · · · · · · · · · · · · · · · · ·			0.00714
			0.00705
			0.01015
			0.00860
			0.00347
			0.01185
			0.00363
• •			0.00520
			0.00029
			0.00023
			0.01017
	6-167 6-215 6-229 6-231 6-247 6-248 6-253 6-262 6-280	6-167 0.928 6-215 1.181 6-229 5.999 6-231 3.245 6-247 5.290 6-248 10.72 6-253 2.699 6-262 2.970 6-280 1.590 6-294 1.850 6-309 1.145 6-310 4.800 6-319 15.91 6-338 4.091 6-340 1.771 6-344 1.869 6-398 1.495 6-415 2.770 6-433 7.821 6-435 3.300 6-436 3.550 6-443 8.599 6-459 0.000 6-476 0.482 6-491 4.441 6-494 0.494 6-498 0.631 6-502 1.320 6-503 1.663	6-167       0.928       0.6839         6-215       1.181       0.6758         6-229       5.999       0.6750         6-231       3.245       0.6822         6-247       5.290       0.6792         6-248       10.72       0.6822         6-253       2.699       0.6820         6-262       2.970       0.6791         6-280       1.590       0.6784         6-294       1.850       0.6790         6-309       1.145       0.6798         6-310       4.800       0.6778         6-319       15.91       0.6821         6-338       4.091       0.6764         6-340       1.771       0.6872         6-344       1.869       0.6801         6-398       1.495       0.6828         6-415       2.770       0.6846         6-433       7.821       0.6825         6-435       3.300       0.6759         6-436       3.550       0.6809         6-443       8.599       0.6821         6-459       0.000       0.6754         6-476       0.482       0.6770         6-491       4.

TABLE 2 UNCERTAINTIES AT 95% LEVEL FOR  $A_s$ ,  $B_s$  and  $k_s$ 

$\overline{A}_s \ [m/rev]$	$E_{A_{ullet}}$ [%]	$\overline{B}_s \ [m/s]$	$E_{B_s} \ [\%]$	$\overline{k}_s \ [s/rev]$	$E_{k_s} \ [\%]$
0.6783	0.302	0.01217	49.686	3.721	90.606
0.6788	0.246	0.00930	40.848	3.375	132.541
0.6791	0.110	0.00740	68.553	2.497	211.245
0.6817	0.272	0.00683	35.196	1.583	113.0855
0.6829	0.242	0.00480	74.280	1.298	196.191

TABLE 3 UNCERTAINTIES AT 95% LEVEL FOR  $A_b$ ,  $B_b$  and  $k_b$ 

$\overline{A_b} \ [m/rev]$	$E_{A_b} \ [\%]$	$\overline{B_b} \ [m/s]$	$E_{oldsymbol{B_b}}\ [\%]$	$rac{\overline{k_b}}{[s/rev]}$	$egin{aligned} E_{k_b} \ [\%] \end{aligned}$	
0.68037	1.1971	0.009269	105.97	3.5564	188.77	

TABLE 4 UNCERTAINTIES AT 95% LEVEL FOR  $\mu_{A_s}$ ,  $\mu_{B_s}$  and  $\mu_{k_s}$ 

$\overline{A}_s \ [m/rev]$	$E_{\mu_{A}}$ , $[\%]$	$\overline{B}_s \ [m/s]$	$E_{\mu_{B_{m{s}}}}$ $[\%]$	$\overline{k}_s \ [s/rev]$	$E_{\mu_{k}}$ , $[\%]$				
 0.6783	0.101	0.01217	16.562	3.721	30.202	_			
0.6788	0.082	0.00930	13.616	3.375	43.847				
0.6791	0.037	0.00740	22.851	2.497	70.415				
0.6817	0.091	0.00683	11.732	1.583	37.695				
0.6829	0.081	0.00480	24.760	1.298	65.397				

TABLE 5 UNCERTAINTIES AT 95% LEVEL FOR  $\mu_{A_b}$ ,  $\mu_{B_b}$  and  $\mu_{k_b}$ 

$\overline{A}_b \ [m/rev]$	$E_{\mu_{m{A}_b}} \ [\%]^{b}$	$\overline{B}_b \ [m/s]$	$E_{\mu_{B_b}} \ [\%]$	$\overline{k}_b \ [s/rev]$	$ar{E}_{\mu_{m{k_b}}} \ [\%]$
0.68037	0.1942	0.009269	17.203	3.5564	30.532

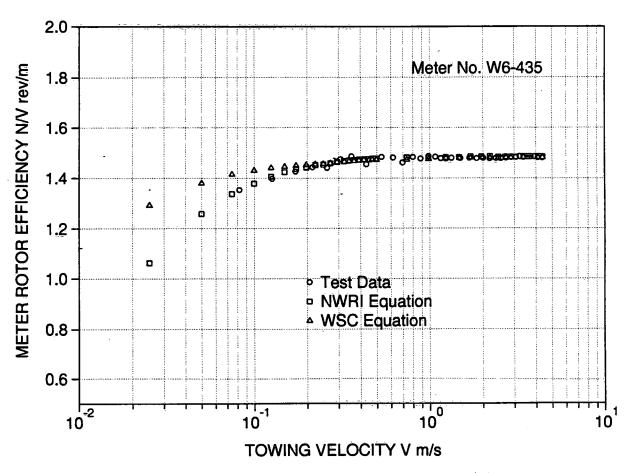


Figure 1. Calibration curves for meter no. W6-435

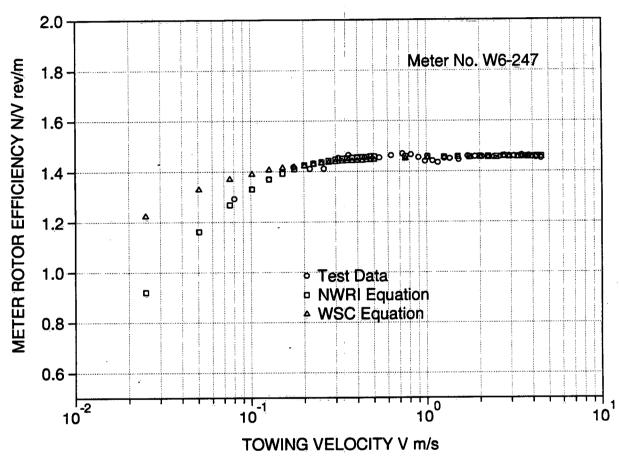


Figure 2. Calibration curves for meter no. W6-247

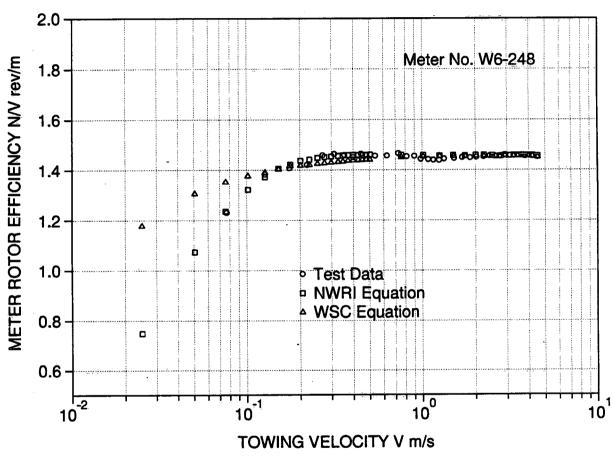


Figure 3. Calibration curves for meter no. W6-248

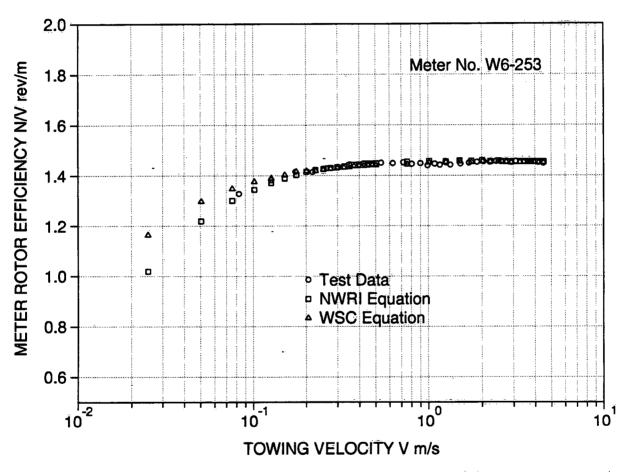


Figure 4. Calibration curves for meter no. W6-253

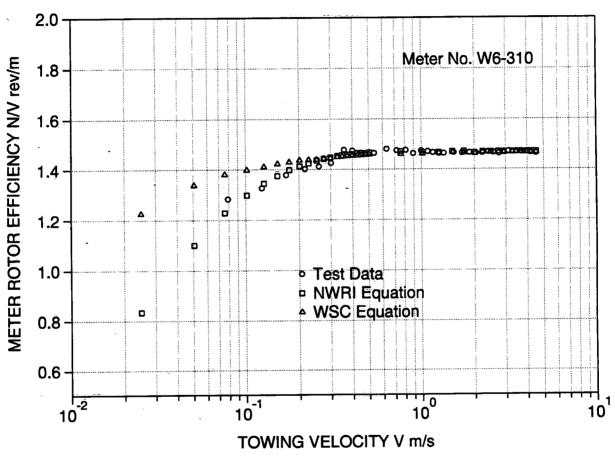


Figure 5. Calibration curves for meter no. W6-310

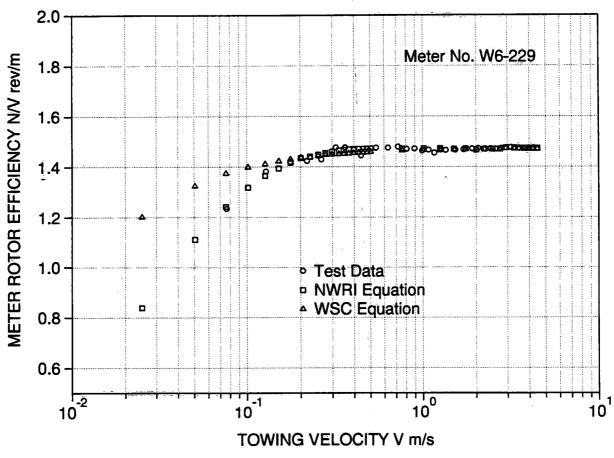


Figure 6. Calibration curves for meter no. W6-229

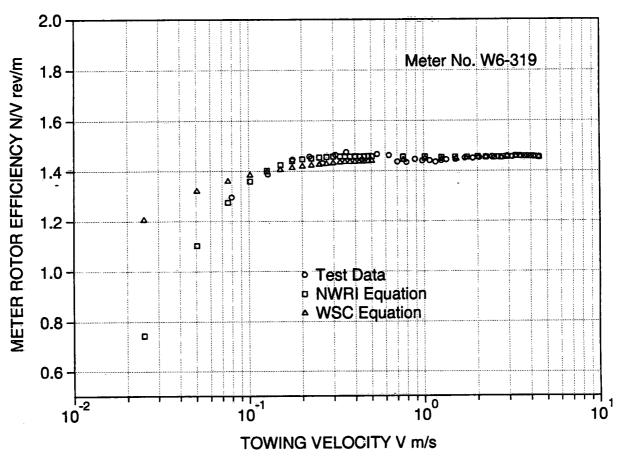


Figure 7. Calibration curves for meter no. W6-319

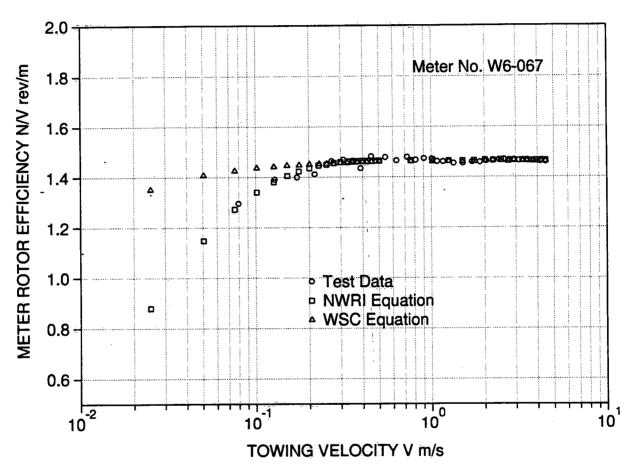


Figure 8. Calibration curves for meter no. W6-067

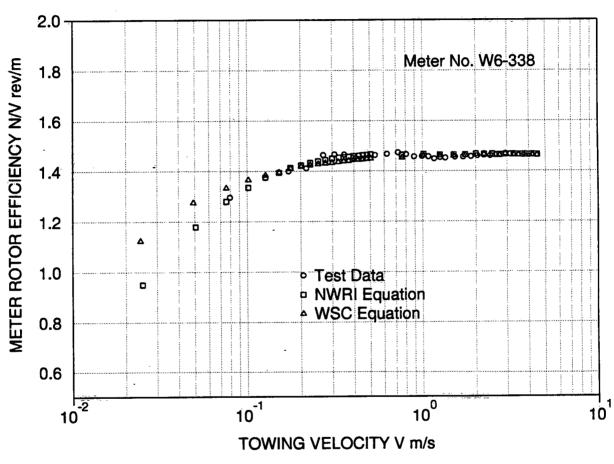


Figure 9. Calibration curves for meter no. W6-338

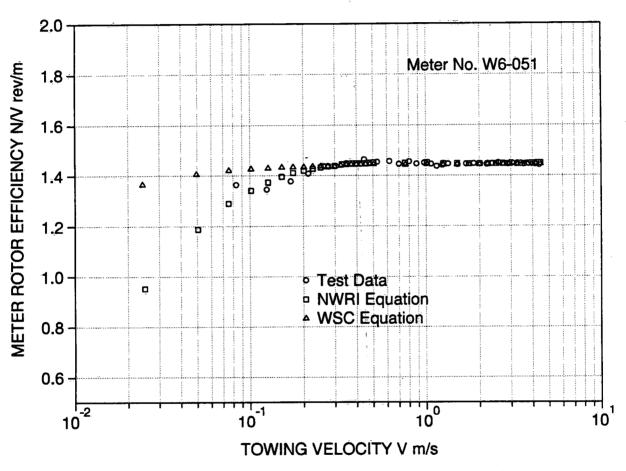


Figure 10 Calibration curves for meter no. W6-051

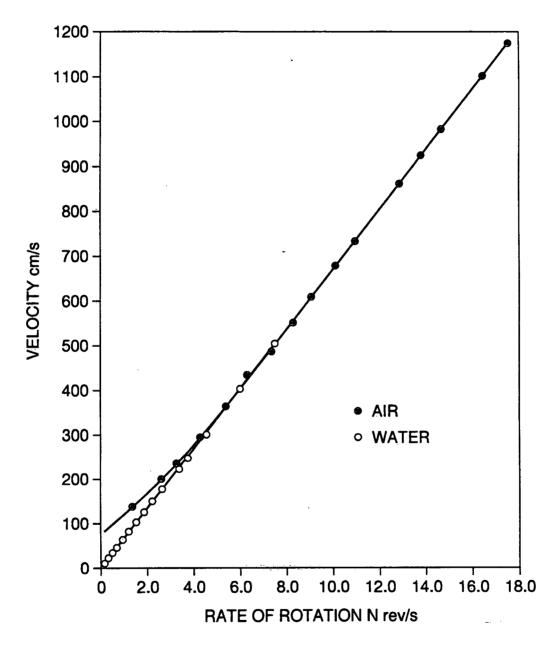


Figure 11. Calibration curves for Price meter in air and water

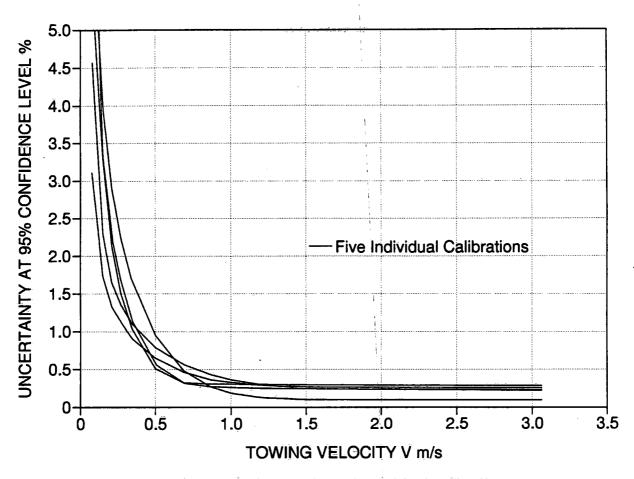


Figure 12. Uncertainty of individual calibrations

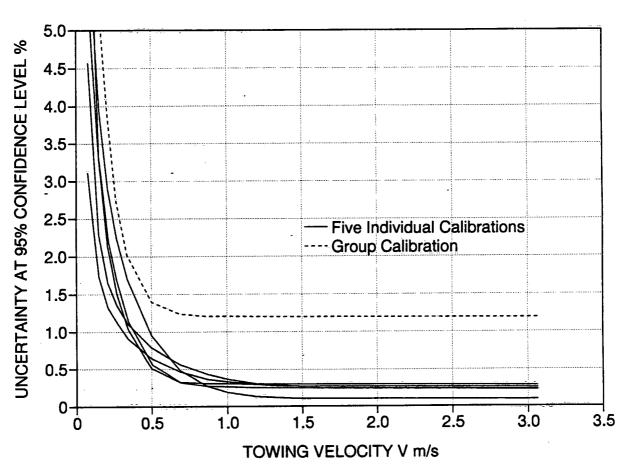


Figure 13. Uncertainty of individual calibrations

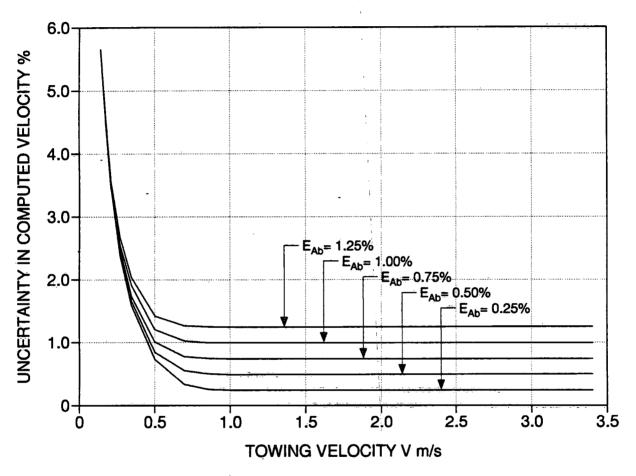


Figure 14. Effect of uncertainty in A at 95% level

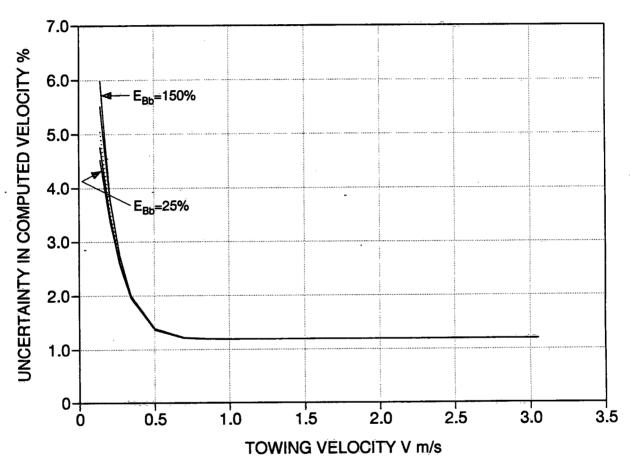


Figure 15. Effect of uncertainty in B at 95% level

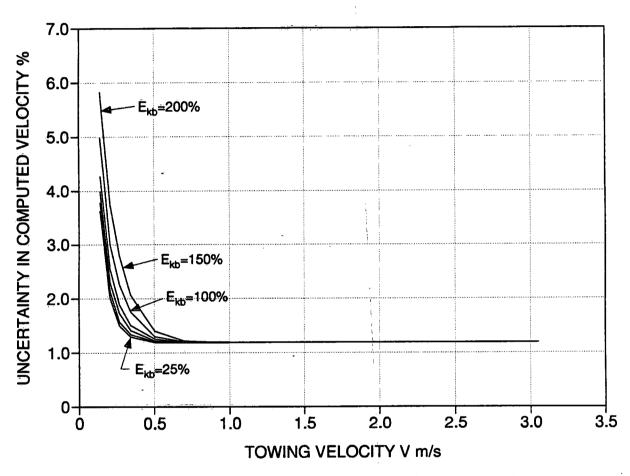


Figure 16. Effect of uncertainty in k at 95% level

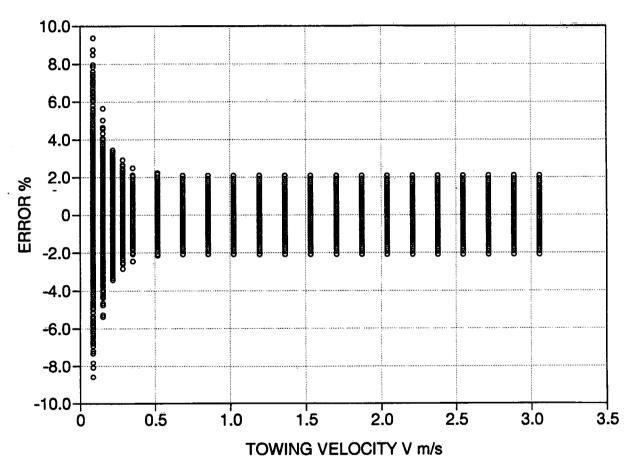
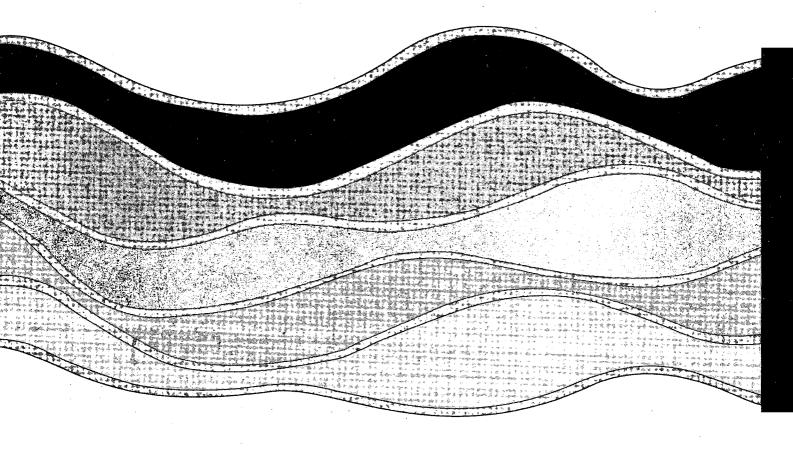


Figure 17. Errors due to interchanged calibrations





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