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MANAGEMENT PERSPECTIVE

Fine sediments in a river system play an important role in the transport of contaminants and hence form an essential component of modelling systems that have been developed for the prediction of transport, fate and bio-accumulation of contaminants in aquatic environments. The existing models of fine sediment transport assume that the fine sediments behave in the same manner as their counterpart, coarse grained sediments. But, recent research has shown that there are fundamental differences in the transport characteristics of the two types of sediments. The new algorithm proposed in this paper treats the transport of fine sediments realistically and hence is an improvement over the existing models.

SOMMAIRE À L'INTENTION DE LA DIRECTION

Les sédiments fins d'un système hydrographique jouent un rôle important pour le transport des contaminants et constituent donc une composante essentielle des systèmes de modélisation élaborés pour prévoir le transport, le devenir et la bioaccumulation des contaminants dans les milieux aquatiques. Dans les modèles existants de transport de sédiments fins, on suppose que ces derniers se comportent de la même façon que leur contrepartie, les sédiments à grains grossiers. Mais, des recherches récentes ont montré qu'il existe des différences fondamentales dans les caractéristiques de transport de ces deux types de sédiments. Le nouvel algorithme proposé dans ce document traite de façon réaliste du transport des sédiments fins et il représente donc une amélioration par rapport aux modèles existants.

ABSTRACT

A new algorithm is proposed for modelling fine sediment transport in rivers under steady flow conditions. The algorithm is based on empirical relationships developed from laboratory experiments of sediment entering a control segment of the river and allows the calculation of the suspended sediment concentration in the water column and the amount of sediment depositing to the bed under a given bed shear stress. The algorithm can be easily incorporated into existing models of contaminant fate and food chain.

RÉSUMÉ

Un nouvel algorithme est proposé pour la modélisation du transport de sédiments fins dans les rivières, l'écoulement étant stationnaire. L'algorithme est fondé sur des relations empiriques élaborées à partir d'expériences en laboratoire portant sur l'érosion des sédiments et leur dépôt dans un conduit circulaire rotatif. L'algorithme exprime le bilan massique des sédiments entrant dans un segment de contrôle de la rivière et permet de calculer la concentration de sédiments en suspension dans la colonne d'eau ainsi que la quantité de sédiments déposés dans le lit sous une contrainte de cisaillement donnée de ce lit. L'algorithme peut être facilement incorporé dans les modèles existants de devenir des contaminants et de chaîne alimentaire.

INTRODUCTION

Existing models of chemical transport and fate treat the transport of fine sediments in the same manner as the coarse grained sediments and allow simultaneous erosion and deposition to take place when the sediment is transported under a particular bed-shear stress. Recent research on fine sediment transport had shown that simultaneous erosion and deposition may occur only for certain range of bed-shear stresses. For shear stresses outside of this range, there could only be sediment deposition or sediment erosion, but not both simultaneously. Therefore, there is a need for a new algorithm for fine sediment transport for these models. In this note, a new algorithm for modeling the Athabasca River sediment is proposed. The new algorithm is based on experiments that were carried out in the Rotating Flume of NWRI for the Athabasca River sediment Details of this algorithm are described below:

BASIS OF THE NEW ALGORITHM

The deposition experiments carried out in the Rotating Flume for the Athabasca River sediment (See Krishnappan and Stephens (1995)) show that for a particular bed shear stress, the steady state suspended sediment concentration is a fixed percentage of the initial concentration and it varies only as a function of the bed shear stress. From these measurements, the fraction of the sediment that would deposit under different bed shear stresses can be established and plotted as shown in Fig. 1. In this figure, the bed shear stress is shown in terms of the critical shear stress for deposition (i.e. the shear stress below which all the initially suspended sediment would eventually deposit). From this figure, it can be seen that when the bed shear stress is at or below the critical shear stress for deposition, the fraction that would deposit takes a value of unity and the fraction deposited decreases as the bed shear stress increases and when the bed shear stress is about six times that of the critical shear stress for deposition, the fraction deposited becomes zero and all the initially suspended sediment stays in suspension. A power law relationship between the fraction deposited and the ratio of bed shear stress to the critical shear stress for deposition was fitted and is shown as solid line in Fig.1. The relationship takes the following form:

$$
\begin{aligned}
f_d &= 1.0 - 0.32(\tau_0/\tau_{cd} - 1)^{0.70} \; for\{1 < \tau_0/\tau_{cd} < 6\} \\
f_d &= 1.0 \; for\{\tau_0/\tau_{cd} < 1\} \\
f_d &= 0 \; for\{\tau_0/\tau_{cd} > 6\}
\end{aligned}
\qquad (1)
$$

where f_d is the fraction deposited, τ_0 is bed shear stress and τ_{cd} is the critical shear stress for deposition.

The erosion experiments carried out in the same flume for the same sediment (See Krishnappan and Stephens (1995)) showed that the critical shear stress for erosion (i.e. shear stress at which the top layer of the deposited sediment begins to erode) is about

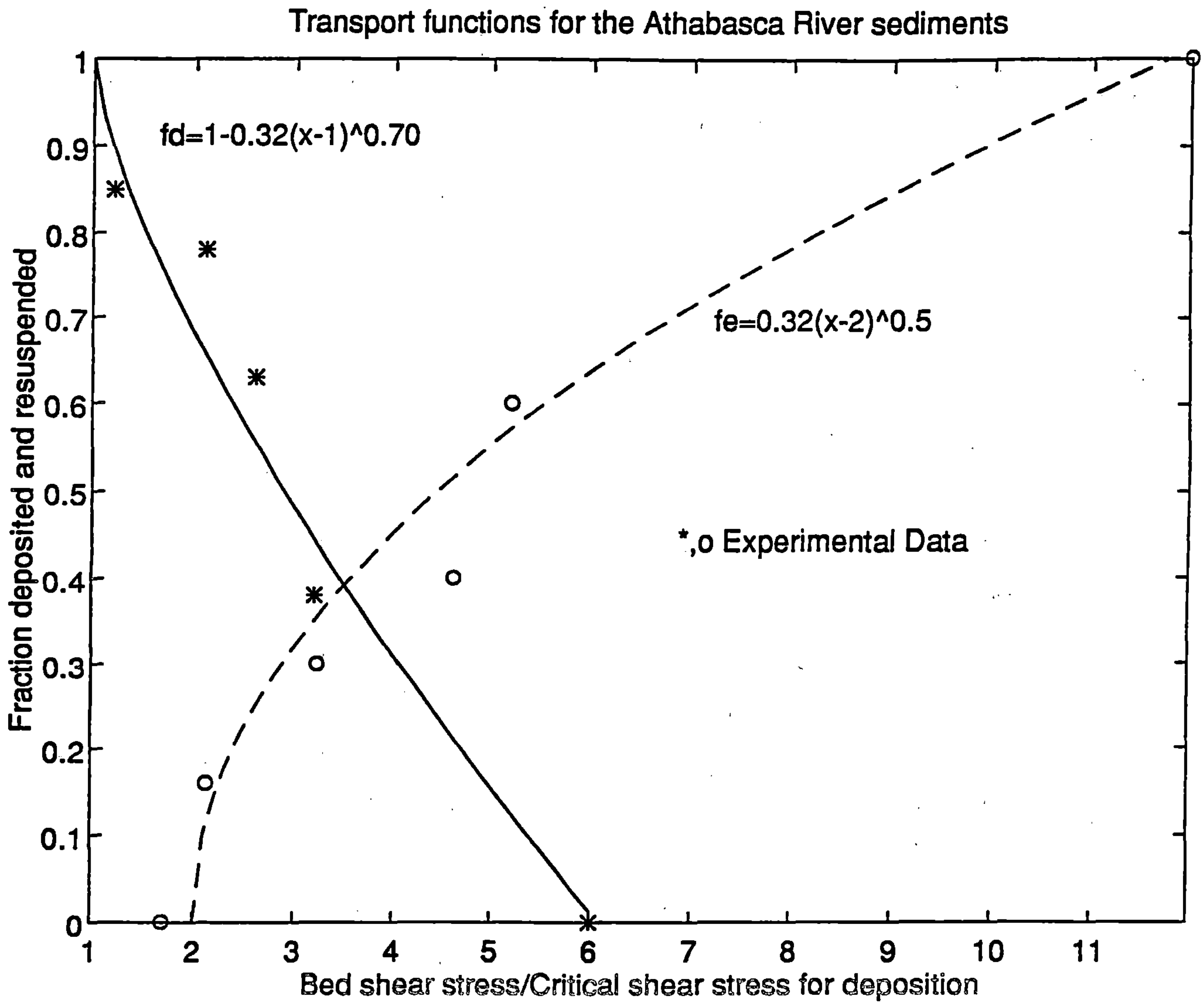

Fig. 1 Fractions of sediment deposited and eroded as a function of bed shear stress

twice as large as the critical shear stress for deposition and the fraction of the sediment coming into suspension increased as a function of the bed shear stress. It was noticed that at the shear stress which maintained 100% of the sediment in suspension during deposition could erode and suspend only 60% of the sediment. To bring all the deposited sediment into suspension, a shear stress 12 times that of the critical shear stress for deposition was needed. From the erosion experiments, the fraction of re-suspension was determined for various bed shear stresses and plotted as shown in Fig. 1 as circles. A power law relationship was fitted through the experimental points. The form of the power law is shown below:

$$\begin{aligned} f_e &= 0.32(\tau_0/\tau_{cd} - 2)^{0.5} \, for\{2 < \tau_0/\tau_{cd} < 12\} \\ f_e &= 0 \, for\{\tau_0/\tau_{cd} < 2\} \\ f_e &= 1 \, for\{\tau_0/\tau_{cd} > 12\} \end{aligned} \tag{2}$$

where f_e is the fraction of sediment re-suspended. Using the two functions given by equations (1) and (2), the transport of the Athabasca River sediment can be modeled as follows:

DETAILS OF THE NEW ALGORITHM

Step 1: Basic information:

The river reach is divided into number of river segments. The flow rate in each segment is considered to be steady. Let the flow rate of the control segment be Q (m^3/s). For a varying flow, a quasi-steady state is assumed and the flow hydrograph is approximated by a stair-case type function. Let the bed surface area of the segment be BSA (m^2). The average bed shear stress (τ_0) as a function of the flow rate for each river segment has to be determined. This can be done in a number of ways depending upon the available flow data and the sophistication of the model. A simple method proposed for the Athabasca River is outlined in the Appendix-A. The critical shear stress for deposition (τ_{cd}) has to be specified. This information comes from the rotating flume experiments.

Step 2: Calculation of sediment on the bed:

Let the concentration of the sediment entering the control segment be C_i (in mg/l) and consider a time interval of Δt. The amount of sediment entering the control segment (q_{su}) is:

$$q_{su} = QC_i\Delta t \tag{3}$$

Out of this amount, the amount that would deposit under the current flow condition will be:

$$q_{sd} = q_{su} f_d \tag{4}$$

Let us assume that the sediment that has been deposited previously on the bed is P kg/m^2. Adding the sediment that is deposited during the current time step with the sediment that is already on the bed gives the total sediment on the bed (prior to re-suspension during the current time step). Therefore, the total sediment on the bed (q_{sb}) becomes:

$$q_{sb} = P * BSA + q_{su} f_d \tag{5}$$

Step 3: Calculation of sediment in suspension:

Consider the re-suspension during the current time step. The amount of sediment that would be re-suspended under the current flow condition will be q_{sr} which is equal to:

$$q_{sr} = q_{sb} f_e \tag{6}$$

Note that q_{sr} is zero if the ratio between the bed shear stress to the critical shear stress is less than two and is equal to q_{sb} if the ratio is greater than 12.

Step 4: Calculation of bed sediment for next time step:

The amount of sediment left behind after the re-suspension phase is q_{sbn} given by:

$$q_{sbn} = q_{sb}(1 - f_e) \tag{7}$$

Therefore, the value of P for the next time step is: $P = q_{sbn}/BSA$.

Step 5: Calculation of sediment in suspension:

The amount of sediment coming into suspension q_{ss} becomes:

$$q_{ss} = q_{su}(1 - f_d) + q_{sb} f_e \tag{8}$$

Knowing the amount of sediment in suspension, the concentration of sediment in suspension C_o can be calculated as:

$$\frac{q_{ss}}{Q\Delta t} = \frac{q_{su}}{Q\Delta t}(1 - f_d) + \frac{q_{sb}}{Q\Delta t} f_e \tag{9}$$

The above expression gives the concentration of the sediment leaving the control segment. This also becomes the concentration that will enter the next control volume downstream.

Step 6: Calculation of deposition and scour velocities:

The traditional parameters such as the deposition velocity and scour velocity that are often used in water Quality models such as WASP can be calculated as follows:

The rate of sediment coming into suspension can be approximated as: $q_{ss}/\Delta t$. From eqn. (9), the rate of sediment coming in suspension becomes:

$$\frac{q_{ss}}{\Delta t} = \frac{q_{su}}{\Delta t} - \frac{q_{su}}{\Delta t} f_d + \frac{P * BSA}{\Delta t} f_e + \frac{q_{su}}{\Delta t} f_d f_e \tag{10}$$

The first term on the right hand side gives the rate of sediment entering from the upstream segment. The second term gives the rate of sediment deposition, the third term gives the rate of erosion of the previously deposited sediment and the fourth term gives the rate of erosion of the sediment that is deposited at the current time step. Therefore, the deposition velocity for sediment can be derived from the second term and the erosion velocity for the sediment can be derived from the third and fourth terms.

Step 7: Deposition Velocity:

Expressing the second term of eqn. (10) as the product $BSA*W_D*C_o$, the deposition velocity, W_D can be calculated as:

$$W_D = \frac{Q}{BSA} \left[\frac{q_{su} f_d}{q_{su}(1 - f_d) + q_{sb} f_e} \right] \tag{11}$$

Step 8: Scour Velocity:

Expressing the third and fourth terms of eqn. (10), as the product: $BSA*W_R*C_o$, the scour velocity, W_R can be evaluated as:

$$W_R = \frac{Q}{BSA} \left[\frac{P * BSA * f_e + q_{su} f_d f_e}{q_{su}(1 - f_d) + q_{sb} f_e} \right] \tag{12}$$

When evaluating the above velocities, care should be taken to assign the proper ranges for the transport functions f_d and f_e.

SUMMARY

A new algorithm for the transport of fine sediments of the Athabasca River is formulated based on laboratory experiments in a rotating circular flume. The algorithm is formulated for steady flows and may be used for gradually varied flow by considering the flow rate as quasi-steady. This algorithm is an improvement over the existing sediment transport models as it is based on realistic behavior of fine sediment transport.

REFERENCES

Krishnappan, B. G. and Stephens, R. 1995. Critical shear stresses for erosion and deposition of fine suspended sediment from the Athabasca River, Report submitted to NRBS (Northern River Basins Study), Edmonton, Alberta.

Appendix-A

Calculation of average bed shear stress:

Assuming that the Leopold-Maddock approach used by Golder Associates is valid for the Athabasca River reach to calculate the average velocity, depth and width as a function of the flow rate, the following method can be used to calculate the average bed shear stress:

Let U and H be the average velocity (in m/s) and flow depth (in m) calculated using Leopold-Maddock equations for a particular flow rate Q (m^3/s). Let D_{65} (in m) be the representative bed material size for the river segment under consideration. The bed shear velocity, U_* (in m/s) can be calculated as:

$$U_* = \frac{U}{2.50\ell n\left[11.0H/(2.50D_{65})\right]} \qquad \text{(A1)}$$

Knowing U_*, the bed shear stress τ_0 in N/m^2 can be calculated using the following conversion:

$$\tau_0 = 1000U_*^2 \qquad \text{(A2)}$$

The equation (A1) assumes that the flow is in rough turbulent regime which may be a reasonable assumption for the Athabasca River. When the flow is ice-covered, a 50% reduction in U_* values can be assumed as a first approximation. Further refinement can be carried out if it is warranted.