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ON THE STRATEGY OF CURRENT METER CALIBRATION

P. Engel

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P. Engel

Research Associate

Aquatic Ecosystem Protection Branch

National Water Research Institute

Burlington, Ontario L7R 4A6

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MANAGEMENT PERSPECTIVE

Increased awareness of river pollution and the importance of water quality monitoring has made it necessary to re-examine the accuracy of discharge measurements. One of the factors contributing to the error in flow velocity measurements is the uncertainty in the current meter calibration itself. Present practice in Canada is to calibrate each current meter individually. A second method, used in the United States, is to develop an average calibration equation, known as a group calibration, based on a large number of current meters of the same type. In this paper, the two strategies are examined to provide basic information for the review of present calibration methods.

ABSTRACT

A single, continuous equation, which takes into account the linear and non-linear components of the Price meter rotor response, was used to examine two current meter calibration strategies. Results showed that the uncertainty of group calibrations is substantially greater than calibrations of individual meters. The difference has been attributed to rotor fabrication variances for velocities greater than 0.3 m/s and residual velocities in the towing tank at velocities less than 0.3 m/s.

SOMMAIRE À L'INTENTION DE LA DIRECTION

Une sensibilisation de plus en plus grande à l'égard de la pollution des rivières et du besoin de surveillance de la qualité de l'eau a rendu nécessaire la réévaluation de l'exactitude des mesures des rejets. L'un des facteurs contribuant aux erreurs dans les mesures du débit est l'incertitude liée à l'étalonnage des moulinets comme tel. Au Canada, chaque moulinet est généralement étalonné individuellement. Une seconde méthode, utilisée aux États-Unis, consiste à déterminer une équation moyenne d'étalonnage, c.-à-d. à étalonner en groupe un grand nombre de moulinets de même type. Dans le présent document, les deux stratégies sont examinées pour obtenir l'information de base permettant de vérifier les méthodes actuelles d'étalonnage.

RÉSUMÉ

Une équation simple, continue, tenant compte des composantes linéaire et non linéaire de la réaction de l'hélice d'un moulinet Price, a servi à examiner deux stratégies d'étalonnage employées actuellement pour les moulinets. Les résultats ont montré que l'incertitude des étalonnages en groupe est sensiblement plus élevée que celle des étalonnages individuels des moulinets. Cette différence a été attribuée aux variances de fabrication des hélices pour les vitesses supérieures à 0,3 m/s et aux vitesses résiduelles dans le réservoir de touage à des vitesses inférieures à 0,3 m/s.

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ON THE STRATEGY OF CURRENT METER CALIBRATION

by

P. Engel[1]

INTRODUCTION

Price current meters used by the Water Survey of Canada (WSC) for streamflow measurements are rated individually at the National Water Research Institute (NWRI) by towing the meters at known velocities in a tank of still water and recording the rates of revolution of the rotors. The current meter calibrations are normally expressed as a linear equation from which calibration certificates are prepared. An economically attractive procedure is to adopt an average equation known as a group calibration, based on a large number of meters, if it can be shown that the uncertainty of the group calibration is not significantly greater than calibrations of individual meters (Charlton, 1978). Based on tests of standard Price current meters in groups as large as 70 meters, Smoot and Carter (1968) concluded that group calibrations provide the same accuracy as individual meter calibrations. Similar results from tests of Price Pygmy meters were reported by Schneider and Smoot (1976). In both cases linear calibration equations were used. It has been shown by Engel (1989) and Engel and Wiebe (1993) that linear equations do not provide an adequate fit to the calibration data for velocities less than 0.30 m/s where the rotor response is decidedly non-linear. As a result, conclusions regarding group calibrations, based on linear equations, may be misleading. In this paper, individual and group calibrations are examined by using a single, continuous calibration equation (Engel, 1989), which combines the linear and nonlinear components

1. Research Associate, Aquatic Ecosystems Protection Branch, National Water Research Institute, Canada Centre for Inland Waters, Burlington, Ontario, Canada, L7R 4A6.

of the rotor response. Data from individual calibrations of 39 rod suspended Price current meters (Engel, 1994), and data from individual calibrations of 5 meters, each repeated 10 times (Engel and Wiebe, 1993), are used to compare the uncertainties obtained with individual meter and group calibrations.

CALIBRATION EQUATION

The calibration equation is given as

$$V = AN + Be^{-kN} \tag{1}$$

where A, B and k are coefficients to be determined by calibration in a towing tank. Typical examples of the goodness of fit of equation (1) can be seen in Figure 1 in which curves of equation (1) are superimposed on the plotted data for 2 of the 39 meters tested. The data are plotted as $\frac{N}{V}$ vs. V. The ratio $\frac{N}{V}$ was used because of its high sensitivity to changes in V. It represents the steady state rotation of the meter rotor for each metre of distance travelled in the towing tank and can be considered to be a form of meter rotor efficiency. The curves fit the data quite well over the full range of velocities tested. Superimposed on the plots are the curves obtained from linear calibrations presently used by (WSC). Agreement is good for velocities greater than 0.30 m/s. As velocities decrease below 0.30 m/s, the difference between the curves increases with the WSC curves giving larger values of $\frac{N}{V}$. Better results are obtained with the United States Geological Survey (USGS) method which employs two linear equations. One equation is used for $N \leq 1.0$ rev/s and the other for $N \geq 1.0$ rev/s (Smoot and Carter, 1968). Once again the fit is the same as with equation (1) when $V > 0.30$ m/s. When $V < 0.30$ m/s the fit is better than that obtained with the WSC method but not as good as with equation (1). Clearly, linear calibration curves should not be used if low velocity accuracy is important. Therefore, only equation (1) is used as a basis for further analysis of calibration uncertainties in this paper.

UNCERTAINTY EQUATION

It can be shown that, in accordance with the format of equation (1), the error in the computed velocity may be expressed as

$$\delta V = \left\{ \left[\frac{\partial V}{\partial A}\delta A\right]^2 + \left[\frac{\partial V}{\partial N}\delta N\right]^2 + \left[\frac{\partial V}{\partial B}\delta B\right]^2 + \left[\frac{\partial V}{\partial k}\delta k\right]^2 \right\}^{\frac{1}{2}} \tag{2}$$

The partial derivatives are obtained by differentiating equation (1), substituting into equation (2) and, after rearranging, the relative error in the velocity is given as

$$\frac{\delta V}{V} = \left\{ \frac{1}{(1+\beta)^2} \left[\beta^2 \left(\frac{\delta A}{A}\right)^2 + \left(\frac{\delta B}{B}\right)^2 + k^2 N^2 \left(\frac{\delta k}{k}\right)^2 + (\beta - kN)^2 \left(\frac{\delta N}{N}\right)^2 \right] \right\}^{\frac{1}{2}} \tag{3}$$

in which $\beta = \frac{AN}{Be^{-kN}}$ is the ratio of the linear component to the non-linear component of equation (1) and as such is a measure of their relative importance for a given value of N. The relative error ratios $(\frac{\delta V}{V})$, $(\frac{\delta A}{A})$, $(\frac{\delta B}{B})$, $(\frac{\delta k}{k})$ and $(\frac{\delta N}{N})$ can be expressed as ratios of the standard deviation to the corresponding mean and as such become coefficients of variation (Herschy, 1978). The coefficient of variation is a basic measure of the uncertainty in the value of the variable it represents. Uncertainties of A, B, k and N in equation (1) can be computed by using small sample theory. For example, the uncertainty in determining the true value of A, obtained from n different observations of A, may be expressed as

$$E_A = t_{0.975} C_A \tag{4}$$

where E_A = the uncertainty of A at the 95% confidence level, $t_{0.975}$ = the confidence coefficient at the 95% confidence level from Student's t distribution for $(n-1)$ degrees of freedom (Spiegel, 1961), n = the number of values of A composing the limited sample and C_A = the coefficient of variation. Similarly, the uncertainties for B, k and N can be computed as E_B, E_k and E_N. Replacing the relative errors in equation (3) with the corresponding uncertainties, one obtains

$$E_V = \left\{ \frac{1}{(1+\beta)^2} \left[\beta^2 E_A^2 + E_B^2 + k^2 N^2 E_k^2 + (\beta - kN)^2 E_N^2 \right] \right\}^{\frac{1}{2}} \tag{5}$$

in which E_V = the uncertainty of the computed velocity at the 95% confidence level. For all practical values of N, $\beta \gg kN$ and therefore equation (5) can be further simplified to give

$$E_V = \left\{ \frac{1}{(1+\beta)^2} \left[\beta^2 (E_A^2 + E_N^2) + E_B^2 + k^2 N^2 E_k^2 \right] \right\}^{\frac{1}{2}} \tag{6}$$

Finally, it is known that for the NWRI calibration facility, $E_N \ll E_A$ and therefore, for engineering purposes, E_N can also be omitted from equation (6) resulting in

$$E_V = \left\{ \frac{1}{(1+\beta)^2} \left[\beta^2 (E_A^2) + E_B^2 + k^2 N^2 E_k^2 \right] \right\}^{\frac{1}{2}} \tag{7}$$

A useful feature of equation (7) is that it emphasizes the relative importance of E_A, E_B and E_k. Over the normal operating range of Price current meters, $\beta \approx 10$ when $N = 0.10$ rev/s, rising rapidly to a very large value when $N = 4.5$ rev/s. As a result the effects of E_B and E_k are negligible for velocities greater than 0.3 m/s with the total uncertainty being accounted for by E_A. Equation (7) is used to examine the uncertainties obtained with individual meter and group calibrations.

INDIVIDUAL METER CALIBRATION

Mean values of the calibration coefficients given as $\overline{A}_s$, $\overline{B}_s$ and $\overline{k}_s$ and the corresponding uncertainties E_{A_s}, E_{B_s} and E_{k_s} for each of 5 meters, calibrated 10 times, are given in Table 1. Uncertainties in the computed velocities, given as E_{V_s}, were computed for different values of the rate of meter rotor rotation N for each of 5 meters tested. The results are plotted in Figure 2 as E_{V_s} versus V. The curves clearly show that repeatability of a given calibration is very good and better than 0.3% for towing velocities greater than 1 m/s. For velocities less than that, the uncertainty increases, with the rate of change increasing, reaching values greater than 5% at velocities less than 0.1 m/s. Considering that geometric properties of each meter are constant throughout the tests, the uncertainties must be attributed to experimental error. The uncertainties of individual calibrations represent the standard against which all other calibration strategies should be compared.

GROUP CALIBRATION

The mean values for the coefficients of the 39 calibrations, expressed as $\overline{A}_b$, $\overline{B}_b$ and $\overline{k}_b$ and the corresponding uncertainties E_{A_b}, E_{B_b} and E_{k_b}, are given in Table 2. The uncertainty in A is about 1.2%. The difference between E_{A_b} and E_{A_s} can be interpreted to be the uncertainty in the group calibration largely due to manufacturing variances. In contrast to this, E_{B_b} and E_{k_b} are very large but are only important at velocities less than 0.30 m/s whereas, E_{A_b} affects the calibration at velocities greater than 0.30 m/s. This is why it is important to use equation (1) instead of linear equations to compare group calibrations and individual meter calibrations.

Uncertainties in the computed velocity given as E_{V_b} were computed for given values of the rate of meter rotor rotation N by substituting the uncertainties from Table 2 in equation (7). The results are plotted in Figure 2 as E_{V_b} versus V superimposed on the uncertainties for individual meter calibrations. It can be seen at once that E_{V_b} is greater than E_{V_s}. For velocities greater than 0.7 m/s, E_{V_b} is constant at about 1.2% representing an increase over individual calibration uncertainty by a factor of about 4 which is virtually the same as the uncertainty in A. This result differs substantially from the findings of Smoot and Carter (1968) that there is no significant difference between individual and group calibrations and is attributed to the use of equation (1) which provides a more accurate representation of the rotor response at low velocities than linear equations. As velocities decrease from 0.7 m/s, the uncertainty increases with the rate of change increasing. In all cases, uncertainty for group calibrations is greater than for individual calibrations.

SUMMARY AND CONCLUSIONS

The best calibration results for rod suspended Price meters are obtained by calibrating each meter individually. For velocities greater than 1 m/s, the uncertainty in the computed velocity is about 0.3% at the 95% confidence level. For group calibrations, the uncertainty increases to 1.2%

for velocities greater than 0.7 m/s. At velocities less than 0.7 m/s, the difference in uncertainty of the two calibration strategies becomes less significant. The difference between individual and group calibrations is mainly due to the uncertainty in A expressed as the difference between E_{A_b} and E_{A_s} which is due to fabrication variances in rotor geometry. Given the present standards of meter fabrication, it does not seem likely that sufficient improvement to reduce E_{A_b} to E_{A_s} can be obtained. Therefore, the choice of calibration strategy is a matter of required velocity measurement accuracy. Tests should be conducted to see if reductions in the uncertainty of B and k can be obtained by changes in meter towing procedures.

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APPENDIX II. NOTATION

The following symbols are used in this paper

A = calibration coefficient;

$\overline{A_s}$ = mean value of A for individual calibrations;

$\overline{A_b}$ = mean value of A for group calibration;

B = calibration coefficient;

$\overline{B_s}$ = mean value of B for individual calibrations;

$\overline{B_b}$ = mean value of B for group calibrations;

C_A = coefficient of variation of A;

e = base of Naperian logarithms;

E_A = uncertainty at 95% confidence level of A;

E_B = uncertainty at 95% confidence level of B;

E_k = uncertainty at 95% confidence level of k;

E_N = uncertainty at 95% confidence level of N;

E_V = uncertainty at 95% confidence level of V;

E_{V_s} = uncertainty at 95% confidence level of V_s;

E_{V_b} = uncertainty at 95% confidence level of V_b;

k = calibration coefficient;

$\overline{k_s}$ = mean value of k for individual calibrations;

$\overline{k_b}$ = mean value of k for group calibration;

N = rate of rotation of meter rotor;

n = number of samples;

$t_{0.975}$ = 95% uncertainty factor from Student's t distribution;

V = flow or towing velocity;

V_b = velocity for group calibration;

ΔV = difference in velocity for a given N of two calibrations;

$\beta = \frac{AN}{Be^{-kN}}$

δ = differential operator;

∂ = partial derivative operator;

TABLE 1 UNCERTAINTIES AT 95% LEVEL FOR A_s, B_s and k_s

$\overline{A}_s$ $[m/rev]$	E_{A_s} $[\%]$	$\overline{B}_s$ $[m/s]$	E_{B_s} $[\%]$	$\overline{k}_s$ $[s/rev]$	E_{k_s} $[\%]$
0.6783	0.302	0.01217	49.686	3.721	90.606
0.6788	0.246	0.00930	40.848	3.375	132.541
0.6791	0.110	0.00740	68.553	2.497	211.245
0.6817	0.272	0.00683	35.196	1.583	113.0855
0.6829	0.242	0.00480	74.280	1.298	196.191

TABLE 2 UNCERTAINTIES AT 95% LEVEL FOR A_b, B_b and k_b

$\overline{A_b}$ $[m/rev]$	E_{A_b} $[\%]$	$\overline{B_b}$ $[m/s]$	E_{B_b} $[\%]$	$\overline{k_b}$ $[s/rev]$	E_{k_b} $[\%]$
0.68037	1.1971	0.009269	105.97	3.5564	188.77

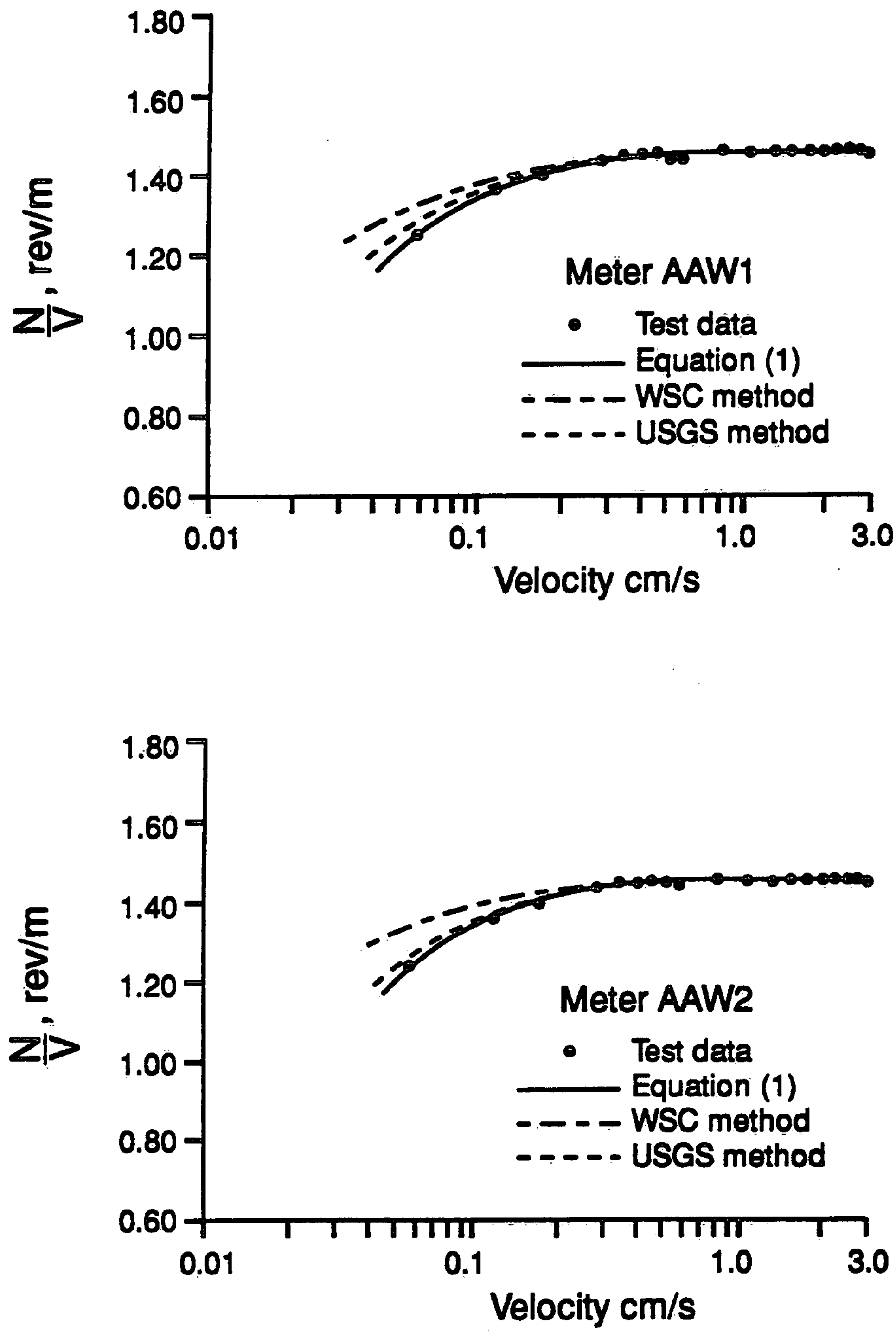

Figure 1. Typical Calibration Results.

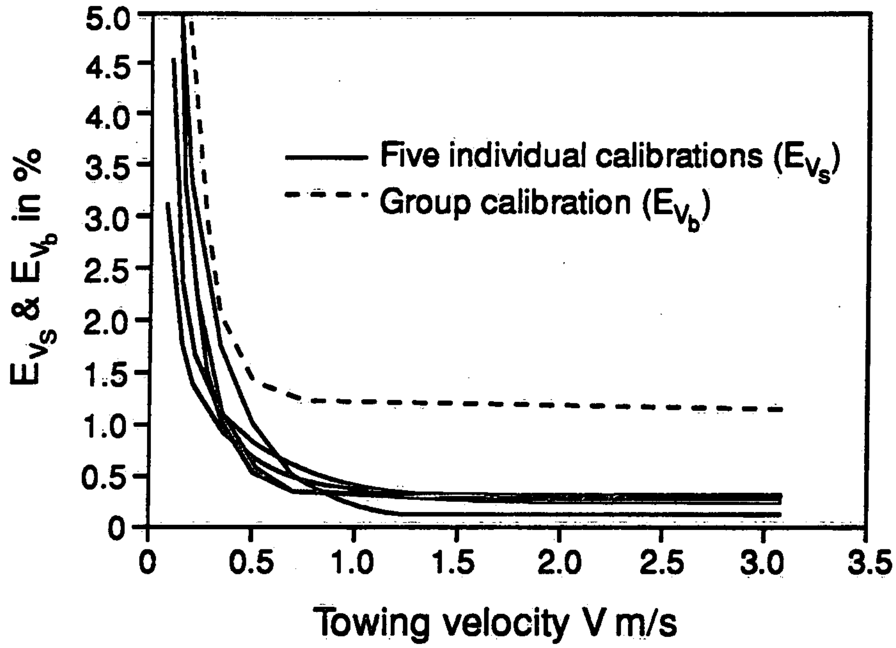

Figure 2. Calibration Uncertainty at 95% Confidence Level.

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