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REASON FOR REPORT: To document wave parameters measured during suspended sediment measurements at Van Wagner's Beach, Lake Ontario, from 1977 to 1979.

### 1.0 INTRODUCTION

Wave measurements were made at Van Wagner's Beach, in conjunction with the suspended sediment experiments by Coakley (1980). The location of the three gauge arrays was 245 m from the baseline, in about 5 m of water (Figure 1). This location was outside the surf zone for all the sled experiments.

The purpose of this note is to present the parameters of the significant waves as determined at the array, and to present the best estimate of these parameters at the breaker line in the vicinity of the sled-way. More detailed analysis of the directional spectra of the waves is being prepared and will be presented elsewhere.

### 2.0 THE MODEL

The simple models of longshore transport commonly used relate the amount of material transported to the wave energy and direction at the breaker line. For example, in the Shore Protection Manual (Anon., 1977), the relation is (equation 4-40, SPM)

$$
\begin{equation*}
\mathrm{Q}=7500 \mathrm{P}_{\ell \mathrm{s}} \tag{1}
\end{equation*}
$$

where $\quad Q$ is the longshore sediment transport (in $\mathrm{yd}^{3} / \mathrm{yr}$ )
and

$$
\begin{equation*}
P_{\ell s}=\frac{\rho g}{16} H_{b}^{2} c_{g b} \sin 2 a_{b} \quad \text { (in ft-lb/sec/ft) } \tag{2}
\end{equation*}
$$

where $\rho \quad$ is the density
$g$ is the acceleration due to gravity
$H_{b}$ is the (significant) breaker wave height
$c_{g b}$ is the breaker group velocity
$a_{b}$ is the angle between the wave rays and the normal to the shore.
This relation is based on the assumptions of a shoreline with parallel contours and waves with single frequency, height, and direction. None of these assumptions hold exactly in a real situation, and the question of just how to relate the real data to the model must be addressed.

Real waves do not break at one depth because their energy is distributed in frequency and direction, and because the bathymetry is, in general, irregular. In these experiments, waves are simplified by representing them by the peak frequency, the significant height and the direction of the peak frequency. The breaker line was not chosen by applying one of the common breaking criteria to this wave. The breaker line was taken to be the outer limit of the surf zone, that is, the offshore location where the largest waves break (see Longuet-Higgins, 1972, Galvin and Vitale, 1976). The wave parameters, as defined by the significant wave height and peak period at this location, were used to define the breaking conditions to be used in Equation 1.

### 3.0 WAVE MEASUREMENTS

The waves were measured either just prior to or during a sled experiment. The time series were analyzed to obtain the spectra, from which the peak period and characteristic (significant) wave height were calculated. From phase differences among the wave gauges, the direction of the peak frequency was determined. These data have been summarized in Table 1.

In addition, using bathymetric charts produced by Coakley (personal communication), the angles the wave rays made with the local contour normal in the vicinity of the array were calculated and are listed in Table 1. Other pertinent data are also listed, such as date and time of the experiment, local depth, date of the bathymetric survey, and the correction of the depth relative to the datum used for the charts ( 74.8 m above sea level, in contrast to the hydrographic charts which use 74.0 m ).

### 4.0 ESTIMATION OF THE BREAKER LOCATION

Coakley (personal communication) made visual estimates of the outer edge of the surf zone during most of the experiments. These were referenced to a number of fixed locations such as the wave gauges, the outer pile of the sledway, and buoys marking fixed current meters. Using these estimates along with the wave parameters, it was possible to identify likely features of the bathymetry where the outer edge of the surf zone was located for each experiment. It turned out that, for the five largest storms, the breaker line could be identified with the outer bar, (at about 175 m offshore from the
baseline) and, for the remainder, it could be identified with the (somewhat less distinct) inner bar (about 85 m from the baseline).

It was then a simple matter to identify the orientation of the contours of the appropriate bar, in the vicinity of sled-way. The orientation is $\theta_{b}$ in Table 2, and the bar is given in the Comments column.

### 5.0 REFRACTION

To determine the wave parameters at the breaker line in the vicinity of the sled-way, refraction and shoaling had to be considered. A refraction program by Dobson (1967) was used to do this.

Using the bathymetry data from the charts supplied by Coakley. regular depth arrays made up of grid squares with sides of 10 m length, were generated for each of the bathymetric surveys used. This involved some linear interpolation between points, but the changes in bottom slopes were typically fairly gentle, so that little error was introduced.

The Dobson refraction program was modified to allow wave rays to be started in intermediate water depths. This involved calculating the reference group velocity and wave length in the main program at line 160 , using the dispersion relation. The reference group velocity (CGO) was transferred between subroutines by adding it to the COMMON/1/DEP... statement. The subroutine HEIGHT was found to contain two errors, which cancelled each other as long as rays were started in deep water. The term 0.5 was left off $\mathrm{c}_{\mathrm{g}}=0.5(1+2 \mathrm{kh} / \mathrm{sinh}$ $2 \mathrm{kh}) \mathrm{c}$ and the shoaling coefficient was given as $\left(\mathrm{c}_{\mathrm{o}} / \mathrm{c}_{\mathrm{g}}\right)^{1 / 2}$ instead of $\left(\mathrm{c}_{\mathrm{go}} / \mathrm{c}_{\mathrm{g}}\right)^{1 / 2}$. These were corrected.

With these changes, the program was run using the wave parameters of Table 1. The rays were started in the vicinity of the gauge array, and the starting points were distributed densely enough to give good coverage in the region of the sled-way. Plots of the rays superimposed on the bathymetry for each experiment are shown in Figures 1-13. The wave parameters were listed at frequent intervals by the program, and the values of these at the location that most closely corresponded to the breaker line on the sled-way have been listed in Table 2.

## 6.0

 DISCUSSIONIn every case, the wave direction at the breaker line (Table 2) indicates longshore sediment transport according to the model would produce a northward movement. This is in contrast to the transport that would have been predicted if the values at the wave gauge array were used (Table 1). In the latter case, two of 13 experiments would have predicted southward movement.

Current measurements obtained from three sets of ducted impeller meters located across the surf zone provide partial verification. The data return from the meters was generally poor, but some indication of direction was discernable in 11 of the 13 experiments. Of these 11 , six cases showed currents going both north and south, with the northward current dominating. Of the remaining five, four indicated a northward current, and one southward. The recorded mean current in the last case was only about 1 cm , which is inconclusive with respect to direction. In general, the wave direction and current direction measurements tend to be supportive, but further comment is not possible because of the poor quality of the current measurements.

Coakley (1980) has recorded the longshore currents across the surf zone on the sled with considerably more success. The net flow as indicated by the longshore discharge (his Table 2) is towards the north except in experiment 15-1, where it is southward. These results agree with those inferred from the wave directions except for experiment 15-1. Closer examination of Coakley's current data indicates that the current direction is northward in the surf zone except very near the shore, so that the agreement between the two data sources is good.

It is not possible to quantify the uncertainty introduced by the many approximations and assumptions used in this note. However, confidence limits can be placed on the estimate of the wave direction at the wave gauge array, assuming that the orientation of the array is known exactly. The degrees of freedom of the squared coherency and phase calculations for the wave data are 80. Typically, the squared coherency at the spectral peak is greater than 0.9 , so that, using Figure 9.3 of Jenkins and Watts (1968), the 95 percent confidence limits on the phase are at worst $\pm_{2}{ }^{\circ}$.

The direction relative to the array is related to the phase by:

$$
\begin{equation*}
\psi=\sin ^{-1} \frac{\phi}{\mathrm{kD}} \tag{3}
\end{equation*}
$$

where
$\psi \quad$ is the direction relative to the array
$\phi \quad$ is the phase angle between gauges
k is the wave number of the spectral component
D is distance between the gauges.
So the error in direction due to the phase is given by:

$$
\begin{equation*}
\Delta \psi=\frac{1}{1-\left(\frac{\phi}{\mathrm{kD}}\right)^{2} \mathrm{I} / 2} \frac{1}{\mathrm{kD}} \Delta \phi \tag{4}
\end{equation*}
$$

Typically
$\phi<5^{\circ}$ or 0.09 radians
$\mathrm{k}>0.162 \mathrm{~m}^{-1}$
D > 3.5 m
$|\Delta \phi|<2^{\circ}$
Therefore $\quad|\Delta \psi|<3.6^{\circ}$
That is, the 95 percent confidence limits for the direction are not greater than $\pm 3.6^{\circ}$.

This result is not very satisfying because in only seven of the 13 experiments is the absolute value of the angle, $\alpha_{b}$, greater than $3.6^{\circ}$ (Table 2). Therefore the direction of inferred transport cannot be resolved in the remaining cases, to the 95 percent confidence level.

The effects of using considerable detail in the bathymetry ( 10 m grid) and of the possible errors in the bathymetry on the calculated wave rays have not been examined. For example, it may be more appropriate to use a smoothed bathymetry. These questions will be examined at a later date.

### 7.0 SUMMARY

The wave data collected during the sled experiments have been summarized and tabulated in Table 1. Reasonable estimates of the breaker line wave height and angle have been made and are given in Table 2. In only seven of the 13 experiments can the inferred direction of transport be determined, to the 95 percent confidence level.

### 8.0 ACKNOWLEDGEMENTS

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### 9.0 REFERENCES

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TABLE 1 WAVE CONDITIONS AT THE WAVE GAUGE ARRAY, APPROXIMATELY 245 M FROM THE BASELINE ON LINE 6

| Experiment | Year | $\begin{aligned} & \text { Date } \\ & \text { Day } \end{aligned}$ | Hour ${ }^{1}$ | Period <br> s | Wave <br> Height <br> m | $\begin{aligned} & \text { Direction }^{2} \\ & \psi_{\mathrm{g}},{ }^{\circ} \mathrm{T} \\ & \hline \end{aligned}$ | Water Depth at Gauges m | Shoreward Normal of Contour at Gauges $\theta_{\mathrm{g}},{ }^{\mathrm{o}} \mathrm{~T}$ | Angle Between Normal and Wave Direction ${ }^{3}$ $\alpha$, degrees | Date of Bathymetric Survey | Depth Correction m |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 2-2 | 77 | 324 | 19 | 4.93 | 0.85 | 236.4 | 5.1 | 235.5 | - 0.9 | 25/10/77 | -0.1 |
| 3-1 | 77 | 327 | 18 | 6.10 | 1.01 | 241.0 | 5.2 | 235.5 | - 5.5 | 25/10/77 | 0.0 |
| 3-2 | 77 | 327 | 20 | 6.10 | 0.87 | 240.2 | 5.2 | 235.5 | -4.7 | 25/10/77 | 0.0 |
| 4-1 | 77 | 329 | 15 | 3.56 | 0.61 | 260.3 | 5.1 | 235.5 | -24.8 | 25/10/77 | -0.1 |
| 4-2 | 77 | 329 | 18 | 4.13 | 0.60 | 244.0 | 5.1 | 235.5 | - 8.5 | 25/10/77 | -0.1 |
| 6-1 | 77 | 339 | 14 | 6.10 | 1.86 | 234.9 | 5.2 | 235.5 | 0.6 | 25/10/77 | 0.0 |
| 9-1 | 78 | 317 | 15 | 4.93 | 0.82 | 239.5 | 4.3 | 236.4 | - 3.1 | 8/11/78 | -0.4 |
| 9-2 | 78 | 317 | 18 | 4.93 | 0.73 | 237.7 | 4.3 | 236.4 | - 1.3 | 8/11/78 | -0.4 |
| 10-1 | 78 | 321 | 15 | 6.10 | 1.04 | 243.5 | 4.3 | 236.4 | - 7.1 | 8/11/78 | -0.4 |
| 11-1 | 78 | 327 | 18 | 6.10 | 0.90 | 237.6 | 4.3 | 236.4 | - 1.2 | 8/11/78 | -0.4 |
| 13-1 | 79 | 145 | 18 | 4.93 | 0.79 | 240.2 | 4.8 | 239.6 | - 0.6 | 2/ 5/79 | $+0.3$ |
| 15-1 | 79 | 289 | 18 | 4.93 | 0.81 | 239.3 | 4.9 | 239.6 | 0.3 | 16/9/79 | -0.1 |
| 16-1 | 79 | 313 | 20 | 3.56 | 0.60 | 243.6 | 4.8 | 239.6 | -4.0 | 16/9/79 | -0.2 |

[^0]| Experiment | Period <br> S | Height <br> m | Direction$\psi_{\mathrm{s}},{ }^{\mathrm{o}} \mathrm{~T}$ | Water Depth <br> m | Shoreward Normal of Contour at Breaking$\theta_{\mathrm{b}}, \quad{ }^{\mathrm{O}_{\mathrm{T}}}$ | Angle Between Normal and Wave Direction$\alpha_{b}, \text { degrees }$ | Comments on Position of Breaker Lines |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  |  |  |  |
| 2-2 | 4.93 | 0.65 | 235.7 | 2.3 | 227.4 | -8.3 | Inner bar (no field observation) |
| 3-1 | 6.10 | 0.97 | 240.8 | 3.8 | 236.6 | - 4.2 | Outer bar |
| 3-2 | 6.10 | 0.85 | 239.9 | 3.8 | 236.6 | - 3.3 | Outer bar |
| 4-1 | 3.56 | 0.49 | 255.6 | 2.8 | 227.4 | -28.2 | Inner bar. Poor estimate of wave parameters due to angle. |
| 4-2 | 4.13 | 0.46 | 239.8 | 2.5 | 227.4 | -12.4 | Inner bar |
| 6-1 | 6.10 | 1.78 | 237.3 | 3.7 | 236.6 | - 0.7 | Outer bar |
| 9-1 | 4.93 | 0.78 | 240.5 | 2.1 | 234.4 | -6.1 | Inner bar |
| 9-2 | 4.93 | 0.60 | 242.8 | 1.8 | 234.4 | - 8.4 | Outer bar |
| 10-1 | 6.10 | 1.08 | 242.6 | 3.2 | 236.5 | -6.1 | Outer bar |
| 11-1 | 6.10 | 0.92 | 239.1 | 3.2 | 236.5 | - 2.6 | Outer bar |
| 13-1 | 4.93 | 0.79 | 237.2 | 2.7 | 235.9 | -1.3 | Inner bar (no field observation) |
| 15-1 | 4.93 | 0.43 | 239.5 | 2.2 | 236.4 | -3.10 | Inner bar. Adjacent rays to south and north are markedly different. (no field observation). |
| 16-1 | 3.56 | 0.36 | 241.4 | 2.1 | 236.4 | - 5.0 I | Inner bar |

[^1]
Wave rays at Van Wagner's Beach: wave parameters at the wave gauge array ${ }^{(*)}$ are given in Table 1;
the sled-way is marked $7-7$; the depth contours are relative to a datum of 74.8 m a.s.1. Experiment $3-1$.
Figure 2


$\begin{array}{ll}\text { Figure } 3 & \text { Wave rays at Van Wagner's Beach: wave parameters at the wave gauge array (*) are given in Table 1; } \\ \text { the sled-way is marked 7-7; the depth contours are relative to a datum of } 74.8 \mathrm{~m} \text { a.s.l. Experiment 3-2. }\end{array}$


Figure 4 Wave rays at Van Wagner's Beach: wave parameters at the wave gauge array (*) are given in Table 1;

Figure $5 \quad$ Wave rays at Van Wagner's Beach: wave parameters at the wave gauge array (*) are given in Table 1;
the sled-way is marked $7-7$; the depth contours are relative to a datum of 74.8 m a.s.l. Experiment 4-2.

Figure 6 Wave rays at Van Wagner's Beach: wave parameters at the wave gauge array (*) are given in Table 1;


Figure $8 \quad \begin{aligned} & \text { Wave rays at Van Wagner's Beach: wave parameters at the wave gauge array (*) are given in Table 1; } \\ & \text { the sled-way is marked } 7-7 \text {; the depth contours are relative to a datum of } 74.8 \mathrm{~m} \text { as. }\end{aligned}$

Wave rays at Van Wagner's Beach: wave parameters at the wave gauge array (*) are given in Table 1;
the sled-way is marked 7-7; the depth contours are relative to a datum of 74.8 m a.s.1. Experiment $10-1$.
Figure 9

Figure 10 Wave rays at Van Wagner's Beach: wave parameters at the wave gauge array (*) are given in Table 1; the sled-way is marked 7-7; the depth contours are relative to a datum of 74.8 m a.s.l. Experiment $11-1$.

Figure 11 Wave rays at Van Wagner's Beach: wave parameters at the wave gauge array (*) are given in Table 1;
1

Figure 12 Wave rays at Van Wagner's Beach: wave parameters at the wave gauge array (*) are given in Table 1;:

Figure 13 Wave rays at Van Wagner's Beach: wave parameters at the wave gauge array (*) are given in Table 1;


[^0]:    ${ }^{1}$ GMT
    ${ }^{2}$ Direction in which waves are travelling, clockwise from true north.
    

[^1]:    ${ }^{1}$ Negative angle indicates northward generated currents: $\alpha_{b}=\theta_{b}-\psi_{b}$

