HYDRAULICS DIVISION
TECHNICAL NOTE

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TITLE: The Maximum Speed for Profiling

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REASON FOR REPORT: In response to a request from M. Charlton, AED NWRI

CORRESPONDENCE FILE: HD83-389(497)

1.0 INTRODUCTION

Mr. M. Charlton, of the Aquatic Ecology Division of the National Water Research Institute, asked how quickly an oxygen sensor could pass through a water column and still measure the oxygen within a chosen accuracy. Since this is a general problem with profilers, with the oxygen profile being a special case because the sensor's time constant is a function of temperature, the answer has been generalized in this note.

It is important to realize that the accuracies of the readings from many standard profiling instruments for oceanography and limnology are severely reduced when the speed of profiling is not matched to the gradients in the water column. In some cases the speeds have to be surprisingly low, even for temperature and conductivity instruments, to restore the desired accuracy to the readings.

2.0 PROBLEM AND ASSUMPTIONS

All profilers, regardless of their speed of response, expressed as a time constant, will produce an error when the parameter they measure varies in time or space. This happens because a finite difference between the stimulus and the response must exist to drive the sensor for a period of time. In the application of sensors, the amplitude of this difference is important because it represents an error in the measurement.

The speed at which the profile is made is one parameter in the error equation. It is useful to know what is the maximum permissable speed for a given allowance for error.

The full analysis of this problem can be simplified if some assumptions are made about the variable being measured and the duration of the measuring time. The first assumption is that the true variations can be closely approximated by linear ramp functions. The second assumption is that the sensor spends sufficient time traversing each ramp to reach a stable signal that tracks the ramp. This is the equivalent of saying that the sensor experiences a given ramp for at least four time constants. The third assumption is that the sensor's response is close to a simple exponential response to a step function in the stimulus.

The first assumption is reasonable in many applications, such as measuring environmental parameters, since step functions (the time limit of a ramp function) are rare because of the turbulence that usually produces gradients between the boundaries of two distinct zones. The second assumption is valid because the operator, making the profile, must slow the profiling to the point that the major ramps are closely approximated. Regardless of how slowly the profile is taken there is a scale of measurements that will be missed because the ramps are too fine to be detected. In the limit, the size of the sensor and its location becomes more important than the time response of the sensor.

As to the third assumption, a large class of sensors exhibit a simple single-pole, low-pass response. Others are somewhat more complex such as having delay functions or multiple exponential (nonlinear) responses. Regardless, the dominant component can be approximated by an exponential response in the majority of sensors used in environmental measurements.

3.0 THEORY

Sensors may be represented and analysed through electrical analogues. If the sensor can be represented by a single-pole low-pass filter, as confirmed by its response to a step function, the behaviour of the sensor can be analysed for the case where it encounters a ramp function in the stimulus. A single-pole, low-pass filter can be made with the combination of a series resistor followed by a parallel capacitor between the driving voltage and the output voltage. The driving voltage in this case is a ramp:

$$v_i(t) = at$$

where a is the slope (V/s)
t is the time (s)

As well, circuit analysis shows that

$$v_i(t) = Ri(t) + \int\frac{dt}{C} + K$$

where R is the resistance of the series resistor (Ω)
C is the capacitance of the parallel capacitor (F)
K is the initial voltage on the capacitor (V)
i(t) is the current in the circuit (A)

The integral equation has the solution in terms of the current,

$$i(t) = aC\ (1-e^{-t/RC})$$

since the output voltage

$$v_o(t) = v_i(t) - Ri(t)$$

then

$$v_i(t) - v_o(t) = Ri(t) = aRC(1-e^{-t/RC})$$

The above difference of voltages is the error of the sensor because ν_i is the true input analogue and v_o is the sensed analogue output.

In a profiling situation the slope variable, a, is related to three variables as

$$a = GUS$$

where G is the gradient of the variable being measured (such as °C/m or mg/L•m

U is the speed of the profile (m/s)

S is the transfer coefficient of the sensor (such as V/°C or V• L/mg)

The product, RC, is the time constant of the circuit which is a direct analogue of the time constant, τ, of the sensor. This is the time in seconds the sensor takes to reach 63% of the final value after being stimulated by a step function.

The error of the sensor's output is

$$E = GUS\tau\ (1-e^{-t/\tau}),\ \text{(volts)}$$

By dividing both sides by S the resulting equation expresses the error as a difference of the parameter being measured, that is

$$D = GU\tau(1-e^{-t/\tau}),\quad \text{(°C or mg/L)}$$

If the sensor is given the equivalent time of four time constants to measure the ramp, the exponential term may be ignored without introducing more than 2% error in the estimate of the error.

Using this assumption and solving for U_m the maximum speed for a given error D is

$$U_m = \frac{D}{G\tau}, \quad (m/s)$$

In the case of an oxygen sensor the time constant is a function of temperature. This function is specific to the membrane and electrode materials. In one case an oxygen sensor (YSI 5740) was thoroughly measured and found to follow the relationship $\tau = 10^{(1.45 - 0.022T)}$. This is a simplification because the τ is affected by a delay function and a compound exponential response. However, for the purposes of most oxygen profiling, the simplification is adequate. T is the temperature in degrees celsius.

4.0 APPLICATION

Using the error relationship, the operator taking a profile can either estimate the error for a given speed of profiling or estimate the maximum speed at which a profile may proceed for a given error. The time constant and the gradient must be known ahead of time. The first is known from laboratory tests. The second can be obtained in two ways. A profile can be made at a constant high speed, then the gradients measured from the graph. This sets the first approximation of the maximum speeds for subsequent profiles. The number of iterations depends upon the severity of the ramp functions encountered. In another way, the profile can be taken as before, then the data processed with a correction function described in the unpublished report, "The Demonstration of a Correction Technique for Oxygen Profiles", Report ES-558 of the National Water Research Institute. The corrected data give a good estimate of the gradients which will govern the maximum speeds in the second and final profile.

Figure 1 is provided to assist the operator in choosing the profiling speeds. It is a nomogram for the general relationship. Figure 2 is the nomogram for the special relationship for a specific oxygen probe, in this case a YSI type 5740. To use Figure 1, the operator starts with the sensor's time constant (0.2 s as an example for a temperature sensor) and draws a line to the difference (error) he wishes to stay within (say 0.01°C). Where the line crosses the pivot line, a second line originates to extent to the gradient (say 1°C/m). Where the line intersects the speed axis (0.05 m/s) is the maximum speed at which the sensor should traverse that gradient zone.

If in the example, the profile were taken at 1 m/s as is normal with field work, the error in temperature rises to 0.2°C in the thermocline. An example of high gradients at the thermocline is given in Figure 3. In this case if the profile were taken at 1 m/s the error would have been about 2°C.

In Figure 2, a temperature variable is incorporated because the oxygen sensor's time constant is dependent upon the water temperature. In this case, the operator draws a line from the start cross-line, through the lower temperature of the water in the gradient (to be conservative) to where it intersects the time constant axis. The nomogram is used from there in the same way as in Figure 1. In the example shown, the temperature is 10°C, the maximum error chosen is 0.2 mg/L and the gradient of interest is 10 mg/L•m. The maximum speed is slightly over 1 mm/s. If the miminum practical speed is 5 cm/s then the sharpest gradient must not be greater than about 0.3 mg/L•m. If that speed (5 cm/s) were used on a gradient of 3 mg/L•m the errors would exceed 2 mg/L while the sensor passed through the gradient.

The general equation can be used in many applications providing the sensor and the variables behave according to the assumptions made earlier. If the sensor behaves like an under-damped servo-system then the first approximation for τ must be used with great caution because it will underestimate the errors.

The speed selection process might be made automatic for oxygen profiling by using a microcomputer to set the winch speed while it gathers the data. The program would measure the temperature gradient as the first indication to slow down the winch because the temperature sensor's time constant is faster and temperature and oxygen features usually coincide. Once the oxygen sensor begins to sense the gradient correctly 70 seconds later (at 10°C) the winch speed can be set more correctly to complete the measurement of the feature. This may be practical enough to reduce the number of profiles to one but more knowledge must be gained in the variability of oxygen and temperature features in the profiles.

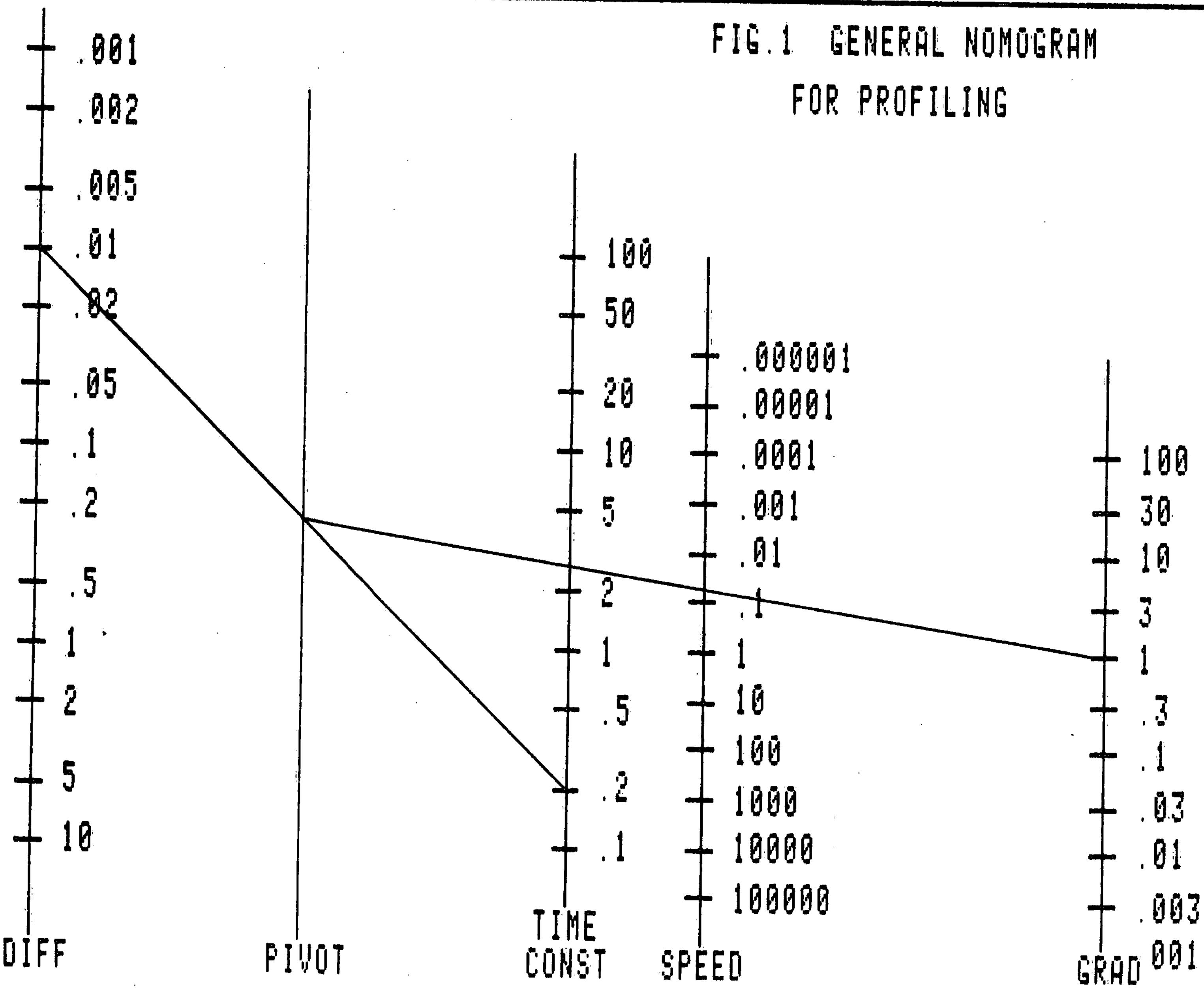

FIG.1 GENERAL NOMOGRAM FOR PROFILING

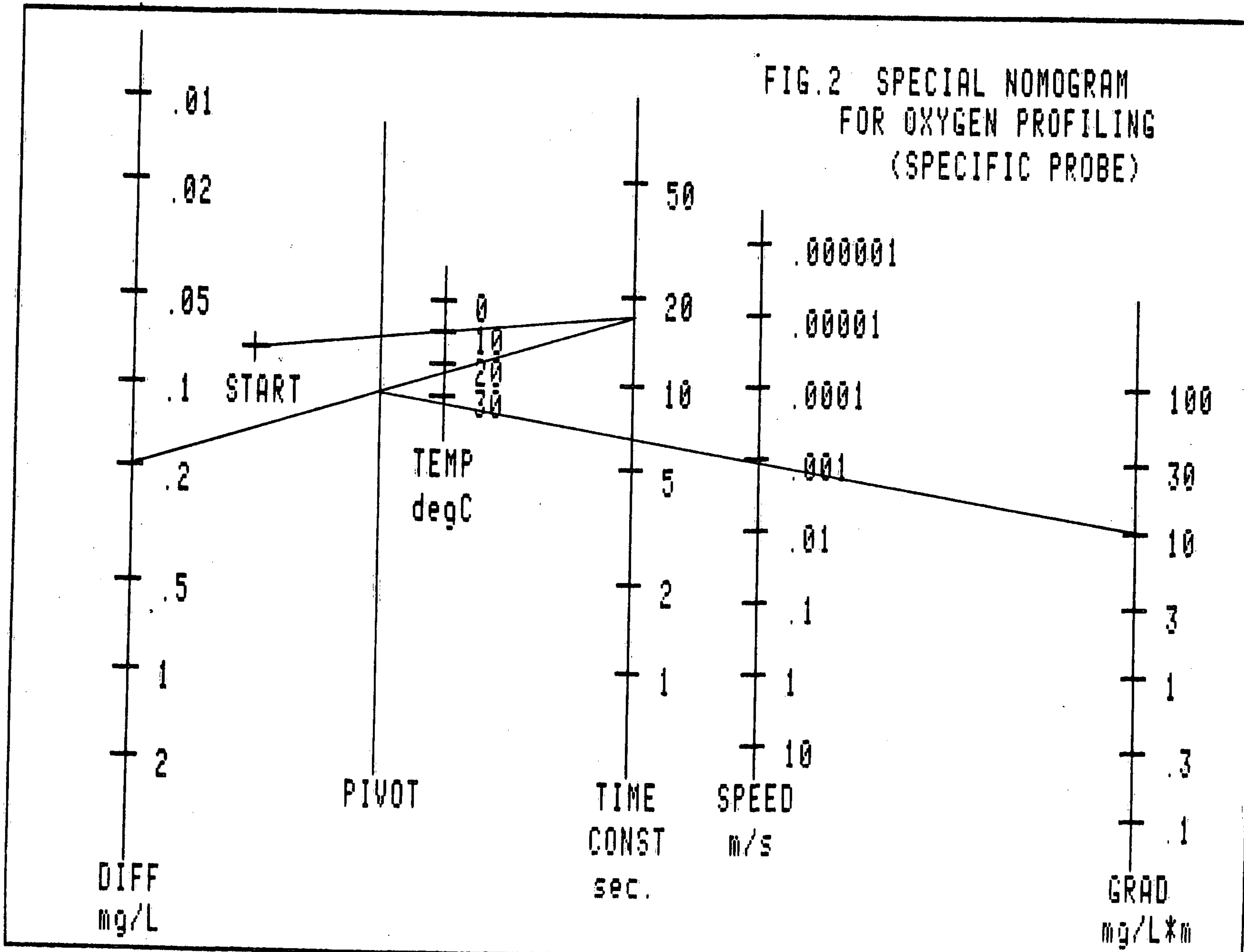

FIG. 2 SPECIAL NOMOGRAM FOR OXYGEN PROFILING (SPECIFIC PROBE)

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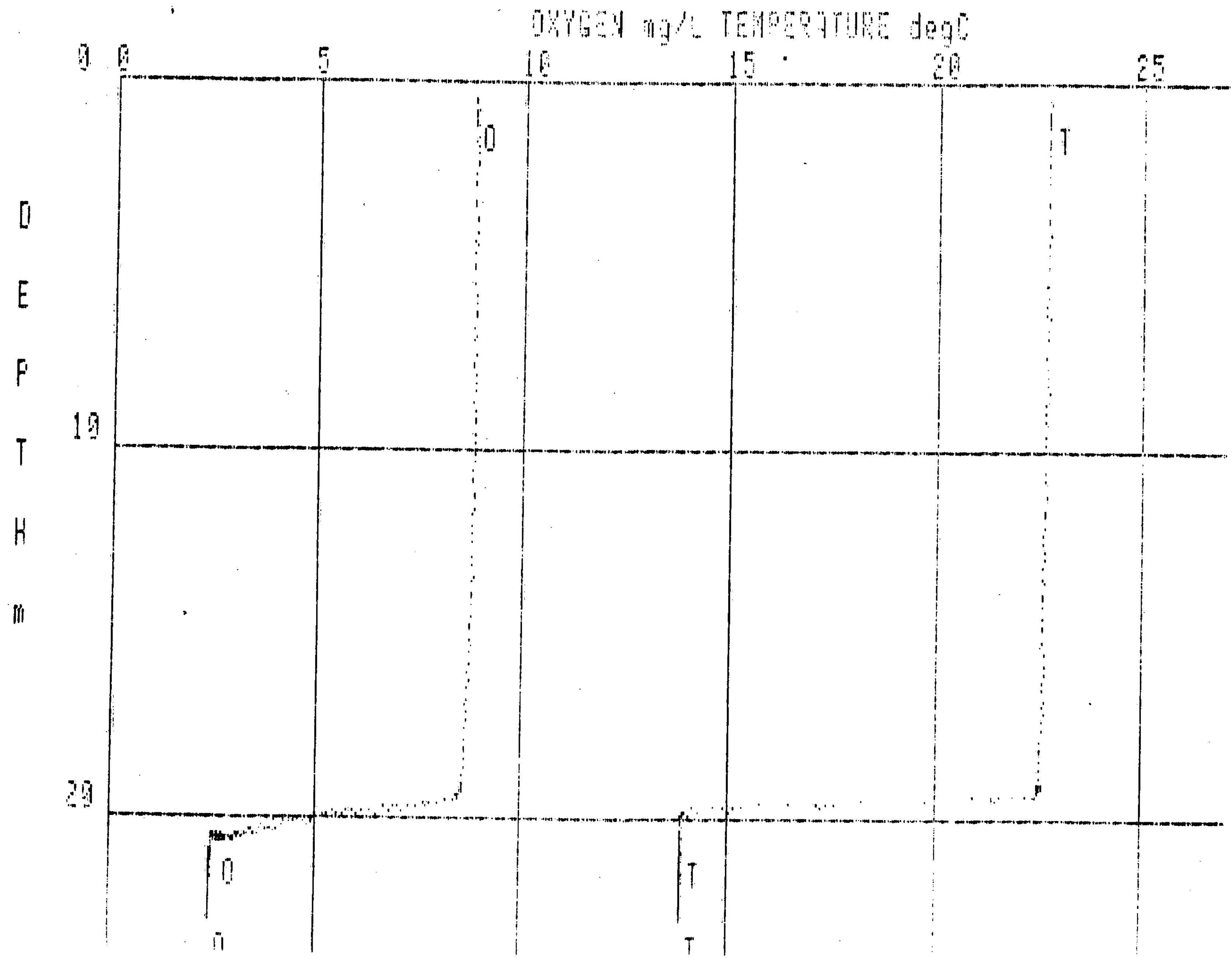

FIG 3 EXAMPLE OF HIGH GRADIENTS IN LAKE ERIE