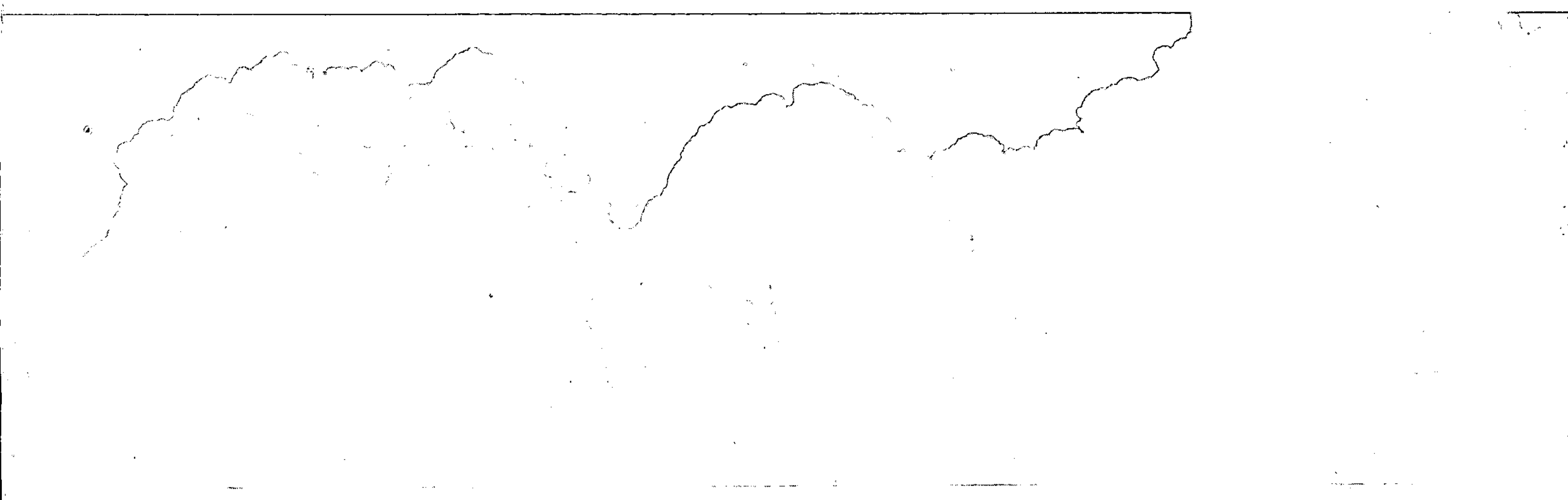
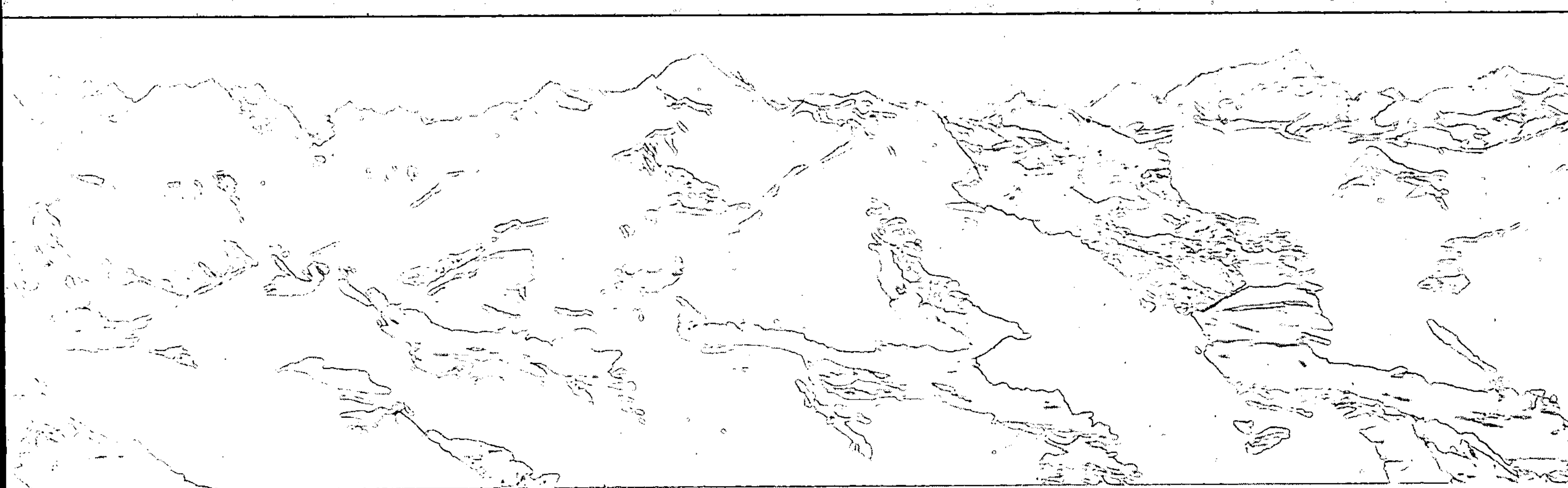
Environment Canada | Environnement Canada

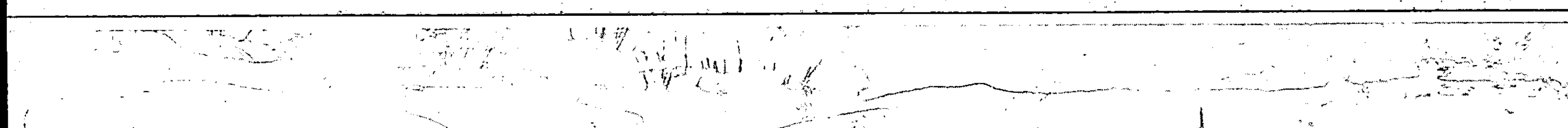
Computer Programs in Use in the Water Quality Branch, Vol. 5

Water Quality Data: Power Spectrum (Harmonic) Analyses and Frequency

Adrian Demayo and David A. Green







REPORT SERIES NO. 38
(Résumé en français)

INLAND WATERS DIRECTORATE,
WATER QUALITY BRANCH,
OTTAWA, CANADA, 1974.

Environment Canada
Environnement Canada

Computer Programs in Use in the Water Quality Branch, Vol. 5

Water Quality Data: Power Spectrum (Harmonic) Analyses and Frequency

Adrian Demayo and David A. Green

REPORT SERIES NO. 38
(Résumé en français)

INLAND WATERS DIRECTORATE,
WATER QUALITY BRANCH,
OTTAWA, CANADA, 1974.

Cat. No.: En 36-508/38-5

Contract # KL327-4-8069
THORN PRESS LIMITED

Contents

Illustrations

Abstract

Time and harmonic frequency analyses have been applied to detect cyclical components in water quality data. This application is explained through a series of examples using synthesized and real data.

Résumé

On a fait des analyses temporelles et harmoniques de la fréquence pour déceler les composantes cycliques dans les données sur la qualité des eaux. On en explique l'application par une série d'exemples basés sur des données synthétisées et réelles.

Computer Programs in Use in the Water Quality Branch, Vol. 5

Water Quality Data: Power Spectrum and Frequency (Harmonic) Analyses

Adrian Demayo and David A. Green

INTRODUCTION

This report describes a computer program that performs power spectrum and frequency analyses of time series water quality data. Several examples are presented.

The purpose of both kinds of analysis is to find the magnitude of those components of the total variance of a record that recur at constant time intervals.

The frequency (harmonic) analysis calculates those components that have a fixed, well-defined periodicity given by the length of the period of time analyzed divided by the number of the harmonic (1,2,3, . . .).

The power spectrum analysis calculates those components that are spread over a band of periodicities rather than a single, discrete periodicity value.

The two types of analysis complement each other. For example, it is recommended first to analyze the record by harmonic analysis to see whether it contains any strong discrete periodicities. If it does, then those periodicities are subtracted from the original record (Thomann, 1967; Sarin and Lee, 1973) and the resulting record is then analyzed by power spectrum analysis.

This approach is possible when the strong periodicities are known in advance. Otherwise all the suspected and possible strong harmonic components must be calculated to detect the significant ones. This approach has the disadvantage of increasing the computation time.

Because, in most cases, only the strong components are sought, in this report the original record is analyzed concomitantly by the harmonic and power spectrum analyses. This approach was found to be useful in the interpretation of the data because the results obtained tend to complement and confirm each other (see examples).

POWER SPECTRUM AND FREQUENCY ANALYSES OF TIME SERIES WATER QUALITY DATA

Power Spectrum Analysis

The mathematical procedure of this analysis has been described by Demayo (1969). Only one slight change has been made in this procedure. In the original report, the calculation of the autocorrelation coefficients is performed as follows:

$$C_r = \frac{1}{n-r} \sum_{i=1}^{n-r} (P_i P_{i+r} - P^2_{av})$$

$r = 0,1,2, \ldots, m$

In the present report, this has been changed to:

$$C_r = \frac{1}{n-r} \sum_{i=1}^{n-r} \left[(P_i - P_{av})\ (P_{i+r} - P_{av}) \right]$$

$r = 0,1,2, \ldots, m$

For completeness the other formulas used in the programs described here are restated:

The Fourier transform at lag r is given by:

$$V_r = \frac{k}{m} \left[C_0 + C_m \cos(r\pi) + 2 \sum_{q=1}^{m-1} C_q \cos\left(\frac{qr\pi}{m}\right) \right]$$

$r = 0,1, \ldots, m$

where k is a constant having the values:

$k = 1/2$ for $r = 0$ and $r = m$
$k = 1$ for $r = 1,2, \ldots, m-1$

The smoothing is done by using the relations:

$$U_0 = 0.54V_0 + 0.46V$$
$$U_r = 0.23V_{r-1} + 0.54V_r + 0.23V_{r+1}$$
$$U_m = 0.46V_{m-1} + 0.54V_m$$
$$r = 1, 2, \ldots, m-1$$

where $U_0, U_1, U_2, \ldots, U_m$ are power spectrum estimates corresponding to lags 0,1,2, . . . , m respectively.

The period corresponding to each lag is determined by the lag number and the sampling interval via the relation:

$$t_r = \frac{2m\Delta t}{r}$$

where t_r is the period corresponding to lag r and Δt is the sampling interval.

The equivalent width, or in other words, the range in the vicinity of the nominal value is given by:

$$W_e = \frac{1}{m\Delta t}$$

The frequency corresponding to a certain lag is obtained from:

$$f_r = \frac{r}{2m\Delta t} \text{ (cycles/time)}$$

where r is the lag number, m the highest lag number, and Δt the sampling interval.

The frequency band corresponding to lag r is in the range:

$$\frac{1}{2m\Delta t}(r-1) \text{ to } \frac{1}{2m\Delta t}(r+1) \text{ in cycles/time}$$

Note: At lag 0 the frequency is $f_0 = 0$. Thus the spectral estimate at lag 0 includes all the record variance that does not recur during the length of the record used in the analysis. Therefore, it includes:

(a) any truly random fluctuations in the record,
(b) any linear trends in the record,
(c) any periodic components in the record that are of such a low frequency that they appear as linear trends in the record.

The "% contribution" of each lag to the total variance of the record is calculated from:

$$(\%\,\text{contrb})_{\text{lag } r} = \frac{V_r}{\sum_{i=0}^{m} V_i}$$

for the unsmoothed transforms, and:

$$(\%\,\text{contrb})_{\text{lag } r} = \frac{U_r}{\sum_{i=0}^{m} U_i}$$

for smoothed values.

Harmonic Analysis

The computer program follows the mathematical procedure described by Thomann (1967). A summary is given here.

The amplitude of a harmonic k is given by

$$H_k = \left(A_k^2 + B_k^2\right)^{1/2}$$

The phase shift of a harmonic k is given by:

$$\theta_k = \arctan\left(A_k/B_k\right)$$

where

$$A_k = \frac{2}{N}\sum_{j=1}^{N-1} P_j \sin(2kj/N)$$

$$B_k = \frac{2}{N}\left(\sum_{j=1}^{N-1} P_j(\cos 2kj/N) + (P_0+P_N)/2\right)$$

N is the total number of points and P_j is the value of the parameter analyzed at point j.

If we use the total variance of the record σ^2 as a measure of the dispersion of the data around the average value, then the contribution of harmonic k to the total variance is obtained from:

$$(\%\,\text{contrb})_k = H_k^2/(2\sigma^2) \quad \text{when } k<N/2$$
$$(\%\,\text{contrb})_k = H_k^2/\sigma^2 \quad \text{when } k = N/2$$

THE COMPUTER PROGRAM AND OPERATING INSTRUCTIONS

The listing of the program is shown in Figure 1. The program is written in FOCAL, EXT 1970, for a PDP-8/E–8K computer or any other PDP–8 family computer with 8K of memory and capable of running the FOCAL.

The "FOCAL, EXT. 1970" is a version of the FOCAL–69 compiler. This new version includes two new functions, FPUT (N,X) and FGET (N), which allows the programmer to store a number X at a specified core location N (FPUT) and then retrieve it (FGET). The array for storing these numbers occupies core locations 6000 to

7677 of memory field 1. Up to 298 numbers can be stored in it.

The steps for loading FOCAL, EXT. 1970 and the program tape are:

1. Load FOCAL–69, 4K.
2. Respond to the initialization dialogue deleting the extended functions.
3. Load tape labelled "FOCAL, EXT. 1970" starting at address 7777.
4. Manually load the following address/instructions:

ADDRESS	INSTRUCTION	COMMENTS
0063	1354	
0064	2414	
2732	5336	
2762	7000	
1217	7600	Deletes : sign
6002	7600	Deletes = sign
5272	6000	
5273	6201	
6375	6000	
6376	5777	
2576	2004	

5. Start FOCAL, EXT. 1970 at address 00200.
6. Read in the program tape.
7. Start the program by giving the command "GO".

The program is divided in two main parts. The first part calculates the power spectrum; the second performs the harmonic analysis.

Input

In response to the "GO" command, the program types out a series of questions. The questions and the correct answers are given in the table below.

After all new values have been entered, the program goes again to question 16, thus allowing the operator to further replace the harmonics that are of no interest.

In Figure 3, the input statements are pointed out by arrows with numbers that correspond to the above explanations. For example, at arrow 17, the operator specified that two of the values entered at arrow 16 should be replaced. At arrow 18, the operator specified the sequential numbers of the two harmonics to be replaced (2 and 3 in this case) and then he gave the new harmonic numbers.

QUESTION	ANSWER AND EXPLANATION	
1. DATE	DD/MM/YY	Day, month, year. Used for identification only.
2. SAMPLING INTERVAL	I	I represents the time interval between two data points.
3. UNITS	sec., min., . . . month, etc.	For information only.
4. NO. OF SAMPLES	NS	NS represents the number of data points.
5. NO. OF LAGS REQUIRED	NL	The greater the NL, the greater will be the resolution of the details of the resultant power spectrum.

Notes: (a) When selecting the number of lags, it is convenient to select a number that will provide a logical and useful time period when calculated. The relation used in this selection is:

$P = 2(NL - 1) \times I/J \qquad J = 1,2,3,\ldots, NL - 1$

where P is the period corresponding to lag no. J.

For example, if periods of 24 and 12 hours are desired and/or expected; then the number of lags required (NL) must be 13 or 25 depending on the number of data points available. For a sample interval of 1 hour (I = 1), 13 lags (NL = 13) will yield periods of 2X(13-1)X1/J=24,12,8 . . . hours; while NL = 25 will yield P = 48, 24, 16, . . .

(b) The sum NS + 3NL $\leq$ 298. This restriction is due to the limited storage available in the computer.

QUESTION	ANSWER AND EXPLANATION
6. ARE THE DATA ARRANGED AS X, Y?	If the data are entered in pairs X, Y type Y. If only the points Y_1, Y_2, . . . are entered, type *any* keyboard character.

In both cases only the Y values are used in calculation, since it is assumed that the X values (time axis) are equally spaced and the time interval is known.

7. LOAD DATA	At this point the X, Y or Y values (depending on the answer to question 6) are loaded via teletype keyboard or, if the date has been already punched on a paper tape, via punched paper tape reader.

Notes: (a) The data must represent samples taken at equal time intervals within ±10% of the interval. When data are missing, a linear interpolation can be used to obtain the missing value.

(b) The 298 available memory spaces are divided as follows:

(i) Data: 1 to NS; (ii) Coefficients C:NS+1 to NS+NL; (iii) V values: NS+NL+1 to NS+2NL; and (iv) U values: NS+2NL+1 to NS+3NL.

This completes the input for the power spectrum analysis. After the spectrum is calculated, the computer asks:

8. IS A HISTOGRAM OF THE TRANSFORMS REQUIRED?	If such a histogram is desired, type Y. Otherwise, type N or *any* other keyboard character.

If the answer to the previous question was Y, the following questions are typed next; otherwise the program goes directly to "Harmonic Analysis", (question 13).

9. SMOOTHED (U) OR UNSMOOTHED (V) VALUES:	Type U or V depending on which values the operator wants plotted. Normally both histograms for smoothed and unsmoothed values are obtained (see question 12).
10. ENTER MAXIMUM ABSOLUTE VALUE OF TRANSFORM 11. ENTER MINIMUM ABSOLUTE VALUE OF TRANSFORM	The first time around, these values are obtained from one of the output tables which lists the values of the transforms for each lag. If certain parts of the histogram require magnification, then the histogram can be repeated (see question 12) with different numbers for the maximum/minimum absolute values of the transforms.

After the computer outputs the respective histogram, the next question is:

12. IS ANOTHER HISTOGRAM OF THE TRANSFORMS REQUIRED?	Answer Y or N. If the answer is Y, then the questions 9 to 11 follow. If the answer is N (or any other keyboard character), the program goes to "Harmonic Analysis", (question 13).
13. HARMONIC ANALYSIS?	Answer Y if an harmonic analysis of the data is to be performed. Otherwise type N or any other keyboard character, and the program will terminate.
14. NO. OF HARMONICS TO BE CALCULATED	Give the number of harmonics to be calculated. It can range between 1 and NS/2. Theoretically, if all NS/2 harmonics are calculated, the original record could be reproduced exactly by adding them up.
15. HARMONICS TO BE CALCULATED (No., Harm. No., Period)	At this point the operator must specify exactly which harmonics are to be calculated. After the HARM. NO. is entered the computer calculates the respective period from NSXI/HARM. NO. Harmonic No. 1, the fundamental harmonic, should always be calculated.

QUESTION	ANSWER AND EXPLANATION
Note: The number of harmonics and the exact harmonics to be calculated are decided by analysis of the "% CONTRIBUTION" table of the POWER SPECTRUM ANALYSIS. The lags making the major contributions to the record are identified, and then their respective periods (nominal and limits) are obtained from the respective table of the POWER SPECTRUM ANALYSIS. It is recommended that *all* the harmonics whose periods fall within these limits be calculated.	
16. ARE THESE VALUES OK?	If the periods of each of the specified harmonics are of interest, then the answer must be Y. This terminates the input to the program. If one or more of the harmonics specified are of no interest, e.g., the period does not correspond to any major contribution lag, and the operator wants to replace them, the answer must be N (or any other keyboard character).
17. HOW MANY NEW VALUES?	This allows the operator to specify how many of the harmonics specified in question 15 he wants to replace.
18. NO., HARM. NO.	The operator must then specify the line he wants replaced and the new HARM. NO.

Output

The output consists of the following values, tables and figures.

(a) MEAN: the average of Y values.

(b) MEAN SQUARE DEVIATION: the mean square deviation of Y values.

(c) TABLE containing the lag number and the values of coefficients C and the smoothed (U) and unsmoothed (V) transforms.

(d) TABLE of lags whose % contribution to the sum of the transforms is greater than 1%.

(e) TABLE of nominal value and limits of periods, and corresponding nominal frequency for each lag.

(f) HISTOGRAMS of smoothed or unsmoothed transforms.

(g) MEAN: the average value of Y values except last one.

Note: The reason for this new average value is the fact that in harmonic analysis the number of points (see INPUT no. 4) is considered to be a complete record, i.e., the last point is the beginning of a new fundamental (longest) cycle of the record, or in other words, the record has come back to its starting point. From the input data of example 1 (Fig. 3) it can be seen that, because of the absence of any random components or any linear trends in the expression used, the value of the last point is indeed equal to that of the first point. Thus the "mean" value calculated here represents the average value of a single, longest cycle. When dealing with data collected from real life, it is important that the first point and last point correspond in time. For example, if monthly data for a period of several years are analyzed, then the first and last values (in chronological order) should have been collected in the same month, several years apart; or if hourly data for a period of several days are analyzed, then the first and last points must have been collected at the same hour of the day.

(h) CORRESPONDING MEAN SQUARE DEVIATION is calculated with respect to the new average value calculated in (g).

(i) TABLE with the values of AMPLITUDE, PHASE SHIFT and % CONTRIBUTION for all harmonics requested (see INPUT nos. 14 and 15).

The Phase Shift is expressed in two kinds of units: (i) when expressed in degrees, the complete record has been considered to correspond to 360 degrees (a full circle), (ii) when expressed in "specified units," i.e., the units specified in input (INPUT no. 3), it shows the shift in time with respect to the first point of the record.

Timing

The time for running the program is directly proportional to the number of points analyzed, number of lags specified, and the number of harmonics calculated. Several examples follow:

Power Spectrum Analysis

No. of points	No. of lags	Execution time (min.)
73	7	7
73	13	10
73	25	15
49	13	10

Harmonic Analysis

No. of points	Execution time per harmonic (sec.)
72	50
48	35

Thus if a complete power spectrum (13 lags) and harmonic (4 harmonics) analyses are done for a 73-point record, then execution time is approximately 15 min.

When the computer is equipped with a medium- or high-speed printer, this execution time is considerably shorter.

Trigonometric Functions

The computation requires the calculation of time, cosine, and arctangent functions. Those functions were calculated using the methods described by Digital Equipment Corporation (1972), which are summarized here.

Sin (X)

$$\text{Sin}(\pm X) = \pm (A_1Y + A_3Y^3 + A_5Y^5 + A_7Y^7 + A_9Y^9)$$

where

$A_1 = 1.5708$; $A_3 = -0.646$; $A_5 = 0.07969$

$A_7 = -0.004674$; and $A_9 = 0.0001515$

Y is given by

Quadrant 0	Y = F
Quadrant 1	Y = 1–F
Quadrant 2	Y = –F
Quadrant 3	Y = F–1

where F is the fractional part of 2X/180, and X is the angle in degrees.

Cos (X)

$$\text{Cos}(X) = \sin(X/180 + 0.5)$$

where again X is in degrees.

Arctangent (X)

$$\text{Tan}^{-1}(X) = X\left(B_0 + \cfrac{B_1}{X^2+B_2+\cfrac{B_3}{X^2+B_4+\cfrac{B_5}{X^2+B_6}}}\right)$$

where

$B_0 = 0.174655$; $B_1 = 3.70926$; $B_2 = 6.76214$

$B_3 = -7.10676$; $B_4 = 3.31634$; $B_5 = -0.264769$;

$B_6 = 1.44863$

X, the argument, is the absolute value of A/B, where A and B are known (see "Harmonic Analysis").

If $X > 1$, $\tan^{-1}(X) = \pi/2 - \tan^{-1}(1/X)$

when:
- (i) A and B positive: the values calculated above stand, i.e., $\tan^{-1}(A/B) = \text{Tan}^{-1}(X)$
- (ii) A and B negative: $\tan^{-1}(A/B) = \tan^{-1}(X) + \pi$
- (iii) A positive and B negative: $\tan^{-1}(A/B) = \pi - \tan^{-1}(X)$
- (iv) A negative and B positive: $\tan^{-1}(A/B) = 2\pi - \tan^{-1}(X)$

Examples

Several sets of data calculated from several mathematical expressions were analyzed with this program to see how well it detects periodicities known to exist in the data.

The synthesized data were obtained from the following two general expressions:

$$Y_j = C \times \text{RAN} + \sum_{i=a,b,c,\ldots} A_i \times \sin(i\phi j - P_i)$$

or

$$Y_j = C \times j + \sum_{i=a,b,c,\ldots} A_i \times \sin(i\phi j - P_i)$$

where:
- i is the harmonic number and it can take any number of discrete, positive, integer values;
- ϕ corresponds to the sampling interval and it is given by 360/number of points per cycle (360 degrees corresponds to one complete cycle);
- P_i is the phase shift in degrees and it can take any values between 0 and ±360.

A_i is the amplitude; it can take any value;
C is a constant, which can take any value;
RAN is a random number, generally between 0 and 1.

When the constant C is zero, the two expressions generate sinusoidal curves. When C is non-zero, a random component is added to each point in the first relation and a linear component in the second one. The listings of the two programs used to generate the test data are shown in Figure 2. The same programs can be used to recreate an observed record by using the results of the power spectrum and harmonic analyses (see examples 6 and 7).

Example 1

The function used was:

$$Y_i = 598 + 356 \sin(3i\phi + 29) + 188 \sin(6i\phi - 41) + 212 \sin(12i\phi + 73)$$

where $\phi = 360/n$ and $i = 0,1,2,\ldots,n$; n is the number of points per cycle. In this example, n = 72 was used; therefore, the function is a combination of three sinusoidal curves with periods of 120, 60 and 30 degrees. The results of the harmonic analysis show that these three harmonics were calculated correctly (Fig. 3). The results of the power spectrum analysis confirm these results.

Note the spread of the contribution of each of the sine functions over several lags in the power spectrum analysis. This is an inevitable result of the overlap that occurs between the period limits of the neighbouring lags. Also, notice in Figure 3 the much larger spread in the smoothed transforms as compared with the unsmoothed values as a result of the weighting operation.

Large contributions from two adjacent lags indicate that one or more strong components have periods that lie in the overlapping zone of the two lags. To demonstrate this, the previous analysis was repeated with 24 lags instead of 25. The results of the harmonic analysis are summarized in the table below. Notice the spread of the sinusoidal component with a nominal period of 120 degrees between the lags 1 and 2; and of the component with a 30-degree period between lags 7 and 8.

Example 2

The function used to generate the data was:

$$y_i = (598 + R) + 356 \sin(3i\phi + 29) + 188 \sin(6i\phi - 41) + 212 \sin(12i\phi + 73)$$

where ϕ and i have the same significance as before; R was a positive random number with values ranging from 50 to 100. The results of the harmonic analysis (Fig. 4) show again that the three harmonic components were detected practically without any distortions. The calculated function was:

$$y_i = 670 + 355 \sin(3i\phi + 29) + 189 \sin(6i\phi - 40.7) + 209 \sin(12i\phi + 72.7)$$

There was no significant change from the previous example in the results of the power spectrum analysis.

Example 3

The same function was used as in example 2 except that in this example R had values between 500 and 1000. With the random component larger than any of the three sinusoidal components, the calculated harmonics show

Power Spectrum Analysis						Harmonic Analysis	
No. of lags used in the computation	Lag no.	Period (degrees) Nominal	Period (degrees) Limits	% Contribution (unsmoothed transform)		Period (degrees)	% Contribution
7	0	∞	∞	39.7		120	61.2
	1	60	∞-30	38.3		60	17.7
	2	30	60-20	15.5		30	21.7
13	1	120	∞-60	59.9		120	61.2
	2	60	120-70	14.1		60	17.7
	4	30	40-24	21.7		30	21.7
25	1	240	∞-120	4.2			
	2	120	240-80	55.5		120	61.2
	7	60	80-40	15.1		60	17.7
	8	30	34.3-26.7	20.3		30	21.7
24	1	230	∞-115	10.6			
	2	115	230-76.6	38.6		120	61.2
	4	57.5	76.6-45	10.6		60	17.7
	7	32.9	38.3-28.8	5.4			
	8	78.8	39.2-25.6	13.6		30	21.7
	9	75.6	28.8-23.0	3.5			

some distortions (Fig. 5). The calculated function was:

$$y_i = 1322 + 343 \sin(3i\phi + 30.7) + 176 \sin(6i\phi - 36.4) + 184 \sin(12i\phi + 70.8)$$

and it only accounted for approximately 82% of the variance of the data.

From the power spectrum results the difference to 100% seems to be located in components with a small period (or high frequency). The power spectrum analysis shows some significant components at high lag numbers, introduced probably by the function used to generate the value of R.

Example 4

The following function was used to generate data:

$$y_i = (598 + i) + 356 \sin(3i\phi + 29) + 188 \sin(6i\phi - 41) + 212 \sin(12i\phi + 73)$$

where the symbols have the same significance as in example 1. The term "i" in the first set of brackets introduces a linear component in the data. The results (Fig. 6) show that the harmonic analysis detected the known components with almost no distortion. The calculated function was:

$$y_i = 633 + 349 \sin(3i\phi + 29.6) + 188 \sin(6i\phi - 41.8) + 211 \sin(12i\phi + 73.4)$$

and it accounted for 99.8% of the variance of the data. The results of the harmonic analysis showed no significant change from examples 1 to 3, although the contributions of lags 0 and 1 would have been expected to increase because they contain the "infinity" period (or the "zero" frequency) corresponding to the linear component.

Example 5

The function used was:

$$y_i = (598 + 10i) + 356 \sin(3i\phi + 29) + 188 \sin(6i\phi - 41) + 212 \sin(12i\phi + 73)$$

which is identical to the function used to generate the data for example 4, the only difference being the linear factor, which was increased by a factor of 10. The data are shown in a plotted form in Figure 7. This increase is reflected in the results (Fig. 8). The harmonic analysis gave the following function:

$$y_i = 953 + 229 \sin(i\phi) + 292 \sin(3i\phi + 36.2) + 161.7 \sin(6i\phi - 49.7) + 208 \sin(12i\phi + 77.5)$$

which accounted for 90.6% (23, 37.3, 11.4 and 18.9%, respectively, for each sinusoidal component) of the total variance of the data. Therefore, the relatively large linear component became the fundamental harmonic.

The power spectrum analysis showed a large increase, from 3 to 21%, in the contribution of lag 0 (frequency 0), which was expected.

The following two examples deal with water quality data.

Example 6

The laboratory results for specific conductance of samples collected from the North Saskatchewan River at Prince Albert (00SA05GG001) between February 1967 and February 1973 were considered for this example. The predominant periodicity detected by the two types of analysis had a period of 12 months, which corresponds to a yearly cycle (Fig. 9).

Data was synthesized, with the following expression accounting for approximately 82% of the variance of the record:

$$\text{Specific conductance} = 457 + 35 \sin(i\phi - 120) + 152 \sin(6i\phi - 256) + 41 \sin(12i\phi - 168)$$

where $i = 0,1,2,\ldots n$; $\phi = 360/n$; and n is the number of points in the record. The constant multiplying the "$i\phi$" term corresponds to the harmonic number.

To convert the angles in the arguments of the trigonometric functions from degrees to actual time units, the following equality must be used:

360 degrees = no. of samples × sampling interval = total length of the record in time units

The observed and calculated data are plotted in Figure 10.

Synthesizing data with eight harmonics improved the fit (Fig. 11). The expression used was:

$$\text{specific conductance} = 457 + 35 \sin(i\phi - 120) + 18 \sin(4i\phi - 93) + 152 \sin(6i\phi - 256) + 20 \sin(7i\phi - 82) + 41 \sin(12i\phi - 168) + 23 \sin(18i\phi - 98 + 29 \sin(24i\phi - 76) + 78 \sin(36i\phi + 90)$$

Notice that a complete record, i.e., one in which the data start and end in the same month (February 1967 to February 1973), was analyzed here. To obtain logical periods for various lags and harmonics complete records only must be analyzed. For example, if the sampling is done quarterly, then the number of samples must be a multiple of four; if the sampling is done weekly, a multiple of 26; if the sampling is done daily, a multiple of 7; if the sampling is done hourly, a multiple of 24; and so on.

Example 7

The laboratory results for specific conductance of samples collected monthly between January 1967 and January 1973, at Battle River near Unwin, were analyzed by power spectrum and harmonic analysis techniques (Fig. 12). The predominant periodicity (42% contribution) has a period of one year.

The following expression accounted for approximately 65% of the record's variance:

$$\text{specific conductance} = 899 + 142 \sin (i\phi - 12.3) + 251.2 \sin (6i\phi - 49.3) + 80.5 \sin (12i\phi + 51) + 82.9\ (24i\phi - 174)$$

The observed and calculated data using this expression are shown in Figure 13.

REFERENCES

Demayo, A. 1969. The computation and interpretation of the power spectra of water quality data. Technical Bulletin No. 16, Department of Energy, Mines and Resources, Ottawa.

Digital Equipment Corporation. 1972. 27 Bit Floating Package User's Manual, DEC-08-NFPEA-A-D.

Sarin, S. C. and E. Stanley Lee. 1973. Time series analysis of air pollution data. Kansas State University Bulletin, Special Report No. 92.

Thomann, Robert V. 1967. Time series analyses of water quality data. Journal of the Sanitary Engineering Division, American Society of Civil Engineers, Vol. 93 (SA1), p. 1.

Illustrations

```
C-FOCAL EXT, 1970

01.04 T !!!"AUTOCORRELATION - POWER SPECTRUM AND HARMONIC ANALYSES"!!
01.05 E
01.08 S S=0; S PI=3.14159; A !!"DATE : ",D,D,D,!
01.09 A !"SAMPLING INTERVAL = ",I,"     UNITS : ",D
01.11 A !"NO. OF SAMPLES = ",A,!"NO. OF LAGS REQUIRED = ",T
01.13 I (T*3+A-299)1.16;T !!"NO. OF SAMPLES + 3*NO. OF LAGS > 298"
01.14 A ".  RESTART ? (Y OR N) : ",D;I (D-0Y)6.3,1.11,6.3
01.16 A !!"IS THE DATA ARRANGED AS (X,Y) ? (Y OR N) : ",D
01.17 I (D-0Y)1.18,1.2;C
01.18 T !!"LOAD DATA :"!;F J=1,A;A P;S Y=FPUT(J,P);S S=S+P
01.19 G 1.21
01.20 T !!"LOAD DATA :"!;F J=1,A;A P,PU;S Y=FPUT(J,P);S S=S+P
01.21 S M=S/A;T !!!"MEAN = ",%,M;S T1=T-1;F J=0,T1;D 3
01.31 S C0=FGET(A+1);S CT=FGET(A+T);S T2=T-2;S S=1;F J=1,T2;D 12
01.33 S S=.5;F J=0,T1,T1;D 12
01.44 S V0=FGET(A+1+T);S VT=FGET(A+2*T);S V1=FGET(A+2+T)
01.46 S VL=FGET(A+2*T-1);S U0=.54*V0+.46*V1;S UT=.46*VL+.54*VT
01.48 F J=A+2+T,A-1+2*T;D 17
01.49 T !"MEAN SQUARE DEVIATION =";S MS=FGET(A+1);T FSQT(MS)
01.50 S Y=FPUT(A+1+2*T,U0);S Y=FPUT(A+3*T,UT)
01.51 T !!!" LAG          COEFFICIENT";S JJ=16;D 1.9;T "TRANSFORM"!
01.52 S JJ=30;D 1.9;T "SMOOTHED              UNSMOOTHED"!
01.53 T "  R                C(R)                U(R)               V(R)"
01.55 F J=0,T1;S K=A+1+J;D 25
01.56 S S=0;S SS=0;S JJ=A+1+T;S JK=A+2*T;F J=JJ,JK;D 18
01.60 T !!!" LAG     CONTRIB. GREATER THAN 1%"
01.61 T !"  R      SMOOTHED         UNSMOOTHED"
01.62 S AB=-1;F J=JJ,JK;D 19
01.66 T !!!"  LAG";S JJ=24;D 1.9;T "PERIOD LIMITS";S JJ=14;D 1.9;T "FRE"
01.67 T "QUENCY"!"             NOMINAL              UPPER              LOWER"!,%2,0
01.68 T "      INFINITY";S JJ=42;D 1.9;T 0;F J=1,T1;D 20
01.69 T !!!"IS A"
01.70 D 4.9;I (D-0Y)6.01,1.8,6.01
01.80 D 4
01.83 T !!!"IS ANOTHER";G 1.7
01.90 F J=1,JJ;T " "

02.10 S CA=0;S CB=0;F K=1,N-1;D 7
02.22 F K=0,N,N;D 7.1;S CB=CB+Y*CX/2
02.24 S A=2*CA/N;S B=2*CB/N;S C=FSQT(A*A+B*B);S VL=100*C*C/(2*MS)
02.26 D 8;S PH=PH*180/PI
02.30 T !,%2,J,"  ",%5.02,N*I/J,"  ",%,C," ",%5.02,90-PH,"   "
02.31 T I*(90-PH)*N/360,"       ",VL

03.03 S CC=A+1+J;S N=A-J;S P1=0;F K=1,N;D 10
03.07 S C=P1/N;S Y=FPUT(CC,C)

04.07 A !"SMOOTHED (U) OR UNSMOOTHED (V) VALUES ? : ",D
04.08 S JJ=A+1+T;S JK=A+2*T;I (D-0V)4.09;G 4.1
04.09 S JJ=JJ+T;S JK=JK+T
04.10 T !!"ENTER MAX";D 4.8;A MX
04.12 T !"ENTER MIN";D 4.8;A MN;S SC=MX-MN
04.16 S TS=SC/64;T !!"HIGHEST RESOLUTION IS = ",%,TS
04.17 S CT=0;T !!!!!!"LAG"
04.18 F J=JJ,JK;D 5
04.70 T !!!;R
04.80 T "IMUM ABSOLUTE VALUE OF TRANSFORM : "
04.90 A " HISTOGRAM OF THE TRANSFORMS REQUIRED ? (Y OR N): ",D

05.05 S PG=FGET(J);S PP=FABS(PG);S TT=(PP-MN)*64/SC;I (TT-65)5.1;S TT=64
05.10 T !,%2,CT," ";F K2=1,TT;T "*"
05.14 S CT=CT+1

06.01 A !!!"HARMONIC ANALYSIS ? (Y OR N) : ",D;I (D-0Y)6.3,6.02,6.3
06.02 S P=FGET(A);S MS=(MS*A-(M-P)*(M-P))/(A-1);S M=(M*A-P)/(A-1)
06.03 T !!"MEAN (CONSIDER COMPLT. RECORD OF NO.SAMPL.-1) = ",%,M
06.04 T !"CORRESPONDING MEAN SQ. DEV. = ",FSQT(MS)
06.06 S N=A-1;T !"NO. OF ";D 6.9;A T,!!;D 6.9;D 6.91
06.07 F J=1,T;T !%2,J,"    ";A D;S P=FPUT(A+J,D);T "  ",%6.01,N*I/D
06.08 A !!"ARE THESE VALUES OK ? (Y OR N) : ",D;I (D-0Y)6.09,6.12;C
06.09 A !"HOW MANY NEW VALUES : ",T1,!
06.10 F J=1,T1;A J1,"   ",D;S P=FPUT(A+J1,D);T "  ",N*I/D,!
06.11 G 6.08
06.12 T !!"NO.   PERIOD     AMPLITUDE            PHASE             % CONTRIB."
06.13 T !"                                  DEG     SP. UNITS"
06.20 F J1=1,T;S J=FGET(N+1+J1);D 2
06.30 T !!!;Q
06.90 T "HARMONICS TO BE CALCULATED : "
06.91 T !"NO.  HARM.NO. PERIOD"

07.10 S Y=FGET(K+1);D 9;D 22;D 9;D 23
07.20 S CA=CA+Y*SX;S CB=CB+Y*CX

08.05 S X=FABS(A/B);I (X)6.3,8.5,8.3
08.30 I (X-1)8.5,8.5;S TR=PI/2;S CT=-1;S X=1/X;G 8.6
08.50 S TR=0;S CT=1
08.60 S PH=7.10676/(X*X+3.31634-.264769/(X*X+1.44863))
08.61 S PH=X*(.174655+3.70926/(X*X+6.76214-PH))
08.62 S PH=TR+CT*PH
08.70 I (A/C)8.75;I (B/C)8.72;R
08.72 S PH=PI-PH;R
08.75 I (B/C)8.8;S PH=2*PI-PH;R
08.80 S PH=PI+PH;R

09.50 S XA=2*K*J/N

10.10 S B=FGET(K);S BB=FGET(K+J);S P1=(B-M)*(BB-M)+P1

12.05 S CA=0;F K=1,T2;D 16
12.07 S XA=J;D 22;S V=S*(C0+CT*CX+2*CA)/T1;S Y=FPUT(J+1+A+T,V)

15.05 S GG=FGET(A+K+1);S XA=K*J/T1;D 22;S CA=GG*CX+CA

16.05 S XA=K*J/T1;D 22;S GY=FGET(A+K+1);S CA=GY*CX+CA

17.05 S XX=FGET(J-1);S YY=FGET(J);S ZZ=FGET(J+1)
17.07 S U=.23*(XX+ZZ)+.54*YY;S Y=FPUT(T+J,U)

18.04 S ST=FGET(J);S S=FABS(ST)+S
18.08 S ST=FGET(J+T);S SS=FABS(ST)+SS

19.05 S Y=FGET(J);S AN=100*FABS(Y)/S;S Y=FGET(J+T);S ST=100*FABS(Y)/SS
19.07 I (1-AN)19.1;I (ST-1)19.2;C
19.10 S AB=AB+1;T !%2,AB,"  ",%,ST,"    ",AN;R
19.20 S AB=AB+1;R

20.05 S DN=2*T1*I;S HL=(J-1)/DN;S HH=(J+1)/DN;S HN=J/DN;S PN=DN/J
20.07 T !%2,J,"  ",%,PN,"    ";I (HL)6.3,20.1;T 1/HL;G 20.3
20.10 T "      INFINITY"
20.30 T "   ",1/HH,"    ",HN

22.10 S XA=XA+.5;D 23;S CX=SX;R

23.05 S SG=1;I (-XA)23.1,23.12;S XA=-XA;S SG=-1
23.10 I (2-XA)23.2,23.12,23.3
23.12 S SX=0;R
23.20 S XA=XA-2;G 23.1
23.30 I (XA-1.5)23.5,23.32;S XA=XA*2;D 23.8;S XA=XA-1;G 23.9
23.32 S SX=-SG;R
23.50 I (XA-1)23.6,23.12;S XA=2*XA;D 23.8;S XA=-XA;G 23.9
23.60 I (XA-.5)23.7,23.62;S XA=2*XA;D 23.8;S XA=1-XA;G 23.9
23.62 S SX=SG;R
23.70 S XA=XA*2;G 23.9
23.80 I (1-XA)23.81;R
23.81 S XA=XA-1;G 23.8
23.90 S D=XA*XA*(.004674-.0001515*XA*XA)
23.91 S SX=SG*XA*(1.5708-XA*XA*(.646-XA*XA*(.07969-D)))

25.01 S Y0=FGET(K);S Y1=FGET(K+T);S Y2=FGET(K+2*T)
25.07 T !%2,J,"        ",%,Y0,"     ",Y2,"     ",Y1
*
```

Figure 1. Power spectrum and harmonic analyses listing (Program A00018).

a

```
C-FOCAL, 1969

01.10 E
01.20 A !!!"NO. OF SINUSOIDAL COMPONENTS : ",I,!"POINTS/CYCLE :",N
01.21 A !"NO. OF CYCLES :",NC,!"AVERAGE VALUE : ",S1
01.22 T !!"NO.  AMPLITUDE  PHASE  HARMONIC NO."
01.25 F J=1,I;T !%2,J,"  ";A "     ",A(J),"     ",P(J),"          ",H(J)
01.26 T !!"RANDOM COMPONENT :"!,I+1,"     ";A R;T !!
01.30 F J=1,N*NC/5+1;T !;F J1=1,5;S ID=(J-1)*5+J1-1;D 1.4;T %6.02,S
01.35 T !!!;Q
01.40 S S=S1+R*FRAN();F L=1,I;D 1.5
01.50 S S=S+A(L)*FSIN(H(L)*ID*6.283/N+P(L)*.01745)
*
```

b

```
C-FOCAL, 1969

01.10 E
01.20 A !!!"NO. OF SINUSOIDAL COMPONENTS : ",I,!"POINTS/CYCLE :",N
01.21 A !"NO. OF CYCLES :",NC,!"AVERAGE VALUE : ",S1
01.22 T !!"NO.  AMPLITUDE  PHASE  HARMONIC NO."
01.25 F J=1,I;T !%2,J,"  ";A "     ",A(J),"     ",P(J),"          ",H(J)
01.26 T !!"LINEAR COMPONENT :"!,I+1,"     ";A R;T !!
01.30 F J=1,N*NC/5+1;T !;F J1=1,5;S ID=(J-1)*5+J1-1;D 1.4;T %6.02,S
01.35 T !!!;Q
01.40 S S=S1+R*ID;F L=1,I;D 1.5
01.50 S S=S+A(L)*FSIN(H(L)*ID*6.283/N+P(L)*.01745)
*
```

Figure 2. Listings of programs used to synthesize data for power spectrum and harmonic analyses.
(a) Program A00019: Sinusoidal curves with a random component.
(b) Program A00019: Sinusoidal curves with a linear component.

AUTOCORRELATION - POWER SPECTRUM AND HARMONIC ANALYSES

```
1 ——> DATE : 19 09 1973

2,3 ——> SAMPLING INTERVAL = 5     UNITS : DEG.
4 ——> NO. OF SAMPLES = 73
5 ——> NO. OF LAGS REQUIRED = 25

6 ——> IS THE DATA ARRANGED AS (X,Y) ? (Y OR N) : N

7 ——> LOAD DATA :
   849.96   964.51   916.73   879.38   983.39
  1168.80  1235.43  1045.10   672.61   351.60
   264.60   381.75   504.82   470.01   306.53
   194.99   271.45   477.85   612.64   532.92
   305.88   155.24   251.97   553.75   849.81
   964.50   916.78   879.36   983.27  1168.68
  1235.47  1045.33   672.88   351.75   264.58
   381.64   504.78   470.10   306.65   195.01
   271.33   477.70   612.62   533.05   306.05
   155.27   251.81   553.51   849.65   964.49
   916.84   879.34   983.15  1168.57  1235.51
  1045.55   673.16   351.91   264.55   381.53
   504.74   470.19   306.78   195.03   271.21
   477.55   612.59   533.19   306.21   155.30
   251.65   553.27   849.49

MEAN =  0.601440E+03
MEAN SQUARE DEVIATION = 0.320859E+03
```

LAG	COEFFICIENT	TRANSFORM	
		SMOOTHED	UNSMOOTHED
R	C(R)	U(R)	V(R)
0	0.102951E+06	0.201149E+04	-0.355255E+03
1	0.877627E+05	0.169667E+05	0.478984E+04
2	0.534777E+05	0.343214E+05	0.628780E+05
3	0.250187E+05	0.166808E+05	-0.319271E+04
4	0.162691E+05	0.837002E+04	0.171432E+05
5	0.184148E+05	0.361122E+04	-0.665174E+03
6	0.111820E+05	-0.261408E+03	0.119468E+03
7	-0.147578E+05	0.490744E+04	-0.751871E+03
8	-0.470164E+05	0.122732E+05	0.229825E+05
9	-0.633170E+05	0.536943E+04	0.154647E+03
10	-0.540389E+05	0.321972E+02	-0.221354E+00
11	-0.326568E+05	-0.615204E+01	-0.141390E+02
12	-0.232242E+05	-0.687191E+01	0.666927E+01
13	-0.370179E+05	-0.137198E+02	-0.313971E+02
14	-0.614261E+05	-0.107286E+02	0.739453E+01
15	-0.721168E+05	-0.143334E+02	-0.326100E+02
16	-0.564533E+05	-0.109250E+02	0.684896E+01
17	-0.250363E+05	-0.136787E+02	-0.309701E+02
18	-0.220072E+03	-0.104440E+02	0.639063E+01
19	0.653286E+04	-0.130229E+02	-0.294427E+02
20	0.554033E+04	-0.995038E+01	0.611458E+01
21	0.170799E+05	-0.124166E+02	-0.281758E+02
22	0.488712E+05	-0.953551E+01	0.605208E+01
23	0.857962E+05	-0.127923E+02	-0.274922E+02
24	0.102670E+06	-0.110932E+02	0.287630E+01

LAG	CONTRIB. GREATER THAN 1%	
R	SMOOTHED	UNSMOOTHED
0	0.191642E+01	0.313638E+00
1	0.161648E+02	0.422872E+01
2	0.326993E+02	0.555119E+02
3	0.158924E+02	0.281869E+01
4	0.797442E+01	0.151349E+02
5	0.344054E+01	0.587250E+00
7	0.467550E+01	0.663790E+00
8	0.116931E+02	0.202901E+02
9	0.511565E+01	0.136530E+00

LAG	PERIOD LIMITS			FREQUENCY
	NOMINAL	UPPER	LOWER	
0	INFINITY			0
1	0.240000E+03	INFINITY	0.120000E+03	0.416667E-02
2	0.120000E+03	0.240000E+03	0.800000E+02	0.833333E-02
3	0.800000E+02	0.120000E+03	0.600000E+02	0.125000E-01
4	0.600000E+02	0.800000E+02	0.480000E+02	0.166667E-01
5	0.480000E+02	0.600000E+02	0.400000E+02	0.208333E-01
6	0.400000E+02	0.480000E+02	0.342857E+02	0.250000E-01
7	0.342857E+02	0.400000E+02	0.300000E+02	0.291667E-01
8	0.300000E+02	0.342857E+02	0.266667E+02	0.333333E-01
9	0.266667E+02	0.300000E+02	0.240000E+02	0.375000E-01
10	0.240000E+02	0.266667E+02	0.218182E+02	0.416667E-01
11	0.218182E+02	0.240000E+02	0.200000E+02	0.458333E-01
12	0.200000E+02	0.218182E+02	0.184615E+02	0.500000E-01
13	0.184615E+02	0.200000E+02	0.171429E+02	0.541667E-01
14	0.171429E+02	0.184615E+02	0.160000E+02	0.583333E-01
15	0.160000E+02	0.171429E+02	0.150000E+02	0.625000E-01
16	0.150000E+02	0.160000E+02	0.141177E+02	0.666667E-01
17	0.141177E+02	0.150000E+02	0.133333E+02	0.708333E-01
18	0.133333E+02	0.141177E+02	0.126316E+02	0.750000E-01
19	0.126316E+02	0.133333E+02	0.120000E+02	0.791667E-01
20	0.120000E+02	0.126316E+02	0.114286E+02	0.833333E-01
21	0.114286E+02	0.120000E+02	0.109091E+02	0.875000E-01
22	0.109091E+02	0.114286E+02	0.104348E+02	0.916666E-01
23	0.104348E+02	0.109091E+02	0.100000E+02	0.958333E-01
24	0.100000E+02	0.104348E+02	0.960000E+01	0.100000E+00

```
8 ——> IS A HISTOGRAM OF THE TRANSFORMS REQUIRED ? (Y OR N):  Y
9 ——> SMOOTHED (U) OR UNSMOOTHED (V) VALUES ? : U

10 ——> ENTER MAXIMUM ABSOLUTE VALUE OF TRANSFORM : 6.3E4
11 ——> ENTER MINIMUM ABSOLUTE VALUE OF TRANSFORM : 0

        HIGHEST RESOLUTION IS =  0.984375E+03

        LAG
          0 **
          1 *****************
          2 **********************************
          3 ****************
          4 ********
          5 ***
          6 *
          7 ****
          8 ************
          9 *****
         10 *
         11 *
         12 *
         13 *
         14 *
         15 *
         16 *
         17 *
         18 *
         19 *
         20 *
         21 *
         22 *
         23 *
         24 *

12 ——> IS ANOTHER HISTOGRAM OF THE TRANSFORMS REQUIRED ? (Y OR N): Y
13 ——> SMOOTHED (U) OR UNSMOOTHED (V) VALUES ? : V

14 ——> ENTER MAXIMUM ABSOLUTE VALUE OF TRANSFORM : 6.3E4
15 ——> ENTER MINIMUM ABSOLUTE VALUE OF TRANSFORM : 0

        HIGHEST RESOLUTION IS =  0.984375E+03

        LAG
          0 *
          1 ****
          2 ***************************************************************
          3 ***
          4 *****************
          5 *
          6 *
          7 *
          8 ***********************
          9 *
         10 *
         11 *
         12 *
         13 *
         14 *
         15 *
         16 *
         17 *
         18 *
         19 *
         20 *
         21 *
         22 *
         23 *
         24 *

12 ——> IS ANOTHER HISTOGRAM OF THE TRANSFORMS REQUIRED ? (Y OR N): N

13 ——> HARMONIC ANALYSIS ? (Y OR N) : Y

        MEAN (CONSIDER COMPLT. RECORD OF NO.SAMPL.-1) =  0.597995E+03
        CORRESPONDING MEAN SQ. DEV. =  0.321755E+03
14      NO. OF HARMONICS TO BE CALCULATED : 4

15 ——> HARMONICS TO BE CALCULATED :
        NO.  HARM.NO.  PERIOD
         1       1      360.0
         2       2      180.0
         3       5       72.0
         4      12       30.0

16 ——> ARE THESE VALUES OK ? (Y OR N) : N
17 ——> HOW MANY NEW VALUES : 2
18       2       3      120.0
         3       6       60.0

16 ——> ARE THESE VALUES OK ? (Y OR N) : Y
```

NO.	PERIOD	AMPLITUDE	PHASE		% CONTRIB.
			DEG	SP. UNITS	
1	360.00	0.190670E-01	-117.94	-117.94	0.00
3	120.00	0.355990E+03	28.98	28.98	61.21
6	60.00	0.188013E+03	- 41.03	- 41.03	17.07
12	30.00	0.211999E+03	72.92	72.92	21.71

Figure 3. Program A00018. Input/output, example 1.

```
AUTOCORRELATION - POWER SPECTRUM AND HARMONIC ANALYSES


DATE : 29 07 1973

SAMPLING INTERVAL = 5      UNITS : DEG
NO. OF SAMPLES = 73
NO. OF LAGS REQUIRED = 13

IS THE DATA ARRANGED AS (X,Y) ? (Y OR N) : N

LOAD DATA :
  911.76 1039.57 1007.90  934.74 1050.64
 1250.47 1334.64 1105.35  745.78  440.47
  318.58  447.31  584.44  566.72  365.26
  266.33  358.09  530.47  676.55  610.55
  400.16  212.50  321.52  638.22  901.10
 1026.80  992.46  971.27 1039.09 1236.48
 1317.82 1095.34  733.62  425.52  354.18
  436.05  570.87  550.37  404.15  254.22
  343.24  565.06  665.66  597.49  384.31
  250.32  309.54  623.62  934.81 1016.21
  979.65  955.63 1075.81 1224.84 1303.86
 1128.57  723.58  413.15  338.92  471.86
  559.60  536.82  387.71  293.33  330.90
  550.06  700.66  586.67  371.17  234.20
  347.48  611.46  920.17


MEAN =  0.673473E+03
MEAN SQUARE DEVIATION = 0.319707E+03


 LAG        COEFFICIENT                  TRANSFORM
                                 SMOOTHED          UNSMOOTHED
  R            C(R)                U(R)               V(R)
  0         0.102213E+06       0.303236E+05       0.337701E+04
  1         0.870628E+05       0.376690E+05       0.619566E+05
  2         0.533471E+05       0.222832E+05       0.149380E+05
  3         0.252150E+05       0.836711E+04      -0.144982E+03
  4         0.162916E+05       0.118314E+05       0.217812E+05
  5         0.177576E+05       0.519673E+04       0.447276E+03
  6         0.103614E+05       0.250111E+02      -0.236801E+03
  7        -0.150360E+05       0.242519E+02       0.217435E+03
  8        -0.465544E+05      -0.168197E+02      -0.168255E+03
  9        -0.622296E+05      -0.139875E+02       0.104470E+03
 10        -0.530623E+05      -0.173771E+02      -0.137837E+03
 11        -0.321171E+05       0.308655E+02       0.143595E+03
 12        -0.232363E+05       0.308992E+02      -0.651009E+02


 LAG    CONTRIB. GREATER THAN 1%
  R     SMOOTHED        UNSMOOTHED
  0   0.261794E+02    0.325594E+01
  1   0.325209E+02    0.597354E+02
  2   0.192378E+02    0.144025E+02
  3   0.722361E+01    0.139784E+00
  4   0.102144E+02    0.210003E+02
  5   0.448651E+01    0.431241E+00


 LAG                        PERIOD LIMITS                  FREQUENCY
         NOMINAL            UPPER           LOWER
  0      INFINITY                                               0
  1   0.120000E+03        INFINITY     0.600000E+02    0.833333E-02
  2   0.600000E+02    0.120000E+03    0.400000E+02    0.166667E-01
  3   0.400000E+02    0.600000E+02    0.300000E+02    0.250000E-01
  4   0.300000E+02    0.400000E+02    0.240000E+02    0.333333E-01
  5   0.240000E+02    0.300000E+02    0.200000E+02    0.416667E-01
  6   0.200000E+02    0.240000E+02    0.171429E+02    0.500000E-01
  7   0.171429E+02    0.200000E+02    0.150000E+02    0.583333E-01
  8   0.150000E+02    0.171429E+02    0.133333E+02    0.666667E-01
  9   0.133333E+02    0.150000E+02    0.120000E+02    0.750000E-01
 10   0.120000E+02    0.133333E+02    0.109091E+02    0.833333E-01
 11   0.109091E+02    0.120000E+02    0.100000E+02    0.916666E-01
 12   0.100000E+02    0.109091E+02    0.923077E+01    0.100000E+00
```

```
IS A HISTOGRAM OF THE TRANSFORMS REQUIRED ? (Y OR N): 1-Y
SMOOTHED (U) OR UNSMOOTHED (V) VALUES ? : U

ENTER MAXIMUM ABSOLUTE VALUE OF TRANSFORM : 6.5E4
ENTER MINIMUM ABSOLUTE VALUE OF TRANSFORM : 1E3

HIGHEST RESOLUTION IS =  0.100000E+04


LAG
  0 *****************************
  1 ************************************
  2 *********************
  3 *******
  4 **********
  5 ****
  6 *
  7 *
  8 *
  9 *
 10 *
 11 *
 12 *


IS ANOTHER HISTOGRAM OF THE TRANSFORMS REQUIRED ? (Y OR N): Y
SMOOTHED (U) OR UNSMOOTHED (V) VALUES ? : V

ENTER MAXIMUM ABSOLUTE VALUE OF TRANSFORM : 6.5E4
ENTER MINIMUM ABSOLUTE VALUE OF TRANSFORM : 1E3

HIGHEST RESOLUTION IS =  0.100000E+04


LAG
  0 **
  1 ************************************************************
  2 *************
  3 *
  4 ********************
  5 *
  6 *
  7 *
  8 *
  9 *
 10 *
 11 *
 12 *


IS ANOTHER HISTOGRAM OF THE TRANSFORMS REQUIRED ? (Y OR N): N


HARMONIC ANALYSIS ? (Y OR N) :Y

MEAN (CONSIDER COMPLT. RECORD OF NO.SAMPL.-1 =  0.670047E+03
CORRESPONDING MEAN SQ. DEV. =  0.320604E+03
NO. OF HARMONICS TO BE CALCULATED : 4

HARMONICS TO BE CALCULATED :
NO.  HARM.NO. PERIOD
 1     1      360.0
 2     3      120.0
 3     6       60.0
 4     12       30.0

ARE THESE VALUES OK ? (Y OR N) : Y

NO.   PERIOD     AMPLITUDE          PHASE           % CONTRIB.
                                DEG    SP. UNITS
  1   360.00   0.152469E+01   81.50     81.50          0.00
  3   120.00   0.354641E+03   28.97     28.97         61.18
  6    60.00   0.188757E+03 - 40.65   - 40.65         17.33
 12    30.00   0.209308E+03   72.74     72.74         21.31
```

Figure 4. Program A00018. Input/output, example 2.

```
AUTOCORRELATION - POWER SPECTRUM AND HARMONIC ANALYSES


DATE : 29 07 1973

SAMPLING INTERVAL = 5      UNITS : DEG
NO. OF SAMPLES = 73
NO. OF LAGS REQUIRED = 13

IS THE DATA ARRANGED AS (X,Y) ? (Y OR N) : N

LOAD DATA :
 1483.24 1733.67 1850.95 1446.73 1672.49
 2005.77 1743.72 1662.47 1422.45 1262.35
  817.70 1053.53 1320.76 1461.05  908.38
  926.00 1159.32 1017.05 1267.55 1328.37
 1272.02  741.97  964.61 1419.32 1375.46
 1602.95 1692.24 1821.23 1555.26 1863.42
 2079.29 1557.78 1295.29 1107.73 1182.78
  939.26 1182.06 1292.71 1305.80  801.79
 1008.31 1372.84 1156.23 1193.32 1108.00
 1129.31  843.34 1271.98 1722.29 1494.44
 1560.51 1661.15 1932.72 1745.24 1935.93
 1896.27 1189.80  979.42 1026.71 1307.24
 1066.92 1153.01 1136.13  698.69  882.95
 1220.57 1515.05 1081.25  971.88  963.81
 1233.66 1149.64 1573.84


MEAN =  0.132574E+04
MEAN SQUARE DEVIATION = 0.332645E+03

 LAG        COEFFICIENT                  TRANSFORM
                                  SMOOTHED          UNSMOOTHED
  R            C(R)                 U(R)               V(R)
  0         0.110653E+06        0.281741E+05       0.355221E+04
  1         0.738255E+05        0.350156E+05       0.570780E+05
  2         0.417036E+05        0.207881E+05       0.146802E+05
  3         0.307739E+05        0.720161E+04      -0.116146E+04
  4         0.255906E+05        0.100345E+05       0.193581E+05
  5         0.706418E+04        0.493687E+04      -0.659898E+03
  6         0.110437E+04        0.441419E+04       0.365587E+04
  7        -0.150409E+04        0.640180E+04       0.112687E+05
  8        -0.448061E+05        0.174026E+04      -0.227884E+04
  9        -0.655809E+05        0.483043E+03       0.164802E+04
 10        -0.451626E+05        0.150142E+04       0.509768E+03
 11        -0.191779E+05        0.194940E+04       0.368303E+04
 12        -0.330603E+05        0.132633E+04      -0.681232E+03

 LAG    CONTRIB. GREATER THAN 1%
  R     SMOOTHED         UNSMOOTHED
  0   0.227271E+02     0.295488E+01
  1   0.282459E+02     0.474799E+02
  2   0.167691E+02     0.122116E+02
  3   0.580930E+01     0.966149E+00
  4   0.809445E+01     0.161029E+02
  5   0.398240E+01     0.548931E+00
  6   0.356078E+01     0.304111E+01
  7   0.516411E+01     0.937375E+01
  8   0.140381E+01     0.189564E+01
  9   0.389655E+00     0.137089E+01
 10   0.121114E+01     0.424046E+00
 11   0.157251E+01     0.306370E+01
 12   0.106990E+01     0.566677E+00

 LAG                         PERIOD LIMITS                   FREQUENCY
        NOMINAL           UPPER            LOWER
  0     INFINITY                                                  0
  1   0.120000E+03      INFINITY       0.600000E+02      0.833333E-02
  2   0.600000E+02    0.120000E+03     0.400000E+02      0.166667E-01
  3   0.400000E+02    0.600000E+02     0.300000E+02      0.250000E-01
  4   0.300000E+02    0.400000E+02     0.240000E+02      0.333333E-01
  5   0.240000E+02    0.300000E+02     0.200000E+02      0.416667E-01
  6   0.200000E+02    0.240000E+02     0.171429E+02      0.500000E-01
  7   0.171429E+02    0.200000E+02     0.150000E+02      0.583333E-01
  8   0.150000E+02    0.171429E+02     0.133333E+02      0.666667E-01
  9   0.133333E+02    0.150000E+02     0.120000E+02      0.750000E-01
 10   0.120000E+02    0.133333E+02     0.109091E+02      0.833333E-01
 11   0.109091E+02    0.120000E+02     0.100000E+02      0.916666E-01
 12   0.100000E+02    0.109091E+02     0.923077E+01      0.100000E+00
```

```
IS A HISTOGRAM OF THE TRANSFORMS REQUIRED ? (Y OR N): 1-Y
SMOOTHED (U) OR UNSMOOTHED (V) VALUES ? : U

ENTER MAXIMUM ABSOLUTE VALUE OF TRANSFORM : 5.6E4
ENTER MINIMUM ABSOLUTE VALUE OF TRANSFORM : 1E3

HIGHEST RESOLUTION IS =  0.890625E+03



LAG
  0 ******************************
  1 **************************************
  2 **********************
  3 ******
  4 **********
  5 ****
  6 ***
  7 ******
  8 *
  9 *
 10 *
 11 *
 12 *


IS ANOTHER HISTOGRAM OF THE TRANSFORMS REQUIRED ? (Y OR N): Y
SMOOTHED (U) OR UNSMOOTHED (V) VALUES ? : V

ENTER MAXIMUM ABSOLUTE VALUE OF TRANSFORM : 5.6E4
ENTER MINIMUM ABSOLUTE VALUE OF TRANSFORM : 1E3

HIGHEST RESOLUTION IS =  0.890625E+03



LAG
  0 **
  1 **************************************************************
  2 ***************
  3 *
  4 ********************
  5 *
  6 **
  7 ***********
  8 *
  9 *
 10 *
 11 ***
 12 *


IS ANOTHER HISTOGRAM OF THE TRANSFORMS REQUIRED ? (Y OR N): N


HARMONIC ANALYSIS ? (Y OR N) : Y

MEAN (CONSIDER COMPLT. RECORD OF NO.SAMPL.-1 =  0.132229E+04
CORRESPONDING MEAN SQ. DEV. =  0.333668E+03
NO. OF HARMONICS TO BE CALCULATED : 4

HARMONICS TO BE CALCULATED :
NO.  HARM.NO. PERIOD
  1      1      360.0
  2      3      120.0
  3      6       60.0
  4     12       30.0

ARE THESE VALUES OK ? (Y OR N) : Y

NO.   PERIOD     AMPLITUDE           PHASE              % CONTRIB.
                                  DEG     SP. UNITS
  1   360.00   0.836210E+01 - 51.26   - 51.26             0.03
  3   120.00   0.343378E+03   30.71     30.71            52.95
  6    60.00   0.176345E+03 - 36.37   - 36.37            13.97
 12    30.00   0.184416E+03   70.77     70.77            15.27
```

Figure 5. Program A00018. Input/output, example 3.

```
AUTOCORRELATION - POWER SPECTRUM AND HARMONIC ANALYSES


DATE : 29 07 1973

SAMPLING INTERVAL = 5      UNITS : DEG
NO. OF SAMPLES =73
NO. OF LAGS REQUIRED = 13

IS THE DATA ARRANGED AS (X,Y) ? (Y OR N) : N

LOAD DATA :
   849.96   965.51   918.73   882.38   987.39
  1173.80  1241.43  1052.10   680.61   360.60
   274.60   392.75   516.82   483.01   320.53
   209.99   287.45   494.85   630.64   551.92
   325.88   176.24   273.97   576.75   873.81
   989.50   942.78   906.36  1011.27  1197.68
  1265.47  1076.33   704.88   384.75   298.58
   416.64   540.78   507.10   344.65   234.01
   311.33   518.70   654.62   576.05   350.05
   200.27   297.81   600.51   897.65  1013.49
   966.84   930.34  1035.15  1221.57  1289.51
  1100.55   729.16   408.91   322.55   440.53
   564.74   531.19   368.78   258.03   335.21
   542.55   678.59   600.19   374.21   224.30
   321.65   624.27   921.49


MEAN =  0.637440E+03
MEAN SQUARE DEVIATION = 0.316911E+03


 LAG        COEFFICIENT                     TRANSFORM
                                 SMOOTHED              UNSMOOTHED
  R            C(R)                U(R)                   V(R)
  0         0.100433E+06       0.293275E+05        0.222970E+04
  1         0.851915E+05       0.368911E+05        0.611380E+05
  2         0.510658E+05       0.219273E+05        0.146248E+05
  3         0.229108E+05       0.843483E+04       -0.138228E+03
  4         0.145154E+05       0.121477E+05        0.223730E+05
  5         0.170108E+05       0.532201E+04        0.426544E+03
  6         0.101254E+05       0.717449E+01       -0.235246E+03
  7        -0.154539E+05      -0.451785E+01        0.156966E+03
  8        -0.473628E+05      -0.228141E+02       -0.152926E+03
  9        -0.633897E+05      -0.766704E+01        0.102885E+03
 10        -0.539666E+05      -0.224292E+02       -0.121966E+03
 11        -0.325507E+05       0.530701E+01        0.859518E+02
 12        -0.231116E+05       0.888756E+01       -0.567598E+02

 LAG     CONTRIB. GREATER THAN 1%
  R      SMOOTHED          UNSMOOTHED
  0   0.256968E+02      0.218936E+01
  1   0.323240E+02      0.600317E+02
  2   0.192127E+02      0.143601E+02
  3   0.739060E+01      0.135727E+00
  4   0.106438E+02      0.219681E+02
  5   0.466314E+01      0.418826E+00

 LAG                          PERIOD LIMITS                    FREQUENCY
        NOMINAL             UPPER              LOWER
  0     INFINITY                                                    0
  1   0.120000E+03        INFINITY        0.600000E+02        0.833333E-02
  2   0.600000E+02      0.120000E+03      0.400000E+02        0.166667E-01
  3   0.400000E+02      0.600000E+02      0.300000E+02        0.250000E-01
  4   0.300000E+02      0.400000E+02      0.240000E+02        0.333333E-01
  5   0.240000E+02      0.300000E+02      0.200000E+02        0.416667E-01
  6   0.200000E+02      0.240000E+02      0.171429E+02        0.500000E-01
  7   0.171429E+02      0.200000E+02      0.150000E+02        0.583333E-01
  8   0.150000E+02      0.171429E+02      0.133333E+02        0.666667E-01
  9   0.133333E+02      0.150000E+02      0.120000E+02        0.750000E-01
 10   0.120000E+02      0.133333E+02      0.109091E+02        0.833333E-01
 11   0.109091E+02      0.120000E+02      0.100000E+02        0.916666E-01
 12   0.100000E+02      0.109091E+02      0.923077E+01        0.100000E+00
```

```
IS A HISTOGRAM OF THE TRANSFORMS REQUIRED ? (Y OR N): 1-Y
SMOOTHED (U) OR UNSMOOTHED (V) VALUES ? : U

ENTER MAXIMUM ABSOLUTE VALUE OF TRAMSFORM : 6E4
ENTER MINIMUM ABSOLUTE VALUE OF TRAMSFORM : 1000

HIGHEST RESOLUTION IS =  0.921875E+03


LAG
  0 ********************************
  1 *****************************************
  2 ************************
  3 *********
  4 *************
  5 ****
  6 *
  7 *
  8 *
  9 *
 10 *
 11 *
 12 *


IS ANOTHER HISTOGRAM OF THE TRANSFORMS REQUIRED ? (Y OR N): Y
SMOOTHED (U) OR UNSMOOTHED (V) VALUES ? : V

ENTER MAXIMUM ABSOLUTE VALUE OF TRAMSFORM : 6E4
ENTER MINIMUM ABSOLUTE VALUE OF TRAMSFORM : 1E3

HIGHEST RESOLUTION IS =  0.921875E+03


LAG
  0 *
  1 ***************************************************************
  2 **************
  3 *
  4 ***********************
  5 *
  6 *
  7 *
  8 *
  9 *
 10 *
 11 *
 12 *


IS ANOTHER HISTOGRAM OF THE TRANSFORMS REQUIRED ? (Y OR N): N


HARMONIC ANALYSIS ? (Y OR N) : Y

MEAN (CONSIDER COMPLT. RECORD OF NO.SAMPL.-1 =  0.633495E+03
CORRESPONDING MEAN SQ. DEV. =  0.317344E+03
NO. OF HARMONICS TO BE CALCULATED : 4

HARMONICS TO BE CALCULATED :
NO.  HARM.NO. PERIOD
  1     1       360.0
  2     3       120.0
  3     6        60.0
  4     12        30.0

ARE THESE VALUES OK ? (Y OR N) :Y

NO.    PERIOD      AMPLITUDE           PHASE              % CONTRIB.
                                   DEG     SP. UNITS
  1    360.00    0.229147E+02     0.04       0.04            0.26
  3    120.00    0.349368E+03    29.58      29.58           60.60
  6     60.00    0.185215E+03 - 41.78    - 41.78           17.03
 12     30.00    0.211497E+03    73.37      73.37           22.21
```

Figure 6. Program A00018. Input/output, example 4.

```
MAX. = 1800
MIN. =  300
ONE DIV. ON Y SCALE =  0.250000E+02
IS THIS OK ? (Y OR N) : Y

HOW MANY POINTS ? :73

LOAD DATA :
  849.96  974.51  936.73  909.38 1023.39
 1218.80 1295.43 1115.10  752.61  441.60
  364.60  491.75  624.82  600.01  446.53
  344.99  431.45  647.85  792.64  722.92
  505.88  365.24  471.97  783.75 1089.81
 1214.50 1176.78 1149.36 1263.27 1458.68
 1535.47 1355.33  992.88  681.75  604.58
  731.64  864.78  840.10  686.65  585.01
  671.38  887.70 1032.62  963.05  746.05
  605.27  711.81 1023.51 1329.65 1454.49
 1416.84 1389.34 1503.15 1698.57 1775.51
 1595.55 1233.16  921.91  844.55  971.52
 1104.74 1080.18  926.78  825.03  911.21
 1127.55 1272.59 1203.19  986.21  845.30
  951.65 1263.27 1569.49
```

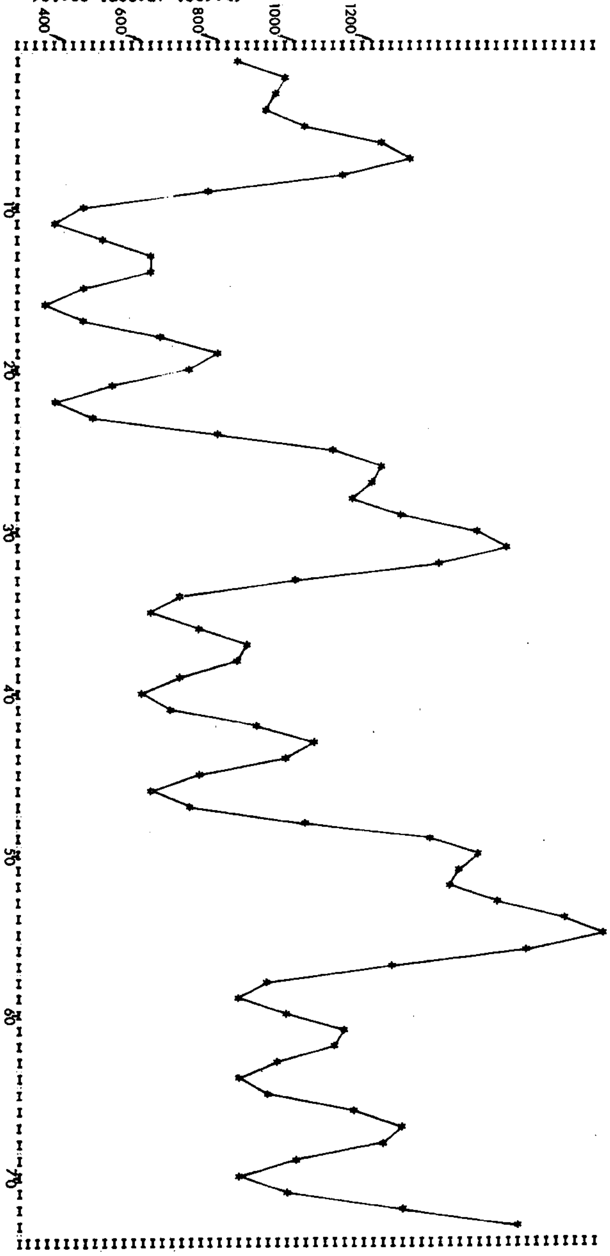


Figure 7. Data used for example 5.

AUTOCORRELATION - POWER SPECTRUM AND HARMONIC ANALYSES

DATE : 29 07 1973

SAMPLING INTERVAL = 5 UNITS : DEG
NO. OF SAMPLES = 73
NO. OF LAGS REQUIRED = 13

IS THE DATA ARRANGED AS (X,Y) ? (Y OR N) : N

LOAD DATA :

849.96	974.51	936.73	909.38	1023.39
1218.80	1295.43	1115.10	752.61	441.60
364.60	491.75	624.82	600.01	446.53
344.99	431.45	647.85	792.64	722.92
505.88	365.24	471.97	783.75	1089.81
1214.50	1176.78	1149.36	1263.27	1458.68
1535.47	1355.33	992.88	681.75	604.58
731.64	864.78	840.10	686.65	585.01
671.33	887.70	1032.62	963.05	746.05
605.27	711.81	1023.51	1329.65	1454.49
1416.84	1389.34	1503.15	1698.57	1775.51
1595.55	1233.16	921.91	844.55	971.52
1104.74	1080.18	926.78	825.03	911.21
1127.55	1272.59	1203.19	986.21	845.30
951.65	1263.27	1569.49		

MEAN = 0.961440E+03
MEAN SQUARE DEVIATION = 0.343120E+03

LAG	COEFFICIENT	TRANSFORM	
		SMOOTHED	UNSMOOTHED
R	C(R)	U(R)	V(R)
0	0.117731E+06	0.386103E+05	0.248134E+05
1	0.100901E+06	0.384141E+05	0.548065E+05
2	0.670692E+05	0.200936E+05	0.135283E+05
3	0.404801E+05	0.862398E+04	0.795110E+03
4	0.340716E+05	0.123484E+05	0.221005E+05
5	0.384844E+05	0.559725E+04	0.100545E+04
6	0.334654E+05	0.262712E+03	-0.125276E+03
7	0.984037E+04	0.185651E+03	0.430901E+03
8	-0.202404E+05	0.124388E+03	-0.792279E+02
9	-0.351544E+05	0.127150E+03	0.295928E+03
10	-0.258058E+05	0.919568E+02	-0.627344E+02
11	-0.549555E+04	0.114501E+03	0.251173E+03
12	0.256078E+04	0.998016E+02	-0.291445E+02

LAG	CONTRIB. GREATER THAN 1%	
R	SMOOTHED	UNSMOOTHED
0	0.309641E+02	0.209708E+02
1	0.308068E+02	0.463192E+02
2	0.161144E+02	0.114333E+02
3	0.691613E+01	0.671978E+00
4	0.990299E+01	0.186780E+02
5	0.448880E+01	0.849748E+00

LAG	NOMINAL	PERIOD LIMITS UPPER	LOWER	FREQUENCY
0	INFINITY			0
1	0.120000E+03	INFINITY	0.600000E+02	0.833333E-02
2	0.600000E+02	0.120000E+03	0.400000E+02	0.166667E-01
3	0.400000E+02	0.600000E+02	0.300000E+02	0.250000E-01
4	0.300000E+02	0.400000E+02	0.240000E+02	0.333333E-01
5	0.240000E+02	0.300000E+02	0.200000E+02	0.416667E-01
6	0.200000E+02	0.240000E+02	0.171429E+02	0.500000E-01
7	0.171429E+02	0.200000E+02	0.150000E+02	0.583333E-01
8	0.150000E+02	0.171429E+02	0.133333E+02	0.666667E-01
9	0.133333E+02	0.150000E+02	0.120000E+02	0.750000E-01
10	0.120000E+02	0.133333E+02	0.109091E+02	0.833333E-01
11	0.109091E+02	0.120000E+02	0.100000E+02	0.916666E-01
12	0.100000E+02	0.109091E+02	0.923077E+01	0.100000E+00

IS A HISTOGRAM OF THE TRANSFORMS REQUIRED ? (Y OR N): Y
SMOOTHED (U) OR UNSMOOTHED (V) VALUES ? :U

ENTER MAXIMUM ABSOLUTE VALUE OF TRANSFORM : 5.5E4
ENTER MINIMUM ABSOLUTE VALUE OF TRANSFORM : 1000

HIGHEST RESOLUTION IS = 0.843750E+03

```
LAG
  0 *********************************************
  1 *********************************************
  2 ***********************
  3 **********
  4 **************
  5 *****
  6 *
  7 *
  8 *
  9 *
 10 *
 11 *
 12 *
```

IS ANOTHER HISTOGRAM OF THE TRANSFORMS REQUIRED ? (Y OR N): Y
SMOOTHED (U) OR UNSMOOTHED (V) VALUES ? :V

ENTER MAXIMUM ABSOLUTE VALUE OF TRANSFORM : 5.5E4
ENTER MINIMUM ABSOLUTE VALUE OF TRANSFORM : 1000

HIGHEST RESOLUTION IS = 0.843750E+03

```
LAG
  0 *****************************
  1 ******************************************************************
  2 ***************
  3 *
  4 **************************
  5 *
  6 *
  7 *
  8 *
  9 *
 10 *
 11 *
 12 *
```

IS ANOTHER HISTOGRAM OF THE TRANSFORMS REQUIRED ? (Y OR N): N

HARMONIC ANALYSIS ? (Y OR N) : Y

MEAN (CONSIDER COMPLT. RECORD OF NO.SAMPL.-1 = 0.952994E+03
CORRESPONDING MEAN SQ. DEV. = 0.337981E+03
NO. OF HARMONICS TO BE CALCULATED : 4

HARMONICS TO BE CALCULATED :

NO.	HARM.NO.	PERIOD
1	1	360.0
2	3	120.0
3	6	60.0
4	12	30.0

ARE THESE VALUES OK ? (Y OR N) : Y

NO.	PERIOD	AMPLITUDE	PHASE DEG	PHASE SP. UNITS	% CONTRIB.
1	360.00	0.229049E+03	0.01	0.01	22.96
3	120.00	0.291875E+03	36.22	36.22	37.29
6	60.00	0.161725E+03	- 49.74	- 49.74	11.45
12	30.00	0.207573E+03	77.49	77.49	18.86

Figure 8. Program A00018. Input/output, example 5.

```
AUTOCORRELATION - POWER SPECTRUM AND HARMONIC ANALYSES


DATE : 24 09 1973

SAMPLING INTERVAL = 1      UNITS : MONTH
NO. OF SAMPLES = 73
NO. OF LAGS REQUIRED = 25

IS THE DATA ARRANGED AS (X,Y) ? (Y OR N) : N

LOAD DATA :
535 482 504 386 396 317 295 250 308 422 607 694 538 495 376 401
363 338 314 290 327 442 685 654 550 529 615 434 435 312 322 294
358 500 668 608 590 576 517 373 443 339 335 351 464 501 772 690
609 620 588 310 440 339 301 288 386 478 648 670 530 493 508 378
404 282 326 418 424 447 491 536 447


MEAN =  0.456932E+03
MEAN SQUARE DEVIATION = 0.125372E+03
```

LAG	COEFFICIENT	TRANSFORM	
		SMOOTHED	UNSMOOTHED
R	C(R)	U(R)	V(R)
0	0.157181E+05	0.241264E+03	0.223101E+03
1	0.110377E+05	0.230608E+03	0.262585E+03
2	0.571840E+04	0.921702E+02	0.163038E+03
3	-0.536300E+03	0.272460E+04	-0.244630E+03
4	-0.579714E+04	0.664306E+04	0.122574E+05
5	-0.103304E+05	0.300298E+04	0.349230E+03
6	-0.105400E+05	0.789431E+02	-0.209087E+02
7	-0.101116E+05	0.203719E+03	0.430908E+02
8	-0.592694E+04	0.513657E+03	0.805475E+03
9	0.928529E+02	0.340271E+03	0.299087E+03
10	0.628256E+04	0.814223E+02	-0.282399E+02
11	0.111465E+05	0.113288E+03	0.121226E+03
12	0.140844E+05	0.187584E+03	0.236178E+03
13	0.102013E+05	0.128256E+03	0.139854E+03
14	0.470046E+04	0.564867E+02	-0.689583E+01
15	-0.959940E+03	0.172196E+03	0.121931E+03
16	-0.650717E+04	0.281765E+03	0.469301E+03
17	-0.107849E+05	0.106097E+03	0.129582E+01
18	-0.110224E+05	0.444591E+01	-0.110531E+02
19	-0.105447E+05	0.285684E+02	0.439849E+02
20	-0.607412E+04	0.391327E+02	0.319948E+02
21	-0.348241E+02	0.555281E+02	0.510392E+02
22	0.626146E+04	0.771048E+02	0.896004E+02
23	0.105438E+05	0.117169E+03	0.738329E+02
24	0.131930E+05	0.167063E+03	0.246482E+03

LAG	CONTRIB. GREATER THAN 1%	
R	SMOOTHED	UNSMOOTHED
0	0.153795E+01	0.136525E+01
1	0.147003E+01	0.160687E+01
3	0.173681E+02	0.149699E+01
4	0.423466E+02	0.750081E+02
5	0.191427E+02	0.213708E+01
7	0.129862E+01	0.263690E+00
8	0.327434E+01	0.492903E+01
9	0.216908E+01	0.183023E+01
12	0.119577E+01	0.144527E+01
15	0.109767E+01	0.746142E+00
16	0.179613E+01	0.287185E+01
24	0.106495E+01	0.150832E+01

LAG		PERIOD LIMITS		FREQUENCY
	NOMINAL	UPPER	LOWER	
0	INFINITY			0
1	0.480000E+02	INFINITY	0.240000E+02	0.208333E-01
2	0.240000E+02	0.480000E+02	0.160000E+02	0.416667E-01
3	0.160000E+02	0.240000E+02	0.120000E+02	0.625000E-01
4	0.120000E+02	0.160000E+02	0.960000E+01	0.833333E-01
5	0.960000E+01	0.120000E+02	0.800000E+01	0.104167E+00
6	0.800000E+01	0.960000E+01	0.685714E+01	0.125000E+00
7	0.685714E+01	0.800000E+01	0.600000E+01	0.145833E+00
8	0.600000E+01	0.685714E+01	0.533333E+01	0.166667E+00
9	0.533333E+01	0.600000E+01	0.480000E+01	0.187500E+00
10	0.480000E+01	0.533333E+01	0.436364E+01	0.208333E+00
11	0.436364E+01	0.480000E+01	0.400000E+01	0.229167E+00
12	0.400000E+01	0.436364E+01	0.369231E+01	0.250000E+00
13	0.369231E+01	0.400000E+01	0.342857E+01	0.270833E+00
14	0.342857E+01	0.369231E+01	0.320000E+01	0.291667E+00
15	0.320000E+01	0.342857E+01	0.300000E+01	0.312500E+00
16	0.300000E+01	0.320000E+01	0.282353E+01	0.333333E+00
17	0.282353E+01	0.300000E+01	0.266667E+01	0.354167E+00
18	0.266667E+01	0.282353E+01	0.252632E+01	0.375000E+00
19	0.252632E+01	0.266667E+01	0.240000E+01	0.395833E+00
20	0.240000E+01	0.252632E+01	0.228572E+01	0.416667E+00
21	0.228572E+01	0.240000E+01	0.218182E+01	0.437500E+00
22	0.218182E+01	0.228572E+01	0.208696E+01	0.458333E+00
23	0.208696E+01	0.218182E+01	0.200000E+01	0.479167E+00
24	0.200000E+01	0.208696E+01	0.192000E+01	0.500000E+00

```
IS A HISTOGRAM OF THE TRANSFORMS REQUIRED ? (Y OR N): Y
SMOOTHED (U) OR UNSMOOTHED (V) VALUES ? : U

ENTER MAXIMUM ABSOLUTE VALUE OF TRANSFORM : 1.3E4
ENTER MINIMUM ABSOLUTE VALUE OF TRANSFORM : 10

HIGHEST RESOLUTION IS =  0.202969E+03


LAG
  0 *
  1 *
  2 *
  3 *************
  4 ********************************
  5 **************
  6 *
  7 *
  8 **
  9 *
 10 *
 11 *
 12 *
 13 *
 14 *
 15 *
 16 *
 17 *
 18 *
 19 *
 20 *
 21 *
 22 *
 23 *
 24 *


IS ANOTHER HISTOGRAM OF THE TRANSFORMS REQUIRED ? (Y OR N): Y
SMOOTHED (U) OR UNSMOOTHED (V) VALUES ? : V

ENTER MAXIMUM ABSOLUTE VALUE OF TRANSFORM : 1.3E4
ENTER MINIMUM ABSOLUTE VALUE OF TRANSFORM : 10

HIGHEST RESOLUTION IS =  0.202969E+03


LAG
  0 *
  1 *
  2 *
  3 *
  4 ************************************************************
  5 *
  6 *
  7 *
  8 ***
  9 *
 10 *
 11 *
 12 *
 13 *
 14 *
 15 *
 16 **
 17 *
 18 *
 19 *
 20 *
 21 *
 22 *
 23 *
 24 *
```

(Figure 9 continued on next page)

Figure 9. Program A00018. Input/output, example 6.

```
IS ANOTHER HISTOGRAM OF THE TRANSFORMS REQUIRED ? (Y OR N): Y
SMOOTHED (U) OR UNSMOOTHED (V) VALUES ? : V

ENTER MAXIMUM ABSOLUTE VALUE OF TRANSFORM : 1000
ENTER MINIMUM ABSOLUTE VALUE OF TRANSFORM : 10

HIGHEST RESOLUTION IS =  0.154686E+02



LAG
  0 *************
  1 ****************
  2 *********
  3 ***************
  4 ********************************************************************
  5 *********************
  6 *
  7 **
  8 ****************************************************
  9 ******************
 10 *
 11 *******
 12 **************
 13 ********
 14 *
 15 *******
 16 *****************************
 17 *
 18 *
 19 **
 20 *
 21 **
 22 *****
 23 ****
 24 ***************



IS ANOTHER HISTOGRAM OF THE TRANSFORMS REQUIRED ? (Y OR N): N

HARMONIC ANALYSIS ? (Y OR N) : Y

MEAN (CONSIDER COMPLT. RECORD OF NO.SAMPL.-1) =  0.457069E+03
CORRESPONDING MEAN SQ. DEV. =  0.126234E+03
NO. OF HARMONICS TO BE CALCULATED : 10

HARMONICS TO BE CALCULATED :
NO.  HARM.NO.  PERIOD
  1      1        72.0
  2      2        36.0
  3      3        24.0
  4      4        18.0
  5      6        12.0
  6      7        10.3
  7     12         6.0
  8     18         4.0
  9     24         3.0
 10     36         2.0

ARE THESE VALUES OK ? (Y OR N) : Y

NO.   PERIOD      AMPLITUDE             PHASE             % CONTRIB.
                                    DEG     SP. UNITS
  1    72.00    0.351642E+02 -119.90    - 23.98            3.88
  2    36.00    0.490799E+01 - 66.63    - 13.33            0.08
  3    24.00    0.123409E+02 -245.71    - 49.14            0.48
  4    18.00    0.180075E+02 - 92.98    - 18.60            1.02
  6    12.00    0.152271E+03 -256.35    - 51.27           72.75
  7    10.29    0.203059E+02 - 82.07    - 16.42            1.29
 12     6.00    0.410402E+02 -168.21    - 33.64            5.29
 18     4.00    0.228830E+02 - 97.89    - 19.58            1.64
 24     3.00    0.292277E+02 - 76.48    - 15.30            2.68
 36     2.00    0.275278E+02   90.00      18.00            2.38
```

Figure 9 (cont'd)

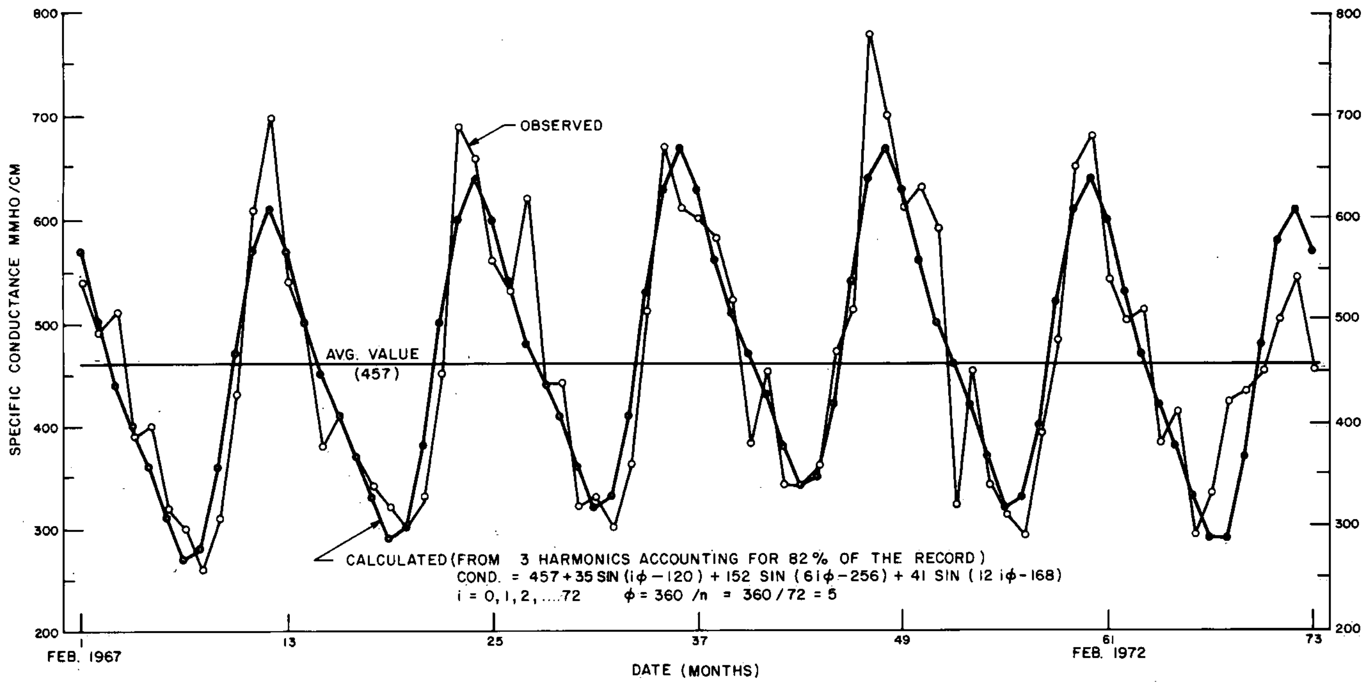


Figure 10. Specific conductance observed and calculated (3 harmonics) values, North Saskatchewan River at Prince Albert (00SA05GG0001), February 1967 – February 1973.

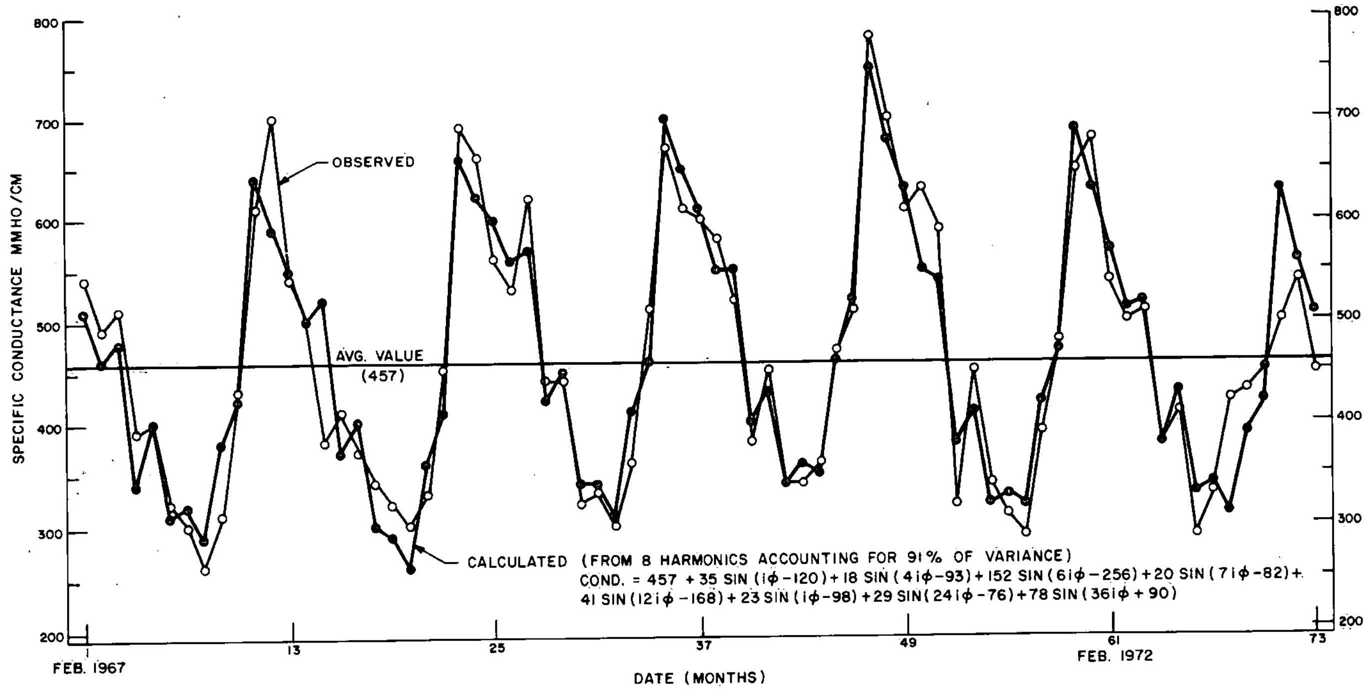


Figure 11. Specific conductance observed and calculated (8 harmonics) values, North Saskatchewan River at Prince Albert (00SA05GG0001), February 1967 – February 1973.

AUTOCORRELATION - POWER SPECTRUM AND HARMONIC ANALYSES

```
DATE : 3 08 1973

SAMPLING INTERVAL = 1      UNITS : MONTH
NO. OF SAMPLES = 73
NO. OF LAGS REQUIRED = 25

IS THE DATA ARRANGED AS (X,Y) ? (Y OR N) : Y

LOAD DATA :
 956 589 1033 573 1006 662 900 568 490 312 544 352 646
466 804 436 1032 572 1043 570 977 654 1550 903 1380 908 1310
862 1540 1019 498 315 796 497 1031 764 1082 671 815 510 950 589
679 426 868 558 1344 891 1152 735 1072 634 1029 616 1161 721
459 279 838 519 1030 632 992 583 1089 659 911 582 1075 670 1260
780 1193 779 1110 783 1106 684 1110 621 325 204 659 430 346 215
744 452 878 522 839 535 1103 734 1054 725 1010 680 992 658 992
651 385 234 421 267 583 380 770 484 595 362 665 427 790 483 1010
627 1100 720 1100 710 811 490 565 360 543 334 507 308 572 343
682 425 687 441 921 514 1155 588 975 635 1089 700 983 684


MEAN =  0.900165E+03
MEAN SQUARE DEVIATION = 0.271156E+03
```

LAG	COEFFICIENT	TRANSFORM	
		SMOOTHED	UNSMOOTHED
R	C(R)	U(R)	V(R)
0	0.735257E+05	0.469908E+04	0.557012E+04
1	0.387191E+05	0.422306E+04	0.367655E+04
2	0.217452E+05	0.317036E+04	0.415911E+04
3	0.659954E+04	0.826634E+04	0.342748E+03
4	-0.108957E+05	0.177155E+05	0.309768E+05
5	-0.172693E+05	0.911843E+04	0.395324E+04
6	-0.171296E+05	0.107766E+04	-0.612968E+03
7	-0.203860E+05	0.171574E+04	0.217135E+04
8	-0.165104E+05	0.296270E+04	0.297476E+04
9	0.859654E+04	0.291999E+04	0.372573E+04
10	0.167894E+05	0.156529E+04	0.973468E+03
11	0.344497E+05	0.861563E+03	0.794339E+03
12	0.375825E+05	0.940136E+03	0.907489E+03
13	0.254140E+05	0.787348E+03	0.116258E+04
14	0.946150E+04	0.616713E+03	-0.213775E+03
15	-0.146181E+04	0.183380E+04	0.202069E+04
16	-0.134148E+05	0.258904E+04	0.344260E+04
17	-0.281823E+05	0.166946E+04	0.115338E+04
18	-0.235511E+05	0.959506E+03	0.110798E+04
19	-0.184791E+05	0.534860E+03	0.417039E+03
20	-0.935447E+04	0.847640E+03	0.238362E+03
21	0.112382E+05	0.172654E+04	0.270872E+04
22	0.194295E+05	0.138321E+04	0.908726E+03
23	0.300648E+05	0.794535E+03	0.117170E+04
24	0.269295E+05	0.428192E+03	-0.205163E+03

LAG	CONTRIB. GREATER THAN 1%	
R	SMOOTHED	UNSMOOTHED
0	0.640144E+01	0.736893E+01
1	0.575297E+01	0.486385E+01
2	0.431890E+01	0.550224E+01
3	0.112610E+02	0.453435E+00
4	0.241334E+02	0.409804E+02
5	0.124218E+02	0.522990E+01
6	0.146806E+01	0.810919E+00
7	0.233731E+01	0.287257E+01
8	0.403601E+01	0.393542E+01
9	0.397782E+01	0.492892E+01
10	0.213235E+01	0.128784E+01
11	0.117369E+01	0.105086E+01
12	0.128072E+01	0.120055E+01
13	0.107259E+01	0.153803E+01
15	0.249814E+01	0.267325E+01
16	0.352698E+01	0.455435E+01
17	0.227426E+01	0.152585E+01
18	0.130711E+01	0.146579E+01
20	0.115472E+01	0.315338E+00
21	0.235202E+01	0.358348E+01
22	0.188431E+01	0.120219E+01
23	0.108238E+01	0.155008E+01

LAG		PERIOD LIMITS		FREQUENCY
	NOMINAL	UPPER	LOWER	
0	INFINITY			0
1	0.480000E+02	INFINITY	0.240000E+02	0.208333E-01
2	0.240000E+02	0.480000E+02	0.160000E+02	0.416667E-01
3	0.160000E+02	0.240000E+02	0.120000E+02	0.625000E-01
4	0.120000E+02	0.160000E+02	0.960000E+01	0.833333E-01
5	0.960000E+01	0.120000E+02	0.800000E+01	0.104167E+00
6	0.800000E+01	0.960000E+01	0.685714E+01	0.125000E+00
7	0.685714E+01	0.800000E+01	0.600000E+01	0.145833E+00
8	0.600000E+01	0.685714E+01	0.533333E+01	0.166667E+00
9	0.533333E+01	0.600000E+01	0.480000E+01	0.187500E+00
10	0.480000E+01	0.533333E+01	0.436364E+01	0.208333E+00
11	0.436364E+01	0.480000E+01	0.400000E+01	0.229167E+00
12	0.400000E+01	0.436364E+01	0.369231E+01	0.250000E+00
13	0.369231E+01	0.400000E+01	0.342857E+01	0.270833E+00
14	0.342857E+01	0.369231E+01	0.320000E+01	0.291667E+00
15	0.320000E+01	0.342857E+01	0.300000E+01	0.312500E+00
16	0.300000E+01	0.320000E+01	0.282353E+01	0.333333E+00
17	0.282353E+01	0.300000E+01	0.266667E+01	0.354167E+00
18	0.266667E+01	0.282353E+01	0.252632E+01	0.375000E+00
19	0.252632E+01	0.266667E+01	0.240000E+01	0.395833E+00
20	0.240000E+01	0.252632E+01	0.228572E+01	0.416667E+00
21	0.228572E+01	0.240000E+01	0.218182E+01	0.437500E+00
22	0.218182E+01	0.228572E+01	0.208696E+01	0.458333E+00
23	0.208696E+01	0.218182E+01	0.200000E+01	0.479167E+00
24	0.200000E+01	0.208696E+01	0.192000E+01	0.500000E+00

```
IS A HISTOGRAM OF THE TRANSFORMS REQUIRED ? (Y OR N): 1-Y
SMOOTHED (U) OR UNSMOOTHED (V) VALUES ? :U

ENTER MAXIMUM ABSOLUTE VALUE OF TRANSFORM : 3.1E4
ENTER MINIMUM ABSOLUTE VALUE OF TRANSFORM : 1E3

HIGHEST RESOLUTION IS =  0.468750E+03


LAG
  0 *******
  1 ******
  2 ****
  3 ***************
  4 ************************************
  5 *****************
  6 *
  7 *
  8 ****
  9 ****
 10 *
 11 *
 12 *
 13 *
 14 *
 15 *
 16 ***
 17 *
 18 *
 19 *
 20 *
 21 *
 22 *
 23 *
 24 *


IS ANOTHER HISTOGRAM OF THE TRANSFORMS REQUIRED ? (Y OR N): Y
SMOOTHED (U) OR UNSMOOTHED (V) VALUES ? :V

ENTER MAXIMUM ABSOLUTE VALUE OF TRANSFORM : 3.1E4
ENTER MINIMUM ABSOLUTE VALUE OF TRANSFORM : 1E3

HIGHEST RESOLUTION IS =  0.468750E+03


LAG
  0 *********
  1 *****
  2 ******
  3 *
  4 **************************************************************
  5 ******
  6 *
  7 **
  8 ****
  9 *****
 10 *
 11 *
 12 *
 13 *
 14 *
 15 **
 16 *****
 17 *
 18 *
 19 *
 20 *
 21 ***
 22 *
 23 *
 24 *


IS ANOTHER HISTOGRAM OF THE TRANSFORMS REQUIRED ? (Y OR N): N

HARMONIC ANALYSIS ? (Y OR N) : Y

MEAN (CONSIDER COMPLT. RECORD OF NO.SAMPL.-1 =  0.899014E+03
CORRESPONDING MEAN SQ. DEV. =  0.272858E+03
NO. OF HARMONICS TO BE CALCULATED : 6

HARMONICS TO BE CALCULATED :
```

NO.	HARM.NO.	PERIOD
1	1	72.0
2	3	24.0
3	6	12.0
4	8	9.0
5	12	6.0
6	24	3.0

ARE THESE VALUES OK ? (Y OR N) :Y

NO.	PERIOD	AMPLITUDE	PHASE		% CONTRIB.
			DEG	SP. UNITS	
1	72.00	0.142424E+03	- 12.29	- 2.46	13.62
3	24.00	0.409214E+02	- 78.36	- 15.67	1.13
6	12.00	0.251215E+03	-246.84	- 49.37	42.38
8	9.00	0.320399E+02	1.47	0.29	0.69
12	6.00	0.805164E+02	50.97	10.19	4.35
24	3.00	0.828593E+02	-174.04	- 34.81	4.61

Figure 12. Program A00018. Input/output, example 7.

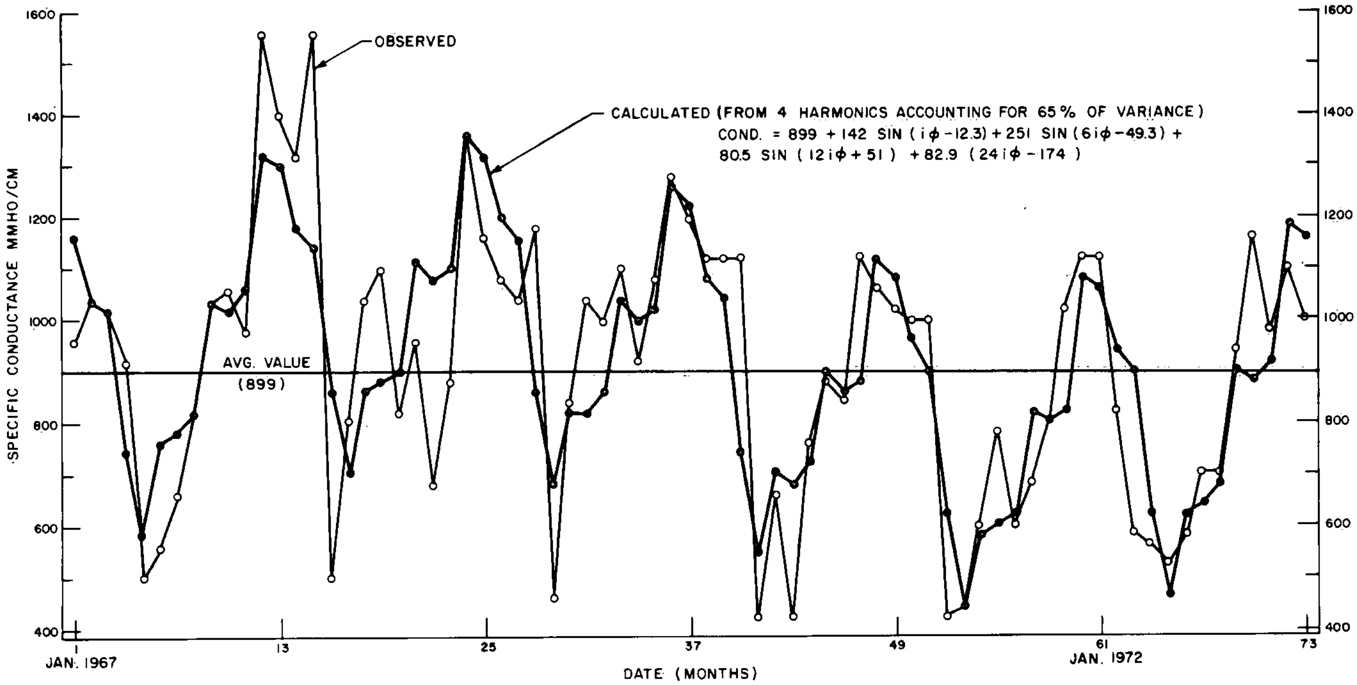


Figure 13. Specific conductance observed and calculated (4 harmonics) values, Battle River near Unwin (00SA05FE0001), January 1967 – January 1972.