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by

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ABSTRACT

A computer programme to calculate wave refraction, wave height and resulting longshore sediment transport is described. Linear theory is used to describe the waves up to the breaker zone. The longshore sediment transport rate is then evaluated using the wave characteristics at breaking. The waves need not originate in deep water, a useful feature when the offshore wave data are hindcasted for shallow lakes. The computed wave direction and height for a plane sloping bathymetry compare favourably to analytical results and to the results of another programme (Dobson, 1967).

RESUME

On trouvera, dans les pages qui suivent, la description d'un programme d'ordinateur destiné à calculer la réfraction des vagues, leur hauteur et les quantités de sédiments qu'elles déposent sur les rives. L'auteur a adopté la théorie linéaire pour décrire les caractéristiques des vagues jusqu'à la zone des brisants. Il a ensuite déterminé les quantités de sédiments déposés sur la rive, d'après les caractéristiques des vagues au moment où elles se brisent. Les vagues ne prennent pas toujours naissance en eau profonde; it est bon de s'en rappeler lorsqu'on fait des prévisions à posteriori sur les vagues échouant sur les rives d'un lac peu profond. Les résultats des calculs effectués pour déterminer la direction et la hauteur des vagues sur des fonds en plan incliné coincident admirablement avec les données obtenues par des analyses ainsi qu'avec les résultats d'une autre méthode de calcul (Dobson, 1967).

1. Introduction

For many problems in coastal engineering, it is necessary to determine the wave climate where the effects of bottom topography are important. Usually only the offshore wave climate is known or estimated and recourse to analytical or empirical techniques is necessary to establish the shallow water wave climate. In recent years a number of papers have been published on the subject of the numerical calculation of refraction, and are reviewed briefly in a paper by Skovgaard et al (1975).

This report describes a programme for calculating not only wave refraction, but also wave height and resulting longshore sediment transport rates. The waves do not have to originate in deep water. This feature is useful when waves have been hindcasted in a shallow lake, where the depth used for hindcasting is specified when the programme is run so that at the start of the refraction calculations the waves have the correct characteristics.

2. Assumptions

First order linear theory is used. There are no currents, and friction and percolation are neglected. Reflection and diffraction effects are also neglected, leaving an initial value problem. The mathematical model assumes that all the wave energy is transmitted along the wave rays (i.e. none is transmitted parallel to the wave crests), so that the problem of crossed rays and caustics cannot be handled.

3. Governing Equations

The dispersion relation for gravity waves is:

$$\omega^2 = gk \tanh k h \quad , \qquad 1.$$

where ω is the frequency in rad/sec, g is the acceleration of gravity, k is the wave number, and h the depth.

The conservation of energy shows that the energy density (proportional to the square of the wave height) changes as a function of group velocity and refraction along a wave ray:

$$\frac{Ec_g}{E_o c_{go}} = \frac{1}{\beta} \quad , \qquad 2.$$

where E is the energy density, c_g the group velocity, β the wave ray separation, and subscript $_o$ refers to the initial conditions.

The ray equations are:

$$\frac{dx}{dt} = c \cos\alpha \quad ,$$

$$\frac{dy}{dt} = c \sin\alpha \quad , \qquad 3.$$

$$\frac{d\alpha}{dt} = - \frac{Dc}{Dn} \quad ,$$

where x and y are the position coordinates along the ray, t is the travel time, c is the phase speed, and α is the angle with respect to the x-axis. $\frac{D}{Dn}$ is differentiation with respect to arc length along the wave front.

It can be shown that $dt = \frac{1}{c}$ Ds, so that equations 3 can be written in terms of s, the arc length along the wave ray:

$$\frac{Dx}{Ds} = \cos\alpha \quad ,$$

$$\frac{Dy}{Ds} = \sin\alpha \quad , \qquad 4.$$

$$\frac{D\alpha}{Ds} = \frac{1}{c}\, \frac{Dc}{Dn} \quad .$$

The operators $\frac{D}{Ds}$ and $\frac{D}{Dn}$ can be expanded:

$$\frac{D}{Ds} = \cos\alpha \frac{\partial}{\partial x} + \sin\alpha \frac{\partial}{\partial y}$$

5.

$$\frac{D}{Dn} = -\sin\alpha \frac{\partial}{\partial x} + \cos\alpha \frac{\partial}{\partial y} \quad .$$

The equation for the ray separation, derived by Munk and Arthur (1952) is:

$$\frac{D^2}{Ds^2}\beta + p\frac{D}{Ds}\beta + q\beta = 0 \quad , \qquad 6.$$

where $p(s) = \frac{1}{c}(\cos\alpha \frac{\partial c}{\partial x} + \sin\alpha \frac{\partial c}{\partial y})$, 7.

and $q(s) = \frac{1}{c}(\sin^2\alpha \frac{\partial^2 c}{\partial x^2} - 2\sin\alpha\cos\alpha \frac{\partial^2 c}{\partial x \partial y} + \cos^2\alpha \frac{\partial^2 c}{\partial y^2})$ 8.

The phase velocity is a function of position coordinates only through its dependence on the depth, so that the chain rule may be used to evaluate its derivatives in equations 7 and 8. Furthermore, as explained below, the bathymetry is approximated locally by a plane so that only the first derivatives of the depth need be retained, allowing equations 7 and 8 to be written:

$$p(s) = -\frac{1}{c}\frac{\partial c}{\partial h}(\frac{\partial h}{\partial x}\cos\alpha + \frac{\partial h}{\partial y}\sin\alpha) \quad , \qquad 9.$$

$$q(s) = \frac{1}{c}\frac{\partial^2 c}{\partial h^2}(\sin\alpha \frac{\partial h}{\partial x} - \cos\alpha \frac{\partial h}{\partial y})^2 \quad , \qquad 10.$$

where $\frac{\partial c}{\partial h} = \frac{c}{h}(\frac{G}{1+G})$,

and $\frac{\partial^2 c}{\partial h^2} = -\frac{\partial c}{\partial h}(\frac{2g}{c^2(1+G)^2})$, 11.

with $G = \frac{2kh}{\sinh 2kh}$.

The progress of a wave ray is determined by solving equations 4. Equation 6 is solved simultaneously and hence equation 2 can be solved. The wave height can then be found because:

$$E = \frac{1}{8} \rho g H^2 \quad , \qquad 12.$$

where ρ is the density and H is the wave height.

If the root mean square wave height is used to define the wave then equation 12 gives the root mean square energy density. Longuet-Higgins (1972) showed that the longshore sediment transport rate can be related to the root mean square energy density flux at the breaker zone. Then the longshore transport rate can be expressed as (Skafel, 1975):

$$S = 1.533 \times 10^4 \, f \, H^2_{rms} \, c_g \, \sin 2\theta \quad , \qquad 13.$$

where S is in $m^3 yr^{-1}$, θ is the acute angle the breakers make with the normal to the beach, and f is the frequency of occurence per year in percent. The wave height and group velocity are evaluated where the wave breaks, which is given by the condition (Noda, 1974):

$$\frac{H}{L} \geq 0.12 \tanh kh \quad , \qquad 14.$$

where L is the wavelength.

4. Numerical Solutions

The bathymetry must be entered as an array with the depths specified at the intersections of a grid of equally spaced points, for example every kilometre. Then the depth at any location within a grid square is estimated by fitting a plane to the depths at the local coordinates defined by the grid square, using a least squares fit. The local coordinates of the grid square and the depth are normalized by the grid size, simplifying the evaluation of the relations in the regression procedure. The resulting normalized equation for the plane is of the form:

$$ax + by + cz = d \quad , \qquad 15.$$

where a, b, and c are the direction cosines and d is the perpendicular distance to the local origin. The derivatives of the depth with respect to x and y are the slopes in the x and y directions, which are simply:

$$\frac{\partial h}{\partial x} = -\frac{a}{c} \text{ and } \frac{\partial h}{\partial y} = -\frac{b}{c} \quad , \qquad 16.$$

and the second derivatives of the depth are zero.

To ensure that the proper plane parameters are evaluated for a grid square containing the shoreline, it is necessary to assign elevations landward of the shore that are mirror images of the adjacent depth values. For example, if the overlay of the grid square on the hydrographic chart shows that the shore intersects the square at (x,y) = (0,0) and (1,1) and and that the depth at (x,y) = (1,0) is 3 metres, then at (x,y) = (0,1) the land elevation must be 3 metres.

When the wave breaks the acute angle between the wave ray and the normal to the beach (directed towards the water) has to be found. The direction of the wave ray is known from the solution of the ray equations (4). The normal to the beach is found from the following considerations. The direction (δ) of the line of steepest ascent up the plane containing the beach differs from the normal to the beach by 180° and is thus the direction needed, for the difference ($\delta - \alpha$) is the required angle. The steepest ascent line is in the same vertical plane as the line through the origin perpendicular to the plane, directed towards the plane. The projection of the latter line onto the x-y plane is the angle δ^1:

$$\delta^1 = \tan^{-1} b/a \quad . \qquad 17.$$

There is an ambiguity of 180°. If the direction cosine in the z direction (c) is negative, them

$$\delta = \delta^1 \qquad 18.$$

If c is positive, then:

$$\delta = \delta^1 + 180^\circ \quad .$$

When $(\delta - \alpha)$ is positive the transport is to the right, looking from the water towards the beach, and to the left when $(\delta - \alpha)$ is negative.

The dispersion relation, equation 1 is solved by the Newton iteration method. If a function is of the form $f(x) = 0$, then the next approximation to the root is:

$$X_n = X_{n-1} - \frac{f(X_{n-1})}{f^1(X_{n-1})} \quad ,$$

where the prime denotes the derivative. It is convenient to rewrite equation 1 in terms of phase speed, $c = \omega/k$:

$$f(c) = c - \frac{g}{\omega} \tanh \frac{\omega}{c} h \quad ,$$

Then $$f'(c) = 1 + \frac{gh}{c^2} \operatorname{sech}^2 \frac{\omega}{c} h$$

Thus the phase speed is found:

$$c_n = c_{n-1} - \frac{c_{n-1} - \frac{g}{\omega} \tanh \frac{\omega h}{c_{n-1}}}{1 + \frac{gh}{c^2_{n-1}} \operatorname{sech}^2 \frac{\omega h}{c_{n-1}}}$$

The initial estimate used to start the iteration is the deep water value or the previously calculated shallow water value. The iterations are stopped when:

$$|c_n - c_{n-1}| \leq 0.0001\, c_n \quad .$$

The wave ray equations 4 and the refraction parameter, equation 6, are solved simultaneously using a Runge-Kutta technique. Write equations 4 in the form:

$$\frac{Dx}{Ds} = \cos \alpha$$

$$\frac{Dy}{Dx} = \sin \alpha$$

$$\frac{D\alpha}{Ds} = \frac{1}{c} \frac{\partial c}{\partial h} \left(\sin \alpha \frac{\partial h}{\partial x} - \cos \alpha \frac{\partial h}{\partial y} \right) \quad ,$$

and equation 6 as two first order equations:

$$\frac{D\beta}{Ds} = \gamma$$

$$\frac{D\gamma}{Ds} = - q\, \beta - p\, \gamma \; .$$

Then let the increments in s be Δ, in x be r, in y be σ, in α be t, in β be n, in γ be n, and f_i (s, x, y...) be the general representation of $\frac{Dx}{Ds}$ through $\frac{D\gamma}{Ds}$. The general form for the 5 first order differential equations to be solved simultaneously is shown in the following table.

S	X	$r = \Delta \cdot f_i$
s_o	x_o	$r_1 = \Delta \cdot f_i \, (s_o, x_o, \ldots)$
$s + \frac{\Delta}{2}$	$x + \frac{r_1}{2}$	$r_2 = \Delta \cdot f_i \, (s_o + \frac{\Delta}{2}, x_o + \frac{r_1}{2}, \ldots)$
$s + \frac{\Delta}{2}$	$x + \frac{r_2}{2}$	$r_3 = \Delta \cdot f_i \, (s_o + \frac{\Delta}{2}, x_o + \frac{r_2}{2}, \ldots)$
$s + \Delta$	$x + r_3$	$r_4 = \Delta \cdot f_i \, (s_o + \Delta, x_o + r_3, \ldots)$

$$R = \frac{1}{6} (r_1 + 2\, r_2 + 2\, r_3 + r_4)$$

And finally: $x(s_o + \Delta) = x_o + R \quad .$

The necessary values of p, q, h and its derivatives, and c and its derivatives are evaluated at each step.

The increment in s is varied as a function of the depth to wavelength ratio, and the depth. If, at the start of the ray, the wave is in deep water the increment size is equal to the grid size until $h/L \leq 0.5$. Then the Runge Kutta scheme is entered and the increment size is ten times the depth when $h/L \geq 0.15$, and is twice the depth when $h/L < 0.15$.

The total distance travelled along the ray is calculated, and after the ray enters water of depth such that $h/L \leq 0.5$, parameters such as location, direction, wave height are printed out at distance intervals about equal to the grid size.

When the wave breaks according to equation 14 all relevant parameters including the longshore sediment transport rate are printed. The longshore transport rate must then be multiplied by the frequency of occurrence per year in percent to yield the correct result (see equation 13).

5. Programme Testing

The programme (RAY4) was run using a plane sloping bottom as test bathymetry for which analytical solutions are readily available for wave height and ray direction. Another programme (Dobson, 1967) was run with the same bathymetry and similar wave parameters so the results could be compared.

The bathymetry was defined by $z = - 0.004x$, for $0 \leq x \leq 18000$ m. The rays in each programme were started at: $(x_o, y_o) = (16000, 2000)$. The wave parameters for this programme (RAY4) were: $f = 0.112$ Hz (Period = 8.93 s); $H_o = 2.0$ m; $\alpha_o = 160^o$ (acute angle with the beach normal = 20^o). The wave parameters for the Dobson programme were: Period = 8.95 s ($f = 0.1117$ Hz); $H_o = 6.50$ ft (1.98 M); $\alpha_o = - 200^o$ (acute angle with beach normal = 20^o, i.e. the same as in programme RAY4).

The step size and display increments differ in the two programmes, so listed results did not correspond. To overcome this the wave rays were plotted on the same map and no differences were discernible in the trajectories of the two rays. In addition the trajectories were compared to the trajectory obtained by graphical methods. The differences were very small, less than the likely error in using the graphical method.

At (x,y) = (2310, 6530), the following parameters from the Dobson programme were compared to the analytical results:

	Dobson Prog.	Analytical	Error
Depth	30.33 ft.	30.315 ft.	+ 0.015 ft.
Angle	12.46^{o}	12.254^{o}	+ 0.206^{o}
KS	0.9642	0.9641	+ 0.0001
KR	0.9820	0.9810	+ 0.001
β	1.0391	1.040	- 0.0009
H/H_o	0.9462	0.9458	+ 0.0004

(KS is the shoaling coefficient and KR is the refraction coefficient).

At (x, y) = (2709, 6445), the following parameters from the programme RAY4 were compared to the analytical solutions:

	RAY 4	Analytical	Error
Depth	10.83 m	10.836 m	- 0.006 m
Angle	13.3^{o}	13.30^{o}	0.0^{o}
KS	0.9455	0.9457	- 0.0002
KR	0.9801	0.9827	- 0.0026
β	1.041	1.0356	+ 0.0054
H/H_o	0.9267	0.9293	- 0.0026

At (x, y) = (783.7, 6815), where the wave broke, the following parameters from the programme RAY4 were compared to the analytical solutions:

	RAY4	Analytical	Error
Depth	3.13 m	3.135 m	- 0.005 m
Angle	7.6^{o}	7.629^{o}	- 0.029^{o}
KS	-	1.1671	-
KR	-	0.9737	-
β	-	1.074	-
H/H_o	1.122	1.1364	- 0.0144

The errors associated with both programmes are small. The error in the angle from the Dobson programme did not show up in the trajectory plot as a shift in the location of the trajectory, suggesting that it is too small to be significant. The error in the wave height from the programme RAY4 increases as the ray progresses into shallower water. This error is due to the error in evaluating β, and is such as to underestimate the wave height slightly. The magnitude of this error is not sufficiently large to be of concern.

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Munk, W. H. and Arthur, R. S. 1952 Wave intensity along a refracted ray. In: Gravity Waves: National Bureau of Standards Circular 521, pp 95-108.

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Appendix A

The input data required for the programme RAY4 are described in this appendix. A flow chart of the programme is shown in figure A1, and a listing of the programme is at the end of this appendix.

Card 1. Parameters: MM, NN, GS, II, D

MM,NN are the upper limits of the depth array in the x and y directions respectively. The depth array is in the first quadrant so the region covered by the depth array is:

$$0 \leq x \leq MM \cdot GS,$$

$$0 \leq y \leq NN \cdot GS$$

The total number of bathymetry points is $(MM + 1) \cdot (NN + 1)$. The subscripts of arrays in Fortran start at one instead of zero. This difference is taken care of within the programme so need not concern the user.

GS is the size of the grid squares of the depth array and is in metres.

II is the number of frequencies for which wave rays are to be calculated.

D is the depth, in metres, used when the wave parameters were hindcasted. If the waves were hindcasted in deep water, enter a large number up to 999.9.

The format for this card is:

FORMAT (I2, 1X, I2, 1X, F6.1, 1X, I2, 1X, F5.1)

Cards 2 to (1 + N). The depth array on N cards.

The data are to be arranged so that, starting at y=0, all the x values are read in for each y successively. Currently the format, set in statement 5, is FORMAT (16F5.1), so there are 16 x values for each y value. The depths, in metres, are positive numbers, locations above mean water level are negative.

Card N + 2. Parameter: DIFF

Parameter DIFF (in metres) allows the depths to be changed from the datum used for the depth array. If the water level is increased DIFF is a positive number.

The format is: FORMAT (F5.1).

Cards N + 3 to N + 2 + II. Parameters: W, AD, HRMS, XI, YI, DYI, YF.

(where II is the number of wave frequencies as entered on the first card). The parameters on these cards are the wave frequency in Hz, the initial direction of the wave rays in degrees measured counter clockwise from the positive x-axis, the wave height in metres, and the starting location of the first ray, the increment in y for the start of the next ray and the maximum y (for the last ray). The last 3 parameters are in metres divided by the grid size. If a different way of starting the rays is required (for example varying x with constant y), these data and statement number 110 and the next statement can be changed. The format for these cards is:

FORMAT (F6.4, 1X, F5.1, 1X, F6.3, 1X, F4.1, 1X, F4.1, 1X, F4.1, 1X, F4.1).

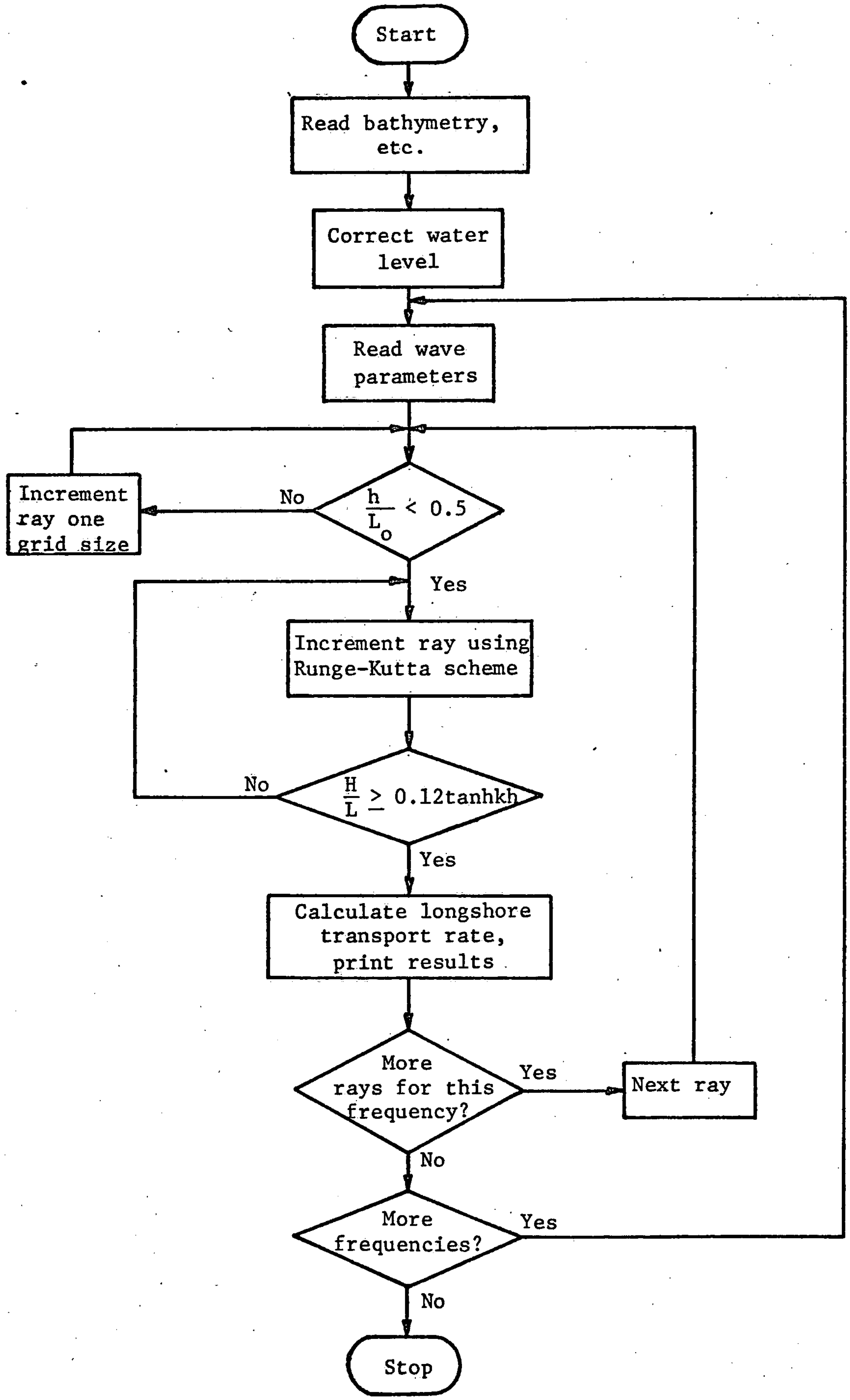


Figure A1. Flow chart for the wave refraction programme (RAY4)

```
      PROGRAM RAY4
C     WAVE RAY PROGRAM- ANY BATHYMETRY. THE ROOT MEAN SQUARE WAVE HEIGHT
C     IS CALCULATED ALONG A NUMBER OF RAYS UP TO THE LOCATION WHERE THE
C     WAVE BREAKS. (CRITERION- H/L.GT.0.12*TANH(D*2*PI/L)).
C     THE BOTTOM FOR EACH GRID SQUARE IS APPROXIMATED BY A PLANE SURFACE.
C     THE DEPTH ARRAY IS IN THE FIRST QUADRANT. IF THE DEPTH AT THE START
C     IS LESS THAN 0.5*WAVELENGTH, A MESSAGE IS PRINTED AND CALCULATIONS
C     CONTINUED. AT THE WAVE BREAKING LOCATION THE WAVE PARAMETERS AND
C     THE LONGSHORE SEDIMENT TRANSPORT RATE ARE LISTED. THE LATTER MUST
C     BE MULTIPLIED BY THE FREQUENCY OF OCCURRENCE (IN PERCENT) TO GIVE
C     THE CORRECT VALUE OF TRANSPORT RATE IN METRES CUBED PER YEAR.
C     DEPTH ARRAY MAXIMUM DIMENSIONS CURRENTLY (21,46).
      REALZ(21,46),K,L,M1,N1
C     PRINT TABLE HEADING, APPLIES TO FORMAT STATEMENT 18 ONLY.
      PRINT 3
    3 FORMAT(1H1,5X,7HFREQ,HZ,10H START X,M,5X,4HY, M,4X,17HA,DEG   HRMS
     1  M   ,8HSTOP X,M,7X,3HY,M,6X,17HA,DEG      HRMS  M,4X,3HH,M,4X,6HCG
     1,M/S,4X,6HDA,DEG,2X,9HSVOL,M3/Y)
      PRINT12
   12 FORMAT(1H0)
      G=9.8044
      PI=3.141592654
C     READ IN DEPTH ARRAY DIMENSIONS (MM FOR X, NN FOR Y ), GRID SIZE (METRES),
C     CURRENTLY SET TO 1000, NUMBER OF FREQUENCIES, DEPTH IN METRES
C     USED IN HINDCASTING WAVES - IF DEEP WATER ENTER A LARGE NUMBER
C     UP TO 999.9.
C     THE NUMBER OF BATHYMETRY POINTS IS (MM+1)*(NN+1)
      READ(60,2)MM,NN,GS,II,D
    2 FORMAT(I2,1X,I2,1X,F6.1,1X,I2,1X,F5.1)
      IF(IFEOF(60).EQ.-1)GOTO450
    5 FORMAT(16F5.1)
      M7=MM+1
      N7=NN+1
C     READ IN BATHYMETRY,IN METRES. MEAN WATER LEVEL IS ZERO,POSITIVE
C     DOWNWARDS, PROGRAM CONVERTS TO POSITIVE UPWARDS. GRID POINTS
C     ONSHORE AND ADJACENT TO SHORELINE MUST BE ≠MIRROR IMAGES≠ ABOUT
C     WATER LEVEL OF ADJACENT OFFSHORE GRID POINTS TO ALLOW RAYS TO BE
C     PROPAGATED INTO THOSE GRID SQUARES. DEPTH ARRAY IS ASSUMED TO BE
C     IN THE FIRST QUADRANT. FORMAT IN STATEMENT 5, ONE Y VALUE PER READ.
      DO7J=1,N7
      READ(60,5)(Z(I,J),I=1,M7)
      IF(IFEOF(60).EQ.-1)GOTO450
    7 CONTINUE
      DO600I=1,M7
      DO600J=1,N7
      Z(I,J)=-Z(I,J)
  600 CONTINUE
C     BASE BATHYMETRY ON CHART DATUM. ENTER CHANGE FROM DATUM (DIFF) IN
C     METRES, POSITIVE NUMBER FOR INCREASE IN DEPTH.
      READ(60,6)DIFF
      IF(IFEOF(60).EQ.-1)GOTO450
    6 FORMAT(F5.1)
```

```
      IF(DIFF.EQ.0.)GOTO1
      DO1I=1,M7
      DO1J=1,N7
      Z(I,J)=Z(I,J)-DIFF
    1 CONTINUE
      DO470J5=1,II
C     READ WAVE CHARACTERISTICS. FREQ.(HZ),ANGLE(DEGREES),HRMS(M)
C     START LOCATION FOR FIRST RAY (XI,YI), INCREMENT IN Y FOR NEXT RAY,S
C     START (DYI), AND MAX Y (YF), IN METRES DIVIDED BY GRID
C     SIZE. (E.G. GS=1000, XI, ETC. IN KILOMETRES.)
      READ(60,4)W,AD,HRMS,XI,YI,DYI,YF
      IF(IFEOF(60).EQ.-1)GOTO450
    4 FORMAT(F6.4,1X,F5.1,1X,F6.3,1X,F4.1,1X,F4.1,1X,F4.1,1X,F4.1)
      W1=W
      W=W*2.*PI
      L=2.*G*PI/W**2
      A=AD*PI/180.
      A2=AD
      CP=W*L/(2.*PI)
  500 W2=W*D/CP
      AA=CP-(G/W)*TANH(W2)
      BB=1.+(G*D/CP**2)*(4./(EXP(W2)+EXP(-W2))**2)
      C1=CP-AA/BB
      IF(ABS(C1-CP).LT..0001*C1)GOTO510
      CP=C1
      GOTO500
  510 CP=C1
      WW=2.*W*D/CP
      CG=.5*CP*(1.+WW/((EXP(WW)-EXP(-WW))/2.))
      CPI=CP
      CGI=CG
      HRMSI=HRMS
      M=0
      N=0
  800 M=M+1
      IF((MM-M).LT.0)GOTO430
      M1=M
      IF(XI.LE.M1)GOTO810
      GOTO800
  810 N=N+1
      IF((NN-N).LT.0)GOTO430
      N1=N
      IF(YI.LE.N1)GOTO19
      GOTO810
   19 X=XI-M1+1.
      I2=0
      Y=YI-N1+1.
      XSN=XI
      YSN=YI
C     DEEP WATER ENERGY INTENSITY PARAMETER FOR REFRACTION CALCULATIONS,
C     BET=1. WAVE FRONTS ARE STRAIGHT, SO DERIVATIVE W.R.T. RAY, DBET=0.
C     SEE MUNK AND ARTHUR (1952). 2ND ORDER D.E. IS SOLVED AS 2 FIRST
```

```
C       ORDER D.E.S, ALONG WITH WAVE RAY SYSTEM OF EQUATIONS
        BET=1.
        DBET=0.
C       GET NORMALIZED DEPTHS FOR GRID SQUARE CONTAINING STARTING POINT
C       X,Y ARE THE COORDINATES  RELATIVE  TO THIS GRID SQUARE
C       CALCULATE DEPTH USING LEAST SQUARES APPROX. TO PLANE
     14 AP=Z(M+1,N)/GS+Z(M+1,N+1)/GS
        B=Z(M+1,N+1)/GS+Z(M,N+1)/GS
        C=Z(M,N)/GS+Z(M+1,N)/GS+Z(M,N+1)/GS+Z(M+1,N+1)/GS
        AN=-(4.*AP-(4.*B-2.*C)+4.*(B-C))/4.
        BN=-(2.*(4.*B-2.*C)+2.*(C-2.*B))/4.
        PN=-(4.*(C-B)-(C-2.*B)-2.*AP)/4.
        RT=SQRT(1.+AN**2+BN**2)
        AN=AN/RT
        BN=BN/RT
        CN=1./RT
        PN=PN/RT
        DR=-PN/CN-AN*X/CN-BN*Y/CN
        DN=ABS(DR)
        SX=-AN/CN
        SY=-BN/CN
        DD=DN*GS
        K=0.
C       COMPARE DEPTH WITH DEEP WATER WAVELENGTH AT START OF RAY.
        IF(I2.GT.0)GOTO11
        I2=1
        IF(DD/L.LT..5)GOTO23
        GOTO22
     11 CONTINUE
C       COMPARE DEPTH WITH DEEP WATER WAVELENGTH AND MEAN WATER LEVEL
        IF(DR.GT..0)GOTO99
        IF(DD/L.LT..5)GOTO20
        GOTO22
     23 CONTINUE
        PRINT620,SX,SY,DD,XSN,YSN,A,M,N
    620 FORMAT(1H0,18HD/L.LT..5 AT START,6E10.4,2I3)
C       STORE PARAMETERS FOR LATER START OF FINITE DEPTH RAY
C       AND AMPLITUDE CALCULATIONS
     22 XT=XSN
        YT=YSN
        DT=DN
        AT=AN
        BT=BN
        CT=CN
        PT=PN
        MO=M
        NO=N
        AO=A
        SXO=SX
        SYO=SY
        XSN=XSN+COS(A)
        YSN=YSN+SIN(A)
```

```
      IF(XSN.LT.0.)GOTO430
      IF(YSN.LT.0.)GOTO430
      M=0
      N=0
    8 M=M+1
      IF((MM-1).LT.0)GOTO430
      M1=M
      IF(XSN.LE.M1)GOTO9
      GOTO8
    9 N=N+1
      IF((NN-N).LT.0)GOTO430
      N1=N
      IF(YSN.LE.N1)GOTO10
      GOTO9
   10 X=XSN-M1+1.
      Y=YSN-N1+1.
      GOTO14
C     D/L.LT.0.5 SO BACK STEP ONE INCREMENT AND GO TO FINITE DEPTH
C     ROUTINE
   20 CPO=CP
      CGO=CG
      HRMSO=HRMS
      XSN=XT
      YSN=YT
      M1=MO
      N1=NO
      DN=DT
      DD=DN*GS
      SX=SXO
      SY=SYO
      W2=W*DD/CPO
C     CH AND CHH ARE FIRST AND SECOND DERIVATIVES OF PHASE SPEED W.R.T.
C     DEPTH
      G9=4.*W2/(EXP(2.*W2)-EXP(-2.*W2))
      CH=CP*G9/(DD*(1.+G9))
      CHH=-CH*2.*G/(CP*(1.+G9))**2
C     DELR IS DISTANCE TRAVELLED ALONG RAY, DELP IS A COUNTER FOR PRINTING
C     VARIABLES AS THE RAY PROGRESSES IN SHALLOW WATER, CURRENTLY SET
C     TO ONE GRID SQUARE
      DELR=SQRT((XSN-XI)**2+(YSN-YI)**2)
      IDEL=DELR
      DELP=IDEL
C     START OF RUNGE-KUTTA SCHEME FOR FINITE DEPTH CALCULATIONS
   21 IF(DD*W/(2.*PI*CP).LT..15)GOTO26
      DEL=10.*DN
      GOTO28
   26 DEL=2.*DN
   28 XO=XSN+1.-M1
      YO=YSN+1.-N1
      R1=DEL*COS(A)
      S1=DEL*SIN(A)
      T1=DEL*K*GS
```

```
C     FIND P AND Q FOR MUNK AND ARTHUR D.E.
      PE=-(COS(A)*SX+SIN(A)*SY)*CH/CP
      QU=CHH*((SX*SIN(A))**2-2.*SIN(A)*COS(A)*SX*SY+(SY*COS(A))**2)/CP
      U1=DEL*DBET*GS
      V1=-DEL*GS*(QU*BET+PE*DBET)
      X1=XSN+R1/2.
      Y1=YSN+S1/2.
      A1=A+T1/2.
      BET1=BET+.5*U1
      DBET1=DBET+.5*V1
C     LOCATE POSITION FIND DEPTH SLOPES
      IF(X1.LT.0.)GOTO430
      IF(Y1.LT.0.)GOTO430
      M=0
      N=0
   34 M=M+1
      IF((MM-M).LT.0)GOTO430
      M1=M
      IF(X1.LE.M1)GOTO36
      GOTO34
   36 N=N+1
      IF((NN-N).LT.0)GOTO430
      N1=N
      IF(Y1.LE.N1)GOTO38
      GOTO36
   38 X=X1+1.-M1
      Y=Y1+1.-N1
      AP=Z(M+1,N)/GS+Z(M+1,N+1)/GS
      B=Z(M+1,N+1)/GS+Z(M,N+1)/GS
      C=Z(M,N)/GS+Z(M+1,N)/GS+Z(M,N+1)/GS+Z(M+1,N+1)/GS
      AN=-(4.*AP-(4.*B-2.*C)+4.*(B-C))/4.
      BN=-(2.*(4.*B-2.*C)+2.*(C-2.*B))/4.
      PN=-(4.*(C-B)-(C-2.*B)-2.*AP)/4.
      RT=SQRT(1.+AN**2+BN**2)
      AN=AN/RT
      BN=BN/RT
      CN=1./RT
      PN=PN/RT
      DR=-PN/CN-AN*X/CN-BN*Y/CN
      DN=ABS(DR)
      SX=-AN/CN
      SY=-BN/CN
      DD=DN*GS
C     FIND CP ETC FOR NEW ESTIMATE OF CURVATURE. FIRST ESTIMATE OF CP IN
C     ALGORITH FOR CP IS CURRENT VALUE
   40 W2=W*DD/CP
      AA=CP-(G/W)*TANH(W2)
      BB=1.+(G*DD/CP**2)*(4./(EXP(W2)+EXP(-W2))**2)
      C1=CP-AA/BB
      IF(ABS(C1-CP).LT..0001*C1)GOTO42
      CP=C1
      GOTO40
```

```
   42 CP=C1
      W2=W*DD/CP
      G9=4.*W2/(EXP(2.*W2)-EXP(-2.*W2))
      CH=CP*G9/(DD*(1.+G9))
      CHH=-CH*2.*G/(CP*(1.+G9))**2
      K=(1./CP)*CH*(SIN(A1)*SX-COS(A1)*SY)*(-1.)
      R2=DEL*COS(A1)
      S2=DEL*SIN(A1)
      T2=DEL*K*GS
      PE=-(COS(A1)*SX+SIN(A1)*SY)*CH/CP
      QU=CHH*((SX*SIN(A1))**2-2.*SIN(A1)*COS(A1)*SX*SY+(SY*COS(A1))**2
     .)/CP
      U2=DEL*DBET1*GS
      V2=-DEL*GS*(QU*BET1+PE*DBET1)
      X1=XSN+R2/2.
      Y1=YSN+S2/2.
      A1=A+T2/2.
      BET1=BET+.5*U2
      DBET1=DBET+.5*V2
C     LOCATE POSITION,FIND DEPTH SLOPES
      IF(X1.LT.0.)GOTO430
      IF(Y1.LT.0.)GOTO430
      M=0
      N=0
   44 M=M+1
      IF((MM-M).LT.0)GOTO430
      M1=M
      IF(X1.LE.M1)GOTO46
      GOTO44
   46 N=N+1
      IF((NN-N).LT.0)GOTO430
      N1=N
      IF(Y1.LE.N1)GOTO48
      GOTO46
   48 X=X1+1.-M1
      Y=Y1+1.-N1
      AP=Z(M+1,N)/GS+Z(M+1,N+1)/GS
      B=Z(M+1,N+1)/GS+Z(M,N+1)/GS
      C=Z(M,N)/GS+Z(M+1,N)/GS+Z(M,N+1)/GS+Z(M+1,N+1)/GS
      AN=-(4.*AP-(4.*B-2.*C)+4.*(B-C))/4.
      BN=-(2.*(4.*B-2.*C)+2.*(C-2.*B))/4.
      PN=-(4.*(C-B)-(C-2.*B)-2.*AP)/4.
      RT=SQRT(1.+AN**2+BN**2)
      AN=AN/RT
      BN=BN/RT
      CN=1./RT
      PN=PN/RT
      DR=-PN/CN-AN*X/CN-BN*Y/CN
      DN=ABS(DR)
      SX=-AN/CN
      SY=-BN/CN
      DD=DN*GS
```

```
      C    CP,K AT X+R2/2, ETC
        50 W2=W*DD/CP
           AA=CP-(G/W)*TANH(W2)
           BB=1.+(G*DD/CP**2)*(4./(EXP(W2)+EXP(-W2))**2)
           C1=CP-AA/BB
           IF(ABS(C1-CP).LT..0001*C1)GOTO52
           CP=C1
           GOTO50
        52 CP=C1
           W2=W*DD/CP
           G9=4.*W2/(EXP(2.*W2)-EXP(-2.*W2))
           CH=CP*G9/(DD*(1.+G9))
           CHH=-CH*2.*G/(CP*(1.+G9))**2
           K=(1./CP)*CH*(SIN(A1)*SX-COS(A1)*SY)*(-1.)
           R3=DEL*COS(A1)
           S3=DEL*SIN(A1)
           T3=DEL*K*GS
           PE=-(COS(A1)*SX+SIN(A1)*SY)*CH/CP
           QU=CHH*((SX*SIN(A1))**2-2.*SIN(A1)*COS(A1)*SX*SY+(SY*COS(A1))**2
          .)/CP
           U3=DEL*DBET1*GS
           V3=-DEL*GS*(QU*BET1+PE*DBET1)
           X1=XSN+R3
           Y1=YSN+S3
           A1=A+T3
           BET1=BET+U3
           DBET1=DBET+V3
      C    LOCATE POSITION DEPTH SLOPES
           IF(X1.LT.0.)GOTO430
           IF(Y1.LT.0.)GOTO430
           M=0
           N=0
        54 M=M+1
           IF((MM-M).LT.0)GOTO430
           M1=M
           IF(X1.LE.M1)GOTO56
           GOTO54
        56 N=N+1
           IF((NN-N).LT.0)GOTO430
           N1=N
           IF(Y1.LE.N1)GOTO58
           GOTO56
        58 X=X1+1.-M1
           Y=Y1+1.-N1
           AP=Z(M+1,N)/GS+Z(M+1,N+1)/GS
           B=Z(M+1,N+1)/GS+Z(M,N+1)/GS
           C=Z(M,N)/GS+Z(M+1,N)/GS+Z(M,N+1)/GS+Z(M+1,N+1)/GS
           AN=-(4.*AP-(4.*B-2.*C)+4.*(B-C))/4.
           BN=-(2.*(4.*B-2.*C)+2.*(C-2.*B))/4.
           PN=-(4.*(C-B)-(C-2.*B)-2.*AP)/4.
           RT=SQRT(1.+AN**2+BN**2)
           AN=AN/RT
```

```
      BN=BN/RT
      CN=1./RT
      PN=PN/RT
      DR=-PN/CN-AN*X/CN-BN*Y/CN
      DN=ABS(DR)
      SX=-AN/CN
      SY=-BN/CN
      DD=DN*GS
C     CP,K AT X+R3/2, ETC
   60 W2=W*DD/CP
      AA=CP-(G/W)*TANH(W2)
      BB=1.+(G*DD/CP**2)*(4./(EXP(W2)+EXP(-W2))**2)
      C1=CP-AA/BB
      IF(ABS(C1-CP).LT..0001*C1)GOTO62
      CP=C1
      GOTO60
   62 CP=C1
      W2=W*DD/CP
      G9=4.*W2/(EXP(2.*W2)-EXP(-2.*W2))
      CH=CP*G9/(DD*(1.+G9))
      CHH=-CH*2.*G/(CP*(1.+G9))**2
      K=(1./CP)*CH*(SIN(A1)*SX-COS(A1)*SY)*(-1.)
      R4=DEL*COS(A1)
      S4=DEL*SIN(A1)
      T4=DEL*K*GS
      PE=-(COS(A1)*SX+SIN(A1)*SY)*CH/CP
      QU=CHH*((SX*SIN(A1))**2-2.*SIN(A1)*COS(A1)*SX*SY+(SY*COS(A1))**2
     .)/CP
      U4=DEL*DBET1*GS
      V4=-DEL*GS*(QU*BET1+PE*DBET1)
C     NEW COORDINATE FROM R-K SCHEME IS-
      XSN=XSN+(1./6.)*(R1+2.*R2+2.*R3+R4)
      YSN=YSN+(1./6.)*(S1+2.*S2+2.*S3+S4)
      A=A+(1./6.)*(T1+2.*T2+2.*T3+T4)
      BET=BET+(1./6.)*(U1+2.*U2+2.*U3+U4)
      DBET=DBET+(1./6.)*(V1+2.*V2+2.*V3+V4)
C     LOCATE THIS COORDINATE GET DEPTH SLOPES
      IF(XSN.LT.0.)GOTO430
      IF(YSN.LT.0.)GOTO430
      M=0
      N=0
   64 M=M+1
      IF((MM-M).LT.0)GOTO430
      M1=M
      IF(XSN.LE.M1)GOTO66
      GOTO64
   66 N=N+1
      IF((NN-N).LT.0)GOTO430
      N1=N
      IF(YSN.LE.N1)GOTO68
      GOTO66
   68 X=XSN+1.-M1
```

```
      Y=YSN+1.-N1
      AP=Z(M+1,N)/GS+Z(M+1,N+1)/GS
      B=Z(M+1,N+1)/GS+Z(M,N+1)/GS
      C=Z(M,N)/GS+Z(M+1,N)/GS+Z(M,N+1)/GS+Z(M+1,N+1)/GS
      AN=-(4.*AP-(4.*B-2.*C)+4.*(B-C))/4.
      BN=-(2.*(4.*B-2.*C)+2.*(C-2.*B))/4.
      PN=-(4.*(C-B)-(C-2.*B)-2.*AP)/4.
      RT=SQRT(1.+AN**2+BN**2)
      AN=AN/RT
      BN=BN/RT
      CN=1./RT
      PN=PN/RT
      DR=-PN/CN-AN*X/CN-BN*Y/CN
      DN=ABS(DR)
      SX=-AN/CN
      SY=-BN/CN
      DD=DN*GS
C     CP,K,CG AT NEW XSN,ETC
   70 W2=W*DD/CP
      AA=CP-(G/W)*TANH(W2)
      BB=1.+(G*DD/CP**2)*(4./(EXP(W2)+EXP(-W2))**2)
      C1=CP-AA/BB
      IF(ABS(C1-CP).LT..0001*C1)GOTO72
      CP=C1
      GOTO70
   72 CP=C1
      WNO=W/CP
      WW=2.*WNO*DD
      CG=.5*CP*(1.+WW/((EXP(WW)-EXP(-WW))/2.))
      W2=W*DD/CP
      G9=4.*W2/(EXP(2.*W2)-EXP(-2.*W2))
      CH=CP*G9/(DD*(1.+G9))
      K=(1./CP)*(-SIN(A)*SX+COS(A)*SY)*CH
      AO=A
      HRMS=HRMSO*SQRT(ABS(CGO/(CG*BET)))
      IF(HRMS*W/(CP*2.*PI).GT..12*TANH(DD*W/CP))GOTO99
C     STOP RAY IF ON SHORE
      IF(DR.GT..0)GOTO99
      R9=((1./6.)*(R1+2.*R2+2.*R3+R4))**2
      S9=((1./6.)*(S1+2.*S2+2.*S3+S4))**2
      R9=R9+S9
      DELR1=SQRT(R9)
      DELR=DELR+DELR1
      IF(DELR.GT.DELP)GOTO750
      GOTO760
  750 AD=180.*A/PI
      PRINT77,W1,XI,YI,XSN,YSN,AD,HRMS,DD,CP,CG,BET,DELR,DELP
   77 FORMAT(1H ,F5.3,2X,F6.3,2X,F6.3,2X,F6.3,2X,F6.3,2X,F5.1,2X,F7.4,2X
     .,F5.2,2X,F5.2,2X,F5.2,2X,E10.4,2X,E10.4,2X,E10.4,2X)
      DELP=DELP+1.
  760 CONTINUE
      GOTO21
```

```
   99 AD=180.*A/PI
C     FIND ANGLE OF STEEPEST ASCENT UP PLANE (THE NORMAL TO THE BEACH),
C     AND ACUTE ANGLE WITH WAVE RAY.
      PS=ATAN2(BN,AN)*180./PI
      IF(CN.GT..0)PS=PS+180.
      IF(PS.LT.0.)PS=PS+360.
      PZ=PS-AD
      PZZ=PZ*PI/90.
      SVOL=15330.0*CG*SIN(PZZ)*HRMS**2
      XS=XSN*GS
      YS=YSN*GS
      XR=XI*GS
      YR=YI*GS
C     PRINT LONGSHORE TRANSPORT RATE, ETC., FOR THIS RAY.
      PRINT18,W1,XR,YR,A2,HRMSI,XS,YS,AD,HRMS,DD,CG,PZ,SVOL
   18 FORMAT(1H ,5X,F6.4,2X,E10.4,1X,E10.4,1X,F5.1,2X,F7.4,5X,E10.4,1X,E
     110.4,2X,F5.1,3X,F7.4,4X,F5.2,3X,F5.2,4X,F6.1,2X,E10.4)
C     NEXT WAVE RAY
  110 YI=YI+DYI
      IF((YF-YI).LT.0.)GOTO470
      N=0
  820 N=N+1
      IF((MM-M).LT.0)GOTO430
      N1=N
      IF(YI.LE.N1)GOTO830
      GOTO820
  830 M=0
  112 M=M+1
      IF((NN-N).LT.0)GOTO430
      M1=M
      IF(XI.LE.M1)GOTO114
      GOTO112
  114 A=A2*PI/180.
      L=2.*G*PI/W**2
      CP=CPI
      CG=CGI
      HRMS=HRMSI
      GOTO19
  430 AD=A*180./PI
      PRINT76,W1,XI,YI,XSN,YSN,AD,HRMS,DD,CP,CG,BET
   76 FORMAT(1H ,6X,F5.3,2X,F6.3,5X,F6.3,4X,20H  OUTSIDE ARRAY AT   ,
     .F6.3,3X,F6.3,8X,F5.1,3X,F7.4,4X,F5.2,3X,F5.2,5X,F5.2,2X,E10.4)
      GOTO110
  470 CONTINUE
      GOTO490
  450 PRINT78
   78 FORMAT(1H-,10X,11HEND OF FILE)
  490 CONTINUE
      STOP
      END
```

Appendix B

The Dobson wave refraction programme was written primarily to plot wave rays. It is well documented by Dobson (1967). The version that is available at the CCIW has been modified from the original, but is not clearly documented. The programme has been amended to allow more than one depth array so that, for example, a nearshore situation can be examined using an expanded scale. Since this programme may be more useful for some problems than the one described in this report, it is included in this appendix. The documentation on the modifications is reproduced below, followed by some notes on the format for the input data. A complete listing is given at the end of the appendix.

Modification to the Dobson Wave Refraction Program

Dobson, R.S. 1967 Some Applications of a Digital Computer to Hydraulics Engineering Problems. Dept. of Civil Engineering, Stanford University, Stanford, California, Tech. Rept. No. 80.

PROGRAM:

This outline is not the be all and end all for wave refraction. It is assumed that the reader knows the theory of wave refraction and is familiar with wave refraction diagrams. For a review and references see the MSc. thesis of E. A. Bryant, Dept. of Geography, McMaster University, 1972.

The program calculates refraction of waves giving co-ordinates and angle of a select point on a wave crest as it moves through a grid of depth values. It also calculates the depth of water for each point, the maximum error of this depth for each point, the standard deviation of the least squares surface in calculating this depth. The wave length, speed, height, and refraction and shoaling coefficients for each point are also calculated. It also produces a rough printer plot of the results.

MODIFICATIONS:

(1) Corrected by Dave Ingram to fit the CDC 6400.

(2) Modified from modifications, by Professor G. F. G. Ratzer, McGill University Computing Center, to the Wilson program. Modification allows for a small grid to be incorporated into the program and the coordinates and angle of the wave are calculated and printed.

(3) Modified by Ted Bryant to take the coordinates and angle of the wave from above, along with Beta values calculated for these points in the HEIGHT subroutine, so that the orthogonal can be run through the smaller grid after the orthogonal has been put through the larger grid. Any number of grids can be put in. The limitation being a maximum of 20 points or orthogonals. For this modification NOSETS has been modified to be the number of grids in the program. This becomes the first data card.

(4) The limitation on the output for calculations near the boundary of the grid has been modified slightly by changing

```
CARD 198* from 22 IF(X.GE.RHS.OR.X.LE.1.5) GO TO 23
                            to
               22 IF(X.GE.RHS.OR.X.LE.1.1)GO TO 23
* Line 70, Subroutine REFRAC

CARD 199* from    IF(Y.GE.TOP.OR.Y.LE.1.5) GO TO 23
                            to
                  IF(Y.GE.TOP.OR.Y.LE.1.1) GO TO 23
* Line 71, Subroutine REFRAC
```

CARD 69* from RHS=RHS-1.5
to
RHS=RHS-1.99

* Line 99, Program DOBW

CARD 71* from TOP=TOP-1.5
to
TOP=TOP-1.99

*Line 101, Program DOBW

(5) The program has been modified to incorporate storm surge and tide compensation for a grid. This modification is only good where the storm surge and tide are of equal height across the grid.

(6) The program has been modified to produce the point at which a wave is breaking and the wave energy at the breaking point in lb/ft of wave crest/wave length. The former is the point at which water depth to wave height equals 1.28. The equation of wave energy is from Munk and Traylor 1947 Refraction of Ocean Waves. Jour. Geol. v. 55 pg. 24, modified from CERC Tech. No. 4 pg. 2 and 3, 1966.

TESTING:

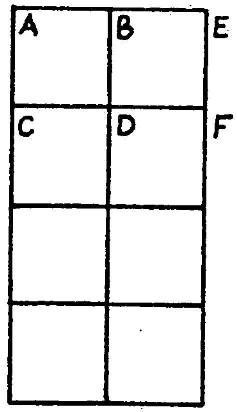
Part 1 and 4 of the modifications has been run against the original data in the Dobson publication and the results were found to be within rounding error of the computer. Part 2 and 3 have been tested by rerunning the calculated wave co-ordinates, angle and Beta values, through the same grid as the original wave was run through. The results were identical between the two calculations.

QUESTIONABLE PARTS:

CARD 318 in the ERROR subroutine of the original program had IF (NPT.LE.3). This statement may generate a mode four error for it appears that the number in this line must equal the value of NPRINT inputted into the program. The logic behind this card lies with the original programmer. No modification was attempted. No error resulted in the printed output by setting the value in the statement equal to NPRINT.

GRID SETUP:

The program is based on a rectangular grid of depth values. The grid was positioned over the beach so that the shoreline in question was at least two grid points away from the edge of the grid.



The grid unit represented by ABCD must be square. This grid represents a 3 x 5 grid. The coordinate of a point on a wave crest to be run through the grid must be at least one grid unit from the border of the grid to enable interpolation of depth values. Thus it is legal to give D as the coordinate of a point but not A, B, C, E, or F.

N.B. In the grid of depth values, point D has coordinates (2,4) since the grid is a matrix, and D is represented by matrix coordinates. However, when inputting the coordinates of D as wave inputs use Cartesian coordinates. Thus D is inputted as (1,3) if it was a point on the wave crest from which refraction was to be calculated. The program converts this to matrix coordinates. The output comes out as cartesian coordinates. Land values are negative and calculated the same as depth values for the sea, by interpolation onto the grid intersects. The grid should be detailed enough to allow for most detail on a bathymetric chart.

VARIABLE	COLUMN	FORMAT	DESCRIPTION
Card 1			
NOSETS	1-5	integer	The number of grids to be run.
Card 2			
MI, MJ	1-5 6-10	integer	Must not exceed 100. The x and y dimensions respectively of the grid of depth values in grid units. For example illustrated above MI 3,MJ=5
IGRON	11-15	integer	Identifies whether the grid units are in feet, miles or meters. 1 = feet 2 = miles 3 = meters
LIMNPT	16-20	integer	Gives the maximum number of points to be calculated for each ray. Use 2000.
NPRINT	21-25	integer	Gives the frequency of printed output for each orthogonal. Usually 10 is suitable. N.B. Questionable Parts.
GRID	26-35	floating point	The width of one grid unit in feet, miles or meters to agree with the value of IGRCON.

VARIABLE	COLUMN	FORMAT	DESCRIPTION
DCON	36-45	floating point	The conversion factor to convert depth values, used in the grid, to feet.
DELTAS	46-55	floating point	The minimum step, in grid units, for which refraction is to be calculated in shallow water. Use .005.
GRINC	56-65	floating point	The minimum step, in grid units, for which refraction is to be calculated in deep water. Use .05.
ZA	66-69	floating point	Controls whether or not a smaller grid or a grid card for a smaller grid is to be used in the program. ZA ≠ 1.0 just the one grid ZA = 1.0 at least one more grid in the program or wave data is to be calculated for a wave entering smaller grid. If just the latter with no execution of wave orthogonal in a smaller grid, set NOSETS = 1.
ZP	70-73	floating point	The interval in grid units for which extra rays will be put on the smaller grids. IF ZA ≠ 1.0, ZP set at 0.0 IF ZA = 1.0, ZP is set in grid units and will allow the program to calculate extra rays on each side of the ray coming into the second grid of the program. Since rays are usually diverged as refracted into a concave shoreline then few rays will enter a small grid. ZP allows more rays to be put in so that computer time may be saved by not having to run the program a second time with added rays as input to get the refraction pattern in the smaller grid. This step is subject to some error since the angle is kept the same as the main ray. If ZP is kept reasonable the error will be minimal. The method is not the best. A better one would be to calculate extra rays half way between two main ones. The programming for this becomes very involved and has limitations.
MXX	80	integer	Set equal to 1 for plot.

VARIABLE	COLUMN	FORMAT	DESCRIPTION
Card 3			
FMT	1-12	alpha	Put in the format in which the depth values of the grid are to be read. Example: (16F10.3). This allows a varied format without changing the program.
Card 4+x			
DEP(I,J)		FMT	The depth values for the grid. The total number must equal MIxMJ. The values start at the lower left hand corner of the grid. All x values are read in before any change in a y value.
Card 5+x			
TITL	1-72	alpha	The title of the beach or program.
Card 6 +x			
NORAYS	1-3	integer	The number of rays or orthogonals or points on the wave crest to be run.
T	4-11	floating point	The wave period in seconds.
HO	12-17	floating point	The wave height in feet.
Card 7+x			
SURGE	1-10	floating point	The tide or surge height in units to agree with depth values in the grid.
Card 8+x			
SCALE	1-10	floating point	Scale is given as numerator of ratio I:X. Omit this card if MXX$\neq$1. E.g. Scale: 1:4204, the real scale is 1"=4204 ft x 12 $\frac{in}{ft}$ = 50448" or $\simeq$ 1:50000
Card 9+x	(Only if ZA = 1.0 otherwise skip)		
X1, X2	1-10, 11-20	floating point	The minimum and maximum x coordinates of the small grid on the larger one.
Y1, Y2	21-30 31-40	floating point	Minimum and maximum y coordinates of the small grid on the larger one.
XSC	41-50	floating point	The dimension of the small grid in the X direction minus one grid unit.
YSC	51-60	floating point	The dimesion of the small grid in the Y direction minus one grid unit.

VARIABLE	COLUMN	FORMAT	DESCRIPTION
Card 10+x	(one card for each point)		
X	1-7	floating point	X coordinate of the original point on the ray.
Y	8-14	floating point	Y coordiante of the original point on the ray. Both X and Y must be 2 grid unit from boundary.
A	15-21	floating point	The angle that the wave is travelling, measured negatively clockwise from positive X in degrees. E.g. If positive x-axis is east, then the angle of a ray travelling south is - 90°.

If a grid card has been used and a second grid of values is to be worked out, then repeat cards 2 to 4+x.

OUTPUT:

POINT	gives the point on the ray for which data is printed out.
X,Y	are the x and y coordinates of this point.
ANGLE	is the angle from positive X that the wave is travelling.
DEPTH	is the calculated depth of water of this point.
MAX DIF	is the maximum error in DEPTH as a percent.
FIT	is the standard deviation of the least squares surface.
LENGTH	is the length of the wave at the point.
SPEED	is the wave velocity (ft/sec).
HEIGHT	is the wave height in feet at the point.
KR	is the refraction coefficient.
KS	is the shoaling coefficient.

Input Data Format for the Dobson Programme

Card 1.	NOSETS (line 78, PROGRAM DOBW) FORMAT (I5)	
Card 2.	MI, MJ, IGRCON, LIMNPT, NPRINT, GRID, DCON, DELTAS, GRINC, ZA, ZP, MXX (line83, PROGRAM DOBW) FORMAT (5I5, 4F10.5, 2F4.1, 6X, I1)	
Card 3.	FMT (line 94, PROGRAM DOBW) FORMAT (6A6)	
Cards 4+x	DEP (I,J) (line 96, PROGRAM DOBW) FORMAT (FMT)	
Card 5+x	TITL (line 127, PROGRAM DOBW) FORMAT (8A10)	
Card 6+x	NORAYS, T, HO (line 131, PROGRAM DOBW) FORMAT (I3, F8.2, F6.2)	
Card 7+x	SURGE (line 132, PROGRAM DOBW) FORMAT (F10.3)	
Card 8+x	SCALE (line 10,SUBROUTINE LABEL) FORMAT (F10.3)	
Card 9+x	X1, X2, Y1, Y2, XSC, YSC (line 147, PROGRAM DOBW) FORMAT (8F10.3)	
Card 10+x	ZX(NORAY), ZY(NORAY), AQ(NORAY) (line 218, PROGRAM DOBW) (1 card for each ray) FORMAT (3F7.2)	

(The line numbers refer to the locations of the READ statements in the programme or subroutine.)

```
            PROGRAM DOBW
      C     DOBW
      C     BY R.S.DOBSON
      C     A PROGRAMME TO CONSTRUCT REFRACTION DIAGRAMS AND COMPUTE WAVE
      C     HEIGHTS FOR WAVES MOVING INTO SHOALING WATER.
      C     STANFORD UNIVERSITY.
      C     MARCH, 1967.
      C     CORRECTED FOR THE IBM 7040 COMPUTER BY DAVE INGRAM-1970.
      C     INPUT PARAMETERS.
      C     MI          = MAX. VALUE FOR I SUBSCRIPT, NOT TO EXCEED 100.
      C     MJ          = MAX. VALUE FOR J SUBSCRIPT, NOT TO EXCEED 100.
      C     IGRCON      = GRID UNIT IDENTIFER, 1 = FEET, 2 = MILES. 3 = METRES.
      C     LIMNPT = MAX. NUMBER OF RAY COMPUTATION POINTS.
      C     NPRINT = FREQUENCY OF PRINTED OUTPUT FOR EACH RAY.
      C     ALL IN FORMAT I5.
      C     GRID        = NUMBER OF GRID UNITS PER GRID DIVISION.
      C     DCON        = MULTIPLIER TO CONVERT DEPTH UNITS TO FEET.
      C     DELTAS      = MINIMUM STEP LENGTH ALONG RAY IN SHALLOW WATER.
      C     GRINC       = STEP LENGTH ALONG RAY IN DEEP WATER.
      C     ALL IN FORMAT F10.5.
      C     FMT         = FORMAT FOR DEPTH DATA.  EG. = (10F8.3).  (2A6).
      C     DEP(I,J) = DEPTH AT GRID POINTS.
      C     NOSETS      =NO OF GRIDS TO BE RUN.  (I3).
      C     TITL        = IDENTIFING TITLE FOR EACH SET.  (12A6).
      C     NORAYS      = NO. RAYS IN EACH SET. (I3).
      C     T           = WAVE PERIOD. SECS.  (F8.2).
      C     H           = DEEP WATER WAVE HEIGHT.  (F6.2).
      C     X, Y        = STARTING COORDINATES.  (F7.2).
      C     A           = INITIAL DIRECTION OF RAY.  (F7.2).
      C     ZP IS THE INTERVAL IN GRID UNITS FOR WHICH EXTRA RAYS WILL BE
      C                             PUT ON THE SMALLER GRIDS.
      C     IF ZA IS NOT EQUAL TO ONE THAN NO GRID CARD IS NEEDED
      C     IF ZA EQUALS ONE THAN A GRID CARD WILL BE READ IN.
      C     IF A PEN PLOT IS REQUIRED SET MXX EQUAL TO 1
      C     IF MXX EQUALS 1 THEN A SCALE CARD MUST BE INSERTED BEFORE
      C        THE RAY DATA.
      C     IF MXX=1 AND NOSETS ≠ 1 THEN THE PROGRAM WILL NOT WORK.
      C     THE PROGRAM CAN ONLY PLOT ONE DIAGRAM.
      C
      C                                EXPLANATION OF THE GRID CARD
      C     THE GRID CARD IS PLACED BEFORE THE RAY CARDS.
      C     X1 AND X2 ARE THE X COORDINATES OF THE SMALLER GRID ON THE
      C                                LARGER GRID.
      C     Y1 AND Y2 ARE THE Y COORDINATES OF THE SMALLER GRID ON
      C                                    THE LARGER GRID.
      C     XSC IS THE MI VALUE OF THE SMALLER GRID MINUS ONE GRID UNIT.
      C     YSC IS THE MJ VALUE OF THE SMALLER GRID MINUS ONE GRID UNIT.
      C     BOTH OF THE LATTER TWO VALUES ARE IN DECIMALS.
      C     IF RAYS ARE TO BE RUN THROUGH A SMALLER GRID, NOSETS MUST BE
      C             GREATER THAN 1.
      C     AFTER THE RAYS IN THE DATA DECK PUT IN THE PLOT CARD FOR THE
      C     SMALLER GRID, FOLLOWED BY THE DEPTH VALUES FOR THE SMALLER GRID.
```

```
C        BE CAREFUL BECAUSE THE PLOT CARD FOR THE SMALLER GRIDS WILL BE
C        DIFFERENT FROM THE ORIGINAL PLOT CARD.
C        IF MORE THAN ONE EXTRA GRID IS USED THEN FOLLOW THE SAME
C                     INSTRUCTION FOR THE GRID CARD.
C        CARD 116 REQUIRES THAT A VALUE BE READ IN TO CORRESPOND TO A
C             TIDE  AND/OR A STORM SURGE FOR THE DEPTH GRID.
C
         COMMON /GR/ GX1,GX2,GY1,GY2,XSC,YSC,X1,X2,Y1,Y2
         COMMON /GRID2/ PANGLE,ZX,ZY,ZB1,ZB2
         COMMON/1/DEP(100,100),D(12),E(6),B1,B2,CO,CXY,DCDH,DCON,DELTAS,DRC
        1,DTGR,DXY,GRINC,HO,IGO,JGO,LIMNPT,NPRINT,NPT,PHX,PHY,RCOO,RHS,RK,
        2SIG,SK,TOP,V,WL,WLO,LL,NGR,JGR,T,MXX,SCAL,MXY
         DIMENSION TITL( 8),FMT(6),ZFEET(90),ZFEET2(90),ZX(90),ZY(90),
        1PANGLE(90),AQ(90),ZB1(90),ZB2(90)
         DATA IFEET,IMILES,IMETRE/4HFEET,5HMILES,6HMETRES/
C
         ZZ=0.0
         LL=0
         WRITE(6,60)
      60 FORMAT(1H1,9X,41HWAVE REFRACTION BY METHOD OF R.S. DOBSON,/1H0,9X,
        150HCIVIL ENGINEERING DEPARTMENT, STANFORD UNIVERSITY./)
         WRITE(6,54)
      54 FORMAT(1H ,9X,63HBENSON-LENHER PLOTTING ADDED AND PROGRAM MODIFIED
        1 BY TED BRYANT,//,9X,41HGEOGRAPHY DEPARTMENT, MCMASTER UNIVERSITY/
        2//)
         READ(5,56) NOSETS
      56 FORMAT(I5)
         DO 111 NOSET=1,NOSETS
C            READ BASIC DATA

         READ(5,51) MI,MJ,IGRCON,LIMNPT,NPRINT,GRID,DCON,DELTAS,GRINC,ZA,ZF
        1,MXX
      51 FORMAT(5I5,4F10.5,2F4.1,6X,I1)
         IF(MXX.EQ.1.AND.NOSETS.NE.1)WRITE(6,65)
         IF(MXX.EQ.1.AND.NOSETS.NE.1)STOP
      65 FORMAT(1H ,43HPROGRAM CAN NOT PLOT MORE THAN ONE DIAGRAM./,1X,
        1 60HANY ERROR IN THE SECOND SET OF DATA WOULD KILL ALL PLOTTING./
        2 1X,94HTHE CHANCE OF ERROR AND THE COST INVOLVED DO NOT MAKE MORE
        3THAN ONE PLOT PER PROGRAM FEASIBLE./1X,16HPROGRAM STOPPED.)
         IF(ZA.NE.1.) NGR=2
         IF (MI .GT. 100 .OR. MJ .GT. 100) GO TO 10
         READ(5, 52) FMT
      52 FORMAT(6A6)
         READ(5,FMT)((DEP(I,J),I=1,MI),J=1,MJ)
         WRITE(6,FMT)((DEP(I,J),I=1,MI),J=1,MJ)
         RHS = MI
         RHS=RHS-1.99
         TOP = MJ
         TOP=TOP-1.99
         UNIT = GRID
         GO TO (16,17,18), IGRCON
      16 IGRCON = IFEET
```

```
      GO TO 19
   17 GRID = GRID*6080.21
      IGRCON = IMILES
      GO TO 19
   18 GRID = GRID*3.281
      IGRCON = IMETRE
   19 CONTINUE
      WRITE(6,63) MI,MJ,NPRINT,UNIT,IGRCON,GRINC,DCON
   63 FORMAT(1H0,9X,21HPROGRAMME PARAMETERS,//25H  GRID LIMITS, ABSCISSA
     1 =,I4,12H, ORDINATE =,I4,1H,/27H  PRINTED OUTPUT INTERVAL =,I4,8H
     2POINTS./19H  GRID SIZE, UNIT =,F9.4,1X,A6,1H./31H  DEEP WATER INCR
     3EMENTAL STEP =,F7.3,12H GRID UNITS./49H  DEPTH CONVERSION, DEP(I,J
     +) TO FEET, MULTIPLY BY,F6.3)
      IF(LL.GE.1) GO TO 25
      GO TO 26
   25 DO 50 J=1,LL
      ZFEET(J)=(ZX(J)-X1)*GRID
      ZFEET2(J)=(ZY(J)-Y1)*GRID
      AQ(J)=PANGLE(J)
   50 CONTINUE
   26 IF(LL.GE.1) GO TO 466
C          READ WAVE DATA
      READ(5,57) TITL
   57 FORMAT(8A10)
  466 IF(LL.GE.1) GO TO 2
      LL=0
      READ(5,58) NORAYS, T, HO
      READ(5,41) SURGE
41    FORMAT(F10.3)
   58 FORMAT(I3,F8.2,F6.2)
      IF(MXX.EQ.1) CALL LABEL(NORAYS,T,HO,SURGE,TITL,MI,MJ,GRID,SCAL)
      DO 40 I=1,MI
      DO 40 J=1,MJ
      IF(DEP(I,J).LT.0.)DEP(I,J)=DEP(I,J)-(3.0/DCON)
      DEP(I,J)=DEP(I,J)+(SURGE/DCON)
   40 CONTINUE
      WRITE(6,478)SURGE
  478 FORMAT(1H ,34HSTORM SURGE AND HIGH TIDE VALUE IS,1X,F10.3)
      IF(MXX.EQ.1)WRITE(6,68)
   68 FORMAT(1H ,59HWAVE REFRACTION DIAGRAM WILL BE PLOTTED IF INPUT IS
     1CORRECT)
    2 IF(ZA.NE.1.) GO TO 1
      READ(5,105) X1,X2,Y1,Y2,XSC,YSC
  105 FORMAT(8F10.3)
      IF(XSC.EQ.0.0) GO TO 1
      DELTA=(X2-X1)/XSC
      WRITE(6,106)X1,X2,Y1,Y2,DELTA,XSC,YSC
  106 FORMAT(1H0, 22HGRID CARD VALUES  X,S=,2F10.2,3X,4HY,S=,2F10.2,
     1 3X,6HDELTA=,1F10.2,3X,13HX - Y SCALE =,2F10.2//)
      GX1=X1+DELTA
      GX2=X2-DELTA
      GY1=Y1+DELTA
```

```
      GY2=Y2-DELTA
    1 IF(LL.GE.1) GO TO 69
      SIG=6.28318531/T
      CO = 5.1204062*T
      WLO = CO*T
      DRC = WLO*0.6
      DTGR = GRINC/CO
      UNIT = DTGR*GRID
   69 IF(LL.GE.1) GO TO 22
      GO TO 499
   22 NORAYS = LL
      ZZ=1.
      LL=0
      DO 20 NORAY = 1, NORAYS
      X = ZFEET(NORAY)/GRID
      Y = ZFEET2(NORAY)/GRID
      A = AQ(NORAY)
      QX1 = X+ZP
      QY1 = Y
      QX2=X-ZP
      QY2 = Y
      Z = 1.
      FMM = MI
      FMM = FMM-1.
      GO TO 23
   23 NPT = 1
      JGR = 2
      CXY = CO
      WL = WLO
      B1 = ZB1(NORAY)
      B2 = ZB2(NORAY)
      RK = 1.
      SK = 1.
      WRITE(6,24)       NOSET,T,NORAY,UNIT,NPT,X,Y,A
   24 FORMAT(1H1,    /8H SET NO.,I3,10H, PERIOD =,F7.2,7H SECS.,,8H RAY
     1NO.,I3,13H, TIME STEP =,F8.4,6H SECS.//1H ,3X,5HPOINT,5X,1HX,8X,1H
     2Y,6X,5HANGLE,5X,5HDEPTH,4X,7HMAX DIF,4X,3HFIT,5X,6HLENGTH,4X,5HSPE
     3ED,5X,6HHEIGHT,5X,2HKR,8X,2HKS,//1H ,I7,3F9.2)
      CALL RAYCON(X,Y,A)
      IF(ZP.EQ.0.0) GO TO 20
      IF(Z.EQ.1.) GO TO 14
      IF(Z.EQ.2.) GO TO 31
      IF(Z.GT.2.) GO TO 20
   14 X=QX1
      IF(X.GE.FMM) GO TO 20
      Y=QY1
      A =AQ(NORAY)
      Z = Z+1.
      GO TO 23
   31 X=QX2
      IF(X.GE.FMM) GO TO 20
      Y=QY2
```

```
      A = AO(NORAY)
      Z=Z+1.
       GO TO 23
   20 CONTINUE
C
C     TEST RAY DATA TO SEE IF IT IS PUNCHED CORRECTLY.
C
  499 DO 112 NORAY=1,NORAYS
      IF(ZZ.EQ.1.) GO TO 500
      READ(5,59)ZX(NORAY),ZY(NORAY),AQ(NORAY)
   59 FORMAT(3F7.2)
      X=ZX(NORAY)
      Y=ZY(NORAY)
      IF(X.GE.RHS.OR.X.LE.1.1.OR.Y.GE.TOP.OR.Y.LE.1.1)WRITE(6,70)NORAY
      IF(X.GE.RHS.OR.X.LE.1.1.OR.Y.GE.TOP.OR.Y.LE.1.1)STOP 111
   70 FORMAT(1H ,30X,10H**********,/,30X,29HRAY DATA LOOKS FISHY, RAY NO
     1.,I5,/,30X,10H**********)
  112 CONTINUE
      WRITE(6,71)
   71 FORMAT(1H ,19HRAY DATA READS O.K.)
  500 DO 110 NORAY =1,NORAYS
      IF(ZZ.EQ.1.0) GO TO 110
      IF(ZA.NE.1.) GX1=0.0
      IF(ZA.NE.1.) GY1=0.0
      MXY=1
      X=ZX(NORAY)
      Y=ZY(NORAY)
      A=AQ(NORAY)
      IF(X.EQ.0.0) STOP
      IF(MXX.EQ.1)CALL GRAPH(X,Y,SCAL,MXY)
      MXY=2
      NPT = 1
      JGR = 2
      CXY = CO
      WL = WLO
      B1 = 1.
      B2 = 1.
      RK = 1.
      SK = 1.
      WRITE(6,61) TITL,NOSET,T,NORAY,UNIT,NPT,X,Y,A,WLO,CO,HO
   61 FORMAT(1H1,8A10/8H SET NO.,I3,10H, PERIOD =,F7.2,7H SECS.,,8H RAY
     1NO.,I3,13H, TIME STEP =,F8.4,6H SECS.//1H ,3X,5HPOINT,5X,1HX,8X,1H
     2Y,6X,5HANGLE,5X,5HDEPTH,4X,7HMAX DIF,4X,3HFIT,5X,6HLENGTH,4X,5HSP
     3ED,5X,6HHEIGHT,5X,2HKR,8X,2HKS,//1H ,I7,3F9.2,29X,3F10.2)
      CALL RAYCON(X,Y,A)
  110 CONTINUE
  111 CONTINUE
      WRITE(6,62) NOSETS
   62 FORMAT(39H0 ALL SETS COMPLETED,  NUMBER OF SETS =,I4)
      IF(MXX.EQ.1) CALL FINIS(MI,SCAL)
      IF(MXX.EQ.1)WRITE(6,53)
   53 FORMAT(1H ,32HWAVE REFRACTION DIAGRAM PLOTTED.)
```

```
      GO TO 12
   10 WRITE(6,64) MI,MJ
   64 FORMAT(1H0,62HPROGRAMME STOPPED, MI OR MJ GREATER THAN 100 NOT AL
     1OWED, MI =,I4,7H,  MJ =,I4)
   12 STOP
      END
```

```
C     RAYCOT DECK
      SUBROUTINE RAYCON(X,Y,A)
      COMMON/1/DEP(100,100),D(12),E(6),B1,B2,CO,CXY,DCDH,DCON,DELTAS,DRC
     1,DTGR,DXY,GRINC,HO,IGO,JGO,LIMNPT,NPRINT,NPT,PHX,PHY,RCCO,RHS,RK,
     2SIG,SK,TOP,V,WL,WLO,LL,NGR,JGR,T,MXX,SCAL,MXY
      COMMON/COMA/XP,YP
      ANG = A
      TED=0.
      A = A*0.0174532925
      COSA = COS(A)
      SINA = SIN(A)
      H = HO
      IGO = 1
   10 PX = X
      PY = Y
      X = COSA*GRINC+X
      Y = SINA*GRINC+Y
      CALL DEPTH(X,Y)
      NWRITE = 1
      IF (DXY .LE. 0.) GO TO 24
      IF (DXY .LT. DRC) GO TO 11
      NPT = NPT + 1
      IF (NPT .GT. LIMNPT) GO TO 26
      IF(X.GE.RHS.OR.X.LE.1.5) GO TO 31
      IF(Y.GE.TOP.OR.Y.LE.1.5) GO TO 31
      IF(NPT/NPRINT*NPRINT-NPT.NE.0) GO TO 10
      GO TO 21
   31 NWRITE=5
      GO TO 21
   11 X = PX
      Y = PY
      CALL CURVE(X,Y,A,FK)
   12 NPT = NPT+1
      IF (NPT .GT. LIMNPT) GO TO 26
      NWRITE = 1
      CALL REFRAC(X,Y,A,FK,I15)
      GO TO (18,22,23,24,25,27),I15
   22 NWRITE = 2
      GO TO 18
   23 NWRITE = 3
      GO TO 18
   24 NWRITE = 4
      GO TO 13
   25 NWRITE = 5
      GO TO 13
   26 NWRITE = 6
      GO TO 13
   27 NWRITE = 7
      GO TO 13
   18 CALL HEIGHT(XP,YP,A,H)
      IF(TED.EQ.1.) GO TO 3
      IF(DXY/H.GT.1.28) GO TO 3
```

```
      TED=1.
      ANG=A*57.29577951
      WRITE(6,4)X,Y,ANG,DXY,WL,CXY,H
    4 FORMAT(1H0,16HWAVE IS BREAKING/6X,3F9.2,F11.2,18X,3F10.2)
      ENERGY = (64.0/WL)*(DXY*H*4./3.)**1.5
      ENERGY = (64.0/WL)*(DXY*H*4./3.)**1.5
      WRITE(6,5) ENERGY
    5 FORMAT(1H ,30HWAVE ENERGY AT BREAKING POINT=,F10.2,1X,
     1   31HLB/FT OF WAVE CREST/WAVE LENGTH/)
    3 IF(NWRITE.GT.1) GO TO 13
      IF(MXX.EQ.1) CALL GRAPH(X,Y,SCAL,MXY)
      IF (NPT/NPRINT*NPRINT-NPT .NE. 0) GO TO 12
   13 ANG = A*57.29577951
   21 CALL WRITER(X,Y,ANG,H,NWRITE)
   14 GO TO (10,12,19), IGO
   19 RETURN
      END
```

USASI FORTRAN DIAGNOSTIC RESULTS FOR RAYCON

NO ERRORS

FOLLOWING ARE COMMON BLOCK NAMES OR NAMES NOT ASSIGNED STORAGE

OMA

UNREFERENCED STATEMENT LABELS

0014

```
C     REFHAT DECK
      SUBROUTINE REFRAC(X,Y,A,FK,I15)
      COMMON/1/DEP(100,100),D(12),E(6),B1,B2,CO,CXY,DCDH,DCON,DELTAS,DRC
     1,DTGR,DXY,GRINC,HO,IGO,JGO,LIMNPT,NPRINT,NPT,PHX,PHY,RCCO,RHS,RK,
     2SIG,SK,TOP,V,WL,WLO,LL,NGR,JGR,T,MXX,SCAL,MXY
      COMMON /GR/ GX1,GX2,GY1,GY2,XSC,YSC,X1,X2,Y1,Y2
      COMMON /GRID2/ PANGLE,ZX,ZY,ZB1,ZB2
      DIMENSION PANGLE(90),ZX(90),ZY(90),ZB1(90),ZB2(90)
      IF(GX1.EQ.0.0.AND.GY1.EQ.0.0) NGR=2
      NCUR = 1
      IF(JGR.EQ.1) GO TO 200
      IF(NGR.EQ.2) GO TO 200
      IF(X.GE.GX1.AND.X.LE.GX2.AND.
     1 Y.GE.GY1.AND.Y.LE.GY2) GO TO 201
      GO TO 200
  201 JGR=1
      LL=LL+1
      WRITE(6,202)
  202 FORMAT(1H0,26HRAY HAS ENTERED SMALL GRID)
      ZX(LL)=X
      ZY(LL)=Y
      SX=((X-X1)/(X2-X1))*(XSC+1.)
      SY=((Y-Y1)/(Y2-Y1))*(YSC+1.)
      PANGLE(LL)=A*57.29577951
      HI=HO+RK*SK
      ZB1(LL) = B1
      ZB2(LL) = B2
      WRITE(6,203) PANGLE(LL),ZX(LL),ZY(LL),HI
  203 FORMAT(1H , 50X, 4F9.2)
      WRITE(6,250)SX,SY
  250 FORMAT(1H ,59X,2F9.2)
  200 CONTINUE
      GO TO (11,12,10), IGO
   11 FKM = FK
      IGO = 2
   12 DS = CXY*DTGR
      IF (DS .GE. DELTAS) GO TO 20
      I15 = 6
      RETURN
   20 RESMAX = 0.00005/DS
   13 DO 110 I=1,20
      DELA = FKM*DS
      AA = A+DELA
      AM = DELA*0.5+A
      XX = COS(AM)*DS+X
      YY = SIN(AM)*DS+Y
      CALL CURVE(XX,YY,AA,FKK)
      IF (DXY .GT. 0.) GO TO 21
      I15 = 4
      RETURN
   21 GO TO (111,16), NCUR
  111 FKM = (FK+FKK)*0.5
```

SI FORTRAN(2.3)/MASTER   INTEGER WORD SIZE = 1 , * OPTION IS OFF , 0 OPTION IS

```
            IF (I .EQ. 1) GO TO 110
            IF (RESMAX .GT. ABS(FKP-FKM)) GO TO 16
            IF (I .EQ. 18) FK18 = FKM
        110 FKP = FKM
            IF (RESMAX .GT. ABS(FK18-FKM)) GO TO 15
            I15 = 3
            RETURN
         15 FKM = (FKM+FK18)*0.5
            NCUR = 2
            GO TO 13
         16 X = XX
            Y = YY
            A = AA
            FK = FKK
            IF (NCUR .NE. 2) GO TO 22
            I15 = 2
            RETURN
         22 IF(X.GE.RHS.OR.X.LE.1.1) GO TO 23
            IF(Y.GE.TOP.OR.Y.LE.1.1) GO TO 23
         10 I15 = 1
            RETURN
         23 I15 = 5
            RETURN
            END
```

USASI FORTRAN DIAGNOSTIC RESULTS FOR REFRAC

NO ERRORS

FOLLOWING ARE COMMON BLOCK NAMES OR NAMES NOT ASSIGNED STORAGE

R   GRID2

```
C     CURVER DECK
      SUBROUTINE CURVE(X,Y,A,FK)
      COMMON/1/DEP(100,100),D(12),E(6),B1,B2,CO,CXY,DCDH,DCON,DELTAS,DRC
     1,DIGR,DXY,GRINC,HO,IGO,JGO,LIMNPT,NPRINT,NPT,PHX,PHY,RCCO,RHS,RK,
     2SIG,SK,TOP,V,WL,WLO,LL,NGR,JGR,T,MXX,SCAL,MXY
      COMMON/COMA/XP,YP
      GO TO (10,11), IGO
   11 CALL DEPTH(X,Y)
      IF (DXY*200. .GT. WL) GO TO 10
      IF (DXY .LE. 0.) RETURN
      JGO = 2
      ARG = 32.1725*DXY
      CXY = SQRT(ARG)
      DCDH = 16.08625/CXY
      GO TO 14
   10 CI = CXY
      JGO = 1
      DO 120 I=1,50
      ARG = (DXY*SIG)/CI
      CXY = CO*TANH(ARG)
      RESID = CXY-CI
      IF (ABS(RESID) .LT. 0.0001) GO TO 13
  120 CI = (CXY+CI)*0.5
   13 RCCO = CXY/CO
      SCMC = (1.-RCCO*RCCO)*SIG
      V = SCMC*DXY+RCCO*CXY
      DCDH = CXY*SCMC/V
   14 PHX = E(4)*2.*XP+E(5)*YP+E(2)
      PHY = E(6)*2.*YP+E(5)*XP+E(3)
      FK = (SIN(A)*PHX-COS(A)*PHY)*DCDH*DCON/CXY
      RETURN
      END
```

USASI FORTRAN DIAGNOSTIC RESULTS FOR CURVE

NO ERRORS

FOLLOWING ARE COMMON BLOCK NAMES OR NAMES NOT ASSIGNED STORAGE

OMA

```
C       DEPTHI DECK
        SUBROUTINE DEPTH(X,Y)
        COMMON/1/DEP(100,100),D(12),E(6),B1,B2,CO,CXY,DCDH,DCON,DELTAS,DR
       1,DTGR,DXY,GRINC,HO,IGO,JGO,LIMNPT,NPRINT,NPT,PHX,PHY,RCCO,RHS,RK,
       2SIG,SK,TOP,V,WL,WLO,LL,NGR,JGR,T,MXX,SCAL,MXY
        COMMON/COMA/XP,YP
        DIMENSION SXY(72)
        DATA (SXY(J),           J=1,72)/0.30861241,0.23684207,0.21770331,
       10.23684207,-0.08492823,2*-0.05143541,-0.08492823,0.00598086,2*0.1
       2038277,0.00598086,0.05322964,0.19677030,0.14413872,0.10586122,0.0
       3031100,-0.06758374,-0.03349283,0.03349282,-0.18241626,-0.34031099
       4-0.12440190,0.12440190,0.05322964,0.10586122,0.14413872,0.1967703
       5,0.03349282,-0.03349283,-0.06758374,0.09031099,0.12440190,-0.1244
       6191,-0.34031099,-0.18241625,4*-0.12499998,2*0.125,2*0.,2*0.124999
       79,2*-0.,0.05263157,-0.05263157,0.05263158,-0.05263157,-0.15789473
       82*0.15789474,2*-0.15789473,2*0.15789473,-0.15789473,4*-0.12499998
       92*0.,2*0.125,2*-0.,2*0.12499999/
        I = X+1.
        J = Y+1.
        XP = AMOD(X,1.)
        YP = AMOD(Y,1.)
        IF (NPT .EQ. 1) GO TO 11
        IF (IP .NE. 1) GO TO 11
        IF (JP .EQ. J) GO TO 14
     11 IP = I
        JP = J
        D(1) = DEP(I,J)
        D(2) = DEP(I+1,J)
        D(3) = DEP(I+1,J+1)
        D(4) = DEP(I,J+1)
        D(5) = DEP(I+2,J)
        D(6) = DEP (I+2,J+1)
        D(7) = DEP(I+1,J+2)
        D(8) = DEP(I,J+2)
        D(9) = DEP(I-1,J+1)
        D(10) = DEP(I-1,J)
        D(11) = DEP(I,J-1)
        D(12) = DEP(I+1,J-1)
        DO 110 K=1,6
        E(K) = 0.
        DO 110 L =1,12
        LK = L+12*(K-1)
    110 E(K) = E(K)+D(L)*SXY(LK)
     14 DXY = (E(1)+E(2)*XP+E(3)*YP+E(4)*XP*XP+E(5)*XP*YP+E(6)*YP*YP)*DCO
        RETURN
        END
```

SI FORTRAN(2.3)/MASTER INTEGER WORD SIZE = 1 , * OPTION IS OFF , O OPTION IS

```
C       HIGHTS DECK
        SUBROUTINE HEIGHT(X,Y,A,H)
        COMMON/1/DEP(100,100),D(12),E(6),B1,B2,CO,CXY,DCDH,DCON,DELTAS,DRO
       1,DTGR,DXY,GRINC,HO,IGO,JGO,LIMNPT,NPRINT,NPT,PHX,PHY,RCCO,RHS,RK,
       2SIG,SK,TOP,V,WL,WLO,LL,NGR,JGR,T,MXX,SCAL,MXY
        WL = WLO*RCCO
C       GN ROUNDED OFF FROM 12.5663706144        RDG, 03/12/74
        GN = 12.566370614*DXY/WL
        SINEH=(EXP(GN)-EXP(-GN))/2.0
C       CG=(1.0+GN/SINH(GN))*CXY
        CG=(1.0+GN/SINEH)*CXY
        IF (CG .LT. 0.) RETURN
        SK = SQRT(CO/CG)
        RK = ABS(1./B2)
        RK = SQRT(RK)
        H = HO*SK*RK
        GO TO (11,12), JGO
     11 U = -2.*SIG*RCCO*CXY/(V*V)
        GO TO 10
     12 U = -0.5/DXY
     10 U = U*DCON
        DCDH = DCDH*DCON
        COSA = COS(A)
        SINA = SIN(A)
        P = -(COSA*PHX+SINA*PHY)*DCDH*DTGR*2.
        Q = ((E(4)*2.+U*PHX*PHX)*SINA*SINA-(E(5)+U*PHX*PHY)*2.*SINA*COSA
       1         +(E(6)*2.+U*PHY*PHY)*COSA*COSA)*DCDH*CXY*DTGR*DTGR*2.
        B3 = ((P-2.)*B1+(4.-Q)*B2)/(P+2.)
        B1 = B2
        B2 = B3
        RETURN
        END
```

USASI FORTRAN DIAGNOSTIC RESULTS FOR HEIGHT

NO ERRORS

SI FORTRAN(2.3)/MASTER   INTEGER WORD SIZE = 1 , * OPTION IS OFF , 0 OPTION IS

```
C       ERRORF DECK
        SUBROUTINE ERROR(FIT,DIFMAX)
        COMMON/1/DEP(100,100),D(12),E(6),B1,B2,CO,CXY,DCDH,DCON,DELTAS,DRC
       1,DIGR,DXY,GRINC,HO,IGO,JGO,LIMNPT,NPRINT,NPT,PHX,PHY,RCCO,RHS,RK,
       2SIG,SK,TOP,V,WL,WLO,LL,NGR,JGR,T,MXX,SCAL,MXY
        DIMENSION DP(4)
        IF(NPT.LE.10) GO TO 11
        IF (EP .EQ. E(5)) GO TO 12
     11 DP(1) = E(1)
        DP(2) = E(1)+E(2)+E(4)
        DP(3) = E(1)+E(2)+E(3)+E(4)+E(5)+E(6)
        DP(4) = E(1)+E(3)+E(6)
        DIFMAY = 0.
        SUM = 0.
        DO 110 I=1,4
        DIF = ABS(D(I)-DP(I))
        DIFMAY = AMAX1(DIF,DIFMAY)
    110 SUM = DIF*DIF+SUM
        DIFMAY = DIFMAY*DCON
        SUM = SUM*0.25
        FIT = SQRT(SUM)
        EP = E(5)
     12 DIFMAX = DIFMAY/DXY*100.
        RETURN
        END
```

USASI FORTRAN DIAGNOSTIC RESULTS FOR ERROR

NO ERRORS

SI FORTRAN(2.3)/MASTER   INTEGER WORD SIZE = 1 , * OPTION IS OFF , 0 OPTION IS

```
C       WRITES DECK
        SUBROUTINE WRITER(X,Y,ANG,H,NWRITE)
        COMMON/1/DEP(100,100),D(12),E(6),B1,B2,CO,CXY,DCDH,DCON,DELTAS,DRO
       1,DTGR,DXY,GRINC,HO,IGO,JGO,LIMNPT,NPRINT,NPT,PHX,PHY,RCCO,RHS,RK,
       2SIG,SK,TOP,V,WL,WLO,LL,NGR,JGR,T,MXX,SCAL,MXY
     61 FORMAT(1H ,I7,3F9.2,F11.2,F10.2,F8.2,3F10.2,2F10.4)
     62 FORMAT(29H  CURVATURE AVERAGED AT POINT, I4)
     63 FORMAT(1H ,42HRAY STOPPED, NO CONVERGENCE FOR CURVATURE.)
     64 FORMAT(1H ,32HRAY STOPPED, REACHED SHORE.  X =,F7.2,6H,   Y =,F7.2)
     65 FORMAT(1H ,35HRAY STOPPED, REACHED BOUNDARY.  X =,F7.2,6H,   Y =,
       1F7.2)
     66 FORMAT(1H ,55HRAY STOPPED, NUMBER OF POINTS EXCEEDS MAXIMUM.  LIMI
       1T =,I4,13H POINTS.  X =,F7.2,6H,   Y =,F7.2)
     67 FORMAT(1H ,51HRAY STOPPED, INCREMENT DISTANCE ALONG RAY LESS THAN,
       1F6.3,17H GRID UNITS.  X =,F7.2,6H,   Y =,F7.2)
C
        CALL ERROR(FIT,DIFMAX)
        WRITE(6,61) NPT,X,Y,ANG,DXY,DIFMAX,FIT,WL,CXY,H,RK,SK
        GO TO (11,20,21,22,23,24,25), NWRITE
     20 WRITE(6,62) NPT
        GO TO 11
     21 WRITE(6,63)
        GO TO 12
     22 WRITE(6,64) X,Y
        GO TO 12
     23 WRITE(6,65) X,Y
        GO TO 12
     24 WRITE(6,66) LIMNPT,X,Y
        GO TO 12
     25 WRITE(6,67) DELTAS, X,Y
     12 IGO = 3
        IF(MXX.EQ.1) CALL GRAPH(X,Y,SCAL,MXY)
        IF(MXX.EQ.1) CALL PLOT(0.0,0.0,3)
     11 RETURN
        END
```

USASI FORTRAN DIAGNOSTIC RESULTS FOR WRITER

NO ERRORS

```
      SUBROUTINE LABEL(N,T,H,SURGE,TITL,MI,MJ,GRID,SCALE)
      DIMENSION TITL(8)
      CALL SYMBOL(0.0,0.0,0.49,12HGILLIE  R.D. ,90.0,12)
      CALL SYMBOL(2.0,0.0,0.35,10HSED.PROC.  ,90.0,10)
      CALL DOY(RDT)
      CALL SYMBOL(4.0,0.0,0.35,RDT,90.0,10)
      CALL PLOT(6.,0.,-3)
      X=MI
      Y=MJ
      READ(5,1)SCALE
    1 FORMAT(F10.3)
C
C     SCALE IS GIVEN AS NUMERATOR OF RATIO 1:X
      X=(X*GRID)/SCALE
      Y=(Y*GRID)/SCALE
      IF(Y.GT.28.0)WRITE(6,10)Y
      IF(Y.GT.28.0) STOP 22
   10 FORMAT(1H ,31HPLOT EXCEEDS PAPER,  HEIGHT IS ,F7.1,5X,15HINCREASE
     1SCALE./)
      WRITE(6,20)X,Y,SCALE
   20 FORMAT(1H ,7HPLOT IS,F7.3,1X,12HINCHES LONG,,F7.3,1X,11HINCHES WID
     1E,/,1X,11HSCALE IS 1:,F7.0)
      X=X+2.0
      Y=Y+2.0
      CALL PLOT(2.0,2.0,3)
      CALL PLOT(2.0,Y,2)
      CALL PLOT(X,Y,2)
      CALL PLOT(X,2.0,2)
      CALL PLOT(2.0,2.0,2)
      CALL PLOT(0.0,0.0,3)
      I=SCALE
      ENCODE(6,2,I)I
    2 FORMAT(I6)
      X=X-1.92
      CALL SYMBOL(X,1.8,0.14,10HSCALE = 1.,0.0,10)
      X=X+1.2
      CALL SYMBOL(X,1.8,0.14,I,0.0,6)
      SCALE=GRID/SCALE
      CALL SYMBOL(2.0,1.2,0.14,TITL,0.0,80)
      ENCODE(3,3,N)N
    3 FORMAT(I3)
      ENCODE(8,5,T)T
      ENCODE(8,5,H)H
      ENCODE(8,5,SURGE)SURGE
    5 FORMAT(F8.2)
      WRITE(61,40) T,H,SURGE
   40 FORMAT(10X,3A8)
      CALL SYMBOL(2.0,0.7,0.14,11HNO. OF RAYS,0.0,11)
      CALL SYMBOL(2.0,0.5,0.14,9HPERIOD = ,0.0,9)
      CALL SYMBOL(3.6,0.7,0.14,N,0.0,3)
      CALL SYMBOL(3.04,0.5,0.14,T,0.0,5)
      CALL SYMBOL(3.7,0.5,0.14,4HSEC.,0.0,4)
```

```
      CALL SYMBOL(5.0,0.7,0.14,14HWAVE HEIGHT = ,0.0,14)
      CALL SYMBOL(6.7,0.7,0.14,H,0.0,5)
      CALL SYMBOL(7.4,0.7,0.14,3HFT.,0.0,3)
      CALL SYMBOL(5.0,0.5,0.14,14HTIDE HEIGHT = ,0.0,14)
      CALL SYMBOL(6.7,0.5,0.14,SURGE,0.0,5)
      CALL SYMBOL(7.4,0.5,0.14,3HFT.,0.0,3)
      DECODE(3,3,N)N
      DECODE(8,5,T)T
      DECODE(3,5,H)H
      DECODE(8,5,SURGE)SURGE
      WRITE(61,50) T,H,SURGE
   50  FORMAT(10X,3F8.2)
      RETURN
      END
```

USASI FORTRAN DIAGNOSTIC RESULTS FOR LABEL

NO ERRORS

NSI FORTRAN(2.3)/MASTER   INTEGER WORD SIZE = 1 , * OPTION IS OFF , 0 OPTION IS

```
      SUBROUTINE GRAPH(A,B,SCALE,MXY)
      X=A
      Y=B
      X=(X*SCALE)+2.0
      Y=(Y*SCALE)+2.0
      IF(MXY.EQ.1)CALL PLOT(X,Y,3)
      IF(MXY.NE.1)CALL PLOT(X,Y,2)
      RETURN
      END
```

USASI FORTRAN DIAGNOSTIC RESULTS FOR GRAPH

NO ERRORS

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