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A Review of the Dynamics of Contained Oil Slicks in Flowing Water

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A REVIEW OF THE DYNAMICS OF CONTAINED OIL SLICKS IN FLOWING WATER

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ABSTRACT

A review of published papers dealing with the dynamics of contained oil slicks in flowing water was made. It was found that the various writers all assumed one-dimensional flows and used the continuity and momentum equations to obtain the thickness of the oil slick. The analysis of Wilkinson was considered to be the most reliable because it was the only one in which finite flow depths were considered and it is mainly the effect of finite depth which causes the inequilibrium of the oil slick. Errors were discovered in the analysis of Cross and Hoult as well as that of Wicks. The stability analysis of Jones was not useful for prediction of boom failure. It was concluded that more work is required to answer many of the questions related to oil spill containment and control. Recommendations are made for further research.

RESUME

D'après une étude des publication sur la dynamique des nappes d'huile à la surface des eaux courantes wui ont été circonscrites, tous les auteurs utilisent les équations de la continuité et de la quantité de mouvement pour mesurer l'épaisseur de ces nappes d'huile et présument que l'écoulement est unidimentionnel. L'analyse de Silkinson a été jugée la plus fiable parce qu'elle était la seule à envisager des profondeurs d'écoulement limitées; ou c'est surtout le caractère fini de la profondeur qui cause le déséquilibre de la nappe d'huile. On a relevé des erreurs dans les études de Cross, de Hoult et de Wicks. L'analyse de la stabilité effectuée par Jones s'est révélée inutile pour ce qui est de prévoir la rupture des "estacades". On a conclu qu'il fallait encore trouver des résponses à de nombreuses questions concernant la régulation et l'importance des nappes d'huile sur les eaux. On recommande donc que d'autres travauz de recherche soient entrepris.

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Successful containment of an oil spill in a river by means of mechanical devices such as booms or barriers requires knowledge of the fundamental dynamic behaviour of an oil slick on flowing water.

In order to combat an oil spill, one needs to know what the hydraulic conditions have to be for containment of an oil slick to be possible. One would also need to know the maximum volume of oil which a boom can retain and the thickness of the oil slick at the boom. These questions cannot be answered unless a good understanding of the dynamics of the slick is achieved.

Other problems, such as boom stability, methods of deployment, etc., are really of secondary importance because if the hydraulic conditions are not favourable, containment is impossible no matter what kind of boom is used.

A survey of the existing literature reveals that, at the time this article is being written, there were only five papers published which dealt with the dynamics of contained oil slicks. This review will relate and compare the various theories, identify the shortcomings and indicate areas requiring further research.

1.1. Definition Sketch and Nomenclature

Previous writers have used different symbols and terms to describe the same quantities but, in order to compare the equations and expressions, the nomenclature will be standardized for this review.

The general shape of a contained oil spill together with some of the basic dimensions are shown in Figure 1. These symbols will be used throughout this article. Other terms, not included in Figure 1, will be defined as they appear in the text.

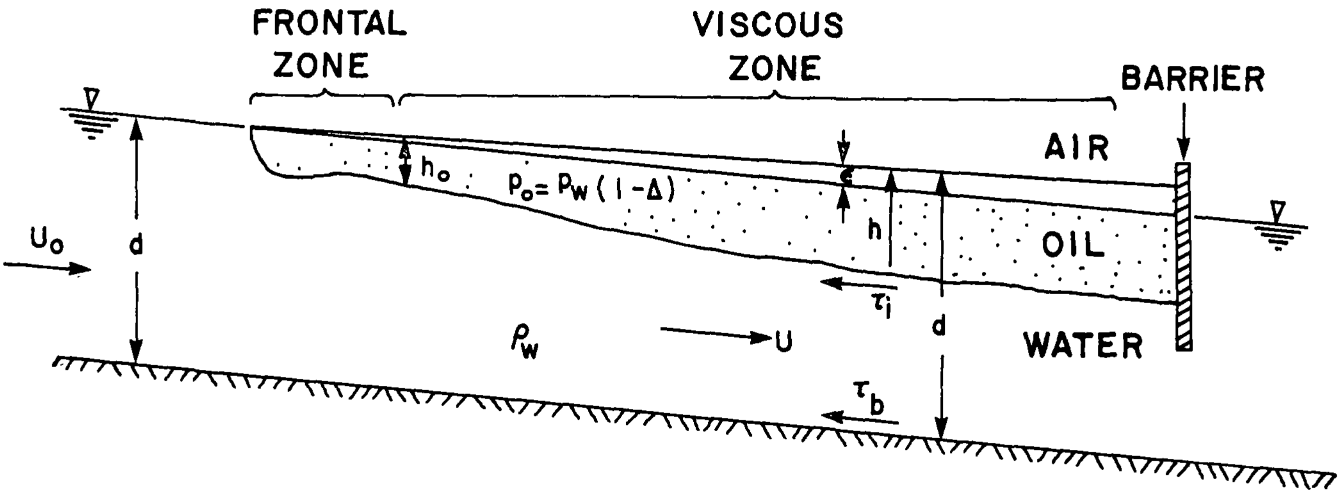

Figure 1. General shape of a contained oil spill and associated symbols

d	total thickness of the oil-water layers $[L]$
d_o	flow depth upstream of the slick $[L]$
F	$U_o/\sqrt{g\Delta d_o}$, densimetric Froude number for the flow upstream of the slick
g	gravitational acceleration $[LT^{-2}]$
h	thickness of the slick in the viscous zone $[L]$
h_o	frontal thickness of the slick $[L]$,
U	mean velocity of the water underneath the slick $[LT^{-1}]$
U_o	mean velocity of the flow upstream of the slick $[LT^{-1}]$
ρ_o	oil density $[ML^{-3}]$
ρ_w	water density $[ML^{-3}]$
τ_i	shear stress at the oil-water interface $[ML^{-1}T^{-2}]$
τ_b	shear stress at the bottom boundary $[ML^{-1}T^{-2}]$
Δ	1-specific gravity of the oil $\frac{1-\rho_o}{\rho_w}$

2. LITERATURE REVIEW

Two authors, namely Wilkinson (1972) and Wicks (1969) recognized the fact that an oil slick can be divided into more than one region. They both considered a frontal zone which is the upstream region of a slick where viscous forces can be neglected in comparison with the dynamic forces and viscous zone downstream of the frontal zone where interfacial shear stresses control the growth of the slick. This review will also consider the two regions separately.

All of the analyses assumed one-dimensional flows which were uniform throughout the depth, and only Wilkinson (1972, 1973) considered the effects of finite depths of flow. The significance of cross-sectional shapes or cross-stream variations in velocity have not been considered.

2.1 Frontal Zone

2.1.1 Wilkinson (1972)

Wilkinson showed that for the upstream region of an oil slick, the shear forces at the oil-water interface was small compared with the dynamic force resulting from the speeding up of the water passing underneath the slick. He found that the ratio of the shear force to the dynamic force was to the order $\frac{xC_f}{2h_o}$ where,

x = downstream distance measured from the leading edge of the slick

h_o = thickness of the frontal zone of the slick

C_f = interfacial shear coefficient = $\frac{\tau i}{\rho_w \frac{U^2}{2}}$

Using the experimental result of Cross and Hoult (1971) that C_f was approximately equal to 0.01, the dynamic forces will exceed the viscous forces by a factor of >10 for $\frac{x}{h_o}$ <20. Wilkinson, therefore, assumed that the length of the frontal zone was approximately equal to $20h_o$.

To determine the thickness, h_o, of the slick, Wilkinson assumed uniform flow conditions at sections 0 and 1, as shown in Figure 2, and applied the one-dimensional momentum equation between these two sections.

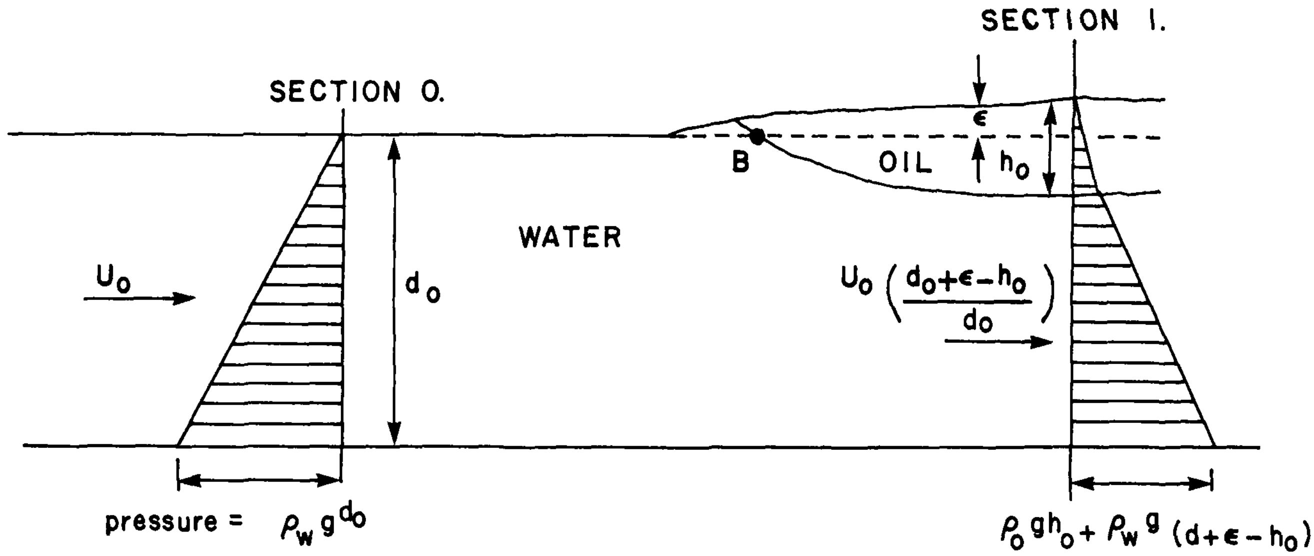

Figure 2. Flow velocity and pressure distribution upstream and in the frontal zone.

This yielded the following equation:

$$\rho w U_o^2 d_o + \frac{\rho_w g d_o^2}{2} = \frac{\rho_w U_o^2 d_o^2}{d_o+\varepsilon-h_o} + \frac{\rho_w (1-\Delta) g (d_o+\varepsilon)^2}{2} + \frac{\Delta\rho_w g (d_o+\varepsilon-h_o)^2}{2} \qquad (1)$$

where,

d_o = depth of the flow upstream of the slick

g = gravitational acceleration

Δ = 1- (specific gravity of the oil)

i.e., $\rho_o = \rho_w\ (1-\Delta)$

ε = difference in elevations of the free surface of sections "0" and "1"

To solve equation (1) which had two unknowns h_o and ε, an additional equation for ε was derived by Wilkinson by assuming the existence of a stagnation point at B (see Figure 2). Equating the stagnation pressure to the hydrostatic pressure of the oil at B, the expression for ε was:

$$\varepsilon = \frac{U_o^2}{2g(1-\Delta)} \qquad (2)$$

Combining equations (1) and (2), and neglecting terms involving $\Delta\varepsilon$ and ε^2 because both Δ and ε were small, Wilkinson derived the following equation for the thickness of the frontal zone:

$$F^2 = \phi(2-\phi)\left(\frac{2\phi}{1-\phi} + \frac{1}{1-\Delta}\right)^{-1} \tag{3}$$

where, $F = \dfrac{U_o}{\sqrt{g\Delta d_o}}$

and, $\phi = \dfrac{h_o}{d_o}$, dimensionless slick thickness.

Equation (3) shows that the thickness of an oil slick in the frontal zone is dependent upon only the upstream flow velocity and depth and the density of the oil. Therefore, given the flow conditions in a river and the kind of oil spilled, the thickness of the slick near the leading edge can be determined from equation (3). Of course, the slick would continue to thicken in the downstream direction but the rate at which it thickens depends upon viscous shear forces and cannot be calculated with the simple analysis used for the frontal zone.

Equation (3) was solved for the cases $\Delta \to 0.0$ and $\Delta = 0.2$, which includes most types of oil. The two solutions were very similar and Figure 3 shows the curve for the case $\Delta \to 0.0$. Of interest is the fact that no solution exists when $F > 0.5$, which indicates that when the densimetric Froude number is too large, the balance between the pressure forces and rate of change of momentum cannot be maintained and the slick becomes unstable. Therefore, Wilkinson arrived at the conclusion that if the upstream densimetric Froude number is larger than about 0.5, containment of oil by a stationary barrier is impossible.

Although there are two solutions for ϕ for every value of F, as shown in Figure 3, Wilkinson showed, by considering the energy loss along a streamline along the oil-water interface, that ϕ cannot be greater than 0.5. Therefore, the lower portion of the curve in Figure 3 is not physically attainable. Since the maximum of the curves for different values of Δ were all approximately equal to 0.3, Wilkinson concluded that the maximum thickness of the <u>frontal zone</u> of any slick is limited to roughly 1/3 of the flow depth. Naturally, an oil slick would continue to thicken downstream of the frontal zone due

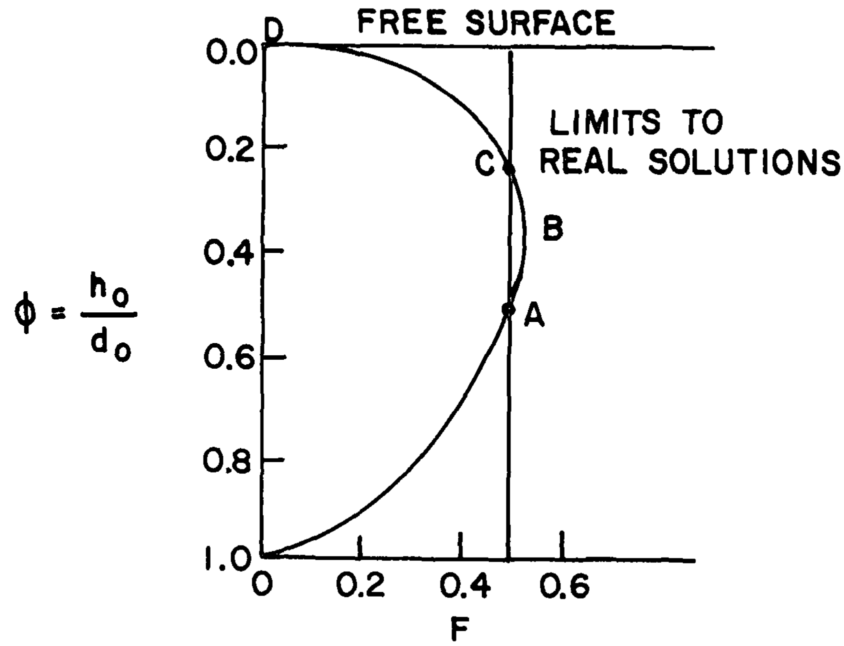

Figure 3. Slick thickness ratio as a Function of Froude Number for the case $\Delta \to 0.0$

to viscous forces.

Based on these findings, it would appear that oil booms should be located in the deeper, slower reaches of a river, where the slick would likely be more stable because of the smaller densimetric Froude number.

For the case of very large flow depth, both F and ϕ tend towards zero and equation (3) has to be rearranged to obtain any meaningful results. Wilkinson introduced a slick Froude number F_s, where

$$F_s = \frac{U_o}{(\Delta g h_o)^{\frac{1}{2}}} = F\phi^{-\frac{1}{2}} \tag{4}$$

Equation (3) was rewritten as

$$F_s^{\,2} = (2-\phi)\left(\frac{2\phi}{1-\phi} + \frac{1}{1-\Delta}\right)^{-1} \tag{5}$$

As $\phi \to 0$, $F_s \to \sqrt{2(1-\Delta)}$,

$$\text{Therefore, } h_o \to \frac{U_o^{\,2}}{2\Delta(1-\Delta)g} \tag{6}$$

Therefore, for the case of very deep current, Wilkinson's analysis shows that the frontal thickness is determined by the flow velocity and the oil density as given in equation (6). It should be noted that for this case, no failure of the slick is predicted, indicating that the inequilibrium of a slick in the frontal zone is caused by the effect of finite flow depth.

Benjamin (1968) showed that as the current becomes very deep, the thickness of an inviscid layer overlaying a flowing layer approaches

$$h_o \rightarrow \frac{U_o^2}{2.g.\Delta} \tag{7}$$

This value for h_o differs from Wilkinson's result by a factor of $\frac{1}{1-\Delta}$ and the difference arises out of the assumption of the location of the stagnation point. Wilkinson assumed the stagnation point to be at point B (Figure 2) which results in equ. (2) for ε. If the stagnation point is assumed to be at the free surface, $\varepsilon = \frac{U_o^2}{2g\Delta}$ and Benjamin's results will be obtained.

Wilkinson made experimental measurements of the frontal thickness of peanut oil (Δ = .087) and hot water slicks ($\Delta \simeq .001$) in a channel 3 inches wide with a working section 7.5 feet in length. The flow depths used were between 15 to 20 centimeters and velocities were between 1 to 27 centimeters per second. The results, shown in Figure 4, are in satisfactory agreement with the theoretical solution given by equation (3), although Wilkinson did observe that as F increased above 0.4, interfacial waves began to develop which sometimes led to emulsification of oil at the interface.

Also shown in Figure 4 are curves of F versus ϕ which would be obtained if the second order terms such as $\Delta\varepsilon$ and ε^2 in equation (1) are retained. The results differ very little from the curve given by equation (3). The simplifications used by Wilkinson therefore seem to be acceptable.

In summary, Wilkinson's analysis of the frontal zone provides the criterion that as long as the upstream densimetric Froude number is larger than about 0.5, containment of oil with a barrier is impossible. However, his experiments indicated that there may be other forms of failure such as emulsification of oil at the oil-water interface which may occur at even smaller values of F. It was also shown that for very deep currents, dynamic equalibrium of the slick could always be maintained and there would be no failure of the slick.

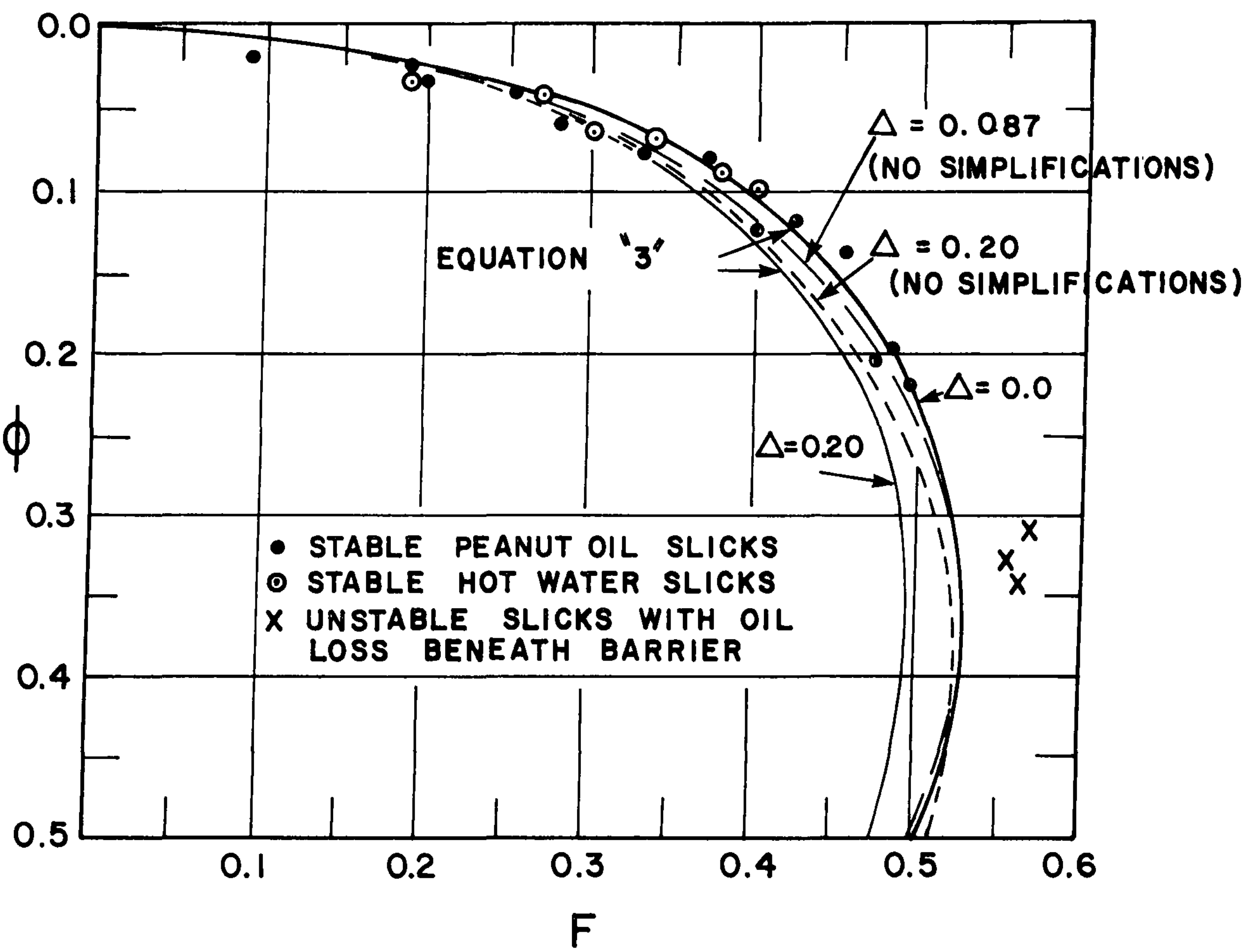

Figure 4. Theoretical and Experimental Relationship between F and ϕ

This analysis also provides the thickness of the frontal zone as a starting point for the calculation of the slick profile in the viscous zone.

2.1.2 M. Wicks, III (1969)

Wicks divides an oil slick into three regions, region I being the frontal zone, region II the viscous zone, and region III the zone near the boom.

Wicks made observations on slick profiles in a tank with a cross section of 6' x 6'. He observed that there was always a thick headwave near the leading edge and that waves and droplets appeared at the lee of the wave. At high water speeds, droplets were torn off and swept downstream. The general picture of the frontal zone which he observed is shown in Figure 5.

Since there was no theory to calculate the length of the frontal zone, Wicks considered the flow past the headwave to be similar to the separation of a boundary layer flow over an abrupt surface roughness. Using the experimental observation of Plate (1964) that a separated boundary layer reattaches itself to the wall about 50 disturbance heights downstream, Wicks chose $50h_o$ as the length of the frontal zone.

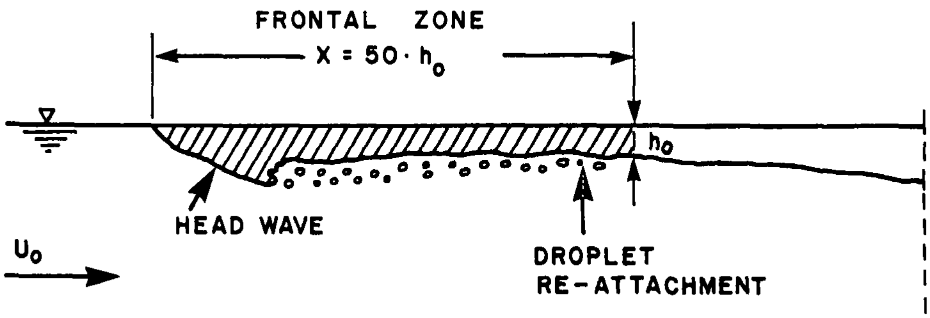

Figure 5. Shape of the Frontal Zone observed by Wicks

Wicks considered the frontal zone to be similar to a gravity wave. Typical examples of this phenomenon are when salt water intrudes into fresh water, or submerged currents of muddy water flow under clearer water, etc.

Von Karman (1940) studied the case of a wedge of heavier fluid penetrating underneath a lighter, infinitely deep fluid, as shown in Figure 6, and calculated that the thickness of the heavy layer was given by

$$h_o = \frac{U_o^2}{2g} \frac{\gamma_2}{\gamma_1 - \gamma_2} = \frac{U_o^2}{2g\frac{\Delta}{1-\Delta}} \tag{8}$$

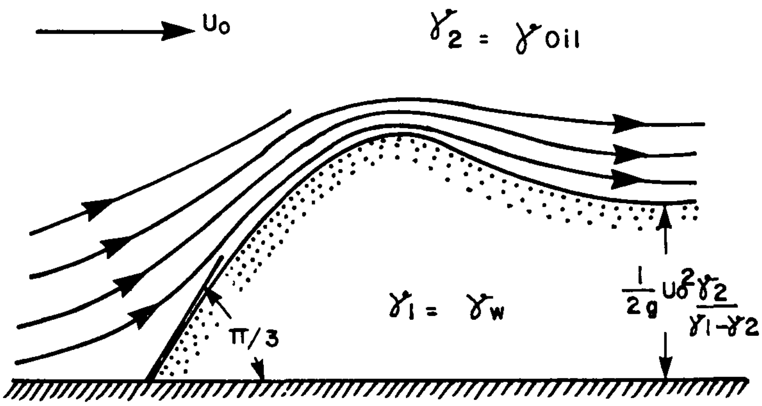

Figure 6. Flow of gravity current under an infinitely deep fluid

Wicks adopted equation (8) for the thickness of the frontal zone. However, this equation is not correct when applied to the present case of an oil slick. Here, it is the lighter fluid which is floating as a layer on top of the heavier fluid. Benjamin (1968) has shown, as already given in equation (7), that for this case the layer thickness should be

$$h_o = \frac{U_o^2}{2g\Delta} \tag{9}$$

Because Wicks applied a theory which was derived from an infinitely deep flowing layer, he did not predict failure of the frontal zone such as the type predicted by Wilkinson, which has been shown to be due to the effect of finite depth.

Wicks observed the separation of oil droplets behind the headwave and used existing theories to predict the flow velocity above which droplets would be detached from the oil. However, detachment of droplets does not necessarily constitute failure as the droplets may reattach themselves to the oil layer further downstream. The length of the viscous zone has to be known before it can be decided whether droplets may escape under the barrier or not.

2.2 Viscous Zone

Downstream of the frontal zone, the viscous shear at the oil-water interface and along the bottom of the river becomes the same order of magnitude as the dynamic forces and can no longer be neglected. The analysis of the slick must, therefore, include both viscous and dynamic forces. This region is termed the viscous zone.

2.2.1 Wilkinson (1973)

Wilkinson's theory of the viscous zone is based on three main assumptions:

a) the momentum flux due to circulation in the oil slick is small compared with interfacial shear forces.

b) the flow beneath the oil slick is steady and uniform over the depth.

c) for equilibrium of the contained oil slick, a change in inertia and pressure force must be balanced by boundary shear stresses.

Considering the equilibrium of forces acting on a combined oil-water layer at length δx, Figure 7, Wilkinson derived the following equation:

$$\frac{\partial}{\partial x}\left(\frac{\rho_w g(1-\Delta)d^2}{2}\right) + \frac{\Delta\rho_w g(d-h)^2}{2} + \frac{\rho_w U_o{}^2 d_o{}^2}{d-h} = -\tau_b \qquad (10)$$

where, x = downstream coordinate measured from the beginning of the viscous zone

d = total depth of the oil-water layer in the viscous zone

h = slick thickness in the viscous zone

τ_b = boundary shear stress at the bottom of the channel

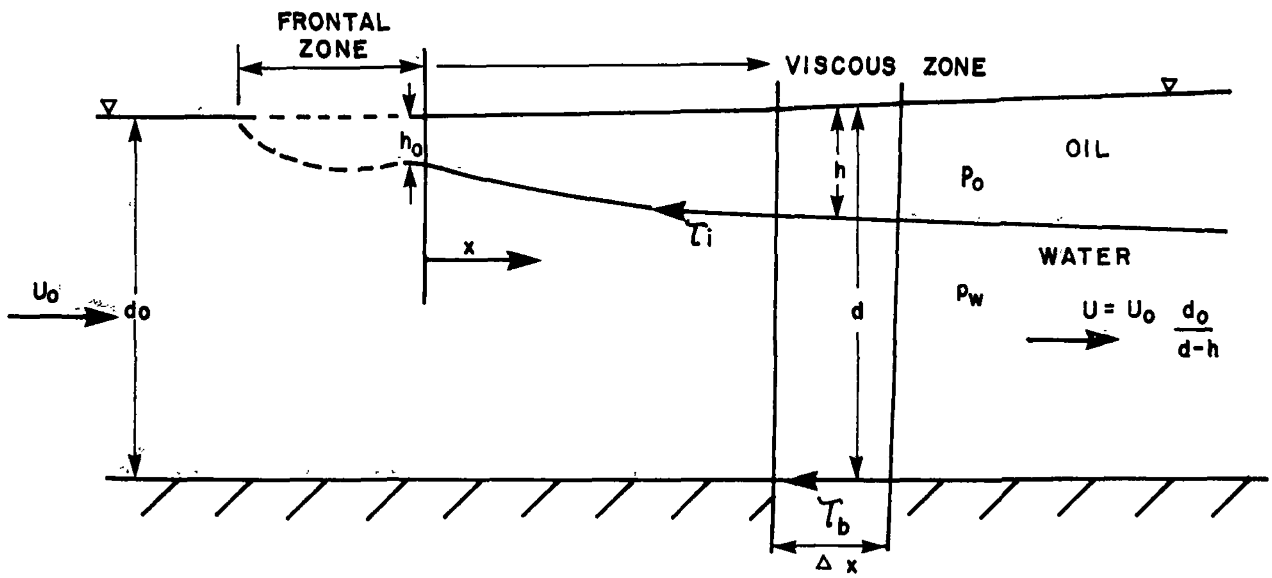

Figure 7. Forces acting on the slick in the viscous zone

Considering the forces acting on the oil layer alone, it can be seen that equilibrium is achieved by the balance of the pressure forces, the shear at the interface and the weight component of the oil acting along the interface. For this equilibrium, Wilkinson derived the following equation:

$$\frac{\partial}{\partial x}\left(\frac{\rho_o g h^2}{2}\right) = \tau_i - \rho_o g h \frac{\partial (d-h)}{\partial x} \tag{11}$$

where τ_i = shear stress at the oil-water interface.

Equations (10) and (11) can be solved simultaneously for the two unknowns d and h, given the upstream velocity U_o and depth d_o, the oil density and values of τ_b and τ_i. Wilkinson expressed the shear stress terms as,

$$\tau_i = \frac{f_i}{4}\ \frac{\rho_w U^2}{2}\ , \quad \text{and}\ , \quad \tau_b = \frac{f_b}{4}\ \frac{\rho_w U^2}{2} \tag{12}$$

where, f_i = interfacial friction coefficient
and f_b = boundary friction coefficient

These expressions for the shear stress terms were subsituted into equations (10) and (11) which were then non-dimensionalized and rearranged to obtain the following two equations:

$$\frac{\partial D}{\partial X}\left[\frac{D}{\Delta} - H - \left(\frac{F}{D-H}\right)^2\right] - \frac{\partial H}{\partial X}\left[D-H - \left(\frac{F}{D-H}\right)^2\right] = -\frac{f_b}{8}\left(\frac{F}{D-H}\right)^2 \tag{13}$$

$$\frac{8H\partial H}{\partial X}\left\{D - \left[H+\left(\frac{F}{D-H}\right)^2\right]\right\} = \left(\frac{F}{D-H}\right)^2 \left[\left\{\frac{f_i}{1-\Delta}\left[D-\Delta\left[H+\left(\frac{F}{D-H}\right)^2\right]\right]\right\} + f_b H\right] \tag{14}$$

where $X = \frac{x}{d_o}$

$D = \frac{d}{d_o}$

and, $H = \frac{h}{d_o}$

Equations (13) and (14) have to be integrated numerically to obtain the slick profile with downstream distance.

From equation (14) it can be seen that when $D = H + \left(\frac{F}{D-H}\right)^2$, $\frac{\partial H}{\partial X}$ becomes infinite. Physically this means that the momentum change in the water layer is so large that equilibrium can no longer be maintained and the slick becomes unstable. This critical thickness ratio is independent of viscous forces. Wilkinson found from his numerical integration that D was very close to unity and therefore the critical thickness ratio could be approximated by

$$H = 1-F^{2/3} \tag{15}$$

i.e., the maximum thickness of an oil slick is approximately

$$h = d_o \left(1 - \frac{U_o}{\sqrt{g \Delta d_o}} \right)^{2/3} \tag{16}$$

Although the maximum thickness of a slick is independent of the viscous forces, the downstream distance at which this thickness is reached and hence the maximum volume of oil containable by a barrier is completely dependent upon viscous forces. Assuming a constant value of f_b=0.03, Wilkinson calculated the maximum slick volumes for various values of F and f_i. The results, shown in Figure 8, shows that the maximum volume containable increases with decreasing densimetric Froude number and interfacial friction coefficient. However, the effect of a change in F is much larger than the effect of a corresponding change in f_i. Therefore, oil booms should be placed in deep, slow reaches of a river where values of F are low in order to contain the maximum volume of oil.

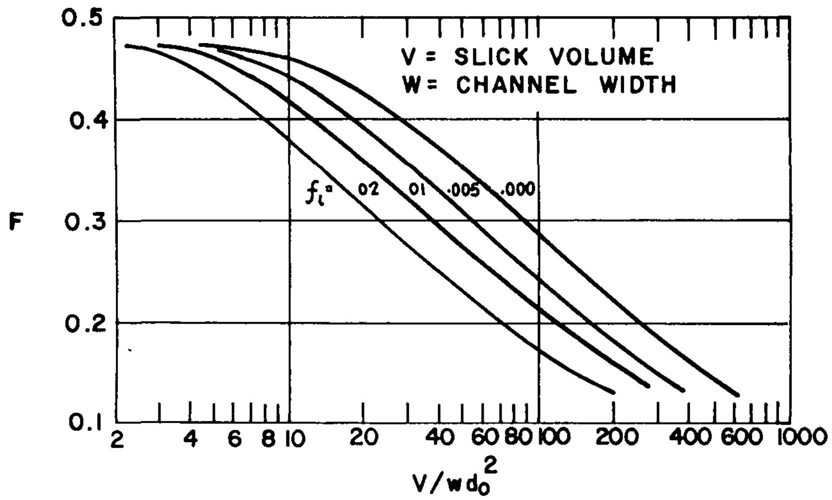

Figure 8. Critical Slick Volume as Function of Froude number and Interfacial Stress (Δ=0.15 and f_b=0.030)

Wilkinson made some experimental measurements of slick profiles and critical thickness ratios. These experiments were carried out in a flume 8cm. wide using flow depths as small as 2.5cm. and velocities of approximately 5cm./sec. The measured slick profiles were compared with the theoretical solutions for various values of f_i as shown in Figure 9.

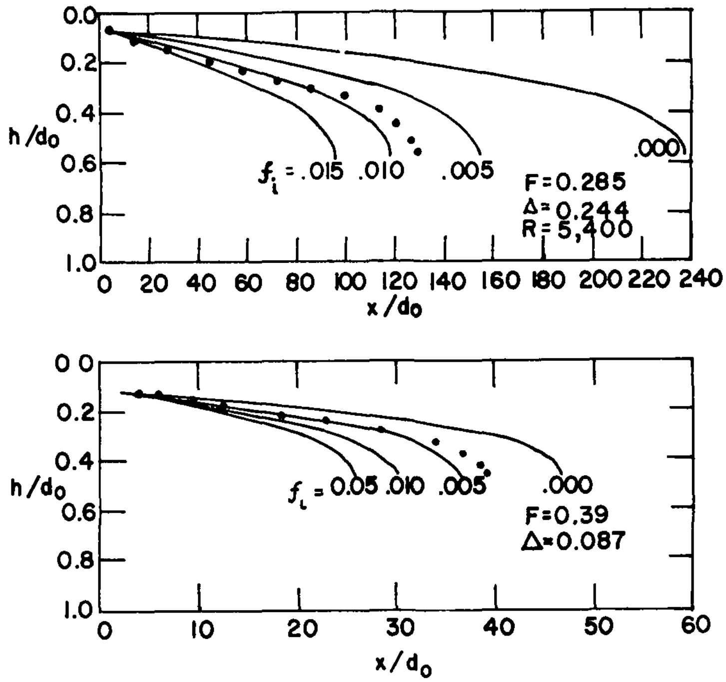

Figure 9. Experimental and Theoretical Slick Profiles obtained by Wilkinson

Agreement between experimental and theoretical slick profiles seemed to be reasonable. Experimental value of f_i were obtained by measuring the interfacial slopes and using these values to calculate f_i from equation (14). However, the values of f_i which Wilkinson obtained were much lower than other published values of interfacial shear coefficient compiled by Dick and Marsalek (1973). In fact, they were even lower than f_i values given by laminar flow theory.

Wilkinson attributed these small values of f_i to the low levels of turbulence in his experiments. However, this cannot explain why the values were lower than that for laminar flow. One possible explanation is that the velocity in the oil was not considered. Wilkinson defined the shear stress to be proportional to the square of the water velocity (equation 12), but the interfacial shear should really depend upon the relative velocity between the water and the oil. Although the velocity in the oil layer may be small enough to permit the momentum of the oil to be neglected when considering a force balance, its effect on the magnitude of the interfacial shear may be appreciable. If τ_i had been defined as being proportional to the square of the relative velocity at the interface, the value of τ_i obtained might have been more realistic. It should be mentioned that the f_i values for some of the arrested thermal wedges given by Dick and Marsalek were also calculated without considering the circulation velocity in the wedge. It is not entirely clear why reasonable values of f_i were obtained for the thermal wedge cases but not for the oil slicks.

Although Wilkinson showed that the densimetric Froude number has a larger effect on the volume of oil retainable than the interfacial shear coefficient, the effect of the variation of bottom shear stress has not been investigated. Wilkinson used f_b = .03 in all his calculations. In real life, f_b may be much larger. From equation (22) it can be shown that the term involving f_b may be much larger than the term involving f_i if f_i is as small as Wilkinson's experimental values.

2.2.2 Cross and Hoult (1971)

This analysis does not divide the oil slick into different regions (frontal zone, viscous zone), but treats it as one unit starting from the leading edge and growing with the distance downstream. The authors considered the equilibrium of the oil layer to be maintained by a balance of pressure forces and shear stresses at the interface as shown in Figure 10. It was assumed that the velocity of the flow under the oil slick remained constant. This assumption limits the application of this analysis only to deep water.

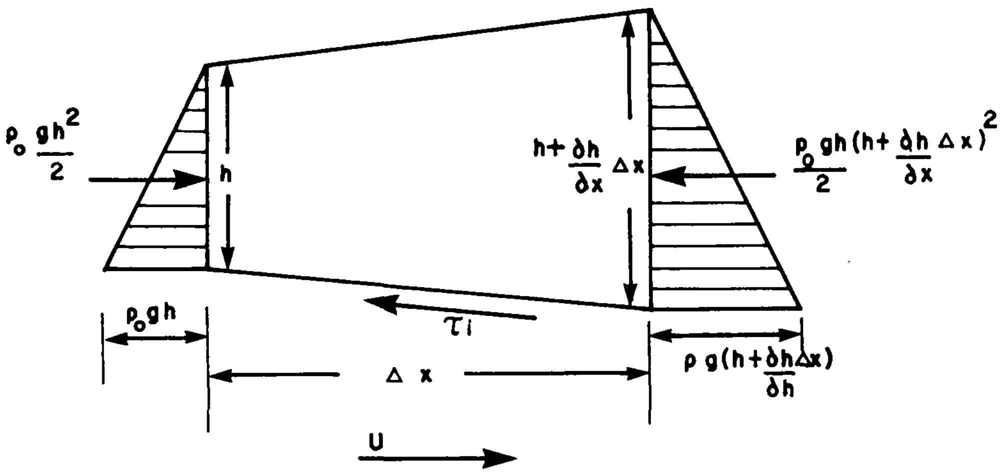

Figure 10. Balance of Pressure Forces and Interfacial Shear Stress

Cross and Hoult made an error when evaluating the force balance and obtained the equation

$$\frac{hg\Delta}{U^2} = C_f^{\frac{1}{2}} \left(\frac{g\Delta x}{U^2}\right)^{\frac{1}{2}}$$

where $C_f = 2\tau_i/\rho_w U^2$ is the shear stress coefficient.

Using the above equation and the experimental data of Robbins (1970) as shown in Figure 11, Cross and Hoult calculated the interfacial friction factors and found them to be

#2 fuel $C_f = 0.005$

soyabean oil $C_f = 0.008$

These values seem to be in agreement with other data of C_f collected by Dick and Marsalek (1973).

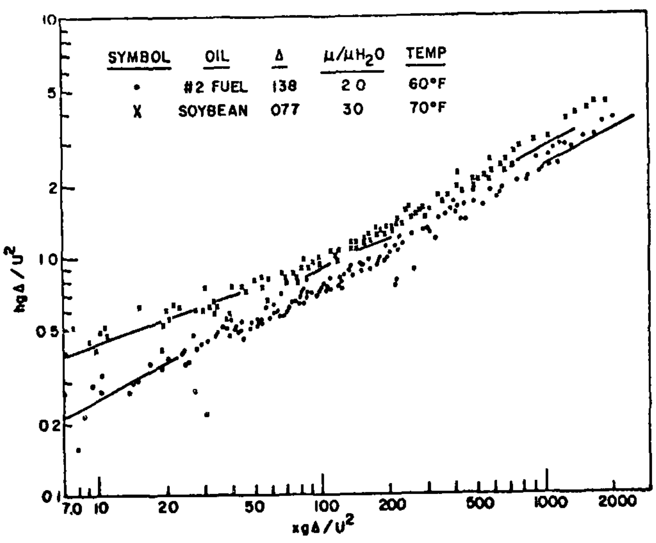

Figure 11. Experimental data of non-dimensional oil thickness versus non-dimensional distance from the leading edge of the slick. (by Robbins)

However, if the correct pressure and shear stress terms are written, the equation for the oil thickness should be

$$\rho_o g h \frac{\partial h}{\partial x} = \tau_i \tag{17}$$

Introducing again the shear stress coefficient C_f, equation (17) reduces to the following equation:

$$\frac{hg\Delta}{U^2} = \left(C_f \frac{\Delta}{1-\Delta}\right)^{\frac{1}{2}} \left(\frac{g\Delta x}{U^2}\right)^{\frac{1}{2}} \tag{18}$$

Using this corrected equation and the data of Robbins, the C_f values obtained are 0.031 and 0.096, respectively. These values of the friction coefficient are higher than those common for rough, solid boundaries and are obviously much too large for the interfacial shear coefficient.

Since there is no evidence to indicate that the data obtained by Robbins was seriously in error, one must then suspect the validity of the analysis which led to these unreasonably high values of C_f.

Referring to Figure 10, it can be seen that the weight component of the oil layer along the interface was not included in the analysis. When this weight component is included in the force balance, the equation for the equilibrium of the oil layer is

$$\rho_o gh\frac{\partial h}{\partial x} = \tau_i - \rho_o gh\frac{\partial (d-h)}{\partial x} \qquad (19)$$

where d is the total thickness of the oil-water layers.

The equation for the equilibrium of the combined oil-water layer is

$$\frac{\partial}{\partial x}\ \frac{\rho_o gd^2}{2} + \frac{\partial}{\partial x}\ \frac{\Delta\rho_w g(d-h)^2}{2} = \tau_b \qquad (20)$$

Equations (19) and (20) are the same as Wilkinson's equations for the viscous zone (equations (10) and (11)), except that the momentum terms are left out. Combining equations (19) and (20) and rearranging, one gets the following equation:

$$\rho_o \Delta gh\ \frac{\partial h}{\partial x} = \tau_i\ \ 1 + (1-\Delta)\frac{h}{d-h}\ + \ \tau_b(1-\Delta)\ \frac{h}{d-h} \qquad (21)$$

For very deep water, $\frac{h}{d-h} \rightarrow 0$ and equation (21) reduces to

$$\rho_o \Delta gh\ \frac{\partial h}{\partial x} = \tau_i \qquad (22)$$

Intrucing the shear stress coefficient and non-dimensionalizing, equation (22) becomes:

$$\frac{g.\Delta.h}{U_o^{\ 2}} = \frac{C_f}{1-\Delta}^{\frac{1}{2}}\ \frac{g.\Delta}{U_o^{\ 2}}\ .\ x^{\ \frac{1}{2}} \qquad (23)$$

Repeating the evaluation of C_f using equation (23) and the data of Robbins, the new values are found to be

#2 fuel : $C_f = 0.0043$
soyabean : $C_f = 0.0074$

These values agree quite well with other published values of interfacial shear coefficient, indicating that the weight component of the slick along the interface must be included. Otherwise, unrealistic results would be obtained.

It can be seen that the analysis of Cross and Hoult is a simplified version of Wilkinson's analysis for the viscous zone but contains an error in leaving out the weight component along the interface. By considering very deep water, the momentum change and the bottom shear stress are neglected, allowing the slick thickness to be obtained in closed form solutions as given in equations (18) and (23).

Because the change in momentum of the water layer is not considered for the case of very deep water, equilibrium of the oil slick can always be maintained and the failure conditions found by Wilkinson do not exist. The slick thickness increases with the downstream distance to the power $\frac{1}{2}$.

2.2.3 M. Wicks (1969)

Using the thickness of the frontal zone h_o, as a starting point, Wicks analyzed the viscous zone in much the same manner as Cross and Hoult except that he included the effect of the circulation in the oil layer.

A balance of pressure forces against interfacial shear for deep water condition was considered. Neglecting the oil weight component acting along the oil-water interface, the equation for this balance is:

$$\rho_o g h \frac{\partial h}{\partial x} = \tau_i \tag{24}$$

The effect of the oil velocity at the oil-water interface is included by defining the interfacial shear stress in terms of the relative

$$\tau_i = C_f \frac{\rho_w (U-u_i)^2}{2} \tag{25}$$

where u_i = oil velocity at the oil-water interface.

The velocity distribution in the oil layer is assumed to be linear, varying from u_i to $-u_i$ as shown in Figure 12. Applying the Navier-Stokes equation in the x-direction and assuming velocity and pressure to be uniform in the x-direction and neglecting vertical velocities, the equation reduces to

$$\frac{\partial^2 u}{\partial y^2} = 0 \tag{26}$$

Integrating equation (26) twice and using the assumed velocity distribution, the velocity of the oil at the interface becomes

$$u_i = \frac{\tau_i}{\mu_o} \frac{h}{2} \tag{27}$$

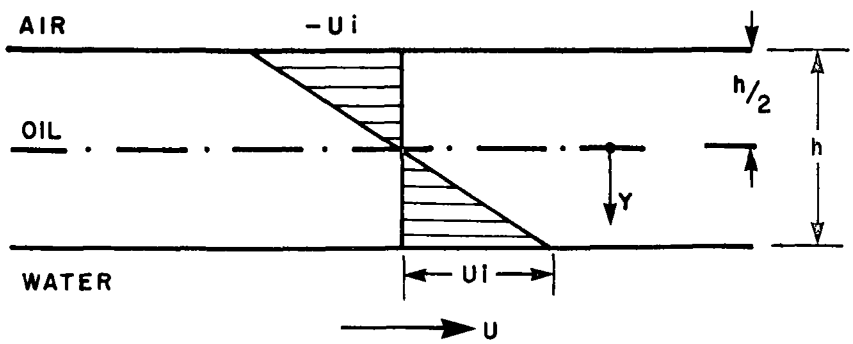

Figure 12. Assumed velocity profile in the oil slick.

Equations (24), (25), and (27) are three equations relating the three unknowns h, u_i and τ_i. Wicks assumed the interfacial friction factor C_f to be a function of the Reynolds number R based on downstream distance.

$$C_f = \left(2.\log_{10} R - 0.65\right)^{-2/3} \tag{28}$$

where $$R = \frac{x.(U-u_i).\rho_w}{\mu_w} \tag{29}$$

With these assumptions, the slick thickness h could not be solved for explicity, but could be obtained by trial and error.

Wicks presented slick profiles for various water velocities and oil properties. In general, for low velocities an obvious increase of h with the distance x downstream was observed. At velocities higher than about one foot per second, the increase in oil layer thickness with x became very small. However, the slicks were always thicker for the higher velocities because of the larger frontal thickness. A few of these profiles are shown in Figure 13.

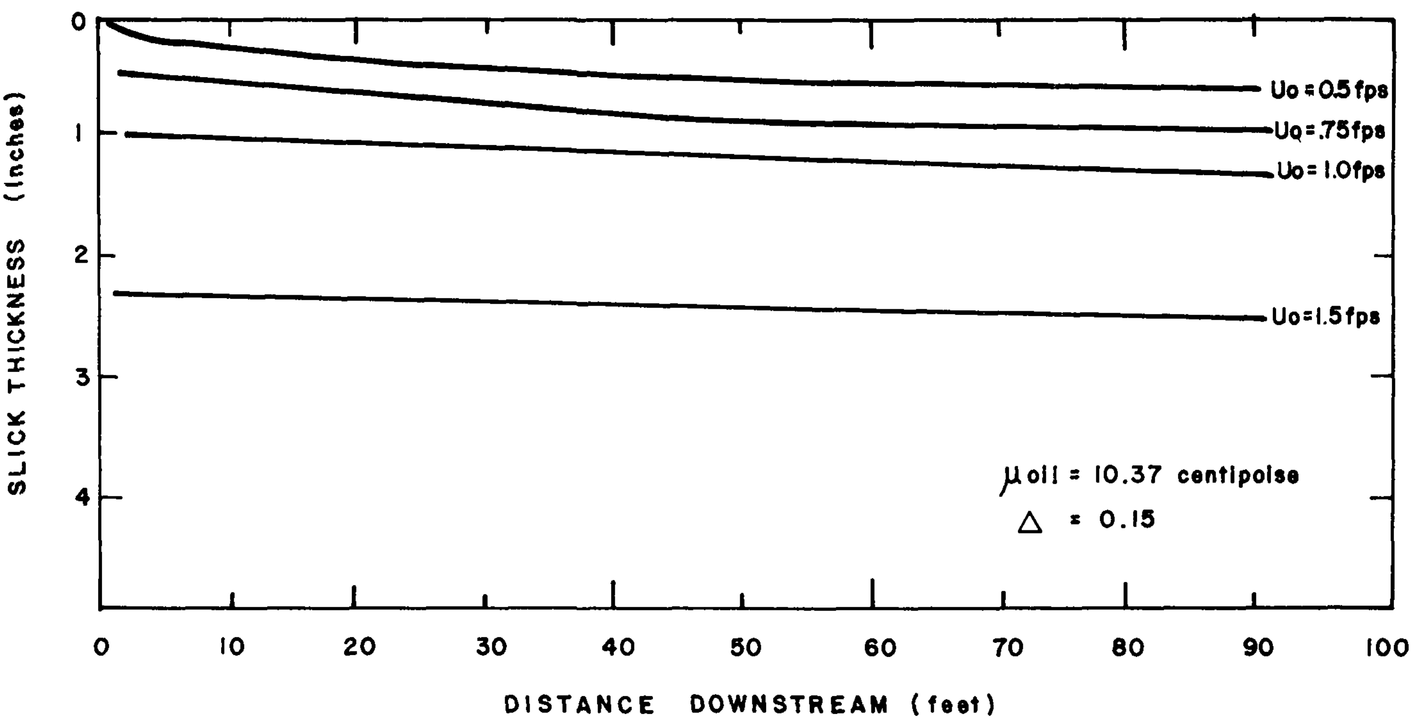

Figure 13. Typical slick profiles calculated by Wicks

Although Wicks made some observations of oil slicks in an experimental tank, he did not present any comparisons of experimental data with calculated slick profiles or frontal thickness.

The analysis of Wicks has two serious drawbacks. The first one is the assumption of the linear velocity distribution in the oil slick. From physical reasoning, one would expect the shear stress at the interface to be smaller as the relative velocity between the water and the oil decreases. This means that, as u_i increases for a given velocity U, the shear stress τ_i should decrease according to equation (25). However, the assumption of a linear velocity distribution in the oil layer results in larger values of τ_i for increasing interfacial velocities since $\tau_i = \mu\frac{\partial u}{\partial y}$ and the velocity gradient in the oil is always equal to $\frac{u_i}{h}$. The use of this linear velocity profile would obviously produce incorrect results for the slick profile.

The other error in Wicks's analysis is in neglecting the component of the weight of the oil in the direction of the sloping interface. As shown in the discussion of Cross and Hoult's work, this assumption is incorrect and leads to erroneous results.

Wicks does not predict any failure due to dynamic inequilibrium because this situation does not occur for deep water. However, his analysis does include a failure mechanism -- droplets passing underneath the barrier. Wicks used the criterion given by Hinze (1955) for estimating the conditions at which droplets would be torn off the head wave, which depends on the Weber number, the ratio of inertia forces to surface tension forces. The criterion is based on the critical value of the Weber number of 22, i.e.,

$$\frac{\rho_w . U^2 . b}{\sigma} = 22 \tag{30}$$

where, b = diameter of droplet

s = interfacial surface tension

Therefore, at a given water velocity, droplets of the diameter $b = \frac{22\sigma}{\rho_w U^2}$ or larger may be torn off.

The size of droplets which may form is based on the equation developed by Christianson and Hixson (1957).

$$b_{max} = \left(\pi \frac{\sigma}{g.\Delta\rho_w}\right)^{\frac{1}{2}} \tag{31}$$

If b_{max} is less than $\frac{22\sigma}{\rho_w U^2}$, then the largest droplet which may form is still smaller than the minimum size required for separation. Therefore, no droplets would be torn off the head wave. If b_{max} is greater than $\frac{22\sigma}{\rho_w U^2}$, then droplets of the size between b_{max} and $\frac{22\sigma}{\rho_w U^2}$ may be detached.

Wicks investigated the motion of the droplets and suggested that only the buoyant force, the weight of the droplet and the drag force opposing the ascent of the droplets needed to be considered. The droplets were also found to reach their terminal rising velocities, V_t, very soon after being torn off the head wave and thus the transient motion did not have to be considered. For equilibrium of the droplet, equation (32) is obtained.

$$1 - \frac{\rho_w}{\rho_o} - \frac{3}{4} \cdot \frac{C_D}{g.b} \cdot \frac{\rho_w}{\rho_o} \cdot V_t \cdot V_t = 0 \tag{32}$$

where, C_D = drag coefficient on the oil drops

Terminal rising velocities for various sizes of droplets were calculated by Wicks using equation (32) using C_D values for droplets in liqui-liquid systems given by Hu and Kintner (1955). Assuming that the droplets will penetrate to an initial depth of twice the thickness of the frontal zone, $2h_o$ beneath the slick, the distance the oil droplets will take to reattach to the oil layer is

$$X = 2h_o \cdot \frac{U}{V_t} \tag{33}$$

Therefore, if the length of the slick were shorter than $2h_o.\frac{U}{V_t}$, the droplets would flow underneath the barrier. Otherwise, they would reattach themselves to the oil and no failure would occur. Practically, this type of failure would not happen except for very short slicks in fast-flowing water. As an example, if $U = 0.65$ metre per second, $\Delta = 0.02$, and $\sigma = 0.00992$ N/M, the smallest droplet which may be detached has a diameter of 0.00063 M and a velocity of ascent of 0.0023 M/S. For these smallest droplets, the slick length required for reattachment is about 4.59 M while the largest droplets would require only about 0.08 metres. Thus droplet failure would not occur for all but the shortest slicks.

In summary, for the viscous zone Wicks considered the case of a slick in deep water in much the same manner as Cross and Hoult. He attempted to include the effect of the circulation velocity in the oil but, unfortunately, used an improper velocity profile. He also neglected to include the component of weight of the slick along the interface. Wicks suggested that one mode of failure may be droplets being torn off the head wave and swept underneath the barrier. However, according to the calculations, this would not occur except for very short slicks in fast-moving water.

2.3 Stability Analysis - Jones (1972)

Jones attempted to explain certain types of boom failures in terms of hydrodryamic instability at the interface. The reasoning is that as instability develops at an oil-water interface, waves would form and grow. When a wave becomes steep enough, interfacial tension causes the crest of the wave to be broken off into droplets which are then entrained into the flow.

Jones considered the case of a slick of constant thickness h at rest on top of an infinitely deep water layer flowing at velocity U. Both layers were considered to be inviscid. An infinitesimal disturbance was then superimposed on the steady flow pattern and classical methods of stability analysis were used to determine the neutral stability curve. From a series of these stability curves, Jones obtained curves of the non-dimensionalized critical velocity $U/\sqrt{gh}$ vs. the parameter $\sigma/(\rho_o gh^2)$ as shown in Figure 14.

For a given value of $\sigma/\rho_o gh$ and Δ, the curve gives the value of $U/\sqrt{gh}$ above which the slick becomes unstable. Sample collections show that thin slicks become unstable at lower velocities than thick slicks.

The results presented by Jones cannot be used for predicting whether a boom would fail to contain oil or not under a given flow condition because the thickness of the slick has to be known before Figure 14 can be

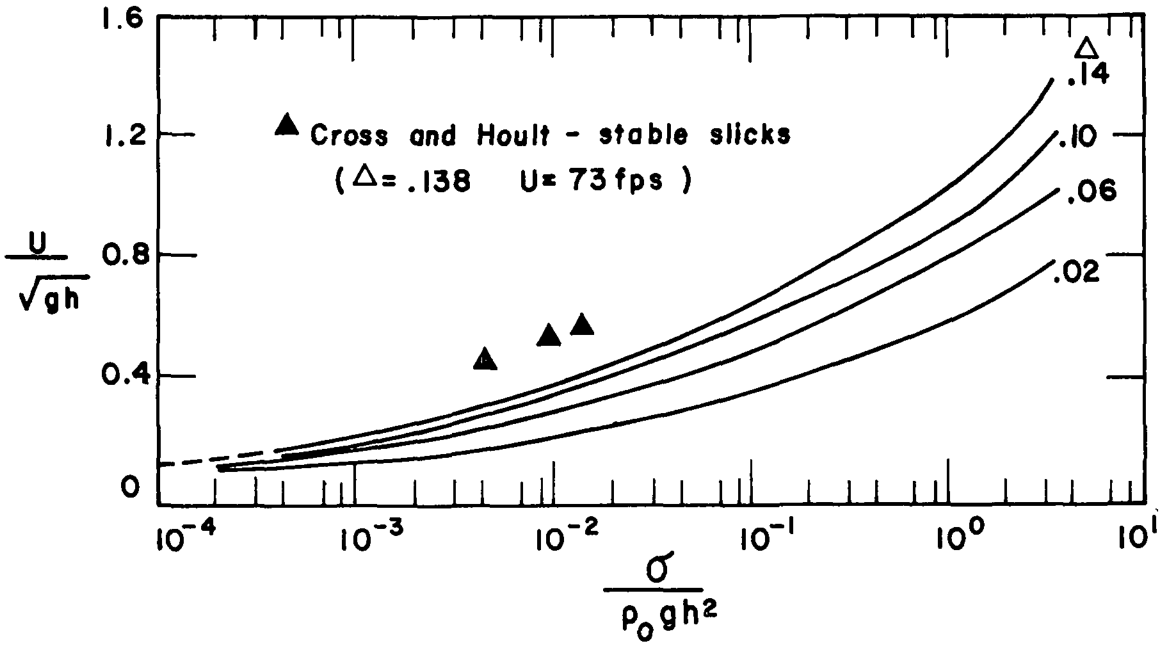

Figure 14. Non-dimensionalized critical velocity curves obtained by Jones

used. It has been demonstrated that slick thicknesses are not constant but increase in the downstream direction and change with flow conditions. However, experimental observations of slick profiles can be used to check the validity of these curves.

Cross and Hoult (1971) reported some measurements of stable slick profiles using #2 fuel oil. Using some of their values of h, Δ, and U, $\sigma/\rho_o gh$ and $U/\sqrt{gh}$ were calculated and plotted in Figure 14. These points lie above the stability curves presented by Jones, i.e., according to Jones the slicks should have been unstable. This seems to indicate that the stability curves obtained by Jones, assuming inviscid fluids and an infinitely deep-flowing layer, give values of critical velocities which are too low.

CONCLUSIONS

Analysis of the dynamics of oil slicks contained by barriers in flowing water have been presented by Wilkinson (1972, 1973), Wicks (1969), Cross and Hoult (1971) and Jones (1972). An examination of these theories and available experimental data indicates that the analysis of Wilkinson is probably the most reliable one. It is also the only one in which the effect of the finite depth of the flow is included. This effect is very important because it causes the change in momentum of the water flowing under the slick which in turn leads to instability of the slick.

Wilkinson used the balance of momentum flux and pressure forces to obtain the thickness of the slick in the frontal zone which he showed to depend only upon the upstream Froude number. The predicted frontal thickness is rather difficult to verify experimentally because the length of the frontal zone is rather ill-defined, being obtained from order of magnitude arguments and being dependent upon the value of the viscous shear coefficient. However, a more important result of his analysis was that equilibrium of the slick could not be maintained whenever the densimetric Froude number based on upstream flow conditions was larger than about 0.5. Wilkinson's experiments showed that the critical Froude number may sometimes be even less than 0.5. As the densimetric Froude number increased beyond 0.4, emulsification of the oil at the interface sometimes caused oil to be swept beneath the barrier. It would seem that failure at the frontal zone could in some way be affected by the turbulence conditions of the river.

For the region of the slick downstream of the frontal zone, Wilkinson used the balance of momentum flux, viscous shear and pressure forces to obtain the equations governing the change of the slick profile with downstream distance. These equations had to be solved numerically. It was shown that for a given set of flow conditions, there is a limiting thickness for the slick above which failure would occur. This limiting thickness is independent of the viscous forces. However, the length of the slick and the slick profile are dependent upon the interfacial and boundary shear stresses. Wilkinson did not investigate the effect of varying of the boundary shear stress on the slick profile. The boundary shear stress may sometimes have a much greater influence on the slick profile than the interfacial shear.

The value of the interfacial shear coefficient obtained by Wilkinson appears to be too low.

Cross and Hoult used the balance of pressure and shear forces to determine the slick profile. However, they left out a term representing the weight component of the slick along the interface and it has been shown that this leads to unreasonable results. Cross and Hoult observed interfacial waves in their experiments in which the densimetric Froude number was about 0.2. These waves were absent in Wilkinson's experiments which were conducted in very shallow water, about 2 cm. It is quite probable that the proximity of the bottom boundary has a significant influence on the development of waves at the interface.

Wicks considered the frontal zone of a slick to be similar to a gravity current and used existing theories for gravity currents in deep water to predict the frontal thickness. For the viscous zone he used the same force balance of Cross and Hoult, but introduced the effect of circulation velocity in the oil. Unfortunately, he used wrong expressions for both the frontal thickness and the oil velocity and, therefore, his calculated slick profiles could not be correct. The effect of oil velocity on the interfacial shear stress is worthy of consideration and it should be possible to solve for the proper velocity distribution and obtain a slick profile for deep water flow which includes this effect. Although an analysis which considers deep water cannot predict any failure of the slick, it would be useful to find out what the flow depth has to be in order that predicted slick profiles would agree resonably well with Wilkinson's solution.

The analysis of Jones attempts to find the conditions when a Kelvin-Helmholtz type instability would occur at the oil-water interface. The results cannot be used to predict the slick profiles. They also appear to predict instability before they should occur because the stable slicks in Cross and Hoult's experiments lie in the region in which instability was forecast.

3. RECOMMENDATIONS FOR RESEARCH

Based on this review, it appears that more work is needed in order that the various questions relating to oil spill containment can be answered with confidence. The topics in which research is required are:

1. Experimental work should be carried out to determine more realistic values of the interfacial shear coefficient since the calculations of slick thickness and maximum volume of oil containable both depend on this coefficient.

2. The effect of different bottom shear stresses on the slick should be investigated theoretically and experimentally.

3. The circulation velocity in the oil may have a significant effect on the interfacial shear stress. Attempts should be made to solve for the velocity distribution in the oil and incorporate this velocity in the prediction of slick profiles.

4. The effect of turbulence in the water on the formation of interfacial waves and detachment of droplets should be investigated.

5. Oil slicks in rivers have reportedly disintegrated and disappeared without a trace. This phenomenon has not been observed in laboratory experiments. Tests should be made to find out the conditions under which this would occur.

6. It may often be necessary to divert oil slicks into areas suitable for containment and clean-up. The use of booms for diverting oil slicks when containment is not possible ought to be investigated.

7. Solutions of slick profiles for deep water can be obtained relatively easily without using numerical integration. It would be useful to compare profiles for deep water with finite depth solutions to see when deep water profiles may be used.

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