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Fourier Analysis for Hovmöller Program at PWC

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INTRODUCTION

Over the winter months Hovmöller data have been collected and analyzed in a variety of formats. 500 millibar height data is averaged over a latitude band for every 5 degrees of longitude. The conventional method of displaying this data has been in a X-T diagram referred to today as the Hovmöller diagram. This diagram illustrates numerous ridges and troughs which facilitate the diagnosis of the significant wave patterns about the hemisphere.

The method of analyzing the wave forms on this particular diagram is the subject of another report and will not be discussed here. However, harmonic analysis has been a popular tool for deciphering the significant wave patterns in the data.

HARMONIC ANALYSIS

When data is periodic in time or in space, the original data set can be transformed into a series of sine and cosine waves. The method described in most statistical and mathematical textbooks is referred to as a fourier, time series or harmonic analysis. The methodology has been well described and will not be repeated here. Instead a very brief descriptive explanation will be given.

The trough and ridge patterns on the Hovmöller diagram can be represented by up to 36 different sine and cosine waves. Each wave is numbered according to its wavelength. For instance, Wavenumber 1 has a wavelength of 360 degrees (which is just a single wave around the hemisphere), while Wavenumber 6 has a wavelength of 360/6 degrees (which has 6 complete waves around the hemisphere). However, Wavenumber Zero has no troughs or ridges and is represented by a circle about the hemisphere. The amplitude for each of these waves is determined by means of a linear combination using the original data and the sine and cosine functions of each wavenumber.

Energy Considerations

The sine and cosine waves for each wavenumber can combine into a single cosine function (using the cosine rule) with a new amplitude or weighting coefficient commonly referred to as a 'C' in the literature. Summing all of these cosine terms over the 36 wavenumbers will reconstruct the original wave. However, we can now consider how the wavenumbers individually or collectively contribute to the original trough and ridge pattern. This information is much more difficult to glean from the Hovmöller diagram.

The relationship between energy and amplitude is not clear cut. Monitoring how the amplitude changes in time, gives an indication of how energy in the latitude band is being transferred from one wavenumber to another. As well, how the amplitudes vary between wavenumbers at one instant in time indicates how the energy in the original wave is being distributed. Statistically, this can be seen by computing the variance for each wavenumber (related to the square of the coefficient 'C') and comparing these values to the total variance (how heights vary about the mean) in the original wave. Therefore, monitoring the change of the various C's in time, may provide some information about the behaviour of the atmosphere.

FOURIER ANALYSIS PROGRAM

A number of coefficients and phase angles for the various wavenumbers are generated by the fourier method. There have been conventional ways of displaying this data. One of the most popular is to divide the wavenumbers into long and short wavelength categories, and then produce a Hovmöller diagram for each.

At the Pacific Weather Centre a different approach is taken where the data is categorized according to wavenumber with a much different graphical approach. Sample output from the computerized fourier analysis is shown in figures 1a and 2a.

At first glance, both figures may appear to look alike, each consisting of three parts. There are three basic components in each figure. In the top left hand corner, there is a contoured table. In the bottom left hand corner, two graphs are presented representing the actual Hovmöller height data as a function of longitude (bottom), and a reconstruction of heights determined from a specific group of wavenumbers (top). Finally, along the right hand side, there are a number of height versus longitude graphs, one for each wavenumber. Other similarities between the figures include a solid line which is drawn vertically along 120 degrees north in all the height diagrams.

There are fundamental differences between the diagrams as well. Figure 1a provides information about how the waves are changing in time. The 'table' is shown in a larger format in figure 1b. It illustrates the change in amplitude (actually coefficient C) in time for wavenumbers 1 through 18. For convenience the high amplitude peaks have been shaded while nothing was done to the low amplitude centres. One notable remark is the oscillatory nature of the amplitudes. The most significant changes in the troughs and ridges are in the low wavenumbers (long waves), while the changes are much less dramatic and appear more sporatic in the shorter wavelengths. During the month of January and early February, the behaviour of wavenumber 2 was very significant to the weather pattern over the Eastern Pacific. However, this will be a topic of another report.

The height graphs are produced for a number of days (which are for those days indicated by the line of ***** on the 'contoured table'), with the latest being the darkest or widest line while the earliest day is depicted by a thin line. Figure 1a indicates the change in the cosine (combined sine and cosine) wave in time for the first 12 wavenumbers (shown beneath the 'table') is compared to the actual Hovmöller resultant wave pattern. This depiction provides a useful interpretation of how much of the wave is 'explained' by the longwave pattern.

The term 'explained' is fully explored in figure 2a. The emphasis in these depictions is on the variance (variation from the mean wave..wave number zero). In particular, the total variance and how much of the resultant wave is explained by wavenumbers 1 to 18 is given in the 'contoured table' (computed by dividing C squared by twice the total variance), also shown in Figure 2b. For example, with wavenumber 1 explaining 38% of the total wave followed by 27% due to wavenumber 3, 14% due to wavenumber 6, and 11% due to wavenumber 2. These wavenumbers are also depicted on the height versus longitude graphs, individually (along the right hand side) and collectively (top panel beneath the 'contoured table'). The summation of these waves explain 90% of the total wave. An interesting comparison can be made to January 18th at 00Z where wavenumber 2 accounted for 53% of the total wave and 86% of this wave was explained by the first 4 wavenumbers. A total variance of 200 dropped to 138 over a period of 5 days. This suggests that the energy was not just transferred from one wave to another within the latitude band, but energy was dispersed outside this band as well.

SUMMARY REMARKS

The advantage the fourier analysis provides is primarily two-fold. It indicates how the various wavelength disturbances are interacting with each other, and how the distribution of energy is changing in time. This form of analysis illustrates the dominant wavenumbers in the circulation pattern more easily than using the Hovmöller analysis. This information used in conjunction with the 500 mb analysis will assist in the meteorologist's assessments of the developing weather patterns.

REFERENCES

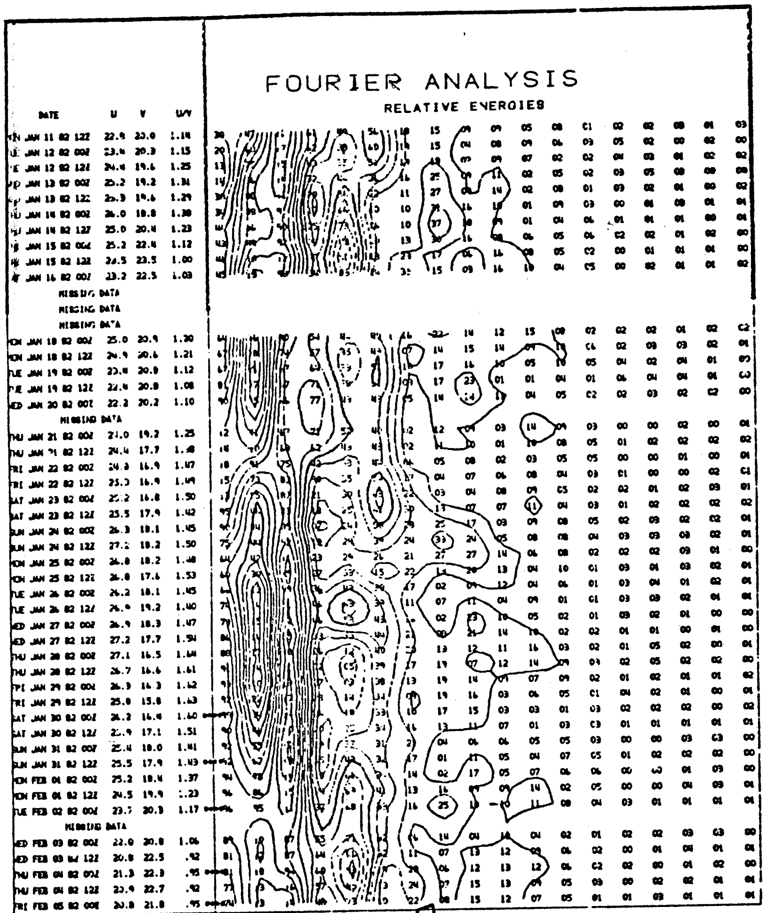
1. PRTN 82-003 Hovmöller Diagram Re-examined at the Pacific Weather Centre - David Grimes

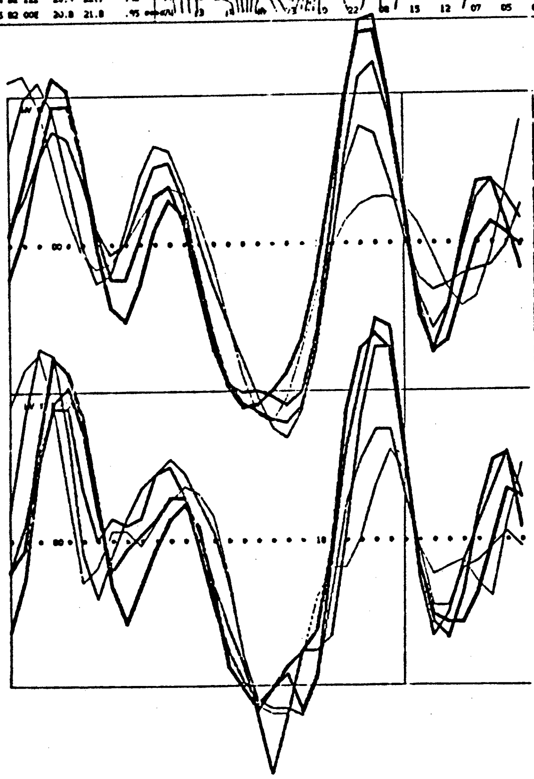
2. Some Applications of Statistics of Meteorology - Panofsky and Brier

FIGURE 1a.

Sample Output from PWC Computerized Fourier Analysis

CONTOURED TABLE





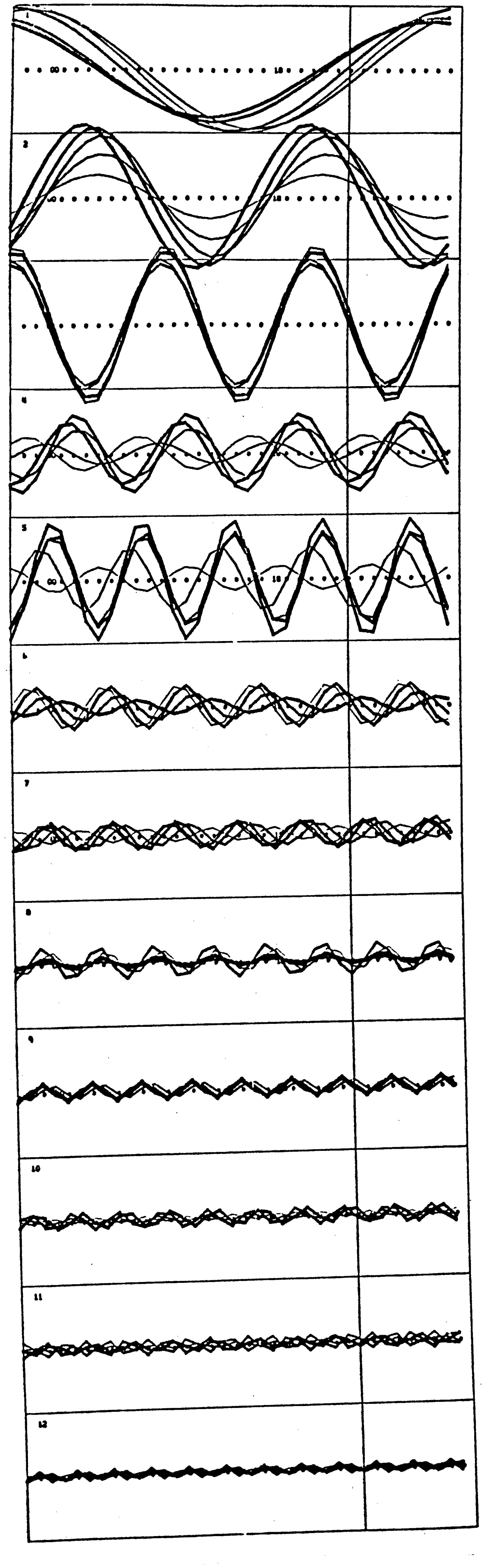


FIGURE 1b.

Enlarged Copy of Fourier Analysis in Figure 1a.

CONTOURED TABLE

DATE	U	V	U/V
MON JAN 11 82 12Z	22.8	20.0	1.14
TUE JAN 12 82 00Z	23.4	20.3	1.15
TUE JAN 12 82 12Z	24.4	19.6	1.25
WED JAN 13 82 00Z	25.2	19.2	1.31
WED JAN 13 82 12Z	25.3	19.6	1.29
THU JAN 14 82 00Z	26.0	18.8	1.38
THU JAN 14 82 12Z	25.0	20.4	1.23
FRI JAN 15 82 00Z	25.2	22.4	1.12
FRI JAN 15 82 12Z	23.5	23.5	1.00
SAT JAN 16 82 00Z	23.2	22.5	1.03
MISSING DATA			
MISSING DATA			
MISSING DATA			
MON JAN 18 82 00Z	25.0	20.9	1.20
MON JAN 18 82 12Z	24.9	20.6	1.21
TUE JAN 19 82 00Z	23.4	20.8	1.12
TUE JAN 19 82 12Z	22.4	20.8	1.08
WED JAN 20 82 00Z	22.2	20.2	1.10
MISSING DATA			
THU JAN 21 82 00Z	24.0	19.2	1.25
THU JAN 21 82 12Z	24.4	17.7	1.38
FRI JAN 22 82 00Z	24.8	16.9	1.47
FRI JAN 22 82 12Z	25.3	16.9	1.49
SAT JAN 23 82 00Z	25.2	16.8	1.50
SAT JAN 23 82 12Z	25.5	17.9	1.42
SUN JAN 24 82 00Z	26.3	18.1	1.45
SUN JAN 24 82 12Z	27.2	18.2	1.50
MON JAN 25 82 00Z	26.8	18.2	1.48
MON JAN 25 82 12Z	26.8	17.6	1.53
TUE JAN 26 82 00Z	26.2	18.1	1.45
TUE JAN 26 82 12Z	26.9	19.2	1.40
WED JAN 27 82 00Z	26.9	18.3	1.47
WED JAN 27 82 12Z	27.2	17.7	1.54
THU JAN 28 82 00Z	27.1	16.5	1.64
THU JAN 28 82 12Z	26.7	16.6	1.61
FRI JAN 29 82 00Z	26.3	16.3	1.62
FRI JAN 29 82 12Z	25.8	15.8	1.63
SAT JAN 30 82 00Z	26.2	16.4	1.60
SAT JAN 30 82 12Z	25.9	17.1	1.51
SUN JAN 31 82 00Z	25.4	18.0	1.41
SUN JAN 31 82 12Z	25.5	17.9	1.43
MON FEB 01 82 00Z	25.2	18.4	1.37
MON FEB 01 82 12Z	24.5	19.9	1.23
TUE FEB 02 82 00Z	23.7	20.3	1.17
MISSING DATA			
WED FEB 03 82 00Z	22.0	20.8	1.06
WED FEB 03 82 12Z	20.0	22.5	.92
THU FEB 04 82 00Z	21.3	22.3	.95
THU FEB 04 82 12Z	20.9	22.7	.92
FRI FEB 05 82 00Z	20.8	21.8	.95

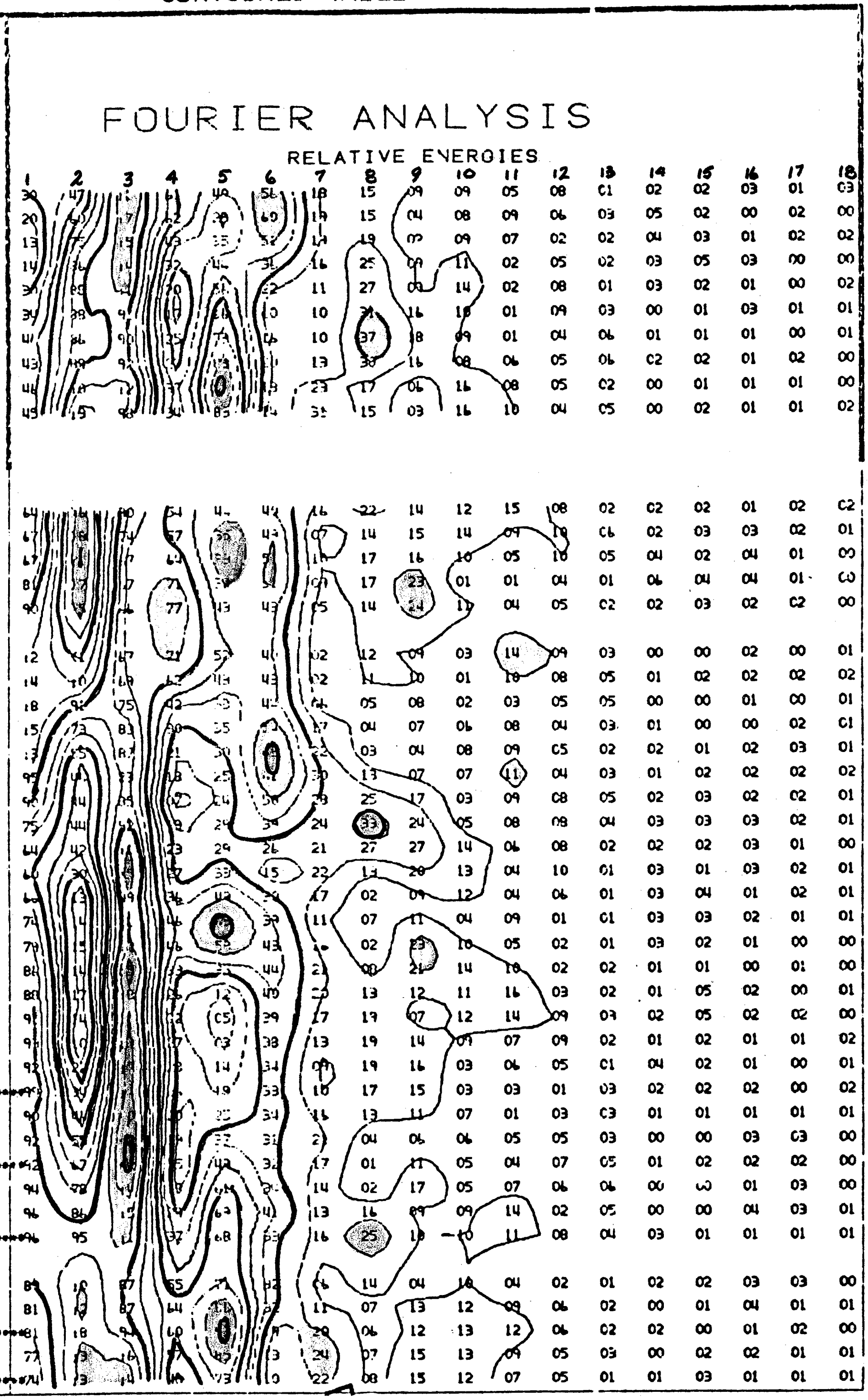
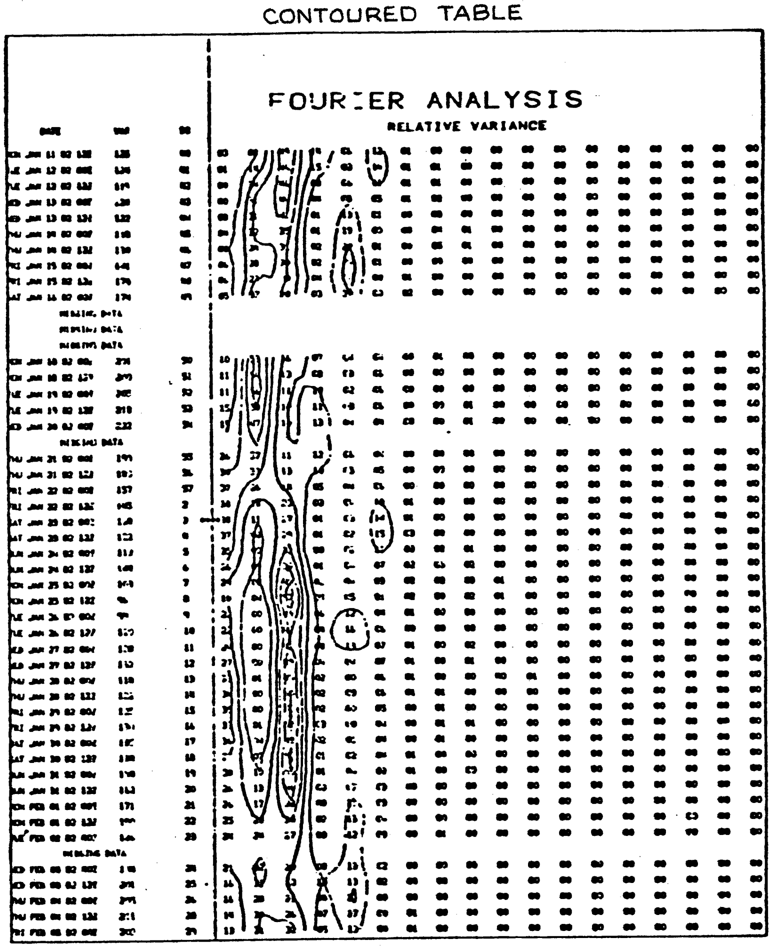
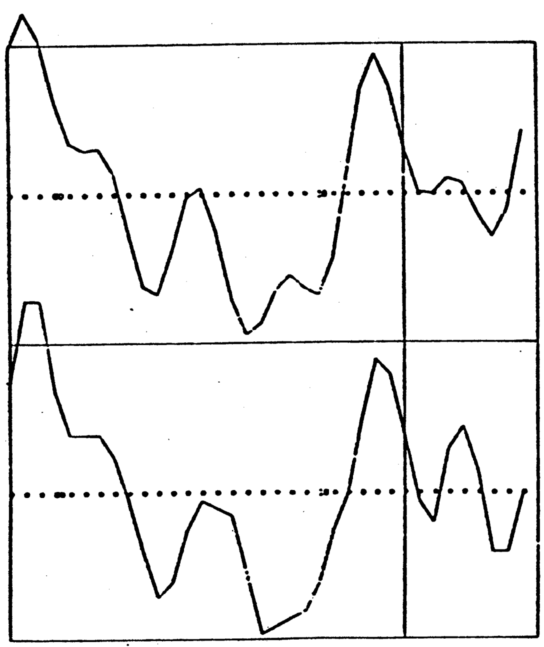


FIGURE 2a.

Sample Output from PWC Computerized Fourier Analysis

CONTOURED TABLE





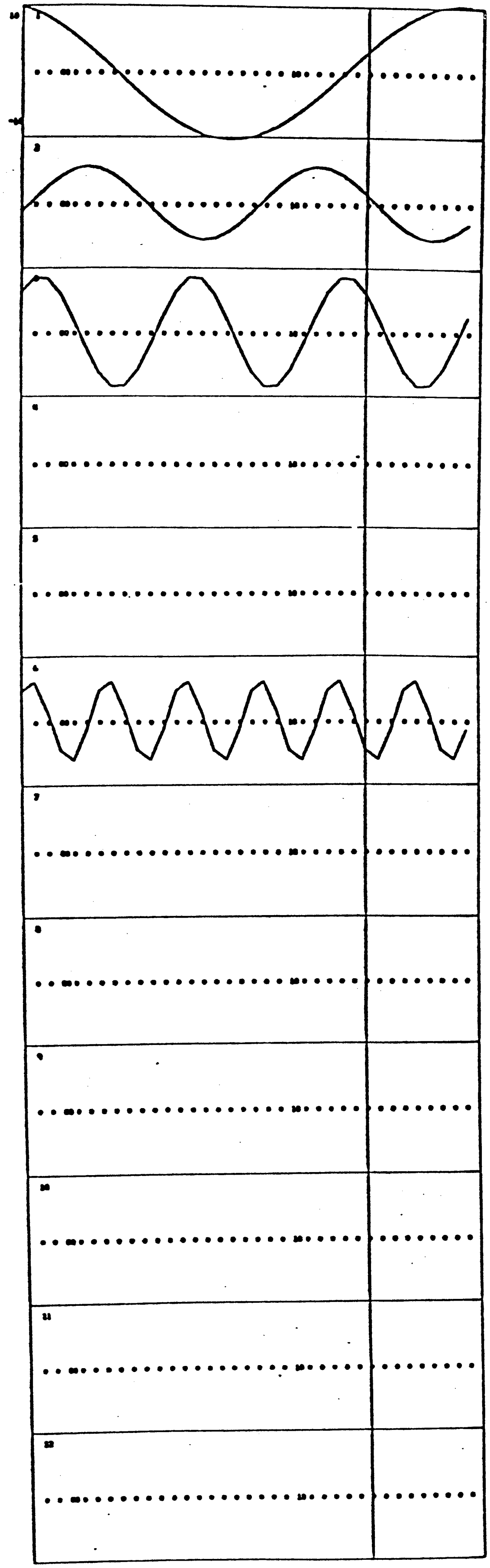


FIGURE 2b.

Enlarged Copy of Fourier Analysis in Figure 2a.

CONTOURED TABLE

Total Variance ↓

FOURIER ANALYSIS

RELATIVE VARIANCE

DATE	VAR	DS	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
MON JAN 11 82 12Z	125	40	03	09	49	14	06	12	01	00	00	00	00	00	00	00	00	00	00	00
TUE JAN 12 82 00Z	124	41	01	14	46	15	03	14	01	01	00	00	00	00	00	00	00	00	00	00
TUE JAN 12 82 12Z	119	42	00	[illegible]	46	08	05	11	01	01	00	00	00	00	00	00	00	00	00	00
WED JAN 13 82 00Z	120	43	00	34	44	04	08	05	01	02	00	00	00	00	00	00	00	00	00	00
WED JAN 13 82 12Z	122	44	03	31	41	01	13	01	00	03	00	00	00	00	00	00	00	00	00	00
THU JAN 14 82 00Z	118	45	04	32	35	01	19	00	00	04	01	00	00	00	00	00	00	00	00	00
THU JAN 14 82 12Z	130	46	08	28	31	02	20	01	00	05	01	00	00	00	00	00	00	00	00	00
FRI JAN 15 82 00Z	141	47	06	28	30	02	24	01	00	03	00	00	00	00	00	00	00	00	00	00
FRI JAN 15 82 12Z	170	48	06	22	30	04	[illegible]	00	01	00	00	00	00	00	00	00	00	00	00	00
SAT JAN 16 82 00Z	174	49	05	37	28	03	26	00	02	00	00	00	00	00	00	00	00	00	00	00
MISSING DATA																				
MISSING DATA																				
MISSING DATA																				
MON JAN 18 82 00Z	201	50	10	53	16	07	04	04	00	01	00	00	00	00	00	00	00	00	00	00
MON JAN 18 82 12Z	200	51	11	[illegible]	13	08	03	06	00	00	00	00	00	00	00	00	00	00	00	00
TUE JAN 19 82 00Z	205	52	11	[illegible]	11	10	02	06	00	00	00	00	00	00	00	00	00	00	00	00
TUE JAN 19 82 12Z	218	53	15	50	16	11	03	06	00	00	01	00	00	00	00	00	00	00	00	00
WED JAN 20 82 00Z	222	54	15	47	10	13	04	04	00	00	01	00	00	00	00	00	00	00	00	00
MISSING DATA																				
THU JAN 21 82 00Z	199	55	26	37	11	12	06	04	00	00	00	00	00	00	00	00	00	00	00	00
THU JAN 21 82 12Z	189	56	30	33	13	10	05	05	00	00	00	00	00	00	00	00	00	00	00	00
FRI JAN 22 82 00Z	157	57	37	26	18	05	04	06	00	00	00	00	00	00	00	00	00	00	00	00
FRI JAN 22 82 12Z	145	2	38	18	23	03	04	10	01	00	00	00	00	00	00	00	00	00	00	00
SAT JAN 23 82 00Z	138	3	38	11	27	01	03	14	01	00	00	00	00	00	00	00	00	00	00	00
SAT JAN 23 82 12Z	122	4	37	08	29	01	02	15	03	00	00	00	00	00	00	00	00	00	00	00
SUN JAN 24 82 00Z	113	5	35	08	33	00	02	11	03	02	01	00	00	00	00	00	00	00	00	00
SUN JAN 24 82 12Z	108	6	26	09	39	01	04	07	02	05	02	00	00	00	00	00	00	00	00	00
MON JAN 25 82 00Z	103	7	28	08	45	02	04	03	02	03	03	01	00	00	00	00	00	00	00	00
MON JAN 25 82 12Z	96	8	18	04	47	04	05	01	02	00	02	01	00	00	00	00	00	00	00	00
TUE JAN 26 82 00Z	99	9	22	00	[illegible]	06	12	04	01	00	00	00	00	00	00	00	00	00	00	00
TUE JAN 26 82 12Z	120	10	22	00	[illegible]	08	16	06	00	00	00	00	00	00	00	00	00	00	00	00
WED JAN 27 82 00Z	128	11	24	00	[illegible]	08	13	07	01	00	02	00	00	00	00	00	00	00	00	00
WED JAN 27 82 12Z	132	12	27	00	[illegible]	04	04	07	01	00	01	00	00	00	00	00	00	00	00	00
THU JAN 28 82 00Z	118	13	33	01	[illegible]	02	00	06	01	00	00	00	01	00	00	00	00	00	00	00
THU JAN 28 82 12Z	122	14	34	00	[illegible]	02	00	06	01	01	00	00	00	00	00	00	00	00	00	00
FRI JAN 29 82 00Z	125	15	35	00	[illegible]	02	00	05	00	01	00	00	00	00	00	00	00	00	00	00
FRI JAN 29 82 12Z	130	16	33	01	[illegible]	03	00	04	00	01	01	00	00	00	00	00	00	00	00	00
SAT JAN 30 82 00Z	135	17	36	04	48	02	01	04	00	01	00	00	00	00	00	00	00	00	00	00
SAT JAN 30 82 12Z	138	18	26	07	52	01	02	04	01	00	00	00	00	00	00	00	00	00	00	00
SUN JAN 31 82 00Z	150	19	28	10	50	01	04	03	01	00	00	00	00	00	00	00	00	00	00	00
SUN JAN 31 82 12Z	162	20	26	13	[illegible]	00	07	03	00	00	00	00	00	00	00	00	00	00	00	00
MON FEB 01 82 00Z	171	21	26	17	38	00	[illegible]	03	00	00	00	00	00	00	00	00	00	00	00	00
MON FEB 01 82 12Z	180	22	25	20	30	02	15	04	00	00	00	00	00	00	00	00	00	00	00	00
TUE FEB 02 82 00Z	186	23	24	24	27	03	12	03	00	01	00	00	00	00	00	00	00	00	00	00
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WED FEB 03 82 00Z	188	24	21	32	25	08	13	02	00	00	00	00	00	00	00	00	00	00	00	00
WED FEB 03 82 12Z	201	25	16	37	[illegible]	10	13	02	00	00	00	00	00	00	00	00	00	00	00	00
THU FEB 04 82 00Z	205	26	16	28	27	08	[illegible]	00	00	00	00	00	00	00	00	00	00	00	00	00
THU FEB 04 82 12Z	211	28	14	30	26	07	17	00	01	00	00	00	00	00	00	00	00	00	00	00
FRI FEB 05 82 00Z	202	29	13	31	32	05	13	00	01	00	00	00	00	00	00	00	00	00	00	00