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EFFICIENCY, EQUITY AND REGULATION:
AN ECONOMETRIC MODEL OF BELL CANADA

Final Report

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EFFICIENCY, EQUITY AND REGULATION: AN ECONOMETRIC MODEL OF BELL CANADA

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March, 1980

The Canadian Department of Communications (DOC), contracted with the Institute of Applied Economic Research (IAER), of Concordia University (contract # 03SU.36100-9-9529) to construct an econometric and policy simulation model of Bell Canada. The work was undertaken during the period July 1st, 1979 to March 31st 1980, by the following research team:

PRINCIPAL INVESTIGATORS: Professor Jon A. Breslaw
Professor J. Barry Smith

RESEARCH ASSISTANT: Gwenn Hughes

RESEARCH ADVISORS: Professor V. Corbo
Professor A.G. Jackson

We would like to thank A. de Fontenay and P. Neogi of the DOC for their cooperation and discussions during this study, as well our thanks go to Jeffrey Rohlf of Bell Laboratories. We also thank Melanie Neufield who provided excellent secretarial assistance throughout this project, and we wish to thank Esther Massa and Mary-Ann Michalak for their help in typing this report.

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Part 1 INTRODUCTION

The research presented in this report reflects two goals. The first is to provide an econometric model of Bell Canada which can be used to study characteristics of demand and the underlying production process. The second goal is to use the econometric model in a social welfare framework in order to address policy questions related to the (socially) optimal pricing of telecommunications services supplied by Bell. Research in these directions has been underway at the IAER for several years. This report includes many of the contributions of the previous reports.

The need to develop a framework in order to ascertain what are "equitable and efficient" prices is one of the outcomes of a Federal-Provincial conference of communication Ministers, Oct. 16-17th, 1979. This working group reached a consensus of policy objectives which must be satisfied in order that the public interest be served. One of the policy objectives stated that:

> Developing and maintaining an efficient telecommunications infrastructure which can provide universal access to a broad range of telecommunications services at economic and equitable rates is a fundamental goal of public policy.

The report has the following structure.

In Part 2 we examine four background studies undertaken at the beginning of the project with the goals of summarizing past research and clarifying the directions to be taken in this project. In Part 3, the results of the Interim Report of this project are summarized; Parts 2 and 3 thereby provide a complete description of the foundation on which this final report rests.

In Part 4, the IAER cost and demand models are specified. The cost model uses a translog function to approximate a two-output three-input production process. The demand models include aggregate and disaggregated service (output) equations for business and residential users.

In Part 5, the results of estimating the cost and demand models are presented. This part includes a discussion of the parameter estimate of the models as well as the deduced characteristics of the underlying production technology.

In Part 6, the accuracy of the models is verified by simulation of the estimated equations over the sample period and subsequent statistical comparisons of the simulated and historic series for the endogenous variables.

In Part 7, characteristics of efficient and equitable telecommunication prices are derived from a formal model of social welfare maximization. This model incorporates the contribution of Ramsey and Feldstein.

In Part 8, the model of Part 7 is simulated under assumption relating to the relative importance of efficiency and equity. The simulated results are compared to historic values and the differences are discussed.

The conclusions of the report are presented in Part 9. Given the limitations of the methodology it still appears reasonable to conclude that there would be a welfare gain associated with small increases in local residential prices and much larger reduction in residential message toll prices. All of the results incorporate a minimum (historic) profit constraint on the operations of Bell Canada.

Part 2 BACKGROUND STUDIES

In this section we report on four background studies which were undertaken at the beginning of this project with the goals of summarizing past research and clarifying the empirical directions to be taken during this project. In these previous works we had simultaneously examined multi-input multi-output cost and production models of the Bell Canada production process. As well, one of the guiding assumptions had been that the regulatory process had had its principal effect through setting the price of local services and that the rate of return constraint faced by Bell was of secondary importance. The background studies summarized here led to the conclusion that continued research effort should not be directed towards estimation of multi-output production functions. As well, considerable support was generated for the assumption that rate of return regulation was not binding.

The discussion of the background work is presented in the following two sections. The major findings and inter-relationships of the studies are summarized. The actual background studies were previously supplied to the Department of Communications and are therefore not included in the final report. Additional copies of these studies are available on request.

I STUDIES RELATED TO THE USE OF COST AND PRODUCTION FUNCTIONS IN THE STUDY OF TECHNOLOGIES

The following two background papers cast light upon the issues involved in the specification of technology:

1) More Pitfalls in the Testing of Duality Theory. (Breslaw and Smith, 1979).

2) A Micro Test of the Neoclassical Production Theory. (Breslaw, Corbo and Smith, 1979).

The Pitfalls paper demonstrated two important results. The first is that one output translog cost functions provide a more general specification of production technologies than do standard one output translog production functions. This result arises from the fact that one output production functions usually explicitly specify output as the dependent variable (ie. output is functionally dependent on inputs). In functional notation, output is the left-hand side variable, and the right-hand side of the production relation consists of a function of the inputs, viz;

(1.1) $q=f(x)$ where: x is a vector of inputs
: q is output
: f is the production function

However, in the above case, output is explicitly separable from inputs. This separability is not encountered in the standard cost function specification:

(1.2) $C=C(r,q)$ where: C is cost
: r is a vector of input prices
: q is output

The importance of the separability issue can only be assessed empirically. To this end, translog-based models corresponding to the separable production model (1.1) and non-separable cost model (1.2) were estimated. It was noted that the cost model was much more robust than the production model. Thus, one should feel much more confident using cost model estimates of technology in the one output case, with this data set. As well, one has now both theoretical and empirical grounds for less concern over the Appelbaum and Burgess results that (separable) production and (non-separable) cost models provide dissimilar estimates of characteristics of the aggregate U.S. economy.

Two important additional results were generated in the <u>Pitfalls</u> paper. The first was that only in the case of one output homogeneous production functions could parameters of a non-separable production function model be estimated uniquely. The second, and related result is that <u>multi-output</u> production functions cannot be reliably estimated. The reasoning behind this result lies in the fact that multi-output production surfaces (and, in fact, non-separable single output production surfaces) must be specified in implicit form as:

wasn't this issue already raised by Diewert? see Christensen on TFP in rail (CDN)

(3) $F(\bar{q},x) = 0$ where: $\bar{q}$ is the output vector
: x is the input vector

However, for practical estimation purposes (for example, a translog approximation of F), a dependent variable must be specified and it is straightforward to show that (a) there is no natural choice of a dependent variable and (b) estimated properties of the underlying technology will change with every different dependent variable. ⇒ this doesn't exclude some optim. choice of the endog. variable - say through non nested test. (granted ∃ no theory for which one

The <u>Micro Test</u> paper empirically demonstrated at the level of a firm (as opposed to the aggregate economy level) that one cannot reject the neoclassical model that the production function is related to the optimal choice of inputs (or, side-order marginal rate of technical substitution optimality conditions). This result is important since it differs from the aggregate U.S. results of Appelbaum and it therefore supplies some support for the

note, the F should only be a scalar!

neoclassical view that production functions and optimization behavior provide a reasonable vehicle for approximating technologies and decision making within firms. These results however can be questioned given that the production function used was separable between inputs and outputs.

II STUDIES RELATED TO THE IMPORTANCE OF RATE OF RETURN CONSTRAINT IN BELL DECISION MAKING

The following two studies examine issues related to the modelling of rate of return constraint and the empirical importance of such a constraint in studying the Bell Canada production process:

1) The Restrictiveness of Flexible Functional Forms in the Modelling of Regulatory Constraint. (Breslaw, Corbo and Smith, 1979).

2) A Direct Test of the A-J Effect: The Case of Bell Canada. (Breslaw, Corbo and Smith, 1979). to be compared to Fuss' model.

In the Restrictiveness paper it is shown that second-order approximations of cost functions are not suitable in general for modelling rate of return constraint. The problems arise from the fact that a rate of return constraint (when it is binding) implies that the optimal factor mix is independent of the user cost of capital The regulated firm will instead make its factor decisions with respect to the allowed rate of return - the maximum allowed cost of capital. However, this independence result implies a set of (derivative) restrictions upon the cost function. Unless factor shares are effectively constant, the standard second-order approximate cost functions will not satisfy these additional constraints.

There are three approaches to the solution of this problem.

<u>First</u>, it can be shown that a third-order approximation is sufficiently flexible to incorporate the additional restrictions. Unfortunately, the number of parameters to be estimated increases geometrically with the order of approximation and this leads to major computational problems given existing technology.

<u>Secondly</u>, it is possible to consider estimating a model of regulatory constraint with the additional restrictions imposed at the mean of the sample only. It was decided that such an approach would not be desirable for this project given that any conclusions drawn would be valid only at the mean.

<u>Finally</u>, it is possible to design a test of rate of return constraint by using a production function approach. The results of this approach are reported in the A-J paper summarized below.

Within a production model of a cost minimizing firm, the effect of a rate of return constraint can be examined in terms of the Lagrange multiplier associated with the constraint. However, it is not a straightforward matter to estimate this Lagrange multiplier from time series data. The problem arises from the fact that the multiplier will differ from year to year and even without taking account of the parameters of the production model, there are as many Lagrange multiplier parameters as data points. Given that it is not reasonable to specify the multiplier as a single (constant) parameter, a modified method was introduced in order to assess the impact of a regulatory rate of return constraint.

An iterative technique (similar to one advanced by Houthakker) was used to estimate the Lagrange multipliers. Since a straight-forward technique was not available for analyzing the individual significance of the multipliers, a series of simulation experiments were designed in order to assess the performance of the model when the inputs were endogeneous and the rate of return constraint was part of the simulated system. Two regimes were utilized - the allowed rate of return being that rate specified by the regulatory authorities, and the rate being the observed rate of return. In every case the tracking of the model which included the specified rate of return constraint was inferior to the tracking of the model when the Lagrange multiplier was constrained to be zero. On the basis of these simulation results, it was concluded that there would be no loss associated with ignoring the rate of return constraint in this project.

Part 3 SUMMARY OF THE INTERIM REPORT

An interim report for this project was forwarded in December, 1979. The purpose of this interim report was to link previous project results (Smith and Corbo, 1979) and (Breslaw, Corbo and Dufour, 1979) with the direction of the new project and to summarize some of the preliminary empirical results which had been generated under the new project.

In the interim report the estimation results of a translog 3 input - 3 output cost and demand model of Bell Canada were presented. Equations corresponding to the demand and cost subsections of the model were estimated simultaneously. Although the specification of the demand model equations showed significant improvement over previous models, serious computational problems were experienced during the simultaneous estimation of the cost and demand model. As well, when detailed properties of the underlying technology were studied it was noted that one of the underlying assumptions of the model - that of profit maximizing behavior with respect to competitive services, was not satisfied. Although we were pleased with the feasibility of estimating the complete model, convergence difficulties and the behavioral properties associated with competitive services indicated that serious problems remained with the specification of the model and the data. Effort was therefore allocated towards solving these problems.

Diagnosis

A detailed examination of the competitive services data (with WATS excluded) isolated the source of the problems. In particular, it was noted that the demand for competitive services was insensitive to price variation. The explanation of this problem lay in the Bell construction of the (1967 constant dollar) output and price series. Bell had constructed these series using a chained Laspeyres price index. Such an index is reasonable when the major underlying components of the output composite bear stable relationships to each other. However, in the case of competitive services

a chained index is not dep. of consid. over time!

where, over time, new services had been added at staggered intervals and had developed revenue shares greater than the services offered before 1967, a chained Laspeyres index did not provide a useful representation of price and output. At the same time, it was determined that Bell did not have the data to provide more satisfactory price and output series. Finally, it was noted that message toll and local services contained a much greater degree of component stability and that the 1967 constant dollar output and price series were thus likely to be more reasonable.

this may be valid but it has not been justified. the problem may be in the high degree of substitution

Action Taken

On the basis of these findings it was agreed (with the Department of Communications) to continue the development of the model but with competitive services excluded from the demand and cost equations. As well, miscellaneous and directory services were excluded from the model. Finally, the factor series for labour, capital and material were adjusted downward to reflect the fact that some of ?what the inputs were necessary to produce the excluded services. The downward adjustment factor was taken as the proportion of total revenue contributed by the excluded services. In 1964 this was 10%.

Part 4 THE IAER MODEL OF THE BELL CANADA PRODUCTION PROCESS

Section 4.1 The Cost Model

In this section we formally describe a three-input two-output cost model of the Bell Canada production process. The inputs include: labour, capital and materials. The outputs are: local services and message toll services, including WATS. It will be noted that profit maximizing behavior is assumed for message toll services. It is assumed as well that regulation results in Bell satisfying demand for local services at the regulated price. The production technology is represented through a cost function.

Specification

A second order logarithmic expansion (translog function) is used to approximate the cost function resulting from the problem of finding that factor mix which minimizes the cost of producing a given output vector. In particular, it is assumed that cost is related to factor prices, output and technology according to equation 4.1. The definitions of all variables introduced can be found in Table 4.1.

There is a set of properties that a cost function must exhibit in order to be consistent with the minimization problem described above. In particular, the cost function must be homogeneous of degree one and concave in factor prices. The concavity property can be expressed in terms of determinants of minors of the factor price Hessian of the cost function. Concavity is not a universal property of translog cost functions and must therefore be verified at each data point.

TABLE 4.1

VARIABLE DEFINITIONS: COST MODEL

C	=	total cost in current dollars = wL+rK+vM
L	=	weighted man hours with weights given by the 1967 wage structure
w	=	wage rate = total wage bill divided by L
K	=	net capital stock in 1967 dollars
r	=	user cost of capital derived using the Hall and Jorgenson (1971) formula and allowing for capital gains
M	=	index of raw materials, supplies and uncollectables in 1967 dollars
v	=	price index of raw materials
T	=	technology index of switching and accessability to the system
QL	=	quantity index of local services in 1967 dollars
PQL	=	price index of local services (1967=1)
QM	=	quantity index of intra territory adjacent, trans-Canada, US and Overseas basic toll services and WATS in 1967 dollars.
PQM	=	price index of QM (1967=1)

CHART 4.1

Equation 4.1 3 INPUT - 2 OUTPUT (SYMMETRIC) TRANSLOG COST FUNCTION

$$
\begin{aligned}
\ell nC = CC_o &+ C_w \ell nw + C_r \ell nr + C_v \ell nv + C_T \ell nT + C_{QL} \ell nQL + C_{QM} \ell nQM \\
&+ \tfrac{1}{2} \ell nw \left[C_{ww} \ell nw + C_{wr} \ell nr + C_{wv} \ell nv + C_{wT} \ell nT + C_{wQL} \ell nQL + C_{wQM} \ell nQM \right] \\
&+ \tfrac{1}{2} \ell nr \left[C_{wr} \ell nw + C_{rr} \ell nr + C_{rv} \ell nv + C_{rT} \ell nT + C_{rQL} \ell nQL + C_{rQM} \ell nQM \right] \\
&+ \tfrac{1}{2} \ell nv \left[C_{wv} \ell nw + C_{rv} \ell nr + C_{vv} \ell nv + C_{vT} \ell nT + C_{vQL} \ell nQL + C_{vQM} \ell nQM \right] \\
&+ \tfrac{1}{2} \ell nT \left[C_{wT} \ell nw + C_{rT} \ell nr + C_{vT} \ell nv + C_{TT} \ell nT + C_{QLT} \ell nQL + C_{QMT} \ell nQM \right] \\
&+ \tfrac{1}{2} \ell nQL \left[C_{wQL} \ell nw + C_{rQL} \ell nr + C_{vQL} \ell nv + C_{QLT} \ell nT + C_{QLQL} \ell nQL + C_{QMQL} \ell nQM \right] \\
&+ \tfrac{1}{2} \ell nQM \left[C_{wQM} \ell nw + C_{rQM} \ell nr + C_{vQM} \ell nv + C_{QMT} \ell nT + C_{QMQL} \ell nQL + C_{QMQM} \ell nQM \right]
\end{aligned}
$$

Alternatively, homogeneity of degree one in factor prices (or equivalently, addition of the derived factor share equations to unity) can be directly imposed by parameter restrictions. These restrictions can be deduced from the factor shares presented as equations 4.2, 4.3, and 4.4. These factor shares reflect Sheppard's Lemma which states that the partial derivatives of a cost function with respect to a factor prices must equal the cost minimizing factor input demands. Vertically adding equations 4.2, 4.3 and 4.4 we note the following seven independent restrictions implied by homogeneity:

$$R_1: \quad C_w + C_r + C_v = 1$$
$$R_2: \quad C_{ww} + C_{wr} + C_{wv} = 0$$
$$R_3: \quad C_{wr} + C_{rr} + C_{rv} = 0$$
$$R_4: \quad C_{wv} + C_{rv} + C_{vv} = 0$$
$$R_5: \quad C_{wT} + C_{rT} + C_{vT} = 0$$
$$R_6: \quad C_{wQL} + C_{rQL} + C_{vQL} = 0$$
$$R_7: \quad C_{wQM} + C_{rQM} + C_{vQM} = 0$$

Profit Maximization

The assumption of profit maximization in the provision of message toll services implies that marginal cost of the service is equated to marginal revenue. A convenient way of writing this condition for a translog cost function is in terms of the value of marginal revenue share equation. This is presented in equation 4.5 where MR_{QM} is the marginal revenue of message toll services.

Summary Information and Statistics from the Cost Function

Following estimation of the entire model and verification of the relevant concavity and profit maximization well-behavedness conditions,

CHART 4.2

DERIVED COST MINIMIZING FACTOR SHARE EQUATIONS

4.2 $$\frac{wL}{C} = C_w + C_{ww}\ln w + C_{wr}\ln r + C_{wv}\ln v + C_{wT}\ln T + C_{wQL}\ln QL + C_{wQM}\ln QM$$

4.3 $$\frac{rK}{C} = C_r + C_{wr}\ln w + C_{rr}\ln r + C_{rv}\ln v + C_{vT}\ln T + C_{rQL}\ln QL + C_{rQM}\ln QM$$

4.4 $$\frac{vM}{C} = C_v + C_{wv}\ln w + C_{rv}\ln r + C_{vv}\ln v + C_{vT}\ln T + C_{vQL}\ln QL + C_{vQM}\ln QM$$

CHART 4.3

DERIVED PROFIT MAXIMIZING VALUE OF MARGINAL REVENUE SHARE EQUATION

4.5 $$\frac{MR_{QM}\cdot QM}{C} = C_{QM} + C_{wQM}\ln w + C_{rQM}\ln r + C_{vQM}\ln v + C_{QMT}\ln T + C_{QMQL}\ln QL + C_{QMQM}\ln QM$$

properties of the cost model are examined with a goal to understanding characteristics of the Bell production process. In particular, marginal costs, cost complementarities, ray scale economies, economies of scope, own and cross factor demand elasticities as well as elasticities of substitution are examined.

The formulae for the summary statistics are given by equations 4.6-4.21. It will be noted that a sufficient condition for economies of scope between two services is that cost complementarities are significantly negative.

CHART 4.4

TECHNOLOGY SUMMARY STATISTICS EQUATIONS

Marginal Cost Equations

4.6 (LOCAL SERVICES) $$MC_{QL} = \left(\frac{C}{QL}\right)\left[C_{QL} + C_{wQL}\ell nw + C_{rQL}\ell nr + C_{vQL}\ell nv + C_{QLT}\ell nT + C_{QLQL}\ell nQL + C_{QMQL}\ell nQM\right]$$

4.7 (MESSAGE TOLL SERVICES) $$MC_{QM} = \frac{C}{QM}\left[C_{QM} + C_{wQM}\ell nw ++ C_{rQM}\ell nr + C_{vQM}\ell nv + C_{QMT}\ell nT + C_{QMQL}\ell nQL + C_{QMQM}\ell nQM\right]$$

$\frac{\partial c}{\partial \ln M} \cdot \frac{1}{M}$

Cost Complementarity Formula

4.8 LOCAL - MESSAGE TOLL $$\frac{\partial MC_{QL}}{\partial QM} = \frac{MC_{QL} \cdot MC_{QM}}{C} - \frac{C_{QMQL} \cdot C}{QL.QM}$$

C^2 $+$?

$$\frac{\partial MC_L}{\partial M} = \left(\frac{C}{Q}\right)\frac{c_{ML}}{M} + \frac{[L]}{L}\frac{\partial c}{\partial M}$$

$$\frac{MC_L}{C} \cdot \frac{[M]}{M}$$

Ray Scale Economies (Ray Cost Elasticity)

4.9 SCALE $$= \left[\frac{MC_{QL} \cdot QL}{C} + \frac{MC_{QM} \cdot QM}{C}\right]$$

Cost Minimizing Factor Demands

4.10 $$L^* = \frac{\partial C}{\partial w} = \left(\frac{C}{w}\right)\left[C_w + C_{ww}\ell nw + C_{wr}\ell nr + C_{wv}\ell nv + C_{wT}\ell nT + C_{wQL}\ell nQL + C_{wQM}\ell nQM\right]$$

4.11 $$K^* = \frac{\partial C}{\partial r} = \left(\frac{C}{r}\right)\left[C_r + C_{wr}\ell nw + C_{rr}\ell nr + C_{rv}\ell nv + C_{rT}\ell nT + C_{rQL}\ell nQL + C_{rQM}\ell nQM\right]$$

4.12 $$M^* = \frac{\partial C}{\partial v} = \left(\frac{C}{v}\right)\left[C_v + C_{wv}\ell nw + C_{rv}\ell nr + C_{vv}\ell nv + C_{vT}\ell nT + C_{vQL}\ell nQL + C_{vQM}\ell nQM\right]$$

CHART 4.4 (continued)

Factor Price Elasticities

$E_{LL} = \frac{\partial L}{\partial w} \cdot \frac{w}{L}$ $\quad L = \frac{\partial C}{\partial w} =$

$= \frac{\partial^2 C}{\partial w} \cdot \frac{w}{L} \overset{?}{=} \frac{\partial^2 C}{\partial w} \cdot 1/\frac{\partial C}{\partial w} \cdot w$? why not?

4.13 (Labour-Labour) $E_{LL} = \frac{\partial^2 C}{\partial w^2} \cdot \frac{w}{L}$

4.14 (Labour-Capital) $E_{LK} = \frac{\partial^2 C}{\partial w \partial r} \cdot \frac{r}{L}$

4.15 (Labour-Materials) $E_{LM} = \frac{\partial^2 C}{\partial w \partial v} \cdot \frac{v}{L}$

4.16 (Capital-Capital) $E_{KK} = \frac{\partial^2 C}{\partial r^2} \cdot \frac{r}{K}$

4.17 (Capital-Materials) $E_{KM} = \frac{\partial^2 C}{\partial r \partial v} \cdot \frac{v}{K}$

4.18 (Materials-Materials) $E_{MM} = \frac{\partial^2 C}{\partial v^2} \cdot \frac{v}{M}$

Elasticities of Substitution

4.19 (Labour-Capital) $S_{LK} = \frac{E_{LK} . C}{r . K}$

4.20 (Labour-Materials) $S_{LM} = \frac{E_{LM} . C}{v . M}$

4.21 (Capital-Materials) $S_{KM} = \frac{E_{KM} . C}{v . M}$

Section 4.2 The Demand Model

The demand model specified in this section provides a significant advance over previous projects from the standpoints of level of aggregation and fit. Data provided by the Department of Communications on local rates by category of service, and on 1979 quantities permitted the construction of a Laspeyres price index for both residential and business local services. (A similar disaggregation of message toll is still not possible). As well, after extensive experimentation with flexible functional forms and revenue functions it was found that the isoelastic functional forms provided the best fit of the DOUBLE LOG data. The assumption of previous projects that outputs and income should be expressed on a per person basis was relaxed and the fit WHY? improved significantly. As well, the use of real personal consumption WHY? expenditures as a proxy for real permanent disposable income helped reduce the spurious serial correlation which had arisen in previous projects. The serial correlation problem had arisen because of the EXPLAIN & JUSTIFY. large transitory swings in real disposable income during the post Korean war expansion and recession.

The demand model is estimated at both levels of aggregation. The demand equations for aggregate local and message toll services are presented in Chart 4.5. In Chart 4.6, the dissagregated demands for business and residential local service are shown. The variable definitions are presented in Table 4.2.

CHART 4.5

AGGREGATE DEMAND FUNCTIONS

Local Services

CONS.

4.22 $$\ln(QL) = A_0 + A_1 \ln\left(\frac{PQL}{CPI}\right) + A_2 \ln\left(\frac{YD}{CPI}\right) + A_3 \ln(POP) + A_4 \cdot \ln(CONV) + AD_1 \cdot D_{59} + AD_2 \cdot D_{70}$$

Message Toll Service

4.23 $$\ln(QM) = B_0 + B_1 \ln\left(\frac{PQM}{CPI}\right) + B_2 \ln\left(\frac{YD}{CPI}\right) + B_3 \ln(POP) + BD_1 \cdot D_{59} + BD_2 \cdot D_{70}$$

CHART 4.6

DISAGGREGATED DEMAND FUNCTIONS: LOCAL SERVICES

4.24 $$\ln(QLR) = RA_0 + RA_1 \ln\left(\frac{PQLR}{CPI}\right) + RA_2 \ln\left(\frac{YD}{CPI}\right) + RA_3 \ln(POP) + RH_4 \ln(CONV) + RD_1 \cdot D_{59} + RD_2 \cdot D_{70}$$

4.25 $$\ln(QLB) = BA_0 + BA_1 \ln\left(\frac{PQLB}{CPI}\right) + BA_2 \ln\left(\frac{YD}{CPI}\right) + BA_3 \ln(POP) + BA_4 \ln(CONV) + BD_1 \cdot D_{59} + BD_2 \cdot D_{70}$$

TABLE 4.2

VARIABLE DEFINITIONS: DEMAND MODELS

CPI	=	consumer price index, Canada (1967=1)
YD	=	disposable income approximated by consumption expenditure
CONV	=	Local conversations per person, Bell network
POP	=	population in the Bell territory
D_{59}	=	dummy variable (= 1, 1959 +)
D_{70}	=	dummy variable (= 1, 1970 +)
QL	=	quantity index of local services in 1967 dollars.
QM	=	quantity index of intra territory adjacent, trans-Canada, U.S. and Overseas message toll services and WATS in 1967 dollars (basic toll services)
PQL	=	price index of local services (1967 = 1)
PQM	=	price index of basic toll services (1967 = 1)
PQLR	=	price of local residential services (1967 = 1)
QLR	=	quantity of local residential services
PQLB	=	price of local business services (1967 = 1)
QLB	=	quantity of local business services

The specified demand equations are seen to depend upon price income and population in a log-linear function. As well, all of the demand equations include two dummy variables D_{59} which corresponds to any taste shifts occasioned by the availability and importance of direct distance dialing and D_{70} which corresponds to the restructuring of rates for long distance calls to a minimum of one minute. Finally, all local demand equations include a conversation per person variable reflecting any continuous restructuring of tastes towards greater reliance upon the local telephone network.

DDD was introd. progressiv.

~~What~~ was the shift in 1 min in '70 or rather in '71?

Part 5 ESTIMATION OF THE COST AND DEMAND MODELS

In the following sections, the parameter estimates and summary statistics from the demand and cost models are presented. In section 5.1, the separate estimation of the demand equations is considered. In section 5.2 the results of the simultaneous estimation of the cost and demand models are examined.

Section 5.1 The Demand Models

We begin by considering the results for the aggregate demands for local and message toll services given by equations 4.22 and 4.23. The equation by equation and system results are presented in Table 5.1.

It is clear that the demand equations fit well. There is no evidence of serial correlation in the residuals. As well, there is no significant changes in the parameter estimates when they are estimated as a seemingly unrelated system as opposed to individually. It should be noted that the demand for aggregate local services is inelastic (-.52) whereas message toll services are elastically demanded (-1.3). These elasticity estimates are reasonably similar to the results of previous models. The better specification of local services seems to have resulted in a higher elasticity estimate for that service whereas the inclusion of WATS services in message toll has resulted in a slightly smaller point estimate of the demand elasticity.

The estimation results for the disaggregated system of local service equations (4.24 and 4.25) is presented in Table 5.2. Three points should be made with respect to these results.

TABLE 5.1
PARAMETER ESTIMATES OF AGGREGATE DEMAND MODEL EQUATION BY EQUATION

LOCAL SERVICES

Parameter	Estimate	Standard Error
A_o	-3.979*	.717
A_1	-.519*	.087
A_2	.438*	.108
A_3	1.042*	.090
A_4	.426*	.136
AD_1	.055*	.012
AD_2	.019	.012
D.W.	1.66	
Log of likelihood function	81.78	

MESSAGE TOLL SERVICES

Parameter		Standard Error
B_o	-5.131*	.893
B_1	-1.292*	.081
B_2	.805*	.106
B_3	.805*	.113
BD_1	.028*	.015
BD_2	.105*	.015
D.W.	1.93	
Log of likelihood function	58.07	

* Significant at the 5% level.

TABLE 5.1 (Continued)

SYSTEM

Parameter	Estimate	Standard Error
A_0	-4.046*	.597
A_1	-.511*	.072
A_2	.439*	.089
A_3	1.038*	.076
A_4	.437*	.108
AD_1	.054*	.010
AD_2	.019*	.009
B_0	-5.054*	.882
B_1	-1.30*	.080
B_2	.797*	.104
B_3	.811*	.113
BD_1	.028*	.015
BD_2	.106	.015
Log of likelihood function	153.185	

Equation	D.W.
Local	1.68
Message	1.94

TABLE 5.2

BUSINESS AND RESIDENTIAL LOCAL DEMAND EQUATIONS

RESIDENTIAL

Parameter	Estimate	Standard Error
RA_0	-3.365*	1.067
RA_1	-.395*	.115
RA_2	.337*	.153
RA_3	.924*	.141
RA_4	.429*	.179
RD_1	.039*	.016
RD_2	.027*	.015
D.W. 1.05	LOG OF LIKELIHOOD 75.068	

BUSINESS

Parameter	Estimate	Standard Error
BA_0	-5.492*	.815
BA_1	-.706*	.104
BA_2	.492*	.126
BA_3	1.140*	.109
BA_4	.434*	.165
BD_1	.062*	.016
BD_2	.028*	.014
D.W. 1.56	LOG OF LIKELIHOOD 77.071	

TABLE 5.2 (Continued)

SYSTEM

Parameter	Estimate	Standard Error
RA_0	-2.936*	.855
RA_1	-.445*	.091
RA_2	.293*	.126
RA_3	.969*	.116
RA_4	.416*	.152
RD_1	.042*	.014
RD_2	.027*	.013
BA_0	-4.949*	.664
BA_1	-.784*	.083
BA_2	.442*	.106
BA_3	1.188*	.091
BA_4	.412*	.142
BD_1	.069*	.014
BD_2	.026*	.012
Log of likelihood function	159.296	

D.W. Residential (.96) Business (1.47)

In the first place, both equations fit well with the business equation performing slightly better. In both the equation by equation results and the system results the Durbin Watson statistic for the residential equation is low - but still in the inconclusive range. An examination of the residuals of this equation suggests that the low statistic arises for only spurious reasons.

Secondly, as in the aggregate demand equations, the move from separate to seemingly unrelated system estimation yields no significant changes in the parameter estimates.

Finally, the business and residential parameter estimates bracket the corresponding aggregate parameter estimates. This result is comforting especially given that the disaggregated system was not constrained to satisfy this condition.

Section 5.2 The Simultaneous Cost and Demand Model

One of the improvements introduced in this project involves the simultaneous estimation of the parameters of the cost and demand models. Given that profit maximization is assumed with respect to message toll services, a simultaneity bias may arise if marginal revenue estimates are used as exogenous information in the cost model. Thus the cost and demand equations were estimated as a simultaneous system with cross-equation parameter constraints. Since it is assumed that the demand for local services will always be satisfied at the existing price (regardless of how it is determined), it was not necessary to include the demand equations for local services in the system. The same result was achieved by simply treating the output of local services as an exogenous variable.

The six equations of the estimated model were given by 4.1, 4.2, 4.3, 4.4, 4.5 and 4.23. The link between the demand and cost models was made by writing marginal revenue in the profit maximization equation (4.5) as:

$$MR_{QM} = P_{QM} \left[1 + \frac{1}{B_1} \right] \tag{4.26}$$

where B_1 is the price elasticity of demand from the demand equation (4.23).

The parameter restrictions implied by homogeneity of degree one of the cost function in factor price were introduced. This resulted in Equation 4.4 (materials share) being dropped. The parameter estimates from this equation were subsequently recouped. An additive random term was affixed to the remaining equations and

and the parameters were estimated using a full information maximum likelihood technique.* The endogenous variables of the model were: cost, labour share, capital share, output of message toll services and price of message toll services. All other variabls were declared exogenous. The parameter estimates were iterated until convergence was achieved and were therefore independent of the deleted equation.

As the iteration process in the estimation algorithm worked itself out, it was noted that three parameters appearing only in the cost function (C_{QLQL}, C_{QLT}, C_{TT}) tended to move in a compensatory fashion having virtually no effect on other parameters or cost. This fact was attributed to the high collinearity between T and QL. Primarily for reasons of computational ease, these coefficients were constrained equal to zero.

The parameter estimates and asymptotic standard errors of the final version of the model are presented in Table 5.3. Additional equation by equation summary statistics are provided in Table 5.4.

An examination of the tables suggests that the fit of the model is quite tight. Approximately 80% of the coefficients are asymptotically significantly different from zero at the 5% level. As well, those coefficients which are not significant are very small. Finally, the sum of squared residuals for each equation is very small.

---

* FIML, implemented on TSP, version 3.4. The estimation involved 5 equations (five endogenous variables), 25 observations (1952-1976), and 24 parameters.

TABLE 5.3

PARAMETER ESTIMATES OF THE SIMULTANEOUS COST AND DEMAND MODEL

Parameter	Estimate	Standard Error
CC_0	-1.295*	.258
C_w	.583*	.073
C_r	.459*	.090
C_v	-.042	.061
C_T	.482*	.069
C_{ww}	-.024*	.025
C_{wr}	.005	.018
C_{wv}	.019	.017
C_{wT}	-.300*	.003
C_{rr}	.064*	.022
C_{rv}	-.069*	.013
C_{rT}	.346*	.038
C_{vv}	.050*	.017
C_{vT}	-.047*	.025
C_{QL}	1.690*	.082
C_{QM}	.246*	.043
C_{QMQL}	-.159*	.014
C_{QMQM}	.089*	.008
C_{QMT}	.037*	.016

TABLE 5.3 (Continued)

Parameter	Estimate	Standard Error
C_{wQL}	$-.058^*$	.022
C_{wQM}	$.085^*$	.009
C_{rQL}	.036	.025
C_{rQM}	$-.083^*$	.009
C_{vQL}	.022	.020
C_{vQM}	-.002	.008
B_0	-4.926^*	.774
B_1	-1.314^*	.067
B_2	$.787^*$	.092
B_3	$.796^*$	.103
BD_1	$.031^*$	.014
BD_2	$.111^*$	.014

* Significant at the 5% level.

Log of Likelihood Function = 127.465

TABLE 5.4

EQUATION BY EQUATION SUMMARY STATISTICS

EQUATION		R^2 +	D-W	SSR
Cost Function	4.1	*	1.161	.004
Labour Share	4.2	.991	1.606	.001
Capital Share	4.3	.986	1.573	.001
Message Profit	4.5	*	1.040	.002
Message Demand	4.23	*	1.900	.006

* Equation estimated in implicit form.

+ The distribution of the statistics presented here are not tabled -thus no significance tests can be performed.

Section 5.3 Analysis of the Estimated Model

(5.3.1) Validation of the Underlying Assumptions

It will be recalled that profit maximization was assumed for message toll services. As well, in order for the cost function to provide an economically reasonable approximation to the underlying production technology, it must be concave in factor prices. At each data point it was verified that the second order conditions of profit maximization $\left(\frac{\partial(MR-MC)}{\partial QM}<0\right)$ and the concavity requirements were satisfied. Thus the data are not in conflict with the underlying assumptions of the model. From an economic and an econometric point of view, these validation results are very encouraging.

(5.3.2) Features of the Demand for Message Toll Services

The demand equation parameter estimates coming from the simultaneous cost and demand model can be usefully compared to the separate estimation of the demand model undertaken in Section 5.1. Comparing Tables 5.1 and 5.3 with respect to B_0, B_1, B_2, B_3, BD_1 and BD_2, it is noted that the point estimates differ by at most 5%. Although this suggests that the gains from simultaneous estimation of the cost and demand models may be limited, it does underline the stability and compatability of the demand and cost models.

(5.3.3) Characteristics of the Underlying Production Process Deduced from the Estimated Cost and Demand Model

The estimated model can be used to gain important insights into the Bell production process. A summary of these results is presented in Tables 5.5 to 5.11. The paragraphs which follow contain a discussion of these tables.

Local Services

In Table 5.5, a six year summary of local services is presented. In the latter period of the sample (1967-1976), price and marginal cost have been moving together quite closely. An examination of the elasticity of marginal cost suggests that the marginal cost curve is slightly downward sloping.

Message Toll Services

Table 5.6 contains some summary information regarding message toll services. The closeness of the marginal revenue and marginal cost numbers attests to the validity of the profit maximization assumption. As well, the Lerner Index of monopoly power suggests that the exploitation of the message toll service market has been stable over time. Finally, the elasticity of marginal cost statistics suggest that the marginal cost of message toll services is virtually constant in the neighbourhood of any data point.

A Comparison of the Marginal Costs with Other Studies

It is possible to make a partial comparison of the marginal costs derived from this study with estimates derived by Bell Canada in an internal study and with estimates derived by Rohlfs using ATT data. To this end, Table 5.7 has been prepared. The numbers presented represent the marginal cost of a dollar of revenue in the given year. An examination of the estimates suggests that the IAER numbers are somewhat lower than the Bell and ATT numbers but, overall, quite similar.

The differences between the methodologies used are discussed below.

TABLE 5.5

LOCAL SERVICES SUMMARY

Year	Price	Marginal Cost*	Elasticity of Marginal Cost $\frac{\partial MC_{QL}}{\partial QL} \cdot \frac{QL}{MC_{QL}}$
1952	.924	1.224	-.057
1957	.933	1.137	-.150
1962	1.000	1.019	-.227
1967	1.000	.939	-.317
1972	1.086	1.049	-.403
1976	1.270	1.215	-.482

* Reference equation 4.6

TABLE 5.6

MESSAGE TOLL SERVICES SUMMARY

Year	Price	Marginal* Revenue	Marginal+ Cost	Lerner# Index	Elasticity of Marginal Cost
1952	1.064	.254	.251	.83	.189
1957	1.062	.254	.244	.83	.212
1962	1.041	.249	.252	.80	.121
1967	1.000	.239	.235	.77	.048
1972	1.102	.263	.278	.75	.006
1976	1.245	.297	.295	.76	.020

*Reference equation 4.26

+ Reference equation 4.7

Lerner Index of Monopoly Power = $\frac{P_{QM} - MC_{QM}}{P_{QM}}$

a) The scaling of costs in the IAER study assumes that the revenue/cost ratio for other toll and miscellaneous is the same as for the aggregate of message toll and local. In fact, the revenue/cost ratio for other toll and miscellaneous is likely to be lower than the revenue/cost ratio for the aggregate of message toll and local. Consequently our estimates may be biased slightly downward. However, given the small revenue share associated with other toll, this bias is likely very small.

b) Both the Rohlf study and the Bell study exclude vertical services when reporting the local service marginal cost. Using Rohlf's data, the difference between the marginal cost of local including vertical services, and excluding vertical services is of the order of 5%.

c) The assumptions relating to the price of capital, and the treatment of tax are different between the studies. This could make a larger difference. Unfortunately, the details, necessary to make the comparison are not available. In addition, differences in revenues and cost calculations between the U.S. and Canada will cause differences between the Rohlf study and the IAER and Bell study.

d) The estimates in all cases are determined using a different methodology. The IAER uses the derivative of the cost function and since the cost function is a long run cost function, it follows that the marginal costs so derived are long run marginal costs. In general, they will be less than short run marginal costs. The Rohlf measure of incremental cost, based on historical data includes "construction for growth".

When compared to the Engineering study for toll, which is certainly long run, one suspects that the .45 historical estimate represents a short run marginal cost, while the .30 represents a long run marginal cost. Finally, the Bell study measures marginal cost as $\frac{\text{Causally related costs}}{\text{Revenue}}$ for a given year. This is closer to an average variable cost than a marginal cost and will overestimate marginal costs if average variable costs are falling.

Bell reports the following cost/revenue ratios for toll such that:

$$\frac{\Delta\ \text{cost}}{\Delta\ \text{revenue}} = \frac{325 - 235}{1115 - 765} = .257$$

For local however, Bell reports increasing cost revenue ratios and this implies:

$$\frac{\Delta\ \text{cost}}{\Delta\ \text{revenue}} = \frac{1000 - 775}{690 - 585} = 2.14$$

This last value seems somewhat extreme, suggesting that Bell's forecast of the future is out of line with historic realization.

TABLE 5.7

COMPARISON OF MARGINAL COSTS PER $ REVENUE

		Local	Message Toll
IAER	(1976)	.96	.24
Bell Canada Study	(1976)	1.32	.31
Rohlf (ATT)	(Historical) 1973-5	1.50	.45
	(Engineering) 1976	N/A	.30

Scale and Scope

Table 5.8 summarizes some of the features of economies of scale and economies of scope in the underlying production process. Although cost complementarities and hence scope economies exist, they are unimportant relative to marginal cost. As well, scale economies exist except for 1952. The trending in the scale economies measure points to the ever-present problem of disassociating the effects of scale and of technology in a complicated production process.

Output Surface Characteristics

Table 5.9 provides additional information regarding the relationship of cost to outputs. The iso-cost output surface demonstrates, for a given level of cost, the manner in which outputs can be transformed. The rate of possible transformation (given by the slope) is quite stable over time. As well, the transformation surface is concave to the origin.

Elasticities of Substitution

Table 5.10 summarizes some of the results concerning factor substitution using the Hicks-Allen (partial) elasticity of substitution measure. The results suggest that the elasticity of substitution between labour and capital has been quite stable over time and is very close to 1. Similarly, the substitution possibilities between labour and materials have been stable. Finally, the capital materials elasticity of substitution estimates suggest that capital and materials are not strongly substitutable and, in fact, showed a complementary property during the early part of the sample.

TABLE 5.8

Scale and Scope Summary

Year	Cost Complementarities (Scope)* $\frac{\partial MC_{QL}}{\partial QM}$	Ray Scale Economies +
1952	-.002	1.024
1957	-.014	.930
1962	-.001	.859
1967	-.0005	.775
1972	-.0005	.694
1976	-.0009	.614

*Reference equation 4.8

+Reference equation 4.9

TABLE 5.9

ISO-COST OUTPUT SURFACE CHARACTERISTICS

Year	Slope $\frac{dQL}{dQM} \quad C=\bar{C}$	Curvature
1952	-.205	concave to origin
1957	-.215	concave to origin
1962	-.247	concave to origin
1967	-.250	concave to origin
1972	-.265	concave to origin
1976	-.242	concave to origin

eg.(1967)

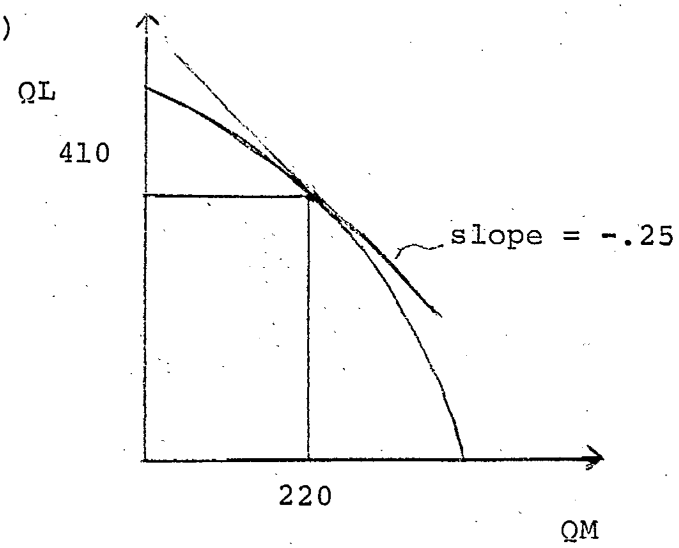

TABLE 5.10

Elasticities of Substitution

Year	labour/capital	labour/materials	capital/materials
1952	1.032	1.250	-.015
1957	1.020	1.270	.073
1962	1.045	1.300	.216
1967	1.019	1.405	.276
1972	1.019	1.315	.312
1976	1.017	1.351	.321

Reference: equations 4.19, 1420, 4.21.

Factor-Price Elasticities

Finally, Table 5.11 illustrates some of the factor price elasticity estimates. All factors are inelastically demanded with labour showing a trend towards greater elasticity. Capital has become somewhat less elastically demanded over time whereas the elasticity of demand for materials has been more or less constant.

TABLE 5.11

COST MINIMIZING OWN FACTOR PRICE ELASTICITIES

Year	Labour	Capital	Materials
1952	-.624	-.444	-.535
1957	-.642	-.440	-.559
1962	-.742	-.396	-.540
1967	-.773	-.369	-.577
1972	-.780	-.354	-.543
1976	-.796	-.342	-.548

Reference: Equations 4.13, 4.16, 4.18.

Part 6 VERIFICATION

Before turning to policy aspects of this report, it is useful to verify the accuracy of the estimated cost and demand model. This involves simulating the model over the historical period and subsequently comparing the actual and simulated variables.*

Two sets of simulations were carried out in the verification process:

(a) Demand Verification

(b) Simultaneous Cost and Demand Verification.

Section 6.1 Demand Verification

The demand model consists of two equations: the demand for local residential services (4.24) and the demand for aggregate toll services (4.23). As mentioned earlier, there was not sufficient data to disaggregate toll services into the business and residential components.

The actual and simulated demand series are presented in Table 6.1. A statistical comparison of the series is provided in Table 6.2. The tracking is very good for both equations. Theil's decomposition of the inequality of the series suggests that almost all of the error is due to different co-variation: thus the demand equations accurately predict the levels of demand within the sample period.

---

* The TSP (3.4) command SIML was used in simulating the model. The procedure was adapted for use on CDC machines, and in addition, the simulation capacity was extended to handle simultaneous equation systems of twelve equations and thirty or more variables.

TABLE 6.1

SIMULATION MODEL

	QLOCR	QLOCRS	QTOL	QTOLS
1952	71.4270	72.5586	52.6097	52.4867
1953	76.8095	77.5123	56.7187	57.2222
1954	82.3187	81.9362	61.2002	61.9023
1955	89.8682	90.1272	70.1569	68.8063
1956	99.4008	100.402	79.0054	77.2081
1957	108.796	107.133	86.2314	85.9669
1958	116.331	114.301	90.3172	93.2821
1959	122.569	123.307	98.6624	99.0600
1960	130.413	129.550	103.748	104.069
1961	138.245	134.709	110.212	110.012
1962	146.935	144.285	130.497	130.279
1963	155.020	152.789	138.740	141.784
1964	159.630	161.316	154.380	155.279
1965	169.208	171.079	175.729	171.597
1966	181.600	184.015	199.910	198.367
1967	192.700	194.314	223.780	221.551
1968	204.800	206.355	244.824	248.833
1969	218.400	220.311	280.929	274.616
1970	234.538	238.686	304.541	300.230
1971	247.499	249.480	320.084	326.462
1972	268.390	267.216	360.755	362.302
1973	288.393	286.166	421.531	411.900
1974	310.657	310.739	485.566	487.426
1975	333.972	328.782	553.000	558.363
1976	342.721	343.440	596.984	604.618

TABLE 6.2

COMPARISON OF ACTUAL AND PREDICTED TIME SERIES

ACTUAL AND PREDICTED VARIABLES...	QTOL	QTOLS
CORRELATION COEFFICIENT =	.9997	
(SQUARED) =	.9995	
ROOT-MEAN-SQUARED ERROR =	3.744	
MEAN ABSOLUTE ERROR =	2.709	
MEAN ERROR =	-.1404	
REGRESSION COEFFICIENT OF ACTUAL ON PREDICTED =		.9943
THEIL"S INEQUALITY COEFFICIENT =		.6987E-02
FRACTION OF ERROR DUE TO BIAS =		.1407E-02
FRACTION OF ERROR DUE TO DIFFERENT VARIATION =		.5265E-01
FRACTION OF ERROR DUE TO DIFFERENT CO-VARIATION =		.9459
ALTERNATIVE DECOMPOSITION (LAST 2 COMPONENTS)		
FRACTION OF ERROR DUE TO DIFFERENCES OF REGRESSION COEFFICIENT FROM UNITY =		.5789E-01
FRACTION OF ERROR DUE TO RESIDUAL VARIANCE =		.9407

TABLE 6.2 (continued)

COMPARISON OF ACTUAL AND PREDICTED TIME SERIES

ACTUAL AND PREDICTED VARIABLES...	QLOCR	QLOCRS
CORRELATION COEFFICIENT =	.9997	
(SQUARED =	.9993	
ROOT-MEAN-SQUARED ERROR =	2.111	
MEAN ABSOLUTE ERROR =	1.750	
MEAN ERROR =	.5264E-02	
REGRESSION COEFFICIENT OF ACTUAL ON PREDICTED =		1.001
THEIL"S INEQUALITY COEFFICIENT =		.5364E-02
FRACTION OF ERROR DUE TO BIAS =		.6215E-05
FRACTION OF ERROR DUE TO DIFFERENT VARIATION =		.3254E-02
FRACTION OF ERROR DUE TO DIFFERENT CO-VARIATION =		.9967
ALTERNATIVE DECOMPOSITION (LAST 2 COMPONENTS)		
FRACTION OF ERROR DUE TO DIFFERENCES OF REGRESSION COEFFICIENT FROM UNITY =		.1933E-02
FRACTION OF ERROR DUE TO RESIDUAL VARIANCE =		.9981

Section 6.2 Simultaneous Cost and Demand Model Verification

The cost and demand model consists of five equations: the cost function (4.1), two factor share equations (4.2, 4.3), the demand for toll services equation (4.23) and the profit maximization equation (4.5). In the verification procedure, the five equations were simultaneously solved for the endogenous variables: cost, labour, capital, price of toll services and quantity of toll services.

The actual and simulated series are presented in Table 6.3 and a comparison of the series is shown in Table 6.4.* Once again it can be seen that the tracking is accurate and unbiased over the sample period.

---

*It will be recalled that the series for factors and hence cost were scaled down when competitive services and miscellaneous outputs were excluded from the model. The results in Tables 6.3 and 6.4 refer to the series rescaled after simulation.

TABLE 6.3

COST MODEL SIMULATION

YEAR	QTOL	QTOLSIM	PTOL	PTOLSIM
1952	52.6097	53.7858	1.06425	1.04463
1953	56.7187	54.4792	1.06551	1.10610
1954	61.2002	57.8608	1.06628	1.12250
1955	70.1569	63.9644	1.06573	1.12658
1956	79.0054	76.0510	1.06491	1.07722
1957	86.2314	88.5941	1.06156	1.03752
1958	90.3172	95.2757	1.07103	1.05393
1959	98.6624	108.318	1.11723	1.04379
1960	103.748	112.203	1.13130	1.06833
1961	110.212	116.115	1.11990	1.07482
1962	130.497	126.860	1.04140	1.06269
1963	138.740	141.199	1.03932	1.04260
1964	154.380	155.970	1.03766	1.03416
1965	175.729	179.957	1.03644	.999590
1966	199.910	201.292	1.00935	.998168
1967	223.780	219.876	1.00000	1.00579
1968	244.824	239.270	.991429	1.02144
1969	280.929	264.596	.994702	1.02324
1970	304.541	308.499	1.07216	1.05022
1971	320.084	322.464	1.08771	1.09796
1972	360.755	341.174	1.10190	1.15346
1973	421.531	408.847	1.12445	1.13084
1974	485.566	490.741	1.13969	1.13383
1975	553.000	549.793	1.18029	1.19427
1976	596.984	612.257	1.24467	1.23283

TABLE 6.3 (continued)

COST MODEL SIMULATION

YEAR	L	LSIM	K	KSIM
1952	44.9000	45.3556	626.600	613.526
1953	46.1000	45.2933	690.400	710.275
1954	48.2000	47.0670	764.900	796.041
1955	51.9000	50.1976	871.300	895.522
1956	55.7000	55.2418	989.900	998.642
1957	57.8000	59.3525	1127.10	1108.22
1958	57.6000	57.5683	1280.00	1266.46
1959	56.5000	58.6924	1429.50	1374.27
1960	54.6000	54.3868	1579.10	1558.75
~~1961~~	~~52.4000~~	~~52.2084~~	~~1721.90~~	~~1725.44~~
1962	52.3000	53.8934	1860.10	1860.81
1963	53.5000	54.4621	2004.40	1988.03
1964	54.4000	53.2731	2150.40	2130.19
1965	55.8000	55.8613	2283.60	2256.16
1966	57.5000	57.1826	2431.20	2411.92
1967	56.6000	58.7693	2585.60	2568.07
1968	55.5000	57.0335	2734.00	2746.12
1969	56.6000	57.6938	2886.00	2919.27
1970	57.8000	59.7921	3054.80	3039.08
1971	58.1000	56.7353	3190.40	3242.48
1972	57.5000	54.0555	3334.90	3391.77
1973	60.4000	58.7691	3494.00	3482.09
1974	63.9000	63.5512	3653.50	3617.36
1975	64.1000	64.5776	3808.90	3825.18
1976	67.3000	65.9881	3978.90	4014.11

TABLE 6.3 (continued)

COST MODEL SIMULATION

YEAR	M	MSIM	COST	COSTSIM
1952	42.4608	41.7625	175.496	174.331
1953	45.9759	47.2488	189.063	190.593
1954	51.1042	52.0978	206.761	208.494
1955	58.3350	59.0338	231.105	230.616
1956	67.9400	65.0275	262.056	259.694
1957	69.9111	71.9437	292.383	295.375
1958	77.1386	76.4566	320.120	318.128
1959	82.0535	81.3483	350.012	348.750
1960	86.2575	85.0542	373.553	369.882
1961	91.1128	90.5109	395.652	395.012
1962	98.0741	97.5315	424.319	428.302
1963	103.402	102.077	458.487	458.290
1964	104.337	103.709	484.499	478.470
1965	113.569	109.156	525.065	518.139
1966	118.468	116.496	580.788	575.634
1967	116.547	125.442	628.030	642.292
1968	122.307	132.125	691.652	709.137
1969	143.302	141.064	791.828	798.558
1970	144.569	152.080	900.246	915.227
1971	168.413	160.310	990.847	982.940
1972	173.292	170.655	1122.67	1110.01
1973	186.739	177.475	1293.03	1268.52
1974	186.361	182.231	1516.85	1500.21
1975	185.056	193.388	1752.27	1774.17
1976	199.898	201.015	2017.83	2018.57

TABLE 6.4

COMPARISON OF ACTUAL AND PREDICTED TIME SERIES

	QTOL	QTOLSIM
CORRELATION COEFFICIENT =	.9988	
(SQUARED =	.9976	
ROOT-MEAN-SQUARED ERROR =	7.706	
MEAN ABSOLUTE ERROR =	5.943	
MEAN ERROR =	.4268	
REGRESSION COEFFICIENT OF ACTUAL ON PREDICTED =		1.000
THEIL'S INEQUALITY COEFFICIENT =		.1441E-01
FRACTION OF ERROR DUE TO BIAS =		.3068E-02
FRACTION OF ERROR DUE TO DIFFERENT VARIATION =		.1164E-02
FRACTION OF ERROR DUE TO DIFFERENT CO-VARIATION =		.9958
ALTERNATIVE DECOMPOSITION (LAST 2 COMPONENTS)		
FRACTION OF ERROR DUE TO DIFFERENCES OF REGRESSION COEFFICIENT FROM UNITY =		.9578E-04
FRACTION OF ERROR DUE TO RESIDUAL VARIANCE =		.9968

	PTOL	PTOLSIM
CORRELATION COEFFICIENT =	.8324	
(SQUARED =	.6930	
ROOT-MEAN-SQUARED ERROR =	.3387E-01	
MEAN ABSOLUTE ERROR =	.2698E-01	
MEAN ERROR =	-.3047E-03	
REGRESSION COEFFICIENT OF ACTUAL ON PREDICTED =		.8143
THEIL'S INEQUALITY COEFFICIENT =		.1569E-01
FRACTION OF ERROR DUE TO BIAS =		.8096E-04
FRACTION OF ERROR DUE TO DIFFERENT VARIATION =		.1449E-02
FRACTION OF ERROR DUE TO DIFFERENT CO-VARIATION =		.9985
ALTERNATIVE DECOMPOSITION (LAST 2 COMPONENTS)		
FRACTION OF ERROR DUE TO DIFFERENCES OF REGRESSION COEFFICIENT FROM UNITY =		.1051
FRACTION OF ERROR DUE TO RESIDUAL VARIANCE =		.8949

TABLE 6.4 (continued)

COMPARISON OF ACTUAL AND PREDICTED TIME SERIES

	L	LSIM
~~CORRELATION COEFFICIENT =~~	~~.9632~~	
(SQUARED =	.9277	
ROOT-MEAN-SQUARED ERROR =	1.387	
MEAN ABSOLUTE ERROR =	1.127	
MEAN ERROR =	-.6835E-04	
REGRESSION COEFFICIENT OF ACTUAL ON PREDICTED =		.9387
THEIL'S INEQUALITY COEFFICIENT =		.1236E-01
FRACTION OF ERROR DUE TO BIAS =		.2430E-08
FRACTION OF ERROR DUE TO DIFFERENT VARIATION =		.8881E-02
FRACTION OF ERROR DUE TO DIFFERENT CO-VARIATION =		.9911
ALTERNATIVE DECOMPOSITION (LAST 2 COMPONENTS)		
FRACTION OF ERROR DUE TO DIFFERENCES OF REGRESSION COEFFICIENT FROM UNITY =		.5181E-01
FRACTION OF ERROR DUE TO RESIDUAL VARIANCE =		.9482

	K	KSIM
CORRELATION COEFFICIENT =	.9997	
(SQUARED =	.9993	
ROOT-MEAN-SQUARED ERROR =	27.37	
MEAN ABSOLUTE ERROR =	23.19	
MEAN ERROR =	-.3364	
REGRESSION COEFFICIENT OF ACTUAL ON PREDICTED =		.9944
THEIL'S INEQUALITY COEFFICIENT =		.5662E-02
FRACTION OF ERROR DUE TO BIAS =		.1511E-03
FRACTION OF ERROR DUE TO DIFFERENT VARIATION =		.3992E-01
FRACTION OF ERROR DUE TO DIFFERENT CO-VARIATION =		.9599
ALTERNATIVE DECOMPOSITION (LAST 2 COMPONENTS)		
FRACTION OF ERROR DUE TO DIFFERENCES OF REGRESSION COEFFICIENT FROM UNITY =		.4511E-01
FRACTION OF ERROR DUE TO RESIDUAL VARIANCE =		.9547

TABLE 6.4 (continued)

COMPARISON OF ACTUAL AND PREDICTED TIME SERIES

	M	MSIM
CORRELATION COEFFICIENT =	.9953	
(SQUARED =	.9906	
ROOT-MEAN-SQUARED ERROR =	4.595	
MEAN ABSOLUTE ERROR =	3.309	
MEAN ERROR =	.5546E-01	
REGRESSION COEFFICIENT OF ACTUAL ON PREDICTED =		1.001
THEIL'S INEQUALITY COEFFICIENT =		.1869E-01
FRACTION OF ERROR DUE TO BIAS =		.1456E-03
FRACTION OF ERROR DUE TO DIFFERENT VARIATION =		.3627E-02
FRACTION OF ERROR DUE TO DIFFERENT CO-VARIATION =		.9962
ALTERNATIVE DECOMPOSITION (LAST 2 COMPONENTS)		
FRACTION OF ERROR DUE TO DIFFERENCES OF REGRESSION COEFFICIENT FROM UNITY =		.1371E-03
FRACTION OF ERROR DUE TO RESIDUAL VARIANCE =		.9997

	COST	COSTSIM
CORRELATION COEFFICIENT =	.9998	
(SQUARED =	.9996	
ROOT-MEAN-SQUARED ERROR =	10.05	
MEAN ABSOLUTE ERROR =	7.117	
MEAN ERROR =	.2107	
REGRESSION COEFFICIENT OF ACTUAL ON PREDICTED =		1.001
THEIL'S INEQUALITY COEFFICIENT =		.5968E-02
FRACTION OF ERROR DUE TO BIAS =		.4392E-03
FRACTION OF ERROR DUE TO DIFFERENT VARIATION =		.1276E-02
FRACTION OF ERROR DUE TO DIFFERENT CO-VARIATION =		.9983
ALTERNATIVE DECOMPOSITION (LAST 2 COMPONENTS)		
FRACTION OF ERROR DUE TO DIFFERENCES OF REGRESSION COEFFICIENT FROM UNITY =		.6581E-03
FRACTION OF ERROR DUE TO RESIDUAL VARIANCE =		.9989

Part 7 EFFICIENCY AND EQUITY

Regulation of Bell Canada (first by CTC and later by CRTC) has served to constrain the activities of this company. Clearly, the intent of such regulation is to serve the public interest given the special (natural monopoly) characteristics of the telecommunications services. Further, notions of both efficiency and equity have played a role in determining the direction of regulation. Historically, both the level of profits (and the rate of return on capital) and the relative price of services have been monitored by the regulatory agencies in order to secure efficiency and equity in the provision of telecommunications services.

In the sections which follow some of the results of Feldstein (1972) are used to formalize a method of examining efficiency and equity in the provision of telecommunications services to residential users in the Bell territory. The model is expressed as a system of equations which is subsequently used to simulate a set of 'optimal' prices and outputs. Various definitions of optimality are examined. On the one hand, purely efficiency regulated (Ramsey) prices are examined. Alternatively, prices which include, in addition, an income-determined weighting of the well-being of different classes of individuals are developed. In all cases, the simulated 'optimal' prices are compared to the historic prices and the differences are discussed.

Section 7.1 The Model

Consider the general problem of chosing output prices so as to maximize social welfare subject to a feasibility (profitability) constraint. Formally, this problem can be written:

$$\text{Max.} \quad W = N\int_{o}^{\infty} V(P_L,P_M,P,y)f(y)dy \qquad (7.1)$$

$$\text{subject to} \quad \Pi(P_L,P_M,y) \geq \Pi_o \qquad (7.2)$$

where:
- N = Number of household in Bell Territory
- V = Indirect utility function of the representative household
- P_L = Residential price of local service (nominal)
- P_M = Residential price of toll service (nominal)
- P = Composite price of all other goods (nominal)
- y = Household income, assumed to be the only difference amongst households
- $f(y)$ = Relative density function of household income
- Π = Profit function of Bell (or the regulated firm in general)
- Π_o = Minimum required profit

In the above problem (which is posed here for residential services only) all households have identical preferences but differ with respect to the amount of income they have. Welfare (W) can been seen as the sum of utilities of consumers over various income classes weighted by the number of consumers in these classes.

Maximization of (7.1) subject to (7.2) is accomplished by first constructing the Lagrange function:

$$\mathcal{L} = N\int_{o}^{\infty} V(P_L,P_M,P,y)f(y)dy + \lambda[\Pi(P_L,P_M,y)-\Pi_o] \qquad (7.3)$$

The first order conditions corresponding to an interior maximimum are:

$$\frac{\partial\mathcal{L}}{\partial P_i} = N\int_{o}^{\infty} \frac{\partial V}{\partial P_i} f(y)dy + \lambda \frac{\partial \Pi}{\partial P_i} = 0 \qquad i = L,M \qquad (7.4)$$

If π represent the producer's U then λ is the measure of the trade off between producer & consumer U

$$\frac{\partial \mathcal{L}}{\partial \lambda} = \Pi(P_L, P_M, y) - \Pi_o = 0 \tag{7.5}$$

The multiplier λ can be interpreted as $\frac{-dW^*}{d\Pi_o}$ >o or the increase in welfare arising from reducing the minimum required profit by a small amount. Thus, Equations (7.4) have the interpretation that price is set where the decrease in welfare arising from raising prices a small amount is just offset by the social value of the change in profits of the regulated firm (Bell).

It is possible to express the first order conditions in a fashion which aids the explanation of the equity aspects of the model.

First, note that by Roy's Identity:

$$\frac{\partial V}{\partial P_i} = -q_i \frac{\partial V}{\partial y} \qquad i = L, M \tag{7.6}$$

? why

As well, the aggregate demand for good i (Q_i) is given by:

$$Q_i = N\int_o^\infty q_i f(y)\,dy \tag{7.7}$$

Finally, it will be noted that:

$$\frac{\partial \Pi}{\partial P_i} = (MR_i - MC_i)\frac{\partial Q_i}{\partial P_i} \qquad i = L, M \tag{7.8}$$

where MR_i and MC_i are respectively the marginal revenue and the marginal cost of service i. It is assumed that cross price elasticities are o and it is further noted that $MR_i = P_i[1 + \frac{1}{\varepsilon_i}]$ where ε_i is the price elasticity of demand for service i.

The distributional coefficient of service i is defined by:

$$R_i = \frac{N}{Q_i}\int_o^\infty q_i \frac{\partial V}{\partial y} f(y)\,dy \tag{7.9}$$

As Feldstein notes:

"the ratio R_i is a weighted average of the marginal social utilities, each household's marginal social utility weighted by that household's consumption of good i. The conventional welfare assumption that $\frac{\partial V}{\partial y}$ declines as y increases implies that the value of R_i will be greater for a necessity $\left(\frac{\partial q_i}{\partial y} \cdot \frac{y}{q_i} < 1\right)$ than for a luxury $\left(\frac{\partial q_i}{\partial y} \cdot \frac{y}{q_i} > 1\right)$. The higher the income elasticity of demand for a good, the lower the value of R_i." (Feldstein, 1972, Page 33).

Substituting (7.9), (7.8), (7.7), (7.6) into (7.4) yields,

$$\frac{(P_L - MC_L)/P_L}{(P_M - MC_M)/P_M} = \frac{\varepsilon_M \; (R_L - \lambda)}{\varepsilon_L \; (R_M - \lambda)} \qquad (7.10)$$

Equation (7.10) represents the relationship of optimal divergences of prices from marginal costs for local and message toll services given <u>both</u> efficiency and equity considerations. Given that equity doesn't matter ($R_L = R_M$) then (7.10) reduces to:

$$\frac{(P_L - MC_L)/P_L}{(P_M - MC_M)/P_M} = \frac{\varepsilon_M}{\varepsilon_L} \qquad (7.11)$$

Equation (7.11) is the Ramsey Rule which states that the percentage divergence of price from marginal cost for a service is inversely related to the price elasticity of demand for the service. Alternatively, the optimal tax on a service is higher as the elasticity is lower. There is an element of discrimination in the Ramsey Rule and it is reminiscent of the well-known result that, under normal conditions, a discriminating monopolist will *ceteris paribus* change a higher price in a less elastic market. charge X

Returning to equation 7.10 it will be noted that the right hand side of this equation is the same as the right hand side of the Ramsey equation (7.11) except for the scaling distributional term $\left(\frac{R_L - \lambda}{R_M - \lambda}\right)$. If equity considerations mitigate the pure efficiency (Ramsey) price relationship then it must be the case that:

$$\left|\frac{R_L-\lambda}{R_M-\lambda}\right| < 1 \quad , \; R_L, R_M, \lambda > o \qquad (7.12)$$

For the case of Bell Canada services, local services have lower price and income elasticities than toll services and thus $R_L > R_M$. It follows then that if (7.12) is satisfied it must be the case the $|R_L-\lambda| < |R_M-\lambda|$ or that R_L is absolutely closer to λ than is R_M.

? Note that if we look as estim. here $\begin{pmatrix} MC_L > P_L \\ MC_M < P_M \end{pmatrix}$ then $\frac{(P_L - MC_L) \cdot P_M}{(P_M - MC_M) \cdot P_L} < 0$

hence either ε_M or $\varepsilon_L > 0$ or their tariffs are not optimal.

Section 7.2 Additional Assumptions

In order to simulate the model described in Section 7.1 and thereby determine the relationships existing between historic, Ramsey and efficiency-equity prices, it is necessary to introduce some additional assumptions. These assumptions are consistent with the foregoing cost and demand models as well as applied economic theory.

The first assumption is that the demands for local and toll residential services can be written in the isoelastic form:

$$q_i = a\, y^{\alpha_i}\, P_i^{\beta_i} \qquad i = L,M \tag{7.13}$$

what is that between p.26 & p.27?

For the case of residential local services this poses no problem. In fact, the estimated parameters are given in Table 5.2 as $\alpha_L = .337$ and $\beta_L = -.395$. It will be recalled however that there was not sufficient data to estimate the residential demand for toll services. Thus, we assume that the price and income elasticities (β_M, α_M) can be approximated by the aggregate toll price and income elasticities. Hence, $\beta_M = -1.314$ and $\alpha_M = .7873$. The price series for residential toll is assumed to be the same as for business toll. Consistent with this latter assumption is the assumption that the historic quantity of residential toll services was proportional to total toll output with the factor of proportionately (lity) determined by the ratio of residential toll revenue to total toll revenue. Data on the business and toll revenues was available.

why not have estm. Res. Dem for MTS on aggregate MTS Price?

The second assumption relates to the marginal utility of money function $\frac{\partial V}{\partial y}$. It is assumed that $\frac{\partial V}{\partial y}$ can be written in the isoelastic form:

$$\frac{\partial V}{\partial Y} = Y^{-\eta} \tag{7.14}$$

Equation (7.14) implies a utility function of the form:

$$U = A - ky^{1-\eta} \quad (7.15)$$

A graph of equation (7.15) is shown in Figure (7.1). For each curve, an arbitrary scaling of U has to occur. The choice made is:

$$y = \$1000 \Rightarrow U = 0 \Rightarrow k = (1000)^{\eta-1}$$

$$y = \infty \Rightarrow U = 1 \Rightarrow A = 1.0$$

For low values of η, utility rises very slowly with income, while with high η it rapidly approaches the bliss level (A). Other studies* suggest that a value of $\eta=1.7$ may be reasonable, and from Figure (7.1) this seems consistent. The simulations in this report are undertaken for a set of η, ranging from 1.0 to 5.0.

?
good

The third assumption relates to the choice of the income distribution function f(y). It is assumed that income is log normally distributed. This provides a reasonable description of income distribution in the Bell territory (Ontario and Quebec) and allows for a straightforward calculation of the equity-related parameters R_M and R_L. In particular, if y is log normally distributed then, (following Theil P.85).

$$\int_0^\infty y^\theta f(y)\,dy = \int_0^\infty e^{\theta \operatorname{Ln}(y)} f(y)\,dy$$
$$= \exp\left[\theta\,\overline{\operatorname{Ln}(y)} + \tfrac{1}{2}\theta^2\sigma^2_{(\operatorname{Ln}(y))}\right] \quad (7.16)$$

where $\overline{\operatorname{Ln}(y)}$ is the mean of the log of y

$\sigma^2_{(\operatorname{Ln}(y))}$ is the variance of the log of y

Using Statistics Canada data (Cat. 98-505, 93-749) for 1961 and 1971 respectively, (total income of households by income of head

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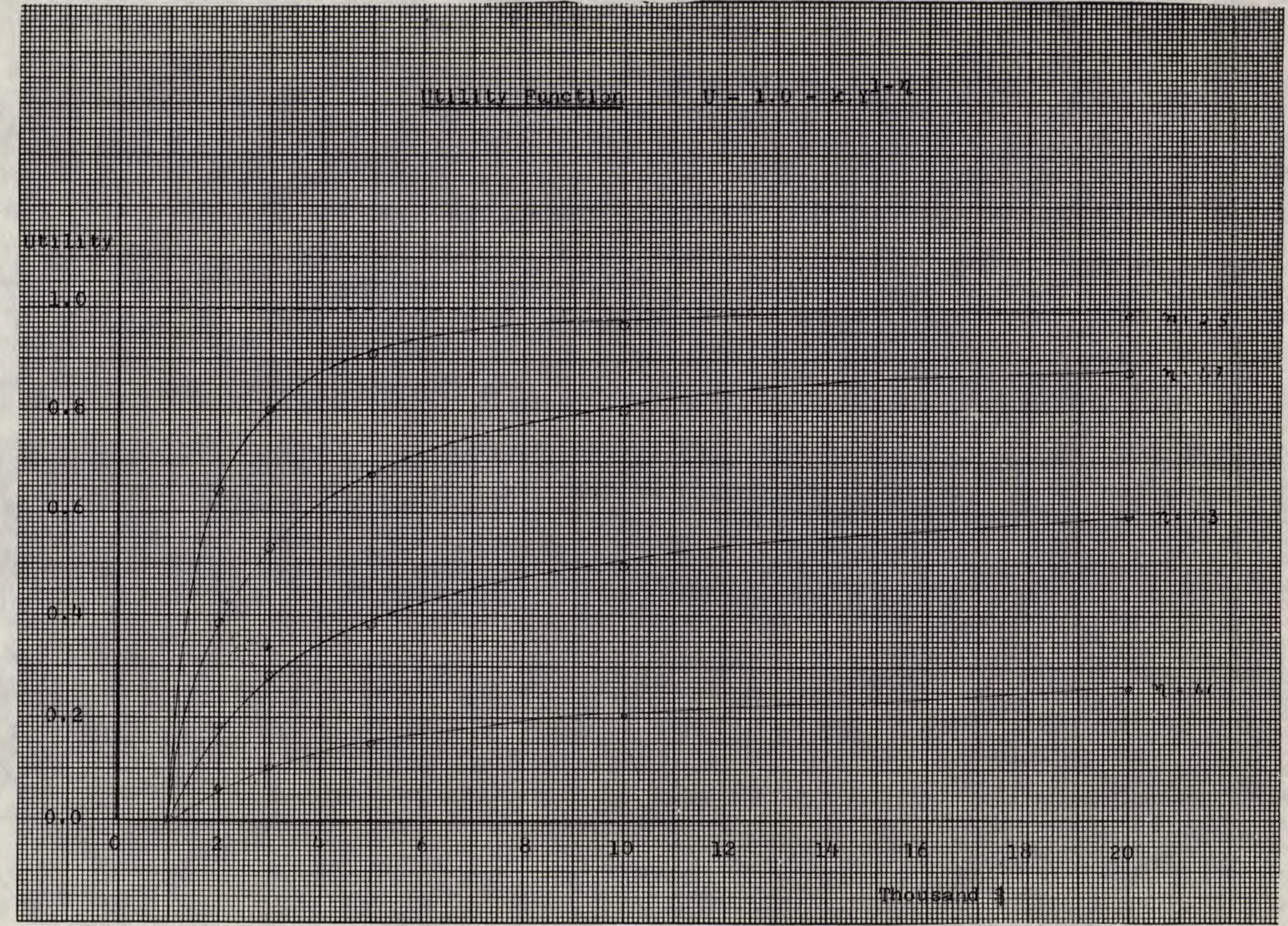
*For discussions on the value of the elasticity of marginal utility with respect to income, see Baumol (1979), Baumol and Bradford (1970), Fellner (1967), Mera (1969), and Powell _et al_ (1968).

FIGURE 7.1


Utility Function
U = 1.0 - x.Y$^{1-\eta}$
Utility
1.0
0.8
0.6
0.4
0.2
0.0
0
2
4
6
8
10
12
14
16
18
20
Thousand $
η = 2.5
η = 1.7
η = 1.3
η = 1.1

for Ontario and Quebec) the mean and variance of the logarithm of income were calculated. Expressed in thousands of dollars, the means were respectively 8.1415 and 8.6139 and the variances were .7203 and .7174. The variance showed little change over time and was assumed constant at the value .72. The means did show some movement and a complete series of means for the sample period 1952-1976 was calculated using the 1961 and 1971 values and the growth rates in personal consumption expenditure to approximate changes in the means.

Using equation (7.16), the series of means and variances and equations (7.13), (7.14), (7.7) it is possible to calculate the distributional coefficients given by equation (7.9). An example set of coefficients for the case $\eta=1.7$ is presented in Table 7.2. It will be noted that $R_L > R_M$ for all years since local services are less of a luxury than toll services ($\alpha_L<\alpha_M$).

The final assumption relates to the form of the profit constraint. In the model it is assumed that residential prices for local and toll services are free to move towards the optimal levels and that business prices and quantities for local and toll services will remain at their historic levels. Thus, the simulations can be viewed as determining optimal residential prices only. In keeping with this, the profit constraint is written:

$$P_L Q_L^R + P_M Q_M^R + \Pi_L Q_L^B + \Pi_M Q_M^B + OR$$
$$-COST\ (Q_L^R + Q_L^B,\ Q_M^R + Q_M^B,\ \rho) \geq \Pi_o \tag{7.17}$$

where Π_L, Π_M, Q_L, Q_M are the historically determined prices and quantities of business local and toll services and OR is the re-

TABLE 7.2

VALUES OF DISTRIBUTION COEFFICIENTS ($x10^{-5}$)

	R_L Local $\alpha_L = .3366$ $\eta = 1.7$	R_T Toll $\alpha_M = .7873$ $\eta = 1.7$
1952	.30574	.17612
1957	.21776	.12544
1962	.1759	.10133
1967	.1106	.06371
1972	.05831	.03359
1976	.02378	.0137

maining revenue of competitive and miscellaneous services fixed at historic levels. Π_o is the historic profit level. ρ is a vector summarizing historic levels of factor prices and technology. Finally, the cost function is such that changes in residential local and toll services determine marginal cost for these services. Of course, the cost function written in (7.17) is the multi-output translog cost function estimated for this project (4.1). As a last point it should be noted that some simulations were undertaken with the assumption that the rate of return to capital and not level of profits formed the binding constraint. The modifications introduced to examine these cases are discussed in the body of the simulation part of this report.

Part 8 SIMULATION

In this part of the report, the results of simulating the policy model introduced in Part 7 are presented. The goal of these simulations was to determine the direction and relative size of movements in residential toll and local service prices where these prices were allowed to optimally diverge from historic levels.

Four simulations were carried out:

a) Pure efficiency (or Ramsey) price simulation with marginal service costs assumed variable and an isoprofit constraint.

b) Efficiency-equity price simulation with marginal service costs assumed constant at the estimated levels and an isoprofit constraint.

c) Efficiency-equity price simulation with marginal service costs assumed variable and an isoprofit constraint.

d) Efficiency-equity price simulation with marginal service costs assumed variable and an iso-rate of return to capital constraint.

In each simulation, the model was solved at five year intervals (1952, 1957, 1962, 1967, 1972, 1976). For each simulation, the historical values are compared to the simulated results where such historic series exist (eg. prices and outputs). In the case of marginal costs the series presented as the 'actual' corresponds to the simulated values of the derivatives of the cost function using historic aggregate quantities and factor prices. In the case of residential toll the 'actual' series represents the results of the scaling of aggregate toll discussed in Part 7. To facilitate the presentation of the simulation results, all of the equations corresponding to the simulated models are presented in Appendix A-1.

Section 8.1 Ramsey Simulation

The results of the Ramsey simulation are presented in Table 8.1. For the years shown the global solution to the model was found. For 1976 (only) local solutions arose and the results are therefore not reported. As well, before turning to the interpretation of the results it is important to recall that income distribution plays no role in determining the optimal price levels in these simulations.

As can be seen from Table (8.1), the prices of residential local services increases and the prices of toll services fall relative to historic levels. Although the changes in message toll prices and quantities appear very large, the changes in message toll revenue are much more modest - an increase of 62% in 1967 is representative. The resulting relative divergences in price from marginal cost are indeed inversely related to the elasticities (see 7.11). Finally, it is interesting to note that local price rise by a smaller percentage than toll prices fall.

TABLE 8.1

RAMSEY SIMULATION

PQLR	ACTUAL	SIMULATED	QLR	ACTUAL	SIMULATED
1952	.8944	1.3137	1952	71.427	62.327
1957	.9032	1.3842	1957	108.796	90.495
1962	.9872	1.4636	1962	146.935	123.482
1967	1.0000	1.5959	1967	192.700	161.528
1972	1.0529	1.8358	1972	268.390	214.495
PQMR			QMR		
1952	1.0643	.1937	1952	21.2819	199.228
1957	1.0616	.2048	1957	36.4276	315.552
1962	1.0414	.2176	1962	56.5409	441.756
1967	1.0000	.2066	1967	97.2000	764.187
1972	1.1019	.2325	1972	171.6310	1331.78
MC LOC			MC TOL		
1952	1.2237	1.1404	1952	.2508	.1860
1957	1.1372	1.0511	1957	.2447	.1900
1962	1.0187	.9348	1962	.2519	.1939
1967	.9387	.8428	1967	.2349	.1773
1972	1.0488	.8442	1972	.2777	.1947

PQLR - Price, local residential
QLR - Quantity, local residential
PQMR - Price, toll residential
QMR - Quantity, toll residential
MC LOC - Marginal Cost, local
MC TOL - Marginal Cost, toll

Section 8.2 Efficiency-Equity Prices with Constant Marginal Costs and an Isoprofit Constraint

Under the constant marginal cost and isoprofit simulation, the marginal costs were taken as constant. This implies that the slope of the marginal cost is zero. Although this is not strictly accurate, the results presented earlier show that the marginal cost elasticities are low. As well, constant marginal costs yield rapid solutions to the model and serve as a useful benchmark for the more complicated variable marginal cost simulations.

The isoprofit constraint requires that the profit level with simulated prices and outputs must be the same as the profit level achieved with historic prices and quantities.

The results are presented in Table 8.2. Column (1) of Table (8.2) shows the historical values. Column (2) shows the quantities of local and toll residential demand that result from simulating the two hypothesized demand equations separately with historic price data. Columns (3) through (7) show the results of solving the model, with the values of η shown. The results show some directional similarity to the Ramsey case, but the price and quantity movements are not as extreme. Residential local service output is reduced, while residential toll output is greatly increased. The effect is largest for low η, and declines as η increases.*

At the η value of 1.7, local services output is reduced by 7 to 8%, while toll output is increased by approximately 400%. The toll increase translates into a revenue increase of only 35%

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* It will be noted that the limiting case $\eta \to 0$ is just the Ramsey case.

TABLE 8.2

EQUITY MODEL SIMULATION, CONSTANT MARGINAL COST, ISOPROFIT

	ACTUAL	DEMAND SIMULATION	Simulation				
			$\eta=1.0$	$\eta=1.7$	$\eta=2.5$	$\eta=3.5$	$\eta=5.0$
PQLR							
1952	.8944		1.0133	.9761	.9451	.9191	.8977
1957	.9032		1.0807	1.0359	.9986	.9986	.9410
1962	.9872		1.1912	1.1425	1.1021	1.0359	1.0408
1967	1.0000		1.2852	1.2266	1.1782	1.1380	1.1048
1972	1.0529		1.2809	1.2082	1.1483	1.0988	1.0853
1976	1.1992		1.5640	1.4534	1.3626	1.2873	1.2255
QLR							
1952	71.427	72.559	69.064	70.093	70.994	71.780	72.451
1957	108.796	107.133	99.798	101.480	102.965	104.273	105.409
1962	146.935	144.285	133.955	136.185	138.136	139.838	141.302
1967	192.700	194.314	175.965	179.243	182.118	184.633	196.805
1972	268.390	267.216	247.290	253.071	258.209	262.752	266.683
1976	342.721	343.440	309.210	318.304	326.533	333.950	340.505
PQMR							
1952	1.0643		.3100	.3579	.4176	.4979	.6215
1957	1.0616		.3136	.3609	.4198	.4983	.6182
1962	1.0414		.3353	.3855	.4475	.5293	.6524
1967	1.0000		.3215	.3689	.4271	.5035	.6176
1972	1.1019		.3785	.4329	.5001	.5890	.7232
1976	1.2447		.4019	.4584	.5283	.6213	.7629
QMR							
1952	21.282	21.232	107.362	88.907	72.575	57.609	43.044
1957	36.428	36.316	180.299	149.874	122.890	98.101	73.895
1962	56.541	56.446	250.219	208.299	171.265	137.360	104.348
1967	97.200	96.232	427.405	356.778	294.333	237.098	181.270
1972	171.631	172.367	701.886	588.284	486.716	392.572	299.737
1976	282.382	285.993	1263.17	1062.62	881.809	712.633	544.116

for 1967. The implications of this are discussed later.

Overall, the equity considerations restrict the rise in local prices over the Ramsey levels. As η rises the marginal utility of money falls more quickly as income increases and the overall result in these simulations is to move prices closer to their historic levels. Nonetheless, the simulated price of toll remains well below the historic level.

Section 8.3 Efficiency-Equity Prices with Variable Marginal Costs and an Isoprofit Constraint

This simulation is similar to the constant marginal cost simulation of Section 8.2 except that the marginal cost of local and the marginal cost of toll are allowed to vary. The isoprofit constraint again restricts the profit to be no less than the historic level. The results are shown in Table 8.3.

A very similar pattern emerges from Table (8.3) as was seen in the constant marginal cost simulation. Residential toll output increases greatly, while residential local output decreases as η increases. For any given η, the effect is slightly greater than in the constant marginal cost case. The reason for this is that even though toll marginal costs are relatively constant, local has declining marginal costs. This can be seen in Table (8.3), for the η=1.7 case.

The effect on revenues and costs of such a large increase in toll quantities are relatively small. For 1967, η= 1.7, costs increased by 12%, as did revenues, since profits are constant. This result arises because the marginal cost of toll is quite small-about .25. It will be recalled that our marginal cost estimates correspond reasonable closely to those of Bell.

TABLE 8.3

EQUITY MODEL SIMULATION - VARIABLE MARGINAL COST, ISOPROFIT

	ACTUAL	DEMAND SIMULATION	Simulation $\eta=1.0$	$\eta=1.7$	$\eta=2.5$	$\eta=3.5$	$\eta=5.0$
PQLR							
1952	.8944		1.0570	.9969	.9527	.9205	.8973
1957	.9032		1.1194	1.0505	1.0005	.9640	.9376
1962	.9872		1.2416	1.1644	1.1084	1.0675	1.0381
1967	1.0000		1.3595	1.2621	1.1915	1.1398	1.1026
1972	1.0529		1.4057	1.2769	1.1824	1.1127	1.0619
1976	1.1992		1.7448	1.5535	1.4126	1.3079	1.2309
QLR							
1952	71.427	72.559	67.899	69.513	70.770	71.738	72.465
1957	108.796	107.133	98.418	100.921	102.887	104.412	105.564
1962	146.935	144.285	131.780	135.169	137.828	139.890	141.446
1967	192.700	194.314	172.098	177.227	181.311	184.515	186.955
1972	268.390	267.216	238.370	247.601	255.240	261.446	266.318
1976	342.721	343.440	296.123	310.036	321.907	331.859	339.915
PQMR							
1952	1.0643		.2703	.3290	.4005	.4924	.6254
1957	1.0616		.2837	.3435	.4161	.5092	.6434
1962	1.0414		.2984	.3607	.4361	.5324	.6707
1967	1.0000		.2801	.3306	.4085	.4990	.6292
1972	1.1019		.3180	.3810	.4584	.5590	.7060
1976	1.2447		.3417	.4064	.4861	.5907	.7455
QMR							
1952	21.282	21.232	128.584	99.304	76.686	58.451	42.697
1957	36.428	36.316	205.621	159.956	126.327	95.346	70.123
1962	56.541	56.446	291.676	227.391	177.187	136.291	100.636
1967	97.200	96.232	512.252	400.145	312.029	239.913	176.897
1972	171.631	172.367	882.290	695.749	545.681	420.425	309.366
1976	282.382	285.993	1563.34	1244.77	983.717	761.557	560.894

TABLE 8.3 (Continued)

MC LOC	ACTUAL	SIMULATION N = 1.7	MC TOL	ACTUAL	SIMULATION N = 1.7
1952	1.2237	1.1503	1952	.2508	.2251
1957	1.1372	1.0741	1957	.2447	.2285
1962	1.0187	.9603	1962	.2519	.2316
1967	.9387	.8767	1967	.2349	.2111
1972	1.0489	.9152	1972	.2777	.2348
1976	1.2150	1.0043	1976	.2951	.2485
COST			LAM		$\times 10^{-5}$
1952	164.046	179.477	1952		.3010
1957	268.111	292.138	1957		.2239
1962	381.722	417.185	1962		.1913
1967	564.106	633.166	1967		.1258
1972	1020.99	1130.52	1972		.0677
1976	1828.54	2053.57	1976		.0280

PQLR - Price, local residential
QLR - Quantity, local residential
PQMR - Price, toll residential
QMR - Quantity, toll residential
MC LOC - Marginal cost, local
MC TOL - Marginal cost, toll
COST - Cost, sealed by ratio
LAM - Language multiplier

Section 8.4 Efficiency-Equity Prices with Variable Marginal Costs and an Iso-rate of return Constraint

In this last simulation, the iso-profit constraint is replaced by an iso-rate of return constraint. The rate of return on capital that Bell makes in this simulation is not permitted to be less than the historical rate of return.

For this simulation, the profit constraint equation is dropped and two additional equations are added.

(1) Side order condition for share of capital - Equation (4.3)

(2) Actual rate of return (RRK)

$$RRK = (P_L Q_L^R + P_M Q_M^R + \Pi_L Q_L^B + \Pi_M Q_M^B + OR - COST + rK)/K \tag{8.1}$$

In this simulation, the level of capital services is simulated in addition to prices and quantities.

The results for this simulation are shown in Table (8.4) for $\eta = 1.7$. It is clear that there is essentially no difference between this simulation and the variable marginal cost and iso-profit simulation. One point of interest is that since toll economizes on capital relative to local (CRQM = -.0826, CRQL = .0360), an increase in toll output and a decline in local output results in less capital being demanded. Of course this is a long run effect.

TABLE 8.4

EQUITY MODEL SIMULATION - VARIABLE MC, ISO-RATE OF RETURN

	PQLR		QLR		PQMR		QMR	
	ACTUAL	SIMULATED	ACTUAL	SIMULATED	ACTUAL	SIMULATED	ACTUAL	SIMULATED
1952	.8944	.9830	71.427	69.898	1.0643	.3270	21.287	100.082
1957	.9032	1.0373	108.796	101.426	1.0616	.3417	36.428	161.042
1962	.9872	1.1375	146.935	136.421	1.0414	.3576	56.541	229.976
1967	1.0000	1.2336	192.700	178.836	1.0000	.3354	97.200	404.297
1972	1.0529	1.2761	268.390	247.658	1.1019	.3809	171.631	695.940
1976	1.1992	1.5631	342.721	309.274	1.2447	.4073	282.382	1241.25

	MC LOC		MC TOL		COSTX		KX	
	ACTUAL	SIMULATED	ACTUAL	SIMULATED	ACTUAL	SIMULATED	ACTUAL	SIMULATED
1952	1.2237	1.1494	.2508	.2246	164.05	180.08	585.72	510.59
1957	1.1372	1.0729	.2447	.2286	268.11	292.91	1033.54	888.31
1962	1.0187	.9581	.2519	.2306	381.72	418.97	1673.37	1529.40
1967	.9387	.8743	.2349	.2103	564.11	635.45	2322.42	2155.09
1972	1.0488	.9151	.2777	.2347	1020.99	1130.62	3032.84	2940.06
1976	1.2150	1.0055	.2951	.2488	1828.54	2051.94	3605.65	3488.20

PLOCR - Price, local residential
QLOCR - Quantity, local residential
PTOLR - Price, toll residential
QTOLR - Quantity, toll residential
MC LOC - Marginal cost, local
MC TOL - Marginal cost, toll
COSTX - Cost, local and message toll output (scaled)
KX - Capital, local and message toll output (scaled)

Note: Simulated values for $\eta=1.7$

Part 9 CONCLUSIONS

In this report, 3 models were presented in order to examine demand, production and socially optimal pricing. The demand and production models yielded price and income elasticity estimates as well as the structure of the cost function from which marginal cost functions could be derived. This information was combined with a range of possible values of a parameter (R) representing distributional considerations of social welfare, based on the choice of the income elasticity of marginal utility of money (η). The resultant model was simulated to determine "optimal" prices for residential users of Bell's services.

All the scenarios yielded the same directional result, but differed with respect to degree. Compared to historic prices, the "optimal" residential local prices were higher, whereas the "optimal" residential toll prices were much lower. The introduction of distributional considerations ($\eta \neq 0$) mitigated the rise in local prices and the fall in toll prices. This dampening effect increased as distributional consideration became more important (η increased). Overall, it seems reasonable to conclude that social welfare would be improved by a small increase in the price of residential local, and a relatively larger decrease in the price of residential toll. This result was derived under the assumption that Bell's historic profit levels were retained.

The conclusions drawn here are consistent with results drawn by Rholf. Rohlf concluded that economically efficieny pricing (in the ATT system) would result in an 80% increase in the price of local services, and a 50% reduction in the price of long distance rates (approximately). These results were based on Ramsey optimality, and excluded equity considerations.

Questions regarding these results may arise for a number of reasons. It may be argued that:

a) The demand elasticities used are incorrect. This is an empirical matter. However it is interesting to note that Rohlf's (similar) results were derived using much lower elasticities.

b) The marginal cost estimates were incorrect. This issue was discussed in the body of the report. A 20% change would not affect the conclusions drawn in this report.

c) The simulation results are outside the sample range. In principle this is a valid criticism. However the substance of the results remain unaffected by restricting the range of possible price movement as in, for example, the economic gradient method of Willig and Bailey. This interesting application of the economic gradient method arises in problems with three or more variables.

d) The prices are not globally optimal. In this report, only residential price changes were considered; business prices were taken as exogenous. The inability to treat rigorously intermediate goods in a welfare model with a production function not characterized by constant returns to scale motivated this approach.

e) The suggested price changes cannot be implemented. Local residential and business billing occur separately, and therefore present no problem. Since the present toll rate structure is the same, irrespective of user, the proposed decrease in residential message toll rates could be implemented by a reduction in the rates of off-peak periods (nights, weekends), where residential usage is proportionally high.

APPENDIX A1

This appendix contains all the equations utilized in the optimal pricing models. As well, a definition of each model in terms of the equation and endogenous variables is provided.

Equation 4.1 3 INPUT - 2 OUTPUT (SYMMETRIC) TRANSLOG COST FUNCTION

COST FUNCTION

4.1

$$
\begin{aligned}
\ell nC = CC_o \; + \; & C_w \ell nw + C_r \ell nr + C_v \ell nv + C_T \ell nT + C_{QL} \ell nQL + C_{QM} \ell nQM \\
+\tfrac{1}{2}\ell nw \; & \left[C_{ww}\ell nw + C_{wr}\ell nr + C_{wv}\ell nv + C_{wT}\ell nT + C_{wQL}\ell nQL + C_{wQM}\ell nQM\right] \\
+\tfrac{1}{2}\ell nr \; & \left[C_{wr}\ell nw + C_{rr}\ell nr + C_{rv}\ell nv + C_{rT}\ell nT + C_{rQL}\ell nQL + C_{rQM}\ell nQM\right] \\
+\tfrac{1}{2}\ell nv \; & \left[C_{wv}\ell nw + C_{rv}\ell nr + C_{vv}\ell nv + C_{vT}\ell nT + C_{vQL}\ell nQL + C_{vQM}\ell nQM\right] \\
+\tfrac{1}{2}\ell nT \; & \left[C_{wT}\ell nw + C_{rT}\ell nr + C_{vT}\ell nv + C_{TT}\ell nT + C_{QLT}\ell nQL + C_{QMT}\ell nQM\right] \\
+\tfrac{1}{2}\ell nQL \; & \left[C_{wQL}\ell nw + C_{rQL}\ell nr + C_{vQL}\ell nv + C_{QLT}\ell nT + C_{QLQL}\ell nQL + C_{QMQL}\ell nQM\right] \\
+\tfrac{1}{2}\ell nQM \; & \left[C_{wQM}\ell nw + C_{rQM}\ell nr + C_{vQM}\ell nv + C_{QMT}\ell nT + C_{QMQL}\ell nQL + C_{QMQM}\ell nQM\right]
\end{aligned}
$$

SIDE ORDER CONDITION FOR CAPITAL

4.3

$$\frac{rK}{C} = C_r + C_{wr}\ell nw + C_{rr}\ell nr + C_{rv}\ell nv + C_{vT}\ell nT + C_{rQL}\ell nQL + C_{rQM}\ell nQM$$

MARGINAL COST OF LOCAL

4.6

$$MC_{QL} = \left(\frac{C}{QL}\right)\Big[C_{QL} + C_{wQL}\ell nw + C_{rQL}\ell nr + C_{vQL}\ell nv + C_{QLT}\ell nT + C_{QLQL}\ell nQL + C_{QMQL}\ell nQM\Big]$$

MARGINAL COST OF TOLL

4.7

$$MC_{QM} = \frac{C}{QM}\Big[C_{QM} + C_{wQM}\ell nw + + C_{rQM}\ell nr + C_{vQM}\ell nv + C_{QMT}\ell nT + C_{QMQL}\ell nQL + C_{QMQM}\ell nQM\Big]$$

RESIDENTIAL LOCAL DEMAND

4.24 $$\ln(QLR) = RA_0 + RA_1 \ln\left(\frac{PQLR}{CPI}\right) + RA_2 \ln\left(\frac{YD}{CPI}\right) + RA_3 \ln(POP) + RH_4 \ln(CONV)$$
$$+ RD_1 \cdot D_{59} + RD_2 \cdot D_{70}$$

PROFITABILITY CONSTRAINT

7.17 $$P_L Q_L{}^R + P_M Q_M{}^R + \Pi_L Q_L{}^B + \Pi_M Q_M{}^B + OR$$
$$-COST\ (Q_L{}^R + Q_L{}^B, Q_M{}^R + Q_M{}^B, \rho) \geq \Pi_\circ$$

RATE OF RETURN CONSTRAINT

8.1 $$RRK = (P_L Q_L{}^R + P_M Q_M{}^R + \Pi_L Q_L{}^B + \Pi_M Q_M{}^B$$
$$+ OR - COST + rK)/K$$

RESIDENTIAL TOLL DEMAND

A1.1 $$\ell n\ (QM^R) = B_0 + B_1 \ell n\left(\frac{PQM}{CPI}\right) + B_2 \ell n\left(\frac{YD}{CPI}\right) + B_3 \ell n(POP)$$
$$+ BD_1 \cdot D_{59} + BD_2 \cdot D_{70} + \ell n\left(\frac{REVMR}{REVMR + REVMB}\right)$$

where REVMR = residential message toll revenue

REVMB = business message toll revenue

1ST ORDER CONDITION FOR RESIDENTIAL LOCAL

$$\text{A1.2} \qquad P_L = LAM.\beta_L \left[P_L \left[1 + \frac{1}{\beta_L} \right] - MC_L \right] R_L^{-1}$$

1ST ORDER CONDITION FOR RESIDENTIAL TOLL

$$\text{A1.3} \qquad P_M = LAM \, . \, \beta_M \left[P_M \left[1 + \frac{1}{\beta_M} \right] - MC_M \right] R_M^{-1}$$

LOCAL IDENTITY

$$\text{A1.4} \qquad Q_L = Q_L^R + Q_L^B$$

TOLL IDENTITY

$$\text{A1.5} \qquad Q_M = Q_M^R + Q_M^B$$

1) Ramsey Model, iso-profit, variable marginal cost

Equations:

Cost function

Marginal cost of local

Marginal cost of toll

Demand for residential local

Demand for residential toll

1st order condition for residential local

1st order condition for residential toll

Profitability constraint

Local identity

Toll identity $(R_L = R_M)$

Endogenous variables:

$$COST, MC_L, MC_M, Q_L^R, Q_M^R, Q_L, Q_M, P_L, P_M, \lambda$$

2) Equity Pricing, Iso-profit, constant marginal cost

Equations:

Cost function

Demand for residential local

Demand for residential toll

1st order condition for residential local

1st order condition for residential toll

Profitability constraint

Local identity

Toll identity

Endogenous variables:

$$COST, Q_L^R, Q_M^R, Q_L, Q_M, P_L, P_M, \lambda$$

3) Equity Pricing, Iso-profit, variable marginal cost

Equations:

Cost function

Marginal cost of local

Marginal cost of toll

Demand for residential local

Demand for residential toll

1st order condition for residential local

1st order condition for residential toll

Profitability constraint

Local identity

Toll identity

Endogenous variables:

$COST, MC_L, MC_M, Q_L^R, Q_M^R, Q_L, Q_M, P_L, P_M, \lambda$

4) Equity pricing, iso-rate of return, variable marginal cost

Equations:

Cost function

Marginal cost of local

Marginal cost of toll

Demand for residential local

Demand for residential toll

1st order condition for residential local

1st order condition for residential toll

Side order condition for capital

Rate of return constraint

Local identity

Toll identity

Endogenous variables:

$COST, MC_L, MC_M, Q_L^R, Q_M^R, Q_L, Q_M, P_L, P_M, K, \lambda$

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