

EFFICIENCY, EQUITY AND REGULATION  
AN ECONOMETRIC MODEL OF BELL CANADA

JON BRESLAW  
and  
J. BARRY SMITH

Discussion Paper 81-01



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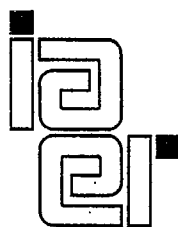
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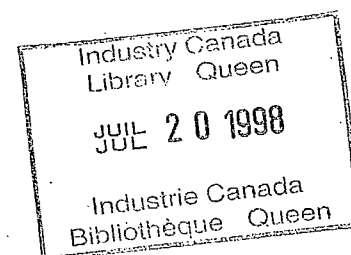
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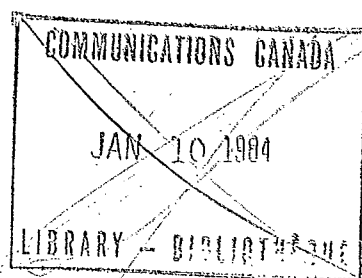
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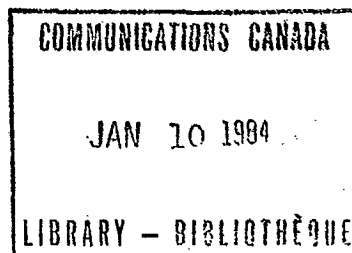
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EFFICIENCY, EQUITY AND REGULATION  
AN ECONOMETRIC MODEL OF BELL CANADA

<sup>1</sup>(Jon A. Breslaw)  
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January 1981



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## 1- Introduction

Over the last decade, the structure and pricing practices of the telecommunications industry in Canada and the United States have been of increasing interest to public policy makers. Recently, a Canadian Federal-Provincial working group of communications ministers achieved a consensus over a range of policy objectives which must be satisfied in order to best serve the public interest. One of these objectives was stated as follows:

"developing and maintaining an efficient telecommunications infrastructure which can provide universal access to a broad range of telecommunications services at economic and equitable rates as a fundamental goal of public policy" [page 2].

To a large extent, this policy statement reflects the provisions of the Railway Act which provides the authority for telecommunications regulation in Canada. Amongst other things, the Railway Act requires that the prices of telecommunications services be "fair and reasonable", although these terms are not rigorously defined.

In this paper we study equity-efficiency pricing issues for the case of Bell Canada - a large telecommunications carrier operating as the sole supplier for almost all of Quebec and Ontario. We begin by formalizing the equity-efficiency issue (and hence the question of "fairness and reasonableness") within a general economic model. The model involves an optimization problem which yields as a solution residential service prices which incorporate both efficiency and equity considerations. We next specify and estimate an econometric multi-input multi-output cost and demand model which is used to study characteristics of the Bell Canada production process. Information

resulting from this empirical model is then introduced into the optimal pricing model. When the pricing model is simulated, sets of efficiency-equity prices result. Differences in prices reflect different efficiency-equity weightings.

Since the model was designed for a public policy application, it was considered important to examine the robustness of the simulation results. To this end, the sensitivity of the model to differing assumptions was examined. An important and robust result following from the analysis is that the historic prices charged by Bell are optimal only if it is socially desirable to weight equity considerations relatively highly. We also demonstrate the fact that it will often be the case that the first adjustment of prices towards optimal prices for residential services will supply the greatest welfare improvement for consumers.

## 2- Efficiency-Equity Pricing

We begin this section by considering the general problem of choosing service prices of a regulated industry so as to maximize the welfare of the non-business consumers of the services. The choice of prices is constrained by the requirement that the regulated industry earn no more than a predetermined level of profit. We conclude this section by developing a model in which an econometric cost model can be combined with the theory to provide a rigorous, consistent and tractable application of the pricing problem. The resulting "efficiency-equity" model is used to simulate socially optimal departures from the historic pattern of prices of telephone

services of Bell Canada.

### A Theoretical Model

The canonical solution to the problem of choosing welfare maximizing prices subject to constraint is attributed to Ramsey (1927). Feldstein (1972a, 1972b, 1972c) extended the analysis to include distributive or equity considerations. The analysis presented here is similar to that of Feldstein (1972a). There are however some interesting differences and extensions. In the first place, the optimality conditions are derived using the (dual) indirect utility function approach. Secondly, a diagrammatic solution to the problem is presented.

The problem considered here can be written:

$$\begin{array}{ll} \text{maximize} & W = N \int_0^{\infty} V(p_1, p_2, p, y) f(y) dy \\ (p_1, p_2) & \end{array} \quad (2.1)$$

$$\text{subject to} \quad \Pi(p_1, p_2, p; K) \geq \Pi_0 \quad (2.2)$$

where  $N$  = number of consumers

$V$  = indirect utility function of the representative consumer assumed quasi-convex in  $(p_1, p_2, p, y)$

$p_1, p_2$  = service prices to be chosen

$p$  = price index for a composite of all other goods

$y$  = household income assumed to be the only difference amongst households

$f(y)$  = relative density function of household income

$\Pi(\ )$  = profit of the regulated firm assumed quasi-concave in prices

$\Pi_0$  = minimum required profit



$K$  = a vector of parameters including characteristics of the income distribution and factor prices.

Two points should be made at the outset. First, this problem is posed for consumers only. It is assumed that the prices faced by firms for the variety of services are unchanged. Secondly, welfare ( $W$ ) can be interpreted as the sum of utilities of consumers of various incomes (or income classes) weighted by the number of consumers in these classes. Class differences, as determined by income, will provide the basis for equity considerations in the model.

The maximization problem is solved by first constructing the Lagrange function:

$$L = N \int_0^{\infty} V(p_1, p_2, p, y) f(y) dy + \lambda (\Pi(p_1, p_2, p; K) - \Pi_0) \quad (2.3)$$

The first order necessary conditions for an interior constrained maximum are given by:

$$\frac{\partial L}{\partial p_1} = N \int_0^{\infty} \frac{\partial V}{\partial p_1}(p_1, p_2, p, y) f(y) dy + \lambda \frac{\partial \Pi}{\partial p_1}(p_1, p_2, p; K) = 0 \quad (2.4)$$

$$\frac{\partial L}{\partial p_2} = N \int_0^{\infty} \frac{\partial V}{\partial p_2}(p_1, p_2, p, y) f(y) dy + \lambda \frac{\partial \Pi}{\partial p_2}(p_1, p_2, p; K) = 0 \quad (2.5)$$

$$\frac{\partial L}{\partial \lambda} = \Pi(p_1, p_2, p; K) - \Pi_0 = 0 \quad (2.6)$$

The second order necessary and sufficient conditions require that the matrix defined below as  $S$  be negative semi-definite at the optimum. This condition reduces to the condition  $\det[S] \geq 0$ .

$$S = \begin{bmatrix} 0 & \Pi_1 & \Pi_2 \\ \Pi_1 & \tilde{V}_{11} + \lambda \Pi_{11} & \tilde{V}_{12} + \lambda \Pi_{12} \\ \Pi_2 & \tilde{V}_{12} + \lambda \Pi_{12} & \tilde{V}_{22} + \lambda \Pi_{22} \end{bmatrix}$$

Where:  $\tilde{V}_{ij} = N \int_0^\infty \frac{\partial^2 V(p_1, p_2, p, y)}{\partial p_i \partial p_j} f(y) dy$

$$\Pi_{ij} = \frac{\partial^2 \Pi(p_1, p_2, p; K)}{\partial p_i \partial p_j},$$

$$\Pi_i = \frac{\partial \Pi(p_1, p_2, p; K)}{\partial p_i}.$$

The multiplier,  $\lambda$ , at the optimum is given by  $\frac{-dW^*}{d\Pi_0} > 0$  or the increase in welfare arising from reducing the minimum required profit by a 'small' amount. Thus, equations (2.4) and (2.5) have the interpretation that at the optimum, price is such that the increase in welfare arising from lowering prices a 'small' amount is just offset by the social value of the associated decrease in profit of the regulated firm.

It is possible to express the first order conditions in a fashion which facilitates their interpretation.

First, it will be recalled that Roy's Identity yields:

$$\frac{\partial V(p_1, p_2, y)}{\partial p_i} = -q_i(p_1, p_2, p, y) \frac{\partial V(p_1, p_2, p, y)}{\partial y} \quad (2.7)$$

Second, the aggregate demand for good  $i$ ,  $Q_i$ , can be defined:

$$Q_i = N \int_0^{\infty} q_i f(y) dy \quad (2.8)$$

Third, the profit derivatives can be re-written:

$$\frac{\partial \Pi(p_1, p_2, p; K)}{\partial p_i} = (MR_i - MC_i) \frac{\partial Q_i}{\partial p_i} \quad (2.9)$$

where  $MR_i$  and  $MC_i$  are respectively the marginal revenue and marginal cost for service  $i$  and  $\frac{\partial Q_1}{\partial p_2} = \frac{\partial Q_2}{\partial p_1} = 0$  by assumption.

Finally, it is convenient to follow Feldstein (1972a) in defining the distributional coefficient of  $i$  as:

$$R_i = \frac{N}{Q_i} \int_0^{\infty} q_i(p_1, p_2, p, y) \frac{\partial V(p_1, p_2, p, y)}{\partial y} f(y) dy \quad (2.10)$$

Feldstein (1972a, p. 33) notes that:

"the ratio  $R_i$  is a weighted average of the marginal social utilities, each household's marginal utility weighted by that household's consumption of good  $i$ . The conventional welfare assumption that  $\frac{\partial V}{\partial y}$  declines as  $y$  increases implies that the value of  $R_i$  will be greater for a necessity than for a luxury. The higher the income elasticity of demand for a good, the lower the value of  $R_i$ ":

Substituting (2.7), (2.8), (2.9) and (2.10) into (2.4) and (2.5) and eliminating  $\lambda$  yields:

$$\frac{(MR_1 - MC_1)/p_1}{(MR_2 - MC_2)/p_2} = \frac{\epsilon_2 R_1}{\epsilon_1 R_2}; \quad \epsilon_i = \frac{\partial Q_i}{\partial p_i} \cdot \frac{p_i}{Q_i} \quad (2.11)$$

Equation (2.11) represents the optimal divergences of marginal revenues from marginal costs for both goods given both efficiency and equity considerations. The case treated by Ramsey ignored equity considerations and can be derived from (2.11) by imposing the restrictions that  $R_1 \equiv 1 \equiv R_2$ . With these restrictions, (2.11) reduces to the familiar Ramsey Rule:

$$\frac{(p_1 - MC_1)/p_1}{(p_2 - MC_2)/p_2} = \frac{\epsilon_2}{\epsilon_1} \quad (2.12)$$

when one makes use of the result  $MR_i = p_i(1+1/\epsilon_i)$ . Equation (2.12) has the interpretation that the optimal percentage divergence of regulated price from marginal cost for a good is inversely related to the price elasticity of demand for the service. Alternatively, the optimal tax on a good is higher the lower is the price elasticity of demand.

There is an element of discrimination in the Ramsey Rule and it is equivalent to the well-known result that, under normal conditions, a discriminating monopolist will charge a higher price in the less elastic of two markets. The unsettling feature of the Ramsey Rule is that, as Atkinson and Siglitz (1972) and Pestieau (1975) point out, less elastic goods are also often necessities and thus the brunt of the 'optimal' tax will be borne by those with lower incomes. Thus, Ramsey optimality may not be distinguishable from regressivity in this context. Equity considerations suggest that whenever a good is a necessity, the optimal tax on the good should be lower than the Ramsey Rule requires. This latter requirement is present in

equation (2.11). The fact that  $R_1$  is smaller for luxuries than for necessities reduces the optimal tax from the levels which would obtain under a Ramsey, or pure efficiency, regime.

This latter fact has been proved by Feldstein (1972a). Thus, rather than reprove the general case, it is useful to describe a specification of a welfare model which can be used in conjunction with an econometric cost model to study optimal efficiency-equity prices for Bell Canada.

#### A Welfare Specification: Initial Considerations

Three sets of constraints arising from empirical, computational and theoretical considerations are important in determining the specified form of the welfare model.

With respect to the constraints placed on the model by data considerations, one of the features of available telecommunications time series data is that double-log demand systems with constant own-price and income elasticity parameters provide a good fit (see for example, Taylor (1980)). Problems including multicollinearity and a small sample size effectively preclude the accurate estimation of cross-elasticity terms or terms which would allow the own-elasticities to vary with price or income.

Easily manipulated functional forms for cost and demand are also desirable from a computational viewpoint. For example, simulation of the system described by the equilibrium conditions (2.4), (2.5), (2.6) (to solve for  $p_1, p_2$  and  $\lambda$ ) is facilitated if closed form expressions of the integrals are available. Also, it is important that computationally attractive features of the profit and distributed

welfare functions should be preserved under differentiation. As shown in equation (2.7) Roy's Identity guarantees that economic theory can be used to simplify some of the derivatives as long as demand and marginal utility of income schedules are tractable.

Theoretical constraints are perhaps the most difficult to satisfy. Both the empirical and computational constraints noted above tend to support the acceptance of double-log demand models with constant own-price and income elasticities. As well, the existing evidence suggests that price and income elasticities differ across goods. Unfortunately, economic theory suggests that the only exact demands consistent with strict constancy of the own-price and income elasticities are everywhere unit elastic and come from Cobb-Douglas utility functions. Although unit elastic demands pass the computation test, they miserably fail on the grounds of observed consumer behavior. As well, from the point of view of ultimate usefulness of the results, it would be pointless to proceed by adopting double-log demands with elasticity parameters different from those consistent with economic theory. Fortunately, there are some conditions under which the demands derived from utility maximization are virtually indistinguishable from double-log demands. Frisch (1959) and Sato (1972) have studied the properties of demands derivable from additive utility functions. They show that if the demand data satisfy certain conditions then the demands will be almost double-log in own-price and income. It is useful to briefly reexamine these results since the Bell Canada data can be shown to satisfy the "almost double-log" conditions.

Using the notation and arguments of Sato (1972) we define:

$q_i$  = quantity of the  $i^{\text{th}}$  good  $i = 1, \dots, n$

$p_i$  = price of the  $i^{\text{th}}$  good  $i = 1, \dots, n$

$y$  = total expenditure =  $\sum_{i=1}^n p_i q_i$

$\theta_i$  = budget share of good  $i$ , =  $\frac{p_i q_i}{y}$ ;  $\sum_{i=1}^n \theta_i = 1$

$\eta_i$  = income elasticity of good  $i$ , =  $\frac{\partial q_i}{\partial y} \cdot \frac{y}{q_i}$   $i = 1, \dots, n$

$\lambda$  = marginal utility of income

$\sigma$  :  $(-1/\sigma)$  is the income elasticity of the marginal utility  
of income =  $\frac{\partial \lambda}{\partial y} \cdot \frac{y}{\lambda}$

Sato shows that if the utility function is additive of the form  
 $V(q) = \sum f_i(q_i)$ , then the price elasticities of demand can be written:

$$\frac{\partial q_i}{\partial p_j} \cdot \frac{p_j}{q_i} = \eta_i \theta_j (\sigma \eta_j - 1) \quad i \neq j \quad (2.13.1)$$

$$\frac{\partial q_i}{\partial p_i} \cdot \frac{p_i}{q_i} = -\sigma \eta_i + \eta_i \theta_i (\sigma \eta_i - 1) \quad (2.13.2)$$

Examining equation (2.13.1) we note that the cross-price elasticity of demand for good  $i$  with respect to the price of good  $j$  can effectively be ignored if the budget share of good  $j$  ( $\theta_j$ ) is small. Similarly, if the own-budget share of a good ( $\theta_j$ ) is small then equation (2.13.2) states that the own-price elasticity of demand will be proportional to the income elasticity of demand with the factor of proportionality given by the inverse of the elasticity of marginal utility of income.

In general, both  $\sigma$  and  $\eta_i$  (and thus the own-price elasticity of demand) will not be constant. It is however possible to constrain each own-price elasticity to be almost constant at the value  $\sigma_i$ . Thus  $\sigma$  and  $\eta_j$  are constrained to satisfy  $\sigma_i = -\sigma\eta_i$  almost everywhere. These constraints implicitly define the utility function:

$$V = \sum_{i=1}^n c_i q_i^{-\rho_i} ; \quad \rho_i = (1-\sigma_i)/\sigma_i ; \quad c_i(1-1/\sigma_i) > 0 \quad (2.14)$$

Further, since Engel's aggregation yields:

$$\sum \eta_i \theta_i = 1$$

it follows that:

$$\sigma = \sum \sigma_i \theta_i \quad (2.15)$$

In general,  $\sigma$  will not be constant. However, if the elasticities  $\sigma_i$  are all close to the same size, or if  $\theta_i$ , the budget shares, are relatively constant, then  $\sigma$  will be effectively constant.

Consider now the special case where utility is defined over three goods  $(q_1, q_2, q_3)$ . Assume that the budget shares of goods  $q_1$  and  $q_2$  are small and stable and that  $q_3$  is a composite commodity. If we adopt the utility function defined by (2.14) and use the results presented in (2.13.1), (2.13.2) and (2.15) we can almost exactly write the demands for  $q_1$  and  $q_2$  in double-log form as:

$$\ln q_i = \ln \alpha_i - \sigma \eta_i \ln(p_i/p) + \eta_i \ln(y/p) \quad i=1,2 \quad (2.16)$$

where  $p$ , the price index, and  $-\sigma$  are defined by:



$$p \approx p_3 \quad (2.17.1)$$

$$\sigma \approx \sigma_3 \quad (2.17.2)$$

Finally, the near constancy of  $\sigma$  implies that the marginal utility of income function can be written:

$$\ln \lambda \approx \ln k - \frac{1}{\sigma} (\ln y - \ln p) - \ln p \quad (2.18)$$

where  $k$  is independent of prices. It will be noted that  $\lambda$  is homogeneous of degree  $(-1)$  in prices and incomes as required.

#### An Exact Specification

In the last section, we provided a demonstration that under some conditions, double-log demands, which are desirable from the point of view of estimation and computation, are also consistent with economic theory. In this section we introduce the results of the last section into the welfare maximization model to derive the final form of the model.

We begin by noting that when (2.16) is substituted into (2.8) and (2.10) the distributional coefficients for goods 1 and 2 can be written

$$R_i = \frac{k p^{(1-\sigma)/\sigma} \int_0^\infty y^{(\eta_i \sigma - 1)/\sigma} f(y) dy}{\int_0^\infty y^{\eta_i} f(y) dy} \quad (2.19)$$

It will be noted that  $R_i$  is independent of  $p_1$  and  $p_2$ . Similarly, the ratio  $R_1/R_2$  is independent of all prices and scale ( $k$ ).

Turning our attention to equation (2.11) we assume that marginal costs for goods 1 and 2 are given by  $c_1$  and  $c_2$ . Equation (2.11) can be then re-written as:

$$\frac{(p_1 + p_1/\epsilon_1 - c_1)/p_1}{(p_2 + p_2/\epsilon_2 - c_2)/p_2} = \frac{\epsilon_2 R_1}{\epsilon_1 R_2} \quad (2.20)$$

where the  $R_1$  are given by (2.19) and the  $\epsilon_1$  are constant and given by equation (2.16) as  $-\sigma\eta_1 = \epsilon_1$ .

At this point it is useful to present a diagrammatic representation of the problem. We will tailor this discussion to the case of Bell Canada so that the numerical results of the sections which follow can be visually interpreted. In particular, we will assume that good 2 (later identified as local residential services) is price inelastic and that good 1 (later identified as toll residential services) is price elastic. Since  $\epsilon_1 = -\eta_1\sigma$ , it follows that good 1 is more of a luxury than good 2 and therefore that  $R_1 < R_2$ . Demands are assumed to be given by (2.16) and for ease of exposition in the diagrammatic case, the marginal costs are assumed constant at the levels  $c_1$  and  $c_2$ .

The equation of the indirect indifference curve for utility level  $\bar{V}$  is given by:

$$N \int_0^\infty V(p_1, p_2, p, y) f(y) dy \equiv \tilde{V}(p_1, p_2, p, N) = \bar{V} \quad (2.21)$$

Since  $\tilde{V}$  is a quasi-convex function of  $(p_1, p_2)$ , the indirect indifference curves in  $(p_1, p_2)$  space are convex to the origin with

direction of improvement towards the origin. Similarly, the preceding arguments guarantee that the iso-profit contour given by  $\Pi(p_1, p_2, p) = \Pi_0$  takes the general form of a parabola with minimum at  $p_1^M = c_1 \epsilon_1 / (\epsilon_1 + 1)$ . This is just the unconstrained profit maximizing price for good 1. The curves  $\Pi_0$  and  $\bar{V}$  are drawn in Figure 1. Higher iso-profit contours lie to the north of contour  $\Pi_0$ . Similarly, that part of the contour  $\Pi_0$  corresponding to prices of good 1 in excess of  $p_1^M$  is unimportant since a rational social manager could move to the left of  $p_1^M$  and lower both prices and thereby raise welfare.

The equilibrium point is given by E where the indirect indifference curve is just tangent to the profit constraint. The second-order conditions require that the iso-profit constraint lie everywhere inside the indirect indifference curve.

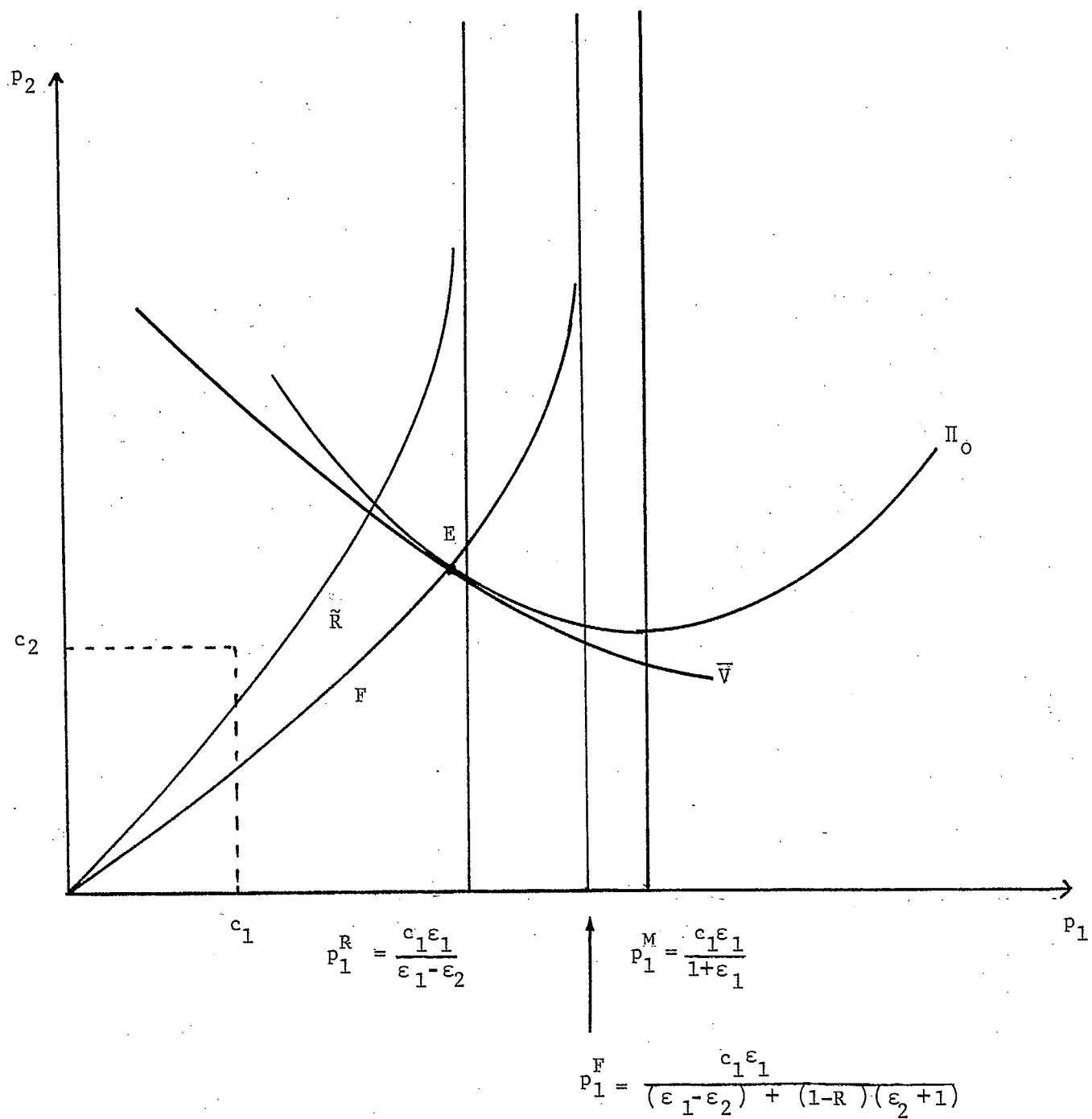
The locus F in Figure 2.1 is the 'efficiency-equity' price locus defined by equation (2.20). At this point, it is useful to re-write (2.20) as:

$$p_2 = \frac{R c_2 \epsilon_2}{(\epsilon_2 - \epsilon_1) + (R-1)(\epsilon_2 + 1) + \epsilon_1 c_1 / p_1} \quad (2.22)$$

where  $R = R_1/R_2 < 1$ . From (2.22) it is clear that the Feldstein locus will go through all equilibrium points such as E and will be asymptotic to the line  $p_1^F = c_1 \epsilon_1 / ((\epsilon_1 - \epsilon_2) + (1-R)(\epsilon_2 + 1))$ . As  $R \rightarrow 0$  or alternatively, as the equity importance of the relative necessity increases or the relative luxury decreases  $p_1^F \rightarrow p_1^M$ . However, as  $R \rightarrow 1$ , equity becomes less important and the Feldstein locus

FIGURE 2.1

FELDSTEIN (EFFICIENCY-EQUITY) AND RAMSEY LOCI



converges to the Ramsey locus given by  $\tilde{R}$ . It may appear counter-intuitive that strong equity weights push one towards the profit maximization point. However, this can be explained by the fact that good 2 is inelastic and the price of good 2 successively decreases as the profit maximization point is approached.

### 3- Cost Model

In this section we discuss the econometric cost model which will be used to estimate the characteristics of the Bell Canada production process. The estimates are later introduced into the pricing model. We begin with a discussion of the reasons behind the choice of a translog specification, and continue with an investigation of the properties of the model and the restrictions placed upon the model by economic theory. We then turn to a brief analysis of how the estimated model can be used to test various hypotheses concerning economies of scale and scope and other properties of the underlying production process. We conclude with a discussion of the estimated model and its properties.

#### Background

In this paper we have chosen to model the Bell Canada production process over the period 1956-1978 with a three-input three-output translog cost function. By selecting a cost function we have made the implicit assumption that Bell Canada will choose inputs of capital, labour and materials so as to minimize the cost of producing

any output vector. We further assume that of the three classes of service outputs of Bell Canada, message toll and toll private line services are supplied to firms and consumers at a rate which maximizes profits whereas local services are supplied to firms and consumers at a rate which just exhausts demand at the regulated price. The implication of our output assumptions is that regulation is effective only for local services and that even though message toll prices are in principle regulated, this regulation does not form a binding constraint.<sup>1</sup> No A-J type rate-of-return constraints are included in the model.<sup>2</sup> We do not model the accumulation of capital in Bell Canada. Rather, we assume that capital service flows are instantaneously optimal at each data point.<sup>3</sup> Finally, we assume that planning and forecasting within Bell Canada are accurate and therefore that all factors adjust to their optimal levels in the year between time series observations. Our estimated cost function is therefore long-run in form.

The translog cost function is sufficiently general to allow testing of restrictions on the functional form. For example, one can directly test whether the cost function is significantly different from more restrictive forms. As well, the translog cost function is linear in factor and output revenue cost shares. This feature is important when the equations of large models such as this are all estimated simultaneously.

#### The Model

The symmetric translog cost function is written:

$$\begin{aligned} \ln(\text{COST}) = & C_0 + \sum_i C_i \ln X_i \\ & + .5 \sum_{ij} C_{ij} \ln X_i \ln X_j \end{aligned} \quad (3.1)$$

where:  $i, j \in (w, r, v, QL, QM, QP, T)$

and  $C_{ij} = C_{ji}$  by assumption.

In the cost function,  $(X_w, X_r, X_v)$  are respectively the factor prices  $(w, r, v)$  for labour, capital services and material. Similarly,  $(X_{QL}, X_{QM}, X_{QP})$  are respectively the outputs  $(QL, QM, QP)$  of local, message toll and private line services. Cost is defined at each point in time by  $\text{COST} = wL + rK + vM$  where  $(L, K, M)$  are the inputs of labour, capital services and material. A complete description of the data can be found in Appendix 1.

$T$  represents a technical change variable and its specification requires some elaboration. Very little is known ex-ante about the way in which technological change has affected costs. Clearly some of the technological improvement is embodied in the capital stock and directly enters the cost function through the user cost of capital  $(r)$ . Unfortunately, there is not sufficient additional information to construct an exact hedonic constant-quality capital index which could be compared to the capital series supplied by Bell Canada. In addition, learning-by-doing type arguments can be introduced to support an argument that all service outputs and factor inputs have had certain amounts of cost savings associated with them over time. As well, it is not unreasonable to suppose that there may have been some Hicks neutral technical change. Finally, proxy

indicators of technical change for all inputs and outputs are not available.

We define our index of technological change as ACCESS - the percent of telephones with access to direct distance dialing. Ex-ante, this variable would appear to incorporate many of the important features discussed above. This variable has also been used by Fuss and Waverman (1980), and Christensen et al (1981).

#### Share Equation for Inputs and Outputs

Logarithmic differentiation of the cost function with respect to inputs and the application of Shephard's Lemma (1953) leads to the following factor share equations:

$$\frac{\partial \ln \text{COST}}{\partial \ln X_i} = S_i = C_i + \sum_j C_{ij} \ln X_j \quad (3.2)$$

$$i \in (w, r, v) ;$$

$$j \in (w, r, v, QL, QM, QP, T)$$

$$S_i = \text{cost share of factor } i$$

Similarly, assuming that the cross-elasticities between service outputs are zero and since profit maximization with respect to message toll and private line services implies  $MR_k = p_k(1+1/\epsilon_k) = MC_k$ , it is possible to write the marginal revenue 'share' equation as:

$$\frac{p_k(1+1/\epsilon_k)q_k}{\text{COST}} = C_k + \sum_j C_{kj} \ln X_j \quad (3.3)$$

$$k \in (QM, QP)$$

$$j \in (w, r, v, QL, QM, QP, T).$$



### Restrictions Arising from Economic Theory

The cost function is assumed to arise from the process of minimizing the cost of producing a given vector of outputs subject to a production function constraint. The minimization guarantees that the cost function will be a (non-strictly) concave function of factor prices. The non-strictness arises from the fact that the cost function must also be homogeneous of degree 1 in factor prices. In terms of the cost function introduced in equation (3.1), the following parameter restrictions are implied by homogeneity:

$$\begin{aligned}\sum_i C_i &= 1 & i \in (w, r, v) \\ \sum_j C_{ij} &= 0 & i, j \in (w, r, v) \\ \sum_i C_{ik} &= 0 & i \in (w, r, v) \\ & & k \in (QL, QM, QP, T)\end{aligned}$$

Since these restrictions are equivalent to having the factor cost shares add to unity, in order to estimate the model it is necessary to 'drop' one of the factor share equations during estimation. It is also customary to re-write the restrictions in terms of the coefficients associated with one of the factors - in this case, materials. The coefficients of the materials variables are later calculated along with their standard errors.

### Well-Behavedness Properties

It was noted above that the cost function must be (weakly)

concave in factor prices. This is not guaranteed by parameter restrictions and must be verified at each data point. Similarly the second-order necessary and sufficient conditions corresponding to the assumption of profit maximization with respect to message toll and private line services must be verified at each data point. These latter conditions require that the Hessian matrix of the profit function in QM and QP be negative definite or equivalently, that the profit function is concave in (QM,QP). It must be stressed that if a cost model violates either of the concavity conditions described above, it can serve no useful purpose for policy analysis.

#### Characteristics of the Technology

The cost model plays an important role in the optimal pricing model. As well, however, the model can be used to study characteristics of the Bell production process. In this section we define some of the important production characteristics in terms of the cost function. In a later section we will examine these characteristics for Bell using the estimated cost model.

The marginal cost for any service is determined by differentiating the cost function partially with respect to the output of that service. The marginal cost formula for service k from the translog cost function is given by

$$MC_k = \frac{COST}{X_k} (C_k + \sum_i C_{ik} \ln X_i) \quad \begin{matrix} k \in (QL, QM, QP) \\ i \in (QL, QM, QP, w, r, v, T) \end{matrix} \quad (3.4)$$

The fact that there are three outputs in the defined technology means that the notation of 'average cost' as present in one-output production models is no longer well defined. Similarly, the elasticity of cost with respect to 'output' is no longer unique. Finally, there can be no unique measures of scale economies in terms of the cost function. These problems have led to the development of an extensive body of Economics literature. Some of the major contributions have been provided by Baumol and Braunstein (1977), and Panzar and Willig (1977a, 1977b, and 1979). At the same time as they extended technical concepts of cost functions to cover multi-output production, they also extended the notions of natural monopoly and competitive industries to the multi-output case. A useful summary of these results can be found in Appendix B of LeBlanc (1979). For our purposes, it is not necessary to redevelop the major results. We simply state the following:

- (a) a monopoly supplying  $n$  services is a natural monopoly if the cost function describing the production process is strictly and globally subadditive. A function  $C(y) = C(y^1 + y^2 + \dots + y^n)$  is strictly and globally subadditive if:

$$C(y^1, y^2, \dots, y^m) < C(y^1) + C(y^2) + \dots + C(y^m), \quad y^j \in R_+^n, \quad j=1, \dots, m$$

- (b) the generally accepted multi-output counterpart of the single output measure of average cost is termed ray average cost and is defined by:

$$RAC(y) = \frac{C(ky)}{k}$$

The ray average cost measure for a given output vector thus involves increasing all outputs by a given factor  $k$ , and subsequently dividing the cost by  $k$ . In the one-output case, there are economies of scale when the long-run average cost function is a decreasing function of output. Geometrically, average costs are declining along a ray from the origin in the multi-output case whenever:

$$C(ky) < kC(y)$$

When  $C(y)$  is differentiable, the ray cost elasticity (RCE) is defined by:

$$RCE = \sum_{i=1}^n y_i (\partial C / \partial y_i) / C(y)$$

Thus,  $RCE < 1$  is equivalent to declining ray average costs for multi-output production. The production counterpart of RCE is the ray scale elasticity defined as:

$$RSCALE = 1/RCE.$$

As in the one-output case, increasing returns to scale are said to hold whenever the ray scale elasticity exceeds unity.

Finally, declining ray average costs are not implied by subadditivity of the cost function.

- (c) A production process is characterized by economies of scope if the cost function is transray convex. A cost function is transray convex if, for any two-output vectors  $y^1, y^2$  satisfying

$\sum_{j=1}^n \delta_j y_j^1 = \sum_{j=1}^n \delta_j y_j^2 = R$ , ( $\delta_j \geq 0$ ), the cost function satisfies:

$$C(ky^1 + (1-k)y^2) \leq kC(y^1) + (1-k)C(y^2), \quad k \in [0,1].$$

An equivalent restriction for twice differentiable cost functions is expressible in terms of cost complementary or cross-partial output derivative terms. In particular,  $C(y)$  is locally transray convex if:

$$\partial^2 C / \partial y_i \partial y_j \leq 0 \quad i, j = 1, \dots, n, \quad i \neq j$$

- (d) a sufficient condition for  $C(y)$  to be subadditive locally is that ray average costs are locally declining and  $C(y)$  is locally transray convex.

In this paper, the above described properties are examined locally for the multi-output cost function estimated for Bell Canada. It is not possible to examine properties such as subadditivity globally since the data set does not contain information on outputs close to or equal to zero. Using this data set, it is not unreasonable to specify the cost function as translog. However, extrapolation to the origin (where any logarithmic function becomes ill-defined) would be characterized by serious predictive errors.

In terms of the cost function defined by (3.1), the cost properties discussed above are given by:

$$RSCALE = 1 / \left( \sum_k MC_k X_k / COST \right), \quad k \in (QL, QM, QP) \quad (3.5)$$

and, for cost complementaries,

$$\frac{\partial^2 C}{\partial X_k \partial X_i} = \frac{MC_i MC_k}{COST} + \frac{C_{ik} COST}{X_i X_k}, \quad i \neq k, \quad i, k \in (QL, QM, QP) \quad (3.6)$$

### The Simultaneity Issue

Referring back for the moment to the marginal revenue share equation (3.3) it will be noted that the own-price elasticities of demand are necessary to estimate these equations. The problem is simplified somewhat if one assumes that the own-price elasticities of demand for message toll and private line services are constant - or that the demand curves are isoelastic. However, the fact remains that the equilibrium condition  $MC_k = MR_k$  has the same econometric implications for simultaneity bias as does any market model of supply and demand. It is therefore desirable in general to estimate the cost model equations with the demand equations to obtain a simultaneous estimate of the price elasticity.

Some preliminary estimates of the joint demand and cost model were produced, using FIML estimation. The results showed that even though the cost model was generally stable, the point estimates of the elasticity of demand for message toll and competitive services varied dependent upon the specification of the demand equations. We were never able to reject the hypothesis that the own-price elasticities were constant. We therefore decided to estimate the cost model alone using a grid of elasticity values for message toll (-1.2, -1.8) and competitive services (-1.25, -5.0) the range of which was determined by the preliminary estimations.<sup>4</sup> The adoption of this strategy significantly reduced the computational complexities and

hence the time necessary to estimate the model. This approach effectively builds a sensitivity analysis into the model.<sup>5</sup>

#### 4- Estimation of the Model and Properties of the Estimates

##### Estimation Results

In this section we report on the estimation of the cost model given by the cost function (3.1), the factor share equations (3.2) and the marginal revenue share equations for message toll and private line services (3.3). We present only the results when the price elasticity of message toll services ( $\epsilon_{QM}$ ) is -1.5 and the elasticity of private line services ( $\epsilon_{QP}$ ) is -2.0. This is referred to as the benchmark case.

The parameter estimates presented here reflect some nested testing which has taken place. Likelihood ratios were used to test the following hypotheses:

- (a) constancy of the cost share of materials
- (b) independence of factor shares from factor prices
- (c) independence of factor shares from outputs
- (d) homogeneity of the cost function in outputs.

The first two hypotheses could not be rejected at the standard 5% level of significance. The latter two hypotheses were rejected. As well, since the quadratic term on Hicks neutral technical change ( $C_{TT}$ ) was never significantly different from zero, it was excluded from the model. Finally, the cost shares of labour and capital were never found to be dependent upon the output of local services.

The parameter estimates of the base model are presented in Table 4.1. Equation by equation results can be found in Table 4.2. Overall, the majority of the parameters have narrow confidence bands. Those coefficients with relatively high asymptotic standard errors are 'small' independent of the scaling of the variables. The equation results suggest that the model explains a large percent of the variation in the data. As well, there is no evidence of serial correlation in the residuals and using a simple 'sign' test, it is not possible to reject the hypothesis that the residuals are indeed random with zero mean.

#### Properties of the Estimated Cost Model

In this section we discuss some important properties of the estimated cost model and the implications for the underlying multi-output production technology.

##### (a) Concavity and the Sufficiency Conditions for Profit Maximization

At each data point it was verified that the cost function was weakly concave in factor prices. As well, the implied profit function was found to be concave in message toll and competitive service outputs thereby guaranteeing satisfaction of the second-order necessary and sufficient conditions for profit maximization at each data point. These results were encouraging since they implied a strong economic foundation for the estimated model. As well, the results supplied indirect support for our arguments concerning the elasticity of demand for message toll and competitive service outputs.



TABLE 4.1

PAREMETER ESTIMATES FROM THE BASE COST MODEL

$$(\varepsilon_{QM} = -1.5, \varepsilon_{QP} = -2.0)$$

<u>Parameter</u>	<u>Estimate</u>	<u>Standard Error</u>
$C_O$	5.538	.506
$C_W$	.390	.013
$C_{WT}$	-.109	.009
$C_{WQM}$	.020	.005
$C_{WQP}$	-.028	.005
$C_r$	.414	.013
$C_{rT}$	.109	.009
$C_{rQM}$	-.020	.005
$C_{rQP}$	.028	.005
$C_v$	.196	.013
$C_{QL}$	-.780	.198
$C_{QLQL}$	.378	.050
$C_{QMQL}$	-.138	.025
$C_{QPQL}$	-.027	.006
$C_{QLT}$	-.277	.028
$C_{QP}$	.198	.019
$C_{QPQP}$	.034	.002
$C_{QMQP}$	-.006	.004
$C_{QPT}$	-.007	.003
$C_{QM}$	.391	.071
$C_{QMQM}$	.092	.014
$C_{QMT}$	.045	.009
$C_T$	1.659	.145

TABLE 4.2

EQUATION BY EQUATION SUMMARY

	<u>R<sup>2</sup></u>	<u>Durbin-Watson</u>	<u>Sum of Squared Residuals</u>
COST FUNCTION	.999	1.327	.0027
LABOUR SHARE	.981	1.484	.0006
CAPITAL SHARE	.979	1.421	.0007
MESSAGE TOLL SHARE	.927	1.192	.0002
PRIVATE LINE SHARE	.987	1.207	.00002

(b) Marginal Costs of the Services

The series of marginal costs of the services are presented in Tables 4.3, 4.4 and 4.5 for selected years. It is noted that the marginal costs for message toll and competitive services track the respectively marginal revenues quite closely. For local services, the marginal costs were close to constant until the early 1970's. Thereafter, they rose quite rapidly reaching a level of \$1.46 in 1978. The marginal cost functions were found to be overall quite inelastic. The Lerner Index suggests that Bell Canada was able to exercise greater monopoly power in message toll as opposed to competitive services.

At this point it is important to note the possibility of making partial comparisons of the marginal costs derived from this study with estimates derived by Bell Canada in an internal (financial/engineering) study and with estimates derived by Rohlfs in an engineering study with AT&T data. Table 4.6 has been prepared to facilitate comparison of these results. The marginal cost values presented by Bell Canada and Rohlfs represent the cost of producing one dollar of revenue in 1976. The corresponding values from our study were constructed by dividing the estimated marginal cost of a service by the corresponding price of the service. Problems associated with comparing the approaches are discussed below. At this juncture it suffices to note that there is strong agreement over the estimates of marginal cost for message toll services. There is less agreement over the cost of a dollar of local revenue. Our results suggest that, at the margin, Bell breaks even. The Bell Canada study

TABLE 4.3

LOCAL SERVICES SUMMARY

<u>Year</u>	<u>Price</u>	<u>Output</u>	<u>Marginal Cost</u>	<u>Elasticity of Marginal Cost</u>
1956	.933	200.500	.747	.235
1962	1.000	321.300	.658	.261
1967	1.000	446.600	.672	.266
1972	1.086	611.700	.828	.264
1978	1.476	921.800	1.460	.244

TABLE 4.4

MESSAGE TOLL SERVICE SUMMARY

<u>Year</u>	<u>Price</u>	<u>Output</u>	<u>Marginal Revenue</u>	<u>Marginal Cost</u>	<u>Elasticity of Marginal Cost</u>	<u>Lerner Index</u> *
1956	1.065	79.010	.355	.361	-.031	.661
1962	1.041	130.505	.347	.361	-.051	.653
1967	1.000	223.800	.333	.329	-.105	.671
1972	1.102	360.785	.367	.356	-.132	.677
1978	1.344	728.986	.448	.450	-.162	.665

\*Lerner Index of Monopoly Power =  $(P-MC)/P$

TABLE 4.5

COMPETITIVE SERVICES SUMMARY

<u>Year</u>	<u>Price</u>	<u>Output</u>	<u>Marginal Revenue</u>	<u>Marginal Cost</u>	<u>Elasticity of Marginal Cost</u>	<u>Lerner Index</u> *
1956	1.017	6.300	.508	.457	2.206	.551
1962	1.017	17.666	.509	.504	.684	.504
1967	1.000	35.220	.500	.485	.280	.515
1972	1.076	62.719	.538	.511	.147	.525
1978	1.636	105.746	.818	.780	.094	.523

\* Lerner Index of Monopoly Power =  $(P - MC) / P$

TABLE 4.6

COMPARISON OF COSTS PER DOLLAR OF REVENUE IN 1976

	<u>Local</u>	<u>Message Toll</u>
THIS STUDY	.974 (.020)	.338 (.003)
BELL CANADA STUDY <sup>+</sup>	1.320	.310
ROHLFS* AT&T ENGINEERING	N/A	.30

Standard errors in parenthesis.

<sup>+</sup>P(CRTC) 26 Jan 78 - 403; appendix II.

\*see Rohlfs (1979).

suggests a \$.32 loss per dollar of revenue in 1976.

We attempted to reconcile the differences between our local marginal cost estimate per dollar of 1976 revenue and that of Bell Canada. The following differences in the studies are likely be responsible for the \$.32 discrepancy:

- 1- This study differs from the Bell Canada study in the way in which estimates of the cost of a dollar of 1976 revenue were derived. In this study, the marginal cost of local services for 1976 was divided by the 1976 price of local services to obtain the required estimate. The marginal cost of local services is determined by evaluating the derivative of the estimated long run total cost function. The Bell estimate is obtained as the ratio of casually related costs of local services to revenue of local services for 1976. The Bell estimate presupposes the legitimacy and accuracy of an allocation of common costs over services. In addition, the Bell estimate is more like an average cost per dollar of 1976 revenue than a marginal cost of a dollar of 1976 revenue. Since any common costs are not 'tied' to given services in the econometric approach, it is not possible to use the cost function to construct a methodologically similar number for comparison purposes. As well, it should be noted that Bell has traditionally allocated costs to services on an incremental basis. In this way almost all common costs are allocated to basic services, which are primary local and message toll services. Non-basic services, including vertical and other competitive services are thus allocated almost no common costs.



- 2- The Bell Canada study excludes vertical services from the measured local service aggregate. Vertical services include options such as coloured sets, special styling and touch tones. The revenue from vertical services accounts for approximately 50% of constant dollar local service output in 1976. Further, since some vertical service options yield a larger revenue stream than black set services with almost no change in the cost of connecting to and remaining connected with the network, it seems reasonable to suppose that the exclusion of vertical services from the measure of output would bias the marginal cost measure upward. This is especially true given the discussion in 1 where it was noted that the Bell measure of marginal cost is more akin to an average cost.
- 3- The assumption relating to the price of capital and the treatment of tax are different in the two studies. Unfortunately, the details necessary to make a useful comparison are not available.

(c) Scale, Scope and Subadditivity

Some of the results regarding scale, scope (cost complementarity) and subadditivity are presented in Table 4.7. The cost complementarity terms (cross-partial derivatives of the cost function with respect to outputs) have negative point estimates at each data point. At the same time, the RSCALE measure indicates that ray average costs are declining at each data point and that the degree of such economies in the Bell Canada production process has been relatively constant. The

TABLE 4.7

SCALE, SCOPE AND SUBADDITIVITY RESULTS

<u>Year</u>	$\frac{\partial^2 C}{\partial Q_L \partial Q_M}$	$\frac{\partial^2 C}{\partial Q_L \partial Q_P}$	$\frac{\partial^2 C}{\partial Q_M \partial Q_P}$	RSCALE
1956	-.0013	-.0044	-.0026	1.438
1962	-.0009	-.0013	-.0007	1.616
1967	-.0005	-.0005	-.0002	1.597
1972	-.0004	-.0003	-.0001	1.565
1978	-.0003	-.0003	-.0001	1.453
1976	-.0003	-.0003	-.00007	1.484
	(.0001)	(.0002)	(.00014)	(.023)

Standard errors in parenthesis

conditions for subadditivity appear to be locally satisfied. However, the results for 1976 suggest that these results must be interpreted with care. The scale measure is significantly greater than unity in 1976. As well, the cost complementarities between local and message toll services, although small, are significantly negative. However, the cost complementarities (at standard significance levels) between competitive services and either of local and message toll services are not significantly different from zero. These results are interesting in that they appear to support the conclusions reached in the recent CRTC regulatory hearing on interconnection between CNCP Telecommunications and Bell Canada. It was argued in these hearings that the Bell Canada natural monopoly did not extend to competitive (private line) services and that another Canadian company, CNCP Telecommunications, should be allowed the right to interconnect with the local switched network of Bell Canada in the competitive provision of these services. [CRTC (1979)].

(d) Technical Changes and Productivity

Given that an index of technical change was used in the estimation of the cost function, it is possible to calculate the responsiveness of cost to technical innovation as measured by the technical change index. The results of this exercise are presented in Table 4.8 for the partial derivative of the cost function with respect to the technology index and for the elasticity of cost with respect to the technology index. One would normally expect a negative relationship between technology and cost. Indeed, this is

TABLE 4.8

RESPONSIVENESS OF COST TO TECHNICAL INNOVATION

<u>Year</u>	$\frac{\partial C}{\partial T}^*$	$\frac{\partial \ln C}{\partial \ln T}$
1956	11.829	.046
1962	-22.608	-.093
1967	-54.819	-.181
1972	-120.599	-.267
1978	-392.343	-.358
1976	-284.873	-.334
	(39.186)	

\*For these calculations  $T = \exp[\text{ACCESS}]$

Standard errors in parenthesis

the case for every year except 1956. The significance testing for 1976 suggests the technology partial derivative (and hence elasticity) is significantly less than zero. The relative constancy and significance of the measure of ray scale economies combined with the significance of the technical change effects suggests that we may have been able to partially disentangle the effects of scale and technology in the Bell Canada production process.

(e) Other Results

We note in passing since the factor cost shares were found to be independent of factor prices, the elasticities of factor substitution will all be unity. Further, it is noted that the isocost output surfaces for all parts of outputs were found to be concave to the origin for each data point.

5- Demand Specification

In this section we briefly review the implications of sections 2 and 3 for the demand equations used within the simulation model. We then derive the final forms of the demand equation for simulation purposes.

Given that the utility function is additively separable, it was shown in section 2 that the demands for services would be almost double-log (with constant income and own-price elasticities) if the expenditure share of the service was small. For Quebec and Ontario (the centre of almost all Bell Canada activities) the share of residential local services in total consumer expenditure is approximately

.005. Residential toll services yield an even smaller share. Thus, one can feel reasonably confident about the double-log specification for residential local and message toll services.

#### Local Residential Demand

Guided by equation (2.16), we write the demand for local residential services as:

$$\begin{aligned} \ln(Q_L^R/POP) = & \alpha_0 + \epsilon_{QL} \ln(P_L^R/P) + \eta_{QL} \ln(y/P \cdot POP) \\ & + \alpha_1 D_1 + \alpha_2 D_2 + \alpha_3 D_3 \end{aligned} \quad (5.1)$$

The variables have the following definitions:

$Q_L^R$  = local residential service output

$P_L^R$  = price of local residential services

$P$  = consumer price index

$y$  = sum of gross provincial outputs for Quebec and Ontario

$POP$  = population in the Bell Canada territory

$D_1$  = step variable for introduction of direct distance dialing in 1959

$D_2$  = step variable for the introduction of the one minute minimum toll call in 1971

$D_3$  = step variable for rate centre shifting in Toronto, 1976.

It will be noted that, consistent with equation (2.16), outputs and income are expressed on a per capita basis. Gross provincial products are used as a proxy for personal disposable income in each province. Finally, it will be noted that  $\epsilon_{QL}$  (the own-price

elasticity) and  $\eta_{QL}$  (the income elasticity) are not free. They are related by the identity:

$$\epsilon_{QL} = -\sigma\eta_{QL} \quad (5.2)$$

where  $(-1/\sigma)$  is the elasticity of marginal utility of money.

In Section 6, the choice of  $\sigma$  will be discussed in greater detail. It is sufficient to note here that the efficiency-equity price simulations are conditional upon a choice from a range of possible values of  $\sigma$ . Given any  $\sigma$ , the simulation begins with the estimation of equation (5.1) subject to the constraint (5.2). This estimation yields point estimates for  $\epsilon_{QL}$  and  $\epsilon_{QL}$  (conditional upon the choice value of  $\sigma$ ) which are subsequently used to determine the optimal efficiency-equity prices.

#### Toll Residential Demand

A time series breakdown of residential message toll prices and quantities is not available in the public domain.<sup>6</sup> In order to derive a message toll residential quantity series from the available message toll residential revenue series, it was assumed that residential message toll prices were equal to the aggregate message toll price at each data point.

It will be recalled that the cost model was to be estimated over a range of aggregate demand elasticities for message toll and private line services. Since we created the residential toll quantity series by assuming that the residential price series was equivalent to the aggregate price series, it did not seem useful to assume that the residential own-price demand elasticity differed

from the price elasticity of demand for the aggregate of message toll services. In addition, since the residential message toll income and own-price demand elasticities are related by:

$$\epsilon_{QM} = -\sigma\eta_{QM} \quad (5.3)$$

it was therefore not necessary to estimate any parameters of toll residential demand. Once the aggregate price elasticity is specified for a given simulation,  $\epsilon_{QM}$  is known and, conditional upon  $\sigma$ ,  $\eta_{QM}$  is known as well. Formally, the residential toll demand can be written:

$$\ln(QM^R/POP) = \alpha(t) + \epsilon_{QM} \ln(P_{QM}^R/P) + \eta_{QM} \ln(y/P \cdot POP) \quad (5.4)$$

where  $Q_M^R$  the quantity of residential message toll services

$P_{QM}^R$  the price of residential message toll services (equal to the price of aggregate message toll services)

$p, y$  and  $POP$  are as defined above.

$\alpha(t)$  is a forcing function which guarantees that at each point in time the right-hand side of (5.4) is equal to the logarithm of historic residential per capita demand.

## 6- Simulation

In this section we begin by noting some additional assumptions which are necessary before simulations can be carried out. We then work through a conceptual simulation experiment and point out where sensitivity tests are conducted. We conclude the section with a set of simulation results which are presented and analysed.



### Additional Assumptions

The following are additional assumptions which have been made.

- 1- It is necessary to choose a range of values for the elasticity of marginal utility of income  $(-1/\sigma)$ . In this paper we have chosen the range  $(0, -2.0)$ . This range seems to be consistent with almost all applied studies with which we are familiar.<sup>7</sup>
- 2- It is assumed that the logarithm of income is distributed as normal with mean  $\overline{\ln y}$  and variance  $\sigma_{\ln y}^2$ . The log normal distribution provides a reasonable description of the distribution of income in Ontario and Quebec. As well, the distributional coefficients  $R_{QL}$  and  $R_{QM}$  can be evaluated without numerical integration. In particular, if  $y$  is lognormally distributed then, for any  $\theta$ ,

$$\begin{aligned} \int_0^{\infty} y^{\theta} f(y) dy &= \int_0^{\infty} \exp[\theta \ln y] f(y) dy \\ &= \exp[\theta \overline{\ln y} + .5\theta^2 \sigma_{\ln y}^2] \end{aligned} \quad (6.1)$$

The mean and variance of the logarithm of household income were calculated for 1961 and 1971 using Statistics Canada data (cat. 98-505, 93-749). The variances of the two years were almost the same and we therefore assumed that variance constant at the level .72. However, there was some increase in the mean from 8.1415 in 1961 to 8.6139 in 1971. A complete series of means for 1956-1978 was created by interpolation and extrapolation using the growth rates of gross provincial product per household to approximate changes in the mean over time.

- 3- The last assumption relates to the way in which the cost model information was introduced into the simulation experiments. A time

series of marginal costs was obtained for the aggregate series of local and message toll services from the cost model. It was assumed that these point estimates of marginal cost were accurate in the neighbourhood of the quantities at which they were estimated. Thus, in the simulation, the point estimates of historic marginal cost were taken as the levels of constant marginal cost. There are two reasons why this assumption is reasonable. In the first place, the estimated cost function can be used to show that, for any given year, marginal costs of services are quite inelastic with respect to both own-service quantities and other service quantities (cost complementarity). Secondly, if the marginal costs are constant at the historic levels then the historic levels of non-residential services remain optimal even if the residential component of local and message toll services changes. Thus, using the constant marginal cost assumption in a neighbourhood of the data points means that it is not necessary to include a reoptimization of non-residential services in the simulations.<sup>8</sup>

#### The Simulation Process - A Conceptual Exercise

In this section we present a brief discussion of the flow of the simulation process for any given simulation experiment in order to facilitate interpretation of the simulation results.

We begin by noting that there is a three dimensional array of initial conditions upon which all simulations are conditional. These conditions arise from the assumptions regarding the ranges of the own-price elasticities of message toll and toll private line services and the elasticity of marginal utility of income. In

particular, it will be recalled that  $\epsilon_{QM}$  was given the range (-1.2, -1.8),  $\epsilon_{QP}$  was given the range (-1.25, -5.0) and  $(-1/\sigma)$  was given the range (0, -2.0). Our benchmark vector  $(\epsilon_{QM}, \epsilon_{QP}, (-1/\sigma))$  was taken as (-1.5, -2.0, -1.5). In this section, we work through the simulations of the benchmark case.

#### Step 1 - Demand Parameter

Given  $(\epsilon_{QM}, \epsilon_{QP}, (-1/\sigma)) = (-1.5, -2.0, -1.5)$ , we begin by using equation (5.3) to calculate the income elasticity of demand for residential message toll services. In this case we arrive at the value  $\eta_{QM} = 2.25$ . Taking  $\epsilon_{QM}$  and  $\eta_{QM}$  we use (5.4) to solve for  $\alpha(t)$  for each series year (1956-1978).

We next use the fact that  $(-1/\sigma) = -1.5$  in (5.2) and estimate the local residential demand equation (5.1) subject to (5.2) to obtain estimates of the price and income elasticities of local residential demand. In this case we obtain  $\epsilon_{QL} = -.445$  and  $\eta_{QL} = .668$ .

#### Step 2 - Distributional Coefficients

Given that the set  $(\epsilon_{QM}, \eta_{QM}, \epsilon_{QP}, \epsilon_{QL}, \eta_{QL}, (-1/\sigma))$  is known, it is possible to use the information on the means and variances of the lognormal distribution of income along with equations (6.1) and (2.19) to compute the distributional coefficients for 1956 to 1978. In Table 6.1 the distributional coefficients corresponding to the benchmark parameter set are displayed.

#### Step 3 - Optimal Price Determination

In this final stage of the simulation process, we compute the

TABLE 6.1

VALUES OF THE DISTRIBUTIONAL COEFFICIENT\* ( $\times 10^{-5}$ )

BENCHMARK CASE

$(\epsilon_{QM}, \eta_{QM}, \epsilon_{QP}, \epsilon_{QL}, \eta_{QL}, (-1/\sigma)) \equiv (-1.5, 2.25, -2.0, -.445, .668, -1.5)$

	<u>R<sub>QL</sub></u>	<u>R<sub>QM</sub></u>
1956	.5564	.1008
1962	.5195	.10941
1967	.3546	.0642
1972	.2383	.0431
1978	.1089	.0197

\*Calculated using equations (6.1) and (2.19)

optimal efficiency-equity prices for residential local and message toll services. The parameter set of Step 1 is augmented by the distributional coefficients (for each year) computed in Step 2. The cost model of section 3 is then estimated conditional upon the parameter set values. The estimated cost function and the demand equations of section 5 are then combined (subject to the constant marginal cost restriction) to define the profit function of equation (2.20). The yearly efficiency-equity prices result as a simultaneous solution to equations (2.20) and (2.6) where the constraining profit level ( $\Pi_0$ ) for any year is the historic level of profits. The results are presented with the optimal efficiency-equity prices and quantities expressed as a ratio of their historic counterparts.

For the Ramsey case the same procedure is followed except that the influence of distributional considerations is removed by the preassignment  $R_{QL} \equiv 1 \equiv R_{QM}$ . It will be noted, however, that because of the demand interaction, the Ramsey case is still conditional upon the choice of  $(-1/\sigma)$ .

#### The Simulation Results

In this section we present the simulation results for the benchmark case. The effects of changes in the benchmark parameterization are then discussed. It might be helpful for the reader to refer back to Figure 2.1 and the related discussion.

The benchmark results are presented in Table 6.2 for the Ramsey case, and for a range of assumptions about the elasticity of marginal utility of income.

One very noteworthy result is that large movements away from

TABLE 6.2

BENCHMARK EQUITY - EFFICIENCY SIMULATION RESULTS

$$\epsilon_{QP} = -2.0, \quad \epsilon_{QM} = -1.5$$

Historic\* Prices and  
Quantities and Mar-  
ginal Costs

Ratio of Optimal Equity-Efficiency Prices and Corres-  
ponding Quantities to Historic\* Prices and Quantities.

	$P_{QL}^R$	$MC_{QL}$	RAMSEY**	$-\frac{1}{\sigma} = -1.0$	$-\frac{1}{\sigma} = -1.5$	$-\frac{1}{\sigma} = -2.0$
1956	.9032	.7404	1.2216	1.0672	1.0100	1.0004
1962	.9872	.6624	1.2348	1.0684	1.0103	1.0004
1967	1.0000	.6698	1.2834	1.0833	1.0126	1.0006
1972	1.0529	.8288	1.3954	1.1198	1.0180	1.0007
1978	1.3399	1.4671	1.5962	1.1907	1.0296	1.0014
	$Q_L^R$					
1956	10.359		.9147	.9639	.9956	.9999
1962	142.704		.9104	.9633	.9955	.9999
1967	195.921		.8949	.9559	.9944	.9998
1972	267.854		.8621	.9382	.9921	.9998
1978	373.393		.8120	.9089	.9871	.9995

TABLE 6.2 (continued)

BENCHMARK EQUITY - EFFICIENCY SIMULATION RESULTS

$$\epsilon_{QP} = -2.0, \quad \epsilon_{QM} = -1.5$$

Historic\* Prices and  
Quantities and Mar-  
ginal Costs

Ratio of Optimal Equity-Efficiency Prices and Corres-  
ponding Quantities to Historic\* Prices and Quantities.

	$P_{QM}^R$	$MC_{QM}$	RAMSEY**	$-\frac{1}{\sigma} = -1.0$	$-\frac{1}{\sigma} = -1.5$	$-\frac{1}{\sigma} = -2.0$
1956	1.0650	.3645	.3793	.5436	.8650	.9593
1962	1.0414	.3588	.3985	.5727	.7901	.9681
1967	1.0000	.3299	.3845	.5495	.7569	.9277
1972	1.1019	.3558	.3708	.5231	.7306	.9055
1978	1.3437	.4481	.3678	.5071	.7290	.9269
	$Q_M^R$					
1956	32.9018		4.2804	2.4953	1.4771	1.0645
1962	56.5391		3.9755	2.3077	1.4240	1.0499
1967	97.2000		4.1947	2.4554	1.5186	1.1294
1972	171.629		4.4286	2.6433	1.6014	1.1606
1978	364.492		4.4837	2.7688	1.6067	1.1206

\* In all cases historic prices were used for comparison purposes. As well, historic quantities of residential message toll services were used. Because the demand equation for residential local services was estimated, the fitted values of this equation were used as the historic values for comparison purposes.

\*\* As mentioned in the text, the Ramsey prices are determined when the distributional coefficients are constrained to equal 1. The demand parameters for the Ramsey case are conditional upon  $-\frac{1}{\sigma} = -1.5$ .

$P_{QL}^R, P_{QM}^R$  refer to the prices of residential local and message toll services respectively.

$Q_{QL}^R, Q_{QM}^R$  refer respectively to the residential quantities of local and message toll services.

$MC_{QL}, MC_{QM}$  are the marginal costs.

away from the historic levels for the price and quantity of local residential services are not optimal when equity considerations are introduced. Not surprisingly, the Ramsey (regressive) case calls for the largest movements. However, maximum movements in price and quantity of residential local services of about 2% are optimal when the elasticity of marginal utility of income is less than or equal to -1.5. As discussed earlier, this range and hence these percentage changes seem to be consistent with most of the equity-weighting Economics literature.

Rather larger movements away from the historic quantities and prices for residential message toll services arise. As in the local case, the greater the equity-weighting, the closer are the optimal prices and quantities to historic levels.

The results for 1978 are studied as a relevant example. Depending upon equity considerations ( $(-1/\sigma) = -1.5$  or  $(-1/\sigma) = -2.0$ ) it is optimal to lower residential toll prices between 27 and 7 percent. It should be noted as well that since residential and business message toll quantities were about equal in 1978, the percentage change in total message toll quantities would be equal to approximately one-half of the percentage change in residential message toll services.

In terms of the graph presented in Figure 2.1, the simulation results suggest that the isoprofit contours (such as  $\Pi_0$ ) are quite flat in a rather large neighbourhood of the historic prices. Thus, it is possible to have large movements in the price of residential message toll services without significant movements in the price of



residential local services.

It is important to investigate the sensitivity of the results to changes in the benchmark assumptions concerning  $\epsilon_{QP}$  ( $=-2.0$ ) and  $\epsilon_{QM}$  ( $=-1.5$ ).

When the elasticity of toll private line services was allowed to vary, there was almost no change in the simulated optimal efficiency-equity prices for residential local and message toll services. The explanation of this result lies in the fact that cost complementarities between toll private line services and local message toll services were negligible.

The sensitivity of the results to changes in the elasticity of message toll services is presented in Table 6.3 for the benchmark case. It will be noted that the less elastic is the demand for message toll, the greater are the price and quantity movements. The explanation of this result is straightforward. Examining equation (5.3) we note that, ceteris paribus, the smaller the price elasticity of demand for residential message toll services, the smaller is the income elasticity for residential message toll services. However, as the message toll income elasticity decreases, so does the spread between the distributional coefficients for local and message toll services. Thus, as the absolute size of  $\epsilon_{QM}$  decreases the induced movement in optimal prices is away from historic levels and towards the Ramsey prices.

## 7- Conclusions

This paper has considered some of the issues which must be

TABLE 6.3

EFFICIENCY-EQUITY PRICES

SENSITIVITY ANALYSIS FOR  $\epsilon_{QM}$

1967

$$\epsilon_{QP} = -2.0 \quad (-1/\sigma) = -1.5$$

<u><math>\epsilon_{QM}</math></u>	<u><math>P_{QL}^R</math></u>	<u><math>QL^R</math></u>	<u><math>P_{QM}^R</math></u>	<u><math>QM^R</math></u>
-1.2	1.0596	190.934	.4286	268.706
-1.5	1.0126	194.830	.7569	147.607
-1.8	1.0030	195.657	.8884	120.295
Historic Value	1.0000	195.921	1.0000	97.200

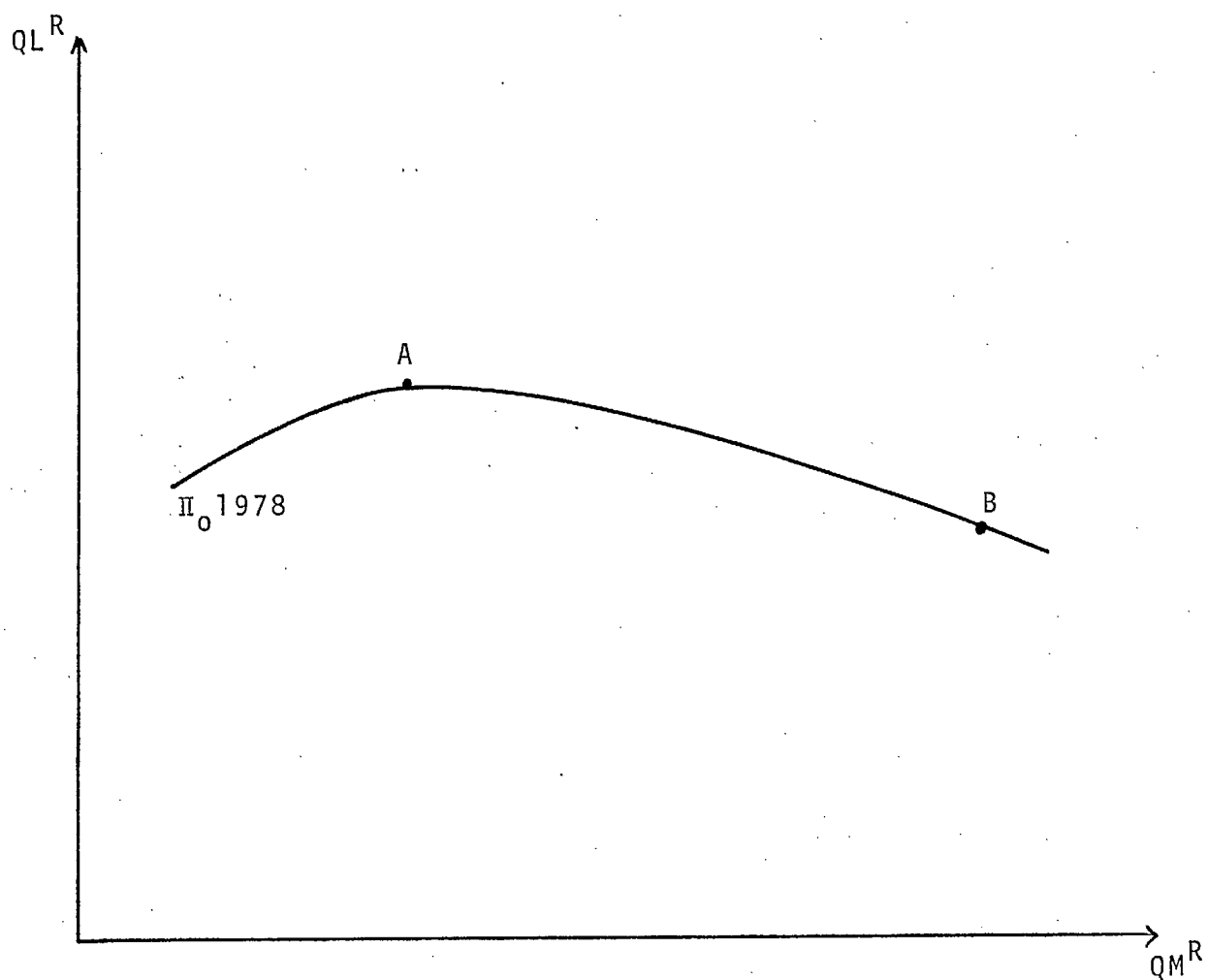
faced in practice if one is interested in introducing both efficiency and equity criteria into the pricing decision of a regulated communications carrier. The results of applying the methodology to Bell Canada suggest that some adjustment in the prices (and hence quantities) of residential local and message toll services should be made. However, the results do not say how these optimal prices should be introduced. It is only reasonable to expect that there would have to be some gradual transition towards any new optimum. It is interesting to note that the model studied in this paper suggests that along some transition paths, the first steps are the most important. Alternatively stated, along some paths to the new equilibrium the welfare of residential consumers of the services will be increasing and the greatest increase in welfare will correspond to the first movements along the transition path.

This result is demonstrated with reference to Figure 7.1. The locus drawn in Figure 7.1 is the isoprofit locus in residential output space for the year 1978. The curve drawn here is the dual to the isoprofit locus in price space ( $\Pi_0$ ) drawn in Figure 2.1. Duality ensures that the historic output vector (A) occurs at a maximum of the isoprofit locus in output space and at a minimum of the isoprofit locus in price space. The Feldstein optimal point in price space is given by E whereas in output space it is given by B. Figure 7.1 is drawn under the benchmark assumption that  $(\epsilon_{QM}, \epsilon_{QP}, (-1/\sigma)) = (-1.5, -2.0, -1.5)$ .

Expressed in output space, the adoption of optimal efficiency-equity prices would involve the transition from point A to point B.

FIGURE 7.1

RESIDENTIAL OUTPUT ISO-PROFIT LOCUS FOR 1978



A = Historic quantities (364.492, 373.393)

B = Feldstein optimal quantities (586.523, 368.524)

We will assume that the transition follows the isoprofit locus from A to B. We will develop the conditions under which consumer welfare is a concave function along the isoprofit locus and therefore the conditions under which the welfare increments associated with successive equal movements along the transition path are decreasing.

To begin, we note that the isoprofit can be written in the general form:

$$Q_L^R = \tilde{\Pi}(Q_M^R) \quad (7.1)$$

Using a prime (') to denote differentiation, we have

$$\tilde{\Pi}'(Q_M^R) < 0 \text{ and } \tilde{\Pi}''(Q_M^R) \leq 0 \text{ in the range AB.}$$

Welfare is in general given by:

$$W = W(Q_L^R, Q_M^R) \quad (7.2)$$

Along the isoprofit constraint, welfare can be written:

$$W = W(\tilde{\Pi}(Q_M^R), Q_M^R) \quad (7.3)$$

We therefore wish to develop the conditions under which welfare given by (7.3) is concave in  $Q_M^R$ . Differentiating the welfare function twice we obtain:

$$W''(Q_M^R) = W_{11}(\tilde{\Pi}')^2 + 2W_{12}\tilde{\Pi}' + W_{22} + W_1\tilde{\Pi}'' \quad (7.4)$$

where, in a standard way, superscript primes refer to total derivatives and subscript numbers refer to partial derivatives with respect to the arguments of the welfare function.

It will be recalled that our simulation results are developed in terms of a welfare function which, in terms of quantities, is additive with the properties  $W_1, W_2 > 0$ ,  $W_{11}, W_{22} < 0$  and  $W_{12} = 0$ . Thus, in terms of our model, the right-hand side of (7.4) is negative and the welfare function is concave over the transition path AB. The interpretation of this result is that, in terms of our model, even though one may be reluctant (for whatever reason) to enforce a movement all the way from A to B, one can remain confident that the first of M equal sized movements along the adjustment path will supply the greatest welfare improvement to residential users of local and message toll services.

FOOTNOTES

- 1) This assumption is consistent with any cross-subsidization goals of regulation whereby profits from message toll services can be used to defray losses incurred in the provision of local services. Given the jointness of production it is extremely difficult (if not impossible) to disentangle the extent to which message toll services subsidize local or any other service. In both 1978 and 1980, Bell was awarded the requested increase in intra-Bell long-distance rates.
- 2) The fact that we do not include a rate-of-return constraint can be justified using both general and Bell - specific arguments. Considering the general arguments first, we can find no empirical A-J study which provides any strong support for the A-J hypothesis. Modelling and measurement errors are simply too large. With regard to Bell, we have demonstrated elsewhere that rate-of-return constraints have never bound Bell Canada (see Breslaw and Smith, 1981). Fuss and Waverman (1980) have examined the same question using a second-order translog cost function. Not surprisingly, the Fuss-Waverman results suggest that regulatory constraints are not consistent with the data. This result of Fuss and Waverman must be tempered with the realization that their use of a second-order translog cost specification is also inconsistent with the existence of a binding regulatory constraint. Breslaw and Smith (1981) have shown that a third-order cost function is necessary to incorporate the constraints imposed by economic theory.

- 3) Although capacity utilization questions may be important, no data were available to adjust the flow of services from measured capital stock. In this paper, it is assumed that in each year the flow of capital services is proportional to the capital stock with a factor of proportionality equal to 1.
- 4) The TSP version of the non-linear SURE estimator was used to estimate the cost model. Given that some data on the measures of technology were not available prior to 1956, the model was estimated for the period 1956-1978.
- 5) In comments upon an earlier draft of this paper as well as other work which we have prepared for the Canadian Department of Communications, Bell Canada has argued that the demand for message toll services is price inelastic. If message toll services are indeed inelastically demanded, unconstrained profit maximization is not a reasonable description of the behaviour of Bell Canada in supplying these services. Thus, equation (3.3) should not be estimated.

The position adopted by Bell is consistent with the service curtailment analyses which Bell has prepared in response to interrogatories before the CRTC. All of these analyses assume that message toll services are inelastically demanded. A critique of the methodology used by Bell in developing price elasticities for message toll services can be found in Breslaw (1980). The results of this critique as well as the work of Breslaw et al (1979a), Fuss and Waverman (1980) and Taylor (1980) all suggest that message toll services are price elastic.



Bell has also argued that a recent experiment in which weekend prices for message toll services were lowered by  $2/3$  has resulted in reduced revenue from these services. These results, they argue, are inconsistent with an elastic demand for message toll services. However, this need not be true. In the first place, demand may be locally elastic but a  $2/3$  price reduction removes price from the elastic neighbourhood. Alternatively, demand may be adapting over time to the new lower prices. Finally, it should be noted that the weekend  $2/3$  price reduction does not cause much of a change to the aggregate message toll price series.

- 6) The Canadian Department of Communications is currently engaged in deriving these price series.
- 7) Many authors have suggested ranges for the elasticity of marginal utility parameters. See, for example, Baumol (1979), Baumol and Bradford (1970), Fellner (1967), Mera (1969), Powell et al (1968), Sato (1972), and Maital (1973). There appeared to be some early agreement on a value of -1.5. More recently, however, Davies (1980) has studied evidence which suggests much higher values of the elasticity of marginal utility of income.
- 8) For comparison purposes, we computed the optimal efficiency-equity prices in the case where marginal cost was variable and no reoptimization was undertaken for business message toll and toll private line. Since the elasticity of the marginal cost functions were never identically zero, there were some movements away from the constant marginal cost solution prices. However,

the movements were not large and they tended to re-enforce the movements away from the historic prices resulting from the constant marginal cost case. As well, there was no important change in the pattern of results when the isoprofit constraint was replaced by a iso-rate-of-return constraint.

## APPENDIX 1

### DATA

The data used in this paper were obtained from the following sources:

Bell Canada, Annual Charts 1935-1978

Statistics Canada CAT 13-213, CAT 93-749, CAT 98-505

Ontario Ministry of the Treasury, Quarterly Time Series 1947-75

Quebec Ministry of Industry and Commerce, Revenues et Depenses 1946-70

Interrogatories in CRTC (Canada) hearings;

BELL (NAPO) 1 FEB 80 - 727

BELL (CAC) 3 APR 80 - 511

P (CRTC) 26 JAN 78 - 403

Department of Communications (personal communication)

Denny et al (1979).

### FACTORS

L Labour, adjusted for quality, excluding construction

K Capital, total average net stock, constant \$1967

M Materials, divisia index of materials, sales taxes and uncollectables, constant \$1967

w wages, (employee expenses and labour tax) ÷ L

r cost of capital, Hall-Jorgenson derivation, real rate-of-return 3.5%

v price, raw materials, Divisia index (see M)

### TECHNOLOGY

ACCESS % phones with access to DDD (see Denny et al (1979) for initial derivation of this series)

SERVICES

- QL Local service + miscellaneous + directory assistance
- QM Message toll output, including WATS (Divisia)
- QP Other toll, excluding WATS
- QL<sup>R</sup> Local residential services
- QL<sup>B</sup> Local business services
- QM<sup>R</sup> Message toll services - residential
- QM<sup>B</sup> Message toll services - business

All quantities are measured in constant \$1967.

Corresponding price series were derived from the quantity and current revenue series.

OTHER

- GPP Real gross provincial product, Ontario and Quebec
- POP Population, Bell Territory

BIBLIOGRAPHY

- Atkinson, A.B., and J.E. Stiglitz, (1972), Indirect taxation and economic efficiency, *Journal of Public Economics* 1, 97-120.
- Baumol, W.J., (1979), Quasi-optimality: The price we must pay for a price system, *Journal of Political Economy* 87, 578-599.
- Baumol, W.J., and D.F. Bradford, (1970), Optimal departures from marginal cost pricing, *American Economic Review* 60, 265-83.
- Baumol, W.J., E.E. Bailey, and R.D. Willig, (1977), Weak invisible hand theorems on the sustainability of prices in a multiproduct monopoly, *American Economic Review* 67, 350-365.
- Baumol, W.J. and Y.M. Braustein, (1977), Empirical study of scale economies and production complementarity: The case of journal publication, *Journal of Political Economy* 85, 1037-1048.
- Box, G.E.P., and D.R. Cox, (1964), An analysis of transformations, *Journal of the Royal Statistical Society, Series B* 26, 211-243.
- Breslaw, J.A., Corbo, V., and J.B. Smith, (1979a), A direct test of the A-J effect: The case of Bell Canada, Working Paper #1979-22, Department of Economics and Institute of Applied Economic Research, Concordia University.
- Breslaw, J.A., Corbo, V., and J.B. Smith, (1979b), A micro test of the neoclassical production theory, Working Paper #1979-21, Department of Economics and Institute of Applied Economic Research, Concordia University.
- Breslaw, J.A., Corbo, V., and J.B. Smith, (1979c), More pitfalls in the testing of duality theory, Working Paper #1979-23, Department of Economics and Institute of Applied Economic Research, Concordia University.

Breslaw, J.A., (1980), Simulations of Bell Canada under various rate scenarios, Canadian Department of Communications, DGCE #151.

Breslaw, J.A., and J.B. Smith, (1981), The restrictiveness of flexible functional forms in the modelling of regulatory constraint: Solving the problem for Bell Canada, Working Paper Department of Economics and Institute of Applied Economic Research, Concordia University.

Christensen, L.R., D. Cummings and P. Schoech, (1981), Econometric estimation of scale economies in telecommunications. Canadian Department of Communications Telecommunications Conference, March 4th, 1981.

Davies, J.B., (1980), Uncertain lifetime, consumption and dissaving in retirement, Journal of Political Economy, (forthcoming).

CRTC (1979), Telecom Decision, CNCP Telecommunications, Inter-connection with Bell Canada, CRTC 79-11, May 17.

Denny, M., M. Fuss, and C. Everson, (1979), Productivity, employment and technical change in Canadian telecommunications: The case for Bell Canada, Report to the Canadian Department of Communications.

Federal-Provincial Conference of Communications Ministers, (1979), Report of the federal-provincial working group on competition/industry structure in the telecommunications industry, Toronto.

Feldstein, M.S., (1972a), Distributional equity and the optimal structure of public prices, American Economic Review 62, 32-36.

Feldstein, M.S., (1972b), Equity and efficiency in public sector pricing: The optimal two-part tariff, Quarterly Journal of Economics, 175-187.

- Feldstein, M.S. (1972c), Pricing of public intermediate goods,  
Journal of Public Economics 1, 45-72
- Fellner, W., (1967), Operational utility: The theoretical background  
and a measurement, in: Ten Economic Studies in the Tradition of  
Irving Fisher, Wiley.
- Frish, R., (1959), A complete scheme for computing all direct and  
cross demand elasticities in a model with many sectors,  
Econometrica 27, 177-196.
- Fuss, M., and L. Waverman, (June 25, 1980), The regulation of  
telecommunications in Canada, Confidential draft of a final  
report to the Economic Council of Canada.
- Hall, R.E., and D.W. Jorgenson, (1967), Application of the theory of  
optimal capital accumulation, in Tax incentives and Capital  
Spending, G. Fromm (ed), The Brookings Institution.
- Houthakker, H.S., (1960), Additive preferences, Econometrica, 224-257.
- LeBlanc, G., (1979), Recent trends in the theory and practice of  
telecommunications regulation in the U.S. and public policy  
implications, Contract #OST 78000073, Canadian Department of  
Communications.
- Maital, S., (1973), Public goods and income distribution: some  
further results, Econometrica 41, 561-568.
- Mera, K., (1969), Experimental determination of relative marginal  
utilities, Quarterly Journal of Economics 83, 464-477.
- Panzar, J.C., and R.D. Willig, (1977a), Free entry and the sustain-  
ability of natural monopoly, Bell Journal 8, 1-22.
- Panzar, J.C., and R.D. Willig, (1977b), Economies of scale in multi-  
output production, Quarterly Journal of Economics 91, 481-94.

- Panzar, J.C. and R.D. Willig, (1979), Economies of scope, product specific economies of scale and the multiproduct competitive firm, Bell Laboratories Economics Discussion Paper #152.
- Pestieau, P., (1975), Regressiveness of efficiency rates in public economics, Canadian Journal of Economics 8, 269-275.
- Powel, A.A., T. Van Hoa, and R.H. Wilson, (1968), A multi-sectoral analysis of consumer demand in the post-war period, Southern Economic Journal 35, 109-120.
- Ramsey, F., (1927), A contribution to the theory of taxation, Economic Journal 37, 47-61.
- Rohlfis, J., (1979), Economically efficient Bell system pricing, Economic Discussion Paper #138, Bell Laboratories.
- Roy, R., (1947), La Distribution du revenue entre les divers biens, Econometrica 15, 205-225.
- Sato, K., (1972), Additive utility functions with double-log consumer demand functions, Journal of Political Economy 80, 102-124.
- Shephard, D., (1953), Cost and production functions, Princeton, Princeton University Press.
- Smith, J.B. and V. Corbo, (1979), Economies of scale and economies of scope in Bell Canada. Report to the Department of Communications.
- Taylor, L.D., (1980), Telecommunications demand: A survey and critique, Ballinger.
- Willig, R.D., (1978), Pareto-superior non-linear outlay schedules, Bell Journal of Economics 9, 56-59.



