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MORE PITFALLS IN THE TESTING OF DUALITY THEORY*

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ABSTRACT

Burgess and Appelbaum have tested neoclassical duality theory and concluded that cost and production models yield different estimates of the properties of the underlying technology. Both of these studies used aggregate US time series and assumed that the underlying production technology was homogeneous of degree one in inputs. In this paper, it is noted that the production model used by Burgess and Appelbaum is separable between inputs and outputs whereas the cost model includes no such separability. It is further shown that it is impossible to employ production models to uniquely estimate properties of a non-separable technology which is not linearly homogeneous. The importance of this separability issue is underlined by a test of duality theory for Bell Canada - a Canadian communications carrier which is unlikely to be characterized by constant returns to scale. In the testing it is found that the separable production model is much less robust than the non-separable cost model. It is concluded that a reworking of the Burgess-Appelbaum papers with non-separability in production may yield important insights into the empirical duality controversy.

INTRODUCTION

The empirical results recently reported in this Journal by Burgess (1975) and Appelbaum (1978) suggest that the estimated characteristics of neoclassical production technologies are not independent of the hypothesized model (primal or dual). For aggregate US data (Burgess) and aggregate US manufacturing (Appelbaum), there was an important difference between the production model and cost model estimates of (partial) elasticities of factor substitution. As well, there were minor differences reported between the substitution estimates for full (cost or production function and implied share equations) and partial (share equations only) models. Both authors concluded that care must be taken in model selection and the interpretation of results. As well, Burgess noted that his production model estimates were more robust.

In their respective studies, both authors constructed aggregate output quantity and price measures as Divisia indices of inputs and input prices. As well, both authors hypothesized homogeneity of degree one in inputs and Hicks neutral technical change as characteristics of the underlying technology. Finally, both Burgess and Appelbaum used more restrictive models on the production side than on the cost side. In particular, both assumed that the production function could be written with output separable from inputs whereas it is possible to implicitly define a non-separable production surface satisfying constant returns to scale and Hicks neutrality.

Two important questions arise. First, do the Burgess/Appelbaum results carry over to the level of the firm where independent input and output series are available and where the assumptions of constant returns to scale and Hicks neutrality can be tested? Secondly, what is the importance of the output separability assumption introduced by Burgess and Appelbaum?

Both of these questions are examined in this paper. Since the separability issue turns out to have rather strong implications for the estimation of both single and multi-output technologies, it is addressed first. A comparison of the dual approaches to modelling production technologies is then undertaken for a large Canadian communications carrier - Bell Canada, for which time series data on a variety of inputs and outputs are in the public domain.

THE SEPARABILITY ISSUE

Both Burgess and Appelbaum define production technology as:

(1) $Q=f(X)$

where X is the appropriate vector of inputs. As such, output is separable from inputs. Consider the following non-separable definition of the production technology.

(2) $F(X,X_{n+1}) = 0$

where $X_{n+1} = Q$. The translog form of F is given by:

(3) $$C_o + \sum_{i=1}^{n+1} \alpha_i \ln X_i + \tfrac{1}{2} \sum_{j=1}^{n+1} \sum_{i=1}^{n+1} \gamma_{ij} \ln X_i \ln X_j = 0$$

where $\gamma_{ij} = \gamma_{ji}$

Clearly, (1) is a special use of (3). As well, it is straightforward to show that production can be constrained to exhibit constant returns to scale. However, since (3) is homogeneous of degree zero in parameters, the estimation requirement that one of $(C_o, \alpha_i, \gamma_{ij})$ be normalized in general leads to the uncomfortable result that either (a) the parameter estimates are arbitrarily dependent upon the chosen scaling of the variables and thus estimates of characteristics of production (including elasticities of substitution) will change with different scalings of the variables or (b) one is forced to choose a dependent variable of limited economic interest.

Suppose α_{n+1} is set to -1 as the arbitrary normalization. Then, if separability holds, equation (3) is identical to equation (1). Any change in scaling of the input, or output will have no effect on the fit, since all the parameters can adjust. If separability does not hold, the normalization $\alpha_{n+1} = -1$ still appears to be the most natural specification since the process of normalization of an implicit function is not distinguishable from the choice of a dependent variable.

However, in the non-separable case, a change of scaling will result in a change of fit, since the parameter adjustment cannot freely take place because of the normalization. Clearly, if all the cross terms between output and inputs were zero (as in equation (1)) there would be no pressure on α_{n+1} to change, and the fit of the model would be independent of the scaling of the variables.

On the other hand, normalization of one of the invariant γ_{ij} parameters (perhaps $\gamma_{n+1,\ n+1}$) instead of α_{n+1} removes the scaling problem, but implies an (effective) dependent variable which is inappropriate and of little economic interest.

The implications of the above for the testing of duality theory are twofold. In the first place, if the guiding assumption is one of constant returns to scale, then the parameter restrictions implied by constant returns to scale (or any other predetermined scale) can all be written in terms of $\left\{\alpha_{n+1},\ \gamma_{i,\ n+1}\right\}$ and a full or partial production model can be estimated with no scaling dependence. That is, a more general version of the Burgess-Appelbaum papers is possible in principle. Secondly, if the assumption of constant returns to scale is not maintained then neither a full nor a partial production model can be uniquely estimated with non-separable production technology. The best that can be done in this case is to assume that production is separable between inputs and outputs. As well, partial production models are of little interest in this situation since the ratio nature of the marginal rate of technical substitution side conditions implies that these side conditions will be homogeneous of degree zero in parameters and yet another parameter normalization will be required. Thus the more general model is the cost model and it would seem reasonable to regard the resulting estimates of technology characteristics with more sympathy.

Finally, it should be noted that the scaling problems virtually eliminate the usefulness of multi-output production models for investigating issues of (ray) scale economies and economies of scope.

DUALITY TEST FOR BELL CANADA

Background

Bell Canada is the largest communications carrier in Canada. It has a virtual monopoly for many classes of service provision in the provinces of Quebec and Ontario. As well, Bell is regulated with respect to price and rate of return. In a previous paper it was shown that rate of return regulation was not binding.[2] Moreover, price regulation appears to be quite important at the aggregate level. The price elasticity of demand for the output index employed in the models which follow was estimated as -.38. Thus, the hypothesis of profit maximization cannot be supported at the one output level. The weaker assumption of cost minimization subject to exogeneously determined (regulated) output was introduced and appears to be quite compatible with the data.

THE PRODUCTION MODEL

The production model is specified in the following way:

(4) minimize $C = wL + vM + rK$

subject to the separable translog production function:

$$(5)\quad Ln\hat{Q} = Ln(Co) + CL\, Ln\,\hat{L} + CK\, Ln\,\hat{K} + CM\, Ln\,\hat{M}$$
$$+ .5\,[CLL(Ln\hat{L})^2 + CKK(Ln\hat{K})^2 + CMM(Ln\hat{M})^2]$$
$$+ CLK\, Ln\,\hat{L}\, Ln\,\hat{K} + CLM\, Ln\,\hat{L}\, Ln\,\hat{M} + CKM\, Ln\,\hat{K}\, Ln\,\hat{M}$$
$$+ CT\, Ln\,\hat{T} + .5(Ln\,\hat{T})^2 + CLT\, Ln\,\hat{L}\, Ln\,\hat{T}$$
$$+ CKT\, Ln\,\hat{K}\, Ln\,\hat{T} + CMT\, Ln\,\hat{M}\, Ln\,\hat{T}$$

where[3] $\hat{X} = X/\bar{X}$ and $\bar{X}$ is the mean of $X\ \varepsilon\,(L,K,M,T,Q)$

C = Total cost, in current dollars

Q = Aggregate output, computed as a Divisia quantity index of local services, telephone message toll and other toll services.

L = Weighted man hours, with weights equal to the wage structure of 1967.

K = Net capital stock, in 1967 dollars.

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2 See Breslaw, Corbo and Smith (1979).

3 The data sources were CRTC (1978) and Bell Canada Annual Reports for the years 1952 to 1976.

M = Quantity index of raw materials, 1967 value equal to raw material cost in that year.

w = Wage Rate (Wage bill divided by L).

v = Unit cost of raw materials (current value of raw materials divided by M).

r = Unit price of capital services in the Hall and Jorgenson (1971) tradition with allowance for capital gains.

T = Technology indicator, measured as a weighted average of new switching technologies and the spread of the new equipment throughout the Bell Canada system.

The formal minimization of (4) subject to (5) yields a system of four first order conditions which includes the production function. The unobservable Lagrange multiplier corresponding to the output constraint is eliminated by expressing the remaining three first order conditions with respect to $\hat{K}$, $\hat{L}$, $\hat{M}$ in ratio form as:

$$(6) \quad \frac{vM}{wL} = \frac{CM + CLM \, Ln \, \hat{L} + CKM \, Ln \, \hat{K} + CMM \, Ln \, \hat{M} + CMT \, Ln \, \hat{T}}{CL + CLL \, Ln \, \hat{L} + CLK \, Ln \, \hat{K} + CLM \, Ln \, \hat{M} + CLT \, Ln \, \hat{T}}$$

and

$$(7) \quad \frac{rK}{wL} = \frac{CK + CLK \, Ln \, \hat{L} + CKK \, Ln \, \hat{K} + CKM \, Ln \, \hat{M} + CKT \, Ln \, \hat{T}}{CL + CLL \, Ln \, \hat{L} + CLK \, Ln \, \hat{K} + CLM \, Ln \, \hat{M} + CLT \, Ln \, \hat{T}}$$

Thus the system to be estimated consists of equations (5), (6) and (7) with added error terms reflecting random optimization errors. The possibility of covariance of errors across equations was entertained. The parameter estimates for the simultaneous system were obtained by a non-linear full information maximum likelihood algorithm. In the estimation (K,L,M) were assumed endogenous and all price variables, output and technology were treated as exogenous. The possibility of a partial production model was not entertained because, treated as a separate system, equations (6) and (7) are homogeneous of degree zero in parameters and require an arbitrary parameter normalization. Unfortunately, this normalization implies that the parameter estimates and hence the estimated production characteristics will be dependent upon scaling of the variables. Finally, it will be noted that technological change is assumed to be fully general.

THE COST MODELS

The cost function was specified in symmetric translog form as:

$$(8) \quad \hat{C} = C_o + C_n \, \ell n\hat{w} + C_v \, \ell n\hat{v} + C_r \, \ell n\hat{r} + C_Q \, \ell n\hat{Q} + C_T \, \ell n\hat{T}$$
$$+ .5 \, [C_{ww} \, (\ell n\hat{w})^2 + C_{vv} \, (\ell n\hat{v})^2 + C_{rr} \, (\ell n\hat{r})^2 + C_{QQ} \, (\ell n\hat{Q})^2 + C_{TT} \, (\ell n\hat{T})^2]$$
$$+ C_{wv} \, \ell n\hat{w} \, \ell n\hat{v} + C_{wr} \, \ell n\hat{w} \, \ell n\hat{r} + C_{wQ} \, \ell n\hat{w} \, \ell n\hat{Q} + C_{wT} \, \ell n\hat{w} \, \ell n\hat{T}$$
$$+ C_{rv} \, \ell n\hat{r} \, \ell n\hat{v} + C_{vQ} \, \ell n\hat{v} \, \ell n\hat{Q} + C_{vT} \, \ell n\hat{v} \, \ell n\hat{T} + C_{rQ} \, \ell n\hat{r} \, \ell n\hat{Q}$$
$$+ C_{rT} \, \ell n\hat{r} \, \ell n\hat{T} + C_{QT} \, \ell n\hat{Q} \, \ell n\hat{T}$$

where a ^ indicates scaling by the mean and all variables are defined as in the production model.

Sheppard's lemma yields the following implied share equations:

$$(9) \quad \frac{wL}{C} = C_w + C_{ww} \, \ell n\hat{w} + C_{wv} \, \ell n\hat{v} + C_{wr} \, \ell n\hat{r} + C_{wT} \, \ell n\hat{T} + C_{wQ} \, \ell n\hat{Q}$$

$$(10) \quad \frac{rK}{C} = C_r + C_{wr} \, \ell n\hat{w} + C_{rv} \, \ell n\hat{v} + C_{rr} \, \ell n\hat{r} + C_{rT} \, \ell n\hat{T} + C_{rQ} \, \ell n\hat{Q}$$

$$(11) \quad \frac{vM}{C} = C_v + C_{wv} \, \ell n\hat{w} + C_{vv} \, \ell n\hat{v} + C_{rv} \, \ell n\hat{r} + C_{vT} \, \ell n\hat{T} + C_{vQ} \, \ell n\hat{Q}$$

Homogeneity of degree one in factor prices implies that the parameters of the cost function must satisfy the following set of independent additional restrictions:

$$
\begin{aligned}
(12) \qquad C_w + C_r + C_v &= 1 \\
C_{ww} + C_{wr} + C_{wv} &= 0 \\
C_{wr} + C_{rr} + C_{rv} &= 0 \\
C_{wT} + C_{rT} + C_{vT} &= 0 \\
C_{wQ} + C_{rQ} + C_{vQ} &= 0
\end{aligned}
$$

For estimation, the restrictions were imposed upon the materials coefficients. As such, the materials share equation could be dropped from the estimating model and the parameters were later recouped for analysis. An error term, again reflecting optimization errors and again allowed to covary across equations was added to each equation. The full cost model thus consisted of equations (8), (9) and (10) whereas the partial cost model was made up of equations (9) and (10).

Once again, $\{K, L, M\}$ were treated as the endogenous variables of the model while the financial variables, output and technology were assumed exogenous. The parameters were estimated using Zellner's technique. The endogeneity of $\{K, L, M\}$ was not an issue for the cost models because the left hand side variables in equations (8), (9) and (10) are monotone functions of $\{K, L, M\}$ and Zellner's technique is asymptotically equivalent to full information maximum likelihood estimation.

PARAMETER ESTIMATES AND WELL-BEHAVEDNESS CONDITIONS

The parameter estimates for the cost and production models are presented in Tables I and II respectively. The asymptotic standard errors are much lower for the cost models. As well, the parameter estimates for the full and partial cost models are quite similar. Finally, hypotheses of constant returns to scale and Hicks neutral technical change were rejected using the likelihood ratio test for both the full cost and production models.

All of the cost and productions functions were well-behaved in every way at each data point thereby guaranteeing satisfaction of the second order conditions. In particular, the estimated production function was quasi-concave with positive marginal products at each data point. Similarly, the cost function was concave with downward sloping constant output factor demands at each data point.

ESTIMATED CHARACTERISTICS OF THE UNDERLYING TECHNOLOGY

The cost and production models suggest markedly different properties for the underlying production technology. Comparing Tables IIIa and IIIc the Allen partial elasticities of substitution between labour and capital (σ_{LK}) and capital and materials (σ_{KM}) differ by up to a factor of 3. As well, the full cost model suggests some complementarity between capital and materials in the early part of the sample. Finally, there is agreement between the models with respect to the partial elasticity of substitution between labour and materials (σ_{LM}).

Alternatively, the full and partial cost models yield quite similar conclusions with respect to the properties of the underlying technology. In particular, a comparison of Tables IIIa and IIIb suggests that the partial elasticity of substitution estimates between labour and capital are virtually the same whereas the elasticities of substitution between labour and materials and capital and materials differ somewhat. These latter differences should be interpreted in light of the fact that the partial cost model was used to estimate 9 different parameters with only 25 data points. As well, there was only one parameter which entered both the labour share and capital share equations. Thus little information was introduced by cross-equation parameter constraints in the partial cost model.

A final comparison of the full cost and production models is made with respect to the scale estimates presented in Table IV. The scale estimates from the production model display a very strong trend. In contrast, the scale estimates from the cost model are quite stable with no evidence of trend. Although both models suggest the existence of economies of scale, the trend in the production model scale estimates is suspicious and suggests that the model may not be able to disentangle the separate contributions of scale and technology.

TABLE I

FULL PRODUCTION MODEL PARAMETER ESTIMATES

PARAMETER	ESTIMATE	STANDARD ERROR
C_O	.8623	.0061
C_L	.5917	.0725
C_K	.8816	.1087
C_M	.3543	.0438
C_{LL}	-.1086	.1089
C_{KK}	-.3762	.1167
C_{MM}	-.0885	.0596
C_{LK}	-.0569	.1134
C_{LM}	-.0968	.0505
C_{KM}	.0708	.0685
C_T	.0810	.2626
C_{TT}	-3.8515	1.3983
C_{LT}	.5215	.2617
C_{KT}	1.7797	.3901
C_{MT}	.3337	.1383

Log of Likelihood function = 204.88

TABLE II

PARAMETER ESTIMATES FROM COST MODELS

Parameter	FULL MODEL Estimate	FULL MODEL Standard error	PARTIAL MODEL Estimate	PARTIAL MODEL Standard Error
C_O	.0450	.0082		
C_w	.3418	.0022	.3416	.0021
C_r	.4663	.0025	.4667	.0025
C_v	.1920	.0015	.1916	.0016
C_Q	.9477	.0623		
C_{ww}	-.0788	.0295	-.0811	.0292
C_{rr}	.0215	.0250	-.0021	.0273
C_{vv}	.0614	.0206	.0616	.0232
C_{QQ}	.9357	.4899		
C_{wR}	.0593	.0212	.0724	.0221
C_{wv}	.0195	.0206	.0087	.0221
C_{rv}	-.0809	.0147	-.0703	.0162
C_{wQ}	.0790	.0158	.0768	.0155
C_{rQ}	-.0933	.0181	-.0913	.0178
C_{vQ}	.0142	.0115	.0146	.0120
C_T	-.9916	.1366		
C_{TT}	3.0817	2.3480		
C_{wT}	-.3293	.0356	-.3237	.0346
C_{rT}	.3704	.0415	.3584	.0412
C_{vT}	-.0411	.0245	-.0347	.0252
C_{QT}	-2.1761	1.0667		
Log of likelihood function	272.64		188.462	

TABLE IIIa

ALLEN PARTIAL ELASTICITY OF SUBSTITUTION ESTIMATES FROM FULL COST MODEL

	σ_{LK}	σ_{LM}	σ_{KM}
1952	1.35196	1.25154	-.183612
1953	1.35689	1.24653	-.145630
1954	1.36095	1.24174	-.124605
1955	1.36707	1.23224	-.119456
1956	1.36991	1.22920	-.108372
1957	1.36746	1.23473	-.937358E-01
1958	1.36639	1.24797	-.306668E-01
1959	1.36501	1.25811	-.617991E-02
1960	1.36793	1.27763	.539007E-01
1961	1.37134	1.28975	.857372E-01
1962	1.37287	1.28740	.912317E-01
1963	1.37506	1.29770	.108974
1964	1.38079	1.31132	.138401
1965	1.38245	1.31211	.144930
1966	1.38430	1.31435	.152277
1967	1.38472	1.31394	.153743
1968	1.38904	1.33382	.169779
1969	1.39034	1.34405	.172957
1970	1.38991	1.35545	.169418
1971	1.39697	1.36215	.189403
1972	1.39935	1.37553	.192088
1973	1.39645	1.37338	.184425
1974	1.39686	1.37442	.185283
1975	1.39881	1.37957	.189128
1976	1.40237	1.38578	.196780

TABLE IIIb

ALLEN PARTIAL ELASTICITY OF SUBSTITUTION ESTIMATES FROM PARTIAL COST MODEL

	σ_{LK}	σ_{LM}	σ_{KM}
1952	1.43229	1.11040	-.289756E-01
1953	1.43684	1.10916	.275330E-02
1954	1.44099	1.10759	.211067E-01
1955	1.44737	1.10403	.258852E-01
1956	1.45075	1.10278	.369546E-01
1957	1.44819	1.10504	.496624E-01
1958	1.44685	1.11106	.104251
1959	1.44548	1.11537	.125486
1960	1.44895	1.12418	.177301
1961	1.45293	1.12977	.204438
1962	1.45466	1.12896	.209208
1963	1.45753	1.13341	.224836
1964	1.46483	1.13944	.251026
1965	1.46706	1.13978	.257271
1966	1.46934	1.14087	.263676
1967	1.46975	1.14081	.264706
1968	1.47504	1.14934	.278390
1969	1.47663	1.15355	.281197
1970	1.47587	1.15813	.277805
1971	1.48410	1.16187	.294033
1972	1.48650	1.16767	.295483
1973	1.48341	1.16613	.290067
1974	1.48455	1.16604	.292511
1975	1.48691	1.16823	.295939
1976	1.49138	1.17090	.302965

TABLE IIIc

ALLEN PARTIAL ELASTICITY OF SUBSTITUTION ESTIMATES FROM FULL PRODUCTION MODEL

	σ_{LK}	σ_{LM}	σ_{KM}
1952	.807644	1.36204	.153518
1953	.803158	1.36093	.153120
1954	.799370	1.36466	.147689
1955	.785365	1.36633	.107999
1956	.789213	1.38065	.122350
1957	.792516	1.37322	.175052
1958	.809839	1.37550	.268759
1959	.816248	1.36759	.324311
1960	.828565	1.35797	.401400
1961	.833298	1.34877	.440366
1962	.832507	1.34707	.448911
1963	.835901	1.34524	.473972
1964	.841404	1.33784	.507994
1965	.842285	1.34135	.518277
1966	.842877	1.34043	.528560
1967	.842754	1.33004	.537249
1968	.847410	1.32519	.562040
1969	.848649	1.33497	.572242
1970	.849508	1.33042	.581642
1971	.849259	1.34028	.585657
1972	.851009	1.33580	.596762
1973	.850800	1.34122	.600936
1974	.852308	1.33900	.610357
1975	.854030	1.33256	.619873
1976	.854179	1.33778	.624782

TABLE IV

SCALE ECONOMY ESTIMATES FROM FULL COST AND PRODUCTION MODELS

	Cost Model Scale*	Production Model Scale**
1952	1.32619	1.23089
1953	1.22522	1.20883
1954	1.14838	1.17850
1955	1.01297	1.09320
1956	.963265	1.08584
1957	.963301	1.13684
1958	1.05622	1.27604
1959	1.10031	1.37223
1960	1.25491	1.55865
1961	1.32100	1.66603
1962	1.22549	1.67633
1963	1.27899	1.75532
1964	1.38138	1.88786
1965	1.33239	1.92388
1966	1.27682	1.95989
1967	1.19811	1.98440
1968	1.27113	2.11283
1969	1.25448	2.17070
1970	1.23914	2.21548
1971	1.22417	2.24011
1972	1.19293	2.30691
1973	1.13180	2.33253
1974	1.11221	2.39662
1975	1.07492	2.45988
1976	1.07216	2.49996

*inverse of scale elasticity

**sum of output elasticities of K,L,M.

CONCLUSION

This paper began with a demonstration of the fact that previous tests of duality theory had disavantaged the production models by assuming that output was strongly separable from inputs. It was also shown that with constant returns to scale in production the models could be estimated without assuming strong separability. Unfortunately, strong separability is required if a production model is to be uniquely estimated with other than constant returns to scale and Hicks neutral technical change.

The paper continued with a test of duality theory based upon published data of a large Canadian communications carrier. It was concluded that the primal and dual models yielded quite different estimates of the properties of the underlying technology. It was noted that the (necessarily) separable production model provides a less robust fit of the data and that the scale estimates were strongly trended. It was concluded that the cost model estimates were more likely to be correct. Notwithstanding the separability issue, it would appear that when technical change is fully general and returns to scale are not constrained, a pattern of results similar to that reported by Burgess and Appelbaum emerges.

An important direction for future research is to examine the empirical significance of the separability issue raised here. It would appear that if the Appelbaum and Burgess production models were reestimated without separability, more insights could be obtained into empirical duality problems.

REFERENCES

Appelbaum, E, 1978, "Testing Neoclassical Production Theory, Journal of Econometrics, 7,1, 87-102.

Appelbaum, E,, forthcoming, "On the Choice of Functional Forms", International Economic Review.

Bell Canada, Annual Reports, 1952 to 1976.

Berndt, E.R. and D. Wood, 1975, "Technology, Prices and the Derived Demand for Energy", Review of Economics and Statistics, 62, 3, 259-268.

Berndt, E.R. and L.R. Christensen, 1973, "The Translog Function and the Substitution of Equipment Structures, and Labor in U.S. Manufacturing 1929-1968", Journal of Econometrics, 1, 1, 81-113.

Berndt, E.R. and L.R. Christensen, 1974, "Testing for the Existence of a Consistent Aggregate Index of Labor Inputs", American Economic Review, 64, 3, 391-404.

Breslaw, J., V. Corbo and J.B. Smith, 1979, "A direct Test of the A-J Effect: The Case of Bell Canada", Mimeo.

Brown, R.S., Caves, D.W. and L.R. Christensen, 1976, "Modelling the Structure of Production with a Joint Cost Function". Manuscript only.

Burgess, D.F., 1975, "Duality Theory and Pitfalls in the Specification of Technologies", Journal of Econometrics, 3, 2, 105-121.

Christensen, L.D., D.W. Jorgenson, and L.J. Lau, 1971, "Conjugate Duality and the Transcendental Logarithmic Production Function" (abstract), Econometrica, 39, 4, 255-256.

Corbo, V., 1976, "Second Order Approximations for Estimating Production Functions", Annals of Economic and Sound Measurement, 5, 1, 65-73.

Corbo, V., and P. Meller, 1978a, "The Translog Production Function: Some Evidence from Establishment Data", Manuscript only.

Corbo, V. and P. Meller, 1978b, "The Substitution of Labor, "Skill" and Capital in Chilean Manufacturing", Mimeo.

CRTC, 1978, Rate Hearings, Bell Canada P(NAPO) 03 March 78-732.

Diewert, W.E., 1974, "Applications of Duality Theory", in M. Intriligator and D. Kendrick (eds), Frontiers of Quantitative Economics, Vol. II, Amsterdam: North Holland Publishing Company.

Fuss, M.A., 1977, "The Demand for Energy in Canadian Manufacturing: An Example of the Estimation of Production Structures with Many Inputs", Journal of Econometrics, 5, 1, 89-116.

Griffin, J.M., 1977, "The Econometrics of Joint Production: Another Approach", Review of Economics and Statistics, LIX, 4, 389-397.

Hall, R.E., 1973, "The Specification of Technology with Several Kinds of Output", Journal of Political Economy, 81, 4, 878-892.

Hall, R.E., and D.W. Jorgenson, 1971, "Application of the Theory of Optimal Capital Accumulation", in G. Fromm (ed.) Tax Incentives and Capital Spending, Washington: The Brookings Institution.

Humphrey, D.B. and J.R. Moroney, 1975, "Substitution Among Capital, Labor and Natural Resource Products in American Manufacturing", Journal of Political Economy, 83, 1, 57-82.

Hall, R.E. and Jorgenson, D.W., "Application of the Theory of Optimum Capital Accumulation" Chapter II in G. Fromm (ed.) Tax Incentives and Capital Spending, (Washington: The Brookings Institution, 1971).

Moroney, R.R., and A.L. Toevs, 1977, "Factor Costs and Factor Use: An analysis of Labor, Capital and Natural Resource Inputs", Southern Economic Journal, 44, 2, 222-239.

Nadiri, M.I., 1978, "Producers Theory", in K.J. Arrow and M.D. Intriligator (eds.), Handbook of Mathematical Economics, Amsterdam: North Holland Publishing Company. (forthcoming).

Pindyck, R.S., 1977, "Interfuel Substitution and the Industrial Demand for Energy: An International Comparison", MIT, Working Paper EL 77-026 WP.

Takayama, A., 1974, Mathematical Economics, Hinsdale, Illinois: The Dryden Press.

Vinod, H.D., 1972, "Nonhomogeneous Production Functions and Applications to Telecommunications", Bell Journal of Economics and Management Science, 3, 2, 531-543.

Vinod, H.D., 1976, "Cannonical Ridge and Econometrics of Joint Production", Journal of Econometrics, 4,2, 147-166.