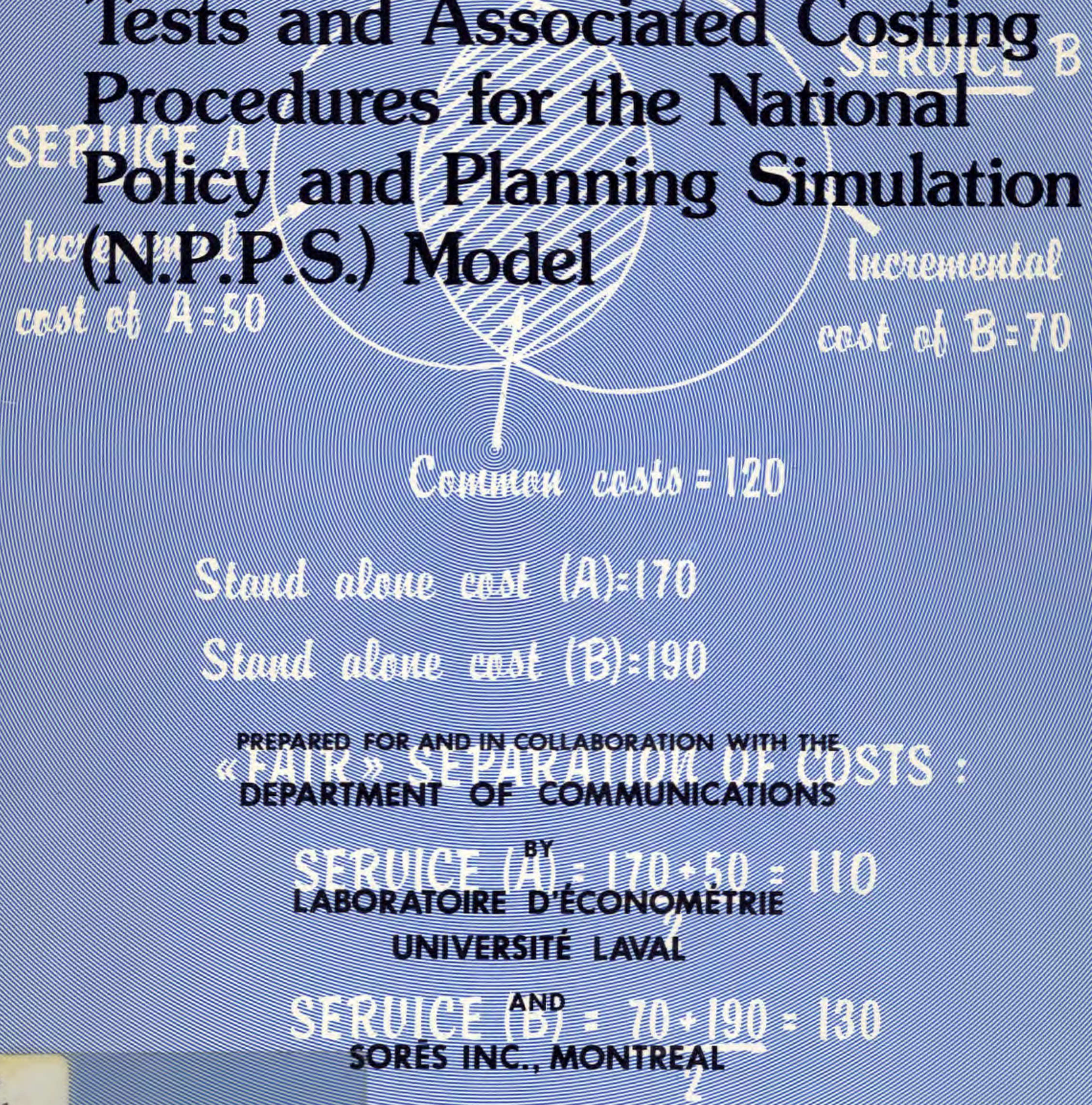
Development and Empirical Evaluation of Cross-subsidy Tests and Associated Costing Procedures for the National Policy and Planning Simulation (N.P.P.S.) Model



PREPARED FOR AND IN COLLABORATION WITH THE
DEPARTMENT OF COMMUNICATIONS

BY

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(1)

DEVELOPMENT AND EMPIRICAL EVALUATION OF CROSS-SUBSIDY TESTS AND ASSOCIATED COSTING PROCEDURES FOR THE NATIONAL POLICY AND PLANNING (N.P.P.S.) SIMULATION MODEL

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SUMMARY

The main objectives of this Report are first to formulate certain cross-subsidy concepts in order to empirically evaluate cross-subsidy tests in the telecommunication industry, second to describe the adjustments needed in the N.P.P.S. model in order to perform those tests, third to report on the simulations done with it, and finally to evaluate the applicability of the proposed tests for this industry. Also the main differences in costing methodology on the various services adopted in the N.P.P.S. model, in the Costing Manual (Costing Inquiry) and the FCC Method 7 are examined.

RESUME

Le présent rapport a pour buts de discuter un certain nombre de concepts ayant trait à l'inter-financement dans l'industrie des communications, de suggérer certains tests qui en résultent, de montrer les modifications qui ont été apportées au modèle N.P.P.S. afin de rendre opérationnels ces tests, de décrire les simulations qui ont été faites avec ce modèle et finalement d'évaluer la pertinence de ces tests pour cette industrie. Les principales différences dans les méthodologies, respectivement dans le projet N.P.P.S., dans le Costing Manual et dans la méthode 7 du FCC sont également examinées.

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1. INTRODUCTION

The central theme of the Report is the empirical evaluation of cross-subsidy tests in the telecommunications industry. This objective is reached by formulating a certain number of tests, tests which are shown from the various definitions of cross-subsidizations, by making the necessary adjustments to the N.P.P.S. model and finally by performing a large number of simulations and evaluating the results. The relationship between the present phase of the project and previous phases is then evident: the N.P.P.S. model is used as the main instrument for evaluating these tests.

Loosely speaking, by cross-subsidization, one means that somebody has to pay in full, or in part, for somebody else's consumption of a particular service. This aspect of who has to pay for whom is always present in our society. In the domain of telecommunications, this latter can be interpreted from many points of view. Among the most important is the possibility of financing a service out of profit generated by supplying some other services. Most recently this problem was posed in the following terms by the Telecommunications Committee, Canadian Transport Commission, in its decision of August 15, 1974. It concerns the expenditures in the Construction Program of Bell Canada for increasing the quality of non-urban services. The Committee said:

> "We fully realize, however, that such expenditures would require substantially more revenues from multi-party services to pay for them than the present rate structure would provide, and that such additional revenues would have to come from a new and higher rate structure for multi-party services <u>or from increased rates for other services offered by Bell, or from both"</u> (emphasis added)

As formulated previously, the cross-subsidization concept is relatively easy to understand. However, things are more complicated once somebody tries to measure it empirically. The problem is even more complex if one has to take into account the characteristics of the telecommunications industry. It is the main objective of this Report to propose some potential empirical tests of cross-subsidization, most of them taken from the existing literature on the subject and to test their validity for the telecommunications industry. An effort is also made to show the various interpretations of cross-subsidy, as well as the adjustments and modifications which had to be made to the N.P.P.S. model for the purpose of implementing these various tests. The modifications mainly concern the problem of dimensioning the switching network and the allocation of circuits in the transmission network, in order to cost the various services provided by the carriers. The main differences between the approach taken for costing the various services in the N.P.P.S. model, the Costing Manual and the FCC Method 7 are also highlighted.

Before formulating these tests and reporting on the various scenarios considered, a section is devoted to the formulation of the cross-subsidization problem, first in general terms and second in relation to the particular domain under study. In particular, some trials are carried out in order to show how certain control variables called tariffs (for the various services) may be utilized in order to achieve certain social, economic and policy objectives. Finally, an intermediate section presents the theoretical basis for the cross-subsidy problem and in particular reviews the principles behind the marginal cost pricing.

To remain consistant with work already presented in the N.P.P.S. Project, the industry is defined as the set of common carriers (mostly regional monopolies) providing telecommunications services. It is noted that no formal consideration is given to the relation between subsidiaries like Northern Telecom, and the parent establishment.

It is immediately apparent when one speaks of cross-subsidy that, implicity or explicitly, the costs associated with the particular services are to be measured in some suitable manner. Service costing has been one of the major intellectual and practical challenges addressed by the project team. The N.P.P.S. "machinery", to a large extent, has been designed to provide solutions in this particular problem area.

The present Report represents the results of a combined effort by three groups whose formal responsibilities were spelled out in the various official documents. In practice, detailed sharing of the tasks was handled by more or less informal exchanges. The tripartite team consisted of the following organizations given here with the names of the specialists involved:

- The Telecommunications Economics Branch, Communications Canada

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2. THE CONCEPT OF CROSS-SUBSIDY IN TELECOMMUNICATIONS

2.1 From Policies to N.P.P.S.

2.1.1 General Considerations

At every period, any society through its representatives assigns to its members a set of objectives (sometimes called priorities) defined in a more or less formal manner. One can remark that it is one thing to enumerate the priorities of a society, while it is often another thing to assign some desired values (or targets) to them. In our free enterprise system, the achievement of these proposed objectives is mostly vested in the individual economic agents (consumers and producers). However, since at least the beginning of the present century, the government, through its expenditures on one hand and regulatory means on the other, has increased gradually its importance in the social and economic domains in such a way that today one can say that it has sufficient power to "contribute" to the realization of the stated objectives.

The big challenge for government is how to utilize its instruments in such a way that the desired values of the stated objectives be approached as near as possible. It is in this context that the NPPS and HERMES models have been developed: to provide an instrument in the hands of the Department of Communications for evaluating, in the NPPS model, the impacts on the carriers financial statements of some modifications of some control variables; and in the HERMES model, an optimal way of expanding the networks once the demands for the various services have been increased. The reader will find in the next sub-section a hierarchy of the objectives for the domain of telecommunications and the respective contributions of both models.

Any social and economic problem can be studied from both an efficiency point of view and from an equity point of view. From an efficiency point of view, the question is of course: is the supplying of a particular good or service done in the most effective way and, if not, is there any incentive to reach this goal. The problem under study here is more complicated because the communications industry is mostly a regulated monopolistic one. Even more importantly, at a more aggregate level, since the various industries compete for the sources of capital, it may be asked if the amount invested in the communications industry is the proper amount, or, if the resources could be more fruitfully utilized in other domains? Generally, the economists say that an allocation is optimal in the Pareto sense when it is not possible to reallocate the resources without penalizing somebody else.

From an equity point of view, one of the questions is to define which means (taxation, regulation, etc...) the government can use in order to ensure that a particular (efficient) state of the economy is achieved. Needless to say, the central problem of cross-subsidization rests on equity grounds since, by definition, there will be some consumers or firms which will have to pay for somebody else. In effect, one can define the cross-subsidy concept as follows: a group of economic agents (consumers or producers) obtain certain service(s) from a producing system for a given period. If the revenues for that service(s) do not cover the value of the corresponding inputs consumed in the producing system, then some other economic agents must bear the difference in value. From a different view point, one can say that studying the problem of cross-subsidization is, then, a way of trying to quantify the "social goals" behind the various tariffs. More generally, the very heart of the problem, at the equity level, is a problem of redistributing the revenues in view of reaching some particular predefined social objectives.

It is evident that studying the problems inherent to the telecommunications industry, this can be done from both view points. In particular, one can ask if the fact of regulating this industry introduces some misallocations of the resources on one hand, and that the tariffs structure is such that there is no cross-subsidization among customers on the other hand. We will briefly discuss these two points of view.

In the context of the telecommunications industry, the efficiency and the equity problems are very difficult to attack for at least three reasons: the definition of the services, the technical characteristics, and because it is regulated.

The concept of services can be studied from at least three various perspectives: from a "public good" perspective, from a "merit good" and finally from a "private good" perspective. Technically, a public good is a commodity such that its consumption by an individual does not prevent its consumption by another agent. In consequence, there is some incentive for some agents to be "free riders", i.e. not paying for this good but still consuming it. By a merit good, one means a private good, the merit of which is viewed sufficiently high by society that it is given free or partly free to certain groups of society. This definition applies to most of the services provided by the telecommunications industry. Finally, a private good is a good where the consumption of a commodity by an agent excludes its consumption by another agent. It can be said that the main results of economic theory are usually only valid for this last class of goods.

Economically speaking, the technical characteristics of the telecommunications industry are now quite well known. As examples, one can mention decreasing average cost in the long run, jointness of supply and finally the existence of certain indivisibilities.

Each of them contributes to certain difficulties in applying the marginal cost pricing approach. In effect, as long term average costs are decreasing, a tariff structure based on this approach will not recover all costs incurred in providing the services. But it is well-known that from an equity point of view, the totality of the services might be self-financing. In consequence, as the industriy is regulated by overall rate of return, some services have to be priced higher than their marginal costs. It can be noted that some authors have questioned the presence of economy of scale particularly for transmission capacity costs where statistically the average transmission cost per channel seems to approach an horizontal asymptote.

By the jointness of supply, we mean that it is cheaper to supply a particular service to a group of customers than to supply it to each customer. However, this characteristic shows that much of the costs will be common and consequently some sharing rule will be necessary to attribute a fraction of the costs of each customer in order to evaluate the presence of cross-subsidization among customers. It seems that one of the main interests in the game theoretical approach, as described below, is precisely to suggest a precise cost separation rule.

The fact that there are certain indivisibilities introduces at least two problems, with respect to the marginal cost approach. The first one is that, in this context, the marginal cost cannot be uniquely defined. The second is that most probably, there will always be excess capacity, even during peak periods. For the interfinancing point of view, this raises the question of which user will pay for this excess capacity. Of course, these three characteristics are strongly related.

Among the other characteristics of this industry, one can mention that each carrier supplies a number of services and this raises the possibility of cross-subsidization among services, the existence of subsidiaries which raises the danger of financing some non-regulated services by regulated ones, the existence of two networks (switching and transmission ones) which can have as a consequence that the non-switched service finances the switched services, or vice versa.

Finally, one can say that a tariff structure to be efficient for a decreasing-cost industry must satisfy the three following criteria:

a) it must enable the total costs of the enterprise to be recovered;

b) it must be so designed that no customer willing to pay at least the marginal cost of serving him is turned away;

c) there should be no sales below marginal cost.

In section 2.1.3, we will look at the possibility of determining such a rate structure.

The telecommunications industry is a regulated industry. From the point of view of regulation, the central tasks can be seen as:

a) to determine who will receive the benefits or burden of regulation;

b) what forms regulation will take;

c) to evaluate the effects of regulation upon the allocation of resources.

Concerning the first point, it can be said that sometimes it is relatively easy to identify who will receive the benefits of a regulation, or who will support the cost of the regulation. However, in the domain of telecommunications, the benefits seem to be distributed to all members of society but not necessarily the cost. As mentioned previously, this diffuse impact of regulation reflects the importance of "externalities" in the telecommunications domain.

The telecommunications industry is mainly regulated by two means; first, by imposing an upper bound on the rate of return the carrier can earn on its investment and second by setting up some barriers to entry for new carriers. The first permits the determination, for a carrier as a whole, of a total revenue requirement which will be the basis to construct a tariff structure.

The second important means of regulation is the control of entry. In other words, the government, for many reasons, controls the degree of competition in this industry, implicitly assuming that if there was full competition, the impact on society would be negative. This issue is not so clear. Some authors have argued for the possible advantages of allowing competition (see, in particular, Welch, J. Workable Alternatives to Regulation, Public Utility Fortnightly, Oct. 23, 1975), while others argued against.

Ideally, the means of regulation must be such that it is a perfect substitute for competition, and consequently, no distortion on the allocation of resources should result. Conversely, Averch and Johnson, at a theoretical level, have shown that if an industry is regulated by allowing a certain rate of return on its capital, the consequence will be an overinvestment in capital.

2.1.2 List of Objectives

A policy is an action pattern for public intervention to solve a public problem in a certain domain. Before determining a policy for rational public intervention, governments need to specify a hierarchy of objectives from higher level principles, almost never reached, to operational targets whose attainability is easier to observe.

This section starts with a listing of objectives drawn from:

- "Proposals for Communications Policy for Canada", March 1973, Department of Communications, Ottawa;
- "Le Québec maître d'oeuvre de la politique des communications sur son territoire", novembre 1973, Ministère des communications, l'Editeur officiel du Québec;
- "Recommended Decision, Chief, Common Carrier Bureau",F.C.C., January 18, 1976, U.S.A.

The main source will be the first document and only token references will be made to the others. A graph (see Figure 2.1) indicates the relationship between the objectives. Three levels have been established but this is opened to re-evaluation if necessary.

2.1.2.1 Higher Level Principles

<u>For Canada</u>

a) In favor of individuals and organizations (objectives 1, 2 3):

   1. protection against any abuses of confidential information;
   2. establishment of conditions for exercising the right to communicate (mainly "necessity of life");
   3. protection against any discrimination in the use of communication services.

b) In favor of society as a functioning whole (objectives 4-8):

   4. preservation of the environment esthetics from the physical communication system impacts;
   5. support of national sovereignty;
   6. help in the definition of collective objectives;
   7. help in the identification of social problems;
   8. harmony of federal and provincial objectives in the communications field.

<u>For the Provinces</u>

The provinces profess the same objectives except that "national sovereignty" is replaced by "provincial identity".

For the F.C.C. (U.S.A.)

The U.S. seems to start from a general consideration of justice and translate it immediately into more concrete objectives as evidenced by the following statements:

- Carriers are obligated to provide interstate and foreign com-communications service upon reasonable request.

- Charges, practices, classifications and regulations shall be just and reasonable (to compare with objective 17 below).

- No unjust or unreasonable preference, advantage or disadvantage is permitted (see objective 17).

- Unjust or unreasonable discrimination in charges, practices, classifications, regulations, facilities or service is prohibited in like communications services (see objective 3).

- To permit the carriers to provide service to any agency of the government "free of charge" in connection with the preparation for the National Defense (see objective 5).

- To permit the carriers to provide free or reduced rate "interconnection" services to non-commercial educational television or radio stations (see objectives 6 and 7).

2.1.2.2 Second Level Objectives in the Communications Policies

In any communication there is informational content and physical support. Accordingly, one can split second level objections as follows:

a) Objectives aiming at the physical communications systems:

9. imposition of good confidentiality rules in handling, storing and transmitting information;

10. accessibility of services (for all) and variety of services (for most);

11. high quality services and reliable networks;

12. efficient and economical systems;

13. Canadian control of the ownership of communications systems.

b) Objectives aiming at the content carried by the systems:

14. provision of fuller and more diverse Canadian sources of information, entertainment and cultural and educational material of excellent quality;

15. active participation of the public in the communications content;

16. avoidance of bad impacts and encouragement of good impacts of the new techniques on 1. - social and cultural values, 2. - the quality of life, 3. - Canadian economy.

2.1.2.3 Third Level Objectives (more concrete goals)

a) Technical goals

18. standardization of technical equipments;

19. help to the introduction of technological innovations in Canada and abroad;

20. help in producing technological innovations.

b) Economic goals

The three preceeding goals are ultimately economic goals to which 3 more goals can be added:

17. obtainment of just and reasonable rates for all interests;

21. allocation of the radio-frequency spectrum;

22. interconnections of the networks and interactions between telecommunication modes within Canada and with the rest of the world.

c) Social goals

23. contribution to the flow and exchange of regional and cultural information;

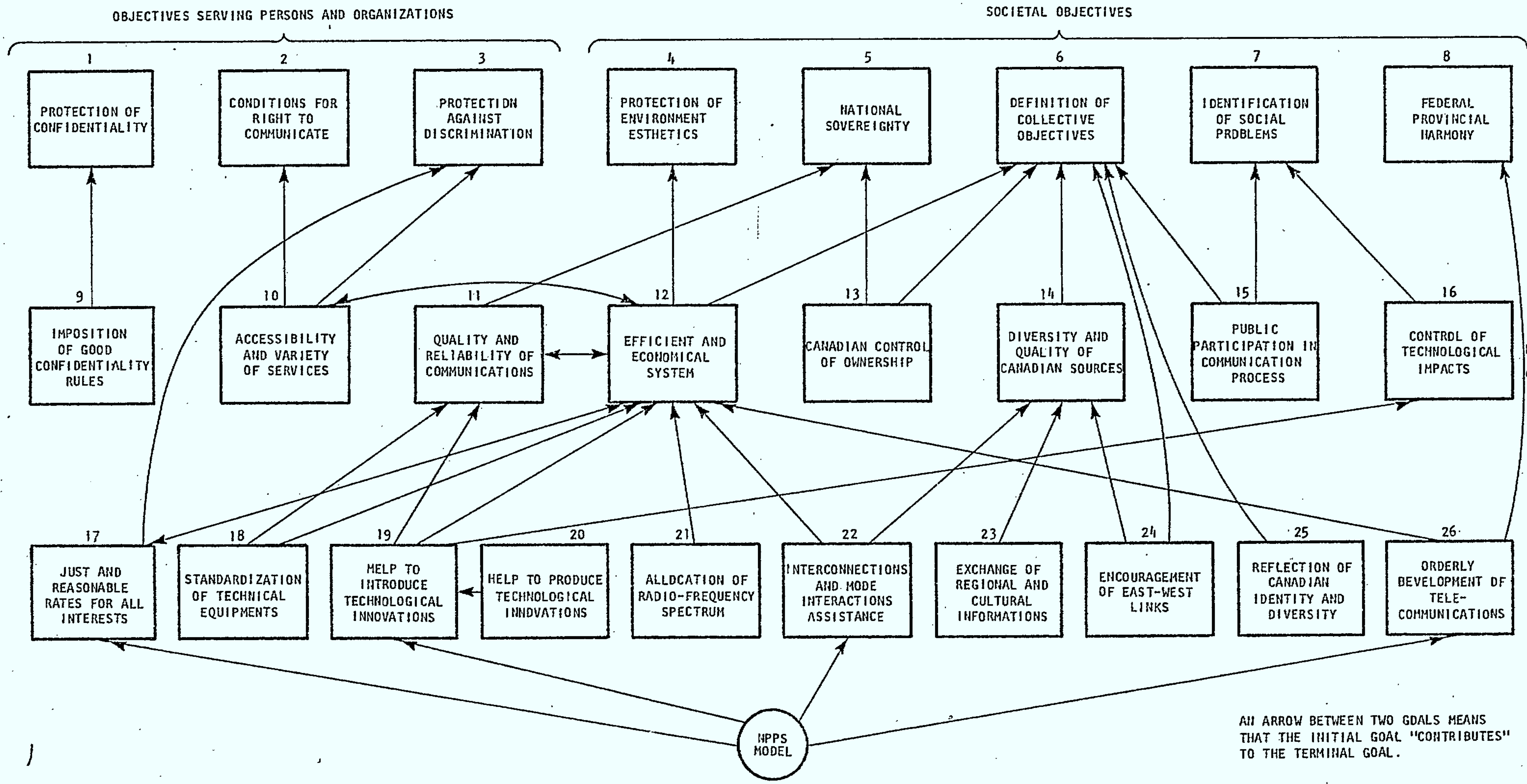
24. encouragement of east-west links;

25. reflection of Canadian identity and diversity of Canadian cultural and social values;

26. orderly development of telecommunications in Canada.

FIGURE 2-1

INTERRELATIONSHIPS BETWEEN OBJECTIVES IN COMMUNICATIONS POLICY



AN ARROW BETWEEN TWO GOALS MEANS THAT THE INITIAL GOAL "CONTRIBUTES" TO THE TERMINAL GOAL.

Comments on the Graph Presented in Figure 2.1

Most arrows are unidirectional, meaning that the objectives at the origin contribute to the one at the terminal end. A few arrows are bidirectional meaning a two way influence. For instance, 11. and 12. interact since better quality may often times be more costly, and efficiency means more resources for better quality or reliability; also 17. and 12. interact since equity and efficiency, apart from an equity based on the Pareto principle, are often antagonistic. It is therefore obvious that the graph is not a hierarchical tree. One will note that objective 12. is a kind of pole with many entering and leaving arrows. This may be due to professional bias; but we are inclined to believe it is a very central objective and that well-managed price systems can help to attain it without too much centralized information.

Finally, arrows start from the words "NPPS MODEL". They indicate the possible contribution of this model and, in following the paths toward the higher levels, a very primitive evaluation of its ultimate contribution can be seen.

2.1.3 Determination of a subsidy-free tariff structure

In sub-section 2.1.1 three criteria were mentioned for a definition of tariff structure to be efficient in the context of a decreasing cost industry. We will see in section 2.2.3.6 how game theory may help in the formulation of such a problem. Unfortunately the formulation presently available rests on the validity of rather stringent hypotheses with regards to demand elasticities and cross-elasticities and further work aiming at relaxing these constraints seems indicated. An iterative approach is for instance suggested in {26}.

We must also note that the existence of such a rate structure depends also upon the definition of the concept of cross-subsidy.

Sandberg {24}, for instance, gives an example where such a rate structure does not exist when using a definition of cross-subsidy different from the one used in the course of this project.

2.2 Theoretical and Critical Bases of Cross-subsidization

2.2.1 The Cross-subsidization Problem

One may initially define cross-subsidization in terms of an advantage drawn by a particular service in burdening other services. The occurence of cross-subsidization is due to market imperfections. Suppose a manager who subsidizes product A by selling product B at higher prices than required. If firms are free to enter into the market, another manufacturer will offer commodity B at lower prices than his competitor's selling prices and force him out of the industry or to produce the commodity more efficiently and sell it at prices eliminating cross-subsidy. The presence of cross-subsidization thus exhibits some restriction of entry into the industry.

We know that a firm subject to rate-of-return regulation may find it profitable to expand output beyond the level at which an unregulated firm would produce. The incentives facing a regulated utility may thus lead to economically inefficient production. Thus, a multiproduct regulated company is inclined to increase the output of each product and even to overproduce some commodities. This production of some services at an accounting loss is possible whenever the total revenue requirement can be satisfied by pricing the remaining services higher than the total average rate of return. Hence, a rate-of-return regulated utility is induced to practice cross-subsidization of non-compensatory competitive services with profits yielded by monopoly services.

The Federal Communications Commission asserts in Docket No 18128 that one fundamental question in regulation is "whether the rate levels for the (telecommunications) services will subject any person or class of persons to unjust or unreasonable preference or advantage to any person, class of persons or locality, or subject any person, class of persons or locality to any undue or unreasonable prejudice or disadvantage...". The motive of such an interest comes to light when the F.C.C. contends that "the public interest is not generally served by cross-subsidization of any one class of services by any other class of services, or by cross-subsidization of one sub-class of services by any other sub-class within the same class of services".

We also find the same concern in the Green Paper of the Government of Canada: "If the carriers are to be permitted to offer unregulated services, one of the essential saveguards is that the public interest be taken fully into account in any circumstances where there is a possibility that the subscribers to one service may bé subsidized by subscribers to another service, particularly if the latter are the general public".

The usual framework in which cross-subsidization is scrutinized is the social welfare theory. One need only to flip through the several dockets of the F.C.C. to find expressions like social optimum, marginal-cost pricing, consumer's surplus, efficiency, etc... which are the very language of welfare theory. For this reason, we shall, in a first section, develop the social welfare approach to cross-subsidization and indicate its merits and disadvantages. In a second section, we shall present a different reference scheme, the game theoretic approach. This approach overcomes many difficult issues of the welfare framework.

2.2.2 The Social Welfare Approach

2.2.2.1 Social Welfare and Marginal Analysis

Since the works of Pareto, the majority of economists of welfare theory based their reasoning upon the assumed value judgment that social welfare is increased if one person is better off while no one else is worse off. The cogency of this assumption rests on the fact that it avoids making interpersonal comparisons of utility. The implementation of Pareto's welfare hypothesis can be realized by the use of marginal analysis.

The marginalist approach is characterized by "the margin", i.e. the search for conditions prevailing when infinitesimal supplementary units of products or factors of production are considered. These marginal conditions are divided into two groups: those related to production, and those to exchange.

Marginal production conditions require that the factors of production be used in the most efficient manner. It must not be possible to produce more of any product without either a reduction of output of some other product or an increase of input of some factor. It must not be possible to use less of any factor without an increase of input for another factor or a reduction of output for some product. The necessary condition for such efficiency is to get a single marginal rate of transformation(1) and a single marginal rate of technical substitution(2) throughout the economy. The marginal conditions of exchange are those insuring that the products supplied in the most efficient way be in compliance with consumers' preference. It is thus necessary that all consumers have the same marginal rate of substitution.

All these necessary conditions mean that the factors used and goods produced be so managed that greater output is impossible without greater cost, and that they be so distributed that greater satisfaction is impossible for one person without less for another.

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(1) Between any two outputs.
(2) Between any two inputs.

It is also essential that the right goods and factors be selected. This requires that the common marginal rate of transformation be equal to the common marginal rate of substitution.

It should be noted that the definition of a social optimum implies the existence of many social optima. In fact, any situation in which the economy has full employment of all available resources in technically efficient production gives a Pareto optimum. The choice of any particular social optimum from among infinite possible alternatives necessitates a value judgment.

2.2.2.2 The Marginal-cost Pricing Principle

The conditions of optimality imply equality between the marginal rate of substitution and the marginal rate of transformation for any two goods. Assuming rationality of behaviour, consumers will equate the marginal rate of substitution to the price ratio while the marginal rate of transformation springs from the monetary system as a ratio of marginal costs.

Setting prices equal to marginal cost satisfies the conditions necessary to obtain a Paretian optimum. Hence, the marginal-cost pricing policy really stems from the theoretical apparatus of welfare economics (and all the hypotheses which support it) (1) and fulfills necessary conditions for improving welfare. The controversy surrounding marginal-cost pricing principles brings out two problems: is it possible to satisfy the marginal conditions and is it sufficient, if possible, to infer that welfare would be increased.

Marginal-cost pricing thus asserts that the most economically efficient allocation of society's resources is achieved when each individual unit of goods or services is priced at the actual marginal or incremental cost of producing that particular unit. Any practical application of this pricing rule necessitates a modification. For reasons of administrative and operating efficiency and for purposes of equity, it is not feasible to charge each customer a different price for essentially the same goods or services. Another limitation imposed by marginal-cost pricing theory lies in the fact that efficiency is attained only if all firms are profit maximizers, if each firm prices all services according to the marginal cost rule and if no total revenue requirement exists for the utility.

A serious subject of dissension about marginal-cost pricing is its application to public utility firms engaged both in monopolistic and competitive lines of production. Since marginal-cost pricing

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(1) Some of "failures": indivisibility, externalities.

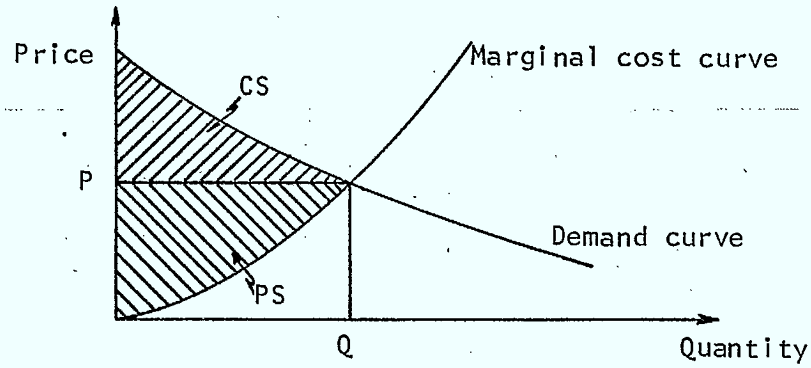
theory holds only if each firm maximizes its profits, it is important to note that in the case of a public utility enterprise, profit maximization can be contrary to public policy. Therefore, the marginal-cost rule may be economically inefficient for a regulated company.

The peak-load pricing problem is a subject closely related to the marginal-cost pricing principle. Many references are made to it in the literature on regulation. The relevance of this subject in the present report lies in the attention paid to demand forecasts when trying to establish tariff structures.

Consumers' surplus is the difference between what consumers would be willing to pay for any given quantity of a good and what they actually pay. It is represented by the shaded area below the demand curve and above the price line in figure 2-2. Producers' surplus is the difference between the revenues necessary to elicit any given quantity of a good and the revenues actually received. It is illustrated by the area above the marginal cost curve and below the price line.

FIGURE 2-2

Marginal Cost Pricing



The peak-load pricing problem is stated via a welfare funcion W defined as follows, with reference to Figure 2-2.

$$W = CS + PS = CS + (TR - TC)$$

where W = net welfare gain

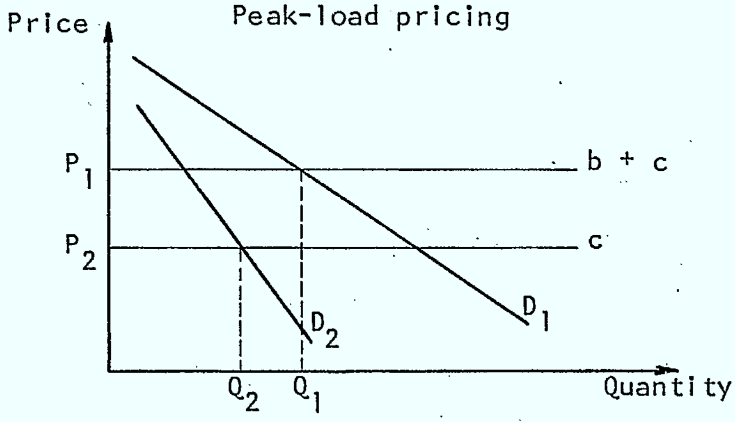
CS = consumers' surplus, PS = producers' surplus,

TR = total revenues, TC = total costs.

The formulation of the peak-load pricing problem illustrated in Figure 2-3 is the following:

FIGURE 2-3

Peak-load pricing



$$\underset{Q_1,\ Q_2,\ K}{\text{Maximize}}\ W = P_1Q_1 + S(P_1) + P_2Q_2 + S(P_2) - c(Q_1 + Q_2) - bK$$

subject to $Q_1 \leq K$

$Q_2 \leq K$

where

Q_1, Q_2 = quantities demanded in peak and off-peak periods respectively,

P_1, P_2 = prices in peak and off-peak periods respectively,

$S(P_1)$, $S(P_2)$ = consumers' surplus in peak and off-peak periods respectively when prices are P_1 and P_2,

c = marginal operating expense,

b = marginal capacity expense,

K = level of capacity.

In posing the problem in such a way, the following hypotheses were assumed (see for example {3}):

i) the firm has a production technology that permits no substitution between capital and variable inputs,

ii) the firm faces a load curve that is invariant with respect to time throughout each period for which price is fixed,

iii) demands are independent from period to period

The separability of the revenue terms was permitted by the hypothesis of no cross-elasticity of demand.

After the optimization is carried out by forming the Lagrangian expression, the first order conditions can be written as

$$P_1 = c + b \qquad (1)$$

$$P_2 = c \qquad (2)$$

$$Q_1 = K,\ Q_2 < K \qquad (3)$$

Condition 1 says that peak users must cover both their marginal operating expenses and the marginal capacity cost while condition 2 requires that off-peak users just pay their marginal operating expenses. Condition 3 states that capacity should be built so as to satisfy the entire peak demand if we hope to reach efficient pricing.

From this model we can then derive the following principles:

1) Efficient peak-load pricing requires that capacity be sufficient to serve all demand.

ii) Efficient peak-load pricing requires that all capacity costs be imputed to peak users, while off-peak users just pay their marginal operating expenses.

iii) Demand elasticities in peak and off-peak periods are irrelevant for efficient peak-load pricing.

We shall now describe some modifications to this model.

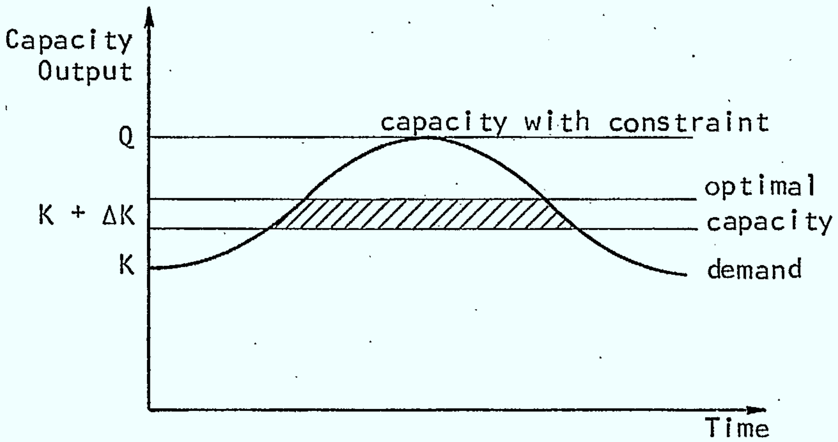
i) Dansby {8} recognized that the empirical data suggest that demands for non-storable public utility services fluctuate within periods having a fixed price. He supposed a demand which is not only a function of price, but also of the time within the period. The formulation is:

$$Q_1^t = D(P_1) + d(t)$$

Dansby obtained the result that if demand varies within periods of fixed price, then it is not efficient to satisfy all demand. It is intuitively clear that it may not be efficient to build capacity to meet high demands of very brief duration. But Dansby has shown that there always exists some excess demand as soon as there are two distinct values of d in a pricing interval. He also demonstrated that the capacity is optimal when the gain in surplus plus revenue generated from adding a small increment of capacity and satisfying some previously unserved demand (the shaded area on figure 2-4) is just equal to the incremental capacity cost.

FIGURE 2-4

Determination of optimal capacity



ii) Panzar {19} described a model where peak and off-peak users both contribute to capacity and operating costs. The larger the demand in any period, the larger the contribution to capacity costs. He assumes that the firm operates with a neoclassical production function (permitting substitution of variable and capacity costs) and defines a variable cost function V(Q,K) giving the non-capital expenditure needed to serve demand in a period, given a specific level K of available capital. He also assumes marginal variable costs were increasing, i.e. $V_{qq} > 0$,(*) with average variable cost always below marginal variable cost.

The formulation is the following (see figure 2-5):

$$\underset{Q_1,\, Q_2,\, K}{\text{Maximize}}\ W = P_1Q_1 + S(P_1) + P_2Q_2 + S(P_2) - V^1(Q_1,K) - V^2(Q_2,K) - bK$$

(*) Note that V_{qq} means the second derivative of V with respect to q.

The first order conditions are:

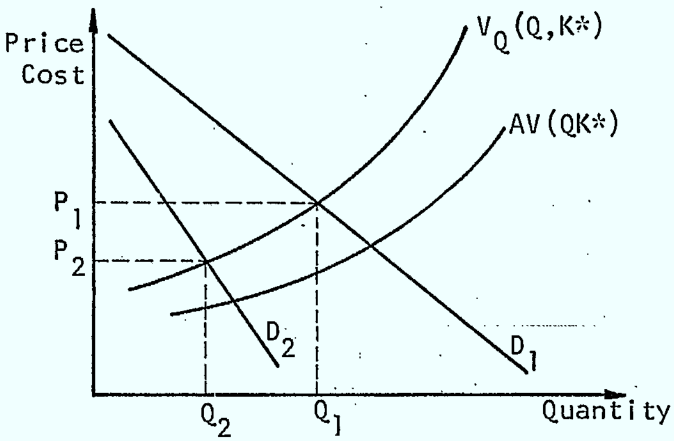
$$P_1 = V^1_{Q_1} \qquad (4)$$

$$P_2 = V^2_{Q_2} \qquad (5)$$

$$- V^1_K - V^2_K = b \qquad (6)$$

FIGURE 2-5

Marginal pricing - Panzar formulation



Conditions 4 and 5 say that the prices in the peak and off-peak periods are set equal to the marginal variable cost. Condition 6 states that the cost of hiring a marginal unit of capital must equal the reductions in variable costs in each period arising when the additional unit of capacity is installed.

Panzar's model shows that if one period's output is larger than that in another period, then its price should also be larger (this is a consequence of the rising average variable cost curve).

Another major result from this model is that all consumers for whom $V_{QQ} > 0$ should contribute to capacity costs. The larger is the demand in any period, the greater will be that period's contribution to capacity costs.

iii) Bailey and White {4} have set up a model to take demand elasticities into account in the search of efficient prices. They studied the case of a regulated enterprise subject to a profit constraint. The formulation is (see figure 2-6):

---

* Note that V_{qq} means the second derivative of V with respect to q.

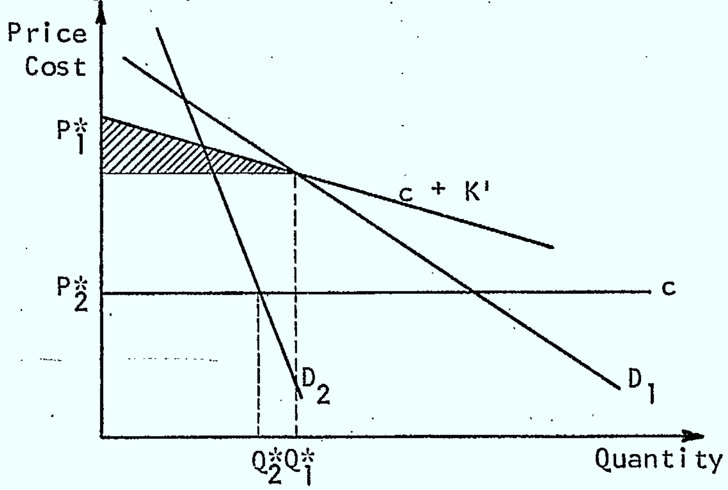
$$\underset{Q_1, Q_2}{\text{Maximize}} \; W = P_1Q_1 + S(P_1) + P_2Q_2 + S(P_2) - c(Q_1 + Q_2) - K(Q_1)$$

subject to $\Pi = P_1Q_1 + P_2Q_2 - c(Q_1 + Q_2) - K(Q_1) \geq 0$

where $K'(Q) > 0, K''(Q) < 0$ (there are increasing returns to scale in the provision of capacity, but constant marginal operating expenses). The constraint forces the firm to cover its total costs.

FIGURE 2-6

Marginal pricing - Bailey and White formulation



With increasing returns, if the firm sells the outputs Q_1^*, Q_2^* where prices equal marginal costs for each period, the firm will incur a loss given by the shaded area. Thus, to break even, the firm must set prices above marginal costs.

The solution given by Bailey and White to the maximization can be summarized by the two first order conditions:

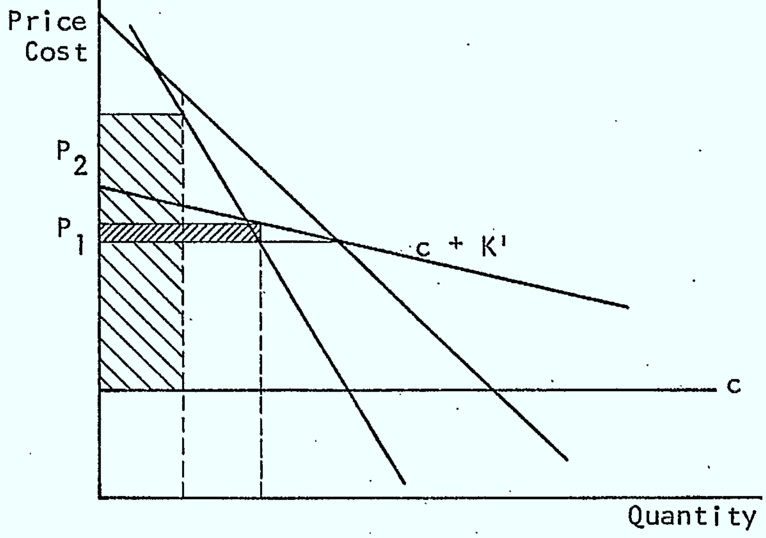
$$P_1 = \frac{(1 + \lambda)\ (b + K')}{1 + \lambda\ (1 - 1/e_1)}, \qquad P_2 = \frac{(1 + \lambda)\ b}{1 + \lambda(1 - 1/e_2)}$$

where e_1 and e_2 are the peak and off-peak price elasticities of demand at the solution point respectively, and λ is the Lagrange multiplier associated with the constraint.

Thus, if peak and off-peak consumers have different elasticities of demand, we might expect them to have prices that are marked up by different amounts over their respective marginal costs.

FIGURE 2-7

Effects of demand elasticities on marginal pricing



If, for example, the off-peak user has the more inelastic demand, then a larger amount of the capacity costs falls upon him, and he will pay more than the peak user (see figure 2-7).

2.2.2.4 Limits of the Welfare Approach

The welfare approach as a way to set up a pricing system (marginal-cost pricing principle) leads to serious difficulties when long-term average costs are diminishing, which is the case in the telecommunications industry. One way to get decreasing costs is the existence of increasing returns to scale which ones can result from indivisibility. In decreasing-cost industries, deficits would arise since revenues obtained from prices equal to marginal costs would be inadequate to meet total costs. The firm then will not be viable unless some subsidies come from elsewhere in the economy.

Various schemes of taxation and price discrimination have been proposed to cover the deficit. Taxes on any marginal transaction will disturb the optimum conditions, and so ideal taxes must accordingly be lump-sum taxes. While perfect discrimination is not possible in practice, several kinds of imperfect discrimination are in use in the form of multi-part pricing. Any way of financing the deficits resulting in decreasing-cost industries will always lead to cope with.

Hence complete, or even partial implementation of the marginal-cost pricing principle, has proven to be very controversial. It has often been said that public utilities should be priced at marginal-cost even though enterprises in the private sector do not. The basis of such argumentation was that it is better to fulfill the optimum conditions somewhere rather than nowhere. The refutation of this reasoning was brought about by the theory of second-best.

The basic argument of disaproval which is simple but devastating is: the marginal conditions of the Paretian optimum are invalid criteria for increasing welfare when they are not all simultaneously satisfied. The dictum that it is preferable to fulfill some optimum conditions rather than none, and the belief that it is better to depart from these conditions in a uniform extent rather than unevenly are both untrue (see {5} and {6}).

Another difficulty complicating the problem is the presence of joint production which signifies that somewhere one of the marginal rates is meaningless. All this relegates us to a situation of second-best that we would like to eschew in view of its complexity and non-malleability. It is the reason why we turn to a game theoretic approach to attack the problem of cross-subsidization rather than adopting a welfare point of view.

Game theory permits the avoidance of some concepts like consumers' surplus and producers' surplus which are difficult to deal with when we do not in fact have the relevant curves. Another interesting feature of game theory lies in the fact that it does not depend on any welfare function, although it refers implicitly to utilities through prices.

2.2.3 The Game Theoretic Approach

2.2.3.1 Introduction

Common costs and economies of scale render difficult the problem of pricing commodities or services produced by a publicly owned or regulated enterprise. In these circumstances, regulators and policy makers are on the horns of the dilemma which arises between efficiency and equity. In fact, common costs and increasing returns call forth a conflict between welfare maximization and subsidization. Economic efficiency argues strongly against subsidization. However, in many cases, governments think equity is more important than efficiency and decide to introduce subsidization.

One question of equity that is raised is the following: does a certain price structure for a multiservice firm unduly favor the consumers of one service at the expense of the consumers of another service, i.e. do the prices result in cross-subsidization? The thorny problem is to define what we mean exactly by cross-subsidy. If those who receive the benefits of an economic process differ from those who bear the costs, there is subsidization. But to calculate the extent of cross-subsidy, we need a much more precise definition. Many authors have proposed different ones leading to several tests. In this section, we will review the more important of these tests.

2.2.3.2 One-service Tests

All the tests quoted in this paragraph are made on a cost-revenue causation basis and examine only one service at a time. Let us consider a firm providing n services, the set of which is denoted $N = \{1, 2, ..., n\}$, with demands q_i and prices p_i. We define a cost function as following: if S is a subset of N, C(S) means the cost of supplying the services in S on a "stand-alone" basis. We make the assumption of economies of joint production, which can be stated:

(1) $C(S \cup T) \leq + C(S) \pm C(T)$

for all subsets S, T of N with $S \cap T = \phi$. This means that supplying services S and T jointly costs no more than supplying these services separately.

Similarly, R(S) represents the revenues gained by the production of this subset of services. We assume the absence of cross-elasticities, that is we assume that quantities demanded are function of their own prices only. We can then write.

$$R(S) = \sum_{i \in S} r_i = \sum_{i \in S} p_i \cdot q_i \quad (p_i).$$

We suppose that the firm's profits must be zero or, equivalently, that revenues just cover total costs, including cost of capital, i.e.

(2) $R(N) - C(N) = \Pi(N) = 0.$

Hazelwood {11} described a way for studying cross-subsidy: "Any ..... subscriber to the service should be able to obtain extra units of the service if he is willing to pay an amount equal to the cost of providing these units". This gave rise to the Incremental Cost Test (ICT), of which one interpretation can be expressed as follows: the firm's prices $(p_1, ..., p_n)$ are subsidy-free if and only if

(3) $R(i) \geq C(N) - C(N - i)$, for any i in N.

This means that the revenues from supplying the service i must at least equal the added costs necessary to provide this service.

Another variant has been proposed by D. Gillette which is called the Stand-Alone Test (SAT): a service of a multiservice enterprise subject to a profit-constraint does not yield a subsidy if the revenues from that service are no greater than the revenues required by a subsidiary firm supplying the same service and bearing the same profit constraint. This can be reformulated in the following terms:

(4) $R(i) \leq C(i)$, for all i in N.

This is a comparison between single revenues and stand-alone costs. Here, we have to point out that C(i) means the cost incurred in supplying individually the service i while R(i) represents the revenues yielded by the service i in the whole coalition of services.

Zajac {30} propounded yet another approach in saying that "no customer group should pay higher prices than it would pay by itself". The difficulty of this approach is in the meaning of the expression "by itself". Zajac proposed two scenarios. The Scenario 1 Test (S1T) stated that "the price for each service does not exceed the service's price if it were the only service offered". He found these minimal prices by setting the prices of all the services but one so high as completely to choke off their demands; it remains just one demand whose minimal price can be easily determined. In the Scenario 2 Test (S2T), no service disappears since it is taken over by an alternative supplier. The existence of many suppliers who act independently makes that scenario an inefficient one.

The last definition of cross-subsidization, we will recall, is that given by Faulhaber {10}: "If the provision of any commodity (or group of commodities) by a multicommodity enterprise subject to a profit constraint leads to prices for the other commodities no higher than they would pay by themselves, then the price structure is subsidy-free". We can conclude from that definition that subsidy-free prices permit to affirm that the supply of each commodity by the firm is "Pareto superior" to non-provision.

All the above definitions or tests look at each service individually; therefore, cross-subsidization involving groups of services may not be detected by these tests. Such tests would be sufficient if a single service were responsible for all costs or if common costs were joint among all services. But when costs are common to a proper subset of the whole set of services, we have to test that subset for cross-subsidy.

2.2.3.3 Generalized Tests

In this section, we will extend some of the preceding tests so that they take into consideration the several groups of services. Loehman and Whinston {17, 18}, in defining incremental costs in the case of joint production, have insisted on the fact that the incremental cost of a service depends upon what it is incremental on. Intuitively, we admit that the incremental cost of a service provided alone may be quite different from the incremental cost of the same service if provided along with other services.

The ICT can be generalized as follows: the firm's prices are subsidy-free if and only if

(5) $R(S) \geq C(N) - C(N-S)$, for all subsets S of N.

The extended form of the SAT is: the firm's prices are subsidy-free if and only if

(6) $R(S) \leq C(S)$, for all subsets S of N.

It is easy to show that the generalized ICT (GICT) is identical in meaning to the generalized SAT (GSAT), by remembering we assumed that the profit constraints must always be satisfied. In fact, if we substract equation 6 from equation 2 and with (6), we have

$$R(N) - R(N-S) \geq C(N) - C(N-S)$$

that is $R(N) - R(N-S) = R(S) \geq C(N) - C(N-S)$, and inversely

This signifies that in the case of a zero-profit constraint, GICT is equivalent to GSAT, i.e. the set of prices satisfying equation 5 (GICT) is identical with the set of prices satisfying equation 6 (GSAT). It is an interesting result since ICT was not equivalent to SAT (Faulhaber {9} has worked out an exemple showing this).

In the first section, we mentioned the interplay between subsidization and restricted entry into the market Cross-subsidy is only possible in a market whose entry is constrained. In a free-entry market, subsidization would be prevented by the threat of a new competitor's entry which could underprice. Faulhaber {10} thus proposed to give a more precise meaning to Zajac's proposition that "no customer group should pay higher prices than it would pay by itself". The new modified version would be: "no customer group should pay higher prices than it would pay if there were free entry into the market".

2.2.3.4 The firm as a cooperative game

Many authors introduced a game theoretic approach for analizing some economic problems. Apparently, the first author who utilized such an approach in the study of cross-subsidization was Faulhaber {9, 10}. The theory of n-person cooperative games yields an easy recognizable structure for the "game" of cross-subsidy.

Let us assume the same hypotheses as at the beginning of section 2.2.3. For given demand levels $q_1, \ldots, q_n$, we can view the consumer groups of the services $N = \{1, \ldots, n\}$ as the "players"; the cost function C(.) is the characteristic function, corresponding to the "value" of the game; the vector of revenues $(p_i q_i)$ is the "payoff" vector; finally, the players can form the "coalition" N whose cost is C(N) or many "subcoalitions" S, where $S \subset N$, with costs C(S).

Expression 1, the assumption of economics of joint production,

$$(1) \quad C(S \cup T) \leq C(S) + (T), \quad \text{for } S, T \subset N, S \cap T = \phi$$

which insures that there is a cost incentive toward cooperation, is the condition os subadditivity.

Equation 2, the zero-profit constraint,

$$(2) \quad R(N) - C(N) = \Pi(N) = 0,$$

represents the condition in game theory that the whole value of the game must be shared among the players by way of the payoff vector.

The set of "imputations" of a game is the set of revenues satisfying the zero-profit constraint and

$$(4) \quad R(i) \leq C(i), \text{ for all } i \text{ in } N.$$

These are the revenues which cover the total costs and for which each consumer group pays no more than its stand-alone cost. We note that the set of imputations is nothing other than the set of revenues passing the stand-alone test.

The "core" of a game is those imputations for which

$$(6) \quad R(S) \leq C(S), \text{ for all } S \subset N.$$

It is the set of revenues covering the total costs and for which no coalition of consumers can pay more than the stand-alone cost of that coalition. Here too we note that the core is nothing other than the set of revenues passing the generalized SAT, that is the generalized ICT.

The reference to the theory of games allows us to apply the results of that theory to the cross-subsidy problem. It is well known from the theory of n-person cooperative games that any game fulfilling the subadditivity condition has a non-empty set of imputations. This implies that, as long as we assume the existence of economies of joint production, there is at least one vector of revenues that passes SAT. This implication is interesting because the hypothesis necessary is not really severe since it corresponds to the notion of a natural monopoly. However, a serious problem crops up with the fact that not every game possesses a core. We will come back to this difficulty later on.

Faulhaber {9} has proved the following very interesting theorem: if we make the assumptions that:

(7) cross-elasticities are zero , i.e. $\delta q_i / \delta p_j = 0$, $i \neq j$,

(8) the prices are not "perverse", i.e. $\delta \Pi(S)/\delta p_i > 0$ for all i in S, then the core of the preceding game is identical to the set of subsidy-free prices.

The theorem signifies that if revenues are in the core of the game, i.e. pass GICT, and if conditions 7 and 8 are satisfied, then no consumer coalition could obtain lower prices. The global coalition N can block all other subcoalitions $S \subset N$. The usefulness of this result as a practical guide stems from the reduction of all cross-subsidization tests to a price test. In the zero-cross-elasticity case, to determine whether prices are subsidy-free or not, we need only calculate revenues and costs of the hypothetical coalition based on the initial fixed price structure and demand levels. There is no need for demand elasticities.

Nevertheless, when cross-elasticities are non-zero, we have to define a more complex game in which the value of the game is now profit and the price vector is the new payoff vector. The profits must be constrained to be non-negative. The core of the new game (the "price" game) is defined as follows: the price vector $p = (p_1, \ldots, p_n)$ belongs to the core if and and only if

a) $\Pi(N, p) = 0$,

b) there does not exist a subset $S = \{i_1. \ldots, i_S\}$ and a price vector $p^* = (p^*_{i_1}, \ldots, p^*_{i_S})$ such that

1) $\Pi(S, p^*) \geq 0$ for any feasible choice of p^*_k, $k \not\subset S$;

2) $p^*_j < p_j$ for all $j \in S$

For any price vector in the core of this game, no incentive exists to form other coalition than N to get lower prices. Faulhaber called such prices stable and gave the following interpretation: if the price vector of a regulated firm is stable, then allowing free entry would not induce any consumer group to desert the global coalition, i.e. the prices must be subsidy-free. He propounded another test for cross-subsidization, called the <u>Stability Test</u> (ST): a regulated firm's price vector is subsidy-free if and only if it belongs to the core of the preceding price game.

Under the hypotheses of zero-cross-elasticities, all the abovementioned tests (GICT, GSAT, SIT, ST) are equivalent. In presence of non-zero cross-elasticities however, these tests are no longer equivalent and the relative stringency of the three relevant tests depends upon the sign of the cross-partial derivatives of the demand relationships, i.e. if the services are substitutes or complements.

2.2.3.5 Imputation of Incremental Cost

Several methods exist, all arbitrary, to separate common costs. Loehman and Whinston {17, 18} have deduced, from a set of axioms, a meaningfull formula of social incremental cost which provides a way of allocating joint costs.

They postulate a service provided from a common facility and distributed to a given set of users. Each user is assumed to face fixed demand. The axioms which they are asked to consider for financing the facility are the following:

1) Charges for use of the facility must cover total costs.

2) Charges imputed to one user must be based only on the incremental costs caused by that user and not on the incremental costs of other users.

3) The charge is independent of the ordering of users, i.e. users with equal demands cause the same incremental costs and hence will pay the same charge.

4) The charge is homogeneous of degree one in the incremental costs, i.e. if all prices increase by a multiple, then the charge will also increase by the same multiple.

These axioms are intended to exhibit some equity in supplying a public service and illustrate an approach for making welfare choices without reference to a welfare function.

From these axioms and assuming that n users with fixed positive demands $K_1, \ldots, K_n$ agree to use a collective facility, Loehman and Whinston demonstrate that individual charges for use of the facility are given by the following formula:

$$F(i) = \sum_{\substack{G \subset n \\ i \in g}} \frac{(n - g)! \quad (g - 1)!}{n!} \{C(G) - C(G-i)\}$$

Where G are subsets of size g of the whole group of users N, and C(G) is the minimum cost in fulfilling demands K_G for the subgroup G. This result signifies that if the supposed users accept the fairness of these axioms and take them as a constitution, they must then also accept the cost-allocation formula F(i).

This pricing system has thus a touch of equity and efficiency since it imposes on each user the need to pay the social incremental costs due to his demands and covers all the costs of supplying a public service. It is also worth noting that the cost-allocation formula derived from the four axioms is the only one that can fulfill all these axioms if we further assume that the function F(i) is twice continuously differentiable for each i (see {18}).

Under the assumptions of perfect competition, the incremental-cost formula shares the costs in the same way as marginal-cost pricing does. However, in the presence of decreasing costs, unlike marginal-cost pricing, the incremental-cost scheme covers the full costs. Moreover, the existence of decreasing costs implies incentives to use and finance a collective facility since $F(i) \leq C(i)$, i.e. a person's charge in a joint facility is no greater than the charge if he had to provide the service by himself.

Those acquainted with game theory will have noticed that the incremental-cost formula is identical to the Shapley value of a game. In fact, Loehman and Whinston {18} have pointed out the parallell between the set of axioms taken by Shapley to derive his formula and the four axioms they used to produce their own scheme. There is a link between the incremental-cost formula and the game theoretic approach which is worth mentioning. If the Shapley value were in the core of the "price" game defined in section 2.2.3.4, this would imply that using the incremental-cost scheme for allocating costs, one could thus obtain subsidy-free prices. Unfortunately, such a result is not yet available and would even be impossible to prove since the core and the Shapley value are two distinct concepts. (Faulhaber {10} asserts that the Shapley value does not need to lie in the core). However, the incremental-cost formula prepresents a useful scheme for allocating costs in a fair manner whether it yields subsidy-free prices or not.

2.2.3.6 Game Theoric Determination of a Subsidy-free tariff Structure

Assume a multiservice firm offering $N = \{1, 2, \ldots n\}$ services and denote by S a subset (a coalition) of N. Denote also by C(S) the (minimal) cost of supplying the subset S. It will be assumed that the C(.) satisfies the following properties:

a) monotonicity: $T \subset S \rightarrow C(T) \leq C(S)$

b) sub-additivity: $C(S \cup T) \leq C(S) + C(T)$, $S, T \subset N$; $S \cap T = \phi$

The interpretation of a) is straightforward. The hypothesis b) means that it does not cost more to provide S and T jointly than to provide them separately.

Denote by R(S) the revenues derived from S. Of course R(.) is additive, i.e.

$$R(S) = \sum_{i \in S} r_i$$

where r_i represents revenues derived from providing the service i. Finally, if we denote by $\Pi(.) \triangleq R(.) - C(.)$, the profit function, it follows from the previous hypothesis about R(.) and C(.) that the profit function is super-additive:

$$\Pi(S \cup T) \geq \Pi(S) + \Pi(T), \quad S, T \subset N; \quad S \cap T \neq \phi$$

The main idea behind the determination of subsidy-free tariff structure is that the tariffs must be such that the gains coming from the economy of scale of providing <u>all</u> services at the same time be not destroyed. To achieve this, one must find some imputations $u = (u_1, \ldots u_n)$ which are in the <u>core</u> of the so-called game, where the core is defined in the following manner

$$\text{Core}(N, \Pi) = \{u \geq 0 \mid \sum_{i \in S} u_i \geq \Pi(S), \sum_{i=1}^{n} u_i = \Pi(N), S \subset N\}$$

So the core is defined as the set of imputations which satisfy the following two constraints: first, the imputation given to any coalition is not less than the profit the coalition can obtain by its own actions, second, the imputation for all the services must add to the maximal profit which the coalition of all services can win. If the core is not empty, it can be obtained by resolving the following standard linear programming problem:

$$\text{Min} \sum_{i=1}^{n} u_i$$

subject to

$$\sum_{i \in S} u_i \geq \Pi(S), \quad \forall S \subset N$$

$$u_i \geq 0 \quad i \in \{1, 2, \ldots n\}$$

Finally, knowing that $\Pi(S) = R(S) - C(S)$, one can then rewrite the first constraints as follows:

$$\sum_{i \in S} u_i \geq R(S) - C(S)$$

Now, if one assumes that the demand for each service is very inelastic to its respective price and that the cross-elasticities of the services are zero, one can redefine the core as follows

$$\text{Core}(N,C) = \{f_i \mid \sum_{i \in S} f_i \leq C(S), \quad f_i \leq r_i, \quad \sum_{i=1}^{n} f_i = C(N)\}$$

by defining $f_i \triangleq r_i - u_i$ and also $r_i \triangleq t_i q_i(t_i)$. It then follows that one can determine subsidy-free tariffs covering the total costs by setting

$$t'_i = f_i / q_i, \qquad i = 1, 2, \ldots n$$

because

$$\sum_{i \in S} t'_i q_i = \sum_{i \in S} f_i \leqq C(S), \; S \subseteq N$$

and

$$\sum_{i=1}^{n} t'_i q_i = \sum_{i=1}^{n} f_i = C(N).,$$

Of course, it is not an easy work to empirically determine this kind of tariff structure, taking into account the lot of information required to apply the previous approach. Moreover, it is evident that the validity of this approach is weakened by the fact that some hypothesis needed are too strong and, in fact, are certainly not true for certain aspects of the telecommunications industry. Hence, we suggest that efforts be directed to improve the theoretical basis and to acquire simultaneously the necessary data. Based on the discussions in the previous two sub-sections, we believe this to be a very worthwhile endeavour.

2.3 Definition of a Testing Methodology to be used with N.P.P.S.

2.3.1 Description of Suitable Tests for Experimental Purpose

Due to the cost of computing a test, only a few tests are proposed. The results will show the possibility of such computations, their costs and their relevance for policy purposes.

For the purpose of empirical calculations, four tests drawn from the game approach appear relevant for testing cross-subsidization. These tests were previously derived and explained and will be expressed here in their mathematical formulation only. A system (economy, carriers,...) producing and distributing a set $N = \{1, \ldots, i, \ldots, n\}$ of n services is supposed. R(.) and C(.) are respectively the revenue and the cost functions defined for a service or a group of services.

The incremental-cost test (ICT) is:

(1) $R(i) \geq C(N) - C(N-i)$, for any i in N.

The stand-alone test (SAT) is:

(2) $R(i) \leq C(i)$, for any i in N.

The generalized incremental-cost test (GICT) is:

(3) $R(S) \geq C(N) - C(N - S)$, for all subsets S of N

The generalized stand-alone test (GSAT) is:

(4) $R(S) \leq C(S)$, for all subsets S of N. It is worth remembering that if the carrier (subset) has to meet a zero-profit constraint, and if cross-elasticities are zero (hypothesis necessarily assumed when no "demand block" exists), then GICT is equivalent to GSAT. Although not a test but a useful "fair" cost-allocation formula, the following will also be needed:

(5) $$F(i) = \sum_{\substack{G \subset N \\ i \in G}} \frac{(n-g)! \quad (g-1)!}{n!} \{C(C) - C(G-i)\},$$

where the symbol meanings can be found in section 2.2.

2.3.2 Qualifications of the Present Cross-subsidy Tests

The theoretical tests proposed in the previous sections involve sets of economic agents and sets of costs. The N.P.P.S. model has been designed to show a fine level of disaggregation for traffic as well as for facility costing. Therefore, it is possible to regroup the demands of the economic agents in a meaningful way and to compute some of the several types of incremental costs used in the cross-subsidization tests. However several points must be stressed in order to show the kinds of interpretations and simplifications which are necessary to implement the theoretical tests.

2.3.2.1 Defining Meaningful Demand Subsets (or Services)

The theories postulate that any individual has perfect knowledge of the alternative subsets he can join and that he has communications and cooperating capacity. Also, any "subset" knows the cost of supplying its own demand. For the problem at hand, it is more realistic to postulate that intermediates (enterprises) regroup individual demands through their offering of services. The meaningful demand subsets are thus characterized by communication-streams involving: origin-destination, types of service, time of day, time of week. For instance a subset could be: "all public message traffic between 100 and 500 miles, from 8 to 18 hours in the business day". The computing cost of tests which present a combinatorial nature will force us to limit the number of demand subsets. Moreover, the regulating agencies already in place impose the regrouping in a limited number of services.

2.3.2.2 Hypothesis on Demand Reactions to Prices and Quality

The demand subsets having been defined, it should be noted that empirical demand functions - that is relationships from tariffs to message units for a given communication quality - are not known to us. For the time being, only requirements (in C.C.S. and number of circuits for public message or in number of circuits only for all other services) for a base period or projected requirements for future periods are available. Tests involving demand elasticities (direct and crossed) are thus beyond our scope. Some simulations using certain subjective price reaction coefficients could be envisaged. However, since the cross-subsidy theory of the game theoretic type is not sufficiently developed to include demand elasticities, these will not be considered in this study.

2.3.2.3 Data Availability for Costing the Services Required by the Demand Subsets.

The cost associated with a given demand subset is theoretically the minimum total cost to supply that subset alone. In real situations, the "initial state" must take care of the physical network and institutional organizations already in place. The "moves" of any subset of economic agents are not built from scratch. The actual network design will impose its structure of nodes and links and most of the incremental cost configurations. For stand-alone tests of relatively small demand subsets, the cost functions available in N.P.P.S. and network configurations will not be satisfactory, since the available network has not been designed for such demands.

2.3.2.4 A Posteriori Testing

From the initial state mentioned in 2.3.2.3, the tests will ask what incremental cost would have been saved if certain demands had been withdrawn. But with cost and technology forecasts, a priori testing could be designed.

2.3.2.5 Dynamic Aspect, Hidden and Explicit

Up to now, the N.P.P.S. model, except the Accounting Block, is a one period model. It computes results for one current year. The cross-subsidy theories above do not have time explictly as a variable. One can always think of a typical year or of a planning horizon during which decisions are made but the computing of costs is quite different in each case. The latter case requires facility expansion features linked to forecast demands. Some conceptual development has been done along that line (see {1}) but no software is available yet.

Even if the one period method is retained, the hidden dynamic characteristics are represented by the existence of excess capacities which can be justified by the indivisibility of installed facilities and other economies of scale combined with growing demands. Therefore, the cost of excess capacities should be either excluded or imputed to the cost of the tested services. Several solutions will be proposed in the next section.

2.3.3 Empirical Test Proposals

In the previous sub-section, it was seen that the networks initial states and the data availability were of paramount importance. Although embedded cost scenarios can be run with the use of the Aging, Indexing and Depreciation programs, the costing concept retained for present computations is the incurred cost based on the reproduction cost. Since our asset valuation functions are of the "fixed cost" type, among many others, two obvious possibilities are available for each existing network element. Average cost or marginal cost (link or node) from which the incremental cost of a "service" (a requirement subset for the entire switching and transmission networks) is computed. The tests will be executed with both concepts (the definition of which is recalled in Figure 4-2).

Four cross-subsidy tests will be presented:

i) Public Messages Versus Private Lines

This is a recurrent question. Private lines should at least pay for their incremental cost.

ii) Origin-destination pairs less than or equal to 1 000 miles apart versus pairs more than 1 000 miles apart.

It is possible that very long lines were favored. Time did not permit the regrouping of mileage bands used in tariff tables using a clustering device as: a new (larger) mileage band is created if the tariffs that form it do not deviate from the average tariff by more than a fixed amount. By such reasoning, long distance calls can be approximately clustered in equi-tariff bands: 0 to 180 miles, 181 to 540 miles, 541 to 1 200 miles, over 1 200 miles.

iii) Regional-adjacent-non-adjacent (including U.S.) traffics

Negociations between carriers distinguish these three types of traffic.

iv) Peak-hour Traffic Versus Non Peak Traffic

A thorny question in economies is whether peak users are subsidized or not by off-peak users. A possible formulation of such a question may be the following: We know that the traffic matrices are dimensioned with respect to the peak demand (rather a kind of average peak demand). If we are given the information that the average demand is about 70% of the peak demand, what is the incremental cost from that average to the full 100%? And does the incremental revenue cover this cost? Alternatively, any percentage down from peak demand could be costed.

v) Full Allocation Versus "Fair" Formula

This is not a cross-subsidy test, but a comparison between two cost allocations: Full allocation based on usage and "fair" formula based on the "fair" postulates enumerated in secion 2.2, a formula which is a weighted average of all possible incremental costs that a service can add when it joins all possible combinations of other services.

2.3.4 Treatment of Excess Capacities

The existence of excess capacities can be explained in several ways: simple planning error, redundancy for survivability, decreasing demand along a cycle or trend, indivisibility of optimal facilities associated with relatively small demands, growth reserve accumulated to protect against any large positive demand variation, growth reserve built to take advantage of economies of scale when the enterprise faces a sustained growing demand. In telecommunication networks, "protection" facilities and indivisibilities leading to economies of scale are frequently mentioned as explanations that we can associate with rapidly growing demand. In other words, in such a dynamic setting, growth reserves will benefit future as well as present customers. It is therefore important to impute at least part of the excess capacities to actual services.

Before imputing the excess capacities, it would be worthwhile to suggest a careful observation of the model network from the excess capacities point of view. Where are the excess capacities? If it is true that an inverse relationship exists between the carried requirements on a link and the excess capacity? Some correlation coefficient could be computed.

In devising several methods to take account of excess capacity when computing incremental costs, we will initially allocate all excess capacity between services, first according to the "fair" formula approach and second, proportionally to utilization. Secondly, keeping in mind that allocation may be made according to game theory or to usage, we will distinguish pure excess capacity and growth reserve by introducing growth rates for services. A last method of treating excess capacity will propose some trade-off between present and future.

The five methods depicted below all obey the same pattern. The incremental cost to be used in the incremental-cost test will be modified. It will be the sum of the previously calculated incremental cost and a term representing a certain part of the excess capacity. Thus, the incremental cost IC(i) for service i will be:

$$IC(i) = C(N) - C(N-i) + EC(i)$$

where EC(i) is the value of excess capacity imputed to service i. Of course, the expression obtained is not a "true" incremental cost, but an "exhaustive" incremental cost in the sense that excess capacity is taken into consideration in the procedure. The methods described obviously may be applied to any service.

In the first two methods, the principle is the same. We admit that the cost of excess capacity must be supported by present customers, whether excess capacity is a growth reserve for the future or an incorrect forecast of future demand. We thus run the model with a specified demand and obtain the magnitude of excess capacity. This excess capacity may be priced on a marginal basis or with average coefficients. This procedure permits the cost of excess capacity to be obtained by link or node.

METHOD A:

With the first method, we want to allocate the cost of excess capacity proportionally to the usage of the element. We then multiply the cost of excess capacity on each link by the relative usage of this link. To obtain the term EC(i) for service i, we proceed in the same way for all links.

This method puts the weight of the cost of excess capacity only on the shoulders of the present generation. Moreover, it is based on the actual relative utilization of the elements and this may be completely out of line with the future usages.

METHOD B

This method adopts the same approach as that employed in method A but allocates excess capacity according to a fairness and game theoretic view. We remember that the cost-allocation formula:

$$F(i) = \sum_{\substack{G \subset N \\ i \in G}} \frac{(n-g)!\ (g-1)!}{n!} \{ C(C) - C(G-i) \}$$

allows a fair separation of common costs. We thus can allocate the cost of excess capacity in proportion to these game theoretic coefficients. The term EC(i) would then equal:

$$EC(i) = \frac{F(i)}{\sum_{i \in N} F(i)} EC(N)$$

where EC(N) is the cost of the total excess capacity for all services over the whole network.

This method, as well, puts the burden for excess capacity, on the present generation only.

The next four methods try to make a distinction between growth reserve which tends to meet an expanding demand as accurately as possible, and what is called pure excess capacity which is the surplus of capacity over the growth reserve. It is probable that the notion of pure excess capacity will require a new interpretation when multiplexing costs are more thoroughly understood. For the moment, we shall accept this concept.

In this perspective, we shall choose a moving horizon of three years since it is admitted that facility installations are anticipated for a period of at least two to six years. Hence, we run the model successively for three years, increasing the demand for each service according to a growth rate particular to each service and determined exogenously. This rate might be of the multiplicative form with $d_i(t) = a_i (1 + r_i)^t$, where $d_i(t)$ represents the demand of service i after a lapse of t years, a_i is the demand of service i presently, and r_i is the growth rate.

The philosophy of these two methods lies in the hypothesis that only growth reserve must be imputed to customers and then allocated between the services. Pure excess capacity must be supported only by the carrier.

Two cases are possible after three years. First, all the links are saturated. In that case, all excess capacity is growth reserve, and all excess capacity has to be separated between services. This possibility reduces to the first two methods previously discussed. In the second case, there is excess capacity on some or all links after having run the model with demand $d_i(3)$. Pure excess capacity is therefore present in the network and must be borne by the carrier. We need only allocate growth reserve in order to execute the cross-subsidy test. This method, however, necessitates some expansion features in the model since after each year some links could be saturated and block future growth even if ample excess capacity still existed on most of the links.

METHOD C

This method allocates the growth reserve only and does it on the basis of present utilization. It represents an improvement on method B since the burden imposed on present consumers corresponds only to their probable growing demand.

METHOD C'

This method is very similar to method C but allocates growth reserve on the basis of future utilization.

METHOD D

This method espouses the same spirit as method C since it attempts to allocate only growth reserve. However, here the principle on which separation is grounded is the fair allocation formula. The new incremental cost would be:

$$IC(i) = C(N) - C(N-i) + \frac{F(i)}{\sum_{i \in N} F(i)} GR(N)$$

where the F(i)'s constitute a fair separation of costs incurred by the present level of utilization and GR(N) is the value of the total growth reserve to be allocated.

METHOD D'

This method is similar to method D but allocates growth reserve on the basis of future utilization.

Another approach which seems fruitful for a future study would be the following:

METHOD E

Here, a service is defined with respect to two criteria: one involving its physical characteristics, the other giving its time dimension. In that manner, the service of private lines produced now would be considered a distinct service from the one produced in one or two years. So, in the case of two services and three different periods of time, we should look at six separate services. The fruitfulness of this type of view lies in the fact that the growth reserve cost will be borne by the present generation and the next one. Each generation benefits from excess capacity and then shares the burden of that cost.

This approach would be treated in the framework of game theory. Many problems crop up, especially the definition of the possible coalitions of services, the weights that must be associated with the services, and the importance of ordering of such services. More thought should be devoted to the approach.

2.4 Comparison with other developments

2.4.1 Introduction

The central theme of the present report concerns the problem of cross-subsidization and, more specifically, to implement empirically and evaluate the limits of some proposed tests (see sub-section 2.3) in the domain of telecommunications. Without going into any detail, it can be said that all the tests compare certain revenues with certain costs for the various services supplied by the carriers. Most of the time, the revenue side causes no problem. However, due to the technological aspects of the telecommunications industry, the computation of incremental costs of the various services is not easy. It can be recalled that by incremental cost, one refers to the costs which are added to the system in response to the addition of specific increments of a service, be it an entire service category or a portion of the demand for a service category. Also, incremental cost can be viewed from three various time dimensions:

- embedded (historical) which refers to costs which have been added to the books of the company by reason of the service having been offered in the past. This embedded incremental cost should then reflect the impact of the services on all past facility additions which are still in plant;

- current, which refers to the cost of acquiring and/or installing units of new plant and equipment in the year during which the study is performed;

- prospective, which refers to the present value of the stream of costs which will be incurred in the future for the plant and facilities needed to provide the service under study. Then, the prospective incremental cost represents the present value of all future additions to facilities and equipment which will be required by the continued (or new) offering of a service.

Needless to say, to compute these costs empirically is not a minor challenge. In the N.P.P.S. Project, some attempts have already been made (and are continuously being improved). For a detailed description of the method, the incremental cost of the services is computed in the N.P.P.S. model, the reader is referred to previous reports and especially to the Interim and Final Reports on the Third Phase (respectively July 31st, 1975, and October 31st, 1975). Of course, some efforts have also been made elsewhere. It is the purpose of the present sub-section to compare the approach employed in the N.P.P.S. Project with the one suggested in the Costing Manual (Vol. 6) and modified in the Costing Manual Clarification of March 23, 1976, on one hand, and with the one presently being discussed in the U.S.A. i.e. the FCC Method 7 on the other hand.

2.4.2 Cost Inquiry Versus N.P.P.S. Project

In the Costing Manual, two general methods are used for computing incremental costs. These are:

i) The synthetic method: the specific facilities and equipment needed to serve a range of circuits or call volume (or other causative variables) are identified and priced out. This approach can be viewed as a causality approach.

ii) the analytical approach: this approach compares actual recorded costs, either investments or expenses, against recorded service volumes for a number of different operating divisions during the same year or for the entire system over a number of years. The approach involves multiple regression.

$$y = a + \sum_{i=1}^{n} b_i x_i + u$$

which allows for the possibility that a number of causative variables may influence costs. Here, u is a random variable which can take all other factors into account. Of course, "a" represents the fixed costs which are not assignable to individual services.

The core of the synthetic approach is the hypothesis that some facilities vary with some variable and that there exist some separation rules which permit the partition of the facilities among the various services. It is well-known that in the domain of telecommunications, fixed costs are relatively important and most of the costs are common (or fungible). By fixed costs, one means those costs which do not vary with changes in the volume of output. By common costs, one means those costs which are incurred in the furnishing of more than one service. It is of course very difficult to imagine a fixed cost which is not a common cost. Consequently, the fixed cost class can be seen as a subset of common costs. As is mentioned in the Costing Manual Clarification Item No 5, these costs are not allocable to services under the incremental costing approaches. Instead, it is suggested that these costs be identified as a separate, unassigned category of costs. Among these costs, the following categories are mentioned (with the corresponding comments):

i) Fixed overheads, like general administration, office buildings, etc...

ii) Fixed facilities which the carriers provide to their customers for access to the network, regardless of the use of that access. The Costing Manual adopts this concept of subscriber access as a discrete service which is distinct from both local and toll exchange services. Of course, some hypothesis concerning the investment in the networks must be made in view of regarding these costs as fixed or variable. For example, during recent decades of rapid increases of demand, most of these facilities expanded in general correspondance with the growth in telephone services.

iii) Growth reserve: many common facilities are installed with a capacity substantially larger than that which is required by current levels of service. Since this growth reserve is the capacity for future rather than present demand, its causation does not necessarily bear any relation to the existing mix of services. Ideally, growth reserve should be allocated among services according to their prospective increases in demand.

Finally, it is noted that economies of scale have as a consequence a total cost curve which is a non-linear function.

The relative importance of these costs are not specified.

In relation to the N.P.P.S. model, the following remarks can be made. It can immediately be noted that in the fore-mentioned N.P.P.S. reports, a number of comments concerning the Costing Manual approach vis-à-vis that employed in the N.P.P.S. project were made. Here, we would like to stress the differences with respect to the Costing Manual Clarifications and in particular with reference to the previous points.

i) As is noted in many places, the philosophy behind the incremental cost in the N.P.P.S. model is completely different from either the synthetic or the analytical approaches. In fact, no formal separation rule is retained because incremental cost is obtained as a difference between two "global" costs.

ii) Up to now, operating costs are a fraction of capital cost. No particular attention has been given to the variables which can "explain" variables like fixed overheads, etc...

iii) In the N.P.P.S. model, as in the Costing Clarification Item 8, two sets of planning units are used for costing the toll facilities. In the switching network, these units are CCS in the busy period and in the transmission network, it is the number of circuit miles.

iv) For an extensive discussion of the alternative treatments of excess capacity, the reader is referred to sub-section 2.3. However, it is important to note that this discussion is closely related to the possible definitions of what can be called the "present capacity" of the network, as extensively discussed in previous reports.

v) In previous reports on the Costing Inquiry, it was suggested that some linear functions be used. At present, the definite choice between linear and non-linear functions does not seem very clear (see Item 5 in the Costing Manual Clarification).

In the N.P.P.S. model, step functions were retained in order to capture economies of scale which are present in the domain of telecommunications. In relation to iv),it is evident that the choice of a particular concept for the capacity of the networks is more important in this context rather than in a linear one.

vi) For the present phase, investment functions are expressed in current (or reproduction) values. In the Costing Block there is an Aging, Indexing and Depreciation/Algorithm for transforming these values and obtaining their historical and/or prospective representations. However, from an economic point of view, retrospective incremental cost is meaningless, contrary to what seems to be the idea in the Costing Manual Clarification (especially Item 2). In particular, we disagree with the idea that under the retrospective incremental cost approach, facilities put in place in response to service growth in one year remain assigned to that service for costing purposes until they are retired, even if they are available for use by other services.

2.4.3 Federal Communications Commission and N.P.P.S.

In this section we will concentrate exclusively on a comparison between the approach taken in the N.P.P.S. model and Method 7 as it is described in "Fully Distributed Cost: Implementation Manual". Essentially, Method 7 is a fully distributed cost method which takes into account the current relative use of the system and its forecast relative use. In consequence, there must be a demand model (at least for MTS and WATS), a translation process which transforms message volumes in terms of facility requirements and a process for distributing all plant dollars comprising the interstate rate base of the Bell System among the various services. In a formal manner, the objective of Method 7 is to calculate, for each service under study, its operating cost and its rate base in such a way that if its revenue is known, the rate of return of the service is obtained. At the end of the process, a comparison among the various rates of return for the services will be made, and with overall rate of return, in order to test for possible cross-subsidization among the services. It can immediately be noted that the way adopted by FCC for evaluating some possible interfinancing among services can also be used in the N.P.P.S. approach.

Contrary to what can be found in the N.P.P.S. model where the growth of the various services is treated as a parameter, in Method 7 there must be a demand model since the forecasts of message volumes and Interexchange Channel (IXC) airline miles will serve as an input for the translation process as well as for the distribution of plant dollars among the services. Moreover these forecasts will be useful in distributing:

a) quantities of facilities associated with future growth to the account "Telephone Plant in Service";

b) growth facilities associated with "Telephone Plant under Construction";

c) growth facilities associated with "Property Held for Future Telephone Use".

For the pertinent details concerning the frequency and time horizon of forecasts, and the general description of forecasting techniques, etc..., the reader is referred to the afore-mentioned reference. However, some comments are in order. First, the method is prospectively-oriented, the forecast period being equal to three years. As a consequence, the present users will currently pay for future costs of the service. In the N.P.P.S. model, as the growth rate is exogenously given, any forecasting period of three years may be retained. However, a strong limitation in both models is the fact that no elasticity, and a fortiori no cross-elasticity, can be computed at the present. Consequently the reaction of demand to various modifications of the tariffs cannot be evaluated. Second, and common to both models, the central theme for forecasting demand is to try to allocate the excess capacity which is present in the networks. Of course excess capacity is strongly dependent on the capacity concept retained. Third, up to now, 16 service categories are considered in Method 7. At first glance, it seems that the definitions of a service is more flexible in the N.P.P.S. model because a service can even be defined as a particular origin-destination pair. Moreover, the level of aggregation of forecasts needed in Method 7 is not unique to interstate service. It seems to vary according to the forecast needs or the availability of historical data (i.e. Long Lines and/or Associated Companies Demands).

The second step in applying Method 7 is that the forecast demands for service categories must be translated in terms of facility requirements. At this level, the Manual is not too explicit except that the Inter-exchange Channel facility requirements are to be calculated on a point-to-point and mileage band basis and also that this transformation process must be either based on "traffic engineering methods" or done by means of "expansion factors" applied to existing network facility quantities. Also, the impacts on plant and facility requirements of busy hour traffic and non-coincident peaks must be taken into account. Finally, with respect to toll switching machine facilities, the appropriate demand units will be in some generally recognized traffic units (erlangs, ccs, call attempts per unit of time). It is interesting to note that the same approaches are present in the N.P.P.S. model. Traffic units are in ccs or circuits requirements, the dimensioning of both networks is done at peak periods, taking the non-coincident peak traffics into account, and finally facilities requirements are obtained first by using the economic ccs rule for dimensioning the switching network and second by using a linear programming approach for dimensioning (routing) the physical network. Finally, the allocation of the circuits is done on a point-to-point basis.

The last step in Method 7 is the association or distribution of plant dollars among service categories in order to obtain the operating cost as well as the rate base per service category. For so doing, two main categories of plant are distinguished: the non-fungibles and the fungibles. By definition, all non-fungible plant will be directly attributable to services based on plant dollars. For fungible plants, some distributive ratios must be calculated. Among the fungible plants, one must distinguish between the inter-exchange plant and non-interexchange plant since both are treated differently in Method 7. The fungible non-interexchange plant dollars per service will be based on experienced proportional use. For the fungible interexchange facilities non-available for future growth, plant dollars are allocated on the basis of the experienced distributions as determined by REDCAP (which is a circuit analysis of message network performed at periodic intervals). Finally, the distributive ratios for that category of plant but available for future growth will be determined by market forecasts. Note that the forecast message volumes for MTS and WATS and the airline miles for private line services have already been expressed in the common denominator of circuit route miles.

It can be recalled that in the N.P.P.S. model, there is no need for computing some distribution ratios as no formal separation procedure is required. In fact, since we have an operational model, it is possible to consider the network as a whole and consequently compute the incremental cost of, say a service, simply by comparing two objective functions. It can also be noted that up to now, the operating expenses are treated as a fraction of the investment costs. For both points, the reader is referred to the last N.P.P.S. reports for more explanations and details.

3

3. ADJUSTMENTS TO THE N.P.P.S. MODEL

In the third phase of the N.P.P.S. project a certain number of shortfalls of the model were pointed out. These had mainly to do with a lack of representativity for interregional and U.S. traffic. Other improvements dealt with the traffic generating module, regional revenue calculations and the dimensioning algorithms. All this streamlining was performed in the early part of this project in order to have a satisfactory tool for cross-subsidy testing.

In addition, while evaluating the results of early cross-subsidy tests, additional difficulties were discovered and led to a revised formulation of certain parts of the model. The three main interventions concerned the following domains:

- formulation of multiplexing problem
- estimation of toll related costs in local network
- derivation of incurred costs starting from reproduction costs

Since these revisions were incorporated progressively to the model during the testing period, the results presented in chapter 4 were obtained with a model in various stages of development and are not always strictly comparable. In this last event, sensitivity analyses have been performed to evaluate the impact of a missing improvement.

3.1 Network Expansion

3.1.1 General

After having experimented with the Maritimes and Bell simulations in the third phase of the N.P.P.S. project, it was recognized that the N.P.P.S. model would yield meaningful results only if a substantial portion of the toll traffic was represented. While regional traffic was fairly adequately considered in both experiments, it was not the case for interregional traffic. In the Bell experiment, for instance, there were 46 Bell demand points which could generate interregional traffic with only 14 non-Bell demand points. Consequently, it was decided to increase the maximum number of demand points acceptable by the model so as to have sufficient interregional demand generating pairs. In order to keep the cost of running the model within resonable limits however, the size of the expanded model was limited at 100 demand points.

In the following paragraphs we shall review:

- the required software adjustments
- the additions to the existing data base
- the comparative results of a 96-node and a 60-node simulations

3.1.2 Software Adjustments

In order to accommodate up to 100 demand points, all programmes were redimensioned. In addition, a few modules had to be redesigned, a simple redimensioning leading to excessive memory requirements or unacceptable computing times.

Major efforts were directed towards the improvement of the transmission facilities allocation module (CIRRES) which was already very close to maximum memory capacity. Satisfactory results were obtained by reducing the problem size prior to allocation and expanding the solution of the reduced problem to original problem size after allocation. Problem reduction is achieved in two phases, namely:

i) the elimination of dangling nodes

ii) the concatenation of chains which do not pass through demand points or junction transmission points.

The principles behind both phases are clearly understandable from the example of figure 3.1.

Implementation of this procedure in N.P.P.S. has shown problem reductions of over 40% and similar savings in computer time.

3.1.3 Extended Data Base

3.1.3.1 Switching Nodes (demand points)

Thirty six new nodes were added to the data base for a total of 96 nodes as follows:

- 3 level 1 nodes
- 7 level 2 nodes
- 25 level 3 nodes
- 61 level 4 nodes

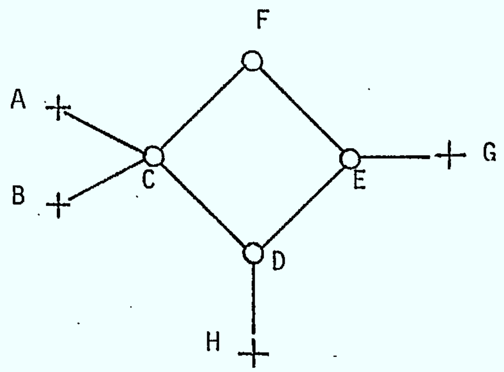
Total: 96 nodes

It must be pointed out that all level 3 nodes but one are included. The only one left out is Abbotsford, B.C. which has too small a population to generate significant traffic. The added new nodes are listed in Table 3.1 by province and by level, altogether with four level 4 nodes which were excluded from Ontario and Quebec.

FIGURE 3-1

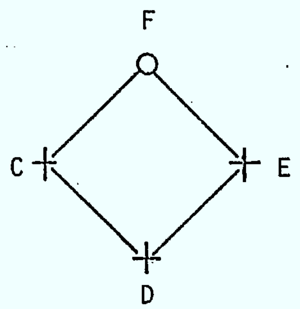
Example of problem reduction prior to allocation

a) Original Problem



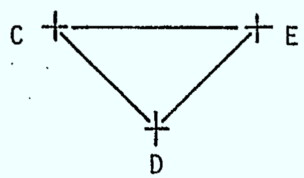
4 demand points (+)
4 transmission points (0)
8 links

b) Problem after Elimination of Dangling Nodes



3 demand points (+)
1 transmission point (0)
4 links

c) Problem after Concatenation of Chains



3 demand points (+)
0 transmission point (0)
3 links

The population of a few nodes was artificially increased to take into consideration significant neighbouring cities. Those artificially increased nodes and their components are listed in Table 3-2. The total population represented in this extended network is of 14.3 M or 66% of the total Canadian population (see Table 3.3) and represents an increase of approximately 12% over the 60-node Bell experiment.

3.1.3.2 Switching Links

The final tree configuration was taken directly from DOC documentation and appears in Figure 3.2. Data on trunking is only partially available. Namely, the number of trunks going to level 4 nodes is not recorded on a node by node basis but rather for groups of level-4 nodes homing on the same higher level node. For instance, it is known that there are 2 362 final trunks linking Toronto to level-4 nodes in its homing section but the actual node-by-node breakdown is not available. The same holds for high usage groups. In this particular instance, the existence of a H.U. was tied to a certain traffic volume and it was assumed that any level-4 node was linked to another node if there was enough traffic between the two points to justify

- 5 trunks if the nodes were in the same code area
- 7 trunks otherwise

Once the potential links were defined and the corresponding overflow plan established, the network was dimensioned. The obtained dimensions were then used to prorate aggregate figures given in the available documentation. The numbers of links obtained by this procedure are shown below and compared to corresponding figures in the 60-node network.

	60 nodes	96 nodes
Full groups:	1	6
High usage groups:	178	271
Final groups:	60	96
Total:	239	373

3.1.3.3 Transmission Network

In order to link the new nodes to the network the following number of facilities were added:

- 38 transmission nodes
- 57 transmission links

TABLE 3-1

Network Modifications
(from 60 nodes to 96 nodes)

Province	Level 3 nodes Added	Level 4 nodes Added	Nodes Deleted
Newfoundland (+1)	Gander		
Nova Scotia (+4)		Truro New Glasgow Kentville Sydney	
New Brunswick (+4)		Edmonton Moncton Fredericton Woodstock	
P.E.I. (+1)		Charlottetown	
Quebec (+1)		Alma Sorel Valleyfield	Ste-Agathe Chibougameau
Ontario (+7)		Belleville Brockville Chatham Fort Erie Guelph St-Thomas Brampton Timmins Welland	Parry Sound Cochrane
Manitoba (+2)	Dauphin Brandon		
Saskatchewan (+4)		Swift Current Moose Jaw Prince Albert North Battleford	
Alberta (+5)		Grande Prairie Vegreville Medecine Hat Lethbridge Red Deer	
British Columbia (+7)	Terrace Prince Georges New Westminster Nanaimo Campbell River	Dawson Creek Victoria	

FIGURE 3-2

Final Tree
Western Canada

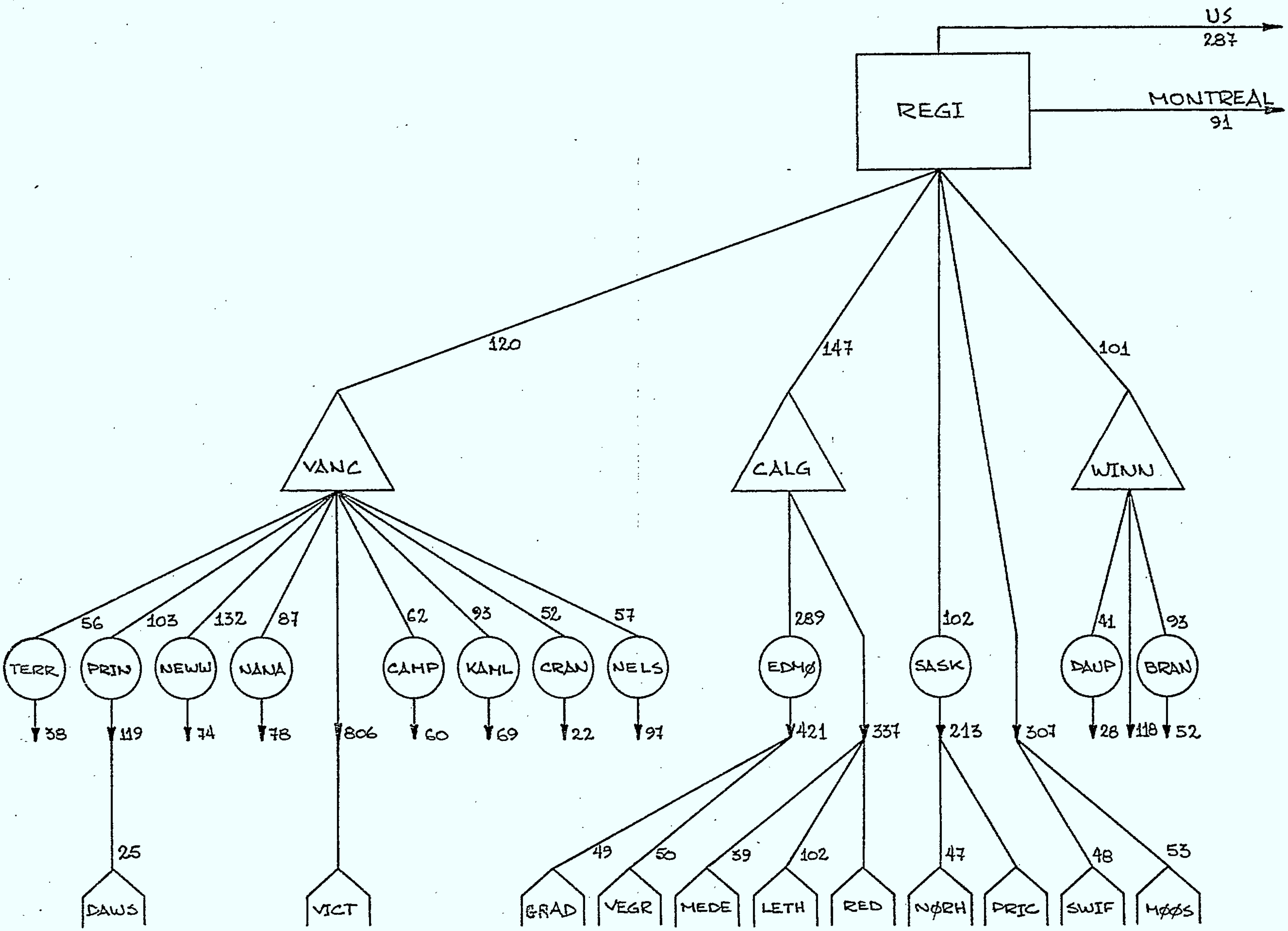
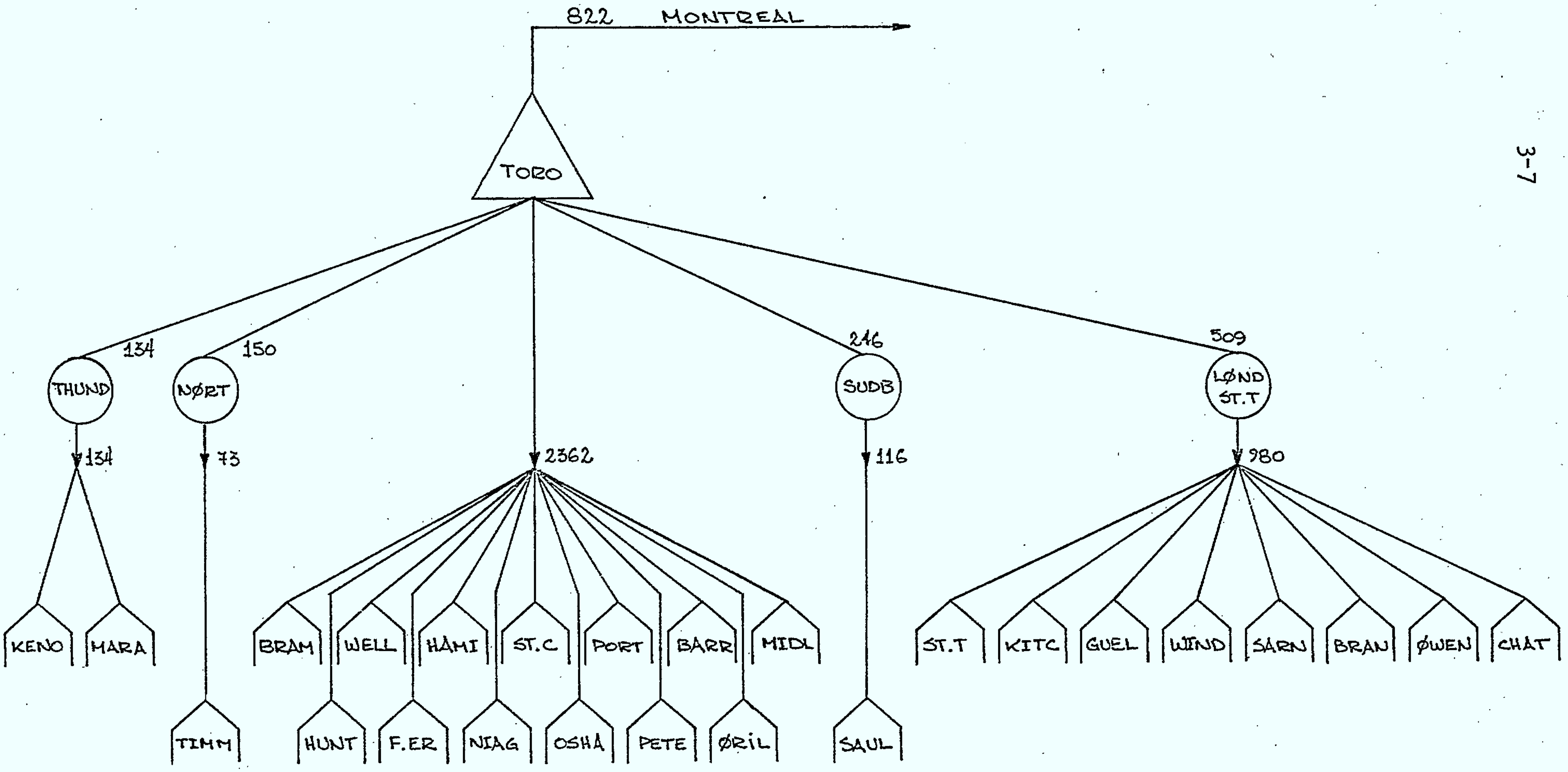


FIGURE 3-2
(cont'd)

Final Tree
Central Canada



3-7

FIGURE 3-2

(cont'd)

Final Tree
Eastern Canada

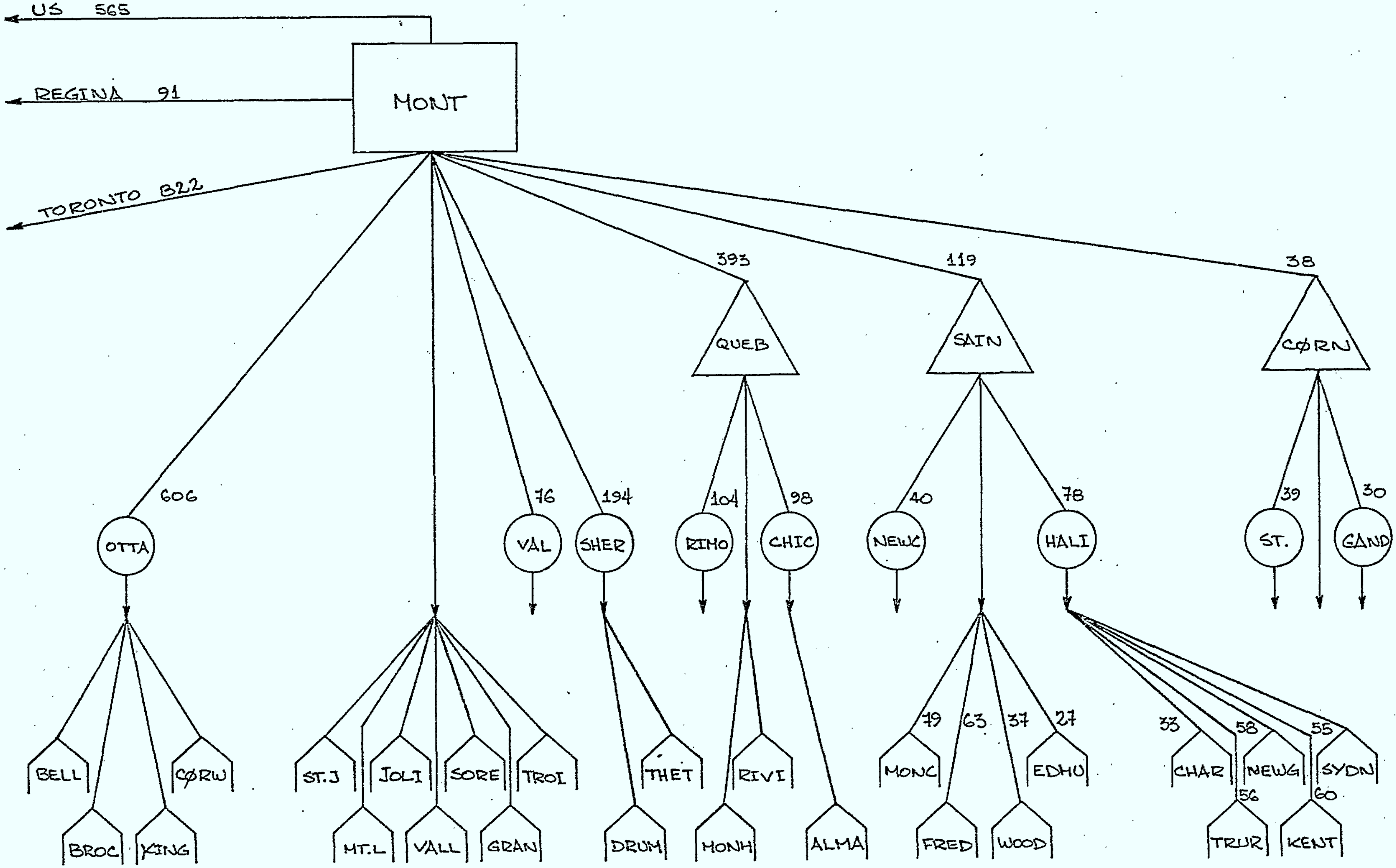


TABLE 3-2

Artificially increased switching nodes

	Nodes		Added population
Newfoundland	Corner Brook	+	Stephenville
New Brunswick	New Castle	+	Bathurst
Quebec	Rimouski	+	Mont-Joli, Matane
	Sherbrooke	+	Magog
	Thetford Mines	+	Mégantic
	Drummondville	+	Victoriaville
	Trois-Rivières	+	Shawinigan + Grand-Mère
	Granby	+	Cowansville + St-Jean
	Sorel	+	St-Hyacinthe
	Valleyfield	+	Ste-Anne de Bellevue
	Mont-Laurier	+	Maniwaki
	Saint Jérôme	+	Sainte-Agathe + Lachute
	Val d'Or	+	Rouyn-Noranda
Ontario	Kitchener	+	Waterloo
	Barry	+	New Market + Aurora
	Peterborough	+	Lindsay
	Port Hope	+	Cobourg
	Oshawa	+	Markham
	Huntsville	+	Parry Sound + Gravenhurst + Bracebridge
	Timmins	+	Cockrane
	Marathon	+	Geraldton
	Kenora	+	Fort-Frances
Alberta	Vegreville	+	St-Paul + Camrose
British Columbia	Terrace	+	Prince Rupert

N.B. St. Thomas was subtracted from London
Markham " " " Toronto
Aurora " " " Toronto
Brampton " " " Toronto
Newmarket " " " Toronto
Ste-Anne de Bellevue " " Montreal

TABLE 3-3

Population by province

	(1) Canadian Yearbook 1971	(2) NPPS	(3) (2)/(1) %
Newfoundland	522,100	175,770	34 %
Nova Scotia	789,000	342,893	44 %
New Brunswick	634,600	256,189	40 %
P.E.I.	111,600	25,253	23 %
Quebec	6,027,800	4,313,880	72 %
Ontario	7,703,100	5,910,652	77 %
Manitoba	988,200	580,303	59 %
Saskatchewan	926,200	355,614	38 %
Alberta	1,627,900	1,026,289	63 %
British Columbia	2,184,600	1,378,545	63 %
Total:	21,515,100	14,365,388	66 %

For a total of:

- 219 transmission nodes
- 239 transmission links

in the data base.

The up-dated transmission network consists of 55.3 millions circuit-miles which can be broken down as follows:

Protection	14.3
T.V. use (pre-assigned):	6.3
Available for other services:	34.7
Total:	55.3

3.1.4 Results

3.1.4.1 Costing Block

A good idea of the network expansion can be obtained by comparing the results of the costing block both for the 60 and 96 node network. Table 3-4 presents the general increase in terms of investment and it can be seen that the plant represented in the model has increased by 25%. The major carrier increase is for B.C.T. for which the number of nodes considered has passed from 3 to 11.

TABLE 3-4

Costing Block Results

Company assets ($ '000)	60 nodes	96 nodes	% increase
B.C.T.	45,937	107,848	135
A.G.T.	75,230	94,929	26
SASK	44,720	57,316	28
MAN. T	31,193	41,232	32
BELL	565,093	652,351	15
N.B.T.	38,726	43,927	13
M.T.T.	33,950	44,671	32
TOTAL:	834,843	1,042,276	25

3.1.4.2 Operating Block

- Demand

The traffic estimation program has been executed using 7%, 10% and 12% peak hour to total day ratios respectively for regional, adjacent and non-adjacent traffic. (Further explanations regarding this procedure are given in section 3.4.1)

The total canadian population in the network has been increased by 12% by the expansion of the network. Consequently, the annual traffic increased by 35%. The Table shown below illustrates the difference, in terms of yearly messages, between the 60-node and the 96-node networks.

1971 Long Distance Calls
Bell Canada
(millions of messages)

	Regional	Adjacent	Non-adjacent
NPPS - 60 nodes	210.	1.9	3.4
NPPS - 97 nodes	291.	2.8	4.2
Benchmark	315.	3.5	6.4

- Switching

The usage of the switching network after the expansion could be summarized as follows:

- Less regional traffic due to the use of 7% peak to total day ratio (instead of 10% in previous simulations).
- Increased number of traffic components carried by the links.
- More traffic between non-adjacent points due to the use of 12% peak to total day ratio. In fact, considering the high usage HALT-REGI, we notice that the primary traffic increased from 70 C.C.S. to 93 C.C.S. and this is attributable to the use of 12% ratio and the increase of traffic components on the link.
- In general, the probability of loss on the final links is negligible (less than 1%).

Table 3-5 illustrates the above comments.

- Transmission

  The allocation model was run using circuit-miles as objective function coefficients. Comparison of the 60 and 96-node experiments is shown in Table 3-6.

3.1.4.3 Sharing Block

The most meaningful points of comparison between the two simulations, however, reside in the results of the sharing block. They are presented on Tables 3-7 and 3-8. Salient results of these printouts are shown in Table 3-9 and compared to available benchmarks when available.

The most interesting improvements are for interregional traffic where incurred costs have increased by 43% and revenues by 28%.

The decrease in U.S. traffic incurred costs was to be expected since the same U.S. traffic was used for both simulations, its relative importance becoming substantially smaller in the 96-node experiment.

TABLE 3-5

Usage of switching network
for certain selected links

	Installed circ.	Type	Prob. of loss	CCS offered	CCS carried	Circ.requi.
(60 nodes)						
HAMI - TORO	271	Final	100 %	12,181	9,756	382
TORO - HAMI	271	"	100 %	11,961	9,756	375
OTTA - MONT	303	"	0 %	3,145	3,145	111
MONT - OTTA	303	"	0 %	3,416	3,416	119
MONT - REGI	43	"	0 %	597	324	7
REGI - MONT	43	"	0 %	96	96	8
MONT - TORO	411	"	0 %	6,882	6,882	224
TORO - MONT	411	"	0 %	7,494	7,494	242
HALI - REGI	2	H.U.	39 %	71	43	4
REGI - HALI	2	H.U.	1 %	4	4	2
VANC - USA	167	H.U.	0 %	3,346	3,346	90
VANC - MONT	15	H.U.	11 %	461	410	16
(96 nodes)						
HAMI - TORO	295	Final	0 %	8,678	8,674	278
TORO - HAMI	295	"	0 %	8,613	8,613	276
OTTA - MONT	303	"	0 %	2,881	2,881	103
MONT - OTTA	303	"	0 %	3,176	3,176	112
MONT - REGI	45	"	0 %	272	272	16
REGI - MONT	45	"	0 %	495	495	24
MONT - TORO	411	"	0 %	6,858	6,858	223
TORO - MONT	411	"	0 %	6,103	6,103	200
HALI - REGI	2	H.U.	48 %	93	48	5
REGI -HALI	2	H.U.	15 %	29	25	3
VANC - USA	167	"	3 %	3,346	3,260	132
VANC - MONT	15	"	45 %	904	501	28

TABLE 3-6

Transmission network allocation

Circuit miles	60 nodes	96 nodes	% increase
Total available for allocation(1)	27,842,816	34,785,280	25
In use	4 651,712	5,005,526	8

(1) After removal of protection and video dedicated channels.

TABLE 3-7

60-node Network

RESULTS OF COMMONWEALTH SHARING SCHEME
FOR PUBLIC MESSAGE SERVICE

NOTE: ALL DOLLAR FIGURES ARE IN THOUSANDS

RESULTS FOR ADJACENT PARTNERS:

	BCT	AGT	SASK	MANT	BCAN	NBT	MTT	TOT
INCURRED COSTS:	2036.	3104.	1828.	1796.	6838.	2400.	680.	18681.
POST SET. REVENUE:	4194.	5457.	2728.	5976.	9573.	1457.	478.	29863.
PRE SET. REVENUE:	3453.	5955.	3014.	6959.	8235.	1969.	278.	29863.
ASSIGNED PLANT INV.:	7736.	14838.	6841.	9022.	21471.	9438.	2668.	72014.
REV/ASSETS:	.54	.37	.40	.66	.45	.15	.18	.41
REV/COSTS:	2.06	1.76	1.49	3.33	1.40	.61	.70	1.60

INCURRED COSTS WHEN ONE PARTNER IS USA:

	BCT	AGT	SASK	MANT	BCAN	NBT	MTT	TOT
INCURRED COSTS:	4331.	2286.	3505.	1340.	27009.	2509.	1442.	42422.
PRE SET. REVENUE:	0.	0.	0.	0.	0.	0.	0.	0.
POST SET. REVENUE:	55.	-1055.	52.	13.	2816.	-735.	700.	1845.
ASSIGNED PLANT INV.:	16533.	10981.	13109.	6642.	84980.	9831.	5597.	147671.
REV/ASSETS:	.00	-.10	.00	.00	.03	-.07	.13	.00
REV/COSTS:	.01	-.46	.01	.01	.10	-.29	.49	.00

RESULTS FOR NON-ADJACENT PARTNERS:

	BCT	AGT	SASK	MANT	BCAN	NBT	MTT	TOT
INCURRED COSTS:	2648.	4580.	5012.	2423.	11263.	1480.	1321.	28727.
POST SET. REVENUE:	5254.	8323.	4804.	3752.	17027.	1323.	2052.	42535.
PRE SET. REVENUE:	8332.	8992.	2303.	3010.	16317.	100.	3483.	42536.
ASSIGNED PLANT INV.:	10045.	21789.	18606.	12010.	35293.	5735.	5180.	108659.
REV/ASSETS:	.52	.38	.26	.31	.48	.23	.40	.39
REV/COSTS:	1.98	1.82	.96	1.55	1.51	.89	1.55	1.48

RESULTS FOR REGIONAL TRAFFIC:

	BCT	AGT	SASK	MANT	BCAN	NBT	MTT	TOT
INCURRED COSTS:	1804.	1563.	50.	0.	121762.	223.	0.	125402.
ASSIGNED PLANT INV.:	6909.	7553.	188.	0.	386680.	894.	0.	402224.

SUBTOTAL FOR ADJ. AND NON-ADJ.

	BCT	AGT	SASK	MANT	BCAN	NBT	MTT	TOT
INCURRED COSTS:	4683.	7684.	6840.	4219.	18101.	3880.	2001.	47408.
POST SET. REVENUE:	9448.	13779.	7532.	9729.	26600.	2781.	2530.	72398.
PRE SET. REVENUE:	11786.	14946.	5316.	9969.	24552.	2070.	3761.	72400.
ASSIGNED PLANT INV.:	17780.	36627.	25448.	21032.	56764.	15173.	7848.	180673.
REV/ASSETS:	.53	.38	.30	.46	.47	.18	.32	.40
REV/COSTS:	2.02	1.79	1.10	2.31	1.47	.72	1.26	1.53

% OF ASS. PLANT INV. BY SERVICE

	BCT	AGT	SASK	MANT	BCAN	NBT	MTT	TOT
ADJACENT:	16.8	19.7	15.3	28.9	3.8	24.4	7.9	8.6
US:	36.0	14.6	29.3	21.3	15.0	25.4	16.5	17.7
NON-ADJACENT:	21.9	29.0	41.6	38.5	6.2	14.8	15.3	13.0
REGIONAL:	15.0	10.0	.4	.0	68.4	2.3	.0	48.2
MISCELLANEOUS:	.1	.2	.2	.3	2.2	9.5	37.3	3.5
TELEVISION:	9.0	7.0	6.4	5.7	3.2	3.3	1.3	4.1
UNASSIGNED:	1.2	19.6	6.7	5.2	1.1	20.3	21.8	5.0

TABLE 3-8

96-node Network

RESULTS OF COMMONWEALTH SHARING SCHEME FOR PUBLIC MESSAGE SERVICE

NOTE: ALL DOLLAR FIGURES ARE IN THOUSANDS

RESULTS FOR ADJACENT PARTNERS:

	BCT	AGT	SASK	MANT	BCAN	NBT	MTT	TOT
INCURRED COSTS:	3266.	5746.	3338.	2742.	7699.	5361.	1763.	29915.
POST SET. REVENUE:	4618.	6856.	3669.	6906.	11452.	5031.	1270.	39802.
PRE SET. REVENUE:	4235.	7193.	3702.	7678.	11040.	5059.	896.	39803.
ASSIGNED PLANT INV.:	12427.	27585.	12613.	13826.	24189.	21286.	6959.	118885.
REV/ASSETS:	.37	.25	.29	.50	.47	.24	.18	.33
REV/COSTS:	1.41	1.19	1.10	2.52	1.49	.94	.72	1.33

INCURRED COSTS WHEN ONE PARTNER IS USA:

	BCT	AGT	SASK	MANT	BCAN	NBT	MTT	TOT
INCURRED COSTS:	3083.	1491.	3070.	1244.	21201.	1616.	721.	32427.
PRE SET. REVENUE:	0.	0.	0.	0.	0.	0.	0.	0.
POST SET. REVENUE:	183.	-941.	164.	-29.	2165.	-682.	201.	1060.
ASSIGNED PLANT INV.:	11775.	7173.	11465.	6168.	66854.	6318.	2798.	112552.
REV/ASSETS:	.02	-.13	.01	-.00	.03	-.11	.07	.00
REV/COSTS:	.06	-.63	.05	-.02	.10	-.42	.28	.00

RESULTS FOR NON-ADJACENT PARTNERS:

	BCT	AGT	SASK	MANT	BCAN	NBT	MTT	TOT
INCURRED COSTS:	7562.	5429.	5979.	2089.	11675.	1590.	3490.	37814.
POST SET. REVENUE:	9444.	9758.	6526.	3597.	17750.	1523.	4400.	52998.
PRE SET. REVENUE:	9898.	9812.	2936.	3318.	21669.	218.	5152.	53003.
ASSIGNED PLANT INV.:	28730.	25944.	22343.	10396.	36597.	6185.	13789.	143985.
REV/ASSETS:	.33	.38	.29	.35	.49	.25	.32	.37
REV/COSTS:	1.25	1.80	1.09	1.72	1.52	.96	1.26	1.40

THE DIFFERENCE OF 5.BETWEEN TOTAL PRE-AND POST-SETTLEMENT REVENUES IS PAID TO QUEBEC-TEL TO COVER THE COSTS IT INCURS AS INTERMEDIATE CARRIER

RESULTS FOR REGIONAL TRAFFIC:

	BCT	AGT	SASK	MANT	BCAN	NBT	MTT	TOT
INCURRED COSTS:	10349.	4450.	2090.	1415.	140666.	1193.	2130.	162293.
ASSIGNED PLANT INV.:	39689.	21799.	7982.	7255.	446461.	4755.	8472.	536412.

SUBTOTAL FOR ADJ. AND NON-ADJ.

	BCT	AGT	SASK	MANT	BCAN	NBT	MTT	TOT
INCURRED COSTS:	10828.	11175.	9317.	4831.	19374.	6951.	5254.	67729.
POST SET. REVENUE:	14062.	16613.	10195.	10503.	29202.	6554.	5671.	92800.
PRE SET. REVENUE:	14133.	17005.	6638.	10995.	32709.	5278.	6047.	92806.
ASSIGNED PLANT INV.:	41158.	53528.	34956.	24222.	60786.	27471.	20749.	262870.
REV/ASSETS:	.34	.31	.29	.43	.48	.24	.27	.35
REV/COSTS:	1.30	1.49	1.09	2.17	1.51	.94	1.08	1.37

THE DIFFERENCE OF 5.BETWEEN TOTAL PRE-AND POST-SETTLEMENT REVENUES IS PAID TO QUEBEC-TEL TO COVER THE COSTS IT INCURS AS INTERMEDIATE CARRIER

% OF ASS. PLANT INV. BY SERVICE

	BCT	AGT	SASK	MANT	BCAN	NBT	MTT	TOT
ADJACENT:	11.5	29.1	22.0	33.5	3.7	48.5	15.6	11.4
US:	10.9	7.6	20.0	15.0	10.2	14.4	6.3	10.8
NON-ADJACENT:	26.6	27.3	39.0	25.2	5.6	14.1	30.9	13.8
REGIONAL:	36.8	23.0	13.9	17.6	68.3	10.8	19.0	51.4
MISCELLANEOUS:	.1	.1	.2	.2	2.1	9.8	26.3	2.9
TELEVISION:	5.6	5.6	4.9	4.5	2.7	2.5	2.0	3.4
UNASSIGNED:	8.5	7.4	.0	3.9	7.5	-.0	-.0	6.4

TABLE 3-9

Salient results of sharing block

Traffic	N.P.P.S.				Benchmark
	Bell		Total		Bell Canada
	60 N	96 N	60 N	96 N	
Adjacent & Non-adjacent					
Incurred costs	18,101	19,374	47,408	67,729	..
Presettlement revenues	24,552	33,709	72,400	92,806	55,450
U.S. traffic					
Incurred costs	27,009	21,201	42,422	32,427	..
Revenues	..	..	..	..	57,467
Regional traffic					
Incurred costs	121,762	140,666	125,402	162,293	..
Revenues	..	..	..	..	293,807
Plant assignment					
Adj. & Non-Adj.	10.0	9.3	21.6	25.2 }	27.7 *
U.S.	15.0	10.2	17.7	10.8 }	
Regional	68.4	68.3	48.2	51.4	72.3
Miscel. & T.V.	5.4	4.8	7.6	6.3	
Unassigned	1.2	7.4	4.9	6.3	

* Split of Bell toll revenues

3.2 Introduction of a Second Artificial U.S. node

In the last phase of the N.P.P.S. project, the introduction of a second U.S. node was suggested in order to give a more realistic representation of the routing of the U.S. traffic. The former and improved configurations of the switching network are given in Figure 3-3 for comparison for a typical example.

3.2.1 Software Improvements

Most of the programs had to be up-dated in order to accommodate the change but no major software adjustment was required.

3.2.2 Data Base

3.2.2.1 Switching Links

Introduction of the second U.S. node allowed a more realistic definition of the switching network. Table 3-11 shows the affected links with their former and improved descriptions.

3.2.2.2 Traffic

Although the two U.S. points could be considered as traffic generators, no traffic has been tagged to the level 1 U.S. node. Our method of traffic estimation is not reliable enough to permit any further desaggregation between the two U.S. nodes. Since U.S. level 1 nodes represent only a small fraction of U.S. population, all traffic has been directed to the U.S. low level node.

Traffic was estimated as in previous simulations and can be compared to the traffic used in the 60-node simulation in Table 3-10.

TABLE 3-10

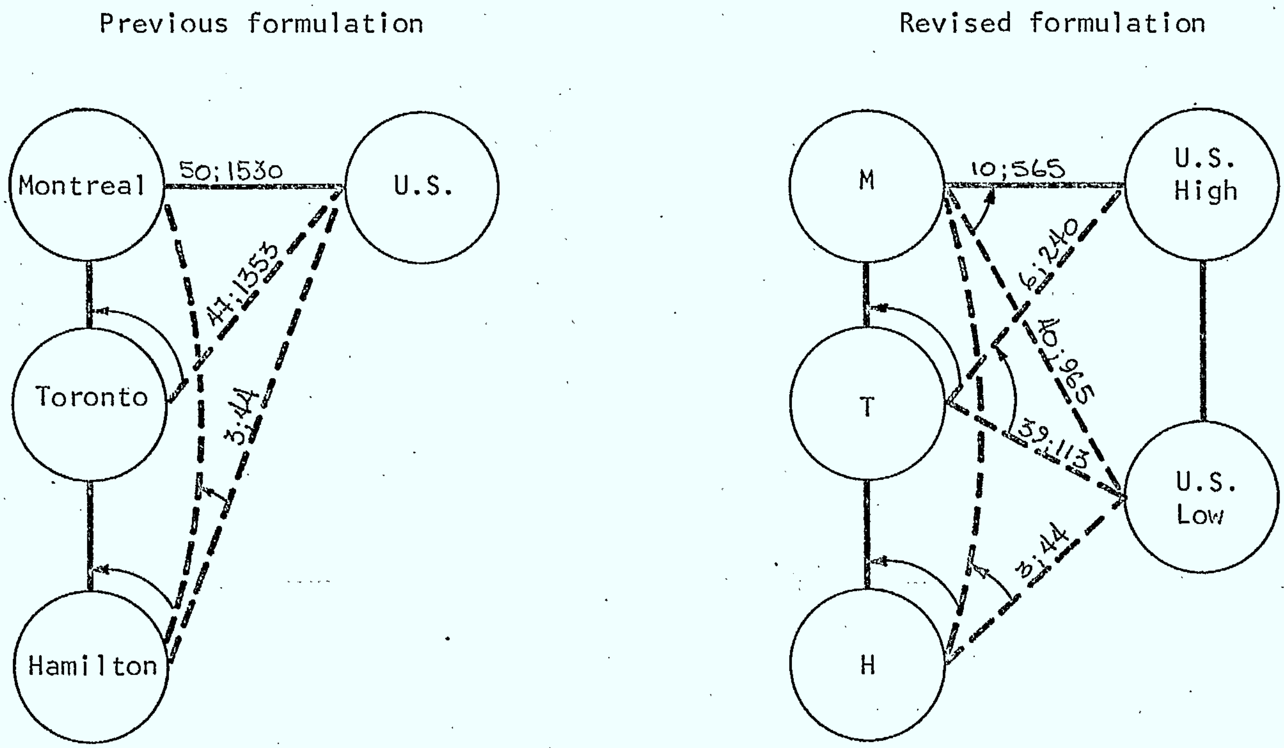
Estimated Peak Hour U.S. Traffic

(C.C.S.)

	Bell Experiment 60 nodes	National Experiment 97 nodes
Bell (Quebec)	12,704	14,950
Bell (Ontario)	18,437	21,636
Other Carriers	8,294	9,254
Total:	39,435	45,840

FIGURE 3-3

Typical representation of switching network
according to number of U.S. nodes



Description of actual Canada-U.S. links

Montreal - U.S. level 1 nodes:		10 final links;	565 circuits
	U.S. lower level nodes:	40 H.U.'s;	965 circuits
Toronto - U.S. level 1 nodes:		8 H.U.'s;	240 circuits
	U.S. lower level nodes:	39 H.U.'s;	1 113 circuits
Hamilton - U.S. level 1 nodes:		none	
	U.S. lower level nodes:	3 H.U.'s;	44 circuits

TABLE 3-11

Description of links affected by additional U.S. node

Canadian node	Former representation		Improved representation		
	Number of circuits	Type	Hierarchy of U.S. node (*)	Number of circuits	Type
Montreal	1,530	Final	H	565	Final
			L	965	H.U.
Regina	354	Final	H	296	Final
			L	58	H.U.
Vancouver	335	H.U.	H	88	H.U.
			L	247	H.U.
Calgary	118	H.U.	H	73	H.U.
			L	45	H.U.
Winnipeg	67	H.U.	H	19	H.U.
			L	48	H.U.
Toronto	1,384	H.U.	H	271	H.U.
			L	1,113	H.U.
Saint John	98	H.U.	H	17	H.U.
			L	81	H.U.
Halifax	36	H.U.	H	7	H.U.
			L	29	H.U.

* H (High): level 1 nodes

L (Low): levels 2, 3 or 4 nodes

3.2.3 Results

Main differences between the 96 and 97-node simulations rest in the routing of the U.S. traffic and consequently the sharing of facilities costs.

The average probabilities of loss and overflow for this simulation are shown on Table 3-12.

Also of interest is the breakdown of traffic carried by the artificial final linking (USA lower level nodes (USAL))-(USA level one nodes (USAH)) since it carries all overflow traffic of H.U.'s from the U.S. and primary traffic to canadian nodes not directly linked to the U.S. It is shown on Table 3-13.

Salient results produced by the sharing block are shown in Table 3-14 where they can be compared to those of the previous simulation.

The improvement brought by the second U.S. node is hard to evaluate from the sharing block results alone because of the principle of full allocation used in this module. It is obvious however that this formulation ensures a better representation of reality from the operating block point of view and should therefore yield more representative results throughout the model.

3.3 <u>Network Dimensioning</u>

3.3.1 Refinement of E.C.C.S. Algorithm

It will be remembered that according to the E.C.C.S. algorithm, the dimension of a link is such that it satisfies the equation

$$M = M'\frac{C}{C} \quad (1)$$

where M and M' are the marginal efficiencies of the direct and alternate routes and C and C' the marginal costs of increasing their respective capacity. In the past, various values have been tested for M' (between 20 and 28) and for the cost ratio C'/C (between 1.3 and 1.8). These values, however, had to be kept constant for all links. Since the cost of an added circuit to any route is made up of a switching cost and a transmission cost, the relative importance of the transmission component becomes larger with distance and consequently the cost factor decreases. The following approximate formula for relating cost ratios to distance is recommended by: A.T.T.:

$$\frac{C'}{C} = M' / 10\, d^{.11} \qquad (2)$$

TABLE 3-12

Usage of 97-node network

AVERAGE PROBABILITY OF OVERFLOW/LOSS :

FULL

	BCT	AGT	SASK	MTS	BCAN	NBT	MTT	NFLT	USA
BCT	.118	.000	.000	.000	.000	.000	.000	.000	.000
AGT	.000	.000	.000	.000	.000	.000	.000	.000	.000
SASK	.000	.000	.000	.000	.000	.000	.000	.000	.000
MTS	.000	.000	.364	-.000	.000	.000	.000	.000	.000
BCAN	.000	.000	.000	.000	.000	.000	.000	.000	.000
NBT	.000	.000	.000	.000	.000	.000	.000	.000	.000
MTT	.000	.000	.000	.000	.000	.000	.000	.000	.000
NFLT	.000	.000	.000	.000	.000	.000	.000	.000	.000
USA	.000	.000	.000	.000	.000	.000	.000	.000	.000

***HU ***

	BCT	AGT	SASK	MTS	BCAN	NBT	MTT	NFLT	USA
BCT	.000	5.024	11.772	21.592	27.056	.000	.000	.000	5.645
AGT	4.493	.000	9.659	4.453	19.377	.000	.000	.000	13.775
SASK	17.031	4.769	.000	.604	17.524	1.403	15.244	.000	69.457
MTS	31.063	2.859	.141	.000	10.347	52.985	.000	.000	14.798
BCAN	10.920	14.622	19.763	5.562	15.945	22.260	13.053	18.120	4.531
NBT	.000	.000	3.625	30.652	26.686	.000	12.646	3.836	5.323
MTT	.000	.000	48.153	.000	21.011	12.520	.000	20.969	8.212
NFLT	.000	.000	.000	.000	12.699	15.968	22.479	.000	.000
USA	5.636	23.657	74.346	12.885	5.302	5.587	9.634	.000	.000

***FIN ***

	BCT	AGT	SASK	MTS	BCAN	NBT	MTT	NFLT	USA
BCT	.000	.000	.000	.000	.000	.000	.000	.000	.000
AGT	.000	.000	-.000	.000	.000	.000	.000	.000	.000
SASK	.000	.000	.000	-.000	.000	.000	.000	.000	.132
MTS	.000	.000	.000	-.000	.000	.000	.000	.000	.000
BCAN	.000	.000	.000	.000	.037	.001	.000	.000	.000
NBT	.000	.000	.000	.000	-.000	.001	.000	.000	.000
MTT	.000	.000	.000	.000	.000	-.000	.001	.000	.000
NFLT	.000	.000	.000	.000	-.000	.000	.000	.000	.000
USA	.000	.000	.143	.000	.000	.000	.000	.000	.000

TOTAL :

=====

FULL.... .091%

HU 14.696%

FIN .... .023%

".000" MEANS NO LINK BETWEEN TWO CARRIERS.

TABLE 3-13

Analysis of traffic carried by the final link USAL ----> USAH

(ALL TRAFFIC FIGURES ARE IN CCS)

LINK #	LINK	NCIR	TYPE	OVERF /LOSS	THEO CIRC	COMP #	COMP	OFFERED TRAFFIC	TRAFFIC CARRIED	RELATIVE CAPACITY
- 590	USAL USAH	###	FIN	.000	175					
						1	USAL CRAN	.36	.36	
						2	USAL KAML	1.51	1.51	
						3	USAL NELS	.29	.29	
						4	USAL VANC	267.73	267.73	
						5	USAL EDMO	200.48	200.48	
						6	USAL CALG	163.19	163.19	
						7	USAL REGI	1605.13	1605.13	
						8	USAL SASK	42.37	42.37	
						9	USAL WINN	94.03	94.03	
						10	USAL LOND	.22	.22	
						11	USAL NORT	.65	.65	
						12	USAL OTTA	1.66	1.66	
						13	USAL SUDB	2.05	2.05	
						14	USAL THUN	.29	.29	
						15	USAL TORO	217.44	217.44	
						16	USAL MONT	2535.19	2535.19	
						17	USAL QUEB	.61	.61	
						18	USAL SHER	.29	.29	
						19	USAL NEWC	.25	.25	
						20	USAL SAIN	14.72	14.72	
						21	USAL HALI	.72	.72	
						22	USAL CORN	3.10	3.10	
						23	USAL ST.	11.41	11.41	
						24	USAL HAMI	.36	.36	
						25	USAL ST.C	.22	.22	
						26	USAL OSHA	2.05	2.05	
						27	USAL PORT	.25	.25	
						28	USAL PETE	1.01	1.01	
						29	USAL BARR	.94	.94	
						30	USAL ORIL	.32	.32	
						31	USAL HUNT	.40	.40	
						32	USAL KITC	.18	.18	
						33	USAL ST.J	2.59	2.59	
						34	USAL MT.L	.68	.68	
						35	USAL VAL	2.23	2.23	
						36	USAL JOLI	1.37	1.37	
						37	USAL DRUM	.22	.22	
						38	USAL TROI	7.60	7.60	
						39	USAL GRAN	4.82	4.82	
						40	USAL CAMP	.36	.36	
						41	USAL DAWS	.36	.36	
						42	USAL NANA	1.30	1.30	
						43	USAL NEWW	1.44	1.44	
						44	USAL PRIN	1.66	1.66	
						45	USAL TERR	.83	.83	
						46	USAL VICT	.47	.47	
						47	USAL GRAD	2.12	2.12	
						48	USAL LETH	6.34	6.34	
						49	USAL MEDE	4.57	4.57	
						50	USAL RED	4.21	4.21	
						51	USAL VEGR	2.45	2.45	
						52	USAL MOOS	10.40	10.40	
						53	USAL NORH	4.46	4.46	
						54	USAL PRIC	9.65	9.65	
						55	USAL SWIF	5.22	5.22	
						56	USAL BRAD	10.40	10.40	
						57	USAL DAUP	.65	.65	
						58	USAL BRAM	.54	.54	
						59	USAL TIMM	.68	.68	
						60	USAL SORE	3.64	3.64	
						61	USAL VALL	2.09	2.09	
						62	USAL FRED	.40	.40	
						63	USAL SYDN	.29	.29	
						64	USAL GAND	.68	.68	
								5264.14	5264.14	173957.90

TABLE 3-14

Principal results of sharing block
for 97 and 96 nodes simulations

	BELL 97 nodes	96 nodes	TOTAL 97 nodes	96 nodes
Incurred Costs ('000)				
Adjacent & Non-adjacent	17,232	19,374	61,542	67,729
U.S. Traffic	26,906	21,207	42,064	32,427
Regional Traffic	136,693	140,666	158,765	162,293
Plant Assignment (%)				
Adjacent & Non-adjacent	8.3	9.3	23.0	25.2
U.S.	13.0	10.2	14.2	10.8
Regional	66.5	68.3	50.4	51.4
TV & Miscellaneous	4.7	4.8	6.2	6.3
Unassigned	7.5	7.4	6.2	6.3

where d is the route mileage of the direct route. When combining (2) to (1), the marginal efficiency of the last circuit on the direct route is:

$$M = 10\ d^{.11} \qquad (3)$$

This last formula was introduced in N.P.P.S., but for simplicity purposes, the route mileage was approximated by the birds' eye distance.

The general effect of this formulation, when compared to a fixed coefficient approach, is to increase the size of short links and decrease it for long distance links. The changes are far from being drastic as can be seen from Table 3-15.

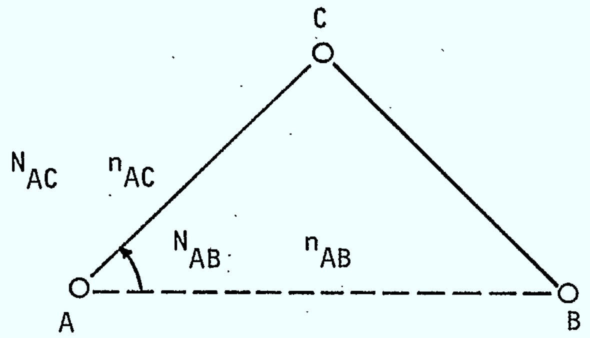
TABLE 3-15

Comparison of E.C.C.S. dimensioning before and after introduction of distance parameter

Links	Number of required circuits	
	Distance, Variable parameter	Fixed parameters
Sample of short distance links		
Hamilton - St. Catherines	55	57
Toronto - Guelph	118	114
Drummondville - Thetford Mines	4	2
Sample of long distance links		
Montreal - Vancouver	71	75
Toronto - St. Johns'	9	12
Toronto - Vancouver	49	55
Total High-usage links	5,428	5,356
Total final links	9,061	9,093

3.3.2 Dimensioning of Network Given Existing Dimensions

The second issue we tried to address was how to dimension the switching network given the existing one, i.e. trying to use as much as possible the existing links. This was done in the following manner. Consider the mini-network shown below where existing dimensions



are shown in upper case character while dimensions yielded by the E.C.C.S. are shown in lower case characters. Assume further that

$N_{AB} > n_{AB}$. The dimension N_{AB} is then retained. Consequently, traffic overflown to link AC is smaller than it would be if dimension n_{AB} had been retained and the E.C.C.S. applied to link AC will result in a dimension $\mathcal{N}_{AC} \leqslant n_{AC}$.

The generalized algorithm is as follows:

i) Dimension link using E.C.C.S.

ii) Compare to existing dimension and retain highest

iii) Calculate overflow to higher level links using retained dimension.

This algorithm obviously does not ensure the optimal trade-off between switching and transmission costs. It simply minimizes additions to the existing switching facilities.

It was tested on the actual network. Due to the underutilization of the existing switching network, it resulted in dimensions which in most cases coincide with the present status of the network.

This algorithm was implemented in N.P.P.S. and is offered as a dimensioning option.

3.4 Minor Streamlining

3.4.1 Estimation of Point to Point Peak Traffic

It is worth reminding that calculations of peak usage in the switching network can be derived by either one of the two following methods:

i) Actual hourly traffic is calculated for various hours of the day using east-west and west-east profiles to take time differentials into account. The usage of the switching network is then calculated for each time period and the maximum usage of each link over the various time periods retained.

ii) Peak hour traffic is defined by the user as a proportion of daily traffic and usage calculations are based on this single matrix.

The first method is obviously more accurate but is extremely lengthy since the programs involved are among the longest in the model. For practical purposes, the project team has mainly worked with the second method. In order to improve its accuracy, however, it was decided to use various peak hour to total day traffic ratios according to the type of traffic considered. Obviously, the larger the time difference between any two points, the higher the ratio should be. Consequently, the single parameter previously introduced by the user has been replaced by three parameters corresponding respectively to regional, adjacent and non-adjacent traffic. All simulations in this project were performed using a ratio of

7% for regional traffic

10% for adjacent traffic

12% for non-adjacent traffic

compared to a single ratio of 10% for all types of traffic in earlier projects using the N.P.P.S. model.

3.4.2 Regional Revenues Calculations

Regional revenues calculations were introduced in the model. Rather than using each carrier rate structure, Bell's tariffs were applied to all carriers in the network. This approximation is fully justified given the importance of Bell Canada in terms of revenues when compared to the whole network.

Results so obtained are compared to available benchmarks in Table 3-16.

TABLE 3-16

Regional Revenues
N.P.P.S. Estimates vs Benchmarks

Carrier	Proportion of total population represented in N.P.P.S. (%)	Regional revenues ($ millions)	
		N.P.P.S. estimates	1971 benchmarks
B.C. Tel.	63%	9.8	59.1
A.G.T.	63%	3.0	43.7
Sask.	38%	.9	14.7
Man. T.	59%	.3	12.0
Bell	75%	283.4	293.8
N.B.T.	40%	.9	8.2
M.T.T.	38%	.8	10.4

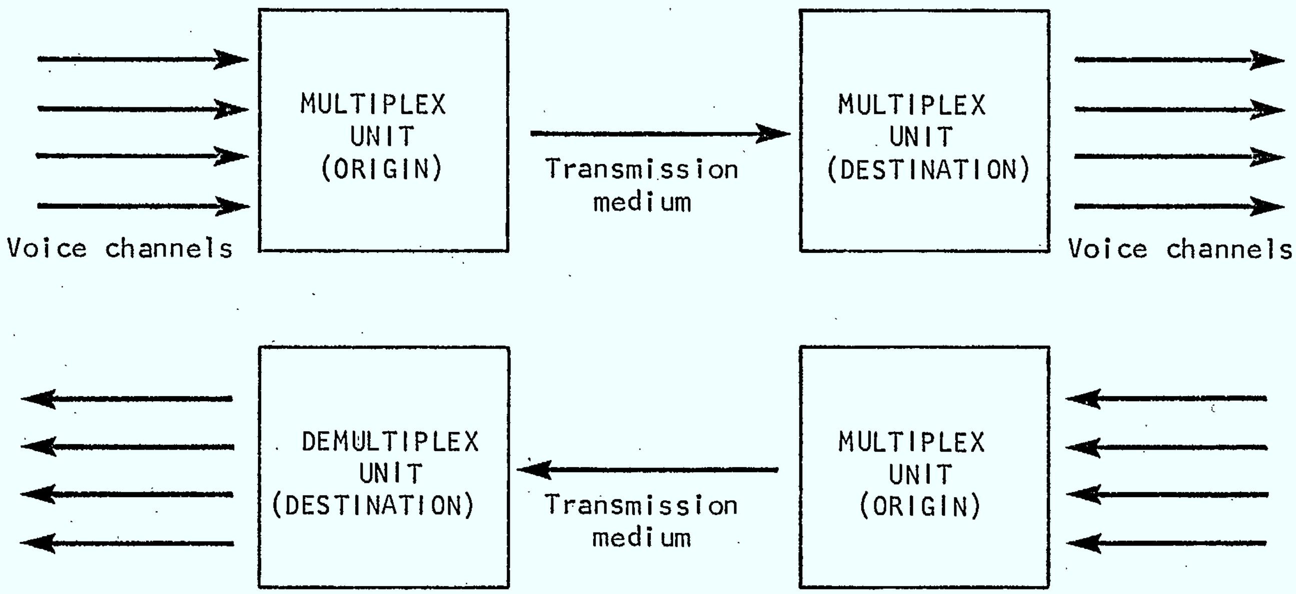
3.5 Formulation of Multiplexing Problem

3.5.1 The Multiplexing Function

Multiplex deals with the transmission of two or more signals simultaneously over a single transmission facility. Thus, if A and B are two demand points linked together by N channels, multiplexing is needed only when traffic requirements exceed N voice circuits. Clearly, each multiplexed signal has to be demultiplexed (i.e. the reverse process) at its destination points (See Figure 3-4).

FIGURE 3-4

Simplified block diagram of a 2-way frequency division multiplex link



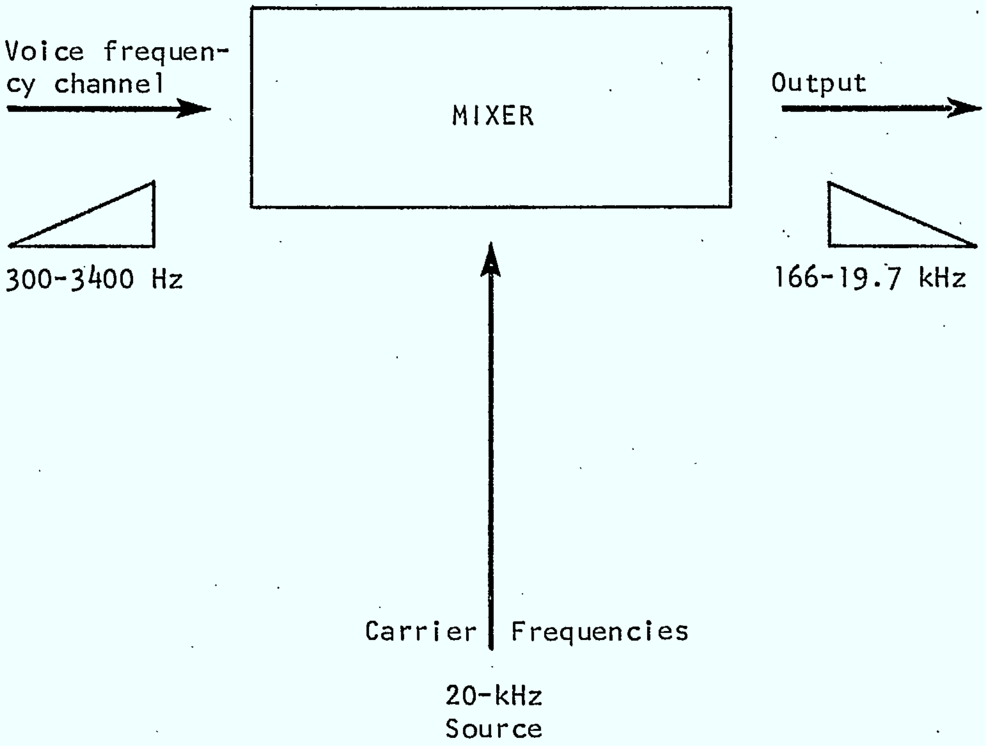
It follows that multiplexing equipments exist necessarily at terminal points of the telephone network, and may exist at junction points where "unbundling" and "re-bundling" is necessary. It is also clear that multiplexing does not exist at regular repeater nodes.

3.5.2 The Technique

The principle consists of mixing signals having the same band of frequencies into one signal with a different band of frequencies. The mixing process of the nominal 4-kHz voice channel (frequencies between 300 and 3 400 Hz) with a 20-kHz frequency signal for instance, yields a spectrum of frequencies between 16.6 and 19.7 kHz as illustrated in Figure 3-5.

FIGURE 3-5

Example of mixing process as used in multiplexing



a) Modulation Plan

A modulation plan sets forth the development of a band of frequencies called the line frequency (i.e., ready for transmission on the line). The modulation plan usually is a diagram showing the necessary mixing input, the carrier frequencies and the line frequency output. The CCITT[1] has recommended a standardized modulation plan with a

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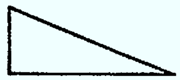
(1) International Consultive Committee for Telephone and Telegraph.

common terminology. This allows large telephone network, on both national and multinational systems, to interconnect.

b) Standard CCITT Group

The standard group as defined by the CCITT occupies the frequency band 60-108 kHz and contains 12 voice circuits. The modulation plan of a standard CCITT group is shown below.

Voice circuit	Input (kHz)	Carrier frequencies (kHz)	Output (kHz)	Final Output (kHz)
1	0-4	108	104-108	
2	0-4	104	100-104	
3	0-4	100	96-100	
4	0-4	96	92-96	
5	0-4	92	88-92	
6	0-4	88	84-88	
7	0-4	84	80-84	60-108
8	0-4	80	76-80	
9	0-4	76	72-76	
10	0-4	72	68-72	
11	0-4	68	64-68	
12	0-4	64	60-64	

c) Standard CCITT Supergroup

A supergroup contains five standard CCITT groups, equivalent to 60 voice circuits. The standard supergroup occupies the frequency band 312-552 kHz.

Each of the five groups forming the supergroup is translated in frequency to the supergroup band by mixing with the proper carrier frequencies. The carrier frequencies are 420 kHz for Group 1, 468 kHz for Group 2, 516 kHz for Group 3, 564 kHz for Group 4, and 612 kHz for Group 5.

d) Mastergroup and Super-mastergroup

The Standard CCITT Basic Mastergroup contains five supergroups, 300 voice circuits. It occupies the spectrum 812-2 044 kHz. The standard CCITT Basic Super-mastergroup contains three mastergroups and occupies the band 8516-12 388 kHz. It is equivalent to 900 voice circuits.

Incidentally, there are other configurations yielding mastergroups with different line frequencies.

The table below gives for each channel size in NPPS data bank the corresponding configuration:

Channel size (circuits)	Configuration SG	MG
120	2	-
300	5	-
480	8	-
600	10	-
900	5	3
960	8	2
1200	10	2
1800	10	3

120 to 600 - channels are mastergroups, and 900 to 1800 - channels are super-mastergroups.

3.5.3 The Planning Task

The multiplexing planning task consists of allocating the circuit requirements on the transmission channels in such a way that the resulting modulation plan optimizes the overall cost.

A telecommunication planner must decide, given the amount of increase in traffic for the next planning period, whether it is more economical to change the channels' loading and hence the modulation plan or to install additional RF channels? The decision here depends obviously on the traffic volume and on the O-D distance. However, telecommunication engineers agree to say: if we can avoid multiplexing, we do not hesitate to do so, because multiplexing is an important cost element in transmission.

One way to avoid or minimize multiplexing costs is to dedicate channels between pairs of demand points. In the case of inevitably non-dedicated channels, (must multiplex at each junction point of the route), the best way is to group the different traffic components separately in the multiplexing hierarchy, and at the highest level possible. That is, in the case of a channel with an equivalent capacity of 300 voice circuits carrying 5 different traffic components requiring 9 circuits each, allocate one supergroup to each O-D instead of putting them all in one supergroup, which is technically feasible. By assigning one SG to each O-D, the DEMUX and MUX process at each junction point is performed at the supergroup level only. This example shows clearly that the optimization process in multiplexing is a source of spare capacity.

The NPPS module for demand allocation on the transmission network has always produced a huge amount of spare capacity which so far could not be totally explained by the growth reserve. In section 3.5.4 we show how we solved the problem of allocating a part of the spare capacity so as to reflect the consequences of optimal multiplexing as shown in the above example.

3.5.4 Adjustments to the N.P.P.S. Model

Determining an overall optimal multiplexing plan for the network is not an easy task. Besides, assuming this global optimum could be obtained, it would be most presumably quite different from the actual status of the network given that in the industry, the multiplexing plan is done area by area and at various points in time.

It was decided therefore not to try defining the optimal multiplexing plan, but rather to estimate how variations in the multiplexing plan affected costs estimated by N.P.P.S. This was done by introducing two additional constraints on demand:

i) It is known that ultimate multiplexing and demultiplexing (i.e. to voice band) occurs only at origin and destination. It follows that circuits required between any two points must be "bundled" into dedicated groups. Consequently, circuit requirements as previously used in N.P.P.S. have been transformed into group requirements by rounding off the number of circuits required to the next multiple of 12.

ii) In addition, to maintain a good quality of service, it is current practice in the industry to avoid a group loading exceeding 75% (i.e. 9 circuits per group). Consequently, group requirements have been further increased to respect this constraint.

The impact of both constraints is evaluated in Table 3-17 where the use of the transmission network is detailed for each of the following working assumptions:

TABLE 3-17

Comparative use of transmission network
according to assumptions on multiplexing

('000 circuit miles)

Allocation method	Circuits	12/group	9/group
Circuits allocated	5 589	7 530	8 813
Idle circuits on used channels	6 769	6 658	6 457
Total used channels	12 358	14 188	15 270
Idle channels	22 219	20 309	19 227
% of channels used	36 %	41 %	44 %
Average channel loading	45 %	53 %	58 %

- demand expressed in circuits (previous formulation)
- demand expressed in groups (up to 12 circuits per group)
- demand expressed in groups having a maximum fill of 75%

It can be seen that the application of both constraints would increase the number of required circuit-miles by almost 60%.

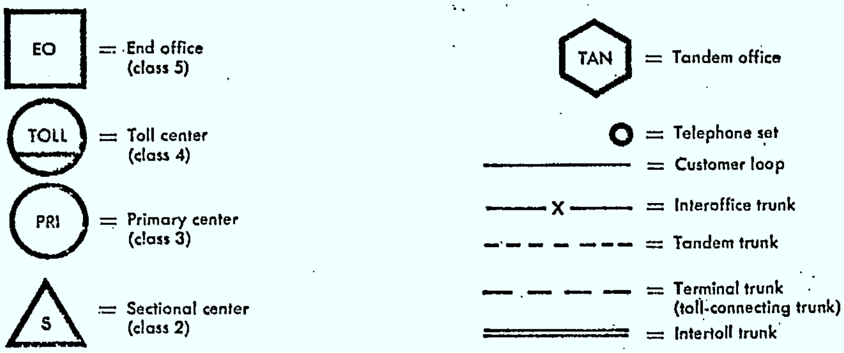
So far, only the possibility of dedicating group has been considered. One could envisage in a latter phase, however, to dedicate super-group and master-group as soon as circuit requirements exceed certain levels, hence further reducing the presently "spare" capacity. (e.g. a super-group has a capacity of 60 circuits but could be dedicated to a single demand as soon as it reaches, say, 30 circuits).

Finally, it must be noted that this formulation of the multiplexing problem was introduced in a later phase of the project so that results shown in section 4.1 do not reflect the improvement while it is taken into account in the results presented in 4.2.

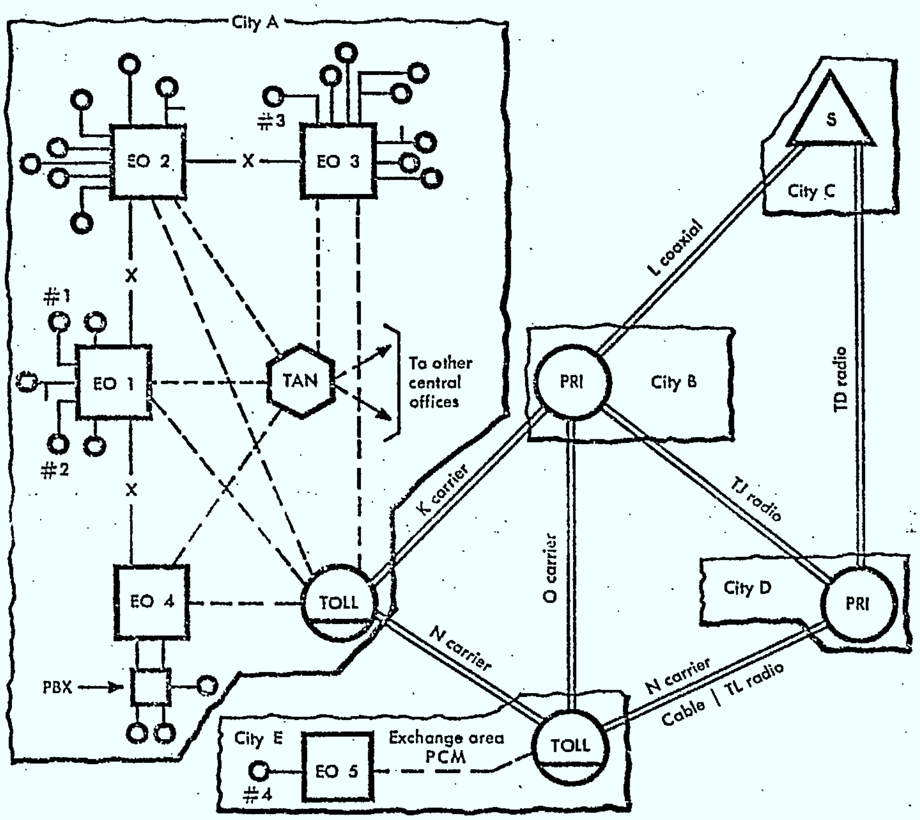
FIGURE 3-6

A Simplified Telephone System

(Extracted from "Transmission Systems for Communications", Fourth Edition, Bell Telephone Laboratories).



For clarity in this diagram, nonstandard symbols are used for the end offices. See Fig. 1-2 for standard usage.



3.6 Estimation of the Related Costs in Local Network.

3.6.1 General

The toll network which the N.P.P.S. model represents constitutes only part of the facilities required to handle a long distance call. Since the final purpose of the project is to determine, compare or interpret costs of long distance services, it is essential that costs attached to facilities not presently considered be incorporated to the model before it can be used for regulatory purposes. In this section, we will describe the part of the telephone plant unaccounted for by N.P.P.S. (we will refer to it as the local network), its characteristics with respect to usage and we will determine the data necessary to obtain at least an approximate idea of the costs associated.

3.6.2 Physical Layout and Characteristics

In its simplest form, the local network is as shown in Figure 3-6, for city E. A certain number of customers each operating a station set are linked to a local office via a local loop. The local office in turn is linked to a toll switching machine through toll connecting trunks.

i) Station set

It accepts a signal from a source and converts it to an electrical signal suitable for transmission or reverses the process. It can be a telephone set, a computer terminal, a telex... Attached to the station set are a certain number of optional so called "vertical services" such as decorator sets, PBX (i.e. customer's own switchboard). The house wiring is also included in this category of equipment.

ii) Local loop

It consists of a pair of wires bundled together with other wires into a cable which connects a set of station sets to the switching machine in the local central office.

iii) Switching machine

It enables connections to be established between a station set and another one in the same office or, through trunks and other switching machines, to any set on the network. A great majority of switching machines presently in service are of the electro-mechanic type but they are replaced gradually by electronic equipment which is more versatile, easier to maintain and much less voluminous.

As soon as a demand center becomes large enough, the configuration gets more complicated as can be seen in Figure 3-6 (city A). There are a certain number of local offices which are interconnected either directly or via a tandem office.

Such a configuration depends on a variety of reasons which may be:

i) economical: trade-off between switching and wiring

ii) technical: proper load balancing

iii) historical: evolution of metropolitan boundaries and population concentration

Two categories of equipment can be distinguished within the local network, whether the size is usage sensitive or not.

Station set and customer loops are not usage sensitive. The latter, however, can become usage sensitive in certain instances. Where suitable calling patterns exist, several loops are grouped into fewer lines by using line concentrators. These can then be considered as lower hierarchy switching machines and the outgoing lines as inter-office trunks.

Central and tandem offices as well as trunks are usage sensitive as is the case of switching machines and trunks in the toll network.

3.6.3 Costs Associated with the Local Network

3.6.3.1 Allocation of costs associated with the local network

It is important to realize that the local network is the most expensive part of a telephone network as shown below in the breakdown of Bell's telephone investments:

	Proportion of total investment
1. Local network	
a) Non usage related (station equipment, local loops)	57%
b) Local and toll usage related (central office switching, exchange trunking)	18%
2. Toll network	15%
3. Support (buildings, vehicles, furniture)	10%

Source: Telecommunications Cost Inquiry, Volume V, p. 66.

Since the subscriber line plant is influenced neither by the size not the mix of the services which use it, there is no usage basis for assigning this plant to the constituent services and any allocation would be purely arbitrary. It must be noted, however, that for revenue sharing purposes, TCTS members assign a portion of their non-usage related equipment (in the local network as well as in support facilities) to toll traffic, the basis of this allocation being the Subscriber's Line Usage (SLU) factor.

If we consider usage related plant, however, costs can be allocated in some objective manner:

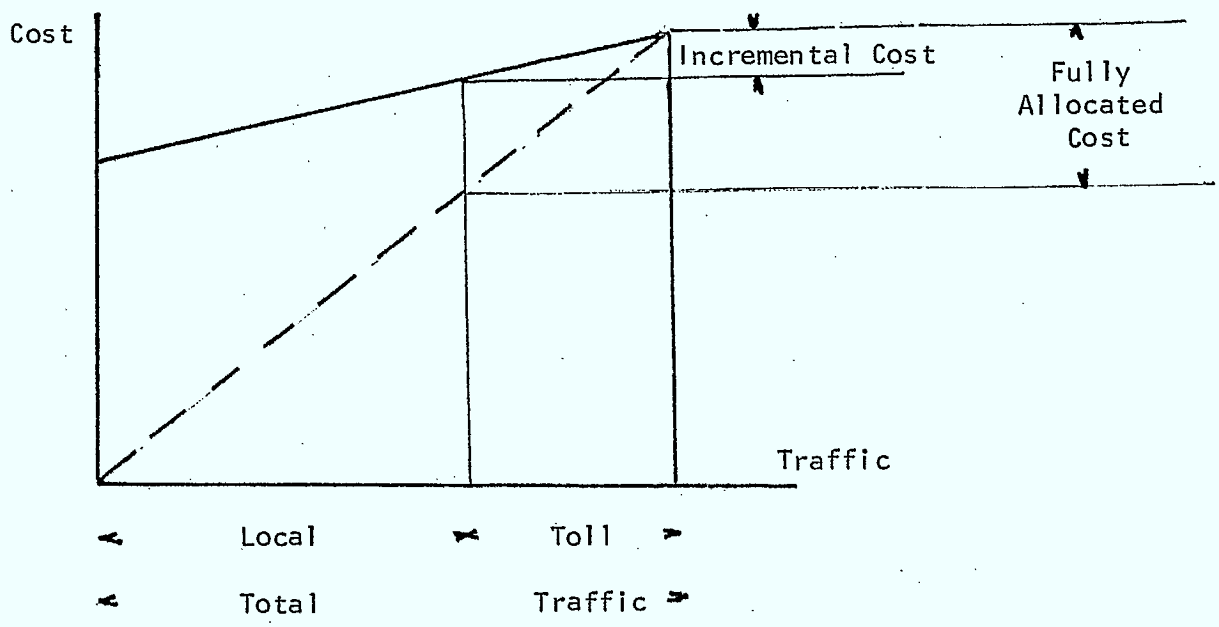
i) Costs associated with inter-exchange trunking can be fully allocated to local traffic (unless the trunks considered go to a tandem office which is connected to the toll network);

ii) Costs associated with toll connecting trunks can clearly be allocated to toll traffic;

iii) There are two ways according to which the costs of central offices can be allocated. The first would consist of a full allocation of costs on a usage basis as is done in the toll network for allocating costs of common plants to regional and inter-regional traffic (see Figure 3-7).

FIGURE 3-7

Allocation of local switching costs



The second, the incremental cost approach, would consist of allocating to toll traffic the additional investment required to handle this extra traffic. It must also be noted that if an incremental approach is used, the costs associated with the subscribers' plant are unambiguously nil.

The proper concept to be used in N.P.P.S. would obviously depend on the ultimate use of the simulation. Since our main concern in this phase of the project is the implementation of cross-subsidy tests, it makes no doubt that the incremental cost concept is to be considered.

3.6.3.2 Cost Data

- Central Offices (Switching)

  Bell Telephone Laboratories price studies have shown[1] that the cost of a local central office varies with the number of lines connected to the machine, peak hour traffic and technology. The following formulation was given:

$$\text{Total Cost} = \alpha + \beta L + \gamma \, IOC$$

where

L : number of lines ($\leqslant$ 30,000)

IOC : number of incoming + outgoing peak hour calls

α, β, γ cost coefficients dependent upon technology

For example, 1969 values of these coefficients for a No 5 crossbar Lama machine were as follows:

α = \$ 347,000.

β = \$ 35.16 / line

γ = \$ 47.39 / peak hour call

(1) Local ESS and No. 5 Crossbar Price Study Results, Case 36279-52.

Consequently, in this case, the added switching cost associated to toll traffic is estimated at $ 47.39 per peak hour call.

- Toll Connecting Trunks (Transmission)

Although radio technology may be used, the most common medium of transmission for toll connecting trunks is cable or twisted pairs. From preliminary investigations, within D.O.C., it appears that coaxial cables are preferred to bundled wires as soon as the distance exceeds a few miles.

The total cost of such trunks can be broken down into:

- termination costs: $ 500. per circuit at each end (D.O.C. estimates)
- multiplexing costs: (for cable and radio technology only) between $ 1040. and $ 1140. per circuit (as presently incorporated in N.P.P.S.)
- cable costs: $8.66 per circuit mile for 9 000 voice circuit cables;

  $7.20 per circuit mile for 18 000 voice circuit cables (D.O.C. information)

It thus follows that a first approximation to the cost of toll connecting trunks would be of the form:

$$\text{Per circuit cost} = \begin{cases} (\$2\ 100) + \$8.00 \times \text{mileage (for cable/radio)} \\ \$1\ 000 + \$8.00 \times \text{mileage (for twisted pairs)} \end{cases}$$

- L.S.C.S. Cost Algorithm

The local Service Costing System used by Bell Canada and presented to CTC for discussion in 1975 provides a slightly different costing basis for No. 5 Crossbar extension costs. In this system, reproduction cost causality is established as shown in Figure 3-8. Since we are only concerned with toll traffic, the following relevant cost components can be extracted:

$ 13./peak hour C.C.S.

$ 22./peak hour outgoing call

$ 14./peak hour incoming call

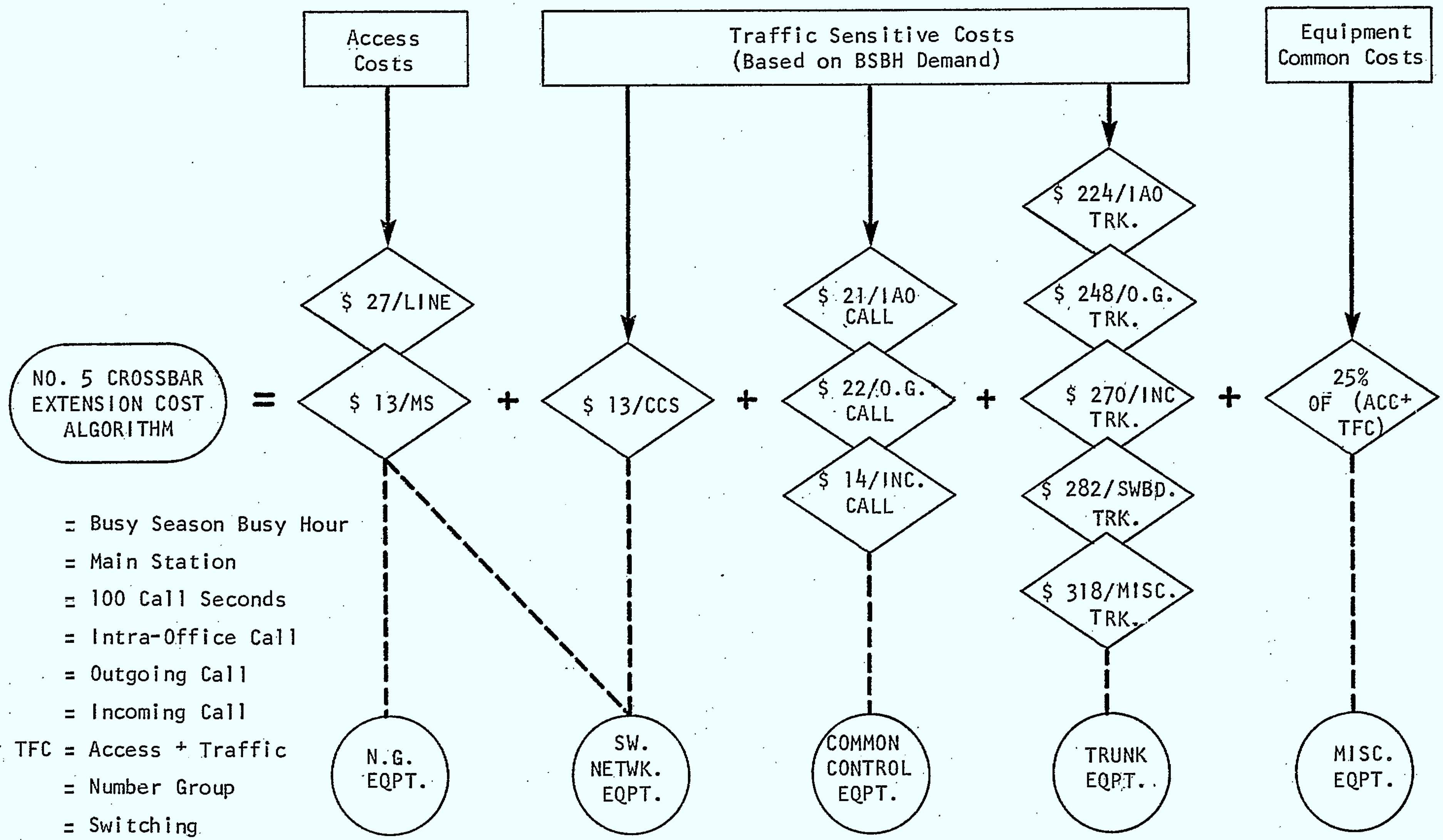
$ 248./peak hour outgoing trunk

$ 270./peak hour incoming trunk

+ 25% loading for power, relays, etc...

FIGURE 3-8

No. 5 Crossbar Extension Cost Algorithm



BSBH = Busy Season Busy Hour
MS = Main Station
CCS = 100 Call Seconds
IAO = Intra-Office Call
OG = Outgoing Call
INC = Incoming Call
ACC + TFC = Access + Traffic
NG = Number Group
SW = Switching

(1) Extracted from: Local Service Costing System. Discussion Notes for Presentation to C.T.C., April 1975.

- . Reconciliation of Available Data

We have presented so far these sources of cost data which we must now reconcile.

Additional switching costs are estimated at \$ 47. per peak hour call in the first one, while valued at \$ 13./peak hour C.C.S. + \$ 22.-14./peak hour call in the third. Given an average call duration of 3.3 C.C.S., this latter method yields an average of \$ 67./peak hour call. If an additional 25% is added for power and relays, the total additional cost per peak hour call amounts to \$ 76.

The resulting difference of 55% between the two estimates may be explained by the following factors:

- both studies were done on different samples (one in the States, one in Canada);

- the first estimate corresponds to 1969 figures while the second is dated 1975. (An inflation rate of 7½% compounded annually over 6 years results in a global increase of 55%).

Additional transmission (trunking) costs are estimated at an average of $\frac{(248 + 270)}{2}$ + 25% = \$ 324. per trunk in the L.S.C.S. algorithm while estimates obtained from D.O.C. show a minimum possible value of \$ 1 000./circuit (distance being quite small for toll connecting trunk, the distance related cost becomes negligible). One possible explanation for this difference is the following: Since the purpose of the L.S.C.S. algorithm is to evaluate extension costs on a No. 5 Crossbar, it most likely does not take into account the costs incurred at the other end of the trunk (i.e. at the toll switch) which are included in D.O.C. estimates.

3.6.4 A First Modelling Effort to Estimate Toll Related Costs in the Local Network.

3.6.4.1 General

Given the size and the complexity of the local network, it is obvious that whatever will be appended to N.P.P.S. to represent the local network will have to be very crude. The fact that the model developed by Bell Canada in the Local Service Costing System requires 3 200 K of data base clearly illustrates this point. This does not mean, however, that no modelling can be done, especially in an incremental cost context. If an upper-bound to the added cost of toll traffic in the local network could be developed, it would already be quite helpful for cross-subsidy tests. In this section we present a first approach to defining the added cost of toll traffic in the local network.

3.6.4.2 Added Switching Cost

Given the cost data shown in 3.6.3.2, this component depends only upon total peak hour C.C.S., peak-hour incoming and outgoing calls, three quantities which are readily available from the N.P.P.S. demand module.

3.6.4.3 Added Transmission Cost

Assume the simplified network of Figure 3-9 consisting of:

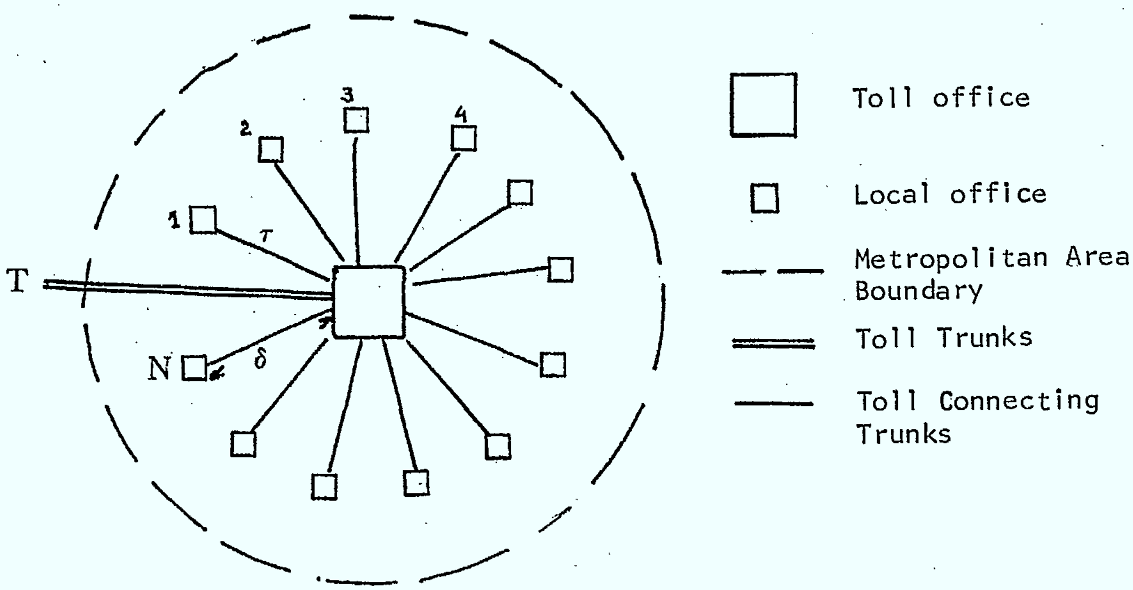
- one toll switching machine
- N local offices.

Further assume that the peak hour toll traffic of T C.C.S. is equally balanced between the N local switches. Toll connecting trunks between the toll switch and any local switch will then have a number of circuits ν sufficient to ensure P_{ol} for a traffic of $\tau = T/N$ at peak hour. ν can be readily determined with the dimensioning procedures for final links used in the toll network.

Incidentally, since both incoming and outgoing traffic are estimated in N.P.P.S., separate calculations can be worked out for incoming and outgoing trunks.

FIGURE 3-9

Hypothetical Metropolitan Node



3.6.4.4 Implementation

Since a few ambiguities still remain with regards to the proper cost functions to be applied, it was decided to use those formulae yielding the higher estimates hence providing an upperbound to toll related costs in the local network. Consequently, added switching costs were estimated using the L.S.C.S. algorithm, while added transmission costs were valued with D.O.C. estimates.

Data on the number of local switches in the cities represented in N.P.P.S. not being available, one local switch for every 10,000 of population was assumed. (The total number of local switches in Bell Canada's territory amounts to 1 200 for approximately 12 millions people served). It was further assumed that the average length of toll connecting trunks varies according to population betwwen 3 miles for the largest city represented and 50 miles for the smallest.

Corresponding results are presented in Table 3-18 where they can be compared to total toll related assets as calculated in N.P.P.S.

This added feature of the N.P.P.S. model was introduced in the very final months of the project and therefore results shown in chapter 4 do not include toll related costs in the local network. The impact of this shortfall is however evaluated in appropriate sensitivity analyses (section 4.3).

TABLE 3-18

Estimation of toll related assets in local network ($ millions)

Company	Additional switching cost	Additional transmission cost	Total cost (1)	Total toll network as per N.P.P.S. (2)	(1) / (2) %
B.C. Tel.	.55	2.74	3.29	107.6	3.1
A.G.T.	.28	1.64	1.92	94.9	2.0
Sask. T.	.18	.83	1.01	57.0	1.8
Man. T.	.15	.87	1.02	41.4	2.5
Bell & Quebec Tel.	8.28	28.73	37.01	672.7	5.5
N.B.T.	.14	.74	.88	44.3	2.0
M.T.T.	.12	.77	.89	44.7	2.0
TOTAL	9.70	36.32	46.02	1062.6	4.3

3.7. Costing Block

3.7.1 Costing Block Flowchart

The previous flowchart of the Costing Block can be found in the Final Report on the Second Phase, December 31, 1974, page 2-42. Some of the modules proposed then have already been implemented and where it has not been completed nor linked, input data for the actual modules are accepted exogenously rather than being computed endogenously by the other modules.

At this point in time, the Costing Block is composed of these three main modules:

- The Asset Valuation Module
- The Aging, Indexing and Depreciation Module
- The Incurred Cost Module.

In Figure 3-10, we can find the flowchart of the Costing Block as it stands now.

3.7.2 The Asset Valuation Module and the Aging, Indexing and Depreciation Module

Those two modules were fully described in previous reports, and with the exception of a correction to the calculation of the first year vintage in the AID module, nothing has been changed in those two modules. The AID is also the only module not linked to the others.

3.7.3 Incurred Cost Module

In this module, all the components have been revised and changed whenever required. The flowchart of this module appears in Figure 3-11.

3.7.3.1 Cost of Capital Rate

The Cost of Capital Rate is calculated using the following equation:

$$C.C. = \frac{1}{1-t} * \{RORC * CEQR + RORP * (1 - CEQR)\} * (1 - DCR) + AIR * DCR \qquad (1)$$

FIGURE 3-10

COSTING BLOCK FLOWCHART

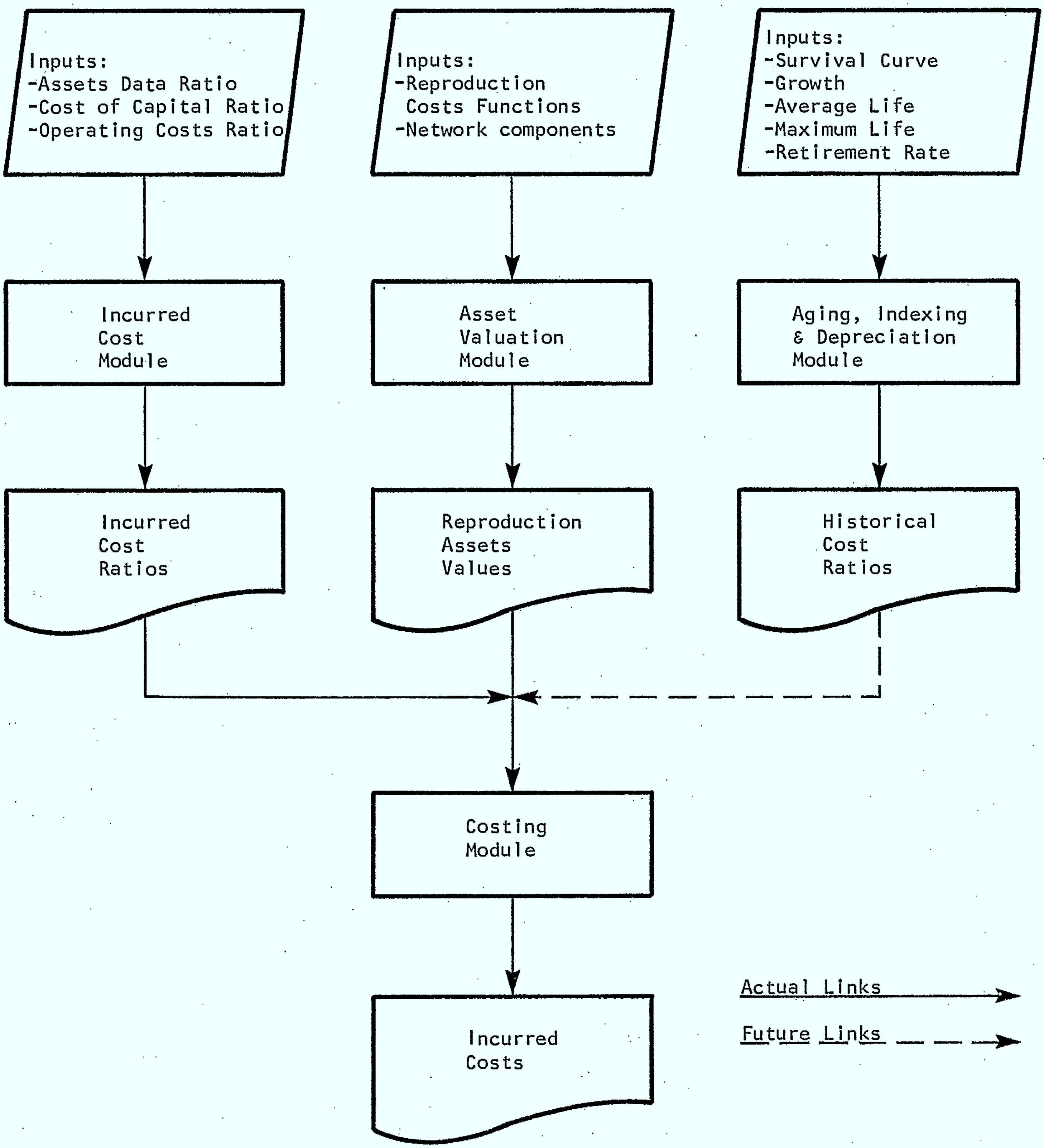
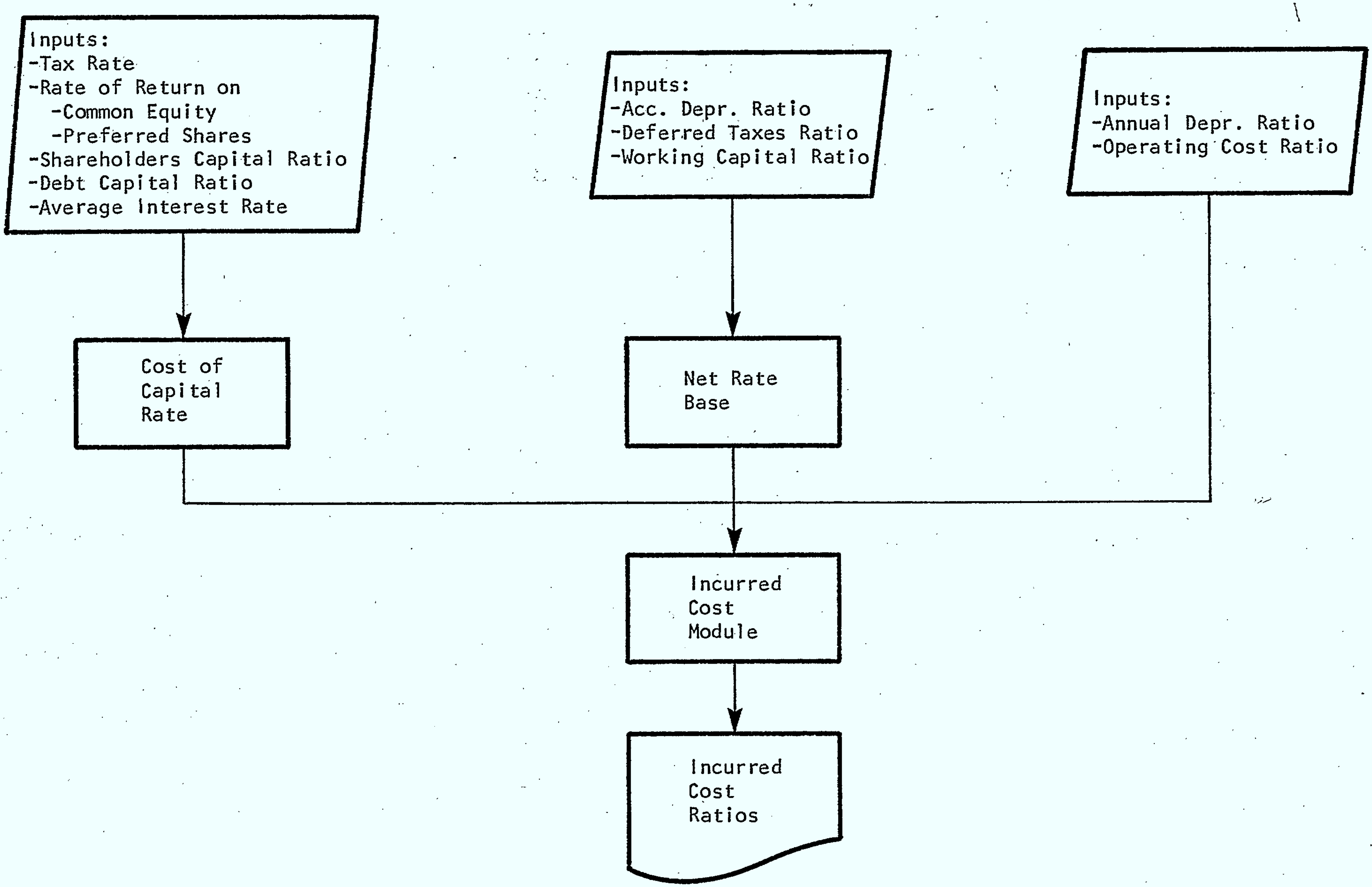


FIGURE 3-11

INCURRED COST MODULE



WHERE: C.C. = Cost of Capital

t = tax rate

RORC = Rate of Return on Common Equity

RORP = Rate of Return on Preferred Shares

CEQR = Common Equity Ratio

DCR = Debt Capital Ratio

AIR = Average Interest Rate on Debt

This Cost of Capital Rate was briefly described in:

- Final Report on Second Phase, page 2-47

- Users' Manual, October 1975, pages 4-9, 4-10, 4-11, 5-4, 5-6.

However, the inputs have been completely revised using the 1971 figures from the Financial Statistics on Canadian Telecommunication Common Carriers, 1973 edition, prepared by the Department of Communications. Whenever the figures were representing end of the year data, we then calculated the mid-year 1971 average (e.g. (end year 70 + end year 71) ÷ 2). Consequently, the input data were calculated according to the following definitions:

- $RORP = \dfrac{\text{Preferred Dividends 1971}}{\text{Preferred Shares Mid-year 1971}}$

- $RORC = \dfrac{\text{Net Income - Preferred Dividends 1971}}{\text{Total Common Shareholders Capital Mid-year 1971}}$

- $CEQR = \dfrac{\text{Common Equity Mid-year 1971}}{\text{Total Equity Mid-year 1971}}$

- $DCR = \dfrac{\text{Total Debt Mid-year 1971}}{\text{Total Debt + Total Shareholders Capital Mid-year 1971}}$

- $AIR = \dfrac{\text{Debt Service Charges 1971}}{\text{Total Debt Mid-year 1971}}$

- $t = \dfrac{\text{Income Taxes 1971}}{\text{OPRV - OPEX - DSC 1971}}$

WHERE: OPRV = Operating Revenues 1971

OPEX = Operating Expenses 1971

= Maintenance + Traffic + Depreciation + Other Expenses + Other Taxes

DSC = Debt Service Charges 1971.

3.7.3.2 Net Rate Base

The rate base upon which the cost of capital is computed is called the Net Rate Base (NRB) and it includes the three following components: the Net Telephone Plant (NTP), the Working Capital (W.C.) and the Deferred Taxes (D.T.). This concept of Net Rate Base was chosen in N.P.P.S. since the capital rate base was not available for the toll network; instead we have a representation of the asset valuation of the regional and inter-regional network.

This Net Rate Base definition is equivalent to the total capitalization concept and it can be explained easily by looking at the balance sheet of a company:

Assets		Liabilities and Equity	
Gross Telephone Plant	(GTP)	Shareholders' Equity	(Eq.)
- Acc. Depreciation		+ Long Term Debt	(Debt)
+ Current Assets	(C.A.)	+ Current Liabilities	(C.L.)
		+ Deferred Taxes	

From the balance sheet, we have the following equation:

GTP - Acc. Depr. + C.A. = Eq. + Debt + C.L. + Def. Taxes (2)

which can be transformed into:

Eq. + Debt = GTP - Acc. Depr. + C.A. - C.L. - Def. Taxes (3)

Eq. + Debt = NTP + W.C. - Def. Taxes (4)

where NTP = Net Telephone Plant = GTP - Acc. Depr.

and W.C. = Working Capital = Current Assets - Current Liabilities

Expressed as a fraction of the Gross Telephone Plant, the Net Rate Base is then defined by the following equation:

NRB = 1 - % Acc. Depr. + % W.C. - % W.C. - % Def. Taxes (5)

with the Accumulated Depreciation, the Working Capital and the Deferred Taxes all calculated at mid-year 1971.

3.7.3.3 Incurred Cost Ratios

The Incurred Costs of operating the telecommunication network can now be expressed as a ratio of the Gross Telephone Plant assets using the following equation:

I.C. ratio = NRB * C.C. + % Depr. + % op. Cost (6)

Where the Incurred Costs is the sum of the Holding Costs

(NRB * C.C. + % Depr.) plus the Operating Costs.

The revised Incurred Cost Matrix is stored on computer in the CINTAB file to be used later as input data for other programs.

On the next page, Table 3-19, we find the three sets of inputs:

- Asset Data
- Cost of Capital Data
- Operating Cost Data

and on the following page, Table 3-20, the three sets of output data:

- Net Rate Base Matrix
- Cost of Capital Vector
- Incurred Cost Matrix

If we compare these results with those previously reported in The User's Manual, April 1975, page 4-19, we can note some significant changes. The coefficients in the Net Rate Base Matrix are now lower as well as different for each carrier. Similarly, the corrections to the data on the cost of capital have produced the true coefficients. Those corrections and improvements are reflected in the incurred cost matrix and have affected mainly the Bell coefficients which we reduced from a previous range of (.307-.343) to a range of (.238-.292) which is more in line with the coefficients of the other carriers. This reduction of approximately .05 in the

TABLE 3-19

Inputs of the Incurred Cost Module

*****BELL*****

ASSET DATA

	SWIT	TRAN	GENL	LAND	BLDG	STAT
DEPN	.04500	.05850	.09640	.00000	.03200	.00000
ADEP	.27200	.29100	.35100	.00000	.25900	.00000
GROW	.12130	.15730	.20520	.06700	.49400	.00000
RETR	.03370	.03950	.11820	.00000	.00000	.00000
NSVR	.00000	.00000	.00000	.00000	.00000	.00000

COST OF CAPITAL DATA

	%WCL	TAXR	RORC	DCR.	IAVG	%D.T	RORP	CEQR
B.C.	.01030	.47320	.09190	.55860	.06880	.07160	.05230	.67210
ALTA	.03730	.00000	.05840	.90030	.05850	.00000	.00000	1.00000
SASK	-.03400	.00000	.27240	.69680	.07250	.00000	.00000	1.00000
MANI	.01700	.00000	.07950	.85990	.06900	.00000	.00000	1.00000
BELL	.01640	.50960	.08840	.46190	.05900	.04870	.06400	.91450
N.B.	.00420	.51490	.10510	.46750	.06690	.09010	.00000	1.00000
MT&T	.02400	.51650	.10200	.49420	.06640	.08570	.07000	.97870
NFLD	.02320	.46790	.10320	.49480	.07690	.00370	.00000	1.00000

OPERATING COST DATA

	SWIT	TRAN	GENL	LAND	BLDG	STAT
B.C.	.12900	.12900	.12900	.12900	.12900	.12900
ALTA	.10900	.10900	.10900	.10900	.10900	.10900
SASK	.09800	.09800	.09800	.09800	.09800	.09800
MANI	.10900	.10900	.10900	.10900	.10900	.10900
BELL	.12050	.12050	.12050	.12050	.12050	.12050
N.B.	.10900	.10900	.10900	.10900	.10900	.10900
MT&T	.10900	.10900	.10900	.10900	.10900	.10900
NFLD	.13400	.13400	.13400	.13400	.13400	.13400

TABLE 3-20

Outputs of the Incurred Cost Module

------ SIMULATION NO. 1 YEAR : 1971

NET RATE BASE MATRIX :

	SWIT	TRAN	GENL	LAND	BLDG	STAT
B.C.	.66670	.64770	.58770	.93870	.67970	.93870
ALTA	.76530	.74630	.68630	1.03730	.77830	1.03730
SASK	.69400	.67500	.61500	.96600	.70700	.96600
MANI	.74500	.72600	.66600	1.01700	.75800	1.01700
BELL	.69570	.67670	.61670	.96770	.70870	.96770
N.B.	.64210	.62310	.56310	.91410	.65510	.91410
MT&T	.66630	.64730	.58730	.93830	.67930	.93830
NFLD	.74750	.72850	.66850	1.01950	.76050	1.01950

COST OF CAPITAL VECTOR :

B.C.	.10455
ALTA	.05849
SASK	.13311
MANI	.07047
BELL	.12196
N.B.	.14665
MT&T	.13881
NFLD	.13603

INCURRED COST MATRIX :

	SWIT	TRAN	GENL	LAND	BLDG	STAT
B.C.	.24371	.25522	.28685	.22714	.23207	.22714
ALTA	.19876	.21115	.24554	.16967	.18652	.16967
SASK	.23538	.24635	.27626	.22658	.22411	.22658
MANI	.20650	.21866	.25233	.18067	.19442	.18067
BELL	.25035	.26153	.29211	.23852	.23893	.23852
N.B.	.24816	.25887	.28798	.24305	.23707	.24305
MT&T	.24649	.25735	.28692	.23924	.23529	.23924
NFLD	.28068	.29160	.32134	.27269	.26945	.27269

Bell incurred cost ratios represents in fact a reduction of some 20% in the Bell incurred costs, from $ 202 millions to $ 165 millions, as can be seen from Table 3-21 and Table 3-22. Incurred costs of other carriers were only marginally affected.

The improved Incurred Cost Module is now fully operational. However, since these modifications were implemented only in the last phase of the project, the corresponding improvement is not reflected in other sections of this report.

Its relative impact on the results shown in chapter 4 will, however, be evaluated in the form of sensitivity analyses.

3.7.4 Linking of the Modules

At this point in time, the Incurred Cost Module is directly linked to the Asset Valuation Module, bypassing completely the Aging, Indexing and Depreciation Module. The basic assumption can be accepted as valid for 1971 since the inflation rates up to 1971 were fairly small and have been offset almost completely by the productivity gains. However, if at a later time we were to use a 1976 network with 1976 costs figures, these assumptions would no longer be valid because of the increased inflation rates since 1971. It would then be essential to link the Aging, Indexing and Depreciation Module to the other modules. Provided with the appropriate inputs, the linking could be easily done simply by expanding the input vector into an input matrix and by providing the appropriate links with the other modules.

TABLE 3-21

Incurred Costs Using 1974 Costing Block

	BCT	AGT	SASK	MANT	BCAN	QUET	NBT	MTT
TOTAL ASSETS	107638386	96536404	57858295	41437574	639083866	15588941	47677249	44222877
INCURRED COSTS								
SWITCHING MACHINES	11328400	5544000	5676000	2003400	72564800	924000	4802000	3886795
TERMINAL REPEATERS	5240038	4093805	3051855	1201608	67099552	775914	3779864	3220945
BRANCHING REPEATERS	5722450	5849039	3599440	3351035	33449920	518907	2281081	3266901
REGULAR REPEATERS	6018029	4589143	3146634	1665682	29079504	2676291	1103693	870760
INC. COSTS BY ELEM. TYPE								
NODES	22998576	16866400	13769469	6963511	179485392	3001745	11101517	10739691
LINKS	5310282	3209582	1704460	1258214	22707856	1893367	865121	505710
TOTAL INCURRED COSTS	28308858	20075982	15473929	8221725	202193248	4895112	11966638	11245401

TABLE 3-22

Incurred Costs Using the 1977 Revised Costing Block

	BCT	AGT	SASK	MANT	BCAN	QUET	NBT	MTT
TOTAL ASSETS	107638386	96536404	57858295	41437574	639083866	15588941	47677249	44222877
INCURRED COSTS								
SWITCHING MACHINES	10882400	5572000	5170000	2194200	58900000	750000	4860800	3886795
TERMINAL REPEATERS	5004398	4093609	2780637	1302283	54942928	635368	3793956	3208117
BRANCHING REPEATERS	5465416	5848855	3279682	3631886	27391008	425116	2289547	3253853
REGULAR REPEATERS	5745791	4587784	2866247	1804011	23802368	2191345	1107204	866842
INC. COSTS BY ELEM. TYPE								
NODES	22027856	16893664	12543981	7569647	146449904	2451539	11183615	10712166
LINKS	5070036	3208579	1552585	1362733	18586176	1550290	867892	503441
TOTAL INCURRED COSTS	27097892	20102243	14096566	8932380	165036080	4001829	12051507	11215607

4

4. EMPIRICAL EVALUATION OF SELECTED CROSS SUBSIDY TESTS

4.1 General

4.1.1 Tests Performed

We present in this section the results obtained for various tests performed during the course of the project. As mentioned earlier, these results were derived with a model in various stages of improvement. Therefore, results are not always strictly comparable. On the other hand, since they appear in the chronological order of testing, the reader will understand better why some model improvements were required and how the final conclusions of this report were eventually reached.

The first serie of tests simply aimed at comparing generated revenues of a service to its stand-alone and incremental costs based on current use of the telephone plant. Obviously, these tests were performed on groups of services where cross-subsidy was suspected, i.e.:

- Public messages / private lines
- Short distance / long distance toll traffic
- Peak traffic / off-peak traffic
- Regional / adjacent / non-adjacent and U.S. traffic

These tests led to the preliminary conclusion that, based on current usage, incremental cost tests were always satisfied given the paramount importance of the plant commonly used by all services and consequently two avenues of research were explored.

- the first one consisted of increasing incremental costs by incorporating the required growth reserve associated with the service. This led to the elaboration of tests based on the prospective use of equipment;
- the second consisted of imposing a definition of cross subsidy much more demanding than the one based on stand-alone and incremental costs alone. This led to the definition of various full cost allocations formulae.

  This second series of tests was performed on a single separation of services, namely, public messages vs private lines.

Finally, the impact of certain known deficiencies of the N.P.P.S. model on the validity of the obtained results was evaluated in the form of sensitivity analyses so as to ascertain the confidence level of these results and enable the project team to reach objective conclusions.

4.1.2 Simulations Procedure

In order to obtain any incremental cost test, two simulations are performed. In a first run, the incurred cost of the network required to support a set of services is calculated and the generated revenues are estimated. In the second run, identical calculations are performed for a subset of these services. Comparison of the results of both simulations yield the incremental cost of the services considered as well as their revenue contribution. Since full allocation of costs is not needed, only part of the N.P.P.S. model is used as shown in diagram 4-1. Traffic for the relevant services is generated and fed to the dimensioning algorithm, the output of which is circuit requirements on the various links of the switching network. These circuit requirements are used first to dimension and cost the switching network then as an input for allocation on the transmission network. Allocation of transmission facilities results in the cost of the transmission network. The final step is to calculate revenues generated by the defined traffic.

4.1.3 Estimation of Costs Incurred in the Transmission on Network.

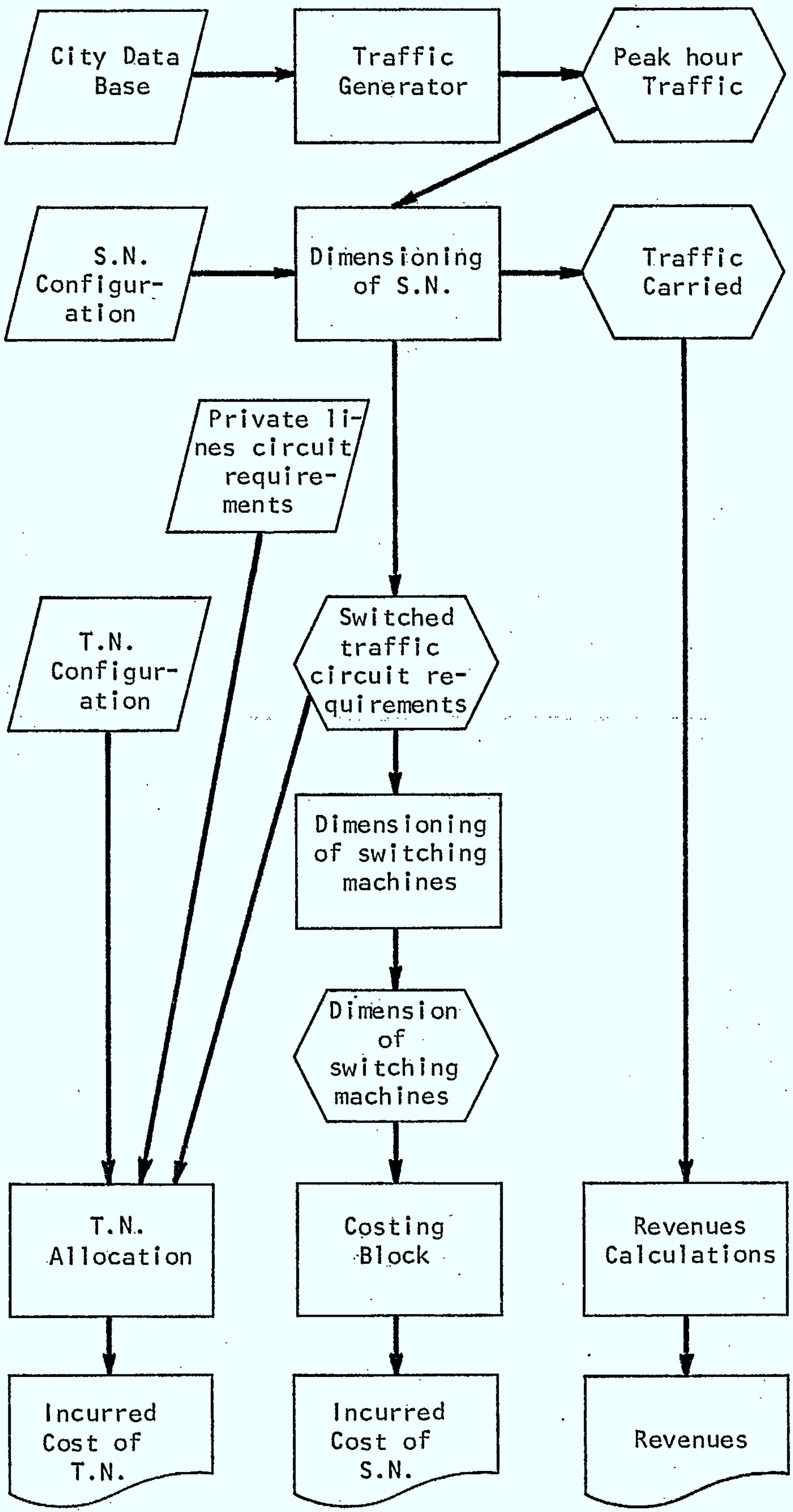
Chapter 3 of this report outlines the physical changes implemented on the N.P.P.S. model. In addition, the way by which some of the model's options are used or some of the results interpreted has been greatly affected through the experience gained by exhaustive empirical testing. The area where this evolution is most evident is certainly the estimation of costs incurred in the transmission network. It will be remembered that during the course of the third phase of the N.P.P.S. project (see Final Report on the Third Phase - Section 4.4.4) two costing concepts to be applied to the transmission network were developed. They were loosely labeled "marginal" and "average cost" concepts although better definitions would be average variable and average capacity costs, respectively. Their definition is recalled graphically in Figure 4-2. One important point to notice is that both definitions are based on installed capacity. Using ultimate capacity would obviously lead to decreases in the average cost while marginal costs would remain more or less unaffected.

Also of importance is the fact that the need for both cost concepts stems from the limitation of the transmission allocation model which accepts linear cost functions only.

Going back to Figure 4-2, one can see that the best approximation to the actual cost of OA circuits (or O'A actual circuits) on a given link, i.e. AE, is given by the average cost AC, the approximation improving with the loading of the link. And this procedure was used in the first serie of tests presented in this section.

Figure 4-1

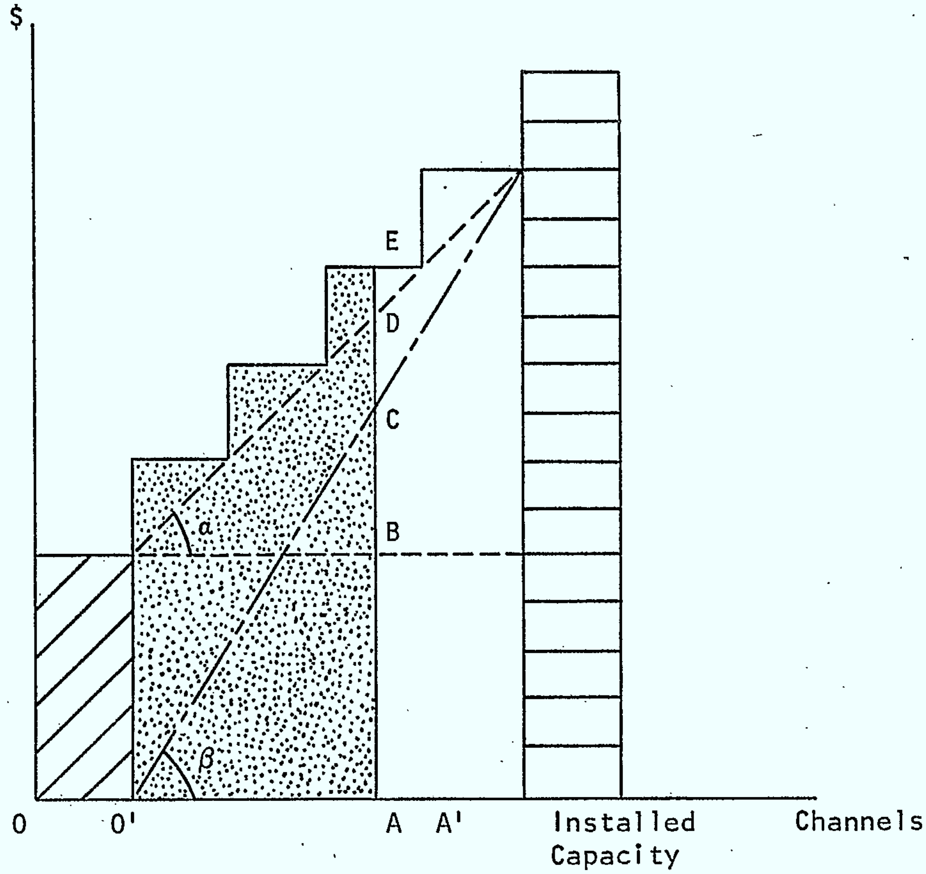
Simulation Flow Chart



T.N.: Transmission Network
S.N.: Switching Network

FIGURE 4-2

Costing of transmission facilities



![hatched] : Protection channel and infrastructure

![lined] : Video dedicated channel(s)

![dotted] : Circuits actually in use.

Ag(α) : Per circuit marginal cost.

Ag(β) : Per circuit average cost.

Actual incurred cost for 0'A operational circuits: AE

Marginal cost for 0'A operational circuits: BD

Average cost for 0'A operational circuits: AC

Fixed cost ± marginal cost: AB + BD = AD

However, a better approximation can be obtained by adding the marginal cost BD to the fixed cost AB. This procedure has the further advantage of giving a degree of approximation which is independent upon the loading of the link. One problem arises however, since the allocation module cannot minimize the sum (fixed cost + marginal cost). Empirical tests have shown that allocations obtained by minimizing average costs yielded a lower (fixed cost + marginal cost) than allocations obtained by minimizing marginal costs. This can be rationalized by the following argument: since the average cost is equal to the marginal cost plus the fixed cost brought back on a per circuit basis, an allocation based on average cost will obviously give some weight to the fixed costs while these are completely ignored in a marginal cost allocation. It follows that the procedure used to cost the transmission network in the second serie of tests was to allocate transmission facilities so as to reduce average cost but to cost the resulting allocation by adding the marginal cost to the fixed cost.

Finally, one could argue that the actual cost associated with supporting O'A circuits is given by the (fixed cost + marginal cost) of O'A' circuits obtained by rounding off O'A to the next higher multiple of the channel size. This procedure has been experimented with and a typical example is given in section 4.4 of this chapter. It presents, however, a serious danger as illustrated by the following example (see Figure 4-3). Assume a link of 3 channels of 300 circuits each. Further assume a fixed cost of $ 10,000. and a marginal cost of $ 10./circuit. Let the circuit requirements on this link be at 200 for traffic messages and 100 for private lines. The corresponding costs given by one method of calculation or the other would be as shown below:

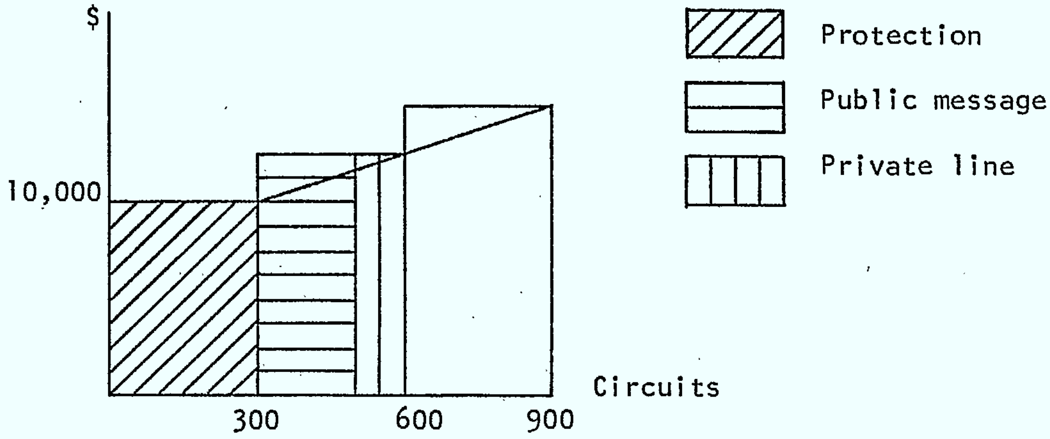
	P.M. alone	Both services	Difference
Circuits required	200	300	100
Fixed cost + marginal cost	12 000	13 000	A = 1 000
Same as above but rounded to higher # of channels	13 000	13 000	B = 0

Assume now a multiplexing plan with a channel loading of 66%. This would be equivalent to circuit requirements of 300 for P.M. and 150 for P.L. and the differences A and B of the previous table would become $ 1 500. and $ 3 000.respectively. If the channel loading is further reduced - say to 50% - A would become 2 000 and B zero. This example clearly indicates that the results could be biased one way or another depending upon which channel loading has been used to approximate the impact of the multiplexing plan and therefore this procedure which consists of costing whole channels only has not been retained for testing purposes.

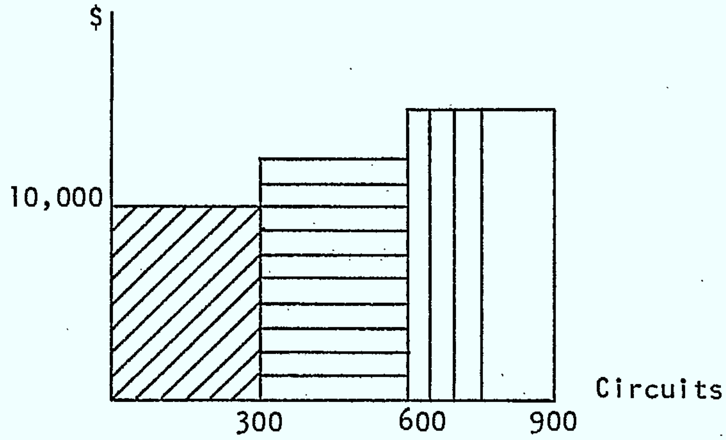
FIGURE 4-3

Costing of transmission network

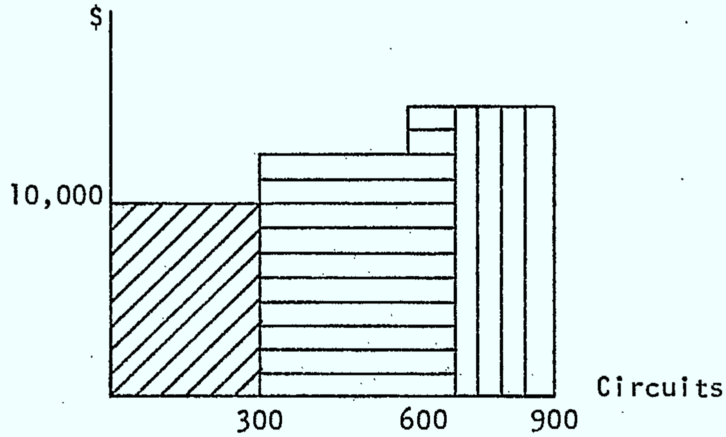
a) 100% channel loading



b) 60% channel loading



c) 50% channel loading



4.2 Tests Based on Current Use of Equipment

4.2.1 Public Messages and Private Lines

Table 4-1 shows the total costs incurred in the switching and the transmission networks required to accommodate first public messages alone and secondly both services. The difference is the incremental cost of private lines. Since this service is not a switched service, there is obviously no incremental cost in the switching network. Also appearing in Table 4-1 are the revenues derived from the services considered as estimated in N.P.P.S. All figures are shown separately for Bell Canada and for the whole network, the relationship between incremental costs and revenues not being always the same at the carrier level.

These comparisons must however be handled very carefully since estimated revenues and costs are not strictly comparable. As a matter of fact, revenues correspond to the part of the service generated in the carrier's territory while costs are those associated with satisfying the whole service over the said territory. For instance, the incremental cost of non-adjacent traffic for the Bell is constituted by the cost of originating, terminating and going through non-adjacent traffic, while calculated revenues are those generated by originating traffic only. We will get back to this issue in more detail in section 5.2 of the present report.

TABLE 4-1

Incremental cost of private lines
(incurred costs and revenues in $ millions)

Service	Carrier	Switching Cost	Transmission Cost (1)	Total Cost	Estimated Revenues (2)
Public Messages	Bell	64.4	16.0	80.4	316.9
	Network	95.8	23.6	119.4	395.2
Both Services	Bell	64.4	21.9	86.3	341.5
	Network	95.8	33.3	129.1	436.7
Incremental Costs & Revenues of Private Lines	Bell	0	5.9	5.9	24.6
	Network	0	9.7	9.7	41.6

(1) Using average cost.
(2) US revenues excluded

4.2.2 O-D pairs < 1 000 miles apart / O-D pairs > 1 000 miles apart

In order to test whether one group of customers cross-subsidized another, three simulations were performed:

- one with all traffic between cities more than 1 000 miles apart;
- one with all traffic between cities less than 1 000 miles apart;
- one with both types of traffic.

Since destination/origin points in the U.S. are not precisely known, U.S. traffic was deliberately omitted from all three simulations.

Table 4-2 is very similar to Table 4-1 and yields the incremental costs of both types of traffic. It can be seen that for pairs > 1 000 miles apart revenues exceed incremental costs by a factor of about 17. For pairs < 1 000 miles apart, the ratio is somewhat lower at about 7.

TABLE 4-2

Incremental cost of O-D pairs more or less than 1 000 miles apart ($ millions)

Simulation and Carrier	Switching Cost	Transmission Cost (1)	Total Cost	Estimated Revenues
Pairs < 1 000				
Bell	60.7	10.2	70.9	298.4
Network	90.7	13.8	104.5	352.9
Pairs > 1 000				
Bell	53.6	2.0	55.6	18.5
Network	83.6	4.7	88.3	42.2
Both Services				
Bell	61.7	11.6	73.3	316.9
Network	94.7	17.2	111.9	395.1
Incremental Costs & Revenues for pairs < 1 000				
Bell	8.1	9.6	17.7	298.4
Network	11.1	12.5	23.6	352.9
Incremental Costs & Revenues for pairs > 1 000				
Bell	1.0	1.4	2.4	18.5
Network	4.0	3.5	7.5	42.2

(1) Using average cost.

4.2.3 Regional / Adjacent / Non-adjacent (and U.S. traffic)

A 3-service experiment gives us the possibility of performing six incremental cost tests and requires seven simulations. Let A, B, C be the services. Then the incurred costs of providing any subset of those services can be computed. They are C_{ABC}, C_{AB}, C_{AC}, C_{BC}, C_A, C_B, C_C. Let R_A, R_B, R_C be the revenues generated by the respective services. If there is no cross-subsidization, then the following six inequalities must be satisfied:

$$R_A \geqslant C_{ABC} - C_{BC}$$

$$R_B \geqslant C_{ABC} - C_{AC}$$

$$R_C \geqslant C_{ABC} - C_{AB}$$

$$R_A + R_B \geqslant C_{ABC} - C_C$$

$$R_A + R_C \geqslant C_{ABC} - C_B$$

$$R_B + R_C \geqslant C_{ABC} - C_A$$

Total incurred costs for each subset of services are shown in Table 4-3. Resulting incremental costs for each service or combination of two services appear on Table 4-4 where they are compared to corresponding revenues. In all cases, revenues are larger than incremental costs, in other words, all tests are passed. The ratio of revenues over incremental costs varies however quite substantially between simulations and between carriers as shown in the last column of Table 4-4.

TABLE 4-3

Three-service Experiment
Total Incurred Costs and Revenues
($ million)

Simulation & Carriers	Switching Cost	Transmission Cost (1)	Total Cost	Estimated Revenues (2)
Reg + Adj + N-Adj + US				
Bell	64.4	16.1	80.5	316.9
All carriers	95.8	23.5	119.3	395.2
Adj + N-Adj + US				
Bell	56.1	7.7	63.8	33.6
All carriers	87.5	14.6	102.1	96.1
Reg + N-Adj + US				
Bell	63.7	15.3	79.0	305.2
All carriers	94.7	21.4	116.1	352.9
Reg + Adj				
Bell	60.7	9.9	70.6	292.2
All carriers	90.7	12.5	103.2	341.4
N-Adj + US				
Bell	56.1	6.9	63.0	21.8
All carriers	86.6	12.4	99.0	53.8
Adj				
Bell	53.6	1.3	54.9	11.8
All carriers	83.6	3.3	86.9	42.3
Reg				
Bell	59.8	9.0	68.8	283.4
All carriers	89.9	10.0	99.9	299.1

(1) Using average cost
(2) Excluding US

TABLE 4-4

Three-service Experiment
Incremental Costs
($ million)

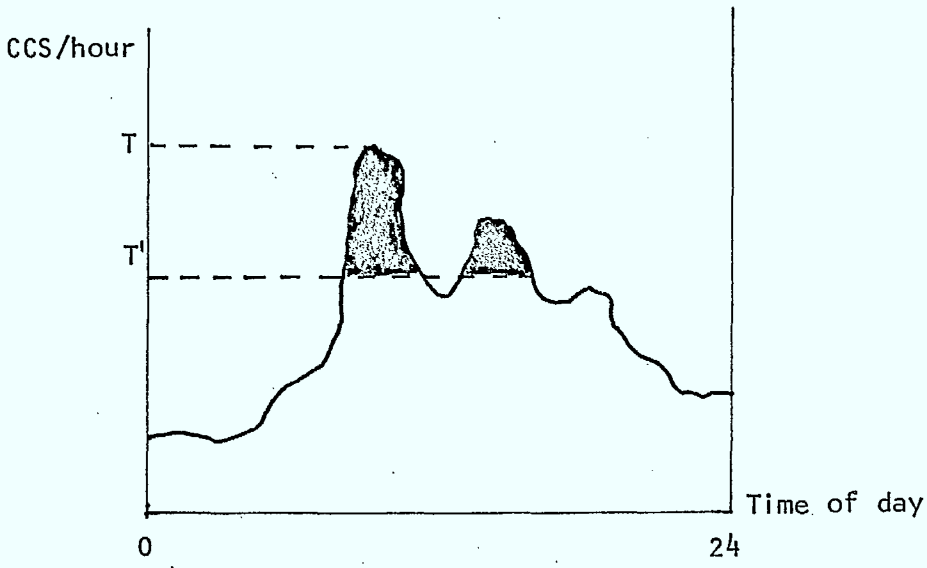
Subset test	Carriers	Incremental cost in switching	Incremental cost in transmission	Total incremental cost (2)	Revenues N.P.P.S. Estimates (1)	Ratio (1) / (2)
Adj +	Bell	4.6	7.1	11.7	33.6	2.9
Non-Adj. + US	All	5.9	13.6	19.5	96.1	4.9
Reg +	Bell	10.8	14.8	25.6	305.2	11.9
Non-Adj. + US	All	12.2	20.3	32.1	352.9	11.0
Reg +	Bell	8.3	9.2	17.5	292.2	16.7
Adj.	All	9.2	13.2	22.4	341.4	15.2
Non-Adj. + US	Bell	3.7	6.2	9.9	21.8	2.2
	All	5.1	11.1	16.2	53.8	3.3
Adj.	Bell	.7	.8	1.5	11.8	7.9
	All	1.1	2.2	3.3	42.3	12.8
Reg.	Bell	8.3	8.4	16.7	283.4	17.0
	All	8.3	9.1	17.4	299.1	17.2

4.2.4 Peak hour traffic / Non-peak traffic

Traffic profiles during an average business day have the general form shown in Figure 4-4.

FIGURE 4-4

Typical Traffic Profile



The network is dimensioned for the peak hour traffic T and costs C(T). Should the peak-hour traffic be smaller, say T', a smaller cost would result C(T'). The test hence consists in comparing the incremental cost of peak-hour traffic (i.e. C(T) - C(T')) to the revenues it generates. These revenues are calculated by multiplying the shaded area of Figure 4-4 by the appropriate tariff. For this experiment, T' was arbitrarily set at 70% of T.

Total incurred costs for peak and reduced peak simulations are presented in Table 4-5. The incremental cost of peak traffic is derived in Table 4-6 and compared to its revenues.

It can be observed that once more incremental revenues largely exceed incremental costs.

TABLE 4-5

Total incurred costs
Peak/off-peak traffic
($ millions)

Simulations & carrier	Switching Costs	Transmission Costs (1)	Total Costs
Peak			
Bell	64.4	16.1	80.5
All carriers	95.8	23.5	119.3
Reduced peak			
Bell	61.5	13.0	74.5
All carriers	92.8	19.3	112.1

(1) Using average cost.

TABLE 4-6

Incremental costs of peak traffic
($ millions)

Carrier	Switching	Transmission	Total (2)	Revenues (1) NPPS Estim. (*)	(1)/(2)
Bell	2.9	3.1	6.0	41.5	6.9
All carriers	3.0	4.2	4.2	51.8	7.2

(*) Excluding US

4.2.5 Preliminary Comments on First Series of Tests

The tests presented so far seem to indicate that the incremental cost is always satisfied. In addition, the ratio of revenues over incremental cost is so large that it could hardly be reduced to values inferior to 1 simply by improving certain approximations of the model.

It appears in certain instances that revenues exceed the stand-alone cost of a service (e.g. public messages). Strictly speaking, the stand-alone cost of a service should be representative of all facilities required to support this service. Consequently, the stand-alone cost of any long distance service would include the cost of the local network. Given its relative importance in the total plant (see section 3.6), it becomes clear that stand-alone cost tests are also satisfied.

The most important point to notice however is the large discrepancy which exists between the cost of the existing transmission network and the part which is allocated to the various services or groups of services tested in this section. The total cost incurred in the toll network (as estimated in N.P.P.S. when costing requirements listed in data base) is compared to the cost allocable to public messages and private lines (as computed in section 4.2.1) in Table 4-7.

TABLE 4-7

Comparison of total cost of plant to cost allocable to Public Messages and Private Lines ($ millions)

	Total incurred cost of plant as estimated in N.P.P.S. (2)	Cost allocated to PM and PL (3)	(3) / (2) %
Switching Network	106.7	94.7	89
Transmission Network (1)	184.8	33.3 (4)	18
Total	291.5	128.0	44

(1) Excluding channels used for video.
(4) It will be seen in Table 4-8 that when using the (fixed cost + marginal cost) formula this value becomes 86.1.

It becomes clear from this table that this difference has to be explained before any further tests are performed and we give below a list of possible contributing factors.

i) Circuit requirements as estimated by dimensioning the switching network are far below those contained in the data base (14 100 vs 23 600). It must be remembered that the dimensioning algorithm is applied to traffic which

- is estimated based on limited data (traffic between 17 cities during two weeks of July 1971);
- does not include WATS, TWX and data transmission;

ii) It was mentioned earlier that costing the transmission network with the average cost formula is a poor appoximation when the link loading is low (see section 4.1.3).

It will be seen, for instance, that costing transmission facilities with the (fixed + marginal) cost approach would result in a total cost of \$ 86.1 millions (see Table 4-8) to be compared to \$ 33.3 millions obtained with the average cost formulation (Table 4-7).

iii) It is known that trade-offs between multiplexing and radio costs result in a channel loading which generally does not exceed 75% (see section 3.5).

iv) A certain amount of unused equipment is included in the plant as a growth reserve.

v) Finally, it must be remembered that the N.P.P.S. allocation procedure does not take survivability constraints into account and therefore yields an allocation which is cheaper than it would be in reality.

4.3 Tests Based on Prospective use of Equipment

4.3.1 General

In view of the results presented in the previous section, a new serie of test was performed. It was decided to concentrate on the appropriate calculation of costs rather than on various splits of the services taken into account. All tests were consequently based on a public message/private lines separation. In order to improve estimation of costs and in line with the observations of section 4.2.5, the following rules were applied:

i) Transmission facilities were costed using the fixed cost + marginal cost approach.

ii) The multiplexing plan was approximated by the formulation suggested in section 3.5 (i.e. circuit requirements constitute integer number of groups, the loaidng of which does not exceed 75%).

iii) Since no precise definition of the growth reserve is available, various policies were tested by which growth reserve was defined as the incremental cost associated with the growth of a service over 1, 2 and 3-year periods.

4.3.2 Description of Simulation Runs and Incremental Cost Tests.

Five simulation runs were performed.

The first one is based on present demand.

The next three consider prospective demand 1, 2 and 3 years from now using:

- a 12% annual growth rate for public messages[(1)]
- an 18% annual growth rate for private lines[(1)]

To test the sensitivity of the results to growth rates, a fifth simulation was performed considering prospective demand in year 3 but with a 10% annual growth rate for private lines.

The results of the five simulations are presented in Table 4-8. It must be remembered that, private lines being a non-switched service, only transmission costs have been analyzed.

One will also notice that the total cost of the transmission network increases with the length of the planning horizon since capacity had to be increased on a certain number of links in order to render the allocation feasible. The corresponding incremental cost of private lines can easily be derived from these results and is shown below.

Basis of calculation for growth reserve	Incremental cost of private lines including growth reserve ($ millions)
No growth reserve	10.1
One year planning horizon	12.9
Two year planning horizon	15.2
Three year planning horizon	16.8
Three year planning horizon (lower growth rate for P.L.)	13.4

---

(1) These rates were applied uniformly to all existing demands and no new demands were considered.

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TABLE 4-8

Simulation results

Incurred costs in $ millions

Simulation	Services considered	Incurred fixed cost	Incurred variable cost[2]	Total incurred cost	Cost of[1] Excess capacity	Total cost of transmission NW (excluding channels used for video)
#1 Present demand	P.M. only	49.4	26.6	76.0	108.8	184.8
	P.L. only	39.1	10.7	49.8	135.0	
	Both services	49.4	36.7	86.1	98.7	
#2 Demand after one year of growth	P.M. only	49.4	28.6	78.0	107.8	185.8
	P.L. only	39.1	12.2	51.3	134.5	
	Both services	49.4	41.5	90.9	94.9	
#3 Demand after two years of growth	P.M. only	49.4	31.1	80.5	109.3	189.8
	P.L. only	39.1	13.7	52.8	137.0	
	Both services	49.4	46.2	95.7	94.1	
#4 Demand after three years of growth	P.M. only	49.4	34.1	83.5	108.2	191.7
	P.L. only	39.1	15.6	54.7	137.0	
	Both services	49.9	50.4	100.3	91.4	
#5 Demand after three years of growth (lower rate for P.L.)	P.M. only	49.4	34.1	83.5	107.8	191.3
	P.L. only	39.1	13.4	52.5	138.8	
	Both services	49.4	47.5	96.9	94.4	

(1) Including $ 9.2 millions for links not used at all.
(2) It can be seen that the incurred variable cost associated with both services is generally lower than the sum of individual variable costs. This results from the rounding procedure which, when applied to both services, results in requirements smaller than the sum of individual rounded requirements.

Although consideration of a 3-year growth reserve increases the incremental cost of private lines by 66%, the revised incremental cost figure still remains much smaller than revenues estimates of $ 41.6 millions.

4.3.3 Tests Based on Full Allocation of Costs

All tests performed so far have shown that the incremental costs of private lines is always covered by generated revenues. If one examines closely the total transmission cost of supporting both private lines and public messages, it can be broken down as follows:

	$ millions
Incremental cost of private lines	10.1
Incremental cost of public messages	36.3
Cost of equipment used jointly	39.7
Total transmission cost (excluding growth reserve)	86.1

If one further considers a 3-year growth reserve and compares all these costs to total costs of the existing transmission network, one obtains a graph of the form shown in Figure 4-5, where surfaces are proportional to costs.

It becomes clear then that a definition of cross-subsidy based on incremental costs alone is not sufficient given the importance of the common costs and other non directly allocable costs and given that total costs must eventually be recovered.

Two questions then arise:

i) How should common costs be allocated;

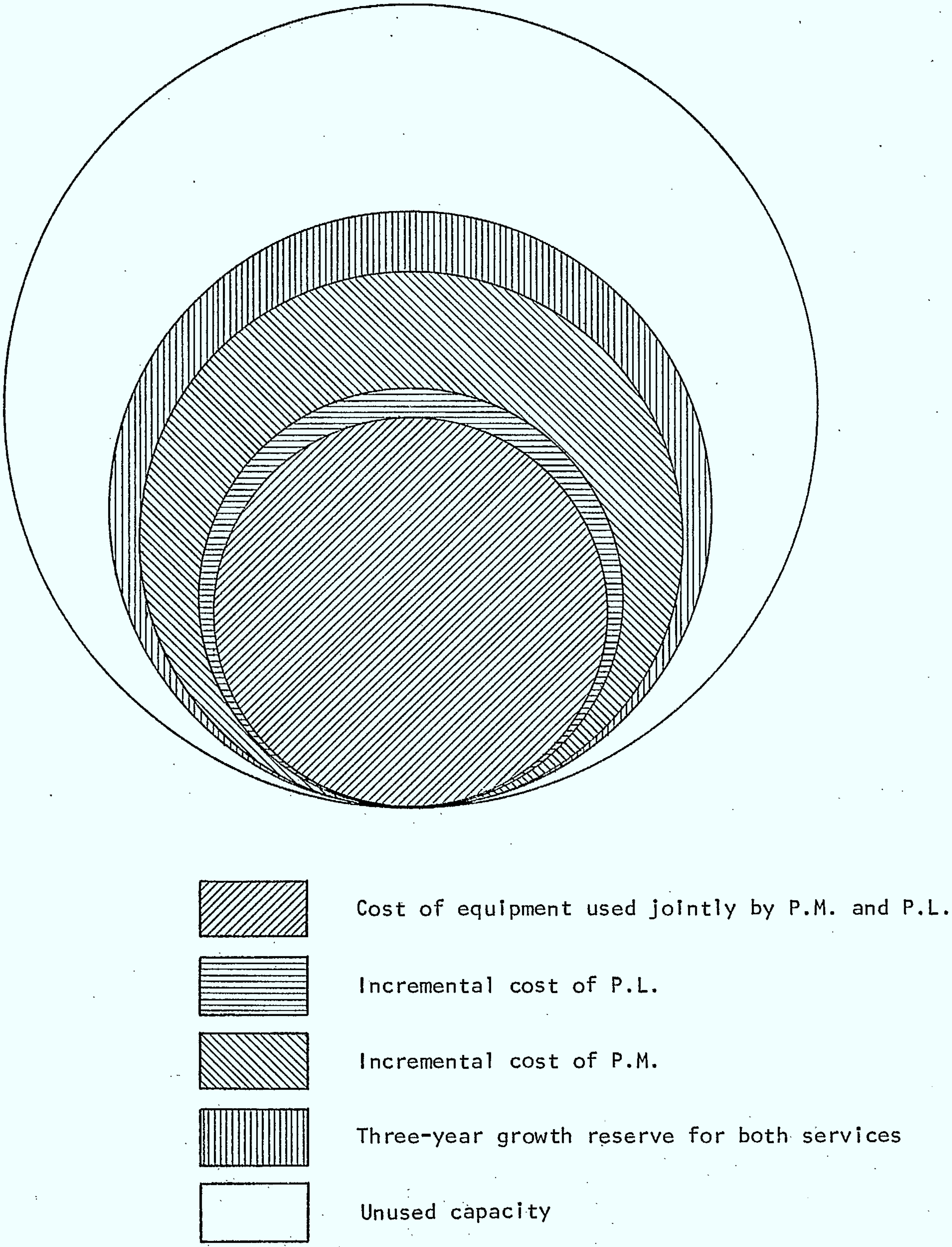
ii) Which common costs should be allocated, namely, should the cost associated with the so called "pure excess" capacity be paid by the consumer or by the carrier. This depends obviously on the origin of this excess which could result from:

- deficiencies of the model (i.e. not enough traffic, no survivability constraints....)

- a larger planning horizon than used in our calculations (i.e. more than three years)

FIGURE 4-5

Allocation of incurred costs in transmission
network as estimated by N.P.P.S.
(Surfaces are proportional to estimated costs)



- a very safe and/or suboptimal planning of the network by the carriers;

- a mixture of the three above-mentioned factors

This leads us to the application of cost separation formulae presented in section 2.3 of this report, the principles of which are recalled below:

METHOD OF ALLOCATION FOR

METHODS	CAPACITY IN USE	GROWTH RESERVE	PURE EXCESS
A	Full allocation . . .	based . . . . on	usage
B	Full allocation . . .	based . . . . on	game theoretic formula
C	Allocation based on usage	Allocation based on present usage	Not allocated
C'	Allocation based on usage	Allocation based on future usage	Not allocated
D.	Gâme theoretic allocation	Game theoretic allocation based on present usage	Not allocated
D'	Game theoretic allocation	Game theoretic allocation based on future usage	Not allocated

Table 4-9 presents all data necessary to calculate cost allocations using the methods described above. The three first columns (Stand-alone cost, incremental cost and "fair" allocation of used capacity) are directly derived from Table 4-8. The allocation based on usage was obtained by the N.P.P.S. model.

Table 4-10 presents cost separations based on methods A and B. It can be noticed that for both methods, full allocated costs of private lines exceed the estimated revenues of \$ 41.6 millions. It can also be seen that the "exhaustive" incremental cost (defined in 2.4.2 as the true incremental cost plus a "fair share" of excess) yielded by method B for private lines also exceeds revenues (i.e. \$ 44.6 vs \$ 41.6 millions).

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TABLE 4-9

Cost allocation of used capacity
($ millions)

Simulations	Services	Stand alone cost	Incremental cost	"Fair" allocation of used capacity	Allocation of used capacity based on usage
#1	P.M.	76.0	36.3	56.1	61.9
Present	P.L.	49.8	10.1	30.0	24.2
#2	P.M.	78.0	39.6	58.8	65.3
One year	P.L.	51.3	12.9	32.1	25.6
#3	P.M.	80.5	42.9	61.7	66.8
Two years	P.L.	52.8	15.2	34.0	28.9
#4	P.M.	83.5	45.6	64.5	68.3
Three years	P.L.	54.7	16.8	35.8	32.0
#5	P.M.	83.5	44.4	64.0	70.9
Three years slower growth for P.L.	P.L.	52.5	13.4	32.9	26.0

TABLE 4-10

Cost allocations based on methods A and B
($ millions)

		Method A		Method B	
		P.M.	P.L.	P.M.	P.L.
Used Capacity	Common Costs	61.9	24.2	19.8	19.8
	Incremental Cost			36.3	10.1
Unused capacity		71.0	27.7	64.3	34.5
Total		132.9	51.9	120.4	64.4

TABLE 4-11

Cost allocations based on methods C and C'
($ millions)

Planning Horizon		Method C		Method C'	
		P.M.	P.L.	P.M.	P.L.
One year	Used capacity	61.9	24.2	61.9	24.2
	Growth reserve	3.5	1.3	3.4	1.4
	Total	65.4	25.5	65.3	25.6
Two years	Used capacity	61.9	24.2	61.9	24.2
	Growth reserve	6.9	2.7	6.7	2.9
	Total	68.8	26.9	68.6	27.1
Three years	Used capacity	61.9	24.2	61.9	24.2
	Growth reserve	10.3	4.0	9.7	4.5
	Total	72.2	28.2	71.6	28.7
Three years (lower growth on P.L.)	Used capacity	61.9	24.2	61.9	24.2
	Growth reserve	7.8	3.0	7.9	2.9
	Total	69.7	27.2	69.8	27.1

Estimated Revenues of Private Lines: 41.6

Cost allocations based on methods C and C' for various planning horizons are exhibited in Table 4-11. Both these methods do not allow for the estimation of an "exhaustive" incremental cost and only full allocations are computed. It can be seen however that revenues of private lines always exceed their fully allocated cost independently of the planning horizon and the method chosen.

Cost allocations based on methods D and D' for various planning horizons appear in Table 4-12. It can be seen that the "exhaustive" incremental cost of private lines never exceeds \$ 15.2 millions while the fully allocated cost varies between \$ 31.7 and \$ 35.1 millions according to the planning horizon and the method selected. One can also notice that fully allocated costs based on the game theoretic approach always disfavour private lines when compared to allocations based on usage. As a matter of fact, a game theoretic allocation splits evenly the costs of the core among participating services while the split is proportional to usage in the other method.

4.4 Sensitivity Analysis

4.4.1 General

In the course of chapters 3 and 4, we have outlined a number of model and formulation qualifications which could affect the outcome of tests performed. It is the purpose of this section to ensure the general validity of our results through appropriate sensitivity analyses. Since the "fair" and full allocation of costs methodologies have not been applied to a full extent, the thrust of our effort will be directed towards incremental cost tests.

4.4.2 Omission of Services

It will be remembered that the demand generated by WATS and TWX are not considered and that facilities required by video are pre-assigned. Assume now all these services are taken into account and the cost of an incremental service is estimated. How would this cost compare with costs determined previously? The addition of switched services would result in an increased efficiency of the switching network and hence a reduced incremental switching cost (see Figure 4-6 (a)). In the transmission network, two situations may occur (see Figure 4-6 (b) and (c)):

i) all links are unsaturated. In which case, the routing of the incremental service is still valid and incremental transmission costs are unchanged;

ii) the presence of new services saturates certain links and forces the routing of the incremental service via more expensive routes in which case the incremental cost of the service is increased.

TABLE 4-12

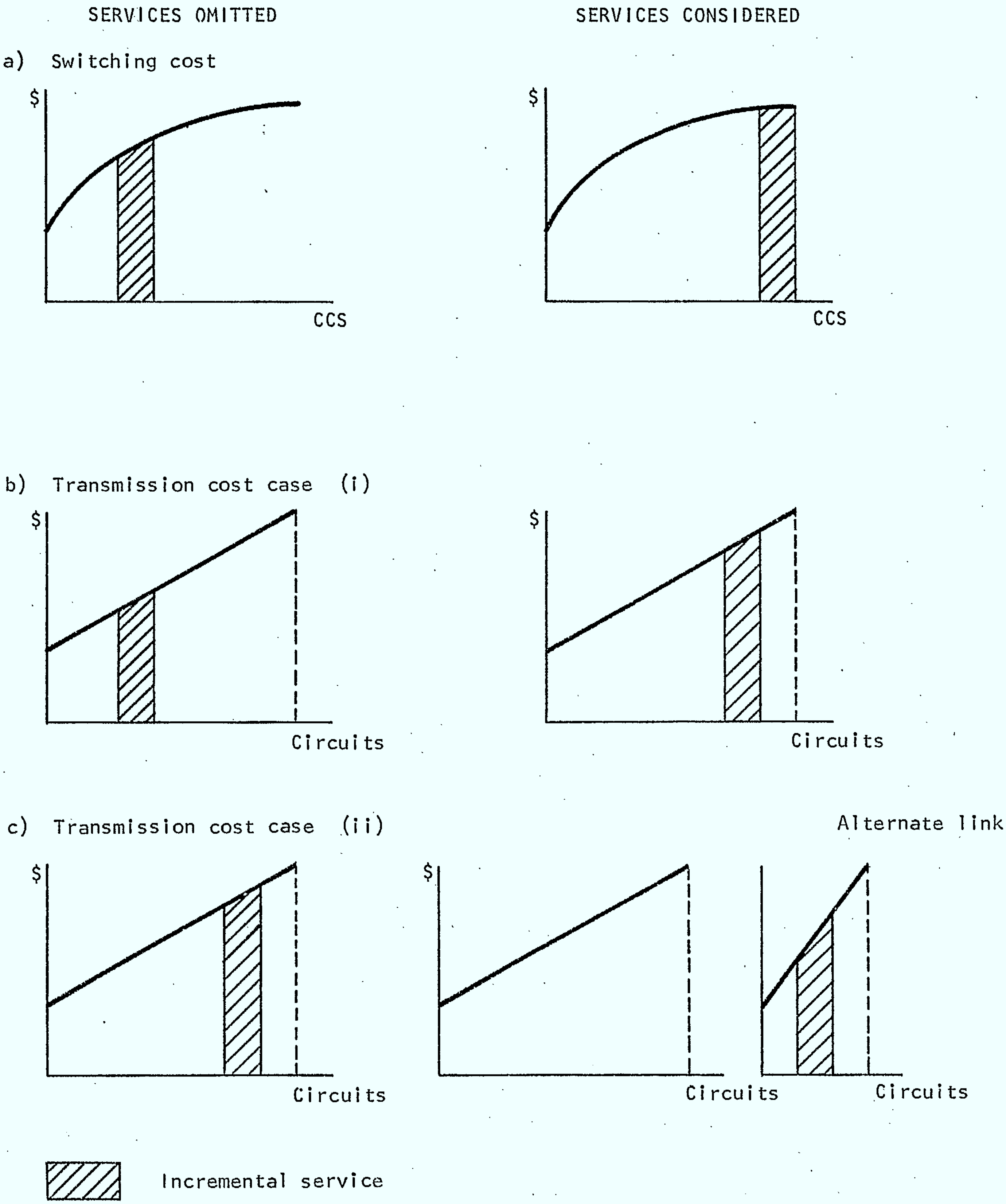
Cost allocation based on methods D and D'
($ millions)

Cost Component	Planning Horizon: One year P.M.	One year P.L.	Two years P.M.	Two years P.L.	Three years P.M.	Three years P.L.	Three years (lower growth P.L.) P.M.	Three years (lower growth P.L.) P.L.
Used capacity (1)								
Common Costs	19.8	19.8	19.8	19.8	19.8	19.8	19.8	19.8
Incremental Cost	36.3	10.1	36.3	10.1	36.3	10.1	36.3	10.1
Total	56.1	30.0	56.1	30.0	56.1	30.0	56.1	30.0
Growth reserve according to D	3.1	1.7	6.3	3.3	9.3	4.9	7.0	3.8
Exhaustive incremental cost according to D	39.4	11.8	42.6	14.4	45.6	15.0	46.3	13.9
Growth reserve according to D'	3.1	1.7	6.2	3.4	9.1	5.1	7.1	3.7
Exhaustive incremental cost according to D'	39.4	11.8	42.5	14.5	45.4	15.2	43.4	13.8
Total allocable cost according to D	59.2	31.7	62.4	33.3	65.4	34.9	63.1	33.8
Total allocable cost according to D'	59.2	31.7	62.3	33.4	65.2	35.1	63.2	33.7

(1) Identical for both methods and independent of planning horizon.

FIGURE 4-6

Impact of omission of services on incremental cost tests



Let us concentrate on case (ii) which may invalidate our results and let us quantify this possibility on the case of private lines. In section 4.3.2, we estimated the incremental cost of private lines three years from now at $ 16.8 millions. This rested on the assumption that public message traffic was increased by 40% and private line requirements by 65%. Had we assumed zero-growth on private lines, the incremental cost would have been obviously lower. In other words, the incremental cost of private lines would increase from $ 10.1 millions, the present estimate, to <u>at most</u> $ 16.8 millions by introducing additional switched services generating an amount of traffic equivalent to 40% of the omitted traffic presently considered. If one further remembers that estimated revenues of private lines are $ 41.6 millions, it can then safely be concluded that, even if omitted services represent more than this 40% proportion, they could not induce an increase in the incremental cost of private lines sufficient to upset the result of our incremental cost test.

4.4.3 Indivisibilities in Transmission Network

The question of indivisibilities in the transmission network was raised in section 4.1.3. We present below an experiment which proves that their existence cannot invalidate the incremental cost test performed on private lines. The experiment consisted in calculating the incremental cost of private lines for present use by costing the transmission network in integer number of channels only. Results appear in Table 4-13 where they are compared to results obtained in section 4.3.2 on a per-circuit basis. It can be seen that this procedure would reduce the incremental cost of private lines by 11%.

It was also mentioned in section 4.1.3 that the results obtained by this costing procedure were dependent upon the group loading factor used to approximate multiplexing (in this instance 75%). If one considers the cost of whole channels required by private lines when this service is alone, one obtains $ 41.0 millions and one can further observe that the average loading of the channels used is of 26%. This clearly indicates that, no matter which multiplexing approximation is selected, the incremental cost of private lines will always remain below $ 41.0 (and hence $ 41.6 millions, the estimated revenues) when estimated on a per channel basis.

4.4.4 Growth Rates and Planning Horizon

Tests based on prospective use of equipment and presented in section 4.3.2 indicate that the incremental cost of private lines increases from $ 10.1 to $ 16.8 millions by inclusion of a 3-year growth reserve assuming 18% annual growth, and $ 13.4 millions when using the lower growth rate of 10%. In other words, the incremental cost of the service is more or less proportional to the assumed volume of service ($16.8 \simeq 10.1 \times 1.18^3$) It can therefore be inferred that more generous assumptions with regards to the growth reserve (say 6 year planning horizon and 20% annual growth) would still not

TABLE 4-13

Impact of indivisibilities in transmission network on general test validity

Incurred costs of transmission network ($ millions)

Services considered	Costing on a per circuit basis (from Table 4-8; simulation #1)	Costing on a per channel basis
Public messages only		
Fixed cost	49.4	49.4
Variable cost	26.6	56.2
Total cost	76.0	105.6
Private lines only		
Fixed cost	39.1	39.1
Variable cost	10.7	41.0
Total cost	49.8	80.1
Both services		
Fixed cost	49.4	49.4
Variable cost	36.7	65.2
Total cost	86.1	114.6
Incremental cost of private lines	10.1	9.0

result in an incremental cost larger than estimated revenues. (e.g. $10.1 \times 1.20^6 = 30.2 \ll 41.6$)

4.4.5 Toll Related Costs in Local Network

Incremental costs derived by the N.P.P.S. model and presented in this chapter pertain only to the toll network. It is recognized, however, that some toll related costs are incurred in the local network. Consequently, to render an incremental cost test fully valid, incremental costs incurred in the local network by a toll service must be accounted for.

A first approximation to incremental toll related costs in the local network presented in section 3.6 showed that they represented roughly 5% of the total costs incurred in the toll network. Since these costs are mainly related to traffic volume (and not to distance of the various O-D's, for instance), it is more appropriate to express them in relation to toll switching costs. The ratio then becomes approximately 15%. It can be seen that an increase of 15% of all incremental switching costs estimated in earlier sections of this chapter would not alter the result of any incremental cost test performed.

4.4.6 Approximation to Multiplexing Problem

The multiplexing problem was approximated in section 4.3 by assuming a maximum group loading of 75%. In other words, actual circuit requirements were multiplied by 1/.75 = 1.33 and rounded up to the next multiple of 12. Consider simulation 5 of section 4.3.2 where demand was increased approximately by 37% (11% compounded over 3 years) and then subjected to the 75% loading rule. This is equivalent to requiring $\frac{1.37}{.75} = 1.82$ times the actual circuits presently required or constraining present demand to a maximum $\frac{1}{1.82} = 55\%$ group loading. Since incremental cost tests were passed in this simulation, we can infer that a substantial reduction of the group maximum loading would not affect the validity of our conclusions.

4.4.7 Model Calibration

There is one apparent calibration problem, namely the difference between circuit requirements and circuits availability in the transmission network. Since estimated costs are compared to revenues derived by the model for the traffic considered, this deficiency does not affect the validity of incremental cost tests. On the other hand, it will obviously bias comparisons between fully allocated costs and revenues.

4.4.8 Survivability Constraint

Although the conceptual formulation of the survivability problem has been resolved, the N.P.P.S. software is still not ready to accept survivability constraints. We will, however, attempt to show, in a very crude way, that the additional cost of attaining suitable survivability is not sufficient to offset the validity of our incremental cost test results.

Let us focus again on the case of private lines. Total costs associated with satisfaction of both services presented in section 4.3.2 can be broken down as follows:

	$ millions
Common fixed costs	49.4
Incurred variable costs	
P.M.	26.2
P.L.	10.5
Total costs	86.1

Since the fixed costs are common to both services and since survivability constraints are also applicable to both services, we need only be concerned with the impact of survivability on variable costs. Assume 50% of private lines to be routed on the cheapest route at a variable cost of 10.5/2 = $ 5.25 millions. Variable cost associated with the remainder of the traffic would have to be larger than (41.6 - 5.25) = $ 36.3 millions to invalidate our incremental cost test since revenues are estimated at $ 41.6 millions. In other words, the alternate routing would have to be, on the average, 36.3/5.25 or seven times more expensive than the direct routing. Given the information we have on the network, this is clearly impossible.

4.4.9 Conclusions

In the previous sections, we have shown that any of the factors considered could not, independently, invalidate the results of our private lines incremental cost test. Due to approximations in the model, we have not been able to evaluate the joint impact of all factors when taken together. Given the large difference observed between incremental costs and revenues, we are fairly confident, however, that the incremental cost test would still be satisfied. As a matter of fact, incremental costs would have to be increased by 300% before any positive test result is encountered.

5

5. ASSESSMENT OF TESTS PERFORMED

5.1 Summary of Results

Going from theoretical test statements to empirical implementation with the logic of the NPPS model and the data at hand, the computations have shown that the generalized incremental tests (GICT) are passed for all partitions of services chosen in the tested examples. Moreover, in each example, if the sub-additivity hypothesis as well as the hypothesis asserting that the revenue of the "grand coalition" equals its cost, are true, the generalized stand alone tests (GSAT) are also passed without having to be computed (see chapter 2). This somewhat reduces the problems since the actual network configuration and its associated costs are often not appropriate for a small service to stand alone. Therefore, if one is willing to accept the notion of cross-subsidy as described earlier, it follows that no such subsidy has been detected in our examples.

Another clear and interesting finding is the fact that incremental costs are often relatively small with respect to common cost. This could explain the large difference observed between the revenue generated by a subset of services and its incremental cost. As a further result, it should be noted that throughout the test series it has been recognized that a relatively large installed excess capacity was present in the network model over a normal three years growth reserve. However, some transmission links had been found to be saturated in the prospective use base tests. Finally, some sensitivity evaluations have shown the results were rather robust.

5.2 Validity in the Context of Telecommunications

The results of the analyses carried out may become a source of encouragement for the policy maker since the hypotheses made to derive the tests are easily acceptable for a careful observer of the telecommunications field (We recommend to read again sub-section 2.3.2 on the qualifications).

The subadditivity hypothesis which states that there can be no total cost increase when the two services are planned to be supported by an integrated system rather than being each on its own, seems plausible in the telecommunications fields. Many forms of economies of scale are treated in the scientific literature (from technical to transactional economies of scale) so it is safe to use that hypothesis. This is also the hypothesis which requires dealing with a global network to generate their economies. In the theoretical chapter, the hypothesis requiring zero price cross-elasticities is mentioned. When is it needed? If one considers that the incremental tests are made for a set of given tariffs to which consumers are fully adapted, the hypothesis is not needed since elasticities are just coefficients of reaction to <u>changes</u> in prices. On the other hand, if one is ready to modify tariffs with core methodologies analogous to what is described in Section 2.1, this hypothesis is crucial. Otherwise, the theory becomes much more complex and the estimations of demand functions become imperative. Therefore, tariff modifications are much more difficult than testing for cross-subsidy since the reactions of customers must be predicted.

The important role of growth reserves is certainly a source of difficulty. To allocate such reserves with a prospective causal responsibility principle in view requires that adequate methods be devised and that services be redefined in a time framework.

Another behavioral hypothesis must be evaluated. It has been seen that the theory assumes a fundamental "liberty" (or egotism) for each service as if a service was a person. For practical purposes, only a few services can be considered at a time and the whole affair may be viewed as a conceptual experiment where each chosen service is personalized. It must also be added that most of the time, the regulatory rules do not permit the creation and dissolution of service coalitions. In any case, the entire apparatus of the game theory can be used by the regulator who, on an "equity basis", can decide to establish tariffs as close as possible to a solution that free economic agents would have naturally reached if they had enough information and economic rationality. If ranges of values for tariffs are available respecting the stand alone and incremental costs inequalities, the regulator can pursue other objectives without feeling the pressure from the group of people consuming regulated services.

The "fair" formula is a weighted average of incremental costs that a "service" should be fairly asked to pay for joining coalitions without taking into consideration the sequence of entries into the coalitions. It is a concensus type formula. If people are willing to accept it, so much the better. The share of a service computed with this formula covers obviously the incremental cost of joining the grand coalition and it is a better bargain than the stand alone cost. One of the disadvantages of this formula however, is that it depends upon the way the services are singled out. In other words, costs allocated to one service will vary according to the number of services into which remaining traffic is broken down. Furthermore, since there are toll related costs in the local network, the fair formula should be applied to the whole telephone network, local services included.

5.3 Applicability to Regulated Companies

5.3.1 Tests at the Carrier Level

All tests performed in the course of this project were done at the level of the national system. The problem which the regulator faces however, involves additional complexity since he is testing for cross-subsidies at the carrier level. It is evident that the non-existence of cross-subsidies among services at the national level does not necessarily imply the same at the carrier level. Furthermore, tests at the national level do not allow the probing of eventual cross-subsidies among carriers. In the course of chapter 4, we have presented partial results at the carrier level but indicated that the costs thereby derived can only be compared to generated revenues if the service considered is regional. Otherwise it should be compared to post-settlement revenues. In the latter case, if any of those incremental costs did exceed corresponding post-settlement revenues, it could indicate either that the sharing principle is disadvantageous to the considered carrier or that the prevailing rate structure is not cross-subsidy free. We present below a methodology which could determine which case applies.

i) First Series of Tests

The first series of tests would compare the incremental cost on the whole network of interregional (i.e. adjacent, non-adjacent, and U.S.) traffic originating in one carrier's territory with (pre-settlement) revenues collected by the considered carrier. If one such test was not passed, it would imply that one set of customers does not pay for its incremental cost and consequently that the interregional rate structure is not subsidy-free.

ii) Second Series of Tests

Assuming all tests to be passed in the first series, the second series of tests would compare the incremental cost incurred by one carrier for all interregional traffic with post-settlement revenues collected by the said carrier. A positive test in this case would imply that the sharing scheme utilized discriminates against the considered carrier.

Given the results obtained in the course of this project, namely the relatively small magnitude of incremental costs when compared to total costs, it is probable that incremental cost tests as previously defined would not lead to any positive conclusion. Therefore, more stringent tests may have to be developed in the same line as those presented in section 4.3 (e.g. taking account of growth reserve or excess capacity).

5.3.2 Practical Applicability

Another problem which arises is the practical implementation of procedures for the carriers and the regulator to evaluate jointly incremental costs. It must be remembered that the incremental cost of a service is the total saving which could have been realized on the network if this service, assumed to be the last one introduced, had not been provided. In this sense, an incremental cost may differ from a directly allocable cost. This is particularly true in the switching network or in the mutiplexing plan where efficiency increases with demand. (e.g. the number of circuits required by a service on a given trunk will increase if the efficiency of the trunk is diminished by decreasing demand from other services).

This implies that costing procedures (should the incremental cost methodology be retained) would have ultimately to be based in dimensioning algorithms allowing the carriers and the regulator to evaluate the total cost of a network with and without a particular service or set of services. These alogorithms would be very similar in nature to those incorporated in the NPPS model in that they would aim at rationalizing total costs while respecting engineering considerations and ensuring a given quality of service.

5.4 Guidelines for Further Work

In the above assessment several unsolved difficulties were mentioned: the role of the cross-elasticities, the treatment of excess capacities in the tariff determination, the inter-carrier cross-subsidies.

The recent literature on multiservice regulated companies frequently discusses the cross-elasticity concept. In a more concrete vein, the empirical effort of the F.C.C. explicitely requires, in Method 7, a role for empirically evaluated elasticities for future tariffs. The NPPS team should as far as possible introduce these price reaction coefficients in some Demand Block for simulation purposes. The introduction can be more or less sophisticated and at different levels of service aggregation but the difficulty is heightened by the lack of data and the need to forecast the elasticities.

Excess capacities can always be apportioned to existing services on a more or less arbitrary basis. The challenge is to redefine services over several periods and to prove that economies of scale are benefiting present and future consumers. Phenomena of technology diffusion must also be taken into account since new equipment is progressively introduced along with retirement plan implementation. Finally, fast growing new service is a problem. What is the best way to finance the required capacity if large indivisible facilities are economically the best choice? It appears that finding the best capital deployment in a multiservice, multiperiod scheme, is certainly a difficult thing, and finding a financing scheme which will be a burden only for the responsible service(s) without intertemporal cross-subsidy is another challenging problem.

The inter-carrier cross-subsidy concept has been examined in section 5.3. The whole issue of determining simultaneously, several tariff structures and levels, combined with the choice of an inter-carrier settlement scheme is a heavy combinatorial problem that should be at least clarified conceptually. Along that line, the methodology of goal programming which has been partially neglected in the present phase, should be revitalized in a multi-carrier context since financial interactions exist throughout the settlement schemes.

Finally, once the prospective costs become an important part in the tariff determination, the whole question of reliability of forecasts has to be introduced in the concept of accountability of the carriers. What are the acceptable errors? Or rather, which are the best methodologies available for forecasting and planning purposes in telecommunications? Who must pay for the errors? These are questions which need a theoretical basis.

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