ira project

interim report on the second phase

PREPARED FOR AND IN COLLABORATION WITH THE

NATIONAL TELECOMMUNICATIONS BRANCH

DEPARTMENT OF COMMUNICATIONS

BY

LABORATOIRE D'ECONOMETRIE

UNIVERSITE LAVAL

AND

SORÈS INC , MONTREAL

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IRA PROJECT

INTER-REGIONAL TELECOMMUNICATIONS ACCOUNTING

INTERIM REPORT ~~ON THE~~ SECOND PHASE

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NATIONAL TELECOMMUNICATIONS BRANCH

DEPARTMENT OF COMMUNICATIONS

by

LE LABORATOIRE D'ECONOMETRIE

de L'UNIVERSITE LAVAL

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SORÈS INC.

Montréal

September 15, 1974

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FOREWORD

This Interim Report on the Second Phase of the IRA Project describes a number of improvements on the IRA model constructed during the First Phase. The ultimate purpose of this operational research model is to simulate the financial consequences of various costing and accounting procedures and of various revenue sharing schemes concerning inter-regional telecommunications, each time with a given state of the network and its operating characteristics.

The main effort of the work done, at this stage, for the present phase, concentrates principally on the following three aspects of the model:

a) improvement of the Operating Block and particularly in the conceptualization and construction of an algorithm to estimate the usage of the physical network;

b) elaboration of the Costing Block principally by studying new cost functions for the transmission facilities, by refining the estimation of capital cost, especially by incorporating more rigorous depreciation algorithms and finally by proposing some unit costing methods;

c) extension of the Accounting Block by disaggregating many financial variables and by adding certain new equations and/or ratios.

The other two blocks, i.e. the Sharing and the Policy Simulation Blocks, have not been our major concern during the present period although we have established a multiperiod simultaneous equation system and have performed some simulations with it. Of course, what we have tried to do is to refine the other blocks in a manner that, we hope, will permit us, in the near future, to answer some questions posed in the previous Reports of the First Phase of the Project.

It is worth reminding the reader that the present Report is not a comprehensive treatise on telecommunications engineering, accounting or, for that matter, operational research. Nor does it presume to provide automatically solutions which could pretend to be "optimal" for the carriers and for the regulatory agencies.

On the other hand, it is to be viewed, in a wider setting, as a contribution towards the formulation of policies aimed at developing an even more efficient telecommunications system, in keeping with the broad social, economic, and political objectives in Canada.

ABSTRACT

This Interim Report on the Second Phase of the IRA Project contains the major improvements in the conceptualization and in the software already completed concerning an Inter-regional Telecommunications model constructed in the First Phase of the Project. Of course, the logic of the IRA model is essentially the same as before, i.e. it still contains five blocks, each one continuing to perform the tasks for which it was constructed. This report contains some discussions of the work to be done for the end of the Second Phase, scheduled for March 31, 1975, most of them in a rather specific manner.

In the Operating Block, the objective for this Report was to conceptualize and develop the corresponding software of an algorithm permitting to estimate the expected usage of the physical network. Consequently, this work was performed in three steps: discussion of the profiles of the demands for the telecommunications services, estimation of the expected usage of the switching network and finally the development of the algorithm already mentioned. It is worth mentioning that this algorithm can treat more than one service, including television service.

In the Costing Block, developments concern: establishment of new asset cost functions for transmission facilities; a refinement of the treatment of capital cost, including depreciation,deferred taxes and the cost of capital; and development of some means to evaluate operating costs. We also discuss and propose some unit costing methods and consider how to approach the problem of joint costs by mathematical programming techniques.

In the Sharing Block, which was not our main concern for this Report, our efforts concentrate on a new approach of defining what we called "streams" in our previous report and on the manner of assigning costs to streams. The interrelationship between this block and the Operating Block is also discussed.

In the Accounting Block, the reader will notice that this Block changes substantially from the previous one in the sense that the Regional Statements will now be the balancing residual between the Company Total and Interregional, instead of being supplied in a exogeneous manner. Also, the Income, Retained Earnings, and Funds Statements are expanded. These expansions and the software needed to take account of the previous modifications are described in the corresponding pages.

Finally, in the Policy Simulation Block, an extension of the previous simultaneous equation system is described and an incursion is made into the multiperiod aspect of the accounting system.

Apart from those mentioned in the ABSTRACT of the Final Report on the First Phase, March 31, 1974, the main intellectual challenges of the work reported on here have been the integration of detailed accounting and engineering data into a simulation model capable of yielding economically useful results, and the taking into account of the dynamic aspects of the problem.

RESUME

Ce rapport intérimaire de la seconde phase du Projet IRA contient les améliorations essentielles apportées, jusqu'à présent, à la conceptualisation et la programmerie du modèle de Télécommunications Inter-régionales conçu lors de la première Phase du Projet. La structure logique du modèle IRA demeure essentiellement la même, c'est-à-dire que le modèle est encore constitué de cinq blocs dont les objectifs demeurent inchangés. Ce rapport comporte aussi certaines remarques spécifiques pour la plupart, quant au travail à faire d'ici la fin de la seconde phase, prévue pour le 31 mars 1975.

En ce qui concerne le "Operating Block", l'objectif pour le présent Rapport était de concevoir et de développer la programmerie d'un algorithme permettant d'estimer l'usage moyen du réseau physique. Pour ce faire, on procède en trois étapes: traitement des profils de demande en services de télécommunication, estimation de l'usage moyen du réseau de commutation et enfin la mise au point de l'algorithme déjà mentionné. Il convient de noter que cet algorithme permet de considérer plus d'un service à la fois, y compris le service de télévision.

Dans le "Costing Block", on trouvera de nouvelles fonctions de coûts en regard des équipements de transmission, un raffinement du traitement du "Capital cost", qui inclue la dépréciation, une discussion de quelques méthodes d'imputation par coûts unitaires et enfin un aperçu de l'approche par la programmation économique du problème des coûts conjoints et communs.

Dans le "Sharing Block", qui n'était pas de nos préoccupations essentielles pour ce Rapport, nos efforts se sont concentrés sur une nouvelle approche de définition des flots ("streams"), tels que mentionnés dans notre rapport précédent, ainsi que sur une technique d'affectation des coûts à ces flots. L'interrelation entre ce bloc et le "Operating Block" est aussi envisagée.

Quant au "Accounting Block", le lecteur notera que ce Bloc diffère sensiblement du précédent en ce sens que les Etats financiers régionaux seront obtenus comme résidus entre le Total d'une compagnie et ses Etats inter-régionaux, plutôt que fournis de façon exogène comme auparavant. En outre les Etats de Revenus et Dépenses, Bénéfices réinvestis et Sources et Usages de Fonds sont désagrégés. Ces raffinements et la programmerie nécessaire à la prise en considération des modifications précédentes sont décrits dans les sections correspondantes.

Enfin, la section "Policy Simulation Block" comporte une description de l'extension du précédent système d'équations simultanées ainsi qu'une incursion dans l'aspect multi-périodes du système comptable.

A part ceux déjà mentionnés dans le RESUME du "Final Report on the First Phase of the Project" du 31 mars 1974, les défis intellectuels principaux qui se sont posés lors des travaux dont traite le présent rapport ont été l'intégration des renseignements comptables et techniques détaillés au sein d'un modèle de simulation capable de fournir des résultats ayant une signification économique ainsi que la prise en compte des aspects dynamiques du problème.

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1. INTRODUCTION

This is an Interim Report on the Second Phase of the building of a simulation model, now called IRA-II, dealing with the financial aspects of inter-regional telecommunications in Canada. Its main purpose is to describe the principal improvements made since the publication of the Final Report on the First Phase. It is to be stressed that it still does not represent the final formulation of the thinking of the team members on this subject.

The work already done is proceeding on schedule established in the Memorandum of Understanding of June 1, 1974, excepting some minor aspects of the Project. Those are reviewed in the Annex E of the present Report.

One other purpose of this Report is to state in a rather specific manner, and sometimes try to evaluate, the various conceptual approaches and options open at this stage: these are discussed in the body of the report in the various passages relevant to the matters concerned.

The present Report represents the result of a combined effort by three participants whose formal responsibilities were spelled out in the various official documents and detailed sharing of the tasks handled by more or less informal exchanges. The tripartite team consisted of the following organizations given here with the names of the specialists involved:

The Corporate and Financial Affairs Directorate of the National Telecommunications Branch:

Mr. J.A. Guérin
Mr. G.G. Henter
Mr. C. Lee
Mr. P. Rogers

Sorès Inc., Montreal:

Mr. J. Cluchey
Miss C. King
Mr. E. Manis
Mr. R. Riendeau

Laboratoire d'économétrie de l'Université Laval:

Prof. C. Autin
Prof. G. LeBlanc
Prof. T. Matuszewski

Being members of le Laboratoire, the following research assistants have contributed to the Project:

Mr. F. Coté
Mr. C. Gilles

Mr. B. Paquet
Miss R. St-Jacques
Mr. G. St-Cyr

Finally, Dr. I. Young, from York University, in the capacity of consultant to the Department of Communications, has contributed in the area of multi-period accounting analysis in the simultaneous equation systems.

2. OPERATING BLOCK

2.1 Introduction

The major effort of the team during the second phase of the IRA Project has concerned until now, the refinement of the Operating Block. The interface of the switching network with the facilities network has been solved, and will be computerized. However, in what follows, the switching network and facilities network are treated separately.

Throughout the following text, reference is made generally to the Canadian inter-regional network but it should be noted that the methodology is in no way tied to the existing network configuration. In principle, any sub-network (e.g. Bell Canada's Network) can be specified and treated by this algorithm in so far as it is remembered that dealing exclusively with a single part of an integrated network introduces factors of sub-optimization.

2.2 Switching Network

2.2.1 Profils de trafic offert

a) Remarques générales

Les profils de trafic sont nécessaire pour le calcul du revenu, et pour l'estimation de l'utilisation du réseau de commutation pour différentes périodes de la journée, ce qui nous permettra de traiter le problème de la non-coincidence des heures de pointes sur le réseau. La détermination des profils types, sujet de cette section, est un exercice rendu nécessaire par le manque des données.

Il s'agit de déterminer, pour différentes paires ordonnées de points de demande, la forme approximative des trafics pour une journée d'affaire moyenne durant la période de pointe. Il n'est pas question pour le moment de considérer d'une part, les trafics de fin de semaine ni d'autre part la distinction entre l'usage d'affaires et l'usage résidentiel des équipements. Ces quelques raffinements, ainsi que les variations saisonnières, pourront faire l'objet d'un traitement ultérieur.

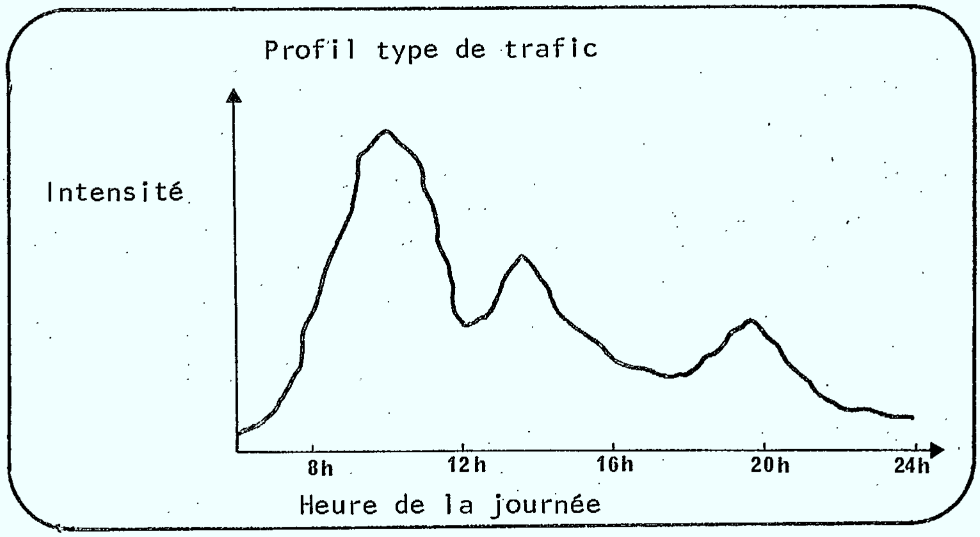
L'étude des profils de trafic pose de sérieux problèmes d'estimation, auxquels vient se greffer celui des fuseaux horaires. On peut se douter que la configuration du trafic entre deux points pendant une journée prend une allure plus ou moins régulière avec des pics et des affaissements à certaines périodes. Plusieurs études de trafic en Suède ont permis d'isoler les caractéristiques suivantes(*):

(*) voir Elldin, A., "Dimensioning for the Dynamic Properties of Telephone Traffic", Ericsson Technics, vol. 23 (1967), no. 3, pp. 315-344.

i) Les profils présentent des variations considérables pendant la journée avec des pointes le matin, l'après-midi et possiblement en soirée. Ces points varient considérablement en intensité et en temps d'occurence.

ii) Le flot maximum de trafic (pointe) n'arrive pas en même temps chaque jour. Parfois il variera de l'avant-midi à l'après-midi. Ceci est particulièrement vrai dans le cas des flux de trafic à faible intensité.

Profil 1



iii) Les trafics de pointe présentent des variations saisonnières considérables, en conformité cependant avec les autres activités humaines.

iv) Les plus hauts trafics de pointe d'une année peuvent se produire juste avant un congé public, mais peuvent aussi bien arriver à n'importe quel jour d'une saison de haute activité. Même les saisons plutôt tranquilles peuvent présenter d'extrêmement hautes valeurs de trafic de pointe. Cependant, il est à noter que les journées où il y a un flux de trafic exceptionnel (telles Pâques, Noël, etc.) sont exclues des calculs du dimensionnement.

v) Certains jours de la semaine peuvent être mis en évidence par des trafics de pointe systématiquement plus élevés que les autres.

vi) Les trafics de pointe ont une tendance générale à augmenter à long terme. Cet accroissement n'est pas uniforme, en général. L'histogramme de la distribution des trafics de pointe d'une année ne sera donc pas celui de l'année précédente, décalé d'un facteur linéaire. Ainsi, la hausse associée à une période définie de l'année peut être différente d'une année à l'autre et son taux sera aussi différent de ceux des autres périodes de la même année. On dispose des statistiques basées sur l'étude des communications interrégionales 1971, donnant les taux d'accroissement annuels du trafic inter-urbain Canadien, du trafic Canada - U.S. et du trafic Canada - Outremer.

vii) Le cycle des variations saisonnières est sans aucun doute d'une année.

viii) La durée moyenne des appels peut aussi varier comme le trafic, quoique de façon moins prononcée,La durée moyenne des appels interurbains pour le Canada est de 5.3 minutes (source: Statistiques Canada). On sait déjà que la durée moyenne varie pendant la journée et que cette durée présente des variations saisonnières considérables pour les heures de pointe. On peut aussi supposer logiquement que les appels résidentiels sont plus longs que les appels d'affaires, bien que cette proposition soit certes sujette à vérification. En supposant une durée moyenne générale, on pourra construire des profils de trafic offert en ccs (ou en erlangs). Toutefois, il serait peut-être souhaitable pour le moment de fonctionner plutôt en nombre d'appels.

b) Traitement des fuseaux horaires

A cause a) de la difficulté de traiter le trafic entre deux points de demande appartenant à des fuseaux horaires différents, b) du besoin de travailler en trafic directionnel pour le calcul des charges hors pointe et c) du désir de traiter simultanément le réseau canadien en son entier, il semble nécessaire de considérer tous les trafics par rapport à une heure de référence commune, celle du Pacifique nous paraissant la plus appropriée pour fin de calculs. Le tableau 1 montre la structure des fuseaux horaires canadiens, en heures de retard sur Greenwich (heures normales).

A titre d'exemple pour les calculs, l'heure de pointe de 10h. à Montréal sera notée à 7h. heure du Pacifique. Nous avons préféré retenir l'heure du Pacifique plutôt que celle de Terre-Neuve à cause de la demi-heure qui risquerait de compliquer inutilement les calculs.

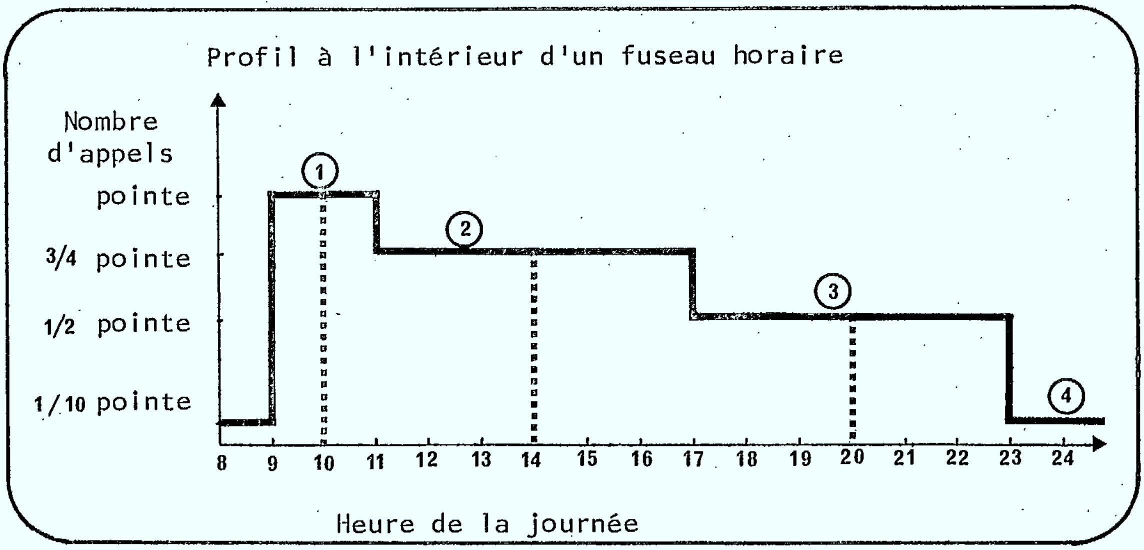
Il est très important de noter que les profils seront directionnels, non pas parce qu'il serait impossible de tenir compte simultanément des deux directions, mais simplement parce que l'objectif ultime, le calcul de l'usage du réseau de commutation et du réseau physique, le requerra.

TABLEAU 1

Région	Nombre d'heures de retard sur Greenwich	Heure des régions à 0h au Pacifique
Pacifique:	8h.	0h.
Rocheuses:	7h.	1h.
Centre	6h.	2h.
Est	5h.	3h.
Atlantique	4h.	4h.
Terre-Neuve	3h.30	4h.30

Pour fin de simplicité, on établit d'abord un profil-type pour les appels entre deux points dans une direction. Le trafic sera distribué en 4 paliers durant la journée. Considérons d'abord le trafic entre deux points d'un même fuseau horaire, en heure locale.

Profil 2



La détermination du nombre de paliers et de leurs durées respectives peut être faite d'une manière générale sans références particulières. Cependant l'importance relative des paliers doit être déterminée de manière empirique à partir de données canadiennes sur le trafic interurbain.

Les quatre paliers du profil 2 font référence à la première remarque tirée de l'expérience suédoise.

Il ne saurait être question, pour le moment, de distinguer les composantes: appels d'affaires et appels résidentiels.

On peut toutefois supposer que les premiers constituent une part importante du trafic pendant les heures d'affaires et les seconds sont plus importants en dehors de ces heures, notamment en soirée. La structure des réductions de tarifs hors pointe nous donne un bon indice des paliers proposés. Le tableau 2 montre quelques données de Bell Canada, pour le Québec, en 1973:

TABLEAU 2

Appels interurbains automatiques: réduction des tarifs		
A l'intérieur du Québec et l'Ontario:		
- Lundi à samedi:	de 6h p.m. à 11h p.m.:	1/3
- Tous les jours:	de 11h p.m. à 8h a.m.:	2/3
- Dimanche:	de 8h a.m. à 6h p.m.:	3/4
	de 6h p.m. à 11h p.m.:	1/2
A l'extérieur du Québec et l'Ontario:		
- Lundi à samedi:	de 6h p.m. à minuit:	30%
- Tous les jours:	de minuit à 8h a.m.:	50%

On fixera ici la pointe pour les affaires à 10h a.m., heure locale et pour les résidences à 8h p.m., et ce toujours à l'intérieur d'un même fuseau horaire. Nous traiterons plus loin le cas du chevauchement des fuseaux. Pour nos quatre paliers, les heures-pivot, ou les points névralgiques seront: 10h, 14h, 20h et 4h, pour les paliers, 1,2,3 et 4 de durée respective de deux, six, six et dix heures. L'espacement entre les heures-pivot sera de quatre, six, huit et six heures dans l'ordre.

Pour les appels couvrant plus d'un fuseau horaire, nous supposerons que les profils sont encore constitués de quatre paliers. Toutefois, les heures-pivot, rajustées à l'heure du Pacifique, ainsi que la durée des paliers pourront varier considérablement.

Nous n'avons pas encore parlé de l'amplitude des paliers. Ne sachant pas jusqu'à quel point le ministère des Communications peut avoir accès à des données détaillées, on pourra proposer certaines mesures des paliers à partir seulement des données de pointe: e.g. palier 2 au 3/4 de la pointe, palier 3 à 1/2 de la pointe et palier 4 à 1/10 de la pointe.

Seule une certaine expérience nous permettra de jauger la valeur de cette dernière méthode, ainsi que les coefficients à utiliser en référence à la période de pointe.

Deux hypothèses supplémentaires nous permettront de déterminer l'allure des profils pour les communications hors fuseau.

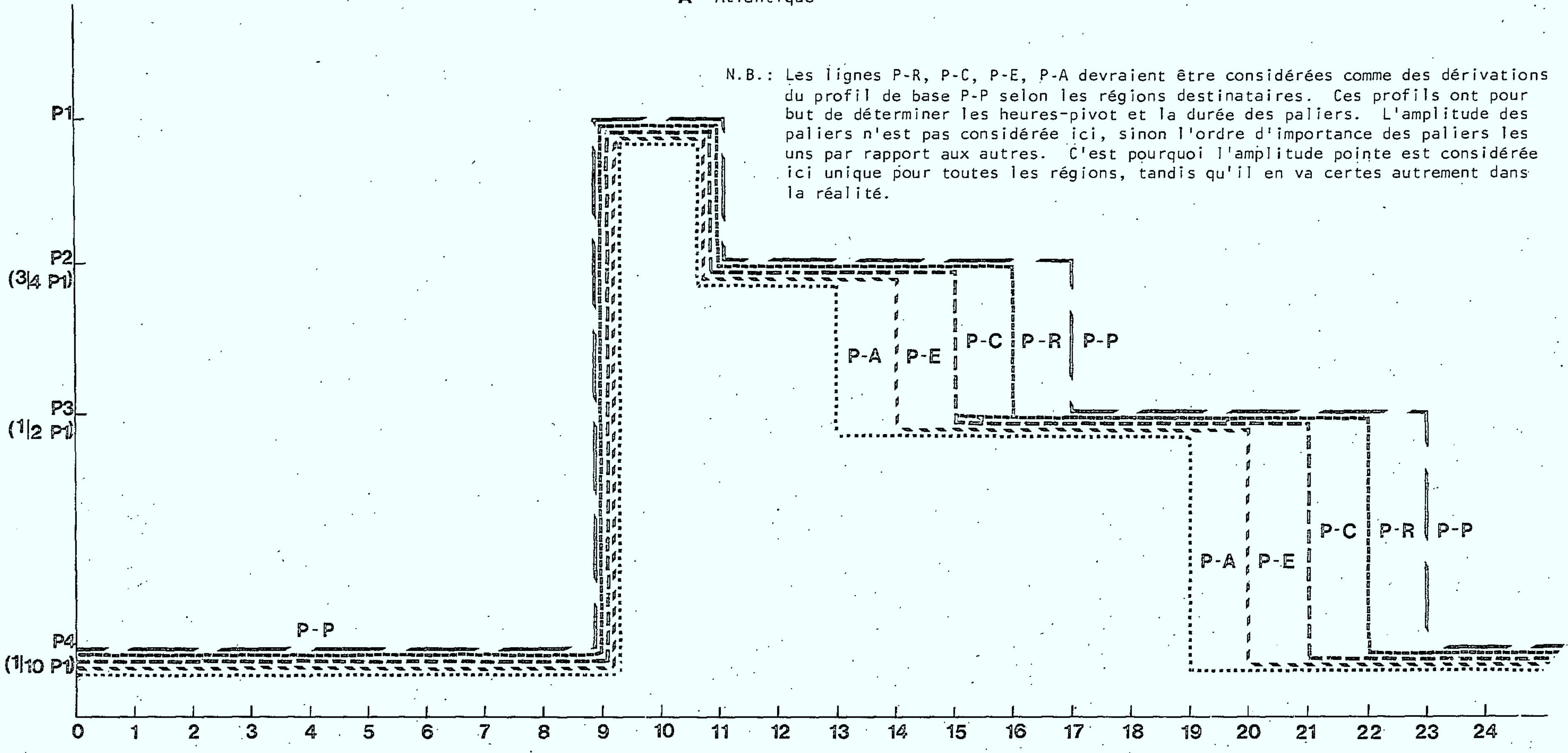
1. Pour les appels d'ouest en est, on peut supposer que les heures-pivot seront celles de la région d'origine, rajustées toujours à l'heure du Pacifique. Cette hypothèse s'appuie sur le fait que si, par exemple, la région d'origine est en pleine activité et s'il est possible d'atteindre la région de destination avant la fermeture des bureaux, alors on tentera d'acheminer les appels. Toutefois, les paliers pourront être tronqués à la droite, en raison du décalage positif d'heure de l'est par rapport à l'ouest. En effet, les bureaux fermant à 5h p.m. à Halifax, il sera impossible pour un homme d'affaires de Vancouver de rejoindre son interlocuteur après 13h, heure du Pacifique (voir le profil 3).

2. Pour les appels d'est en ouest, on peut supposer cette fois que les heures-pivot seront celles de la région de destination, rajustées comme convenu à l'heure du Pacifique. Il s'agirait de supposer que les appels d'affaires ne peuvent être acheminés d'est en ouest avant l'ouverture des bureaux situés dans l'ouest canadien. Par exemple, les appels de pointe originant en n'importe quel point du pays et destinés à la Colombie Britanique ne seront pas acheminés avant 9 heures ou 10 heures du Pacifique. A titre d'exemple, le profil 4 indique la configuration des appels en provenance de l'Atlantique. Dans cette région, l'heure de pointe est fixée à 6h, heure du Pacifique. Il semble donc logique que la pointe vers les régions à l'ouest soit retardée d'une heure par fuseau horaire soit à l'ouverture des bureaux dans les différentes régions. Remarquons que le profil des appels à l'intérieur de cette région est identique à celui du Pacifique, tout au moins quant à la durée des paliers, et ce à une translation près de quatre heures. Quant aux autres paliers, ils suivent sensiblement l'horaire de l'Atlantique, à cause de la fermeture des bureaux et des périodes de tarification dans cette région.

   Rappelons le problème envisagé ici. Il s'agit de diviser la journée de 24 heures en quatre ou cinq périodes suffisamment représentatives. Idéalement, nous aurions tout intérêt à effectuer tous les calculs du Bloc d'Opérations pour chaque

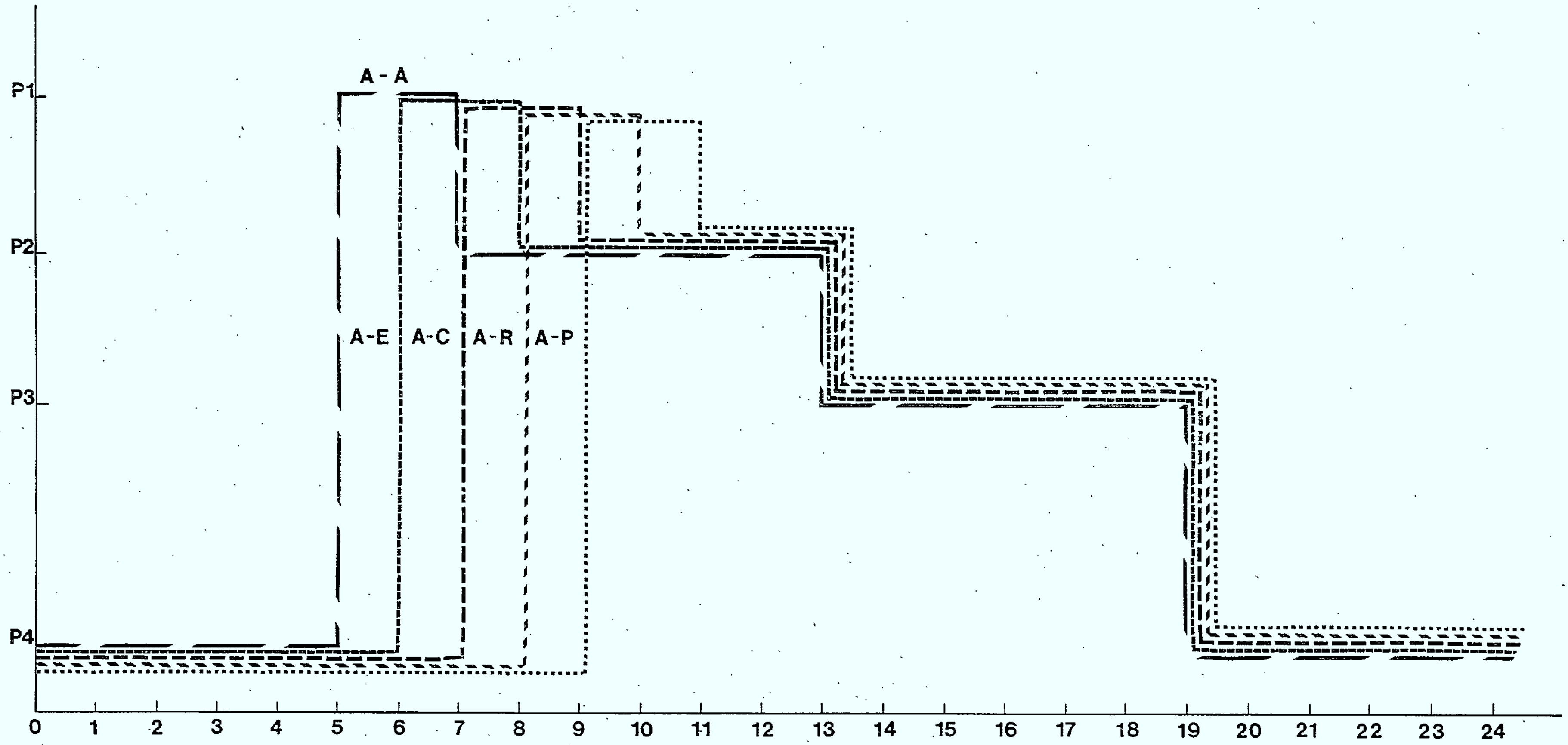
PROFIL 3 En provenance du Pacifique

P Pacifique
R Rocheuses
C Centre
E Est
A Atlantique



N.B.: Les lignes P-R, P-C, P-E, P-A devraient être considérées comme des dérivations du profil de base P-P selon les régions destinataires. Ces profils ont pour but de déterminer les heures-pivot et la durée des paliers. L'amplitude des paliers n'est pas considérée ici, sinon l'ordre d'importance des paliers les uns par rapport aux autres. C'est pourquoi l'amplitude pointe est considérée ici unique pour toutes les régions, tandis qu'il en va certes autrement dans la réalité.

PROFIL 4 En provenance de l'Atlantique



heure de la journée. Mais une telle entreprise serait certes onéreuse . Lorsqu'on veut étendre les calculs à tout le modèle canadien, il arrive malheureusement que tous les paliers n'ont pas lieu en même temps dans toutes les régions, en vertu du décalage horaire. Ainsi, la pointe de l'Atlantique a lieu à 5h. du matin au Pacifique, i.e., au moment où le volume des communications dans cette dernière région est presque nul. Quelles périodes de la journée pourront être sélectionnées (toujours en heures du Pacifique)?

Une première approche consiste à considérer que les profils de point à point pour tout le pays se présenteront entre les deux extrêmes du Pacifique et de l'Atlantique. A la lumière de ces deux séries de profils, quatre périodes de la journée pourront être sélectionnées:

i) 5h. à 11h. (Pacifique): toutes les pointes (palier 1) de toutes les régions inter et intra-fuseaux devraient se situer dans cette période.

La sélection de cette période présente le désavantage que les paliers 1,2 et 4 de l'ensemble des profils sont soit entièrement ou partiellement couverts. Comme on décidera probablement de calculer le trafic moyen sur cette période, il semble évident qu'une moyenne des paliers 1 et 4 pourrait fausser l'aperçu de l'utilisation véritable du réseau de commutation.* De toute façon, il faudra pondérer par la durée des paliers considérés à l'intérieur de cette période.

ii) 11h. à 17h. (Pacifique): cette période couvrira entièrement ou partiellement des paliers 2 et 3, dont l'écart d'amplitude sera plus ou moins prononcé.

iii) 17h. à 23h. (Pacifique): cette période couvrira des paliers 3 ou 4, selon les profils.

iv) 23h. à 5h. (Pacifique): cette période couvrira seulement des paliers de niveau 4. On peut donc prévoir de très bonnes estimations des trafics pendant cette période.

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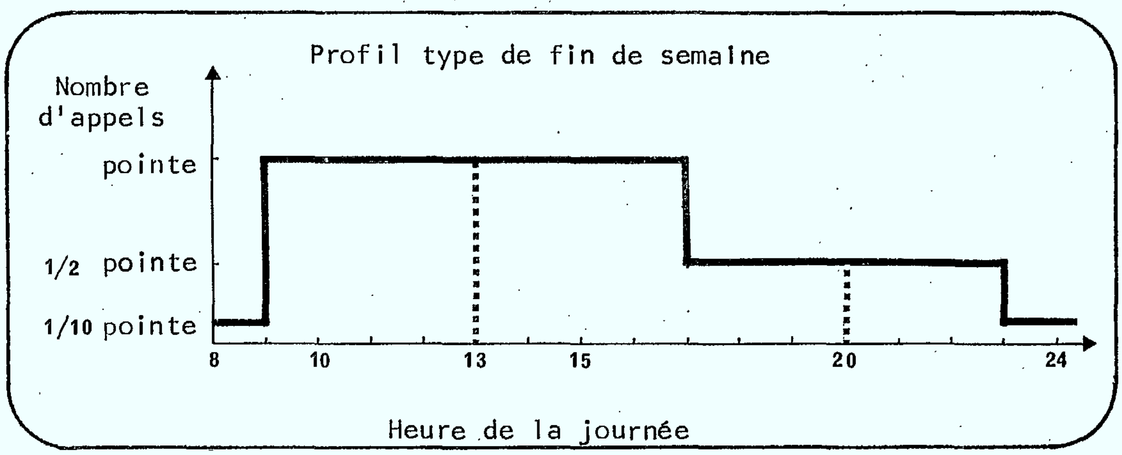
* Il faut ici bien distinguer les deux méthodes suivantes de calcul de l'usage moyen du réseau de commutation: 1) d'abord calculer l'usage pour chaque heure, et faire la moyenne sur les arêtes du réseau ou 2) d'abord faire la moyenne par couple de points de demande, et calculer l'usage du réseau à partir de ces moyennes. La première méthode est certes la plus efficace, mais aussi la plus onéreuse. La seconde a pour but d'estimer la première, dans des limites de confiance raisonnables avec un moindre effort.

Nous venons donc de diviser la journée en quatre périodes de six heures chacune. La durée de périodes de six heures n'a pas été choisie a priori, mais après une étude sommaire des profils de l'Atlantique et du Pacifique.

La seconde approche pour la sélection de périodes-témoins de la journée est sensiblement plus compliquée, mais probablement plus réaliste. Elle consiste à ne plus tenir compte uniquement de la durée des paliers, mais aussi de l'intensité relative des paliers en comparant les couples de points de demande les uns aux autres. Par exemple, s'il arrivait que le profil Montréal - Toronto domine tous les autres profils (i.e. que l'intensité de ce trafic soit supérieure à celle de tous les autres trafics et ce, pour toute heure de la journée), alors on aurait intérêt à donner plus d'importance à ce profil qu'aux autres dans le choix des périodes. On pourrait donc affecter chaque profil d'un coefficient d'importance qui permet de sélectionner plus adéquatement les périodes (ce coefficient serait sans aucun doute fonction de l'intensité). Dans le même ordre d'idée, on pourrait déterminer les périodes en tenant compte simplement du trafic total du réseau pour toutes les paires de points de demande.

Assez peu de choses peuvent être dites a priori sur le trafic de fin de semaine. A titre expérimental, on pourrait proposer le profil-type suivant pour le trafic à l'intérieur de la zone du Pacifique. Il est constitué de simplement 3 paliers, le plus important étant celui de la matinée et de l'après-midi, centré à 13h.; le même profil pourrait s'appliquer le samedi et le dimanche, puisque le palier 1 du samedi, engendré par une certaine activité commerciale, est très certainement maintenu le dimanche par la hausse du trafic d'origine résidentielle.

Profil 5



Pour le moment, il serait difficile de pousser plus loin les considérations logiques du modèle des profils. Un effort empirique permettra de préciser la forme exacte des profils par paire de points de demande.

On s'attend à obtenir dans un proche avenir des données pour des flux de trafic Canada - U.S. qui nous permettront de réévaluer nos profils.

2.2.2 Usage of the Switching Network

a) Outline of the problem

In order to determine cost responsibility and obtain a justifiable separation of costs, we must be able to identify the use of the switching network relative to the demand for communication services. In the case of public message traffic, the principal information required would be a breakdown, for any particular link of the network, of the traffic load carried, under typical conditions, into its components of traffic identified by origin-destination pair. This information would have, for example, the following form, for any typical period.

link <a,b> carries (i,j) traffic $X^{\langle a,b \rangle}_{(i,j)}$ amount

(l,m) traffic $X^{\langle a,b \rangle}_{(l,m)}$ amount

etc.

This information may be transferred for processing to the algorithm treating the transmission network described in Section 2.3 of this report. Furthermore, aside from the use of this algorithm as a basis for allocation of circuit demands on the transmission network certain peripheral analyses may be accomplished.

We may for example, run the algorithm for several typical traffic periods, and by comparison of the usage of circuits in the network, determine the peak and weighted average usage of the switching network. This result permits the establishment of a relationship between the hourly demand profile on a traffic stream (origin-destination pair) and the hourly load profiles on any particular link of the network. At the same time, apparent under-use or congestion of the network, relative to the number of circuits available, may be analysed. Due to the inherent nature of the outputs produced it will be possible to examine the quantity and importance of first routed traffic on any link of the network relative to overflow and transit traffic.

Further, since origin-destination traffic streams can be classified in the categories regional interurban or interregional interurban this algorithm will furnish one basis for separation of costs between these two categories of message. The principle topic of this subsection will be to describe the algorithm developed to achieve these results.

b) Data required

For purposes of the ensuing discussion it will be assumed that the following data is known and available:

i) The configuration of the switching network in terms of the switching hierarchy, the location of switching centers, the configuration of the final tree, (i.e. the homing rules) and location of all installed high usage and full groups. It is assumed, furthermore that the number of circuits installed on each link of the switching network is known.

The routing principles of the network as well as the high usage to final overflow configuration are assumed to be the standard ones relative to hierarchical networks.

ii) A series of square matrices D_t of dimension n^2 where the entries in the matrices represent point to point directional traffic between two traffic sources i and j (i,j=1,...,n) for various typical periods indexed as t.

The (i,j) th entry of D_t will thus be the directional traffic from origin i to destination j for the period t.

The sources of these data are as follows. Data required in item i) concerning the final tree configuration and switching hierarchy are determined according to the particular reduced network model to be used. Data concerning the number of circuits in each direction for a particular link is not directly available but may be obtained for these purposes indirectly from other studies. It is assumed that all circuits are directional, and although this assumption is not verified in reality, certain arguments can be made for maintaining it. Further discussion of this topic will be found in a later section of this Report. Data required in item ii) is to be derived from certain base data sources and from assumptions concerning the traffic profile on long distance traffic streams as discussed in Sections 2.2.1 and 2.5 of this Report. The algorithm which is proposed, it should be noted, is in no way tied to a particular final tree configuration and switching hierarchy, but can be used for any hypothetical hierarchical arrangement which respects the same routing principles.

In summary then, the problem posed is seen to be one of estimating the traffic loads on links of a switching network using a typical demand matrix and data on the configuration of the switching network. It will be noted that no grade-of-service parameter is assumed known, but as indicated later, the grade of service is a result of the application of the algorithm.

c) Comparison to the "CHARGE" module of the HERMES Project

In the HERMES project, the "CHARGE" module was built to attack a somewhat similar problem. In that formulation, the intent was to calculate traffic load on a switching network assuming a known peak demand (point-to-point) matrix, network configuration and (through the hypothesis of simultaneity of peak periods on all parts of the network) a given grade-of-service parameter for each link of the network. In contradistinction to "CHARGE", the problem posed herein uses a typical period demand matrix where the implicit assumption is that peak periods may occur in certain geographical areas of the network, while off peak usage occurs simultaneously elsewhere, and hence the grade of service is a result of and not an input to calculation. The interest of these calculations resides in the fact that overflow probabilities are variable in that they are not fixed in advance by engineering techniques at a maximum level, whereas blocking probabilities are fixed at a maximum value. In this latter case, the interest resides in the calculation of blocking probabilities for off peak periods. It should be noted, in fact, that "CHARGE" addressed itself to a much more complicated problem involving expansion of a network, in which the above mentioned problem is embedded. Despite these distinctions, it was none-the-less possible to extract and use large parts of "CHARGE" in the construction of the present algorithm.

d) Preliminary technical details

Before beginning a detailed description of the workings of the algorithm, we present a brief discussion of certain technical details which are needed in that description.

First we consider the general problem of the calculation of traffic load carried on a group of circuits if the number of circuits in the group is known and the offered traffic is given. The problem can be approached in various ways depending upon the assumptions made about the distribution of traffic arriving at the circuit group. If it is assumed that traffic arriving at a group is distributed according to the Poisson distribution (an acceptable assumption for first routed traffic) then the amount of traffic carried on each circuit of the group can be estimated according to the Erlang formula.

An example of the information provided by this formula is shown in the Table 3.*

Using such tables we can immediately determine the total load carried when "a" erlangs are offered to "c" circuits. The use of this distribution assumes the full availability of the group

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* See Annex A for a statement of the Erlang formula.

TABLE 3

Load carried by each trunk in a full availability group
for 360 ccs offered random load (360 ccs=10 erlangs) assuming sequential access

Trunk Number	Load carried on trunk (erlangs)	Total load by all trunks (erlangs)
1	.9090	.9090
2	.8942	1.8032
3	.8762	2.6794
4	.8539	3.5333
5	.8271	4.3604
6	.7944	5.1548
7	.7547	5.9095
8	.7547	6.6642
9	.6581	7.3223
10	.5863	7.9086
etc.	etc.	etc.

of circuits to the offered traffic, and that for each incoming call, attempts are made to route the call on each circuit consecutively up to the maximum availability at which point the call is lost or overflowed to a final group.

A second alternative is to estimate load carried according to the Poisson distribution which gives results shown in table 4.*

Use of this distribution involves the assumption that calls not serviced immediately return to the trunk group, where as use of the Erlang formula involves the assumption that unserviced calls are cleared from the trunk group. We thus note that the Erlang trunking formula is used on high usage routes, and the Poisson formula for final routes. Furthermore, we note from the tables that for an equal amount of offered traffic the Poisson formula specifies a greater circuits requirement than that specified by the Erlang formula, for the same grade of service (for grades of service of P05, P01 etc.).

* See Annex A for a statement of the Poisson formula.

TABLE 4

Total load carried by trunks in a common final group for 360 ccs offered random load (360 ccs=10 erlangs)

Number of trunks	Load carried on last trunk (erlangs)	Total load carried (erlangs)
1	.9999	.9999
2	.9991	1.9990
3	.9931	2.9921
4	.9713	3.9634
5	.9081	4.8715
6	.8299	5.7014
7	.6788	6.3802
8	.5292	6.9094
9	.3974	7.3068
10	.3162	7.6230
etc.	etc.	etc.

In fact however, statistical theory indicates that the sum of various overflow traffics from a Poisson process (such as total traffic offered to final routes) does not follow the Poisson distribution (where mean (μ) = variance (σ^2). The true nature of this traffic process has variance σ^2 different from the mean μ. In order to obtain better estimates of load carried on final trunks use is made of the Equivalent Random Theory (ERT) of Wilkinson*, the essence of which is as follows**.

"The overflow traffic from one or more first choice groups which are offered independent random traffics is described by two parameters, namely its mean and its variance. The mean of the total overflow traffic is calculated as the sum of the individual means, and the variance as the sum of individual variances". All the first choice groups are then replaced by an equivalent full availability group the total overflow traffic from which has the same mean and the same variance as the total overflow traffic from the individual groups. From these two conditions the number of

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* R.I. Wilkinson "Theories for Toll Traffic Engineering in the U.S.A." Bell System Technical Journal, (vol. 35); pp 421-514, (March 1956).

** From Y. Rapp; "Planning of Junction Network in A Multi-Exchange Area", Ericcson Technics No.1 (1964) pp.113-116.

junctions (circuits) and the (random) traffic offered to the equivalent group are determined. The complex overflow system is thus reduced to a full availability group consisting of the equivalent group and the common overflow group". Using this equivalent full availability group, Wilkinson estimates the number of circuits required on the common overflow group to establish a given grade of service on the whole system. Inversely, we may fix the number of circuits and estimate the traffic load carried and the grade of service with Wilkinson's model.*

A comparison between estimation of traffic load from the Poisson formula and from Wilkinson's theory shows that the Poisson formula tends to be more conservative in that it estimates carried loads for a given number of circuits, and fixed offered traffic, at lower levels than those estimated by E.R.T. Inversely, using the Poisson formula in dimensioning applications for a fixed grade of service tends to indicate a larger circuit requirement than that obtained using E.R.T.

The initial form of the algorithm to be described below uses the Poisson trunking formula due to its simplicity of formulation, but ultimately, both methods will be implemented in the software.

We now turn to the question of availability. It is well known that on some high usage groups, offered traffic overflows even when some circuits remain unused in the group. This question of availability can be treated correctly only if more extensive information on the switching network is available. In the interim, we assume, for purposes of the algorithm, that all trunk groups are fully available to first routed traffic being offered to the group. The impact of this hypothesis on the accuracy of results is unknown, but some estimates of that impact should be made if possible. In any case, the true nature of the switching arrangements is far too complex to treat computationally in full detail, and this, coupled with the use of a reduced network indicates that some hypothesis of this nature will be necessary to permit a computational solution to the problem posed.

Finally, we refer briefly to the "directionality of circuits". The circuits of the switching network are variously accessible from one end only or from both ends, but data on the composition of each link of the switching network regarding directionality is not publicly available. Furthermore, data on the number of circuits in the various links is known only in total (the total of directional circuits from A to B, directional circuits from B to A and bidirectional circuits joining A and B). To obtain the date which is assumed available (see Section b.)) we must make

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* For the statement of the formulae involved in Wilkinson's E.R.T. see Annex A.

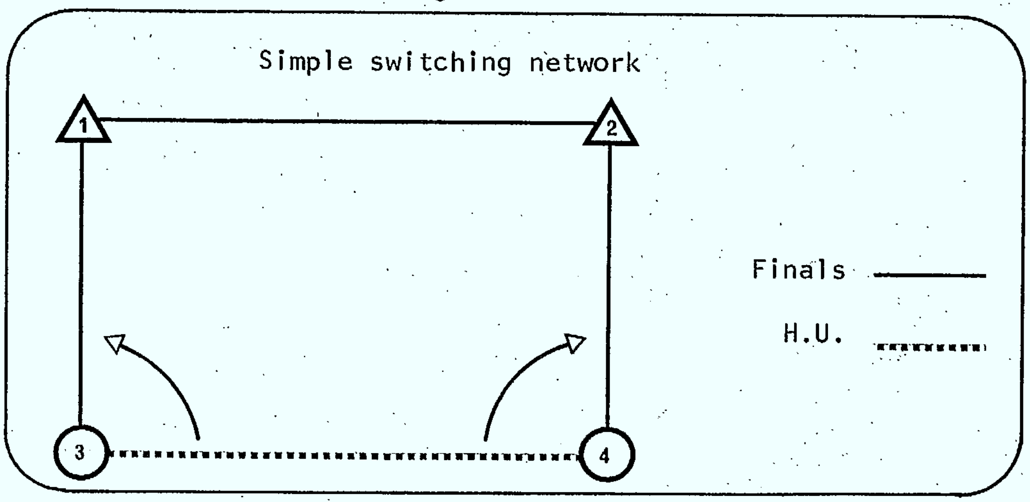
some assumptions regarding the nature of this total. The initial hypothesis used is that the known total number of circuits per group, N, is composed of αN directional circuits from A to B and $(1-\alpha)N$ directional circuits from B to A.

The possible existence of bidirectional circuits is overlooked by this hypothesis. Since the existence of bidirectional circuits involves certain economies of scale in the average load per circuit, we can be sure that our assumption will introduce a conservative bias in the results. That is, we will find less over-capacity, more congestion and lower grades of service everywhere. This assumption is then adopted in the absence of more complete information, partly because of its conservative nature, and partly because it greatly simplifies the software development of the algorithm.

e) General principles of an algorithm for the calculation of average load on a switching network.

In accounting for the load carried on any particular link of the switching network, we must account for first routed traffic or primary traffic, overflowed traffic and transit traffic on each link of the network. Due to the composite nature of the traffic load on any link of the network, it is apparent that accounting for transit traffic and for overflow traffic must begin at the source of this traffic. This claim can be shown with the help of the following diagram of a simple switching network.

Diagram 1



Simple switching network

It can be seen that the traffic on the link <1,2> is composed of first routed traffic from △1 to △2 and from △1 to ④ ; transit from ③ to ② via △1 and traffic from ③ to ④ overflowed from the H.U. route to the final route.

From this example, it is evident that the transit traffic on link <1,2> cannot be estimated until the link <3,1> is examined, and the amount of traffic loss is known on this link. However, the total traffic offered to link <3,1> cannot be known until the link <3,4> is examined and its overflow is estimated. The link <3,4> itself carries only first-routed traffic from ③ to ④ (a consequence of the routing rules), and the first routed traffic offered to link <3,4> is equivalent to the total traffic demand originated in ③ for ④.

This example illustrates that the method of accounting for three types of traffic on any link establishes an implicit hierarchy among the links of the network. If we know only point-to-point traffic demand, we must start our calculations with thoses links whose offered traffic is composed of only first routed traffic amounts to each of the other links of the network. A systematic set of rules for the selection of links in a switching network has been developed and the description of this procedure is given in the following section.

Coupling the method of accounting for offered traffic with the techniques mentioned above for the calculation of traffic load enables us to estimate the probability of overflow or loss consecutively in the links of the network and thereby obtain the distribution of traffic load in the network.

Before providing the details of the algorithm, we attempt to clarify the principles involved with the help of an example.

The following network, a simplified one having a three level hierarchy system, and two number 2 switching nodes, is used to illustrate the principles.

To calculate the offered load on the link <5,4> , we must add:

i) The first routed demand from △5 to △4

ii) The overflow traffic from link <5,1> of the first routed demand from △5 to ①.

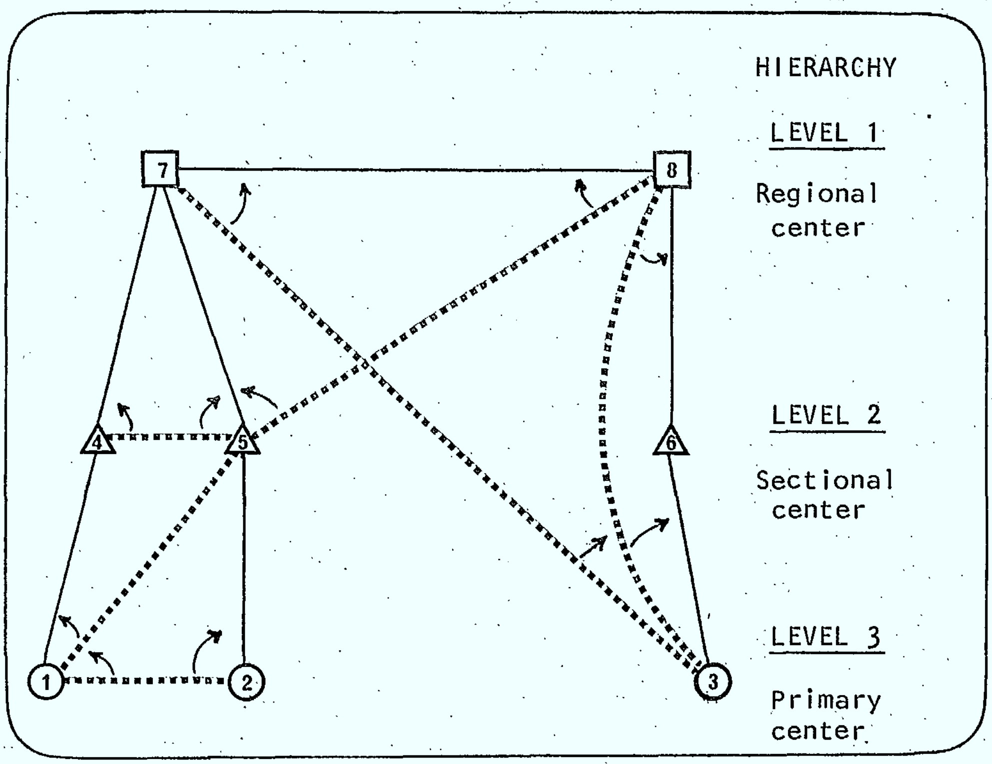
iii) The transit traffic from ② to △4 and from ② to ① which, in the latter case, has overflowed from link <2,1> to link <2,5> and transits via <5,4> to destination.

Thus, we see that the link <5,4> can carry traffic from the following streams (origin—destination pair).

② to ① ; △5 to ① ;

② to △4 ; △5 to △4 .

DIAGRAM 2: EXAMPLE OF ALGORITHM



It is possible to identify the list of these components for any link of the network in the manner described in the following section (section f).

Following the example above, and the previously described rules for accounting for traffic, we can see that the hierarchy of links requires that we treat first the lowest level oriented high usage groups ⟨1,2⟩ and ⟨2,1⟩ before treating the high usage groups going from level 3 to level 2 or level 1 (⟨1,5⟩, ⟨3,7⟩,⟨3,8⟩). These in turn permit us to calculate overflow on the final groups from level 3 to level 2 (⟨1,4⟩ , ⟨2,5⟩ , ⟨3,6⟩). Having determined offered traffic on links ⟨5,2⟩ and the load carried, we may then determine traffic offered to link ⟨5,1⟩ before determining the overflow and hence traffic offered to link ⟨5,4⟩ .

Continuing in this fashion, the following order of accounting may be seen to prevail.

Step 1: ⟨1,2⟩, ⟨2,1⟩, ⟨2,5⟩, ⟨3,7⟩
Step 2: ⟨1,5⟩, ⟨3,8⟩
Step 3: ⟨1,4⟩, ⟨3,6⟩
Step 4: ⟨4,5⟩, ⟨5,1⟩, ⟨5,8⟩, ⟨6,8⟩
Step 5: ⟨4,7⟩, ⟨5,4⟩
Step 6: ⟨5,7⟩,
Step 7: ⟨7,3⟩, ⟨8,5⟩
Step 8: ⟨7,8⟩, ⟨8,7⟩

Step 9: <7,4>, <7,5>, <8,3>
Step 10: <8,6>
Step 11: <4,1>, <5,2>, <6,3>

The steps 9, 10 and 11 represent the oriented final groups which descend the hierarchy, and reflect the fact that any descending final group cannot be treated until all ascending groups potentially feeding traffic to descending groups have been accounted for. A certain amount of indeterminacy is involved in the hierarchy in that, for example, calculations for link <6,8> listed in step 4 produce results used only in link <8,5> of step 7. The systematic procedure presented in the following section provides a generalized method of selection of the links which gives a slightly different order than that shown avove, but which turns out to be a sufficient representation of the hierarchy nonetheless.

f) Details of an algorithm for the calculation of average load on a switching network.

This detailed section is included in the Report for completeness and reading of the section is not essential for the comprehension of the Report.

In the following section, we assume there is a set of traffic centers N, elements of which are represented by lower case letters a, b, c, i, j, etc. A link of the switching network is noted <a,b> ; a traffic stream is noted (a,b) .

For each link a,b we can define a set $C^{\langle a,b\rangle}$ of traffic components (i,j) which can be routed via the link <a,b> . In our previous example, we had $C^{\langle 5,4\rangle} = \{(2,1), (2,4), (5,1), (5,4)\}$

These sets $C^{\langle a,b\rangle}$ are determined in general in the following manner. Given the routing rules in a network and any two demand centers a and b, it is possible to determine all the routes over which traffic (a, b) may travel. This is done by an algorithm developed by M. Hupé of the HERMES Project, which uses the cornerstone principle of the routing pattern that "any switching point used as the intermediate switching point in the alternate route of a high usage group, must lie in the (unique) final route between the terminal offices of the high usage group".*

Hence, for each traffic stream (i,j), we determine the links a,b which may receive part of the load of (i,j). By a simple inversion of this correspondence, we obtain $C^{\langle a,b\rangle}$ for each

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* C.J. Truitt; "Traffic Engineering Techniques for Determining Trunk Requirements in Alternate Routing Trunk Networks"; Bell System Technical J., (Vol. 33), No. 2, (March 1954); p.290.

<a,b> . Having obtained $C^{\langle a,b\rangle}$ we define $O^{\langle a,b\rangle}_{(i,j)}$ as the traffic offered (in ccs or Erlangs) to the links <a,b> due to demand between origin i and destination j. Further, we define $L^{\langle a,b\rangle}_{(i,j)}$ as the amount of (i,j) traffic carried on link <a,b> (a function of the number of circuits $n^{\langle a,b\rangle}$ on link <a,b> and the traffic offered to link <a,b>).

We may note, as an aside at this moment, that all traffic streams (i,j) and all links <a,b> are considered to be oriented. That is, the link <a,b> is not the same as the link <b,a> .

The procedure developed for the selection of links <a,b> in an order permitting the consecutive calculation of offered traffic, load carried and overflow or transit traffic is given below. This procedure uses information on the switching hierarchy, and on the nature (H.U. or final) of the trunk groups.

The procedure is as follows:

I. Ascending the node hierarchy, and relative to a particular level in the switching hierarchy, we select the links in the order defined as follows:

   a) The descending high usage groups going to another ladder and beginning with

      1) Those descending 3 hierarchical levels (if any)

      2) Those descending 2 " " "

      3) Those descending to the next lower hierarchical level (if any);

   b) The high usage groups connecting nodes at the same hierarchical level in any order (if any);

   c) The ascending high usage groups going to another ladder and beginning with:

      1) Those ascending to the next higher hierarchical level (if any)

      2) Those ascending 2 hierarchical levels (if any)

      3) Those ascending 3 hierarchical levels (if any);

      (Note: this procedure is the reverse of I-a)

   d) The ascending high usage groups remaining in the same ladder and taking changes of level into account as in I-a;

   e) The final groups ascending to higher levels.

This entire procedure is repeated for each hierarchical level beginning at the lowest and progressing to the highest.

Next in order, we consider:

II. The final groups joining two nodes of the same hierarchical status (occuring for no. 1 level only); and finally,

III. Descending the node hierarchy and relative to a particular level in the switching hierarchy we select the links in the order defined as follows:

a) The descending high usage groups remaining in the same ladder and taking changes of level into account as in I-a.

b) The descending final groups.

This procedure (i.e. III) is repeated for each hierarchical level beginning at the highest and progressing to the lowest.

Two remarks are necessary. The oriented links falling in the same class of this hierarchy may be treated in any order in the algorithm. Furthermore, full groups may be considered as high usage groups with the exception that overflows to other links may not occur, although transit traffic may use a full group route.

Using this hierarchy, and our previous notation, the algorithm proceeds in the following fashion.

For a particular link <a,b> , we assume that $O_{(i,j)}^{\langle a,b\rangle}$ is known for all (i,j) in $C^{\langle a,b\rangle}$. We then calculate the load carried in the following fashion.

$$\sum_{(i,j)\in C^{\langle a,b\rangle}} L_{(i,j)}^{\langle a,b\rangle} = f\left(n^{\langle a,b\rangle}, \sum_{(i,j)\in C^{\langle a,b\rangle}} O_{(i,j)}^{\langle a,b\rangle}\right)$$

Where the function* f is the Erlang function when <a,b> is a high usage group, the Poisson function when <a,b> is a final group. That is, we calculate the total load as a function of the total number of circuits and the total offered traffic.

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* The functions f and the calculation of probability of overflow are described in the Annex A.

The individual terms of $\sum_{(i,j)} L^{\langle a,b\rangle}_{(i,j)}$ are obtained on a mean value assumption that each $L^{\langle a,b\rangle}_{(i,j)}$ is in the same proportion to $\sum L^{\langle a,b\rangle}_{(i,j)}$ as the corresponding $O^{\langle a,b\rangle}_{(i,j)}$ is to $\sum_{(i,j)} O^{\langle a,b\rangle}_{(i,j)}$

For each (i,j) in $C^{\langle a,b\rangle}$, we then account for transiting and overflowing traffic in the following manner.

The load $L^{\langle a,b\rangle}_{(i,j)}$ has either arrived at destination or continues to another link. If the latter case holds the next link of the chain, (say <c,d>) is determined from the initial routing tableau and we accumulate as follows:

$$O^{\langle c,d\rangle}_{(i,j)} = L^{\langle a,b\rangle}_{(i,j)} + \text{old } O^{\langle c,d\rangle}_{(i,j)}$$

Each component of transit traffic is then accounted for as offered traffic in the next consecutive link to which it may be offered.

The load $L^{\langle a,b\rangle}_{(i,j)}$ must be less than $O^{\langle a,b\rangle}_{(i,j)}$ and it is this difference $O^{\langle a,b\rangle}_{(i,j)} - L^{\langle a,b\rangle}_{(i,j)}$ which is lost when <a,b> is a final group or overflowed when <a,b> is a high usage group. If the later case holds, the overflow link (say <c,d>) is determined from an input overflow table and the accumulation.

$$O^{\langle c,d\rangle}_{(i,j)} = \left[O^{\langle c,d\rangle}_{(i,j)} - L^{\langle a,b\rangle}_{(i,j)}\right] + \text{old } O^{\langle c,d\rangle}_{(i,j)}$$

is made, thus offering overflow traffic to its next alternate link.

In the case of primary or first routed traffic, we simply take

$$O^{\langle a,b\rangle}_{(i,j)} \equiv \text{the demand}$$

as found in the demand matrix.

With this system of calculation and accounting, coupled with the hierarchy of links described, it can be seen that all traffic is systematically accounted for, and actual overflow probabilities are calculated depending on the conditions described by the given demand matrix.

g) Computational experience

The described algorithm has been programmed and although complete data is not available, some results have been obtained indicating that the algorithm is workable in its present form on a 25 to 30 node network. The problems involved in expanding to incorporate a 60 to 70 node network have not been investigated but should not be insurmountable. An approximate method for the incorporation of the routing of Canada-U.S. traffic has been devised and incorporated in the algorithm (see Section 1.3).

The preliminary tests were based on a demand matrix $D = \{ d(i,j) \}$ in which directional splits of demand are unknown and assumed to be equal.

Thus, $d_{ij} = d_{ji}$ and $d_{ij} + d_{ji} = d(i,j)$ which is the only known parameter. Similarly, values $n(a,b)$ for total number of circuits in any group were split evently into oriented circuits in each direction ($n^{\langle a,b \rangle} = n^{\langle b,a \rangle}$ and $n^{\langle a,b \rangle} + n^{\langle b,a \rangle} = n(a,b)$)

With these data, the algorithm gives, except for some cases, rather plausible results indicating that in many cases, the first routed traffic of any link corresponds to 50% to 80% of all traffic carried on the link. Further, the amount of traffic remaining on alternate routes after a series of multiple overflows is very small with respect to the primary traffic on these routes. On the Montreal-Regina final group, for example, the algorithm shows only 16% of the possible 135 traffic streams having any significant overflow to this level.

Further experience with the algorithm should permit us to develop better notions of how the circuit groups can be split directionally in a reasonable fashion, and how much error is introduced due to the various hypotheses.

2.3 Réseau Physique

2.3.1 Rôle d'un constructeur de chaînes sur le réseau physique

La section précédente (et plus précisément la Sous-Section 2.2.2) nous a fourni un algorithme permettant de dimensionner les arêtes du réseau de commutation . La question qu'il faut maintenant aborder est la suivante: comment doit-on utiliser le réseau physique pour supporter le réseau de commutation, i.e. quelles chaînes du réseau physique serviront à acheminer le trafic calculé sur chacune des arêtes du réseau de commutation, et quelle sera la dimension (le nombre de circuits) de ces chaînes, étant donnée la capacité, limitée, des arêtes du réseau physique (rappelons que notre problème en est un de gestion et non d'expansion.

Dès qu'on a un réseau le moindrement maillé l'énumération de toutes les chaînes possibles devient un problème gigantesque de telle sorte qu'il faut un algorithme permettant de restreindre le choix des chaînes à certaines seulement, qu'on qualifiera d'"admissibles" selon un critère à déterminer; c'est là le rôle d'un constructeur de chaînes.

Le programme ROUTE part des fichiers qui contiennent des renseignements sur l'existence des arêtes physiques de transmission (propriétaire (s) de l'arête, longitude et latitude de chaque extrémité, capacité nominale en circuits). Comme le niveau de détail est très fin, le programme grâce à des sous-routines comme CUT et TELESCOPE, réduit l'ensemble des éléments à considérer. Malheureusement CUT n'utilise que la notion de distance pour tailler une région admissible (rectangle ou ellipse). Aucun critère économique explicite n'entre en jeu. La réduction des éléments étant faite, le programme établit l'ensemble des chaînes dont il donne une carte et une tabulation. A partir de cet ensemble, il serait intéressant de trier les chaînes disjointes au sens des noeuds et de greffer des algorithmes cherchant les ensembles d'articulation afin d'évaluer certaines solutions d'affectation des circuits du point de vue de la survivance des communications. En tout état de cause, l'ensemble des chaînes retenues peut être utilisé dans certains modèles d'affectation et/ou d'expansion qui exigent l'énumération explicite des chaînes. Etant donné le grand nombre de chaînes il serait indispensable que les résultats de ROUTE puissent être transférés à partir de leur support informatique.

Le programme CADUCEE III de HERMES III aurait pu aussi nous être utile si notre critère d'admissibilité avait pu s'exprimer en termes de coûts variables d'exploitation, ce qui n'est pas le cas pour notre problème de gestion; si de plus on avait des fonctions de coût continues, il faudrait réécrire CADUCEE sans être sûr de son efficacité dans sa capacité à couper drastiquement dans le nombre de chaînes.

Nous avons donc été amenés à définir des critères d'admissibilité selon une optique d'affectation, et ceux qui nous ont paru les plus probants sont la distance, la survivance, et la propriété. Il semble que l'on ne puisse éviter l'énumération d'au moins une partie des chaînes, ne serait-ce que pour établir entre les couples Origine-Destination jugés importants, un nombre de chaînes disjointes supérieur ou égal à deux afin d'assurer une certaine survivance du trafic téléphonique.

Nous reparlerons plus loin de ces chaînes, car l'algorithme que nous avons développé pour répondre au problème d'affectation permet d'éviter l'énumération complète de toutes les chaînes admissibles, ce qui est un avantage assez évident du point de vue de l'espace disponible en mémoire sur ordinateur, et donc de l'opérationnalité d'un tel algorithme. La section suivante explicite l'approche méthodologique qui nous a semblé la plus fructueuse pour traiter d'une manière qui soit la plus opérationnelle possible le problème déjà mentionné au début de cette section.

2.3.2 Utilisation moyenne du réseau physique

Il est évident que même si nous n'avons pas à énumérer toutes les chaînes possibles, il faut néanmoins que les chaînes satisfassent certaines contraintes: assurément il faut que le nombre de circuits sur chaque chaîne soit tel que la demande (sur le réseau de commutation) soit satisfaite, que la capacité sur chaque arête du réseau physique ne soit pas dépassée, et que les contraintes de survivance soient respectées (qu'à tout le moins deux chaînes disjointes au sens des arêtes ou des sommets assurent le lien entre chaque couple Origine-Destination). Une contrainte porte spécifiquement sur le trafic de télévision: il faut que les circuits soient exclusivement réservés, i.e. il en faut tant, ni plus, ni moins, et il faut qu'ils soient acheminés consécutivement et "tout en bloc" entre l'Origine et la Destination. De plus, lorsque plusieurs destinations doivent être rejointes, les circuits ne cumulent pas sur les arêtes comme ils le font pour la téléphonie. Il s'agit donc d'introduire simultanément la recherche d'un arbre pour la télévision et l'acheminement des circuits pour la téléphonie. On parlera plus loin de contraintes supplémentaires qu'il sera possible de prendre en considération dans le modèle global d'affectation.

Après avoir formulé ces contraintes il nous reste à formuler un critère d'optimisation qui nous permettre de discriminer les chaînes optimales selon le critère choisi; il est clair, du moins théoriquement, que l'affectation des circuits sur le réseau physique devrait être telle que le coût d'exploitation variable soit minimisé; il se trouve que ce coût est pratiquement nul dans l'état actuel de nos connaissances. Nous devons donc nous rabattre sur d'autres critères d'optimisation.

Par exemple, on pourrait chercher les chaînes qui, sous les contraintes déjà citées, maximisent la somme des sur-capacités (en mille de circuits) des arêtes du réseau physique; cet objectif aurait l'avantage évident de nous renseigner sur les arêtes pouvant acheminer un trafic supplémentaire à un coût additionnel négligeable, soit pour répondre à un trafic de pointe anormalement plus élevé en certaines périodes, soit pour répondre à des accroissements futurs de la demande; on peut pondérer les sur-capacités de la fonction-objectif pour tenir compte de facteurs tels que la distance, la vulnérabilité,...

On pourrait également choisir les chaînes qui minimisent le nombre de connexions entre chaque Origine-Destination, sous les contraintes habituelles (demand, capacité, survivance,...), car l'on sait que le coût de la commutation ou du simple passage dans un noeud n'est pas négligeable.

On pourrait également choisir les chaînes qui, sous les contraintes habituelles, maximisent l'intervalle de temps, à partir de maintenant, pendant lequel l'accroissement prévu de la demande pourra être "absorbé" par le réseau physique actuel, ceci en supposant des accroissements de la demande "linéaires" (proportionnels au temps), sinon on peut toujours trouver quelque façon de "linéariser".

Comme dernière illustration de critères d'optimisation mentionnons celui qui consisterait à maximiser la somme des "circuits par $", i.e. le nombre de circuits divisé par la charge comptable annuelle d'un élément physique du réseau; cet objectif favoriserait les chaînes les moins coûteuses pour les utilisateurs lorsqu'on utilise la notion de coût unitaire pour faire payer l'utilisateur.

Remarquons qu'en maximisant la somme des sur-capacités, on minimise dans une certaine mesure le nombre de connexions entre chaque Origine-Destination car pour le premier objectif on a avantage à choisir les chaînes les plus courtes (en effet plus la chaîne est longue moins l'on alloue de surcapacité sur la somme des arêtes; mais ce qui semble intuitivement vrai pour une chaîne peut ne plus l'être lorsque sont considérées simultanément toutes les demandes et toutes les contraintes de capacité).

Le choix des objectifs à formuler relève des instances décisionnelles du D.O.C. Mais pour éprouver notre méthodologie, nous avons tout de même choisi un objectif et quelques contraintes; la suite de ce texte a pour objectif d'expliciter cette méthodologie, et subséquemment on trouvera quelques explications sur ce qui a été résolu, théoriquement et pratiquement, et ce qu'il nous reste à faire.

2.3.3 Formulation mathématique

Si l'on pouvait énumérer toutes les chaînes possibles entre chaque couple (Origine-Destination), on pourrait former le tableau suivant où sont représentées schématiquement les contraintes de demandes et de capacités:

$(O\text{-}D)_i$: i-ème couple Origine-Destination ($i = 1, 2, \ldots, p$) sur le réseau de commutation

v_i : i-ème demande, en circuits

u_j : j-ème capacité, i.e. capacité de l'arête physique j ($j = 1, 2, \ldots, q$)

$(O\text{-}D)_1$	$(O\text{-}D)_2$	...	$(O\text{-}D)_p$	
x_1 x_2	..x_s	...	....	
1011 0	0010 11	...	1010 0	$\leq u_1$
1100 1	1010 01	...	1100 1	$\leq u_2$
....	....	...	....	
0110 0	1101 01	...	0001 0	$\leq u_q$
1111 1	0000 00	...	0000 0	$\geq v_1$
0000 0	1111 11	...	0000 0	$\geq v_2$
0000 0	0000 00	...	: :	.....
: :	: :		: :	
: :	: :		0000 0	$\geq v_{p-1}$
0000 0	0000 00	...	1111 1	$\geq v_p$

Notons F cette matrice formée de 0 et 1 (en nombre très grand car chaque colonne représente une chaîne possible); on l'a construite de la façon suivante:

prenons la s-ème colonne de F et

notons $F[r;s]$ = l'élément en ligne r et colonne s de F; alors

$$F[r;s] = \begin{cases} 1 & \text{si l'arête r est dans la s-ème chaîne, } 1 \leq r \leq q \\ 0 & \text{sinon} \end{cases}$$

$$F[r;s] = \begin{cases} 1 & \text{si la chaîne s sert } (O\text{-}D)_i, \quad q+1 \leq r \leq q+p, \\ 0 & \text{sinon} \end{cases}$$ (On suppose la s-ème colonne sous $(O\text{-}D)_i$)

En lisant F ligne par ligne, les premières contraintes expriment que le trafic (pour toutes les chaînes contenant l'arête j par exemple) ne doit pas dépasser la capacité de l'arête j. Les contraintes suivantes expriment que les chaînes servant à répondre à la demande (pour $O\text{-}D_i$ par exemple) doivent au moins la satisfaire. On verra plus loin comment formuler les contraintes de survivance, mais auparavant un bref rappel de programmation linéaire servira à introduire la méthodologie adoptée pour éviter l'écriture "in extenso" de cette matrice F.

Un problème tel que le nôtre peut se ramener à la forme suivante.
Max $z = CX$ sous les contraintes $AX = b$ et $X \geq 0$

où $X = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$, x_i représentant la i-ème activité (ce sera pour notre exemple, le nombre de circuits porté par la chaîne i, ou le nombre de circuits "retenu" par la variable d'écart associée à la i-ème contrainte $\leq$ ou $\geq$)

$C = [c_1 \; c_2 \; \ldots \; c_n]$ où c_i est le coût associé à x_i (si notre objectif est de maximiser la somme des sur-capacités alors c_i = longueur d'un circuit, pour les x_i associés aux variables d'écart des contraintes de capacité, et $c_i = 0$ pour les autres)

A, une matrice de format m par n où m est le nombre de contraintes et n, le nombre de vecteurs d'activité x_i (y inclus les x_i associés aux variables d'écart).

$$B = \begin{bmatrix} b_1 \\ \vdots \\ b_m \end{bmatrix}, \text{ où } b_i = \begin{cases} u_i & \text{pour } 1 \leq i \leq q & (u_i\text{: i-ème capacité}) \\ v_{i-q} & \text{pour } q+1 \leq i \leq q+p = m & (v_j\text{: j-ème demande}) \end{cases}$$

Partitionnons A en $[B \vdots R]$ (B pour Base, R pour Reste) où B est une sous-matrice inversible de A; les contraintes s'écrivent alors

$BX^B + RX^R = b$ et $X^B \geq 0$, $X^R \geq 0$ et la fonction objectif s'écrit

$z = C^B X^B + C^R X^R$

Puisque B est inversible $BX^B + RX^R = b$ peut s'écrire $X^B = B^{-1}b - B^{-1}RX^R$
d'où $z = C^B B^{-1} b - C^B B^{-1} RX^R + C^R X^R$

Notons $\Pi = C^B B^{-1}$ alors $z = \Pi b - \Pi RX^R + C^R X^R$ où $\Pi = [\Pi_1, \Pi_2, \ldots, \Pi_m]$

En augmentant (ou diminuant) de 1 unité b_i, z changera de Π_i unités (symboliquement: $\frac{\Delta z}{\Delta b_i} = \Pi_i$). On appelle Π_i le prix d'ordre associé à la i-ème contrainte.

Comme les variables hors base sont à un niveau nul, quel serait le changement dans z suite à l'introduction de la j-ème variable x_j^R ?

$$\frac{\Delta z}{\Delta x_j^R} = -\Pi R_j + c_j = c_j - z_j$$ où R_j est la j-ème colonne de R.

Quand notre problème en est un de maximisation, les x_j^R (les chaînes) qu'on aura localement avantage à entrer dans la base sont celles qui augmentent le plus z, i.e. pour lesquelles $c_j - z_j > 0$ est le plus grand possible i.e. $z_j - c_j$ (<0) minimal.

Quand l'objectif choisi est de maximiser la somme des sur-capacités on a $c_j = 0$, pour les j associés aux chaînes; donc la chaîne candidate à entrer dans la base sera celle pour laquelle $z_j \leq 0$ sera minimal; or $z_j = \Pi R_j$ où R_j est le vecteur-colonne de la chaîne, donc le problème de chercher la chaîne qui soit la meilleure candidate à entrer dans la base peut se formuler comme une recherche de la <u>chaîne la plus courte</u> au sens des prix d'ordre. En développant ΠR_j selon les composantes associées aux contraintes de capacités et de demandes, respectivement, on a

$\Pi R_j = \Pi^c R_j^{c \mid (C-D)_i} + \Pi^{(O-D)_i}$ (en supposant que R_j soit une chaîne de $(O-D)_i$)

donc $\min_j \Pi R_j = \min_i \left\{ \Pi^{(O-D)_i} + \min_{j \in A_i} \Pi^c R_j^{c \mid (O-D)_i} \right\}$ où A_i est l'ensemble des indices des chaînes servant à répondre à la demande $(O-D)_i$. Il faudra donc trouver la chaîne minimale par rapport à Π^c pour chaque $(O-D)_i$, puis la chaîne minimale (parmi ces dernières) par rapport à Π^d où $\Pi^d = [\Pi^{(O-D)_1} \ldots \Pi^{(O-D)_k}]$. Remarquons que s'il existe des prix d'ordre négatifs pour quelques-unes des contraintes de capacité, il est avantageux de faire entrer dans la base la ou les colonnes associées aux variables d'écart correspondantes, car premièrement, il y a possibilité (et même grande probabilité!) de cyclage lorsqu'on cherche avec certains algorithmes la chaîne la plus courte au sens des prix d'ordre associés aux contraintes de capacité, et deuxièmement parce que de toutes façons l'objectif s'en trouve amélioré

$$(\text{car } \frac{\Delta z}{\Delta_{x_k} R} = c_k - z_k = c_k - \Pi R_k = c_k - \Pi_k > 0)$$

Quand $\Pi^c \geq 0$ on peut chercher sans crainte la chaîne la plus courte (i.e. $\min_{j \in A_i} \Pi^c R_j^c \mid (O-D)_i$), et il ne dérange en rien que $\Pi^{(O-D)_i}$ soit négatif, au contraire, car l'on veut $z_j \leqslant 0$.

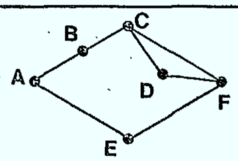
Une fois qu'on a trouvé la chaîne la plus courte au sens des prix d'ordre on doit choisir parmi les variables d'écart (ou les variables artificielles selon le cas) et cette chaîne, celle qui va entrer dans la base, et parmi celles qui sont dans la base celle qui va en sortir, par les critères usuels de la méthode du simplexe (ici on utilise le simplexe revisé car on n'a pas à énumérer physiquement toutes les chaînes possibles au départ, mais on va plutôt chercher à chaque itération celle qui augmentera le plus la valeur de l'objectif si elle entre dans la base).

A cet effet la programmerie a été mise au point pour résoudre le problème de maximiser la somme des sur-capacités sur chacune des arêtes du réseau physique sous les contraintes de demandes et de capacités; quand à l'objectif de maximiser la période de temps à partir d'un état initial où la demande pourra être tout juste "contenue" par le réseau physique, la programmerie mise au point n'aura à subir que de légères modifications si l'on peut déterminer un taux d'accroissement proportionnel au temps, sur chaque arête du réseau de commutation (un taux non proportionnel pourra être approximé par quelques simulations avec divers taux proportionnels!).

Il nous reste maintenant à inclure dans cette programmerie les contraintes de survivance, et d'y insérer les structures permettant de traiter d'un "flot" homogène, soit celui de la télévision. En ce qui concerne les contraintes de survivance le problème est théoriquement résolu lorsqu'on identifie l'ensemble d'articulation minimal et les chaînes disjointes passant par chacun des sommets de cet ensemble; on a alors qu'à imposer que tel ou tel pourcentage du trafic passe par chacune de ces chaînes disjointes; le reste du trafic sera "routé" par le modèle d'optimisation choisi. Il reste néanmoins une possibilité d'arbitraire encore non résolue, et que l'on a illustrée sur le diagramme suivant:

DIAGRAMME No 3

deux ensembles d'articulation minimaux, soit $\{B,E\}$ et $\{C,E\}$; et si on a choisit $\{B,E\}$, deux ensembles de chaînes disjointes sont admissibles, soit



$\langle A, B, C, F\rangle$ et $\langle A, E, F\rangle$

ou

$\langle A, B, C, D, F\rangle$ et $\langle A, E, F\rangle$

On a déjà parlé brièvement de la particularité de flot "télévision"; nous devons traiter la contrainte d'acheminement des circuits en tenant compte que ce flot est homogène, alors que les diverses demandes téléphoniques nous avaient conduit à un problème de multiflot (plusieurs flots différents). Ce genre de contrainte appelle une méthode de résolution propre, déjà résolue théoriquement par le Laboratoire d'économétrie, mais qu'il reste à inclure dans la programmerie déjà au point en ajoutant des contraintes permettant de manier le flot "télévision", et des contraintes de liaison pour tenir compte de tous les flots à la fois.

En résumé, voici ce qui a été résolu:

- le problème de maximiser la somme des sur-capacités sous des contraintes de demande et de capacité; un algorithme en APL est déjà au point;

- identifier un ensemble d'articulation minimal entre chaque couple Origine-Destination; un algorithme en APL est déjà au point qui, en dehors du modèle d'affectation, peut être utilisé avec avantage pour les problèmes de survivance.

Et voici ce qu'on envisage comme extensions:

- à partir de l'algorithme identifiant un ensemble d'articulation minimal, on peut retracer un nombre de chaînes disjointes (autant qu'il y a de sommets dans l'ensemble d'articulation) et imposer comme nouvelles contraintes que tel ou tel pourcentage de la demande en circuits passe par chacune;

- inclure le flot "télévision" dans notre modèle d'optimisation;

- retoucher la programmerie pour pouvoir prendre en compte l'objectif de maximiser la période de temps, à partir de maintenant, pendant laquelle l'accroissement prévu de la demande pourra être "absorbé" par le réseau physique actuel, en supposant des accroissements "linéaires" de la demande en fonction du temps;

- essayer l'algorithme en prenant comme objectif: Maximiser la somme des "circuits par $";

- tenir compte du facteur distance, et du droit de propriété, avec les contraintes que cela implique pour l'acheminement du trafic, et donc sur l'allocation des circuits aux diverses chaînes.

C'étaient là, brièvement énumérées, les questions sur lesquelles nous nous penchons actuellement et dont une partie aura été résolue, nous l'espérons, lors de la parution ce ce rapport.

2.4 Interface with U.S.A. and Overseas Network

The previous Sections 2.2 and 2.3 have described the procedural methods proposed for a simulation of traffic flow on the switching, and on the facilities network. Each of these algorithms is based on the assumption that data (albeit rudimentary) is available for all parts of the network. However, concerning traffic between Canada and the United States or overseas destinations, we are faced with the problem of the connection of the Canadian system to foreign systems. In the case of Canada-U.S. traffic, not only is its quantity relatively large, but the long distance networks of the two countries are fully interconnected. We cannot therefore treat the problem rigorously without taking full consideration of the alternate routing possibilities introduced by this interconnection. Practically speaking, however, within the scope of this study we are not equipped nor required to perform a full auxiliary study of the United States (or overseas) networks. We must therefore seek a compromise.

The central problem to be undertaken relative to Canada-U.S. traffic is to discover how this traffic is routed through the Canadian system until entering the U.S. in order to estimate costs (operating, and capital) as well as investments which are necessitated or incurred by this traffic. Furthermore, in correct accounting for the application of the Commonwealth revenue sharing scheme we must be able to identify the usage of the switching and the facilities network for all traffic streams.

A preliminary solution to this problem can be described in two phases. First, regarding the switching network, we propose a scheme whereby a fictitious traffic source is introduced in the network to represent U.S. destinated and originating traffic. This node (U.S.) is connected to the Canadian final tree via final group routes for Montreal to U.S. and Regina to U.S.. The number of circuits in these groups is estimated as the sum of all final circuits leaving Montreal (or Regina) for any of the 10 United States No. 1 switching centers. For high usage group connections from various Canadian traffic centers to the U.S., we include a fictitious high usage route from each point to the U.S. node, the number of circuits again being estimated as the total of circuits in all high usage groups emanating from the Canadian center to the U.S. With this arrangement, and a minor modification to the algorithm of Section 2.2.2 it is possible to estimate the routing of Canada-U.S. traffic on the schematic switching network.

The second phase of this approach involves the conversion of traffic load information on the switching network to allocation of circuits on the facilities network. The facilities network comprises several cross-over points (or portals) for connection of the Canadian network to the U.S. network. For a total traffic from, say, Vancouver to the U.S. it would be necessary to know how much of this traffic reaches the U.S. network through each cross-over point in order to allocate circuits in Canada for this traffic. At present no firm solution to this problem is available, but it is proposed to separate the total traffic of any Canada-U.S. stream in proportion to the breakdown of the circuits in each of the distinct high usage or final group routes emanating from the Canadian source to the U.S. destination. For U.S. originated traffic the only apparent choice will be to assume a similar distribution on cross-over points as is assumed for Canada-U.S. traffic.

The second problem, regarding overseas traffic, has not been examined in detail but the following points can be made. For routing of overseas traffic two portals (Montreal and Vancouver) will provide connection to the C.O.T.C. system. Overseas messages then remain in the C.O.T.C. system until leaving the country, a circumstance which will necessitate manual routing of these streams in this phase. The above comments concerning Canada-U.S. traffic are valid also for the C.O.T.C. overseas connection, and revenue settlements can be analyzed only if more data is obtained. Settlements between C.O.T.C. and T.C.T.C. however can be analyzed more closely and put on a systematic basis since all cost and revenue information will be, in principle, available.

The possible bias introduced by this hypothesis is unknown but can be examined only in the presence of more complete data on the network. The solution proposed cannot be directly validated and must therefore be judged on indirect criteria.

2.5 Updating and Generating Data

To answer many questions about policy impacts, traffic data must be known at a very fine level of disaggregation. Given the importance of these data for the entire project, the "Laboratoire" accepted to conceive some method to generate at least the semi-realistic data which are needed during the development phase of the project.

The following paragraphs will state the problem and elements of solution will be furnished for the public switched message traffic only.

The actual data developed by the D.O.C. were generated following a method well defined in D.O.C.'s paper entitled " Derivation of Point to Point Traffic Demand, August 30, 1972" (in Progress Report, Project HERMES, September 13, 1972). The first thinking of the "Laboratoire" on this subject appears in "Note pour le calcul de données fictives de trafic, Appendix D, Interim Report on the First Phase of the Project (IRA), December 1st, 1973".

In its full generality, the problem consists in filling a traffic matrix T showing the offered loads for all oriented pairs of demand points. If 50 demand points are envisaged, 50 x 50 - 50 = 2450 (zero diagonal) numbers have to be found. If 200 points are considered, 40,000 - 200 numbers (!) are needed. These loads are for a given period.

It is clear that a priori information must be used to decrease the indeterminacy. The list of what is known in this problem is as follows:

1) the switching network and its hierarchy

2) the routing and accessibility rules in the switching network (S.N.)

3) the relationship between the load, the number of circuits and the probabilities of loss or overflow

4) the numbers of circuits on each link of the S.N.

5) the grade of service on each link during the peak load period

6) the load lower bounds for H.U. creation

7) the distance between the pair of demand points

8) the population or number of business and residence telephones in the zone around each demand point

9) the offered load profiles (not the loads themselves) for each pair of demand points

10) some bench-marks for loads of a set of pairs of points.

To be sure, the above restrictions are not precise enough as expressed; operational definitions should be given.

A first method to estimate the traffic matrix is to create software which reproduces the D.O.C.'s method outlined in its document of August 30, 1972.

A second method is to implement the mathematical program outlined in the Laboratoire's paper of December 1st, 1973. Some statistical computations are now being made in order to possibly find a relationship between the loads as estimated for the 24 node network and some gravity-like variables (for instance the product of the population involved in each Origin-Destination divided by the squared distance separating the same points). The department of Communications is also working on the same problem (see Annex F).

A third method amounts to solve a huge under-determined linear system of equations. As a matter of fact, consider a switching network with N links (Final and H.U.) and K Origin-Destination pairs of demand points. Assume that the number of circuits and the probabilities of overflow and loss are known (or derivable from certain rules on H.U.

creation and time zone profile considerations). The algorithm CHARGE from the HERMES model can compute for one unit of a given original load the fraction which is offered to a given link of the switching network. The system of equations is then derived as follows:

1) Find the offered load for each link of the S.N. using the number of circuits, the probabilities of loss or overflow and the Poisson and Erlang formulae.

2) Note that for a H.U. linking a pending node to another pending node, the computed load in 1) is pure point to point load without any transit or overflow traffic. Therefore the number of loads to be determined has shrunk.

3) Fix any a priori known load of the traffic matrix. This reduces again the number of loads to be estimated.

4) For a given link of the S.N., multiply each load (known or unknown) by the fraction offered to that particular link, add the results for all laods and set this equal to the line load computed in 1). There are as many equations as links in the S.N. and as many unknown variables as variables left after steps 2) and 3). The coefficient matrix A is at most N by K. We suppose A is full rank (that is rank A=N). We have a tremendous under-determination, at most K-N variables can be fixed at will. Gravity models again would be used to reduce the indeterminacy. We rather propose the usage of the generalized inverse to solve that problem. We are presently making some strides in that direction.

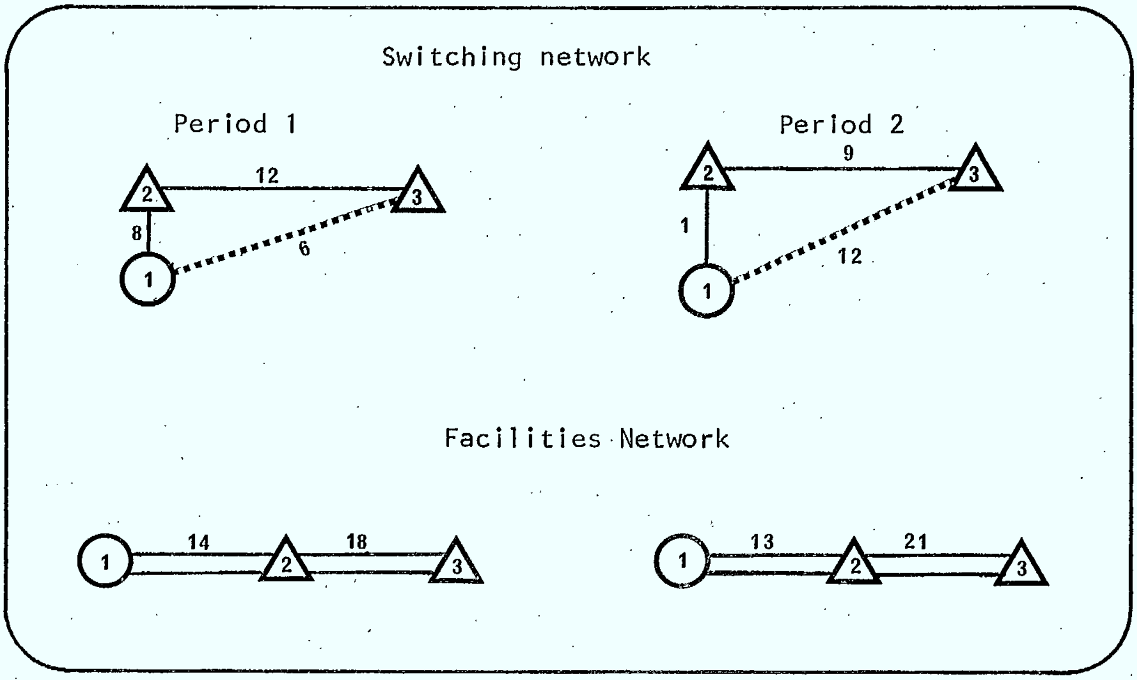
2.6 Treatment of non-simultaneous peak periods

Throughout this chapter it becomes evident that a central problem which we face in dealing with a geographically dispersed network is the non-coincidence or non-simultaneity of peak usage periods in different geographical areas of the network. This occurence is partially but not entirely due to the fact that the network covers different time zones. Inside the same time zone one also observes non-simultaneity of peak periods in different exchanges depending on the residential or commercial business character of the exchange.

With the algorithms described in Sections 1.1 and 1.2 we are now able to calculate the average usage of the switching and the facilities network for any typical traffic period (peak or off peak) and determine thereby the peak usage of any element by selecting its maximum use over all typical periods.

We may, however, at this point raise a methodological issue with the aid of the following simplified example.

Diagram 4

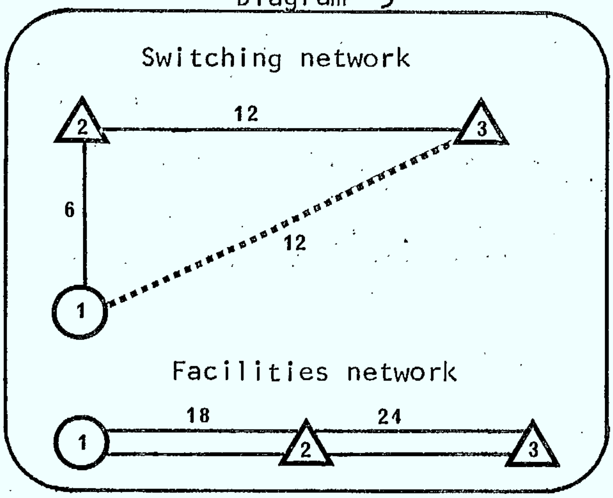


In this example, we note that for a typical traffic matrix in each period, (not shown) the total number of circuits required on each link of the switching network can be derived (as in 2.2.2). Supposing that points ①, △2 and △3 are located geographically in the fashion shown above we might find the allocation and circuit requirements as indicated in the diagram (according to the method of 2.3).

From these results, we note that the facilities joining △2 to △3 have an average peak usage of 21 circuits occuring in period 2.

Using the same example, we may inquire what would happen if we formed, at the level of the switching network, an artificial peak usage diagram by taking the maximum usage on all links of the switching network, and allocated the peak usage to the facilities network.

Diagram 5



We obtain, on the facilities network, a circuit requirement which is superior to the true peak (14 circuits (1) to △2, 21 circuits △2 to △3). Thus we deduce that estimation of peak usage of the facilities network must be done by an independent treatment of the entire system for each typical period, and a maximization which takes place at the end of these calculations.

This necessity holds in so far as any element of the facilities network is the common support of two or more high usage or final groups of the switching network (which is the case). Alternatively, each link of the switching network could be assigned a physical support independently of each other and the problem would be decomposable.

3. COSTING BLOCK

3.1 Introduction

The main purpose of this introduction is to put the Costing Block in its proper perspective. In particular, we will try to show that the general philosophy behind this block is to some extent the same as the one which prevails in Canada in the regulation of the industry of communications. Although many aspects will be considered, those relating to this philosophy will be explained in greater detail in the following sub-sections, and in particular we will propose some means of evaluating or estimating various costs.

The Costing Block as it stands presently, can be divided into the following five sub-sections:

a) The determination and the indexing of the asset cost functions, attempting to take into account the modifications of the technology. The assets considered are the different elements of the facilities network.

b) The evaluation of the capital cost which includes depreciation, income taxes and the cost of capital.

c) The treatment of operating costs which include among other things maintenance, traffic expenses, etc.

d) A review of some costing methods and some approaches to cost estimation and a discussion of the problem of separation of common and joint costs. In the IRA Project, separation is an important aspect, as our main concern is in inter-regional activities. Also, it is important to be able to calculate different unit costs since the results will be used in the Sharing Block and, in the future, in the Policy Simulation Block for studying, among others, the problem of cross-subsidization among services.

e) The handling of the inter-relationships between this block and the Accounting Block, the main purpose being to achieve consistency with the financial statements which are produced by the IRA model.

In Canadian regulatory practice for the telecommunication industry in general, permissible ranges are given for the rate of return, which are then applied to certain rate bases. In other words, knowing the current operating costs, the amount of capital invested (or to be invested) and the allowed rate of return, it is possible to determine the so-called total revenue requirements. Knowing the total revenue requirements, the next problem to be resolved is that of establishing the appropriate tariffs for the services which are regulated so that the carrier can recover the total revenue requirements. It may be noted that in the determination of tariffs for each service cost is only one factor taken into consideration. Many other factors can be taken into account, in particular, the problem of peak demand, the problem of assuring an efficient utilization of the network, some social criteria, and the consideration that businesses must pay more than residential customers. Of course, for some problems already mentioned, it is possible to supply economic rationale to justify these arrangements. For example, those users of the network who are responsible for peak demand ought to pay more because they create a need for excess capacity.

Basically, these regulatory aspects comprise all Costing Block activities. Of course, in the IRA Project, our main concern is in the Inter-regional activities, so we must be able to make a separation between the different sectors or activities considered. For some factors this separation is very difficult to make, and sometimes not meaningful. For example, if the rate of return is defined as the required rate of return (i.e. the cost of capital) by the investors, what is the significance of an Inter-regional required rate of return, since the investor regards the company as a whole. However, the point being made in this introduction is, first of all, that the problems to be resolved for regulation and Costing Block activities are similar, and secondly that the Costing Block can be used to evaluate modifications of policy in the communication industry, once some decisions regarding the separation of common and joint costs between the two sectors (Regional and Inter-regional) are made.

Finally, a number of problems are reviewed and for each, certain solutions are proposed. The first problem is, of course, to find appropriate cost functions for assets (switching and transmission facilities) which, when applied to these facilities, can reflect the amount of capital invested, and consequently, the rate base. In Sub-Section 3.2 we suggest new asset cost functions for the transmission facilities. For switching asset cost functions further development work is required. Evidently, as previously stated, the main objective of these functions is to give an evaluation of the assets put in place at some given point in time. For many reasons this value is different from the present reproduction costs, so a re-evaluation is necessary. This re-evaluation will be done by taking three factors into account: i.e. aging of the assets, price level changes and changes in technology. In the proper sub-sections we provide some motivation for considering these factors.

Capital cost is a delicate question in the sense that there is no unanimity about how to treat it: questions such as what to include in the depreciation, which method to use, and how to compute the cost of capital are posed. For example, there exist depreciation in an accounting sense, and depreciation in an economic sense. Also, the question may be posed whether a certain rate of inflation should be included in the cost of capital or whether it is already included. In Sub-Section 3.3, we supply the appropriate formulae to calculate the depreciation reserve and the annual depreciation rate using the Average Service Life and Equal Life Group methods. We also consider the problem of flowthrough versus normalization of deferred taxes. Finally, we show how to calculate the cost of capital. Further, in Sub-Section 3.4, we propose some means to evaluate the operating costs. One can note at least three points: firstly, these costs are current as opposed to historical, secondly many expenses which appear under this heading are largely discretionary expenses, and finally, it is perhaps at this level that the problem of common costs is more evident.

In Sub-Section 3.5, five types of unit costing are considered, some means of evaluating unit costs are proposed, and certain problems related to cost separation are enumerated. In a last Sub-Section, we try to ensure the consistency between the Costing Block and the Accounting Block, particularly as far as the roles of the cost of capital and deferred taxes are concerned.

It is interesting to note once again that, in the regulatory process, the problems to be solved are basically the same when the carriers apply for a revision of their tariffs. Of course, in regulation there are a number of other factors which must also be considered as, for example, the disparity between the book rate of return and the market rate of return.

3.2 Asset Costs

3.2.1 Asset cost functions

The cost functions used in IRA I were those originally developed for the HERMES series of models. The functions were good approximations at the time they were constructed and were the only ones available for the first phase of the IRA Project. However a considerable amount of work has been done on this subject in the last few years by the Network Development Section of the NTB and by the Communications Research Centre of the DOC*. It was generally felt that the asset valuation procedure of IRA II could be improved by incorporating this new information on asset costs into the model. The new cost function data does not come in a form identical to the format of the IRA I cost functions. Thus some method was required to put the data from the two studies mentioned above on a common basis and then make this compatible with IRA's input requirements. This section of the report outlines the methodology or approach we intend to use to overcome these problems.

We begin with a brief review of the present IRA cost functions. The asset cost functions presently used are of the form

$$F_k\ (x)$$

where x is the number of circuits and
k is the type of asset.

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* Bowen, R.R., Baser, R.V., Walker, E.A., and Hutchison, R., L, "A simplified Model of the Canadian Terrestial Trunk Communications Network"; Communication Research Center, Department of Communications Ottawa, 1974.

DIAGRAM 6

TCTS MICROWAVE
RFC = 960

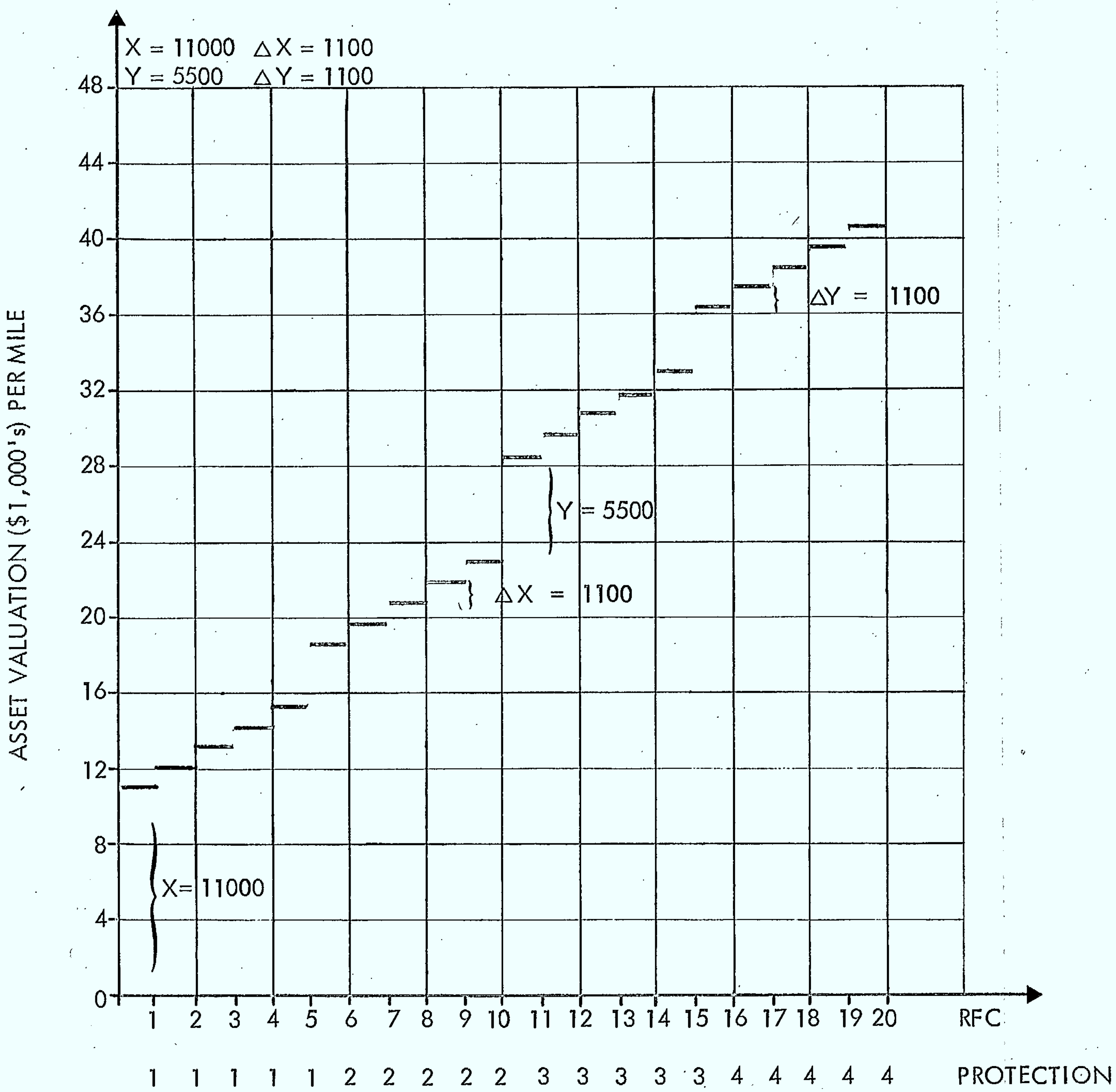
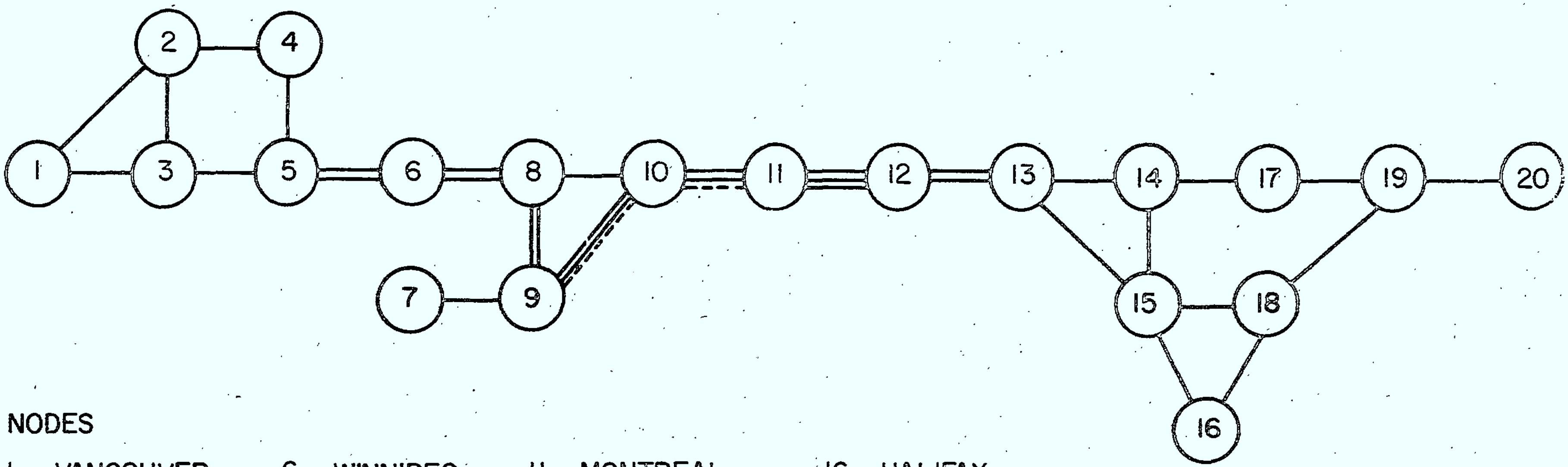


DIAGRAM 7

Source: Bowen et Al.



NODES

1	VANCOUVER	6	WINNIPEG	11	MONTREAL	16	HALIFAX
2	EDMONTON	7	WINDSOR	12	QUEBEC CITY	17	SEPT - ILES
3	CALGARY	8	SUDBURY	13	RIV. DU LOUP	18	SYDNEY
4	SASKATOON	9	TORONTO	14	RIV. BLANCHE	19	CORNERBROOK
5	REGINA	10	OTTAWA	15	MONCTON	20	ST. JOHN'S

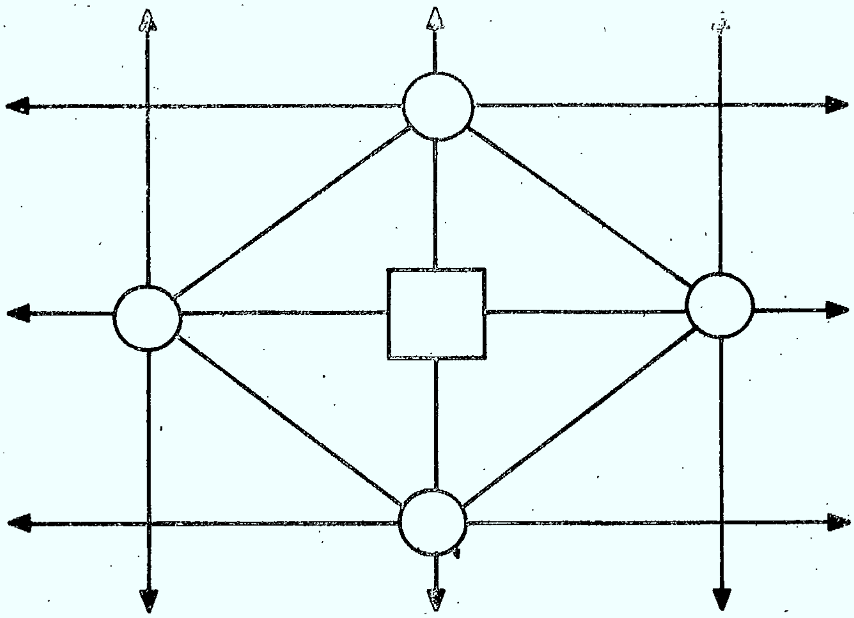
LEGEND

——— 4GHz RADIO SYSTEM

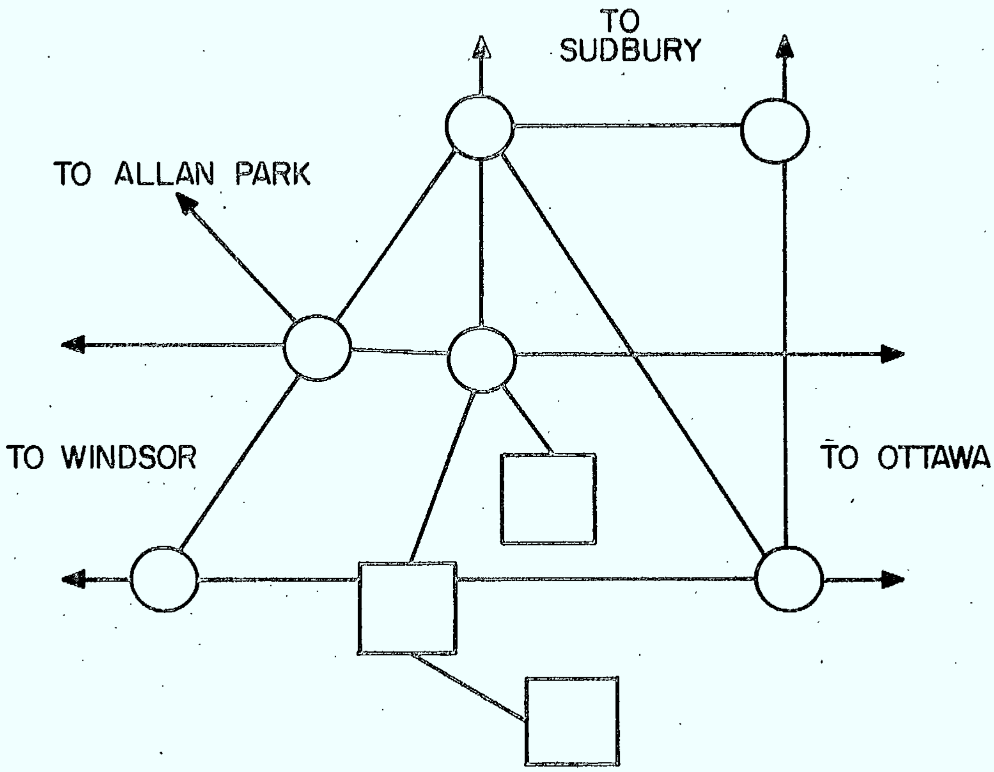
- - - - - - DIGITAL CABLE SYSTEM

SIMPLIFIED TCTS LONG-HAUL TRUNK NETWORK

DIAGRAM 8



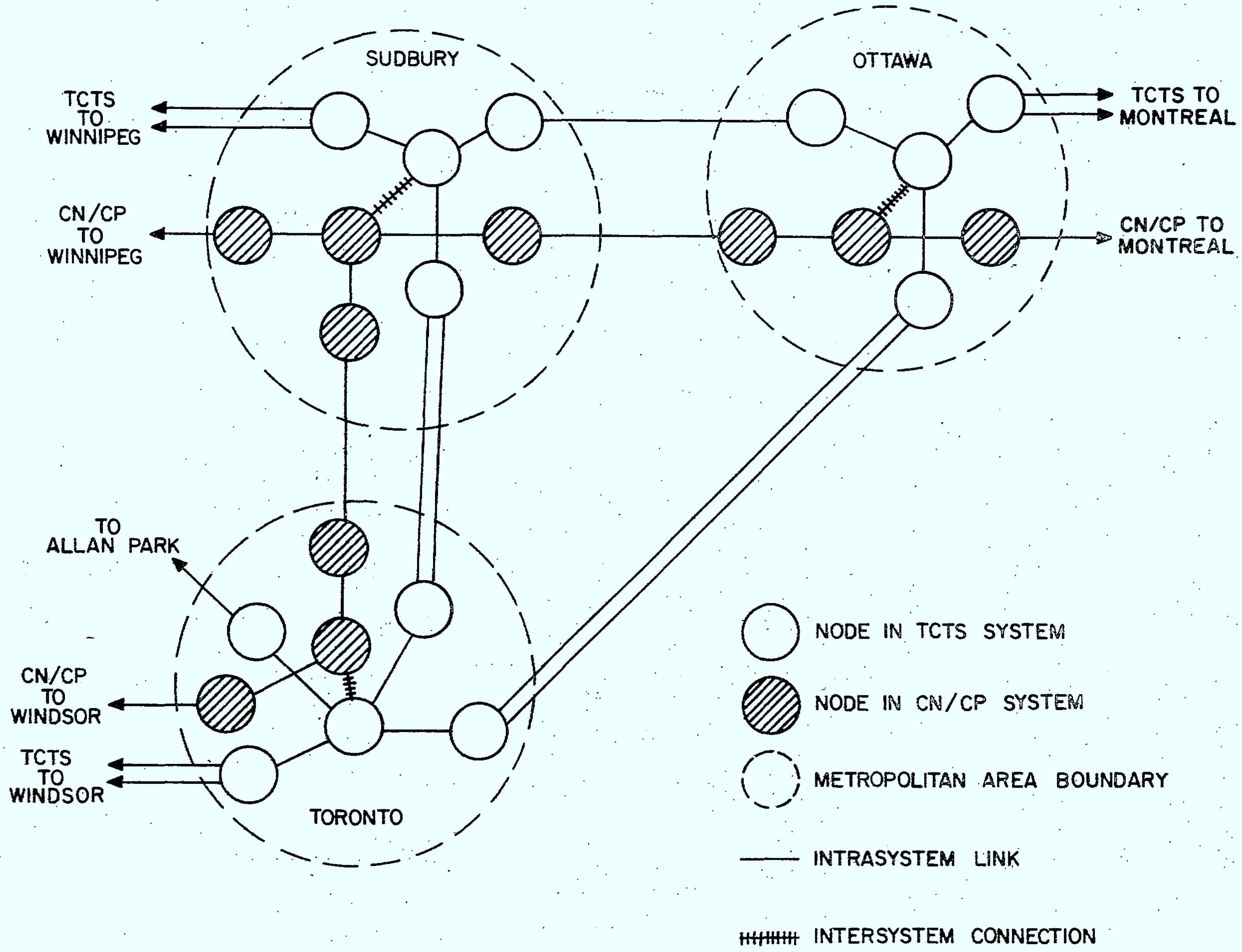
IDEAL MATURE METROPOLITAN NODE



Source: Bowen et Al

METROPOLITAN TORONTO NODE, 1973

DIAGRAM 9



Source: Bowen et Al

$F_k(x)$ gives the dollars investment required for the given number of circuits on 1971 reproduction cost basis. For transmission equipment the function gives total $/mile. This is a step function as illustrated in the diagram no 6. It should be noted that this type of function applies to the elements in the facilities network.

For purposes of comparison we will now examine the form of the new cost functions that have been developed.

The study by the Communications Research Center, directed by R.R. Bowen, produced an extensive examination of the costs of existing and proposed terrestrial systems. We are, of course, concerned with the existing TCTS microwave network and we will focus only on those cost functions that apply to this area.

The Bowen study begins with a simplified model of the TCTS network consisting of 20 nodes (see diagram 7). His purpose is to develop asset cost functions for the links between the metropolitan nodes in his 20-node network. That is, his costing is done on a link basis. The problem that arises is that of dealing with the costs of links connected to major metropolitan nodes. From the diagrams of metropolitan nodes that follow (diagram 8) it can be seen that with a node of this size there are junction repeaters (the circles) and switching machines (the squares). The cost of the junction repeaters must be included in and allocated to the cost of the transmission links.

In order to retain the concept of allocating all costs to transmission links rather than nodes, Bowen developed a model in which each major metropolitan area is represented by a star network, with a branch of the star for each other node that is connected to the node being represented. Bowen's "star" models of the Toronto-Sudbury-Ottawa nodes are shown in diagram 9. Based on a cost analysis of each of the nodes modelled in this way (using 1973 costs), the total cost of each "star" is equal to the actual metropolitan (transmission) cost for that area. The cost of each branch or link is proportional to its number of installed circuits. This is an ingenious and apparently effective method of allocating transmission costs in the major metropolitan nodes to various links.

Having dealt with this problem Bowen is able to develop cost functions for the links between all the nodes in his network. His cost function for a fully implemented link is of the following form:

$$C_F = C_1 + C_6 + C_{11} + 11.5C_N$$

where

C_1 is the cost of the first full-duplex radio channel

C_6 is the cost of the sixth channel

C_{11} is the cost of the eleventh channel

C_N is the incremental capital cost of each other channel except the sixth and eleventh.

A glance at the 20-node network employed by Bowen reveals that it is quite a simplified model of the Canadian toll trunk network. In the IRA Project we are working with a model of the transmission network that is far more detailed and disaggregated. Where Bowen conceives of one link between, for example, Vancouver and Edmonton, IRA has many smaller links which, joined together, make up the Vancouver-Edmonton "link". Thus, for the purposes of IRA it will be necessary to cost out the transmission network at this lower level, i.e. at what we might call a "micro-link" basis. For costing at this level, the cost functions developed by the NTB might be quite useful.

This model is capable of costing any link or set of links in the network providing the functions of all the terminals and repeaters in the sub-network can be specified. The model expresses the total investment cost of any such sub-system as follows:

$$\text{Total} = (A \cdot C_1) + (B \cdot C_2) + (D \cdot C_3) + (A + D) \cdot MUX \cdot i \cdot N$$

where:

- A is the total number of terminals
- B is the total number of repeaters
- D is the total number of branching repeaters
- C_1 is the capital cost per terminal
- C_2 is the capital cost per repeater
- C_3 is the capital cost per branching repeater
- MUX is the cost of multiplexing per voice circuit
- i is the number of voice circuits
- N is the number of R.F. channels

C_1, C_2, C_3 and MUX are provided by the NTB paper. A,B,D,i and N are inputs determined by the system configuration.

NTB's functions do not however incorporate Bowen's technique of handling the transmission costs associated with major metropolitan nodes. The lack of a technique for handling these costs makes the NTB functions less accurate when costing sub-systems or links involving major metropolitan nodes.

It appears that Bowen's cost functions are very accurate for obtaining the investment cost of an entire link or route between metropolitan nodes in his simplified network. However, for the purposes of IRA we must obtain investment costs at a much more disaggregated level, the transmission link level. It may be possible to use Bowen's data on equipment costs (see p. 45-49 of Bowen's paper) to obtain investment costs at the element level. These data are the major inputs or components of his link cost functions. The equipment cost data is provided in tabular form and can easily be translated into step cost functions to be compatible with the present input requirements.

These cost functions will be applied on an element basis to the IRA model of the facilities network. (An explanation of the term element is given below). Once these cost functions have been applied a value can be obtained for all the assets in the network. Furthermore, from our knowledge of the routing and volume of traffic in the facilities network, we can allocate or attribute the capital cost of every element to the various traffic streams using the element in proportion to the volume of the streams.

We can identify four types of elements in the facilities network.

The first type includes all switching machines. This type of element will be valued according to revised cost functions for switching machines which have been prepared by the NTB.

The second type of element includes all transmission equipment which is in a major metropolitan area and connected to a switching machine. For all major metropolitan areas, equipment of this kind will be identified and valued according to the functions provided by Bowen for junction terminals and terminals. Then, the values of this kind of equipment will be summed for each node and then allocated to the various routes or links entering the node in proportion to the number of installed circuits of each such route or link. The resulting costs will represent the costs of transmission facilities associated with a switching node. This costing approach is similar to that followed by Bowen.

The third type of element is usually referred to as a simple repeater. From the length of each transmission link and by assuming an average hop between repeaters we can compute the number of simple repeaters in a given link. These can be valued using Bowen's figures for a regular repeater.

The final type of element is sometimes known as a branching repeater but is also referred to as a transmission node. At a transmission node radio channels are re-routed in several directions. No message switching takes place. The elements will be costed using Bowen's functions for drop repeaters and junction repeaters.

By applying these different cost functions to the four kinds of elements described above we can obtain asset values for the entire facilities network.

The cost functions which we are presently applying to switching machines are still fairly primitive and must be refined to account for the various kinds of switching machines. Work is also continuing in the NTB on the development of better cost functions for transmission facilities. The reader is referred to Section 3.2 which contains more information on costing, particularly the relation between element costs and stream costs.

3.2.2 Aging - Valuation of historical costs

The value of telephone properties calculated by the asset cost functions of Section 3.2.1 represents the reproduction cost of the existing telephone properties in service. In other words, the assets of all vintages are evaluated at say, 1971 prices. The task before us is to convert such reproduction costs into the historical or original costs. In order to do this, we have to calculate the dollars surviving from the vintages, which we call the aging procedure, and to construct price indices relative to such vintages, which we call the indexing procedure. By combining these two procedures we may accomplish at least theoretically, the reconstruction of historical costs of surviving plants in service of various vintages/ages.

The following algorithm is for the calculation of the dollar values of reproduction costs based on a given year's prices of surviving plants by vintages.

The aging of the physical facilities will be based on the Life Formulae. In addition, these computations will be done on a plant category basis in the categories of switching, transmission, station equipment, general equipment and buildings, this being the lowest level of detail for which data is available.

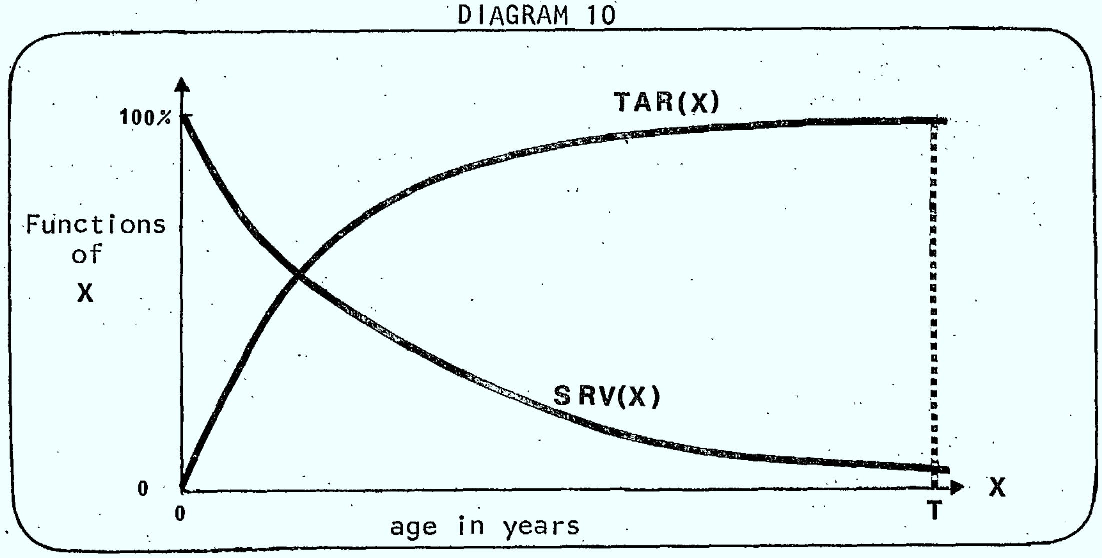
For each of the 5 plant categories, the following input is required:

i) Average Life, L

ii) Maximum Life, T. This is the age of the oldest surviving plant .

iii) A life Survival Curve. This will be based on tables chosen from the Iowa Curves or any other type curves. For a given curve the following graphs are provided with values for discrete points of the graphs:

DIAGRAM 10



where

X = age in years

Y = percentage of age (years) to average service life, L e.g. for 2 years age with average life of 5 years, Y is 40%

SRV(X)= current percentage of surviving plant installed X years ago

TAR(X)= current total accumulated recoupment percentage of plant installed X years ago.

iv) Growth rate, R

R will be calculated in the growth or gross additions module of the Costing Block. In using a single R, the assumption of a constant growth rate is made. If r represents the annual rate of growth, say .10, then R = 1.00 + .10 i.e. R = 1.10. Such an R allows a compounding of growth from year to year using exponentiation.

v) Current Gross Telephone Plant, GTP

The GTP represents the historical book cost of surviving plant.

With these inputs, the values of the surviving plants by vintages can be calculated. It should be noted that any plant installed more than T years ago will not be surviving today by definition of T, and hence is of no interest.

The first calculation is the determination of the gross additions, GA, for the oldest surviving plant (plant installed T years ago):

$$GA = GTP / \sum_{X=1}^{T} \left[R^{(T-X)} \cdot SRV(100 \cdot X/L) \right] \quad \ldots\ldots\ldots (1)$$

This is based on the current value of surviving plant installed X years ago being

$$GA \cdot R^{(T-x)} \cdot SRV(100 \cdot X/L)$$

Where

GA is the initial, oldest installation, e.g. $1,000.

$R^{(T-X)}$ is a compounded growth factor e.g. X = 2 years, T = 14 years.

$R^{(T-X)} = (1.1)^{12} = 3.138$

Hence GA $R^{(T-X)}$ = \$ 1,000 • 3.138 = \$3,138 which is the value of gross additions two years ago. The gross addition have grown from $ 1,000, 14 years ago to $3,138, 12 years later.

SRV(100·X/L) is a percentage surviving which is based on the given Iowa Curve and is also a function of percentage of average service life

e.g. X = 2 years, L = 5 years
SRV(40) = .8095 or 80.95% for $L_{0.0}$ Iowa Curve

The present value of the gross additions installed two years ago is:

$GA \cdot R^{(T-X)} \cdot SRV(40) = \$1,000 \cdot 3,138 \cdot .8095 = \$2,540.$
That is of $3,138 installed two years ago 80.95% or $2,540 has survived until the present.

Hence, GTP is based on surviving plant for all vintages from present to T years ago.

$$GTP = \sum_{X=0}^{T} \left[GA \cdot R^{(T-X)} \cdot SRV\left(100 \cdot X/L\right) \right]$$

$$= GA \sum_{X=0}^{T} \left[R^{(T-X)} \cdot SRV\left(100 \cdot X/L\right) \right]$$

which yields equation (1) above.

It is noted that in the first approximation the given GTP is a reproduction cost but after indexing is applied, historical values will be used and a GTP reflecting them will be used in financial statements.

3.2.3 Indexing

a) Pricing

There are many economic and accounting reasons for indexing the value of cost functions. Three questions must be answered: which functions must be indexed, why and finally how. We can immediately say that in the present sub-section, only the asset cost functions will be considered. Of course, both investment and annual operating expenditures have been increasing for the different carriers. But contrary to the first class of expenditures which are most often evaluated at their original costs without taking inflation and technological improvements into account, the second class already incorporates these elements. We can also envisage indexing the cost of capital (see Sub-Section 3.3.3) however we think the rate of inflation is already taken into account by the investors in their required rate of return.

In the IRA model, the main reason for indexing the asset costs is to reflect the historical costs of these assets instead of the reproduction costs. In the face of the experienced rate of inflation the question has been posed: should the rate of return be applied against the original cost rate base or against a rate base which reflects the effects experienced in inflation? The main argument against the rate base indexation is, as we said before, the fact that the inflation is already incorporated in the required rate of return or cost of capital. However, there are at least two principles upon which the rate base can be developed.

1) The original cost principle: the rate base is taken at its book value

2) The fair value principle, or current principle: the rate base is evaluated taking into full account the effects of inflation.

Finally there exists another reason for indexing: in the application of some of the costing methods (see Section 3.5) we have to know the prospective marginal costs. In other words, we have to know the replacement costs; it has been postulated that by indexing the historical costs that some idea may be obtained about these costs.

b) Changes in technology

As one might suspect, inflation plays a significant role in the increase of investment and operating expenses. Nevertheless, inflation cannot fully explain increases in total company costs and does little to explain any of the fairly substantial structural changes in investment and expenses components over the decade. Moreover, it seems that contrary to what might be expected, technological improvements have not had, from some viewpoints and disregarding quality changes, the effect of cancelling costs increases, or even netting out the effects of inflation. The Volume II "Characteristics of Telecommunications Costs" prepared for the Canadian Transport Commission in the Cost Enquiry, presents some numerical evidence supporting this claim. At the present state, we cannot do better than to refer the reader to this volume. Also, there is an appendix to the Report already mentioned which supplies some examples of the role of technology in communications. Here again, we can only refer the reader to this appendix for some information on the impact of the technological improvements experienced in the last decade and those which may come about in the more or less near future. Of course, one can ask the following question: knowing the costs of these technological improvements, should those be incorporated in the total revenue requirements without any further consideration?

This question cannot be answered without having some precise ideas on policy in the industry of telecommunications.

There is a fundamental problem of deciding who ought to pay for technological improvements, research and development, and for the putting in of "excess capacity" to take advantage of decreasing unit costs due to the presence of important indivisibilities (e.g. the communications satellite or the co-axial cable). This problem is particularly acute in a period of accelerating technical progress. If these costs are incorporated into total revenue requirements, then they are borne by the current users. There are at least two alternatives; government subsidies and/or the floating of common stocks. Only a systematic multiperiod approach could throw useful light on this important policy question which is basically one of cross subsidization over time.

3.3 Capital Cost

3.3.1 Depreciation Algorithms

There are two methods of depreciation to be simulated upon. The first is Average Service Life, ASL, which depreciates on a straight line basis the whole of the vintage group. The second is Equal Life Groups, ELG, which depreciates the smaller equal life groups within the vintage groups over the individual lives of each equal life group. Both of these methods will be tested for each of 5 plant categories. (see 3.2.2 for terminology and symbology).

a) Average Service Life Method (ASL)

The depreciation rate is given as 1/L which implies a straight line method using the average service life L. For example, if L is 5 then the rate is 1/5 or 20% per annum.

The depreciation reserve is calculated as follows:

Firstly, the average age of plant $\bar{X}$ is calculated.

$$\bar{X} = \frac{\sum_{X=0}^{T} \left[SRV(Y) \cdot R^{(T-X)} \cdot X \right]}{\sum_{X=0}^{T} \left[SRV(Y) \cdot R^{(T-X)} \right]} \qquad \ldots\ldots \qquad (2)$$

where $Y = 100 \cdot X / L$

This is a mathematical expectation of X where a density function is expressed by $GA \cdot SRV(X) \cdot R^{(T-X)}$ for all values of X; and GA, a constant, independent of the summation parameter, cancels out for numerator and denominator.

Secondly, knowing the average age, $\bar{X}$, the present life expectancy, EX, at average age $\bar{X}$ is calculated as

$$EX = \sum_{X=\bar{X}}^{T} \left[\left(SRV(Y) + SRV(Y-1)\right)/2 \right] / SRV(\bar{Y}) \qquad \ldots (3)$$

where $\bar{Y} = 100 \cdot \bar{X}/L$

In this equation, we use the information that the plant has average age $\bar{X}$ and sum mid-year survival percentages from this point $\bar{X}$ which may not be integer, to maximum age T.

The components of the summation are divided by the term $SRV(\bar{Y})$, the percentage survivability for the average age of the plant.

Finally, the depreciation reserve, DRASL, is:

$$DRASL = \left(1 - \frac{EX}{L}\right) \cdot GTP \cdot (1 - SAL) \qquad \ldots (4)$$

where ASL is a salvage rate (e.g..10).

b) Equal Life Group Method (ELG)

The annual depreciation rate, ADRELG, is calculated by:

$$ADRELG = \frac{\sum_{X=1}^{T} \left[\left(TAR(Y) - TAR(Y-1)\right) \cdot R^{(T-X)} \cdot GA \right]}{\sum_{X=1}^{T} \left[\left(\left(SRV(Y-1) + SRV(Y)\right)/2\right) \cdot R^{(T-X)} \cdot GA \right]} \qquad (5)$$

where GA is a constant factor which cancels out.

The current depreciation reserve, DRELG, is:

$$DRELG = \sum_{X=0}^{T} \left[\left(TAR(Y) - \left(1 - SRV(Y)\right)\right) \cdot (1 - SAL) \cdot R^{(T-X)} \cdot GA \right] \quad (6$$

3.3.2 Deferred taxes

The algorithms for the calculation of deferred taxes is presented in 5.2.1 step 3 below. The result DETX, deferred taxes is used in equation c, the calculation of cost of capital including taxes payable. (see 3.6.2).

3.3.3 Computation of the cost of capital

a) Introduction

The purpose of the present sub-section is to describe how we will compute the cost of capital which is, by definition, the required rate of return for the investors. We have to make a distinction between privately and publicly owned enterprises. There are some problems with the latter ones. In the following, we will only present the formulae to calculate the cost of capital for privately owned enterprises. We expect to consider the other class of enterprises in the near future. There is also a problem with the carrier seen as a whole (i.e. including the regional activities as well as its subsidiaries) or seen as oriented only towards the inter-regional services, as explained in Sub-Section 3.1.

For a private enterprise, the financing is composed of the following four elements:

- common equity (which includes retained earnings),
- debt,
- straight preferred stock, and
- convertible preferred stock.

Several studies are available to compute these four elements: however there is no unanimity on how to calculate the cost associated with the first, third and fourth. In the following we proceed as in Halpern's and Gordon's studies. For more details, the reader should refer to these studies.

b) Cost of capital associated with the common equity

As we said before, the cost of equity being an opportunity cost, must be based on data which reflects shareholders expectations. Therefore, in measuring the cost of equity we will use market data, (as opposed to book value data) i.e. stock prices on dividends per share.

The measurement of the cost of equity capital will be based on the fact that stockholders purchase future dividends when they buy equity securities. Therefore, the current price per share to the current shareholders is the discounted stream of dividends per share expected to accrue to them. The discount rate used is the required yield on equity.

We will assume that the dividend per share is expected to grow at a rate equal to g. This is a perpetual rate of growth and is derived from the fact that the company is expected to have some long run rate of growth in assets.

With these assumptions, we can simplify the stock valuation model to

$$P_t = \frac{d_{t+1}}{k-g}$$

or

$$k = \frac{d_{t+1}}{P_t} + g$$

where

P_t: current stock price at the end of the year;

d_{t+1}: dividend per share expected to prevail in the next period;

k: required rate on equity;

g: expected growth in dividends per share.

The estimate of g, the expected growth in dividends per share, can be calculated in the following manner

$g = br + vs$

where

b: retention rate;

r: expected rate of return on equity;

v: expected profitability of investment;

s: funds raised from the sale of stock as a fraction of existing common equity.

We now consider the measurement of each variable.

b: this is measured as the ratio of retained earnings per share to total earnings per share. We can take an average of past retention rates.

r: this is measured as an average of past rates of return, i.e. as an average of past return on book equity. Each rate of return on book equity is measured by the ratio of earnings per share to book value of equity per share at the end of the previous year, i.e.

$$\text{book return} \equiv \frac{(\text{earning per share})_t}{(\text{book value per share})_{t-1}}$$

s: this variable reflects the long-run expected stock financing that the company will undertake. One problem in estimating this variable is that stock financing is used sporadically and in very large values. Fortunately in this instance a value of s need not be calculated for the measurement of the cost of equity.

v: this variable is the profitability of the stock investment. It is measured as the divergence between stock price and book value of equity per share divided by the latter, i.e.

$$v = \frac{P_t - e_t}{e_t}$$

where P_t is the market price per share and e_t is the book value of equity per share.

If the company does not issue new equity during the coming year, then $s = 0$ and consequently $g = br$. In the same manner, one would expect that no new equity will be issued if $v < 0$ and consequently g is also equal to br. Therefore, we have the following result

$$k = \frac{d_{t+1}}{P_t} + br$$

However, when <u>new</u> equity is issued there is an underpricing effect associated with the issue costs and market pressure. Let d be the discount. Then the cost of equity capital for <u>new</u> equity issues can be measured by

$$k' = \frac{d_{t+1}}{P_t(1-d)} + g.$$

c) Embedded Cost of Debt

The total debt outstanding at any point in time is a collection of bonds issued at different interest in the past. Then the cost of debt can be thought of as a weighted average of the coupon rates on all outstanding debt as of a particular point in time. This is, in fact, one technique of estimating the embedded cost of debt. In principle, the embedded cost of debt calculated as in the preceeding sentence should be equal to the total interest payments during the year considered (in our case 1971) divided by the debt outstanding at the end of the same year. However, they will be equal if, and only if, all changes in the debt during the year occured at the beginning of the year. In our calculations, we assume that the repayment and the new debt are made in the middle of the year, so we have

$$\text{Cost of debt} = (\text{debt}_0 - 1/2 \text{ repayment}) \cdot (\text{embedded interest rate}) + (\text{new debt}) \cdot (1/2 \text{ new interest rate})$$

d) Embedded Cost of Straight Preferred Shares

We can immediately note that this problem does not apply to Bell Canada for the years already considered since Bell began to issue this kind of stock only in 1973.

The embedded cost of straight preferred shares can be calculated as a weighted average of the dividend yield (based on book value) for the outstanding issues, where

$$\text{Dividend yield} = \frac{\text{dividend}}{\text{book value}}$$

e) Cost of Convertible Preferred Shares

A convertible preferred security is a hybrid financial instrument which has characteristics of both straight preferred and equity. In determining the cost of these securities, there are two questions that must be answered.

i) Should convertible preferred shares be included as equity?

ii) What is the appropriate cost to use?

Answering these two questions, it seems that as there is a leverage effect and a potential dilution effect, then the stockholder is protecting himself by increasing the cost of equity. Consequently, all that the convertible preferred shareholders need to receive is the embedded cost of their shares, i.e. total dividends divided by total book value of convertible preferred shares. However, if there is a high probability that the convertible preferred will be converted in the coming year, the securities should be treated as common equity. We will make the assumption that this will not be the case, so we can compute the cost associated with the convertible preferred shares as done in the preceeding case.

f) Weighted Average Cost of Capital

In order to obtain the overall cost of capital, the specific cost of each form of capital, i.e. common equity, debt, straight preferred securities, convertible preferred securities, must be weighted by the expected proportion of the form of capital to the total capital employed. For so doing, the capital structure ratios are based on the average book value of securities outstanding.

If we let

D: outstanding debt at book value and d its embedded cost;

PS: outstanding preferred stock at book value and p its embedded cost;

CPS: outstanding convertible preferred stock at book value and c its embedded cost;

B: common equity at book value and k the cost associated with it;

$$A = D + PS + CPS + B;$$

then the weighted average cost of capital is given by

$$K = d\frac{D}{A} + p\frac{PS}{A} + c\frac{CPS}{A} + k\frac{B}{A} .$$

3.4 Operating Costs

The purpose of the present section is merely to identify the various types and relative magnitudes of operating costs for facilities, for other than facility usage activities, and to analyse the nature of some factors which are responsible for the causality of these costs. We have to keep in mind that since we work at an Inter-regional level, some separation scheme has to be applied to the data which are only available in aggregated form for both regional and inter-regional activities. This part of our studies is still in a rather exploratory stage.

We present here a break-down of the components of costs, which is in accordance with the format of the carrier classification of accounts, and suggest that this breakout should be used in the further analysis of operating costs.

There are five main types of operating costs: maintenance, traffic, commercial, marketing and general office expenses.

Here in we also supply the relative importance of these five types for Bell Canada and B.C. Tel., for 1971. These data are taken from the reports for the Cost Inquiry and are not restricted to the inter-regional level.

i) Maintenance expenses

This category of expenses can be broken into two components:

a) Costs of moving telephone equipment from place to place. Of course, the factor responsible for this component of maintenance costs is related to population shifts which, in turn, are dependent upon the income status and the age distribution of the population. At the Inter-regional level this component is not applicable for the present.

b) Costs of repairs: this component is a function of traffic usage (for step-by-step switching only) of peak requirements which establish the total amount of facilities to be maintained, and finally of environment (this variable reflecting essentially the choice of technology: step-by-step, cross-bar or electronic switching).

However, it will be very difficult to break down these two components between regional and inter-regional. As a practical solution, since maintenance is directly related to plant, maintenance is expressed as a ratio to book investments in the inter-regional activities. The major plant categories are the following:

- Station Apparatus
- Outside Plant
- Central Office Equipment
- Building and Land

ii) Traffic expenses

This category of operating expenses is most directly related to the volume of operator-assisted toll telephone traffic. The cost impact of this traffic is largely dependent upon the technology used for switching. In step-by-step central offices, an operator must intercept each long distance call for billing purposes while in cross-bar or electronic control offices, the intervention of an operator can be avoided.

This type of costs could be specified as a function of the proportion of operator-assisted toll telephone calls over total toll calls.

$$T.E. = f\left(\frac{S}{T}\right)$$

where T.E. represents total annual traffic expenses and $\frac{S}{T}$ is the proportion of operated-assisted toll telephone calls over total toll calls.

From Vol. II "Characteristics of Telecommunications Costs" (July 15, 1974), the authors provide the following figures for the proportion of Toll Messages Customer-Dialed.

Proportion of toll messages which are customer dialed		
	1962	1972
Bell Canada	25%	71%
British Columbia Telephone	25%	58%

iii) Commercial expenses

These expenses can be related to the number of telephones, total billed revenues and the number of processed service orders.

Let

ce be total annual commercial expenses

T be the number of telephones

R be total billed revenues

S the number of processed service orders.

one could probably write

$$ce = aT + bR + cS.$$

This relation could easily be estimated by means of a regression of ce on T, R and S using the appropriate data.

We suggest that the following two categories, which are largely discretionary expenses, be evaluated as a percentage of Operating Expenses.

iv) Marketing expenses

They include essentially advertising expenses

v) General office expenses

This category of operating expenses has been disaggregated to 9 components in Bell Canada accounts:

- Executive expenses: consist of the pay and other expenses of officials and their secretaries engaged in the general administration.

- Accounting expenses.

- Treasury expenses: include the pay and other expenses of the treasurer and staff, the cost of cheques issued, postage and envelopes used in forwarding these cheques and the expenses of dividend announcements.

- Law expenses: consist of the pay and expenses of Bell's staff and fees and retainers paid.

- Public relation expenses: include internal employee program, news media considerations, printing and distributing certain reports and public affairs activities.

- Personnel expenses: include employee training, labour relations and other employment activities.

- Engineering expenses: are for those charges representing engineering of a general nature which are not associated with specific construction or maintenance projects.

- Operating rents: include payments for building space (exclusive of space occupied by plant), agency office space, circuits, temporary right of way, attachment privileges, inside wiring and telephone equipment.

- Other operating expenses: include payments under service agreements to parents and affiliates, insurance premiums, accidents and damages...

vi) Relative magnitude of Components of Operating expenses

In summary the relative magnitude of various components are shown below

Category of expenses	Bell Canada (1971)	British Columbia Telephone (1971)
Maintenance expenses	39.5%	44.3%
Traffic expenses	13.8%	18.0%
Commercial expenses	10.0%	9.9%
Marketing expenses	6.5%	3.5%
General office expenses	30.2%	24.1%

3.5 Approaches to Costing

3.5.1 Unit Costing Methods

In the present and in the next sub-sections, we will discus different alternative costing approaches for each service and also the problem of the allocation of common and joint costs through the use of the economic programming technique. In the present sub-section we define some costing methods while in the next we formulate some problems which must be resolved to apply the methodology proposed. It is worth noting that both problems mentioned are relevant to the Operating and to the Policy Simulation Blocks. The costing methods proposed can be seen firstly as the means to test the reasonableness of the tariffs of each service considering given the elasticity of the demand, tariff levels affect the usage of the network and secondly as the means for some tests which can be performed for studying, among others, the problem of cross-subsidization among monopoly and competitive services. Finally, it was stated in Section 3.1, some aspects of the present section are to be seen at a more exploratory level than the other sections. More intensive work is planned for the final report of december 31, 1974.

Before presenting the unit costing methods, consider the following: suppose that we know the demand, for a given service, between two points on the switching network. The elements on the facilities network which support the "corresponding" element of the switching network are not necessarily unique; there could exist two routes on the physical network by which the message can be transmitted. In other words, the route unit cost can be non-unique: in this case, which one should we use? A second problem, which is as difficult as the previous one, is the fact that through each element on the facilities network the demand between more than one origin-destination pair could pass. In such cases, if the tariff of each origin-destination pair is a function, at least partially, of the cost of transmitting a message between these given nodes, we have to know the relative demand between each pair of nodes using the element considered. Finally (but the list is surely not exhaustive) due to the hierarchical structure more than one route may exist (nodes and links) on the physical network between a given origin-destination pair, which the given demand can use at different times. This again, is a problem of the non-uniqueness of the route unit cost. Of course, this last problem is behind the costing of the peak-load demand.

Keeping in mind the above observations, and consequently the problem of separation we now review five alternative costing approaches. These approaches are largely taken from the inquiry into Telecommunications Carrier's Costing and Accounting procedures. However as can be seen certain results from the previous sections are necessary to permit the computation or estimation of the costs proposed.

i) Fully Distributed Costs:

All costs are allocated to the respective services on the basis of its current usage. In other words, this is an allocation of the costs in the given year of the revenue requirement to each service in such a manner that the sum of the costs of all services reconcile with the total revenue requirement. This method is difficult to apply since facility assignments on any two days might reveal significantly different patterns of usage among services.

ii) Average Variable Costs: (embedded and current)

By definition, average variable cost is the product of the average book cost for each unit of plant and each expense element which is variable in the long run, and the number of units of plant and expenses elements used, to provide the service in question. Then, the average variable embedded costs reflect the net book investment value and associated expenses which are reasonably related to and variable with the provision of each service. They fully reflect the age and technology distribution of the facilities which are currently used by each service. The average variable current costs present the costs of the facilities variable with each service at current price levels and using technology which is currently being installed. These measures reflect different considerations: the first one indicates the impact of each service on the firm's revenue requirement, the second one shows the ongoing effect of each service on the construction program. Taking into account the information generated by the Operating and Costing Blocks, it should not be too difficult to evaluate these variable costs for the Inter-regional activities.

iii) Prospective Incremental Costs:

These are the present value of the stream of future costs which will be incurred because the service considered is established (new service) or retained (existing service). Of course, in order to assess if the given service is profitable the stream of future revenues will also have to be considered. It is here perhaps that the motivations for indexing the asset functions and trying to evaluate the impact of new technology on the costs of each service arise. But, at least two questions can be asked: which rate of discount must we use and does the problem of inflation pose a growing likehood that prospective costs will not always be lower than the retrospective incremental costs.

iv) Retrospective Incremental Costs:

These are the book costs in a given year of the revenue requirement which would have been avoided had the service in question never been offered.

v) Avoidable Costs:

Finally, we will consider the avoidable costs as the costs in a given year, or the revenue requirement, which could be avoided if, hypothetically, the service in question were abandoned. Of course this last approach is closely related to the preceeding one.

3.5.2 Séparation

Le problème que nous cherchons à résoudre est le suivant: comment allouer les différents coûts d'investissement et d'opération entre les différents services offerts par un transporteur, ces services pouvant utiliser les mêmes installations. Comme nous l'avons vu précédemment, ce problème est directement relié à celui de la détermination des coûts unitaires par services: nous pouvons presque dire que les différentes approches suggérées dans la sous-section précédente sont des solutions possibles au problème de séparation des coûts liés et communs. Toutefois, l'optique que nous adoptons dans la présente sous-section est quelque peu plus spécifique: il s'agit d'étudier comment ce problème peut être résolu à l'aide de la programmation économique. Le présent texte ne reflète que nos premières réflexions sur le sujet: il est bien évident que dans la phase ultérieure un effort plus intensif devra être fait. Compte tenu de ce qui précède, le lecteur trouvera surtout ici une série de questions (ou de problèmes) qui devront être résolus afin de mener à bien notre étude.

L'importance d'une telle séparation est évidente pour les fins d'une politique de communications. En premier lieu, et en supposant que les tarifs soient en partie basés sur les coûts, cette séparation permet de déterminer une structure de tarifs pour les différents services et peut-être aussi pour chaque service pour une certaine période d'observation. En deuxième lieu, et compte tenu de l'élasticité de la demande par rapport aux tarifs, cette structure peut permettre une meilleure utilisation du réseau et, par voie de conséquence, peut en retarder l'expansion. En troisième lieu, et puisque notre approche permet l'établissement des coûts marginaux, notre analyse permet d'étudier l'inter-financement possible entre différents services: cette étude est particulièrement importante lorsqu'une entreprise est règlementée pour certain (s) service (s) et en concurrence pour d'autres services. Nous noterons d'ailleurs que les différentes approches considérées précédemment permettent aussi une telle étude: pour de plus amples renseignements on se référera au Volume III: "The Costing of Telecommunications Services" préparé pour "The Telecommunication Commitee of the Canadian Transport Commission". En fait, les cinq moyens de calculer les coûts unitaires sont différents tests qui permettent d'étudier l'inter-financement possible entre les services.

Le critère général que nous allons adopter est le suivant: l'allocation des coûts est faite selon le critère que le coût marginal de chaque service soit égal au revenu marginal de ce service. Cette approche est d'ailleurs celle qui est utilisée par W.R. Scott dans l'article "Certain Accounting Aspects of Telecommunications Regulation" Cette méthode d'allocation part donc du principe que si par un certain moyen il était possible d'offrir chaque service séparément, le transporteur en offrirait jusqu'au point où, pour chaque service, il égaliserait le revenu marginal à son coût marginal. Toutefois, il se pose plusieurs problèmes que nous allons considérer successivement.

a) La définition de coût marginal (en présence de produits multiples) n'est pas évidente. En effet, elle suppose implicitement l'indépendance entre les différents services. Or, dans le domaine des communications, bien que cette indépendance entre certains services soit vérifiée, il en existe d'autres pour lesquels ceci n'est plus vraie. Dans ces conditions, l'on ne peut définir le coût marginal que pour certains aggrégats de services. Il se pose aussi le problème de l'unité d'observation (nombre d'appels, cas, etc.).

b) Dans le cas des télécommunications, le réseau est dimensionné selon la demande de pointe. Par définition, il existe donc une surcapacité inutilisée à l'extérieur de ces heures, et par conséquent comment définir le coût marginal pour les appels hors-pointe? Formellement, et si l'on admet que le coût marginal d'opération est négligeable, ce coût marginal est nul.

c) Dans l'article mentionné plus haut, le revenu marginal de chaque service est obtenu à partir de la solution d'un programme linéaire d'optimisation. Or, quel critère (linéaire ou non-linéaire) doit-on retenir pour les entreprises réglementées par leur taux de rendement sur le capital investi? On ne peut bien sûr retenir le critère de maximum de profits, bien que ces entreprises offrent des services non-réglementées par l'Etat. Il nous semble, qu'il faudrait considérer un critère qui pondère au moins les objectifs ou intérêts des consommateurs, des transporteurs et des autorités gouvernementales. Bien sûr, les pondérations devraient être données subjectivement.

d) Doit-on chercher à allouer tous les coûts, et en particulier les coûts discrétionnaires. Evidemment, la réponse à cette question n'est pas indépendante des questions auxquelles on cherche une réponse.

e) Le présent point est d'ordre plus technique. Pour utiliser l'approche par la programmation économique, il nous faut connaître les fonctions de coûts par élément: toutefois, rien ne nous assure a priori que ces fonctions auront les propriétés mathématiques qui sont habituellement postulées dans l'emploi de cette technique.

f) L'application de la méthode proposée suppose la connaissance des fonctions de demande pour chaque service. Il est manifeste que dans l'état actuel du projet, nous disposons de très peu d'informations concernant ces différentes demandes. Peut-être pouvons-nous contourner ce problème en utilisant l'information du trafic moyen par élément, obtenu de l'Operating Block.

g) Peut-être que dans la formulation de notre modèle nous devons tenir compte de certaines contraintes additionnelles. Par exemple, il est souvent admis que l'industrie des communications se caractérise, entre autres, par des coûts moyens décroissants, du moins pour ce qui est de la transmission. Compte tenu de cette caractéristique, il nous faut introduire une contrainte budgétaire pour éviter que n'apparaissent des profits négatifs, ce qui serait évidemment contraire à l'observation des faits.

3.6 Inter-relation of the Costing and Accounting Blocks

3.6.1 Inter-relation of cost of capital in the costing block and rates of return in the accounting block

The cost of capital, measured in per cent, is a required rate of return, reflecting an opportunity cost of total capitalization of a company. The cost of capital provides a measure for the regulatory authorities to determine an appropriate allowed rate of return on total capital. The cost of capital so conceived and measured may not always equal the realized rate of return on total capitalization in reality. The concept of cost of capital in the Costing Block represents such an opportunity cost while the rate of return on capital in the Accounting Block is an ex post facto concept. These two measures are interrelated in one sense that the cost of capital would in the longer run be influenced by the realized rate of return. As a more specific relation between accounting facts and the cost of capital the latter is arithmetically influenced by the actual capital sourcing and the resulting embedded costs of Debit and preferred equity components.

3.6.2 Role of deferred taxes in the calculation of cost of capital

Deferred taxes may be reflected in the calculation of the cost of capital. Cost of capital comprises cost of capital per se and taxes which are imposed on the total equity capital. Cost of capital per se, namely, cost of capital after taxes is defined as being oblivious to the tax treatments. It is given and taxes calculated will be added upon it as added capital costs.

The formula for the calculation of the cost of capital is:

i) Cost of capital (after taxes) = $R(I) \cdot$ Net Rate Base

where $R(I) = DCR \cdot i + (1 - DCR) \cdot (p \cdot PS + c \cdot CPS + e \cdot CE)$

and Net Rate Base will be the denominator of either RORBI or RORBE, namely, either (Tel Plant at Cost - Acc Dep + WK - Adj B) or (Tel Plant at Cost - Acc Dep + WK - Adj B - PUC)

There are two kinds of cost of capital including taxes; one is the cost of capital including book taxes, the other being cost of capital including taxes payable only.

Their formulae are:

ii) Cost of Capital including Book Taxes = R(II) · Net Rate Base

where $R(II) = DCR \cdot i + [(1-DCR)/(1-bt)] \cdot (p \cdot PS + c \cdot CPS + e \cdot CE)$;

bt = effective tax rate for book purposes.

iii) Cost of Capital including Taxes Payable R(III) Net Rate Base

where $R(III) = DCR \cdot i + [(1-DCR)/(1-pt)] \cdot (p \cdot PS + c \cdot CPS + e \cdot CE)$;

pt = effective tax rate for taxes payable

$$bt - \frac{Dftx \cdot (1-\alpha)}{NOR - DSC + \text{Other Inc. Txble}}$$

Notation :

DCR : Debt Capitalization Ratio

i : embedded cost of outstanding long-term debt at book value

PS : ratio of outstanding preferred stocks at book value to total equity capital

p : embedded cost of outstanding preferred stock at book value

CPS : ratio of outstanding convertible preferred stock at book value to total equity capital

c : embedded cost of convertible preferred stock at book value

CE : ratio of common equity at book value to total equity capital

e : cost of common equity

R's : weighted average cost of capital

α : flowthrough coefficient

NOR : Net Operating Revenue (net of Other Operating Taxes)

DSC : Debt Service Charges

Dftx: Deferred income taxes

Algorithms for the calculation of deferred income taxes and various embedded cost rates are provided elsewhere in this Interim Report.

4. SHARING BLOCK

4.1 General Considerations

During the present phase, the Sharing Block has not been a priority, the team being occupied with the improvements in the other Blocks. The main change will come from the possibilities of having an algorithm which computes the usage of the switching network and another one - also in the Operating Block - which identifies the physical routes corresponding to the preceding network. As described in the IRA Final Report, March 31, 1974, the routes were given as data and the share of each route for a given traffic stream was also imposed exogenously. This feature should remain as an option, but the routing of a stream is now rather a diffusion in the relevant network see (Section 4.2) due to the overflowing capabilities, at least for the public message traffic. It is therefore the cost and usage oriented sharing schemes which will profit from the new developments.

4.2 Statistical Reports on Unit Costing

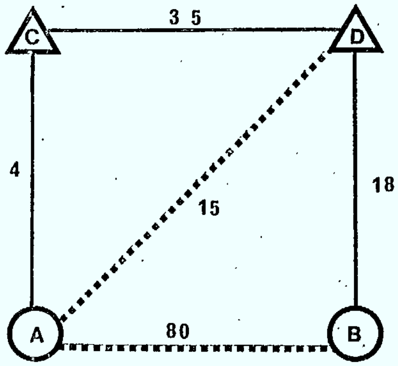
In the following section, we will elaborate upon a comparison, on one hand, of the notion of stream used in IRA-I and the comparable notion to be used in IRA-II, that of a subgraph, and on the other hand of the methods used for assigning costs to streams on a linearized basis. These items form the backbone of the Commonwealth revenue sharing scheme and may be adapted to produce comprehensive statistical reports on comparative stream costs for different areas of the network. The form of these statistical reports will not be discussed, but the basic notions involved are explained below.

In IRA-I, the notion of traffic stream was taken to mean a flow of traffic from an origin to a destination. In IRA-II, this notion has not changed. What has changed however is the notion of the nature of a stream relative to a switching network. In IRA-I, we considered (as an admitted first approximation) that a stream was composed of several alternate routes - which were not necessarily disjoint - upon which the traffic could travel. We assumed that each route carried a certain percentage of the total load on a traffic stream and that the different elements of the same route carried the same traffic throughout. This last assumption is hardly justifiable because conditions vary from link to link and certainly some loss or overflow may occur along a route, meaning that share of traffic on a particular route depends on the conditions prevailing at each switching node, which in turn depend on the competing traffic from other streams.

In IRA-II, this problem is treated in a more satisfactory fashion in that the algorithm (described in Section 2.2.2) enables us to distinguish the amount of traffic on any particular link due to a demand in a particular traffic stream.

A typical example of the output of this algorithm for one traffic stream might be as follows:

DISTRIBUTION OF TRAFFIC ON SWITCHING NETWORK



- Final Groups ————
- High usage groups ------
- Traffic strean A ⟶ B
- Offered traffic 100 ccs

High usage	link	A-B	Offered	100	Carried	80	Overflowed	20
"	"	A - D	"	20	"	15	"	5
Final		A - C	"	5	"	4	Loss	1
Final		C - D	"	4	"	3.5	"	.5
Final		D - B	"	18.5	"	18	"	.5

Total traffic arriving at destination: 98CCS

It is possible to identify routes in this subgraph if so desired, but it serves no particular purpose as will be seen in the following section.

In IRA-I, the usage of the network was calculated based on stream unit costs and on the estimated total traffic travelling on various elements.

In tableau form, these calculations can be represented as follows. Let each column represent an element (in IRA-I; switching node, transmission node or transmission link) and each row represent a route of a stream.

The typical entry x_{ij} of the tableau was obtained in the following way.

When i represents a route of private line circuits, x_{ij} was given exogenously in number of circuits. Where i represents a television stream x_{ij} is the number of radio channels multiplied by the circuits per channel for element j. When i represents a telephone traffic route, the values x_{ij} are expressed in c.c.s. or erlangs. In order to obtain unit costs for each element, the number of circuits of private line and television routes is added to the circuit equivalent (assuming a given grade of service) of the total telephone

TABLEAU OF STREAM COSTING METHODS IN IRA I

		Elements 1	2		j		n	Row Total = Stream or Route Cost.
Unit cost 1		K_1	K_2		K_j		K_n	
Unit cost 2		A_1	A_2		A_j		A_n	
Telephone								
Stream 1	R1	$x_{1,1}$	$x_{1,2}$	…	$x_{1,j}$	…		$\sum_j A_j x_{1,j}$
	R2	$x_{2,1}$			⋮			⋮
	⋮	⋮			⋮			⋮
Stream 2	R1	⋮			⋮			⋮
	R2	⋮			⋮			⋮
	⋮	⋮			$x_{i,j}$	…		
Private Line and television								
Stream 1		$y_{1,1}$	$y_{1,2}$	…	$y_{1,j}$	…		⋮
Stream 2		$y_{2,1}$			⋮			$\sum_j k_j x_{i,j}$
⋮		⋮			$y_{i,j}$	…		⋮
Column Total = total incurred cost per element		…	…	…	$\sum_i k_j x_{i,j} + \sum_i A_j x_{i,j}$	…		$\sum_{ij} k_j x_{i,j} + \sum_{ij} A_j x_{i,j}$

traffic. The unit cost K_j is then the ratio of total incurred cost (estimated from cost functions proportional to asset values) to total used circuits. The unit cost A_j is estimated as a unit cost per c.c.s. (or erlangs) to be applied to telephone traffic. The estimate of this unit cost is as follows:

$$A_j = \frac{\text{\$ incurred cost for element } j}{\text{total circuits element } j} \cdot \frac{\text{number of telephone circuits}}{\text{number of telephone}}$$

Thus, A_j is a cost per c.c.s. linearized over that fraction of total costs which are proportionnally due to telephone traffic.

The calculation of stream costs in IRA-I was done as follows:

- For television or private line i; stream cost $= \sum_j k_j y_{ij}$
- For telephone route i ; route cost $= \sum_j A_j X_{ij}$
- Stream cost $= \sum$ route costs (for all routes of stream i)

The remark that may be made, for telephone traffic, with this tableau is that IRA-I assumed.

$$X_{ij} = \begin{cases} K & \text{if element } j \text{ used in route } i \\ 0 & \text{if not} \end{cases}$$

for all elements of a route.

That is, that traffic was constant on all elements of the same route.

The principal advantage gained in IRA II is that by automatic computerized algorithms we are able to estimate the X_{ij}'s in c.c.s. for any network. In IRA-I, these were developped manually. Furthermore, the notion of subgraph can be easily represented in the same schema. For telephone streams, instead of a list of routes, each having a list of elements, we may simply specify a list of elements, and the amount of traffic on each, the order being irrelevant for calculations of stream cost.

We must, to be sure, consider the column sums (equal to total used circuits per element in IRA-I, and equivalent in IRA-II) carefully for it is here that we encounter the question of apparently unused circuits in an element. But if this question is dealt with satisfactorily, it can be seen that the grand total (lower right of tableau) represents the total system cost, and reading the right column, we see the apportionment of this cost to various streams on a usage basis, while reading the lower (sum) row, we see the total cost of any particular element of the network.

This system is used in IRA-I for the Commonwealth revenue sharing scheme, and if we replace unit incurred costs by unit asset costs (i.e. investment/circuit or/CCS), we can apply a similar apportionment of asset value to streams for statistical reports.

4.3 Sharing Schemes

4.3.1 Full Division of Revenues Schemes

Although the implementation of the Full Division of Revenue (TCTS) Sharing Scheme (FDRSS) is due for the December report, it is appropriate to envisage the implications of such a scheme for the Operating Block. The algorithm which should compute the expected usage of the switching network has not yet been utilized and its efficiency for the 24 node network is unknown. The FDRSS must provide expenses for terminating the service up to the customer. This means the network has to be expanded to the end offices, that is level 5 of switching network. This size increase is a tremendous step and the only hope is some kind of decomposition permitting the computation of the usage for subsets of nodes. It is known that high usage trunks can exist between any pair of nodes of the hierarchy. However, the H.U.'s from a level 5 node to a 3,2 or 1 level node should be rare so that the part of switching network below each number 4 (Toll center) node could be treated separately.

A similar decomposition should be possible for the treatment of the circuit allocation by algorithms coping with the physical support of the switching network. It is quite possible that the facilities linking the level 5 nodes homing on a single level 4 node are separate from the other facilities linking higher level nodes together. The star pattern design as currently used provides a basis for the above assertion. Supposing that size is not a problem, the circuit allocation algorithms use as inputs the number of circuits required for each arc of the switching network and each special demand. Data exist for the present situation and this may be sufficient to answer certain questions on over-capacity.

When the splitting of the usage of the physical facilities are important, data on traffic are necessary, and the more detailed the network, the greater the data collecting effort. Therefore the tasks aiming at updating and generating data on traffic are of paramount importance for the implementation of the Full Division Scheme.

If the extended network is too costly to envisage, the following short cut for implementing the FDRSS is proposed:

The IRA Model can compute the shares of the assigned expenses and assigned plant investments which are related to the part of facilities network treated in the Model. The "terminating" shares can be estimated using the proportion observed between the known and "unknown" shares for a benchmark year (see Trans-Canada Telephone System, "Exhibits from the DR Presentation of April 21, 1972").

4.3.2 Negotiated settlement schemes

The central issue to be resolved in this area is how to apply settlement schemes when the basic data on costs and revenues is not available. This problem arises in treating Canada-U.S. traffic or Canada Overseas traffic specifically regarding the settlement between T.C.T.S. and C.O.T.C. We have two available alternatives. We may attempt to obtain typical data on the results of negotiated revenue settlements between these groups, and use this data in financial results, bypassing the application of any systematic settlement scheme. Alternatively, we may attempt to estimate total costs and revenues of the foreign parties in order to apply certain systematic settlement schemes. Such data would permit the use of any scheme which is applied at an aggregate level (e.g. Old Commonwealth). However, to apply a usage based scheme, such as the New Commonwealth scheme to any of these revenue settlements, we would require more data on networks in foreign countries which is undoubtedly impossible or at least impractical In conclusion, we anticipate, for this stage, the adoption of the first alternative with perhaps an attempt to obtain aggregated cost and revenue data for an application of non-usage based systematic revenue sharing schemes.

4.3.3 Implementation of Mixed Schemes

The logistic and data requirements for the Full Divisions of Revenue (TCTS), Commonwealth and Old Commonwealth Schemes have been well defined since they related toschemes already in existence. However simulations can be made on various combinations and modifications of these schemes which are called Mixed Schemes.

The initial assumption for all Mixed Schemes is that the profit (collected revenue - all incurred costs) is firstly distributed, for example,1/3 to each of the terminal partners (origination and destination) for that stream. This means that this calculation is usage-based i.e. we must know who are the terminal partners and what are the costs of each partner. This scheme is settled stream by stream and not on the basis of total revenues and costs for all streams as is the Old Commonwealth Scheme.

The remaining 1/3 (residual of 1 minus terminal partners shares) of profit can be allocated in several ways and hence there are a multitude of Mixed Schemes. The first choice to be made is to whom to allocate the residual. This residual can be distributed to A) the transit partners of the stream, or B) the transit and terminal partners.

The next decision is on what basis to allocate the residual. The following bases are considered:

i) Asset based (TCTS type). Here the residual is distributed in proportion to asset for the carriers.

ii) Cost based (Old Commonwealth type) Similar to i) but based on incurred costs.

iii) Equally to all carriers involved.

These 3 choices can be applied under A) transit partners only, which implies settlement on a stream by stream basis; or B) all partners, which can be settled stream by stream, but since total costs (assets and incurred) are the linear sum of stream costs, this scheme can be settled on an aggregated basis.

Another basis of distribution is:

iv) Allocated costs (New Commonwealth system) wherein costs are allocated on a stream basis. Each traffic stream is analyzed and the terminal partners of the stream bear 50% of the stream cost each. Hence under alternative A) - transit partners only - the problem of identifying a unique set of terminal partners within transit partners arises. Within the subgraph for an origin destination stream the only unique set of partners for the several alternate routings are those of the terminal partners. Under alternative B) - All partners - the resulting allocation using the parameter 1/3 consists of an initial 1/3 to each terminal partner plus 1/2 of the remaining 1/3 giving a total of 1/2 of the allocation. This scheme therefore reduces to the Commonwealth scheme and is redundant.

The total collected by each member is then their distributed portion of profit and their individual recoupment of incurred costs. This sum is transferred to the Accounting Block.

5. ACCOUNTING BLOCK

5.1 Introduction

The Accounting Block for IRA II will change substantially from that of IRA I. A primary change will be a shift of emphasis away from the provision of fixed Regional financial statements and the Company Totals varying accordingly with the Inter-regional results. IRA II will incorporate the Company Total statements as being fixed and thereby agree in total with those reported by the carriers. The Regional statements will now be the balancing residual between the Company Total and Inter-regional result and this change will provide better data bases which will ensure coherence with Company Totals and better data upon which simulations can be made.

Previously, certain Accounting Block operations such as aging and depreciation, deferred tax calculations, and growth of gross additions were all applied in the Accounting Block as simple ratios. These were transferred to the Costing Block (Table 17) where these ratios were needed for Inter-regional calculations. In this phase, these operations will be expanded in great detail.

The basic structure of the financial statements will remain the same. However expansions will be made in the Income, Retained Earnings and Funds Statement. These expansions are described below.

5.2 Development in the Accounting Logic and Software

5.2.1 Generation of Accounting Block Data

For the static one year accounting model the same logic will be applied for financial statement for:

1. Inter-regional
2. Regional, and
3. Company Total

However, the sources and data development differ for each. Firstly, the Inter-regional (IR) statements will be based in part on endogenous data i.e. data calculated within the IRA Model such as asset and cost data from the Costing Block, and in part on exogenous data provided by the D.O.C. For this exogenous data the D.O.C. will still have to split the Company Totals into an Inter-regional portion and a Regional portion. Based on this data, the accompanying logic and supplementary ratios the Inter-regional financial statements will be generated. The steps for additional and expanded calculations, and the appropriate data requirements for the new accounting logic for IR financial statements are detailed below.

Step 1 Partial Income Statement

Revenues will be transferred from the Sharing Block, and expenses will be obtained for switching and transmission equipment from the Costing Block. Other Income will now be split into Taxable and Non-Taxable portions and a partial Income Statement (up to the entry "Income before income taxes and debt service charges") is completed.

Step 2 Asset Side of the Balance Sheet

The plant asset values, at cost (beginning of year) and accumulated depreciation for the five plant types - switching equipment, transmission equipment, station equipment, general equipment, and buildings - are all provided from the Costing Block. Land, (beginning of year), is also provided from the Costing Block. Plant under construction which was previously exogenous will now be calculated by a ratio based on Company Total plant under construction. This will ensure a more realistic value for this entry.

Investments figures (beginning of year), will be provided exogenously. Current Assets, (beginning of year), will be split into the two exogenous entries Cash and Temporary Investments, and Other Current Assets. Similarly, Deferred Charges will be split into Debt Related charges and Other Charges.

The end-of-year value for plant assets will be calculated as follows:

End of year of cost = Beginning of year at cost
+ Additions (gross)
− retirements

We will first present the generalized formula for the calculation of deferred taxes which may be used for any period of time, present, past or future and the shortened version for the calculation of deferred taxes in the period, namely $t = o$, will follow.

The generalized deferred taxes algorithm is as follows:

For each category of plant

$$CCA(t) = CCAR \cdot GA \cdot \left[\sum_{X=0}^{N} (1-CCAR)^{N-X} \cdot R^{X} \right]$$

where CCA is current year CCA

CCAR is CCA rate

GA is gross addition installed for oldest surviving plant

R = (1 + growth rate), used to compound growth of gross additions (from Cost Block).

EX is the ratio of expensing to gross additions (current); mathematically,

$$EX = \frac{\text{EXPR} * \text{OPXP (net of Depr)}}{\text{GROSS ADDITIONS(CURRENT)}}$$

Beginning of year costs, additions and retirements are supplied endogenously from the Costing Block. The end of year accumulated depreciation is

=Beginning of year figure
+Annual depreciation
-retirements
+Net salvage value

The beginning of year accumulated depreciation and annual depreciation figures will originate in the depreciation algorithm used in the Costing Block. Net salvage value is also provided endogenously from the Costing Block.

Land, (end of year), is provided endogenously from the Costing Block. Plant under construction should be calculated by the same ratio as the ratio used for beginning of year values. Investments, current assets (cash and temporary investments, and other) and deferred charges (debt related and other) are all provided exogenously for end of year figures.

Total assets, end of year, can now be calculated and the Asset Sheet is complete.

Step 3. Calculation of Capital Cost Allowance (CCA) and Deferred Taxes

There are two major categories of deferred taxes, one arising from the differences between the CCA claimed and Depreciation applicable thereto, another being the tax effect of differences arising from the amounts capitalized for book purposes but expensed for tax purposes.

CCA and deferred taxes may now be calculable using either the method used in IRA I (IRA I Final Report, Appendix G, Table "A") or a new, more rigorous method introduced below.

The former method is designed to overcome the absence of the detailed data on CCA in the calculation of deferred taxes. When sufficient data for the deferred taxes calculation is available, the latter procedure is recommended. This new method represents a major revision of the old one since CCA is calculated for each category of plant and is based on the growth of gross additions over the lifetime of the plant.

For each plant category we have

$$CCA = CCAR \cdot (1-EXP) \cdot GA \cdot \sum_{P=0}^{T} \left[(1-CCAR)^{T-P} \cdot R^{P} \right]$$

where

CCA   is current year CCA,

CCAR   is CCA rate,

GA   is gross addition installed for oldest surviving plant,

R   is (1 + growth rate) used to compound growth of gross additions (source: Costing Block),

$$EXP = EXPR \cdot (OPXP - DEPR) \,/\, (GA \cdot R^{T})$$

EXPR   is an expensing rate (%)

OPXP   is the current total annual operating expenses, and

DEPR   is current annual book depreciation.

The total CCA is calculated as

$$\sum_{i} CCA$$

CCA in excess of book depreciation is

$$\sum_{i} CCA - DEPR \qquad \ldots \text{(a)}$$

The tax effect of expensing can be expressed by

$$EXPR \cdot (OPXP - DEPR) \qquad \ldots \text{(b)}$$

Finally total deferred taxes are

$$((a) + (b)) \cdot \text{effective tax rate} \cdot (1 - \text{flow through coeff.})$$

Step 4. Beginning of Year Liabilities

Total liabilities are equated to total assets. Then all components of liabilities will be split in the same proportion as Company Totals.

Step 5. Calculation of certain required items

The following items are required in the next step of solution of simultaneous equations.

Repayment of Long Term Debt is calculated as

$$\text{Co. Total Repayments} \cdot \frac{\text{I.R. Assets}}{\text{Company Total Assets}}$$

Proportionality is applied in a similar fashion for Year End Current Liabilities and other deferred credits.

Finally, deferred taxes (year end) are given as

deferred taxes (beginning of year)
+ current deferred taxes (Step 3)
+ prior deferred taxes (exogenous)

Step 6. Simultaneous Accounting Equations

This system is used in the software to assist in the generation of financial statements. In particular, the total assets and hence to total liabilities, end of year, are known but the breakdown of liabilities is unknown. In solving for the components of capital, the solution for debt and accordingly for debt service charges allows the income statement to be completed. The following section describes the system of simultaneous equations used by Sorês.

1. Net income =

(1 - tax rate) {op. rev. - op. exp - depreciation
- ($debt_o$ - repayment/2) (embedded interest rate)
- (average interest rate on new bond) (new debt/2)}

+ miscellaneous + other income (non-taxable)
+ α deferred taxes (current)

where α coefficient of flow through

2. Equity =

$Equity_0$ + net income - (PRDIV + DIVI)

+ new equity - (share issue expense - other adjustments)

3. Debt = $Debt_0$ + new debt - repayment

4) $\frac{\text{Debt}}{\text{Debt + Equity}}$ = Debt/Capital Ratio (D.C.R.)

5) Total Assets = Total liabilities (year end)
= Equity + Debt + Current liabilities
+ Deferred tax (year end) + other deferred credits.

6) PRDIV = PRRATIO ∘ PR

7) DIVI = OPR ∘ (RNI-PRDIV)

The above equations can be represented as follows for the Interregional Sector. The unknown variables to be solved for are:

RNI =net income

EQ =equity (year end)

total including retained earnings

RNEQ =new equity on capital stock only

D =debt (year end)

RND =new debt

DIVI =common dividends (including transfers to government owners)

PRDIV =preferred dividends

PRRATIO=preferred dividend ratio or preferred equity

DPR =common dividend payout ratio

If the solution for these unknowns yields a negative capital equity change as may happen when net income is very large relative to the increase in total assets or total liabilities, then the following steps are taken.

1) RNEQ is set =0 i.e. no increase or decrease in capital equity

2) DCR equation 4 is ignored

A revised system is then solved with one less equation and one less variable.

If the next solution yields a negative change in this revised system the absolute value is added to cash and temporary investments and total assets =total liabilities are accordingly increased and the revised system is solved.

Step 7. Retained Earnings Statement

Once net income has been calculated the Retained Earnings Statement is also easily completed.

All other basic elements of this statement are

Preferred Share Dividends
Common Share Dividends
Transfer to Government Owner
Share Issue Expense
Other Adjustments (net)

New items for this statement not in IRA I are beginning of year retained earnings and end of year retained earnings (=beginning of year retained earnings + net change in retained earnings).

Step 8. Completion of Liabilities Side of Balance Sheet (end of year)

Total equity was calculated in Step 6. Retained Earnings (year end), was calculated in Step 7. The balance of equity (total equity - retained earnings) is distributed between common and preferred stock in the same ratio as the beginning of year. Total debt was calculated in Step 6 and its components are similarly distributed in the same proportion as for beginning of year figures.

Step 9. Completion of income statement

$$\text{Debt Service Charge} = (\text{Debt}_0 - 1/2 \text{ Repayment}) \cdot (\text{Average interest rate on L.T.D.})$$
$$+ (\text{new debt}) \cdot (1/2 \text{ interest rate on new debt})$$

$$\text{Income taxes} = (\text{net operating revenue} - \text{DSC} + \text{other taxable income}) \cdot (\text{tax rate})$$

With these items, the remaining entries of the Income Statement are calculated to complete this statement.

Step 10. Calculation of miscellaneous and change in working capital (W.C.) for Sources and Uses of funds.

$$\Delta \text{ W.C.} = \text{Change in W.C.} = \text{W.C.}_0 - \text{W.C.}$$
$$(\text{Current Assets} - \text{Current Liabilities})_0$$
$$-(\text{Current Assets} - \text{Current Liabilities})$$

If ΔW.C. is positive:

Reduction in W.C. = Δ W.C. (Source of Funds)
Increase in W.C. = 0

If Δ W.C. is negative:

Increase in W.C. = $|\Delta \text{W.C.}|$ (Use of Funds)
Reduction in W.C. = 0

A Miscellaneous item is calculated to incorporate all other changes in the Balance Sheet:

Miscellaneous = Deferred Charges - Other Deferred Credits
- Net Salvage Value - Other Adjustments (net)
- Share Issue Expenses

where net Inter-regional salvage value is calculated from data of the Costing Block.

Step 11. Funds Statement

All items which appear in this statement have previously been calculated. In this phase of IRA the following new breakdowns are made:

- Depreciation and other non-cash charges (net)

  a) Depreciation as per Income Statement

  b) Other non-cash charges

     1. Depreciation (tools and vehicles) charged to operation

     2. Interest during construction

- Charges not requiring funds consisting of

  a) Interest during construction (I.D.C.) (based on an IDC rate mid-year plant under construction)

  b) Depreciation (tools and vehicles) capitalized

- Plant acquired minus plant sold (purchase price - sale cost)

Gross construction expenditure is based on the gross addition or growth module for the Costing Block.

5.2.2 Mechanization of Computation of Accounting Input

a) Current year

In keeping with the philosophy of mechanizing operations previously done manually the following calculations will be automated. These ratios provided will be based on the Company Total figures which will be provided exogenously. These inputs will be generated in modules to be used in the Costing Block and the Accounting Block.

i) Effective tax rate

$$= \frac{\text{income taxes}}{\text{net operating revenue} \quad \text{other taxable income} - \text{debt service charges}}$$

ii) Rate of return for equity components:

Firstly, preferred shares and corresponding dividends will be split into:

1) Convertible preferred equity

2) Preferred equity

then preferred dividend rate $= \frac{\text{preferred dividend}}{\text{preferred equity}}$

iii) Embedded interest rate on long term debt

$$= \frac{\text{total debt service charges} - \text{new debt service charges}}{\text{total debt}_0 - 1/2 \text{ repayment}}$$

iv) debt/capital ratio

will be calculated based on end of year data.

v) Operating costs will be calculated based on the Company Total Costs/Total Gross Depreciable Plant Assets for the following expense items:

maintenance
traffic
commercial and marketing
other expenses
taxes other than income taxes

These calculations may be altered as discussed in section 3.4.

vi) Additions and retirements are discussed in the various algorithms of 3., Costing Block,and will be provided directly from these modules.

viii)Ratios of type of plant/ inter-regional plant at cost total will continue to be calculated and provided directly by the D.O.C. since these ratios are different from Company Total Ratios.

With respect to pure Accounting Block activities the following calculations will be automated.

i) The former Table 7-Telephone Property and Depreciation-and Table 8-Net Change to Accumulated Depreciation-can be calculated internally for Inter-Regional and Company Totals based on the depreciation and gross additions or growth modules of Section 3. For each plant category this can be done and the asset sheet can be produced for all depreciable plant types, beginning and end of year.

ii) The solution of simultaneous equations (section 5.2.1 Step 6) can be applied to Company Totals and Regional as well as Inter-Regional statements. However, this would serve as a check to verify the simultaneous equation system since base data must be provided (Company Total in this phase) in order to calculate various ratios, for example, interest on long term debt. Once this system is operational, it can generate new values under differing simulation assumptions as for example, partial, complete or no flow through.

b) Accounting Output Data Bases and Linkage with Policy Simulation Blocks.

IRA II will produce a "Summary of Financial Highlights" which will summarize the detailed financial statements which are calculated. Exhibit 1 shows an example of such a summary for the public message service for Company Total activities of B.C. Telephone, 1971. This is based on IRA I models and is included for display purposes only. These summaries serve two primary purposes. Firstly, they allow easier comparison between simulations with a less cumbersome volume of data. Secondly, the summary values correspond to those needed for the single period models used in the Policy Simulation Block. These values for Inter-Regional, Regional and Company Total statements will be stored on keypunch cards and used as required by the single period models. (see section 6.2.1).

The Laboratoire d'économétrie will carry out the multi-period analysis model. However, in order to experiment with output from the Inter-Regional and Regional activities, which are calculated in the main software programs, storage and linkage must be carried out.

From a paper by Dr. Young, the variables required which differ in part from the data requirements of the single period models are:

1) E_o - common equity

2) R_o - retained earnings

3) PR_o - preferred equity

4) L_o - long term debt

5) RL - repayments

6) GTP_o - gross telephone plant (total)

7) AD_o - accumulated depreciation (total)

8) OCA_o - other current assets

9) CL) - current liabilities

These nine variables and other required data will be calculated for 8 TCTS carriers for their Inter-Regional, Regional and Company Total Activities. These values will be stored on keypunch cards in a fixed format and sent to the Laboratoire d'économétrie. With these values a data base will be available for multiperiod projection and analysis. (see Sub-Section 6.2).

EXHIBIT 1

B C TEL
SUMMARY OF FINANCIAL HIGHLIGHTS 1971
COMPANY TOTAL

INCOME AND RETAINED EARNINGS STATEMENT

OPERATING REVENUE		201223
OPERATING EXPENSES EXC. DEPR.	(	91694)
DEPRECIATION	(	34078)
DEBT SERVICE CHARGES	(	19509)
INCOME TAXES	(	26468)
OTHER INCOME		1423
NET INCOME		30897
PREFERRED DIVIDENDS	(	4391)
NET INCOME AVAILABLE(FOR COMMON)		26506
CURRENT INCOME RETAINED		15496
RETAINED EARNINGS(END OF YEAR)		56354

TOTAL SOURCES OF FUNDS	104361
TOTAL USES OF FUNDS	104358

BALANCE SHEET

PROPERTY(NET OF ACCUM. DEP.)	579203	PREFERRED	74678
INVESTMENTS	3423	COMMON & PREMIUM	119745
CURRENT ASSETS	39177	RETAINED EARNINGS	56354
DEFERRED CHARGES	5858	TOTAL DEBT	290592
		CURRENT LIABILITIES	28843
		DEFERRED CREDITS-TAXES	57448
		DEFERRED CREDITS,OTHER	0
	627661		627661

RATIOS

DEBT/CAPITAL RATIO	0.54647
PREFERRED STOCK/TOTAL CAPITAL	0.13804
RATE OF RETURN ON COMMON EQUITY	0.16273
RATE OF RETURN ON CAPITAL	0.05964
PAYOUT RATIO ON COMMON EQUITY	0.35818

6. POLICY SIMULATION

6.1 Introduction

In the previous reports of the First Phase of the IRA Project, a number of issues relevant to the telecommunications industry were identified such as, the rational utilization of the network, the problem of cross-subsidization among services, etc. Also, a preliminary plan of simulations was proposed to obtain some answers to certain questions on these issues. Our objective in this introduction is firstly, to try to evaluate, in very general terms however, the progress made during the present period in that direction, and secondly, to formulate some general considerations about the IRA model, taking the policy view point into account.

At the end of the First Phase, the team recognized that the improvements of the Operating Block were to have a high priority during the Second Phase. The reason, of course is self-evident: many inputs which are necessary for the other blocks originate there, and without strength and flexibility in this block it would be impossible to perform many of the simulations we have in mind. The reader can surely notice that the improvements described previously in the present Report will now permit a good deal of flexibility in the Operating Block.

Similarly, the work planned or already accomplished in the other blocks, represents steps towards our main objectives. We note here as the main achievements, firstly, the extension of the simultaneous accounting equation system to include additional equations and variables and secondly, the development of these equations for multi-period analysis. The idea behind this approach is to forecast the exogeneous variables and resolve the equation system with these new values. This approach also takes into account the conditional pattern of the endogenous variables which are considered.

It is important to note that the proceedings of the CTC Cost Inquiry were closely observed and the information emanating from the inquiry carefully considered in the course of developing the IRA model. From the outset the objective, that the results of the cost inquiry and those of the IRA development work were to be complimentary, was kept in focus. It appears now that the costing concepts and costs proposed by the Cost Inquiry Consultants may be adopted to the IRA logic with no difficulty and moreover, the IRA Model's capability to determine facility utilization as between different services, as the basis of developing costs e.g. incremental, could potentially complement the costing procedures proposed by the consultants in the inquiry.

One basic objective of the IRA model is to furnish information on the consequences of various policy options wich may be envisaged by the Department of Communications. A number of Operational Research type techniques have been employed for this purpose. Among these, goal programming (G.P.), a relatively new technique, proved to be a most effective tool in solving the problems of routing the traffic in the physical network and of finding acceptable compromises or even optimal solutions in the domain of corporate financing. In the latter case, various numerical preference functions or quantitative criteria (in the sense of ordinal rank) were tried in the Phase One of the model. The results of this approach were encouraging, demonstrating the potential usefulness of this technique.

The IRA model has thus undergone a transformation in its nature in the methodological sense. Although it is still basically a simulation model it is equipped with limited local optimization capabilities. A set of input variables can be identified as D.O.C. control (instrument) variables. The variables can be manually changed, the consequences computed according to the logic of the model and the outputs evaluated by D.O.C.

As far as the logic is concerned, the IRA model is a deterministic representation of the annual inter-regional telecommunication activities. The validity of the model can be tested by examining the inter-relationships it describes in all its parts and also by comparing the results it computes with the observed actual results. As a whole the model is, for simulation purposes, a "black box" transforming controllable inputs into outputs. Search procedures are needed if the user aims at some target. From an initial state, small changes in value of a set of jointly moving factors determine the direction of change and the distance to the target can be computed, but we cannot be sure of the convergence toward the target in a finite number of moves. Moreover, certain factors we wish to change are purely qualitative and the notion of "small change" is meaningless.

The IRA model is intended to be capable of handling industry; wide technical and policy-related issues in the domain of inter-regional communications. However, at the present stage of its development the Accounting Block is not yet fully developed as the industry accounting module. This problem will be overcome as the individual company's data are aggregated and the logic of the Block is adjusted accordingly. For the present phase, further development in the Accounting Block is directed towards the refinement of the existing one-period accounting model and associated simulation model, and to the introduction of the multiperiod simulation capability; all these at both the individual corporate level and at the level of industry-wide aggregation.

6.2 Extension of the Accounting Simulation Model

6.2.1 Introduction

The discussion of this section is divided into two parts: section 6.2.2 describes the expansion of the existing one-period accounting simulation model and the role of goal programming in there; section 6.2.3 deals with the development of the multi-period extension of this one-period model. It is devoted to forecasting the key exogenous variables described in the one-period model. Once the forecasting is accomplished, we re-enter the world of the one-period logic and adhere to the accounting equations contained therein.

6.2.2 One Period Model

a) Expansion of One-Period Simultaneous Equations

The accounting simulation system with which we worked in IRA I contains four equations and three ratio conditions, involving ten endogenous variables and eleven exogeneous variables. In the present phase, we disaggregate some variables and introduce more equations. The system, as it now stands, contains seven equations, and seven ratio conditions. We describe below the final model and its properties. One can find in Annex D a glossary of the symbols used. Also, as in the preceding phase of the Project, a program in APL Language is constructed; this program can work in an autonomous manner, that is, without using the information provided by the other blocks of the IRA model. The Program is displayed in Annex B. However, some exogeneous variables are in a more aggregate form than those which appear in the following system. Of course, the APL program will be adapted with these equations.

Eq. 1)
$$\begin{aligned} & GCE + CONST_5 + CONST_{5A} \cdot \Delta PR + DIVI + \Delta CII \\ & + CONST_{1A} \equiv NETINC + DEPR + \\ & \Delta E(I - C_e) + \Delta PR\ (CI - C_p) + NEWDEBT\ (I - C_1) \\ & + [1 - (1-\alpha) \cdot t] \cdot DEPDIF - \alpha \cdot \Delta DEFTX \end{aligned}$$

Eq. 2)
$$\begin{aligned} & NETINC \equiv (1 - t)\ [OPRV - OPXP - DEPR - CONST_2 \\ & - CONST_{2A} \cdot NEWDEBT + \beta \cdot OTHERINC] + \\ & (1 - \beta)\ OTHERINC + \left(\frac{I_c}{1 + I_c}\right) \cdot \gamma \cdot GCE + \\ & \alpha \cdot \Delta DEFTX - [1 - (1 - \alpha) \cdot t] \cdot DEPDIF \end{aligned}$$

Eq. 3)
$$NIA \equiv NETINC - CONST_5 - CONS_{5A} \cdot \Delta PR$$

Eq. 4)
$$E \equiv E_0 + \Delta E$$

Eq. 5)
$$\begin{aligned} & R \equiv R_0 + NIA - C_e \cdot \Delta E - Cp \cdot \Delta PR \\ & \quad - DIVI + CONST_3 \end{aligned}$$

Eq. 6)
$$PR \equiv PR_0 + \Delta PR$$

Eq. 7)
$$L \equiv L_0 + NEWDEBT - CONST_4$$

Eq. 8) $DCR = \dfrac{L_0 + L}{E_0 + E + R_0 + R + PR_0 + PR + L_0 + L}$

Eq. 9) $PCR = \dfrac{PR_0 + PR}{E_0 + E + R_0 + R + PR_0 + PR + L_0 + L}$

Eq. 10) $ROREC = \dfrac{NIA}{1/2 \left[E_0 + E + R_0 + R\right]}$

Eq. 11) $RORC = \dfrac{NETINC + CONST_2 + CONST_{2A} * NEWDEBT}{1/2 \left[E_0 + E + R_0 + R + PR_0 + PR + L_0 + L\right]}$

Eq. 12) $DPR = \dfrac{DIVI}{NIA}$

Eq. 13) $RORBI = \dfrac{NETINC + DSC + ADJI}{TELPLANT - ACCDEPR + WK - ADJB}$

Eq. 14) $RORBE = \dfrac{NETINC + DSC - ADJI}{TELPLANT - ACCDEPR + WK - ADJB - PUC}$

$$CONST_{1A} = \left[-\Delta DCRO - \Delta CL - NSALV + \Delta OÇA + \Delta I + \Delta DCO - AMORT + CONST_4 - CONST_3\right]$$

$$CONST_2 = \left[L_0 - 1/2\ RL\right] \cdot i_0$$

$$CONST_{2A} = 1/2 \cdot i_n$$

$CONST_3$ = other adjustments net

$CONST_4 = RL$

$CONST_5 = PR_0 \cdot P_0$

$CONST_{5A} = 1/2 \quad P_m$

α ≡ Flowthrough coefficient

β ≡ Ratio of taxes on Other Inc to Other Inc.

γ ≡ Ratio of average PUC to GCE

Ic ≡ Interest Rate applied to PUC

t ≡ Effective Tax Rate on Book Taxes

DEPDIF ≡ Change in DEPR due to change in depreciation methods

WK ≡ ϕ*OPXP (net of DEPR) (where ϕ to be determined)

RORBI ≡ required rate of return on rate base including PUC

RORBE ≡ required rate of return on rate base excluding PUC

This is a completely flexible system and can easily be adopted for simulation on past data or forecasting future data. Of the forty-three terms in the model, eighteen must always be provided exogenously at the start:

CE_0, R_0, L_0, PR_0, $Const_{1A}$, $Const_2$, $Const_{2A}$, $Const_3$, $Const_4$, $Const_5$, $Const_{5A}$, α, t, Ce, Cp, C, Ic, γ

Out of the following terms eleven must also be provided exogenously and then the remaining fourteen can be solved using the fourteen equations.

NETINC, NIA, ΔCE, ΔPR, NEWDEBT, CE, R, L, PR, GCE, DEPR, DEPDIF, ΔDEFTX, DIVI, ΔCTI, OPRU, OPXP, OTHERINC, DCR, PCR, ROREC, RORC, DPR, RORBI, and RORE.

Which eleven are given exogenous values depends very much on the questions which are being asked.

b) Goal Programming

For the time being, there is little to say except that the methodology, already developed in the First Phase of the Project, can be applied to the extended version of the equation system. What can be done, is to specify more constraints and the appropriate weight to be included in the objective function in the Goal programming approach, and this causes no problem from the methodological viewpoint. However, and infortunately nothing has been done during the present period, it is possible to study the case where we consider more than one carrier at the time, and look for some common constraints ("liaison" constraints). We still think that there is some interest in this extension and consequently we will try to put some effort, for the final report, in this direction.

6.2.3 Multi-periods simultaneous equation system and projections

In the one-period accounting model exogenous variables must be inputted outside of the model. If a forecasting model, this means that they must be forecasted for the ensuing period.

Once the values of exogenous variables are determined, the calculation of endogenous variables using the one-period accounting logic will enable us to complete the corporate financial statements in a future period. The implications of corporate financial statements derived from such forecasted financial variables can be examined, employing various simulation techniques, including the goal programming technique.

a) Methodology of forecasting

The classical forecasting technique is of course, regression analysis. Thus, if we want to forecast Y which we feel is a function of X and Z we may write

$$Y = a + bX + cZ$$

where a, b and c are determined by past data. Given values of X and Z a value of Y can be obtained.

We could write our functional relationship as

$$Y = bX$$

and it is clear that we are implicity assuming strict proportionality $(a = 0)$ and Y is not a function of other variables $(c = 0)$. We could, of course, estimate by regression techniques but a simpler method is to calculate values of b in only the recent past and assume that the average relationship will hold roughly in the future. Of course, the APL language is written to be able to perform both techniques.

The advantage of the latter method is the great simplicity it brings to the forecasting scheme. Moreover, for short term forecasts the proportional relationship is probably quite good. For longer periods, it is more likely to be inaccurate.

b) The telephone property sector

The change in the telephone property sector in a period can be written as

$$\Delta NTP_t = \Delta GTP - \Delta AD_t$$

and since

$$\Delta GTP_t = GCE_t - RET_t$$

$$\Delta AD_t = DEPR_t + NSALV_t - RET_t$$

we can write

$$\Delta NTP_t = GCE_t - DEPR_t - NSALV_t$$

Since GCE_t, $DEPR_t$ and $NSALV_t$ are needed as inputs to the forecasting model, each must be predicted and ΔNTP_t can be derived as a residual. Although this seems the logical way to do things, it has disadvantages as will be shown below. In fact, of the four items in the equation, prediction of any three would yield the fourth as a residual. Two alternative methods are discussed below.

METHOD I

Forecast GCE_t, $DEPR_t$, $NSALV_t$ and RET_t and derive ΔNTP_t as a residual

The difficulty with this method is the considerable instability in GCE_t. Rates of growth over the past few years were

1969	15%	(Bell)
1970	3%	
1971	17%	
1972	8%	

Such instability would seem to preclude reliable forecasting, but if such a forecast can be developed, this method has great appeal.

METHOD II

Forecast ΔNTP_t, $DEPR_t$, $NSALV_t$ and RET_t with GCE_t determined as a residual

The underlying assumption is that ΔNTP_t is much more stable than GCE_t and therefore easier to forecast. Rates of growth over the past few years were

1969	9.0%
1970	8.0%
1971	9.3%
1972	8.6%

Although the pattern of growth is the same, the magnitude of the changes is much less severe. (Most of the changes in the rate of growth of ΔNTP are due to changes in plant under construction. The growth rate of net plant in service has been remarkably stable).

The difficulty with this indirect estimation is that $DEPR_t$ is used to calculate GCE_t, hence it would imply that a change in the accounting calculation of depreciation would change GCE_t.

Note that if "true depreciation" increased, then we would probably expect a change in GCE_t. The difficulty is with the measurement, not the concept.

It was decided to choose the second method.

The model for the plant sector is

P1) NTP $= (1 + r)\ NTP_0$

P2) RET $= \alpha_1\ GTP_0$

P3) NSALV $= \alpha_2\ GTP_0$

P4) DEPR $= \alpha_3\ GTP_0$

P5) GCE $= NTP - NTP_0 + DEPR + NSALV$

To operate, we need the initial input variables NTP, GTP and estimates of r, α_1, α_2, α_3, α_4. Note that the latter may be estimated from past data or simply selected by the user if he suspects the past is not a good guide to the future. Clearly these equations could be replaced by more sophisticated ones if desired.

c) Operating expenses (OPXP)

We assume that OPXP is a proportional function of average net telephone property during the period. Two equations are therefore needed.

X1) Average NTP $= 1/2\ [NTP_0 + NTP]$

X2) OPXP $= \alpha_4$ Average NTP

To operate, NTP is an initial input, NTP is calculated in the plant sector, and α_5 must be selected by the user.

d) Deferred credits - taxes DCRT

This is an extremely difficult item to predict because institutional influences are very significant. Two equations are formulated

T1) $ALLEXP = \alpha_5 \cdot NTP$

T2) $\Delta DEFTX = (1-\alpha)\ t(ALLEXP - OPXP - DEPR$

Note that an initial estimate of allowable expenses (ALLEXP) if not known can be made using T2

i.e. $$ALLEXP = \frac{\Delta DEFTX}{(1-\alpha)\ t} + OPXP + DEPR$$

Also note that debt service charges are omitted since they will not be much different for tax purposes.

α_6 must be inputted, OPXP, DEPR, NTP are calculated in other section.

e) Other Estimates

a) Current liabilities

01) $CL = \alpha_6\ OPXP$

02) $\Delta CL = CL - CL_0$

b) Other current assets

03) $OCA = \alpha_7$ (average NTP)

04) $\Delta OCA = OCA - OCA_0$

c) $CONST_{ia}$

$= (-\Delta DCRO - \Delta CL - NSALV + \Delta OCA + \Delta I + \Delta DCO - AMORT + RL$ - other adjustments net)

ΔCL, ΔOLA, and NSALV are calculated elsewhere
RL is obtained from an examination of bond maturity data contained in the published reports.

It is suggested that for forecasting purposes, the remainder ΔDCRO, ΔI, ΔDCO, other adjustments net be assumed to equal 0.

AMORT should be given the same value as it had in the previous period.

d) $CONST_2$

$= (L_0 - 1/2\ RL) \cdot i_0$

L_0 and RL are obtained from last published reports

$$i_0 = \frac{\text{Debt service charges in previous period}}{L_0}$$

e) $CONST_{2A}$

$= 1/2\ i_n$

i_n must be forecast directly by the analyst

f) $CONST_3$

= other adjustments net

g) $CONST_4$

= RL

h) $CONST_5$

$= PR_0 \cdot \rho_0$

PR_0 is obtained from the last balance sheet

$$\rho_0 = \frac{\text{Dividend on shares outstanding at beginning of period}}{PR_0}$$

i) $CONST_{5A}$

$= 1/2\ \rho_n$

ρ_n must be forecast directly by the analyst

j) tax rate t

This is estimated from past experience

$$t = \frac{\text{Taxes Paid (Normalization)}}{\text{OPRV} - \text{OPXP} - \text{DEPR} - \text{debt service charges}}$$

k) issue expenses C_e, C_1, c_p

To take C_p as an example

$$C_p = \frac{\text{Issue expenses}}{\text{Gross proceeds of issue}}$$

These again estimated from past experience.

l) interest on construction INTCON

INTCON is incorporated by use of the interest rate I_c which is inputted by the analyst

m) Other income OTHERINC

The simplest way to calculate OTHERINC is to assume it will be the same as last year.

annexes

ANNEX A

TRUNKING FORMULAE

This Annex will state the trunking formulae used (or to be used) in the software of IRA-II. For explanations of derivation or other details, the reader is referred to the references listed at the end of this annex.

<u>Trunking Formulae when Incoming Traffic Density is Known</u>

In general, we assume that for first routed traffic, the incoming traffic process on a particular link can be represented by the Poisson density function (usually referred to as pure random traffic) having a mean 'a' equal to its variance 'a'. This assumption may also be valid when large components of traffic on one link are first routed traffic since the means and variances of traffic from independent sources are additive.

That is - suppose total offered traffic on a link is composed of components wherein the first m are first routed traffic following the Poisson process. These m components have means and variances a_i, σ_i^2 where a_i σ_i^2

The remaining n-m components are non-pure random traffic components (overflow or transit traffic) whose means a_j and variances σ_j^2 are not identical. (usually σ_j^2 a_j meaning that the peakedness factor or variance to mean ratio is larger than 1).

The mean and variance of the total traffic process, assuming independent sources, are $M = \sum a_i$ and $V = \sum \sigma_i^2$. It is evident from this statement that when total pure random traffic on a link is large with respect to non pure random traffic that M and V will be approximately equal. Hence, the following formulae may be used as approximations even when the basic assumptions are not fully verified.

Based on this assumption of a Poisson traffic process, Erlang developed a trunking formulae which is generally accepted as a satisfactory model of loading of circuits. In this model, the formulae for load carried on a group of 'c' fully accessible circuits when 'a' erlangs of traffic are offered is:

$$L = a\left[1 - \left(\frac{a^c}{c!}\right) \Big/ \left(\sum_{x=0}^{c} \frac{a^x}{x!}\right)\right]$$

The probability of overflow is:

$$P_D = \left(\frac{a^c}{c!}\right) \Big/ \left(\sum_{x=0}^{c} \frac{a^x}{x!}\right)$$

This model is predicated on the assumption of sequential access, that is that circuits in a full availability group are numbered so that any incoming call attempt is routed to the first circuit, then to the second and so on. Furthermore, the formulae is used for high usage groups since the traffic process is best approximated by the Poisson distribution. Generally, with this system, we must assume that the average calling rate is not affected by congestion, and that traffic overflowed to common final groups must be handled without delay.

An alternative to this solution which is useful in some cases is to assume simply a Poisson process for the traffic load carried. This process provides the formulae.

$$L = a\sum_{x=0}^{c-2} \frac{e^{-a}a^x}{x!} + c\sum_{x=c}^{\infty} \frac{e^{-a}a^x}{x!}$$

and probability of loss or overflow

$$P_D = \sum_{x\ c}^{\infty} \frac{e^{-a}a^x}{x!}$$

This process is suitable for the commonly termed "lost calls held" assumption wherein it is assumed that call attempts receiving an initial busy signal are resubmitted to the system continually for a period of time equal to the original intended holding time. This trunking formula may be used on final groups as an approximation to the traffic process.

Wilkinson's Equivalent Random Theory

Wilkinson proposes an alternative approach to trunking formulae for final groups. His approach, already mentioned in section 2.2.2 consists basically in modelling a system of several high usage groups and one common overflow group by a single full accessibility group having the same overflow mean and variance as the original system. The formulae used are as follows.

1) The original collection of high usage groups has for group i, a_i units of offered traffic (pure random traffic) and n_i circuits (for each of i = 1, ..., n groups.

The overflow mean and variance of this traffic is as follows:

$$\mu_i = a_i \cdot E(n_i, a_i)$$

where $E(n,a) = \dfrac{\frac{a^n}{n!}}{\sum_{x=0}^{n} \frac{a^x}{x!}}$ (the erlang overflow probability)

and $v_i = \mu_i \left(1 - \mu_i + \dfrac{a_i}{n_i + 1 + \mu_i - a_i}\right)$

2) For the total overflow process offered to the final group, the following formulae determine the mean M and the variance V

$$M = \sum \mu_i$$
$$V = \sum v_i$$

3) The number of circuits n^* in the equivalent group and the traffic offered A^* to the equivalent group may be determined from the following equations

$$M = A^* \cdot E(n^*, A^*)$$

$$V = M \left(1 - M + \frac{A^*}{n^* - 1 - M - A^*}\right)$$

This system has the approximate solution (proposed by Rapp (1)) as follows:

$$A^* = V + 3\frac{V}{M}\left(\frac{V}{M} - 1\right)$$

$$n^* = \frac{A}{q} - M - 1$$

where $q = 1 - \dfrac{1}{M + \frac{V}{M}}$

4) From this equivalent scheme, we form the expression

$$L = A^*(1 - E(n^* + m, A^*))$$

for the load carried by the entire system. For fixed values of m we may determine the load L carried on the entire system and the load M-L carried on the common overflow group. Alternatively, the quality of service $\frac{L}{A^*}$ may be fixed and m determined as an optimal number of circuits on the final group.

REFERENCES

(1) Rapp, Y.; "Planning of a Junction Network in a Multi-Exchange Area. I. General Principles", Ericson Technics 20, (1964), 1; pp. 77-130

(2) Truitt, C.J.; "Traffic Engineering Techniques for Determining Trunk Requirements in alternate routing trunk network"; Bell System Technical Journal, vol. 23 (1954), no 2 (March); pp. 277-302.

(3) Wilkinson, R.I.; "Theories for Toll Traffic Engineering in the U.S.A.". Bell System Technical Journal 35 (1956); pp. 421-514.

ANNEXE B

Le programme SIMEQ2 de résolution du système d'équations simultanées

Le système comporte 12 équations en 28 variables exogènes (dont certaines sont agrégées en constantes pour simplifier la notation) et 17 variables endogènes. Le programme, lorsqu'appelé, énumère ces variables en les numérotant.

1ère étape: Estimation des variables exogènes pour la période de prévision.

Le programme demande les numéros des variables exogènes à estimer (de 1 à 28).

Pour chacune de ces variables, il demande ensuite:

1) le nombre d'observations et le nombre de régresseurs

2) d'entrer les observations sur chacun des régresseurs (si le nombre d'observations fourni pour un régresseur ne correspond pas à celui donné en 1), un message d'erreur est tapé sur la console et on doit recommencer à donner les observations sur ce régresseur à partir de la première)

3) d'entrer les observations sur la variable exogène à prévoir (si le nombre d'observations fourni ne correspond pas à celui donné en 1), un message d'erreur est tapé et il faut recommencer à partir de la première observation sur cette variable)

4) de fournir les valeurs des régresseurs pour la période de prévision. Les 3 étapes précédentes ont permis le calcul des coefficients de régression; on veut maintenant obtenir une prévision à partir de cette régression.

2ème étape: Résolution du système.

Lorsque toutes les variables exogènes spécifiées ont été estimées, le programme demande les valeurs des autres variables exogènes (après les avoir énumérées). Si on a estimé toutes les variables exogènes au moyen de régressions, il passe directement à l'étape suivante.

Il demande ensuite la valeur des charges non amorties sur la dette.

Le système comporte 12 équations en 17 variables endogènes. Il faut donc choisir 5 de ces variables comme prédéterminées.

Le programme demande donc les numéros (de 1 à 17) de ces 5 variables. Puis il demande d'entrer (dans le même ordre) les valeurs de ces variables. Si l'on donne plus ou moins de 5 numéros de variables, un message d'erreur est tapé et il faut recommencer à donner ces numéros; de même, si l'on ne donne pas exactement 5 valeurs pour ces variables.

Le programme résoud alors le système d'équations et fournit les résultats sous la forme des états financiers des compagnies (i.e. Bilan - Etat des revenus et dépenses).

N.B. Il est possible que les valeurs choisies pour les variables prédéterminées ne permettent pas de résoudre le système. Par exemple, une des équations est:

$\Delta E = E - E_o$ où E et ΔE sont endogènes et E_o exogène. Si l'on choisit E et ΔE comme variables prédéterminées, il faut se garder de provoquer une contradiction. Si cela se produit, le programme tape le message. "Les variables prédéterminées choisies ne permettent pas de résoudre le système". Pour recommencer, il suffit de répondre oui à la question "Désirez-vous effectuer d'autres simulations?"

Les variables du système sont:

Variables exogènes

1. GCE Gross Construction expenditures
2. DEPR Depreciation
3. OPXP Operating Expenses
4. OINC Other Income
5. INTCO Interest on Construction
6. E_o Ordinary Stock (beginning of period)
7. R_o Retained earnings (beginning)
8. L_o Long Term Debt (beginning)
9. PR_o Preferred Stock (beginning)
10. T Tax rate
11. α Coefficient of flowthrough
12. ΔDCRT ΔDeferred Credit Taxes
13. ΔDCRO ΔDeferred Credit (others)
14. ΔCL ΔCurrent Liabilities
15. NSALV Net salvage value
16. ΔOCA ΔOther current assets
17. ΔI ΔInvestments
18. ΔDCO ΔOther deferred charges
19. ISSO Issue expenses of new ordinary shares
20. ISSL Issue expenses of new debt
21. ISSP Issue expenses of new preferred shares
22. AMORT Amortization of debt issue expenses
23. RL Debt retired
24. OADS Other adjustments net
25. i_o Interest rate on debt existing at the beginning
26. i_n Interest rate on new debt
27. p_o Dividend rate on preferred existing at the beginning
28. p_n Dividend rate on preferred issued during the period

sorēs inc

Variables endogènes

1.	NETIN	Net Income
2.	NIA	Net income available
3.	E	Ordinary Stock (end of period)
4.	ΔCIT	ΔCash and Temporary Investments
5.	NDEBT	New Debt
6.	DIVI	Dividends
7.	OPRV	Operating Revenue
8.	PR	Preferred Stock (end)
9.	DCR	Debt to Capital Ratio
10.	PRC	Preferred to Capital Ratio
11.	RORE	Rate of return on ordinary
12.	RORC	Rate of return on capital
13.	POR	Payout ratio (ordinary)
14.	ΔE	ΔOrdinary Stock
15.	ΔPR	ΔPreferred Stock
16.	R	Retained earnings (end)
17.	L	Long Term Debt (end)

ANNEXE C

LISTING DU PROGRAMME SIMEQ2

```
      ∇SIMEQ2[⎕]∇
    ∇ SIMEQ2;EN;EX;LK;M;NE;NEN;NURE;PRD;PT;RE;RHS;SS;SYS;V;VAL;V1
[1]   NEN← 17 6 ρ'NETIN NIA   E     ACIT  ENDETT DIVI  OPRV  PR    DCR   PEC   RORE  RORC  POR   ΔE    ΔPR   E     L'
[2]   NE← 28 6 ρ'GCE   PEPR  OPXP  OIFC  INTCO EQ    PG    LC    PEQ   T     α     ΔDCPT ΔDCRO ΔCL   USALT ΔOCA  AI    ADCO  ISSO  ISSL  ISSP  AMORT RL    OADJ  IO    IN    PO    PE    '
[3]   'CE SYSTEME COMPORTE 12 EQUATIONS EN 17 VARIABLES ENDOGENES (NUMEROTEES DE 1 A 17) ET 28 VARIABLES EXOGENES (NUMEROTEES DE 1 A 28).CES VARIABLES SONT:'
[4]   'VARIABLES EXOGENES   VARIABLES ENDOGENES'
[5]   'I3,5X1,6A1,I10,5X1,6A1' ΔFMT(⍳28;NE;⍳17;NEN)
[6]   2 1 ρ' '
[7]   'DONNER LES NUMEROS DES VAR. EXO. QUE VOUS DESIREZ ESTIMER AU MOYEN D''UNE REGRESSION'
[8]   NURE←⎕,0ρ⍞←'(INSCRIVEZ 0 SI VOUS N''EN DESIREZ AUCUNE.)'
[9]   EX←REGEX NURE
[10]  'DONNER LE CHANGEMENT DANS LES CHARGES NON AMORTIES SUR LA DETTE (POUR L''ECRITURE DU BILAN)'
[11]  V←EX[12+⍳16],⎕
[12]  EX←EX[⍳9],(((¯1+EX[11])×EX[12])++/EX[5 23])+-/EX[16 13 17 14 18 15 19 22 20 24]),((EX[8]-0.5×EX[23])×EX[25]),(0.5×EX[26]),(-/EX[21 24 20]),EX[23],(EX[9]×EX[27]),(0.5×EX[28]),EX[10],EX[11],EX[12]
[13]  RHS←((+/EX[1 6 10 15])+(EX[9]×1-EX[16])-+/EX[2 5])
[14]  RHS←RHS,(EX[4]+((EX[17]-1)×+/EX[3 11])+×/EX[18 19]),((×/EX[9 16])-EX[15])
[15]  SYS←(3,8)ρ(1 0 1 1 1 ¯1 0),(1-EX[16]),1,(3ρ0),((1-EX[17])×EX[12],0,¯1),0,¯1,1,(5ρ0),EX[16]
[16]  →52×⍳5≠ρPT←⎕,0ρ⍞←'DONNER LES NUMEROS DE 5 VARIABLES PREDETERMINEES'
[17]  →53×⍳5≠ρV1←⎕,0ρ⍞←'DONNER LES VALEURS DE CES VARIABLES, DANS LE MEME ORDRE'
[18]  V1←V1[⍋PT]
[19]  PRD←(PRD<14)/PRD←(PT≥9)/PT←PT[⍋PT]
[20]  V1[(PT∊ 14 15 17)/⍳5]←V1[(PT∊ 14 15 17)/⍳5]+((14∊PT)/EX[6]),((15∊PT)/EX[9]),(17∊PT)/-/EX[14 8]
[21]  PT[(PT∊ 14 15 17)/⍳5]←(((14∊PT)/3),((15∊PT)/8),(17∊PT)/5)[⍋(PT∊ 14 15 17)/PT]
[22]  SYS←((3+LK←∨/PT=16),8)ρ(,SYS), 0 1 0 0 0 ¯1 0 0
[23]  RHS←RHS,V1[(PT=16)/⍳5]+LK/-/EX[13 7]
[24]  VAL←((PT←(PT≠16)/PT)≥9)/V1←(PT≠16)/V1
[25]  V1←(PT<9)/V1
[26]  PT←(PT<9)/PT
[27]  →34×⍳1>ρPRD
[28]  RE← 9 10 11 12 13 ∊PRD
[29]  RHS←RHS,(VAL×RE/(SS,SS,(EX[7]+0.5×-/EX[6 13]),(0.5×SS←(+/EX[6 7 7 8 8 9])-+/EX[13 14]),0))-RE/((2×EX[8])-EX[14]),EX[9],0,EX[11],0
[30]  SYS←(((3+LK)ρ1),(ρPRD)ρ0)⍀SYS
[31]  M←(0 1 1 0 1 1 0 1)⍀ 5 5 ρ- 1 1 0.5 0.5 1 1 1 0.5 0.5 0 1 1 0 0.5 0 ¯1 ¯1 ¯0.5 ¯0.5 0 1 1 0 0.5 0
[32]  M←⍉(RE/M)×(8,ρVAL)ρVAL
[33]  SYS[3+LK+⍳ρPRD;]←M+RE⌿ 5 8 ρ(4ρ0),1,(10ρ0),1,0,1,(6ρ0),1,(3ρ0),EX[12],(8ρ0),1,0,0
[34]  RHS←RHS-SYS[;(RE←(⍳8)∊PT)/⍳8]+.×V1←V1[⍋PT]
[35]  SYS←(~RE)/SYS
[36]  EN←17ρ0
[37]  →54×⍳2≠ρρM←INV SYS
[38]  EN[⍳8]←((~RE)\M+.×RHS)+RE\V1
[39]  EN[13+⍳4]←(EN[3]-EX[6]),(EN[8]-EX[9]),((-/EN[2 6])+-/EX[7 13]),EN[5]+-/EX[8 14]
[40]  EN[8+⍳5]←((EN[17]+EX[8])÷S),((EN[8]+EX[9])÷S),(EN[2]÷0.5×(+/EN[3 16])++/EX[6 7]),((EN[1]+EX[11]+EX[12]×EN[5])÷0.5×S←(+/EN[3 8 16 17])++/EX[5+⍳4]),÷/EN[6 2]
[41]  3 1 ρ' '
[42]  'LA SOLUTION DE CE SYSTEME EST:'
```

```
[43]  2 1 ρ
[44]  '  VARIABLES EXOGENES            VARIABLES ENDOGENES'
[45]  '6A1,F14.5,X7,6A1,F15.5' ΔFMT(NE;(EX[⍳9],EX[18+⍳3],V);NEN;EN)
[46]  2 1 ρ' '
[47]  OUTPUT
[48]  2 1 ρ' '
[49]  'DESIREZ-VOUS EFFECTUER D''AUTRES SIMULATIONS?'
[50]  →0×⍳'N'=⍞[1]
[51]  →7,ρS←0ρ⍞,⍞←'CHANGER DE PAGE ET TAPER SUR ''''RETURN'''''
[52]  →16,0ρ⍞←'ERREUR DANS L''INTRODUCTION DE CES NUMEROS; VEUILLEZ RECOMMENCER A '
[53]  →17,0ρ⍞←'ERREUR DANS L''INTRODUCTION DE CES VALEURS; VEUILLEZ RECOMMENCER A '
[54]  →49,0ρ⍞←'LES VARIABLES PREDETERMINEES CHOISIES NE PERMETTENT PAS DE RESOUDRE LE SYSTEME: '
    ∇


      ∇REGEX[⎕]∇
    ∇ E←REGEX N;I;J;M;NROB;S
[1]   E←28ρ0×I←1
[2]   →16×⍳∨/0=N
[3]   'COMBIEN D''OBSERVATIONS ET COMBIEN DE REGRESSEURS Y A-T-IL POUR LA VARIABLE ';NE[N[I];]
[4]   M←((NROB←⎕)+ 0 1)ρρρJ←1
[5]   'ECRIRE LES OBSERVATIONS SUR  LE REGRESSEUR NUMERO ';J
[6]   →21×⍳(ρM)[1]≠ρS←,⎕
[7]   M[;J]←S
[8]   →(NROB[2]≥J←J+1)/5
[9]   'ENTRER LES OBSERVATIONS SUR LA VARIABLE ';NE[N[I];]
[10]  →22×⍳(ρM)[1]≠ρS←,⎕
[11]  M[;J]←S
[12]  'DONNER LES VALEURS DES REGRESSEURS DEVANT SERVIR A ESTIMER ';NE[N[I];]
[13]  →23×⍳NROB[2]≠ρS←,⎕
[14]  E[N[I]]←+/(¯3↓((⍳(ρM)[2]) REG M)[;2])×1,S
[15]  →3×⍳(ρN)≥I←I+1
[16]  →0×⍳28=ρN
[17]  'DONNER LES VALEURS DES AUTRES VARIABLES EXOGENES,A SAVOIR,DANS L''ORDRE: ';(~(⍳28)∊N)⌿NE
[18]  →24×⍳(+/~(⍳28)∊N)≠ρS←,⎕
[19]  E[(~(⍳28)∊N)/⍳28]←S
[20]  →0
[21]  →5,ρ⍞←'ERREUR DANS L''INTRODUCTION DE CES OBSERVATIONS; VEUILLEZ RECOMMENCER A '
[22]  →9,ρ⍞←'ERREUR DANS L''INTRODUCTION DE CES OBSERVATIONS; VEUILLEZ RECOMMENCER A '
[23]  →12,ρ⍞←'ERREUR DANS L''INTRODUCTION DE CES VALEURS; VEUILLEZ RECOMMENCER A '
[24]  →17,ρ⍞←'ERREUR DANS L''INTRODUCTION DE CES VALEURS; VEUILLEZ RECOMMENCER A '
    ∇
```

```
      ∇OUTPUT[⎕]∇
    ∇ OUTPUT
[1]   A← 5 23 ρ'REVENUS D''EXPLOITATION DEPENSES D''EXPLOITATIONAUTRES REVENUS          REVENU NET              REVENU NE
      T ATTRIBUABLE '
[2]   '                                         ETAT DES REVENUS ET DEPENSES'
[3]   2 1 ρ' '
[4]   'X20,23A1,X25,BF10.1' ∆FMT(A;(EN[7],EX[3 4],EN[1 2]))
[5]   5 1 ρ' '
[6]   '                                                     BILAN'
[7]   2 1 ρ' '
[8]   '                   ACTIF                                                 PASSIF'
[9]   2 1 ρ' '
[10]  B← 4 23 ρ'∆PROPRIETE TELEPHONIQUE∆INVESTISSEMENTS        ∆ACTIFS                ∆CHARGES DIFFEREES       '
[11]  C← 7 24 ρ'∆PRIVILEGIERS            ∆ORDINAIRES              ∆BENEFICES RETENUS       ∆DETTE A LONG TERME      ∆EXIGIB
      ILITES            ∆TAXES DIFFEREES         ∆AUTRES CREDITS DIFFERES'
[12]  'X8,23A1,X5,BF10.1,X8,24A1,X5,BF10.1' ∆FMT(B;((EX[1]-EX[2]-V[3]),V[5],(EN[4]+V[4]),(V[6]+V[17]));C;(EN[15 14],(EN
      [16]-EX[7]),(EN[5]-V[11]),V[2],EX[12],V[1]))
    ∇
```

ANNEX D

DEFINITION OF SYMBOLS

AD	accumulated depreciation
AMORT	amortization of long term debt issue expenses
CTI	cash and temporary investments
CL	current liabilities
Ce	issue expenses of ordinary shares as a percentage of gross issue proceeds
C	issue expenses of long term debt as a percentage of par value
C_p	issue expenses of preferred shares as a percentage of gross issue proceeds
DCD	deferred charges - debt
DCO	other deferred charges
DCR	debt to capital ratio
DCRO	other deferred credits
DCRT	deferred credits
DEPR	depreciation
DIVI	ordinary dividends
E	common share capital and premiums
GCE	gross construction expenditures
GTP	gross telephone property at cost
I	investments
I_c	interest rate on construction
ISSL	issue expenses of new debt
ISSO	issue expenses of ordinary shares
ISSP	issue expenses of preferred shares
i_n	interest rate on new debt
i_o	interest rate on debt existing at the beginning of a period
L	long term debt
NCE	net construction expenditures
NEWDEBT	par value of long term debt issued during period
NETINC(α)	net income, with α representing the degree of flowthrough
NIA	net income available to ordianry

NTP	net telephone property
OCA	other current assets
OPRV	operating revenue
OPXP	operating expenses (excluding depreciation)
OTHERINC	other income (excluding interest on construction)
PDIVI	preferred dividends
PR	preferred dividends
ρ_o	dividend rate on preferred existing at the beginning of a period
ρ_n	dividend rate on preferred issued during the period
R	retained earnings
RET	retirements of telephone property at cost
RORC	rate of return on capital
ROREC	rate of return on common equity
t	tax rate
INTCON	interest on construction
PCR	preferred to capital ratio
ALLEXP	expenses allowed for tax purposes
ADJI	adjustment in net income
ADJB	adjustment in rate base
TELEPLANT	telephone plant at cost

ANNEX E

STATUS REPORT ON VARIOUS TASKS OF THE MEMORANDUM OF UNDERSTANDING

The following section refers to the Memorandum of Understanding of IRA-I (June, 1, 1974) and compares the statement of work therein with the progress indicated on in this Report.

a) Operating Block

Among the tasks which were to be completed before this Interim Report, the task 3.1.7 (Estimation of Traffic and Expected Usage of the Switching Network, Conception) had been assigned a low level of priority. However, the laboratoire has submitted its thought on this subject covered by section 2.5 of the present Report.

b) Costing Block

No delay has been incurred in the tasks of this block and some ideas are taking shape for each as can be seen in the present Report, section 2.

c) Sharing Block

The tasks 3.3.1 (Statistical Report) and 3.3.4 (Mixed Schemes) have been completed. The tasks 3.3.2 (Full Division of Revenue) and 3.3.3 (Negociated Settlement) have been hardly started since they were not due for the present phase.

d) Accounting Block

All tasks proceed as was intended in the Memorandum of Understanding.

e) Policy Simulation Block

Only the task 3.5.6 (Multi-period Extension for the Simultaneous Equations System (conception) has been partially completed (see section 6.2.2) although it is due for March 31st 1975. The other tasks have been delayed, but implicitly some work has been done on 3.5.1 (Ranges for IRA Variables) in the modeling of the Operating Block. The tasks 3.5.2 (Search Procedures) is difficult to conceive given the state of the entire IRA Model. However, in the Operations and Accounting Blocks, several objective functions have been proposed.

ANNEX F

A Model for Estimating Point-to-Point Traffic

Network Development
September 20, 1974

by

G. Dunn
J. Guérin

I - Introduction

Point-to-point traffic data is of vital importance for the operation of the IRA and HERMES Programmes and also for any in-depth study of the telecommunications sector. Thus far the point-to-point traffic estimates were obtained by converting backward from existing trunks, using Erlang and Poisson formulae and given blocking and overflow probabilities on the trunk groups. The Laboratoire d'économétrie (Université Laval) is currently exploring methods for solving this estimation problem. The work presented here is a complement of their work, not a substitute.

This paper presents the results of an attempt to develop a procedure for the estimation of point-to-point traffic using gravity models. Various estimates were made by regression analysis (ordinary least squares) for a number of different variations in the form of the model. In each estimate the variables remained the same, population and distance. Although all models will be briefly described only those giving the best results will be discussed.

The set of points used in the estimations consisted of eighteen major cities in Canada. This theoretically gives 306 observations for the unidirectional traffic estimation and 153 for the two-way traffic estimation.

II - The Methodology

Various models were examined in order to find the one which best estimates the point-to-point traffic. For the first five models, we have three types of traffic T_{1ij} denotes the number of messages between points i and j, T_{2ij} the number of paid minutes and T_{3ij} the number of CCS. In describing the models we will use only T_{ij} . In the sixth model, unidirectional traffic was used. Here there are two types, t_{1ij} denoting the number of messages from point i to point j and t_{3ij}, the number of CCS. For simplification only t_{ij} will be used.

The models are:

A. $T_{ij} / P_i P_j = K / D^{\alpha}_{ij}$

Since this is a nonlinear equation, it is tranformed to a linear model by taking logarithms to the base 10 giving

$$\log (T_{ij} / P_i P_j) = \log K - \alpha \log D_{ij}$$

where T_{ij} is the traffic

K is a constant to be determined

$P_i P_j$ is the product of the populations of points i and j

D_{ij} is the distance between points i and j

B. $T_{ij} = \alpha + \beta X_{ij} + \gamma D_{ij}$

In this model, traffic is assumed to be a purely linear function of the product or sum of population and of distance. T_{ij} and D_{ij} are as previously defined and X_{ij} is either the product $(P_i P_j)$ or the sum $(P_i + P_j)$ of the populations of points i and j.

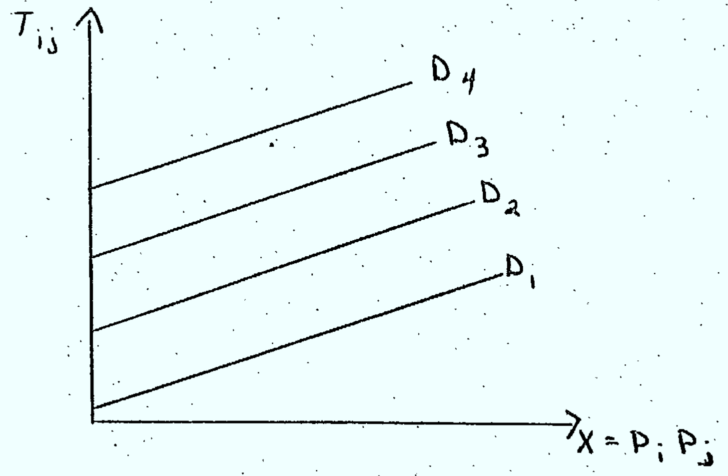
C. $T_{ij} = K (P_i P_j)^{\gamma} / D^{\alpha}_{ij}$

This too, is a nonlinear model and is transformed into

$$\log T_{ij} = \log K + \gamma \log P_i P_j - \alpha \log D_{ij}$$

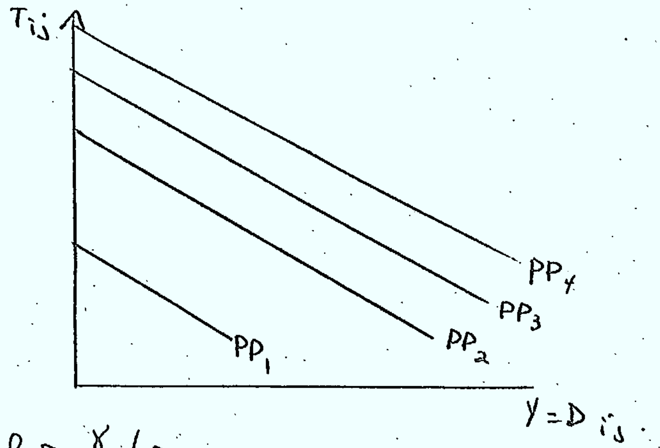
D. $T_i = \alpha_i + \beta_i X_i, \quad i = 1, 4$

This model is based on the hypothesis that within a certain distance band D_i (eg 0-699 miles) the level of traffic is a function of the product of populations. Two functions were used, one giving a linear model and the second a nonlinear model which was then transformed into a linear logarithmic form. The following diagram illustrates the idea



E. $T_j = \alpha_j + \beta_j + Y_j, \; j = 1,4$

This model assumes that within a certain population band, PP_i, the traffic varies only with distance. As with the preceding model, a linear and a nonlinear model were used.



F. $t_{ij} = K P_i^{\beta} P_j^{\gamma} / D_{ij}$

Being a nonlinear model, it is transformed into

$$\log t_{ij} = \log K + \beta \log P_i + \gamma \log P_j - \alpha \log D_{ij}$$

<u>III - The Results</u>

Using the criterions that the final equation should explain more than 75% of the variation in the traffic (i.e. R^2 (adjusted for the degrees of freedom) > 0.75) and all estimated coefficients should be statistically significant at the 95% level, we have these models, C, D and F, which are good estimators of traffic.

In the case of the models that were transformed into a logarithmic form, a number of the observations had to be deleted because the log value of zero is $-\infty$. This removal of observations has reduced the number of degrees of freedom inside each model but this has no detrimental effect on the results since we were working with a very large number of observations.

III.1 Model C

The results of the regression are as follows:

		for T_{1ij}	for T_{2ij}	for T_{3ij}
intercept (log K)		-6.41	-5.53	-8.66
regression coefficients:	γ	1.35	1.31	1.37
	α	1.61	1.51	1.41
T-value:	γ	22.027	22.038	20.718
	α	17.598	16.868	13.920
R^2 (adjusted)		0.845	0.841	0.817
S.E.E. (%)		12.73	9.84	23.70
F-value		285.227	172.411	304.926
d.f.		140	140	136

Thus, our estimator equations are:

$$T_{1ij} \quad (3.89 \text{ X}10 \ ) \ (P_i P_j)^{1.38} / D_{ij}^{1.61} \qquad (1)$$

$$T_{2ij} = (2.95 \times 10^{-6})(P_i P_j)^{1.31} / D_{ij}^{1.51} \qquad (2)$$

and $$T_{3ij} = (2.19 \times 10^{-9})(P_i P_j)^{1.37} / D_{ij}^{1.41} \qquad (3)$$

To show the goodness of fit of these equations, a number of tests are made. The R^2 (adjusted for the degrees of freedom) all fulfill the criterion of being greater than 0.75. Doing an F-test, we see that the calculated F-values are greater than their respective tabulated $F(2,140,0.99) \approx 4.75$ and $F(2,136,0.99) \approx 4.76$. This means that on the basis of a risk of 0.01 the equations are good estimators. Testing the second criterion of statistically significant coefficients, we find that the two coefficients in each regression are significantly different from zero at the 1% level for a two-tail test. For the first two regressions, the T-values are all greater than T(140,0.99) ≈ 2.610 while for the last, the values are greater than T(136,0.99) ≈ 2.611. Thus, it can be said that model C is a good estimator of point-to-point traffic.

III.2 Model D

Of the two functions only the model in the logarithmic form fulfills the specified criteria. The distance bands were D_1(0-699 miles), D_2(700-1299 miles), D_3(1300-1899 miles) and D_4(1900+ miles). The results of the regressions are as follows:

for T_{1ij}

	for D_1	for D_2	for D_3	for D_4
intercept (log Z)	-8.39	-14.32	-13.09	-16.42
regression coefficient: β_i	1.15	1.64	1.48	1.78
T-value: β_1	8.271	15.147	11.054	15.041
R^2 (adjusted)	0.574	0.861	0.802	0.911
S.E.E. (%)	16.24	11.54	14.32	9.47
F-value	68.415	229.426	122.180	226.234
d.f.	50	37	30	22

for T_{2ij}

	for D_1	for D_2	for D_3	for D_4
intercept (log Z)	-7.43	-12.85	-11.67	-15.00
regression coefficient: β_i	1.13	1.58	1.43	1.73
T-value: β_1	8.832	14.598	10.606	13.364
R^2 (adjusted)	0.606	0.851	0.788	0.890
S.E.E. (%)	12.35	8.92	10.97	7.65
F-value	78.009	213.097	112.478	178.587
d.f.	50	37	30	22

The results of the regression using T_{3ij} did not show a very good fit and so the equation can not be used as an estimator of traffic.

Thus our equations are:

$$\text{for } T_{1ij}\ D_1 \qquad T_{1ij} = (4.07 \times 10^{-0})\, P_i P_j^{1.13} \qquad (4)$$

$$D_2 \qquad T_{1ij} = (4.79 \times 10^{-15})\, P_i P_j^{1.64} \qquad (5)$$

$$D_3 \qquad T_{1ij} = (8.13 \times 10^{-14})\, P_i P_j^{1.48} \qquad (6)$$

$$D_4 \qquad T_{1ij} = (3.80 \times 10^{-17})\, P_i P_j^{1.78} \qquad (7)$$

$$T_{2ij},\ D_1 \qquad T_{2ij} = (3.72 \times 10^{-8})\, P_i P_j^{1.13} \qquad (8)$$

$$D_2 \qquad T_{2ij} = (1.41 \times 10^{-13})\, P_i P_j^{1.58} \qquad (9)$$

$$D_3 \qquad T_{2ij} = (2.14 \times 10^{-12})\, P_i P_j^{1.43} \qquad (10)$$

$$D_4 \qquad T_{2ij} = (1.00 \times 10^{-15})\, P_i P_j^{1.73} \qquad (11)$$

The results of these regressions generally indicate that the relationship between population and traffic by "distance" bands is statistically good. However, in the (0-699) band the degree of explanation of the equations is weak. Indeed, approximatively 40% of the vargiations in traffic are not explained by the equations.

In the case of the regressions made within the "population" bands, when traffic is purely a function of distance, the results indicated that both the linear and non-linear equations were statisticaly insignificant.

III-3 Model F

Of the two types of traffic only T_{1ij} is a good estimator. The results of the regression are as follows:

intercept (log K)		-7.25
regression coefficients:	β	1.24
	γ	1.55
	α	1.61
T-value:	β	24.281
	γ	23.991
	α	25.501

R^2 (adjusted)	0.853
S.E.E. (%)	13.89
F-value	553.708
d.f.	285

Thus our estimator equation is

$$t_{ij} = (5.62 \times 10^{-8})\, P_i^{1.24} P_j^{1.55} / D_{ij} \qquad (12)$$

Looking first at the R^2 (adjusted for the degrees of freedom) we see that it fulfills the criterion of R^2 being greater than 0.75. Doing an F-test, we find that the calculated F-value is greater than the tabulated $F(3,285,0.99) \approx 3.90$. Testing the criterion of statistically significant coefficients, all the T-values are found to be greater than the tabulated $T(285,0.99) \approx 2.59$ proving that all the coefficients are significantly different from zero. From this, we can say that this regression is a good estimator of traffic.

IV - Conclusion

This preliminary study on the estimation of point-to-point traffic by means of gravity models has given satisfactory results from a purely statistical point of view. We are aware of the weaknesses of such models for the estimation of traffic demand. No measures of prices and wealth or income are directly taken into account in these models. Tariffs or prices are only indirectly reflected by the "distance" variable. But, since the tariffs are not directly proportional to distance, the "distance" variable only reflects very imperfectly the effect of tariffs on the level of traffic demand.

For the estimation of bothway traffic, equations (1) and (2) constitutes good estimators. For the estimation of unidirectional traffic equation (12) can be used with confidence. Again, even if gravity models have little economic exploratory power, they very often constitute a useful mechanistic procedure of estimation for point-to-point relationships.

We are now in the process of completing this preliminary study. Basicly, the same models will be tested. However, the population variables will be replaced by the total number of telephones or some combinations of business and residence telephones. We will also attempt to introduce some relative measures of wealth (or income) and prices.

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