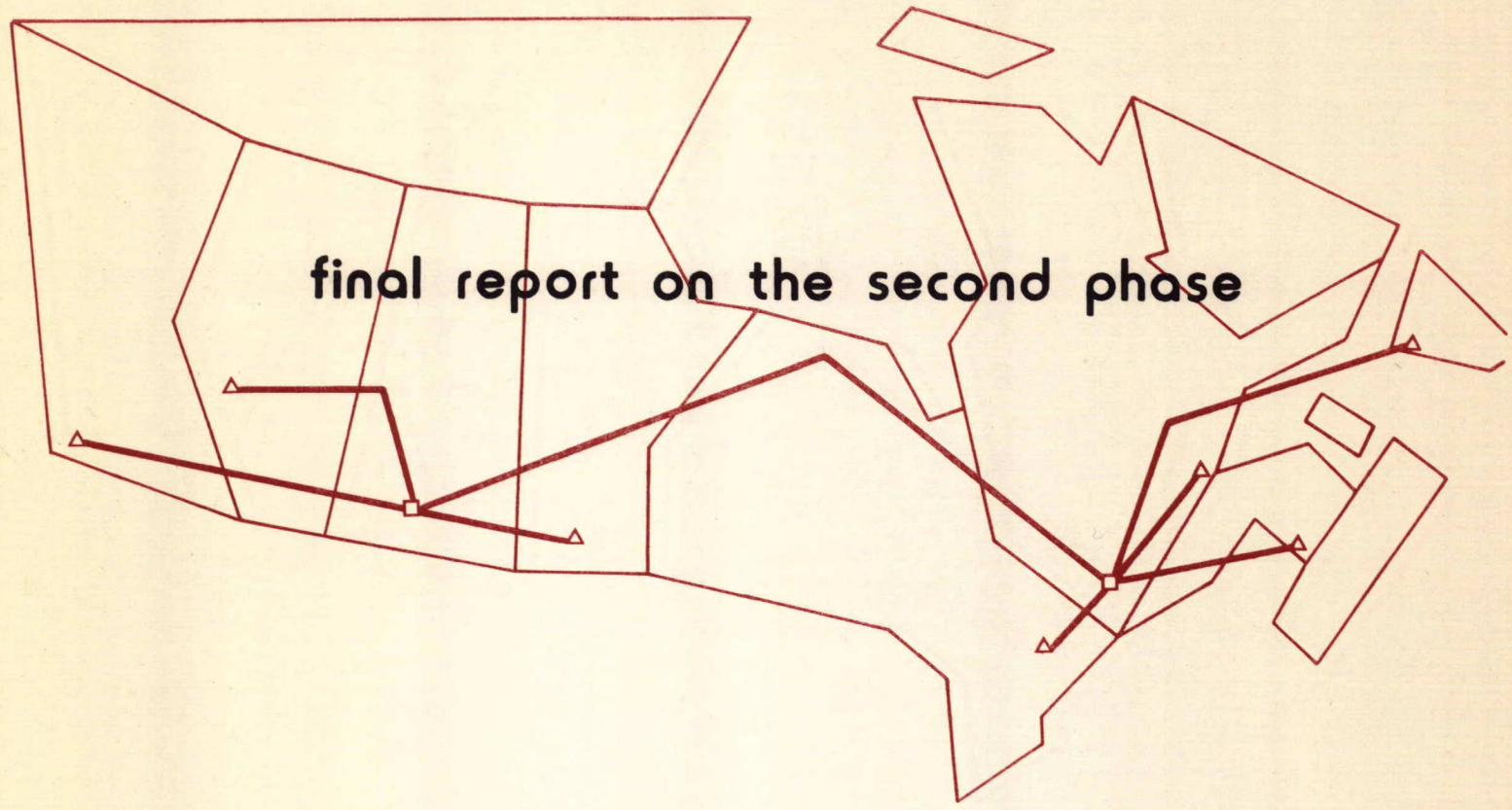


# ira project



PREPARED FOR AND IN COLLABORATION WITH THE

NATIONAL TELECOMMUNICATIONS BRANCH

DEPARTMENT OF COMMUNICATIONS

BY

LABORATOIRE D'ECONOMETRIE

UNIVERSITE LAVAL

AND

SORÈS INC., MONTREAL

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IRA PROJECT

INTER-REGIONAL TELECOMMUNICATIONS ACCOUNTING

second phase

FINAL REPORT ON THE SECOND PHASE

prepared for and in collaboration with the



NATIONAL TELECOMMUNICATIONS BRANCH  
DEPARTMENT OF COMMUNICATIONS

by

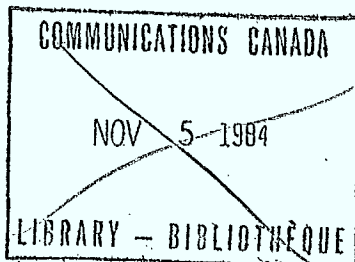
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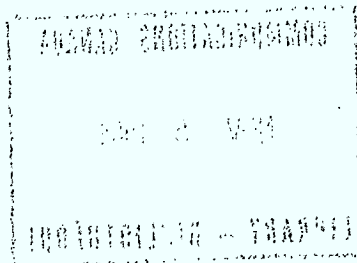
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## FOREWORD

The two main objectives of this Report on the Second Phase of the IRA Project are to report the results on the first test runs made with the algorithms already developed and to establish a plan of simulations which the model can now perform or will be able to perform in the future. The various developments since the Interim Report of September 15, 1974 are also described in details. The reader who is familiar with our previous reports on the Project will notice that the emphasis has been shifted among the various aspects of the Project. In particular, we can now give more emphasis to policy questions aiming at obtaining an efficient allocation of resources in the industry of telecommunications. Of course, the other objectives of the Project are still present, i.e., it is possible to simulate the financial consequences of various costing and accounting procedures and of various revenue sharing schemes concerning inter-regional telecommunications. But these aspects now constitute only a part of the various options that the model can evaluate.

It is worth reminding the reader that the present Report is not a comprehensive treatise on telecommunications engineering, accounting or, for that matter, operational research. Nor does it presume to provide automatically solutions which could pretend to be "optimal" for the carriers and for the regulatory agencies. On the other hand, it is to be viewed, in a wider setting, as a contribution towards the formulations of policies aimed at developing an even more efficient telecommunications system, in keeping with the broad economic and political objectives in Canada.



## ABSTRACT

This Final Report on the Second Phase of the IRA Project consists of an Introduction and of four chapters. A large part of it is devoted to the test runs performed with the different blocks, or parts of them, and with the model as a whole. The second main purpose of the present Report is to propose some scenarios of simulations in view of posing and evaluating some policy questions to which the model now addresses itself in a more precise manner. Although the emphasis is now put on policy questions it is worth reminding the reader that the logic of the model is essentially the same as before, i.e. it contains four blocks each one continuing to perform the tasks for which it was constructed. A fifth block is now being included, and described in Chapter 4, under the more general heading of Simulation.

The first chapter constitutes an Introduction. The second one contains the developments at the conceptual as well as at the software levels, since the Interim Report of September 15, 1974. Each section corresponds to a given block. In the Operating Block, the main innovations concern the treatment of survivability and of the estimation of the usage of the transmission network facilities by message streams. A few examples of the optimal allocation of circuits on this network are also given. In the Costing Block, the three main issues which are discussed concern the introduction of new asset valuation functions for the transmission facilities, the creation of a growth module, the different operations performed on asset valuations, (aging, indexing, depreciation) all these in view of the various unit costing methods envisaged. At the level of the Sharing Block for revenue computations more detailed traffic profiles and holding time distributions are considered. Finally, the Accounting Block has now been totally mechanized, the description of the mechanization is reported herein.

Chapter 3 presents the first test runs made with the different algorithms already developed. In this Report, the term test run means either the validation of the algorithms used or sensitivity analysis. The objective of validation is to prove that the algorithm performs the task for which it was constructed, the time it takes to perform it, and the dimensions of the problems that can be treated, etc... The objective of the sensitivity analysis is to establish the range of significant variations of the different parameters. Of course, these two operations are crucial in order to obtain practical experience with these algorithms, and an idea of the cost of specific simulations.

The last two chapters, Chapter 4 and 5, are at a more tentative level, although some of the ideas expressed in both are rather precise. In Chapter 4, we present our thinking on the formulation of a plan of simulation. Of course, as we said previously, some aspects of this plan are strongly dependent on how the algorithms will perform with problems of relatively large size. It is intended to proceed on a question-answer basis reflecting the man-machine characteristic of the model. In other words, having formulated a broad question like the efficient

allocation of resources in the industry of telecommunications, the problem of competition among carriers, the possible cross-subsidization among services, etc., the policy maker becomes more and more precise in his specification by interacting with the model until the algorithms required to answer the question initially asked are identified. Of course, the kinds of algorithms used will inevitably impose certain questions at different parts of the process. The second part of this chapter presents the results of some simulations performed with the multi-period simultaneous equations system and with the goal programming approach.

Finally, in Chapter 5, some possible future developments including some improvements of the data basis are presented. Some of them will require relatively small effort while some others requiring more substantial resources and time.

Apart from those mentioned in the ABSTRACT of the previous reports, the main intellectual challenges of the work reported on here have been the simultaneous presence of technical, accounting and economic variables, the determination of some trade-off among them, the management of the model of this size and the taking into account of the dynamic aspects of the problem.

## RESUME

Ce Rapport final de la deuxième phase du Projet IRA comprend une introduction et quatre chapitres. Une grande partie du rapport est consacrée aux différents essais effectués à l'aide des différents blocs, ou certaines de leurs parties, et avec le modèle dans son ensemble. De plus, plusieurs types de simulations sont proposés, l'objectif étant de suggérer et d'évaluer certaines questions de politiques des télécommunications. Même si une plus grande emphase est maintenant mise sur les questions de politiques, il est toutefois utile de mentionner que la logique du modèle est essentiellement la même qu'auparavant, c'est-à-dire que le modèle contient encore les mêmes blocs qu'initialement, l'objectif de chacun restant inchangé. Toutefois, le Chapitre 4, Simulation, englobe maintenant le cinquième bloc qui apparaissait dans la formulation initiale du modèle.

L'Introduction constitue le premier chapitre. Le second chapitre décrit les développements, tant au niveau de la conceptualisation qu'au niveau de la programmation, réalisés depuis la publication du Rapport Intérimaire du 15 septembre 1974. Chaque section correspond à un bloc donné. Pour le bloc "Operating", les innovations les plus importantes ont trait à l'introduction de la survivance et à l'estimation, par flux de messages, de l'utilisation des installations du réseau de transmission. De plus, quelques exemples de l'allocation optimale des circuits sur ce réseau sont aussi présentés. Pour le bloc "Costing", les trois sujets discutés concernent l'introduction de nouvelles fonctions d'évaluation des actifs pour les installations de transmission, la création d'un module "croissance" et enfin une description des différentes opérations (dépréciation, indexation, etc) effectuées sur les fonctions mentionnées précédemment, ceci en vue d'appliquer les différentes méthodes de coût unitaire décrites dans le Rapport Intérimaire du 15 septembre 1974. Au niveau du bloc "Sharing", le calcul des revenus est maintenant fait à l'aide de profils de trafic plus détaillés et à l'aide de différentes distributions pour la durée des appels. Finalement, le bloc "Accounting" est complètement automatisé; la description de cette automatisation est ici donnée.

Le Chapitre 3 présente les premiers essais effectués avec les différents algorithmes qui sont actuellement disponibles. Dans le présent rapport, ces essais veulent en premier lieu montrer l'efficacité des différents algorithmes proposés et répondre à certaines questions quant au temps-machine utilisé pour compléter chaque essai, la dimension maximale des problèmes qui peuvent être traités, etc. En deuxième lieu, certaines études de sensibilité sont effectuées afin d'établir les champs de variation significatifs des différents paramètres. Ce genre d'essais est évidemment crucial, entre autres pour obtenir quelques expériences pratiques avec les algorithmes d'une part, et afin d'avoir une idée du coût des simulations proposées d'autre part.

Les deux derniers chapitres, Chapitre 4 et 5 sont à un niveau plus exploratoire bien que certaines idées avancées soient plutôt spécifiques. Dans le Chapitre 4 plusieurs considérations quant à la formulation d'un plan de simulations sont mises de l'avant. Toutefois, comme il a été dit précédemment, certains aspects de ce plan dépendent très fortement de l'efficacité, quant aux dimensions possibles des problèmes traités, des algorithmes utilisés. La procédure suggérée est de formuler les simulations sur la base question-réponse, reflétant la caractéristique homme-machine du modèle. En d'autres termes, une fois qu'une question générale, par exemple l'allocation optimale des ressources dans l'industrie des télécommunications, le problème de la concurrence entre les transporteurs, l'inter-financement entre différents services est formulée, l'agent, par l'interaction avec le modèle, spécifie de plus en plus sa question jusqu'au point où le modèle identifie les algorithmes nécessaires pour répondre à la question posée. Le genre d'algorithmes utilisés nécessite évidemment que l'agent fasse un choix entre plusieurs possibilités, à différentes étapes du processus. La deuxième partie de ce chapitre présente les résultats de quelques simulations faites avec le système d'équations simultanées pour plusieurs périodes, et avec l'approche par programmation par objectif (goal programming).

Finalement dans le Chapitre 5 certaines possibilités de développement futurs sont discutés, incluant l'amélioration des données, certains développements nécessitant relativement peu d'efforts alors que certains autres nécessiteront des ressources plus substantielles, et aussi beaucoup plus de temps.

A part ceux déjà mentionnés dans les RESUME des rapport précédents, les principaux défis intellectuels qui se sont posés lors des travaux dont traite le présent rapport, ont été la présence simultanée de variables techniques, comptables et économiques, la détermination de taux de substitution (trade-off) parmi ces variables, la manipulation de modèles de cette taille, et la prise en compte des aspects dynamiques du problème.



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## 1. INTRODUCTION

This is the Final Report on the Second Phase of the building of a simulation model dealing with many technical, accounting and financial aspects of telecommunications in Canada. A Supplementary Report, for the present Phase is also scheduled for March 31, 1975. The main purposes of the present Report are :

1. To present the main developments since the Interim Report on September 15, 1974;
2. To report the first test runs performed with the different algorithms;
3. To propose a plan of simulation;
4. To suggest some future developments.

It is to be noted that some considerations discussed in the last two points are only at a tentative level: more will be said on them in the Supplementary Report, scheduled for March 31, 1975. This Supplementary Report will also include further improvements regarding the first two points.

It is worth reminding the reader that the purpose of this simulation model is to provide for the Department of Communications, at least in part, the necessary tools and methods for evaluating various policy options and alternatives in the industry of telecommunications. Of course, one way of evaluating them is by examining the quantitative impacts of the scenarios under consideration through the utilization of techniques of simulation. Accordingly, the model has been designed for simulation purposes with a capability of dealing with alternatives and for issues with respect both to the real operations of the telecommunications system and to the financial consequences for the carriers and for the system.

The present Report represents the results of a combined effort by three participants whose formal responsibilities were spelled out in the various official documents and detailed sharing of the tasks handled by more or less informal exchanges. The tripartite team consisted of the following organizations given here with the names of the specialists involved :

The National Telecommunications Branch, Communications, Canada:

Mr. J.A. Guérin  
Mr. G.G. Henter  
Mr. C. Lee  
Mr. P. Rogers

Sorēs Inc. Montréal :

Mr. J. Cluchey  
Miss C. King  
Mr. E. Manis

Laboratoire d'économétrie de l'Université Laval :

Prof. C. Autin  
Prof. G. LeBlanc  
Prof. T. Matuszewski

Being members of Le Laboratoire d'économétrie, the following research assistants have contributed to the Project :

Mr. F. Côté  
Mr. J. Fortin  
Mr. B. Paquet  
Miss R. St-Jacques  
Mr. G. St-Cyr

Finally, Dr. I. Young from York University, in the capacity of consultant to the Department of Communications, has contributed in the area of multi-period accounting analysis in the simultaneous equation systems.

## 2. DEVELOPMENTS SINCE THE INTERIM REPORT

### 2.1 Operating Block

In this block certain additional concepts have been developed since the printing of the interim report and these concepts are outlined in the following sections. We introduce at this point the overall flowchart for the operating block which reflects the organization of software in this version of the IRA model.

In this flowchart, on the left hand side we show the various basic components of input required to run this block. These include the initial states of the switching and transmission network, peak traffic estimators, and hourly traffic profiles for estimation of point-to-point traffic. We also provide as data the routing of television programs exogenously for this phase (concepts have been recently developed to handle this distribution problem simultaneously with the circuit allocation problem see section 2.1.2) and finally the desired survivability constraint for public message services, if any. The main flow of operations begins with the estimation of point-to-point traffic and the derivation of point-to-point traffic matrices by hour of the typical day. The second operation is the estimation of average total ccs loads on switching links and the breakdown of this total into the amounts due to various message streams. This breakdown is used only for calculation of the use of facility chains by message stream.

In parallel, the pre-emption of circuits for television and for survivability (see section 2.1.2 and 2.1.3) takes place, giving a reduced or residual network for which an optimal allocation of circuits to facilities is performed.

The final result of this allocation, and the results of estimation of use of the switching network are combined (see section 2.1.4) to provide the estimate of use of facilities by message stream.

#### 2.1.1 Estimation of Traffic

##### A) Estimation of point-to-point traffic

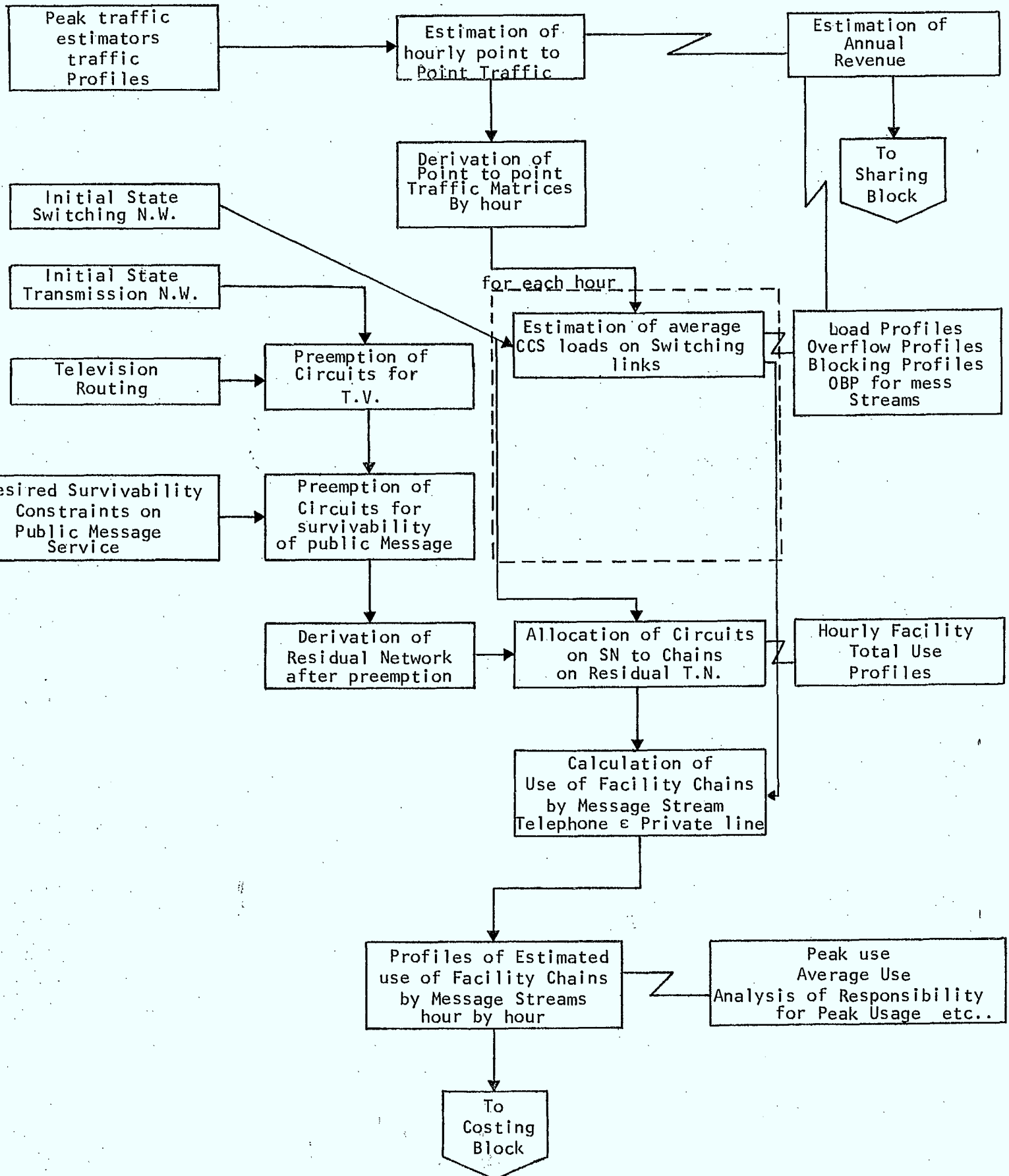
For the estimation of point-to-point traffic in the peak hour for city A to city B there are at least two approaches available.

The first approach is based on the use of erlang and Poisson formulae, given the blocking and overflow probabilities and given the number of existing point-to-point trunks for adjacent switching nodes.

The second approach is a statistical one, based on the estimation of traffic between two points as a function of various parameters such as population, distance between centres, measures of total income per city, tariffs, and number of telephones (business and residence).

## Operating Block Flowchart

sorès inc.



In this phase of IRA-II we have adopted the second approach since the results of regression analysis give relatively satisfactory estimates of traffic between points and since the first method as well as being tedious, involves a larger number of indeterminacies, some of which must be resolved using rules of thumb which may be disputable. Furthermore the estimation of traffic based on explaining variables such as numbers of telephones is logically more satisfactory than estimating traffic from the circuits in place, for two reasons. First since at any particular instant, more than enough circuits may be available as a planned provision for growth in demand in recognition of a 3 to 4 year planning horizon in the telecommunications industry, this may result in an over-estimation of traffic. Second and more importantly, the allocation of the total flow of traffic in any given trunk groups among relevant message streams is rather arbitrary (e.g. primary traffic = 60% of total traffic on final group) under the first method.

B) Estimation of traffic for the average peak hour

Accordingly, certain regressions have been performed (not yet in their final form) which give a satisfactory estimation of total traffic (measured in number of messages) in a ten day period as follows:

$$T_{ij} = \frac{1.99 \times 10^{-7} P_i^{1.32} P_j^{1.35}}{D_{ij}^{1.58}}$$

where  $P_i$  is the population of centre i (city or metropolitan area)

$P_j$  is the population of centre j (city or metropolitan area)

$D_{ij}$  is the distance from i to j

and  $T_{ij}$  is the total number of messages in an average 10 day period originating in centre i destined for centre j. (See Annex F of the Interim Report on the Second Phase for explanations).

Assuming that total traffic is divided evenly among the 10 days and that peak hour traffic represents the proportion  $q$  of daily total traffic \* we obtain the estimate for average number of messages in the peak hour from centre i to centre j as

$$\frac{q \cdot T_{ij}}{10}$$

---

\* estimate of .1133 for  $q$  obtained in IRA-I. Source "Switching Systems" American Telephone and Telegraph Co., N.Y. Reported in the Final Report on the First Phase, 31 March 1974; p. 14.



An example of the results of the application of this formula is shown in Tableau 2.1.1A.

One should note that this model underestimates the Montreal-Toronto traffic because no variable has been introduced to reflect the degree of business activity between pairs of points.

An attempt to enlarge the scope of these gravity models is ongoing.

#### C) Derivation of hourly traffic matrices

In order to take into consideration the non-coincidence of the peak hour throughout the network, we require an estimate of the average traffic in each hour of the day. Accordingly, it is necessary to obtain information regarding the distribution of telephone traffic throughout the day (a traffic profile). In the Interim Report of the Second Phase, the section 2.2.1, Profils de trafic offert, gives an analysis of the various forms taken by these traffic profiles under different circumstances. A procedure is outlined therein whereby it would be possible to derive a typical traffic profile between time zones depending on the number of hours time differential and on the direction of traffic (east to west or west to east).

In this phase of the project, we still have little information in this area, but the following approach has been adopted. We obtain a typical standard traffic profile for long distance traffic within a single time zone derived from published statistics dealing with business and residential traffic. A typical profile is shown in tableau 2.1.1B.

The standard profile is obtained from these data as the average of business and residence traffic standardized on the scale 0-1 where 1 is the height of traffic in the peak hour.

Assuming this profile to be valid for all traffic within time zones (0 hours time differential between centres) we are able to estimate absolute numbers of messages in any hour by the multiplication of  $q T_{ij}/10$  with the profile level for that hour.

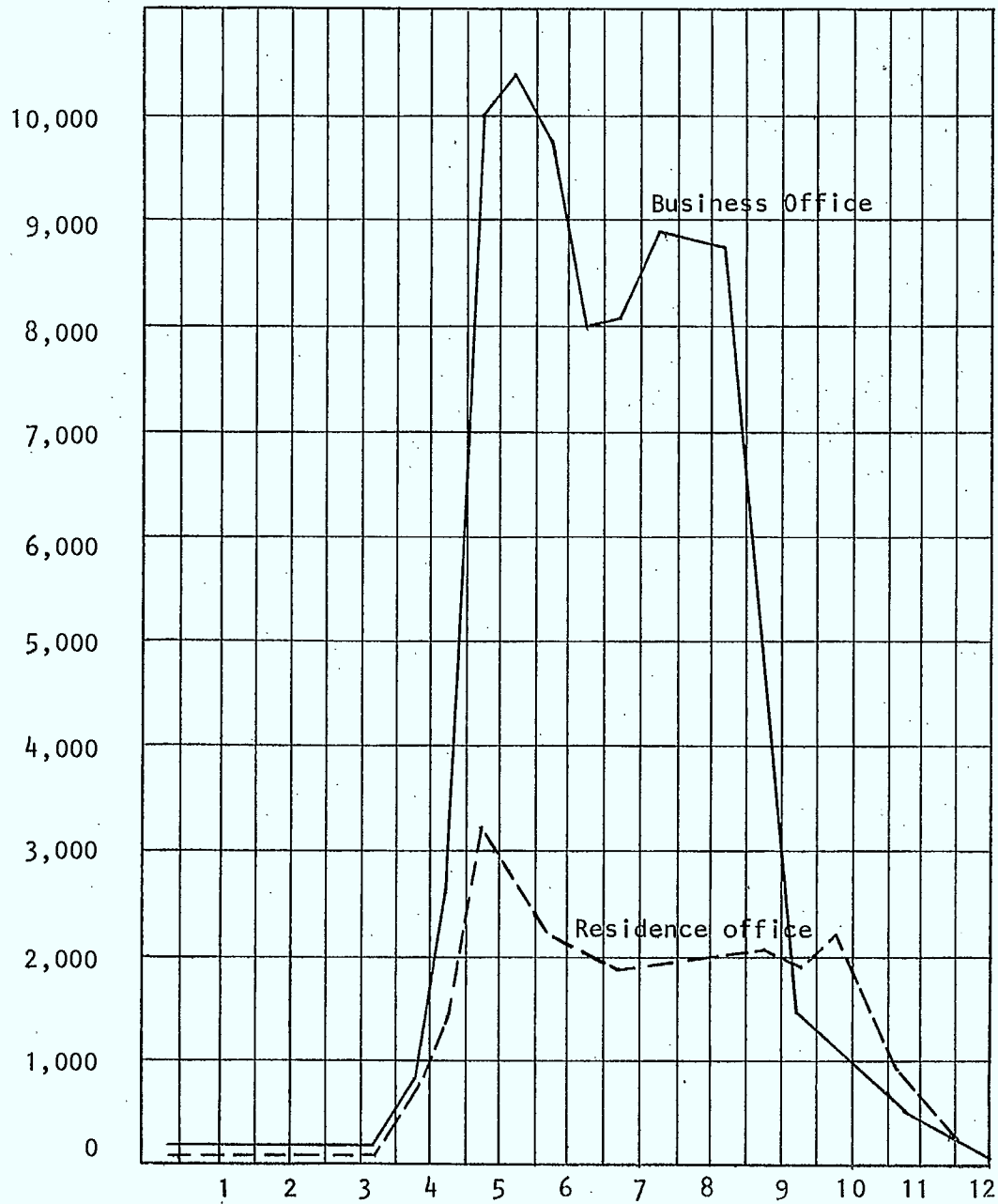
For traffic profiles between time zones, the situation is more complex since for example the profiles of business traffic will be truncated due to the fact that offices do not open at the same hour in each time zone (hour here is measured relative to a reference hour such as GMT or if desired, the hour of any time zone: in this study PST-Pacific standard time has been adopted as the reference hour). Accordingly, the principles described

TABLEAU 2.1.1.A

NOMBRE DE MESSAGES A LA POINTE

		1	2	3	4	5	6	7	8	9	10	11	12
CRAN	1	0.0	0.1	0.2	2.2	2.9	9.9	0.3	0.3	0.3	0.1	0.0	0.1
KAML	2	0.1	0.0	0.1	19.6	6.1	8.1	0.4	0.5	0.5	0.2	0.0	0.2
NELS	3	0.2	0.1	0.0	2.2	1.3	4.5	0.2	0.2	0.2	0.0	0.0	0.1
VANC	4	2.0	18.1	2.0	0.0	132.3	101.2	13.1	13.4	16.5	6.0	0.8	8.0
EDMO	5	2.6	5.6	1.6	132.2	0.0	662.5	37.6	57.6	34.4	8.8	1.2	11.7
CALG	6	8.9	7.4	4.0	161.4	664.1	0.0	36.0	45.3	30.5	7.9	1.1	10.3
REGI	7	0.2	0.4	0.1	13.5	33.9	37.1	0.0	41.2	26.8	3.1	0.4	3.8
SASK	8	0.3	0.5	0.2	13.9	59.8	46.9	41.3	0.0	15.2	2.4	0.3	3.8
WINN	9	0.2	0.5	0.1	16.7	35.0	30.9	26.3	14.9	0.0	10.8	1.5	12.7
LOND	10	0.1	0.2	0.0	6.1	8.9	8.0	3.1	2.3	10.8	0.0	9.2	71.2
NORT	11	0.0	0.0	0.0	0.8	1.3	1.1	0.4	0.3	1.6	9.7	0.0	22.0
OTTA	12	0.1	0.2	0.1	8.0	11.8	10.4	3.8	3.0	12.6	70.5	20.9	0.0
SUDB	13	0.0	0.1	0.0	2.0	3.1	2.7	1.1	0.8	4.1	21.7	18.4	29.2
THUN	14	0.0	0.1	0.0	3.6	6.3	5.5	2.7	1.9	16.0	8.3	1.3	8.8
TORO	15	0.3	0.7	0.2	27.2	39.9	35.5	13.3	10.2	46.5	1273.0	73.2	617.6
MONT	16	0.5	1.2	0.3	47.2	68.9	60.0	21.2	16.9	69.1	291.8	67.8	3925.0
QUEB	17	0.0	0.1	0.0	3.8	5.6	4.8	1.7	1.3	5.3	15.6	3.5	91.2
RIMO	18	0.0	0.0	0.0	0.3	0.4	0.4	0.1	0.1	0.4	0.8	0.2	3.1
SHER	19	0.0	0.0	0.0	1.3	1.8	1.6	0.5	0.4	1.7	6.3	1.3	44.3
NEWC	20	0.0	0.0	0.0	0.0	0.1	0.0	0.0	0.0	0.0	0.1	0.0	0.3
SAIN	21	0.0	0.0	0.0	1.2	1.7	1.5	0.5	0.4	1.4	3.5	0.6	11.6
HALI	22	0.0	0.0	0.0	1.7	2.4	2.1	0.7	0.5	1.9	4.3	0.7	12.2
CORN	23	0.0	0.0	0.0	0.2	0.3	0.3	0.1	0.1	0.2	0.4	0.1	0.9
ST.	24	0.0	0.0	0.0	0.9	1.2	1.1	0.3	0.3	0.9	1.3	0.2	3.0
		13	14	15	16	17	18	19	20	21	22	23	24
CRAN	1	0.0	0.0	0.3	0.6	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
KAML	2	0.1	0.1	0.8	1.4	0.1	0.0	0.0	0.0	0.0	0.0	0.0	0.0
NELS	3	0.0	0.0	0.2	0.4	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
VANC	4	1.9	3.5	27.6	48.7	3.7	0.3	1.2	0.0	1.2	1.6	0.2	0.9
EDMO	5	2.9	6.0	40.5	71.0	5.4	0.4	1.7	0.0	1.6	2.3	0.3	1.2
CALG	6	2.6	5.3	36.1	62.1	4.7	0.3	1.5	0.0	1.4	2.0	0.2	1.0
REGI	7	1.1	2.7	14.0	22.7	1.7	0.1	0.5	0.0	0.5	0.7	0.1	0.3
SASK	8	0.8	1.9	10.8	18.1	1.4	0.1	0.4	0.0	0.4	0.5	0.1	0.3
WINN	9	4.0	15.6	48.0	72.5	5.2	0.3	1.7	0.0	1.4	1.9	0.2	0.9
LOND	10	21.2	8.1	1318.1	307.0	15.5	0.8	6.1	0.1	3.4	4.2	0.3	1.3
NORT	11	18.7	1.3	79.3	74.7	3.7	0.2	1.3	0.0	0.6	0.7	0.1	0.2
OTTA	12	28.1	8.5	633.7	4092.2	89.9	2.9	42.6	0.3	11.2	11.8	0.8	2.9
SUDB	13	0.0	3.9	139.1	115.8	6.3	0.3	2.2	0.0	1.1	1.4	0.1	0.4
THUN	14	3.9	0.0	36.5	46.5	3.2	0.2	1.0	0.0	0.7	1.0	0.1	0.4
TORO	15	130.7	34.5	0.0	22433.8	100.0	4.5	41.1	0.5	19.9	23.5	1.6	6.7
MONT	16	107.2	43.2	2208.3	0.0	1225.1	26.4	983.3	2.7	104.2	100.6	6.1	20.9
QUEB	17	0.2	3.1	104.1	1296.0	0.0	5.9	56.7	0.4	14.1	11.8	0.7	2.2
RIMO	18	0.3	0.2	5.0	29.6	6.2	0.0	1.0	0.1	1.2	1.1	0.1	0.2
SHER	19	2.2	1.0	43.8	1066.6	58.1	1.0	0.0	0.1	4.3	3.7	0.2	0.7
NEWC	20	0.0	0.0	0.6	3.1	0.5	0.1	0.1	0.0	0.5	0.4	0.0	0.0
SAIN	21	1.2	0.8	21.2	112.7	14.4	1.2	4.3	0.5	0.0	27.2	0.4	1.3
HALI	22	1.4	1.0	24.8	107.8	11.9	1.1	3.7	0.4	27.0	0.0	0.8	2.5
CORN	23	0.1	0.1	2.0	6.9	0.7	0.1	0.2	0.0	0.5	0.9	0.0	1.2
ST.	24	0.4	0.4	7.2	22.6	2.2	0.2	0.7	0.0	1.3	2.5	1.1	0.0
STOP		0											
*END													

TABLEAU 2.1.1 B

Hourly Variation in Residential  
and Business Calls

in the forementioned section 2.2.1 of the Interim Report of the Second Phase have been applied to the standard profile for 0 hours time differential to obtain standard profiles for .5, 1.5, 2, 2.5, 3, 3.5, 4 and 4.5 hours time differential in each calling direction (east to west and west to east).

These principles provide the following result for the sample case of a 2 hour time differential and a west-to-east call direction.

The standard profile for a 2 hour time differential is assumed to be the same as that for a 0 hour time differential for the period midnight to 3 P.M.. At 3 P.M. reference time at point of origin, local time in the place of destination will be 5 P.M. and hence for the period 3 P.M. to 5 P.M. at the origin, fewer than normal business calls can be placed due to closing of offices at the place of destination. For this two hour interval the level of the (2 hour time differential) profile is assumed to drop to the average 5 P.M. to 7 P.M. level on the standard (0 hour time differential) profile. That is, afternoon peaks are partially truncated due to closing of offices in destinating areas. Finally, the 2 hour time differential profile is assumed to be equivalent to the zero hour time differential profile for the period 5 P.M. to midnight.

Similar derivations are applied for each of the possible time differentials.

These methods give results which are roughly comparable to certain statistics dealing with Canada-overseas traffic (where the time zone problem also occurs) and hence is satisfactory for this phase but should of course be replaced by precise data when these become available.

Finally, we assume that the standard profile for a particular time differential holds, no matter where the time differential occurs. In other words, the standard profile for Edmonton-Winnipeg traffic (1 hour time differential) would be identical to that for Winnipeg-Toronto traffic (1 hour time differential). The absolute number of messages in each case is obtained by multiplication of the profile level with the factor  $q T_{ij}/10$ .

Finally, in order to derive a matrix of point-to-point traffic for any reference hour  $t$  we calculate the local time  $t'$  at the place of origin and enter into the traffic matrix for time  $t$ , the number of messages obtained from the relevant profile level multiplied by  $q T_{ij}/10$ . (The relevant profile is identified by the direction of call and by the relative time differential between origin and destination).

## D) Estimation of traffic in call seconds

Finally, in order to convert traffic measured in number of messages into traffic measured in call seconds we multiply by an average holding time, which may vary according to the mileage band. The average holding time may vary, of course, with many other factors including for example the type of message (DDD; Station to Station, Operator Handled; Person to Person) but since our traffic estimates are totals, not broken down into type of message, we use the same average holding time by mileage band, for all types of messages.

Data on average holding times by mileage band were obtained from Bell Canada's submission to the CTC in response to a question from the Province of Quebec (P (Q) 14 décembre, 738-71).

These are shown in Tableau 2.2.1C.

TABLEAU 2.1.1C

Average Holding Time per Call  
by Mileage Band

Mileage Band	Average Holding Time (Minutes)
0-10	3.56
11-14	2.86
15-20	3.32
23-30	3.62
31-40	3.86
41-50	4.11
51-60	4.33
61-80	4.67
81-100	4.90
101-130	5.10
131-160	5.08
161-200	5.71
201-250	5.71
251-300	6.37
301-400	5.92
401-500	6.34
501 and over	6.47

From this figure one can observe that average holding time is apparently an increasing function of distance.

The multiplication of average holding times in ccs by peak numbers of messages gives a peak traffic matrix in ccs which is shown in Tableau 2.1.1D.



NOMBRE DE CCS A LA POINTE

		1	2	3	4	5	6	7	8	9	10	11	12
CRAN	1	0.0	0.1	0.2	3.1	4.1	14.1	0.4	0.4	0.4	0.1	0.0	0.1
KARL	2	0.1	0.0	0.2	28.0	8.7	11.5	0.6	0.7	0.7	0.2	0.0	0.3
NELS	3	0.2	0.2	0.0	3.2	2.6	6.4	0.2	0.2	0.2	0.1	0.0	0.1
VANC	4	2.8	25.7	2.8	0.0	188.5	229.7	18.7	19.1	23.5	8.6	1.1	11.3
EDMO	5	3.7	8.0	2.3	188.4	0.0	944.2	53.5	82.2	49.0	12.5	1.7	16.7
CALG	6	12.7	10.6	5.7	230.1	946.6	0.0	51.3	64.5	43.4	11.3	1.5	14.6
REGI	7	0.4	0.6	0.2	19.3	55.4	52.9	0.0	58.7	38.2	4.4	0.6	5.5
SASK	8	0.4	0.7	0.2	19.8	85.3	66.8	58.8	0.0	21.7	3.4	0.5	4.3
WINN	9	0.3	0.7	0.2	23.8	49.9	44.1	37.5	21.3	0.0	15.3	2.1	18.1
LOND	10	0.1	0.2	0.1	8.7	12.7	11.5	4.3	3.3	15.4	0.0	13.2	101.5
NORT	11	0.0	0.0	0.0	1.2	1.8	1.6	0.6	0.5	2.3	13.8	0.0	3.4
OTTA	12	0.1	0.3	0.1	11.5	16.9	14.8	5.3	4.2	18.0	100.5	29.7	0.0
SUDB	13	0.0	0.1	0.0	2.9	4.4	3.9	1.5	1.2	5.8	31.0	26.2	41.6
THUN	14	0.1	0.1	0.0	5.2	8.9	7.9	3.9	2.7	22.8	11.8	1.8	12.5
TORO	15	0.4	1.0	0.3	38.8	56.8	50.6	19.0	14.6	66.3	181.4	104.3	880.3
MONT	16	0.7	1.7	0.5	67.3	98.2	85.6	30.3	24.1	98.5	415.9	96.7	5594.2
QUEB	17	0.1	0.1	0.0	5.4	7.9	6.8	2.4	1.9	7.5	22.2	5.0	130.0
RIMO	18	0.0	0.0	0.0	0.4	0.6	0.5	0.2	0.1	0.5	1.2	0.2	4.4
SHER	19	0.0	0.0	0.0	1.8	2.6	2.2	0.8	0.6	2.5	9.0	1.8	63.1
NEWC	20	0.0	0.0	0.0	0.1	0.1	0.1	0.0	0.0	0.1	0.1	0.0	0.5
SAIN	21	0.0	0.0	0.0	1.7	2.4	2.1	0.7	0.6	2.1	5.0	0.9	16.5
HALI	22	0.0	0.1	0.0	2.4	3.4	2.9	0.9	0.8	2.7	6.1	1.0	17.3
CORN	23	0.0	0.0	0.0	0.3	0.4	0.4	0.1	0.1	0.3	0.5	0.1	1.2
ST.	24	0.0	0.0	0.0	1.3	1.8	1.5	0.5	0.4	1.2	1.9	0.3	4.2
		13	14	15	16	17	18	19	20	21	22	23	24
CRAN	1	0.0	0.1	0.5	0.8	0.1	0.0	0.0	0.0	0.0	0.0	0.0	0.0
KARL	2	0.1	0.1	1.1	1.9	0.1	0.0	0.0	0.0	0.0	0.1	0.0	0.0
NELS	3	0.0	0.0	0.3	0.6	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
VANC	4	2.7	5.0	39.4	69.4	5.3	0.4	1.7	0.0	1.6	2.3	0.3	1.2
EDMO	5	4.2	8.6	57.7	101.2	7.7	0.5	2.5	0.1	2.3	3.3	0.4	1.7
CALG	6	3.7	7.6	51.5	88.5	6.7	0.5	2.1	0.1	2.0	2.8	0.3	1.4
REGI	7	1.5	3.8	19.9	32.3	2.4	0.2	0.8	0.0	0.7	0.9	0.1	0.5
SASK	8	1.2	2.7	15.3	25.8	1.9	0.1	0.6	0.0	0.6	0.8	0.1	0.4
WINN	9	5.6	22.3	68.4	103.3	7.5	0.5	2.4	0.1	2.0	2.7	0.3	1.2
LOND	10	30.2	11.5	1878.6	437.6	22.1	1.1	8.7	0.1	4.9	6.0	0.5	1.8
NORT	11	26.7	1.9	113.0	106.4	5.2	0.2	1.9	0.0	0.9	1.0	0.1	0.3
OTTA	12	40.1	12.2	903.2	5832.4	128.1	4.1	60.7	0.4	15.9	16.9	1.2	4.1
SUDB	13	0.0	5.6	198.2	165.1	9.0	0.4	3.1	0.0	1.6	1.9	0.2	0.6
THUN	14	5.5	0.0	52.0	66.3	4.5	0.3	1.5	0.0	1.1	1.4	0.1	0.5
TORO	15	186.3	49.2	0.0	3198.0	142.5	6.4	58.5	0.8	28.3	33.5	2.5	9.6
MONT	16	152.7	61.6	3147.3	0.0	1746.0	37.7	1401.5	3.8	148.5	143.4	8.7	29.8
QUEB	17	8.8	4.4	148.4	1847.1	0.0	8.4	80.8	0.6	20.0	16.8	1.0	3.1
RIMO	18	0.5	0.3	7.1	42.2	8.9	0.0	1.4	0.1	1.8	1.6	0.1	0.3
SHER	19	3.1	1.5	62.5	1520.2	82.8	1.4	0.0	0.1	6.2	5.3	0.3	0.9
NEWC	20	0.1	0.0	0.9	4.5	0.7	0.1	0.2	0.0	0.8	0.6	0.0	0.1
SAIN	21	1.6	1.1	30.2	160.7	20.5	1.7	6.1	0.7	0.0	38.8	0.6	1.8
HALI	22	1.9	1.4	35.4	153.7	17.0	1.5	5.3	0.5	38.4	0.0	1.2	3.6
CORN	23	0.2	0.1	2.8	9.8	1.1	0.1	0.3	0.0	0.6	1.2	0.0	1.6
ST.	24	0.6	0.6	10.2	32.2	3.2	0.3	0.9	0.1	1.8	3.6	1.6	0.0
STOP		0											
*END													
*GO													

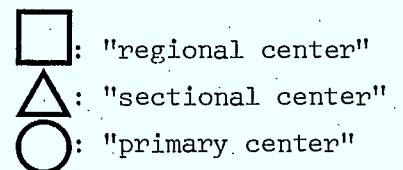
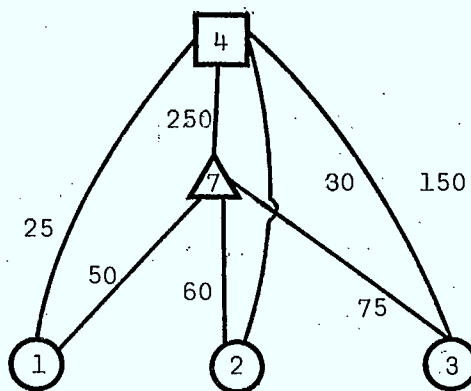
## 2.1.2 Exemple d'affectation optimale des circuits sur le réseau de transmission

### 2.1.2.1 Enoncé du problème

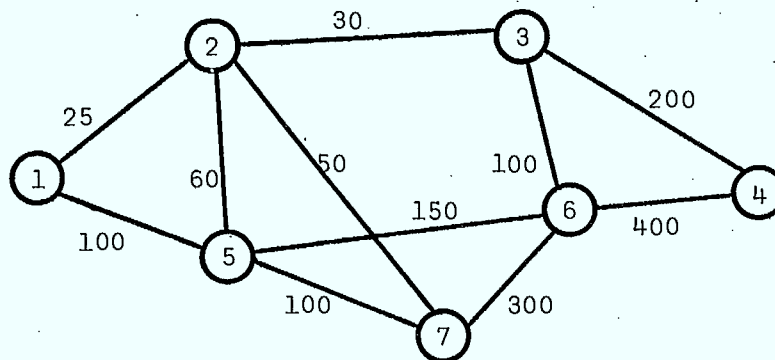
Cette section a pour objet d'explicitier à l'aide d'exemples fictifs la solution du problème d'affectation, telle que décrite dans la section 2.3 du Rapport Intérimaire de septembre 1974 et que nous allons brièvement rappeler ici.

Pour bien situer le problème il faut d'abord distinguer le réseau de commutation ("Switching Network") du réseau de transmission ("Transmission Network"). Voici un exemple de commutation et du réseau de transmission qui lui est associé:

Réseau de  
commutation



Réseau  
physique



Cet exemple montre que les sommets ⑤ et ⑥ du réseau physique ne sont pas des centres d'où originent des besoins mais seulement des sommets de relais pour les divers biens physiques. Les arêtes du réseau de commutation ont certaines exigences de trafic en nombre de circuits à satisfaire ("circuit requirements") et les arêtes du réseau physique ont des capacités limitées en nombre de circuits; ce sont ces nombres qui apparaissent respectivement le long des arêtes sur les deux réseaux. Le problème d'estimer les besoins en circuits sur le réseau de commutation a fait l'objet d'un algorithme tel qu'expliqué en section 2.2 du Rapport Intérimaire du 15 septembre 1974. Le problème qui se pose ensuite est celui de déterminer quelles chaînes du réseau physique seront utilisées pour satisfaire les besoins calculés sur le réseau de commutation; quelles chaînes seront utilisées et combien de circuits chacune portera de telle sorte que les capacités des arêtes du réseau physique ne soient pas dépassées, que les exigences du réseau de commutation soient satisfaites et qu'un objectif fixé soit optimisé (soit par exemple de maximiser la somme des surcapacités (capacités en réserve) sur les arêtes sur réseau physique)? Tel est le problème d'affectation dans sa forme la plus simple.

Nous verrons plus loin comment tenir compte de nouvelles contraintes et/ou objectifs; mais nous y arriverons en compliquant progressivement l'exemple simple déjà énoncé.

Si l'on se donnait la peine d'énumérer toutes les chaînes possibles du réseau physique pouvant satisfaire les exigences d'une arête du réseau de commutation, nous aurions une matrice de la forme suivante:

(Toutes les colonnes n'ont pas été écrites car il y en aurait environ 75).

arêtes sur le réseau de commutation	$[\textcircled{1}, \boxed{4}]$	$[\textcircled{1}, \triangle 7]$	.....	$[\boxed{4}, \triangle 7]$	
arêtes sur le réseau physique	$x_1' x_2' \dots x_m'$		.....	$x_1' x_2' \dots x_m'$	
$\langle 1, 2 \rangle$	11 .... 0			00 .... 1	$\leq 25$
$\langle 1, 5 \rangle$	00 .... 1			00 .... 1	$\leq 100$
$\langle 2, 3 \rangle$	11 .... 0			00 .... 1	$\leq 30$
$\langle 2, 5 \rangle$	00 .... 1			00 .... 0	$\leq 60$
$\langle 2, 7 \rangle$	00 .... 1	...	...	00 .... 0	$\leq 50$
$\langle 3, 4 \rangle$	10 .... 1			00 .... 1	$\leq 200$
$\langle 3, 6 \rangle$	01 .... 1			00 .... 0	$\leq 100$
$\langle 4, 6 \rangle$	01 .... 0			11 .... 0	$\leq 400$
$\langle 5, 6 \rangle$	00 .... 0			01 .... 1	$\leq 150$
$\langle 5, 7 \rangle$	00 .... 0			01 .... 0	$\leq 100$
$\langle 6, 7 \rangle$	00 .... 1		.....	10 .... 1	$\leq 300$
$[\textcircled{1}, \boxed{4}]$	11 .... 1	0 .... 0		0 .... 0	$\geq 25$
$[\textcircled{1}, \triangle 7]$	0 ..... 0	11-----1		0 ..... 0	$\geq 50$
$[\textcircled{2}, \boxed{4}]$	.....				$\geq 30$
$[\textcircled{2}, \triangle 7]$	.....				$\geq 60$
$[\textcircled{3}, \boxed{4}]$	.....				$\geq 150$
$[\textcircled{3}, \triangle 7]$	.....				$\geq 75$
$[\boxed{4}, \triangle 7]$	0 ..... 0	0 ..... 0	.....	11 .... 1	$\geq 250$

Les colonnes représentent des chaînes; par exemple la première colonne décrit une chaîne possible pouvant satisfaire les exigences de  $\boxed{1}, \boxed{4}$  sur le réseau de commutation (voilà le sens du 1 en douzième ligne); elle emprunterait les arêtes  $\langle 1,2 \rangle$ ,  $\langle 2,3 \rangle$ ,  $\langle 3,4 \rangle$  sur le réseau physique, on notera donc le chiffre 1 sur les lignes correspondantes 1, 3 et 6 et zéro sur les autres lignes.

A chaque chaîne  $i$  est associée une variable  $x_i^k$  qui représente le nombre de circuits portés par la chaîne  $i$  pour satisfaire aux besoins de l'arête  $k$  du réseau de commutation.

Les lignes nous permettent de lire les contraintes du problème. La première ligne, par exemple, contient des 1 pour les chaînes qui utilisent l'arête  $\langle 1,2 \rangle$  du réseau physique. La première ligne nous indique donc que les circuits affectés sur l'arête  $\langle 1,2 \rangle$  (c'est la somme des circuits pour toutes les chaînes qui utilisent cette arête) ne doivent pas dépasser 25 circuits. Les lignes 12 à 18 ont trait aux exigences du réseau de commutation; par exemple la dix-huitième ligne indique que toutes les chaînes qui desservent l'arête  $\boxed{4}, \triangle 7$  du réseau de commutation doivent porter, au total, au moins 250 circuits.

#### 2.1.2.2 Maximiser la somme des surcapacités sous contraintes

Avec un réseau physique de 7 sommets on arrive à plus de 70 chaînes; parmi ces chaînes nous allons chercher celles qui, tout en satisfaisant les contraintes ci-haut mentionnées, seront telles que la somme des surcapacités (i.e. la somme des circuits non utilisés) de chaque arête soit maximisée. Comme on l'a déjà expliqué dans le Rapport Intérimaire de septembre 1974 (section 2.3.3), il n'est pas nécessaire d'énumérer à priori toutes les chaînes possibles avant de choisir parmi ces dernières celles qui maximiseront la somme des surcapacités (cette remarque prend tout son sens lorsque le nombre de sommets dépasse 20 ou 50 car alors le nombre de chaînes possibles peut aller jusqu'au million). C'est donc l'algorithme tel qu'expliqué à la section 2.3.3 du Rapport intérimaire dont nous nous sommes servis afin de trouver les chaînes optimales (au sens de l'objectif déjà mentionné).



On obtient la solution suivante:

	[①, ④]	[①, ⑦]	[②, ④]	[②, ⑦]	[③, ④]	[③, ⑦]	[④, ⑦]
<1,2>	0	0	0	0 0	0	0 0	0
<1,5>	1	1	0	0 0	0	0 0	0
<2,3>	0	0	1	0 0	0	0 0	0
<2,5>	0	0	0	0 1	0	0 0	0
<2,7>	0	0	0	1 0	0	0 0	0
<3,4>	0	0	1	0 0	1	0 0	0
<3,6>	0	0	0	0 0	0	1 1	0
<4,6>	1	0	0	0 0	0	0 0	1
<5,6>	1	0	0	0 0	0	0 1	0
<5,7>	0	1	0	0 1	0	0 1	0
<6,7>	0	0	0	0 0	0	1 0	1
# de circuits portés:	25	50	30	<u>50 10</u> 60	150	50 25	250

On s'aperçoit que cette solution optimale est telle que le nombre de circuits disponibles en surcapacité est au total 385, se répartissant comme suit par arête (sur le réseau physique):

arête	<1,2>	<1,5>	<2,3>	<2,5>	<2,7>	<3,4>	<3,6>	<4,6>	<5,6>	<5,7>	<6,7>
surcapacité	25	25	0	50	0	20	25	125	100	15	0

Cette solution est optimale pour le critère choisi (à savoir maximiser la somme des surcapacités). Mais il serait intéressant d'ajouter d'autres contraintes (par exemple, de survivance) et de voir comment la solution optimale sera affectée; on remarque que dans la solution déjà obtenue il n'y a que l'arête  $\textcircled{2}, \triangle$  pour laquelle il y a au moins deux chaînes disjointes au sens des sommets, satisfaisant ainsi nos critères de survivance.

Le traitement des circuits non commutés ne pose aucune difficulté spéciale, à moins que leur nombre soit excessif, car cela ne fait que rajouter des couples Origine-Destination sur le réseau de transmission avec un ensemble de chaînes possibles parmi lesquelles il faut aussi chercher la plus courte au sens des prix d'ordre (cf. Rapport Interiminaire, sept. 1974, page 2-29). Toutefois, pour les couples Origine-Destination des circuits non commutés identiques aux couples adjacents du réseau de commutation, il faut agréger la demande de circuits à celle de la demande commutée afin d'éviter d'avoir deux contraintes dont une redondante. Dans les calculs de coûts d'utilisation, il faudra séparer à nouveau les deux types de demande.

#### 2.1.2.3 Même objectif, mais en rajoutant des contraintes de survivance

Mais si nous voulons être plus exigeants, nous pouvons imposer que certaines arêtes du réseau de commutations soient assurées d'être desservies que par deux chaînes disjointes au sens des sommets, portant chacune une fraction arbitraire du trafic; c'est là une définition restreinte du concept de survivance mais qui peut être modifiée selon les exigences des utilisateurs du modèle. D'abord, il nous faut trouver des

chaînes disjointes au sens des sommets; un algorithme permettant l'identification du nombre maximal de chaînes disjointes entre chaque couple de sommets du réseau physique est exposé ailleurs dans ce rapport (section 2.1.3); ici nous nous contenterons de deux chaînes disjointes quelconques (mais l'algorithme permet de trouver les plus courtes en termes de distance ou en termes de poids associés aux arêtes).

Supposons, par exemple, que l'on impose a priori

- a) pour  $[\square, \triangle]$ , que 80 circuits passent par  $\langle 4,6 \rangle$ ,  $\langle 6,7 \rangle$ ,  
et que 20 circuits passent par  $\langle 4,3 \rangle$ ,  $\langle 3,2 \rangle$ ,  $\langle 2,7 \rangle$ ;
- b) pour  $[\square, \textcircled{3}]$ , que 20 circuits passent par  $\langle 4,3 \rangle$ ,  
et que 30 circuits passent par  $\langle 4,6 \rangle$ ,  $\langle 6,3 \rangle$ .

Donc, sur le réseau de commutation les arêtes  $[\square, \triangle]$  et  $[\square, \textcircled{3}]$  seront diminuées de 100 et 50 circuits respectivement.

Et sur le réseau physique les arêtes  $\langle 4,3 \rangle$ ,  $\langle 3,2 \rangle$ ,  $\langle 2,7 \rangle$ ,  $\langle 4,6 \rangle$ ,  $\langle 6,7 \rangle$ ,  $\langle 6,3 \rangle$  seront diminuées respectivement de  $(20+20)$ , 20, 20,  $(80+30)$ , 80, 30, respectivement. Après avoir enlevé ces circuits nous repassons l'algorithme d'optimisation (avec le même critère, soit de maximiser la somme des surcapacités) et nous obtenons la solution suivante:

	①, ④	①, ⑤	②, ④	②, ⑤	③, ④	③, ⑤	④, ⑤	survivance ④, ⑤ ③, ④			
<1,2>	0	0	0 0	0 0	0	0 0	0	0	0	0	0
<1,5>	1	1	0 0	0 0	0	0 0	0	0	0	0	0
<2,3>	0	0	0 1	0 0	0	0 1	0	0	1	0	0
<2,5>	0	0	1 0	0 1	0	0 0	0	0	0	0	0
<2,7>	0	0	0 0	1 0	0	0 1	0	0	1	0	0
<3,4>	0	0	0 1	0 0	1	0 0	0	0	1	1	0
<3,6>	0	0	0 0	0 0	0	1 0	0	0	0	0	1
<4,6>	1	0	1 0	0 0	0	0 0	1	1	0	0	1
<5,6>	1	0	1 0	0 0	0	0 0	0	0	0	0	0
<5,7>	0	1	0 0	0 1	0	0 0	0	0	0	0	0
<6,7>	0	0	0 0	0 0	0	1 0	1	1	0	0	0
# de circuits portés:	25	50	25 5	25 35	100	70 5	150	80	20	20	30

Remarquons que pour avoir la solution finale il faut rajouter à cet ensemble de chaînes celles qui nous ont servis à assurer une certaine survivance; nous les avons placées à l'extrême droite du tableau. Cette solution nous donne une surcapacité totale de 310 circuits, se répartissant comme suit:

arête	<1,2>	<1,5>	<2,3>	<2,5>	<2,7>	<3,4>	<3,6>	<4,6>	<5,6>	<5,7>	<6,7>
surcapacité	25	25	0	0	0	55	0	90	100	15	0

Le lecteur pourra comparer ce dernier tableau et le précédent avec ceux que nous avons obtenus sans les contraintes de survivance. A remarquer que ces contraintes de survivance, leur nombre, ainsi que leur importance (en nombre de circuits) sont arbitraires et déterminées par l'utilisateur.

2.1.2.4 Maximiser la somme des surcapacités pondérées

Jusqu'ici notre objectif était de maximiser la somme des surcapacités; nous pouvons aisément faire un pas de plus et maximiser la somme des surcapacités, pondérées selon des objectifs prédéterminés par l'utilisateur ou simplement pour tenir compte du fait qu'un circuit de 40 milles n'aura pas le même poids pour nos objectifs qu'un circuit de 150 milles. Par exemple l'utilisateur peut juger que certaines arêtes du réseau physique devraient être moins achalandées que d'autres; pour ce faire, il n'y a qu'à donner un poids plus grand à ces arêtes dans la fonction objectif. Exemple: supposons que les surcapacités des arêtes <1,2> <1,5> <2,3> <2,5> <2,7> <3,4> <3,6> <4,6> <5,6> <5,7> <6,7> reçoivent respectivement les pondérations 1, 3, 1, 1, 2, 1, 1, 3, 1, 2, 3; cet exemple cherche à décongestionner (dans la mesure où c'est possible) les arêtes <1,5> <4,6> <6,7> davantage que les autres. La solution obtenue est alors:

	①, ④		①, ⑦		②, ④		②, ⑦		③, ④		③, ⑦		④, ⑦	
<1,2>	1	1	0		0		0	0	0		0	0	0	
<1,5>	0	0	1		0		0	0	0		0	0	0	
<2,3>	0	0	0		1		0	0	0		0	0	0	
<2,5>	1	1	0		0		0	1	0		0	0	0	
<2,7>	0	0	0		0		1	0	0		0	0	0	
<3,4>	1	0	0		1		0	0	1		0	0	0	
<3,6>	1	0	0		0		0	0	0		1	1	0	
<4,6>	0	1	0		0		0	0	0		0	0	1	
<5,6>	1	1	0		0		0	0	0		0	1	0	
<5,7>	0	0	1		0		0	1	0		0	1	0	
<6,7>	0	0	0		0		0	0	0		1	0	1	
# de circuits portés:	20	5	50		30		50	10	150		50	25	250	

On pourra comparer ce tableau avec le premier de cette section et on remarquera qu'effectivement le routage a été légèrement modifié pour dégager davantage les arêtes <1,5> et <4,6>.

Cet objectif, à savoir: maximiser la somme des surcapacités pondérées, est formellement équivalent à un autre objectif déjà mentionné dans le Rapport Intérimaire, à savoir minimiser, pour l'ensemble des arêtes de transmission, la moyenne harmonique H des coûts par mille de circuit.

Soit  $c_i$  le coût sur l'arête i (coût annuel total de l'équipement installé sur i par exemple),

$x_i$  la somme des circuits affectés sur i,

$m_i$  la longueur de i,

$u_i$  la capacité de i,

$S_i$  la surcapacité,

$$H = \frac{1}{\frac{1}{n} \left[ \frac{1}{c_1/x_1 m_1} + \frac{1}{c_2/x_2 m_2} + \dots + \frac{1}{c_n/x_n m_n} \right]}$$

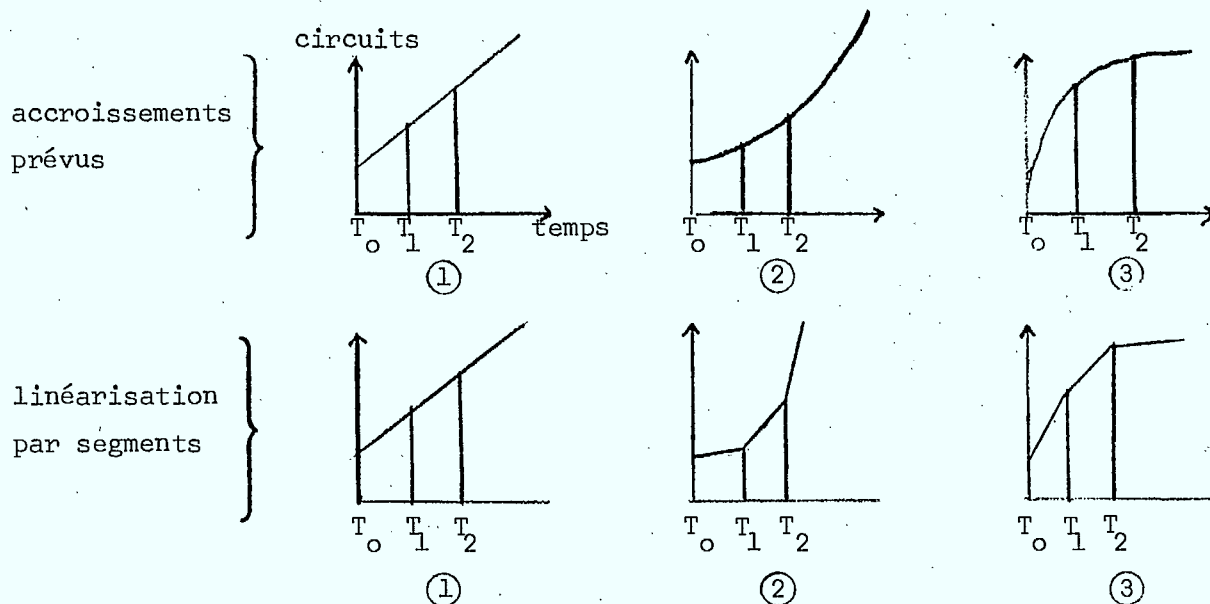
mais minimiser H est équivalent à maximiser

$$\frac{n}{H} = \sum_i \frac{x_i m_i}{c_i} = \sum_i \frac{(u_i - S_i) m_i}{c_i} = \sum_i \frac{u_i m_i}{c_i} - \sum_i \frac{S_i m_i}{c_i} = \text{constante} - \sum_i \alpha_i S_i$$

avec  $\alpha_i = \frac{m_i}{c_i}$ , donc minimiser H revient à maximiser  $\sum_i \alpha_i S_i$ .

#### 2.1.2.5 Maximiser l'intervalle de temps à partir duquel le réseau physique (avec ses capacités limitées) pourra satisfaire les besoins actuels et les accroissements prévus

Ces accroissements sur le réseau de commutation et pour les lignes privées seront des projections que l'utilisateur pourra obtenir par des méthodes de son choix. Une fois ces projections faites on veut envisager surtout trois possibilités: des fonctions d'accroissement linéaires, convexes, concaves. Etant donné que les algorithmes à notre disposition ne peuvent bien traiter que des fonctions linéaires, il nous faudra linéariser par segments. Exemples:



Prenons d'abord un cas simple, celui où une seule arête du réseau de commutation connaît un accroissement linéaire, soit par exemple l'arête  $[(1), [4]]$  du réseau de commutation avec un accroissement d'un circuit par unité de temps. L'algorithme nous donne alors comme solution que l'accroissement peut se poursuivre jusqu'à la cinquantième unité de temps, donc un accroissement possible de 50 circuits. On remarquera sur le réseau physique que seulement 125 circuits peuvent partir de ① ou y arriver; comme il y avait déjà 75 circuits requis (25 pour  $[(1), [4]]$  et 50 pour  $[(1), \Delta]$ ) il ne restait de la place que pour 50 circuits en réserve ( $50 = 125 - 75$ ) et c'est le maximum obtenu.

Nous pouvons compliquer un peu plus cet exemple et considérer des accroissements sur toutes les arêtes du réseau de commutation, en prenant une fonction convexe avec linéarisation par segment. Supposons que l'accroissement prévu soit de 0.5 circuit par unité de temps entre  $T_0$  et  $T_1$ , de 1 circuit par unité de temps entre  $T_1$  et  $T_2$ , et de 2 circuits par unité de temps entre  $T_2$  et  $T_3$ . Il nous faut prendre 3 variables ( $\Delta_1, \Delta_2, \Delta_3$ ) bornées comme suit:

$$0 \leq \Delta_1 \leq T_1 - T_0 = T_1 = 1$$

$$0 \leq \Delta_2 \leq T_2 - T_1 = 2 - 1 = 1$$

$$0 \leq \Delta_3 \leq T_3 - T_2 = 1' \text{infini}$$



Chaque membre droit des contraintes de besoin de circuits devient:

$$(\text{besoin initial}) + .5\Delta_1 + 1\Delta_2 + 2\Delta_3$$

Comme les pentes vont en s'accroissant (0.5, 1, 2) et que l'on veut maximiser  $\sum_{i=1}^3 \Delta_i$ , il est assuré que les variables  $\Delta_1$ ,  $\Delta_2$  et  $\Delta_3$  entreront l'une après l'autre dans la solution.

La solution est que chaque arête du réseau de commutation peut augmenter ses besoins de 3.75 circuits, car dans la solution

$$\Delta_1 = 1, \Delta_2 = 1, \Delta_3 = 1.1$$

d'où

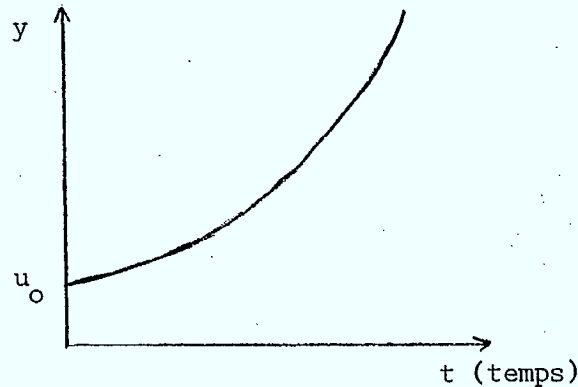
$$.5\Delta_1 + \Delta_2 + 2\Delta_3 = 3.75 \text{ circuits.}$$

Ce problème supposait les mêmes pentes d'accroissement pour chacune des arêtes du réseau de commutation, rien n'empêche d'avoir des pentes différentes.

	[①, ④]	[①, ⑤]	[②, ④]	[②, ⑤]		[③, ④]		[③, ⑤]		[④, ⑤]
<1,2>	0	0	0	0	0	0	0	0	0	0
<1,5>	1	1	0	0	0	0	0	0	0	0
<2,3>	0	0	0	0	0	0	0	0	0	0
<2,5>	0	0	1	0	1	0	0	0	0	0
<2,7>	0	0	0	1	0	0	0	0	0	0
<3,4>	0	0	0	0	0	1	0	0	0	0
<3,6>	0	0	0	0	0	0	1	1	1	0
<4,6>	1	0	1	0	0	0	1	0	0	1
<5,6>	1	0	1	0	0	0	0	0	1	0
<5,7>	0	1	0	0	1	0	0	0	1	0
<6,7>	0	0	0	0	0	0	0	1	0	1
# de circuits portés:	28.75	53.75	33.75	50	13.75	132.5	21.25	46.25	32.5	253.75

### 2.1.2.6 Autre formulation de la maximisation de l'intervalle de temps critique

Soit la fonction d'accroissement suivante:

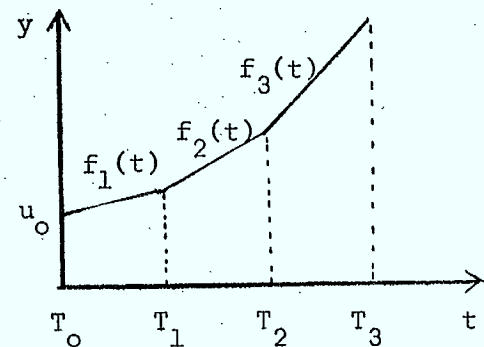


où  $y$  est mis pour le nombre de circuits dont on prévoit avoir besoin sur une arête du réseau de commutation.

On peut écrire cette fonction  $y = f(t)$

Les algorithmes que nous utilisons ne traitant que les cas où les fonctions sont linéaires, nous allons approximer cette fonction par 3 segments linéaires d'équations

$$\begin{aligned}
 \hat{y} &= f_1(t) = m_1 + b_1 t \text{ pour } T_0 \leq t < T_1 \\
 (1) \quad \hat{y} &= f_2(t) = m_2 + b_2 t \text{ pour } T_1 \leq t < T_2 \\
 \hat{y} &= f_3(t) = m_3 + b_3 t \text{ pour } T_2 \leq t \leq T_3
 \end{aligned}$$



Le système d'équations en (1) peut s'écrire de façon plus compacte en remarquant que  $T_i \leq t < T_{i+1}$  peut s'écrire  $t = \lambda T_i + (1-\lambda)T_{i+1}$  avec  $0 < \lambda \leq 1$ , ou encore  $t = \sum_{i=0}^3 \lambda_i T_i$  avec  $\sum_{i=0}^3 \lambda_i = 1$ ,  $\lambda_i \geq 0$  et seulement deux  $\lambda_i$  contigus et non nuls.

Alors on a la formule  $\hat{y} = \sum_{i=0}^3 \lambda_i f_{T_i}$  où  $f_{T_i} = f(T_i)$  et où  $t = \sum_{i=0}^3 \lambda_i T_i$ ,  $\lambda_i \geq 0$ ,  $\sum_{i=0}^3 \lambda_i = 1$  avec seulement deux lambdas contigus non nuls, les autres  $\lambda_i$  étant nuls.

Par exemple, pour  $T_1 \leq t \leq T_2$  on peut écrire  $t = \lambda T_1 + (1-\lambda)T_2$  avec  $0 \leq \lambda \leq 1$  et alors  $\hat{y} = \lambda f(T_1) + (1-\lambda)f(T_2) = \sum_{i=0}^3 \lambda_i f_{T_i}$  avec seulement deux  $\lambda_i$  contigus non nuls, i.e.  $\lambda_0 = 0$ ,  $\lambda_1 = \lambda$ ,  $\lambda_2 = 1-\lambda$ ,  $\lambda_3 = 0$ , avec  $0 \leq \lambda \leq 1$ .

Donc notre objectif consiste à trouver le couple de lambdas contigus non nuls tels qu'ils maximisent  $\sum_{i=0}^3 \lambda_i T_i$  avec des demandes (par exemple pour l'arête j) sous la forme  $\lambda_0 u_0 + \lambda_1 f_{T_1} + \lambda_2 f_{T_2} + \lambda_3 f_{T_3}$  où  $u_0$  est le besoin initial en circuits que l'on avait estimé sur le réseau de commutation pour l'arête j.

Remarquons finalement que lorsque la fonction est convexe, la contiguïté des lambdas nous semble automatique, du fait que, si on ne l'avait pas, on pourrait facilement montrer qu'il existe une meilleure solution. Cependant il faudrait aussi que les accroissements par fonctions convexes "dominent" ceux par fonctions concaves pour continuer à assurer la contiguïté. On peut néanmoins, en pratique, insérer l'exigence de contiguïté comme telle dans l'algorithme (comme on le fait pour certains algorithmes de programmation séparable).

2.1.2.7. Traitement simultané de la télévision et de la téléphonie dans les modèles d'affectation de circuits.

Rappelons que la télévision exige des blocs de circuits (1200 par exemple). Nous voulons, d'autre part, éviter la programmation mathématique mixte. La question est donc: pouvons-nous trouver une formulation mathématique du problème d'affectation des circuits qui garantisse cette indivisibilité bien qu'on utilise la programmation linéaire habituelle?

1. Une forme souvent suffisante serait de formuler le problème sous la forme arête-chaîne comme suit:

Soit  $D(j)$  l'ensemble des indices des chaînes joignant le sommet  $j$  à la source d'émission de la télévision,

soit  $Q$  l'ensemble des indices des sommets devant recevoir la télévision,

soit  $K$  l'ensemble des indices des arêtes susceptibles de porter des circuits de télévision,

soit  $R$  l'ensemble des indices des arêtes du réseau physique ( $K$  est un sous-ensemble de  $R$ ),

soit  $x_{ij}$  le nombre de circuits portés par la chaîne  $i$  de  $D(j)$ ,

soit une variable de Kronecker  $\delta_{ik}$  qui prend la valeur un si la chaîne  $i$  de  $D(j)$  a  $k$  pour arête et prend la valeur zéro dans l'autre cas,

soit  $d$  le nombre de circuits formant un bloc indivisible pour la télévision, en supposant que la capacité des arêtes susceptibles de porter cette télévision, est supérieure à  $d$ ,

on a:

$$(1) \quad \sum_{j \in Q} \sum_{i \in D(j)} \delta_{ik} x_{ij} \leq d \quad k \in K$$

$$(2) \quad \sum_{i \in D(j)} x_{ij} \geq d \quad j \in Q$$

Bien que respectant l'indivisibilité la plupart du temps, cette formulation nous voue à n'avoir que des chaînes disjointes au sens des sommets.

Ainsi, si S est la source et A, B sont les 2 sommets devant recevoir la télévision, le graphe de la figure 1 ne sera jamais choisi pour acheminer les circuits bien qu'il puisse améliorer peut-être la solution globale.

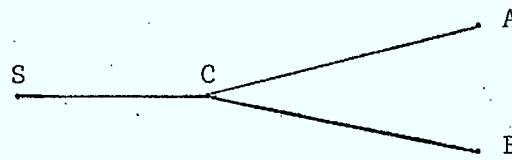


figure 1

D'autre part la figure 2 nous montre un cas où les contraintes sont respectées sans qu'on obtienne l'indivisibilité, où chaque arête porte la moitié de d.

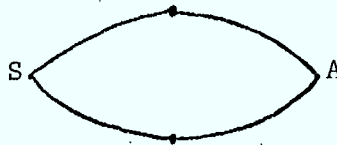


figure 2

Evidemment, dans le cas précédent, si au moins un coût (au sens large) sur une arête est différent des autres coûts et si on minimise les coûts, nous n'aurons qu'une chaîne donc l'indivisibilité.

2. Une autre façon de formuler le problème partirait de la remarque suivante:
  - même si plusieurs chaînes utilisent la même arête, une fois que d circuits sont affectés à cette arête, les autres que l'on affecterait fictivement ne coûteraient rien puisqu'en fait ils ne seraient jamais physiquement installés étant donné que d circuits suffisent pour supporter la télévision. Par exemple, dans la figure 1 on aurait d circuits tout le long des chaînes SCA et SCB mais un seul coût de d circuits sur l'arête SC.

On peut formuler le problème de la façon suivante:

soit les mêmes notations que précédemment et soit  $v_k$  la capacité de l'arête  $k$ , cherchons à maximiser  $\sum_{r \in R} \alpha_r s_r + \sum_{k \in K} \beta_k (y_k^+ - y_k^-)$  (où  $\alpha_r$  et  $\beta_k$  sont des pondérations fixées par l'utilisateur) sous les contraintes suivantes:

$$(3) \quad \sum_{j \in Q} \sum_{i \in D(j)} \delta_{ik} x_{ij} + y_k^+ - y_k^- = d \quad k \in K$$

où  $y_k^+$  et  $y_k^-$  sont des variables d'écart non négatives,

$$(4) \quad \sum_{j \in Q} \sum_{i \in D(j)} \delta_{ik} x_{ij} + (d - y_r^+) + s_r = v_r \quad r \in R$$

où  $s_r$  est la capacité en réserve de l'arête  $r$  après affectation de la téléphonie et de la télévision, et où  $\bar{Q}$  est l'ensemble des indices des paires origine-destination pour la téléphonie; autrement dit, on utilise sur l'arête  $k$  pour la télévision que des circuits à concurrence de  $d$ ; dépassé  $d$ , les circuits fictifs sont gratuits; de plus, on doit avoir

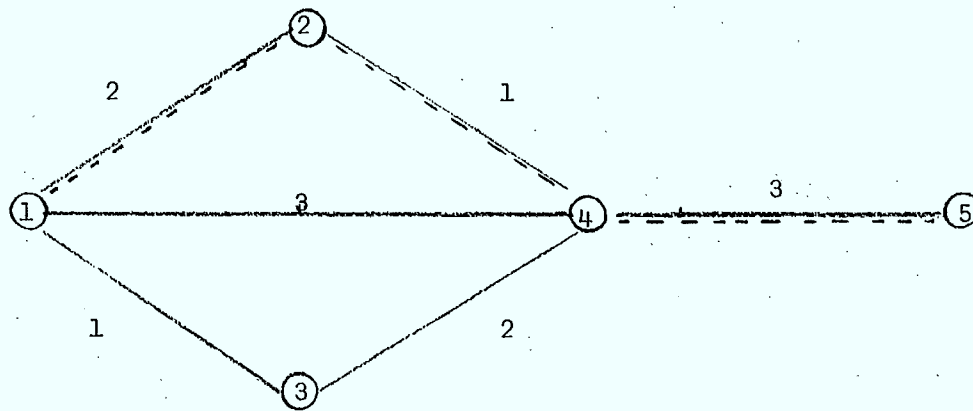
$$(5) \quad \sum_{i \in D(j)} x_{ij} \geq d \quad j \in Q$$

$$\sum_{i \in D(j)} x_{ij} \geq d_j \quad j \in \bar{Q}$$

Cette formulation devrait pousser le choix des chaînes de la télévision vers une utilisation conjointe des arêtes, donc on peut supposer qu'on devrait être proche d'un arbre extrémal et obtenir dans de nombreux cas l'indivisibilité de  $d$  circuits sur les arêtes.

Par exemple, pour le problème suivant réduit à l'acheminement de la télévision seule, on obtient l'indivisibilité:

Soit une demande de  $d=1000$  circuits (nombre purement fictif) de télévision venant de ① et allant vers ②, ④ et ⑤, et soit  $u_i = 3000$  circuits, pour tout  $i$ , la capacité des arêtes de transmission, arêtes pour lesquelles les coûts unitaires par circuit sont indiqués sur le graphique. La solution quand le problème est formulé comme ci-dessus, est identique à une solution qu'un algorithme d'arbre extrémal minimal obtiendrait. Le graphique indique la solution en pointillé.





### 2.1.3 Method of identifying disjoint chains and use for survivability of messages

#### a) Introduction

The problem of survivability in IRA II has been viewed in the context of disjoint chains. The survivability of a message stream is assumed satisfactory if messages are routed physically on at least two node-disjoint chains from origin to destination (the condition of "node-disjoint" applying for intermediate nodes only). Other definitions of survivability are of course possible (e.g. not more than 7 C.S.P.'s in a switched route to guarantee quality of transmission) but these have not been examined in detail for this phase.

The problem of finding disjoint chains in a network can be viewed at two levels. Firstly, one can simply count the number of node-disjoint chains (using minimum cut methods). Secondly one can count the number of node-disjoint chains and enumerate these chains in detail. In this phase of IRA II, algorithms have been developed for both types of problems but only the latter (enumeration of node disjoint chains) has been incorporated into the software, and will be described herein. The disjoint chain algorithm finds the maximal collection of node-disjoint chains having minimal cost, the minimal cost criterion being used to choose among disjoint chains in the case where the maximal set of node disjoint chains is not unique (see the example of page 2-3b in the Interim Report on the Second Phase).

The algorithm uses labelling methods and is based on an enlarged network where each node is converted into two nodes with a directed link between them.

The following paragraphs are intended to describe this algorithm in detail and reading of these paragraphs is not essential to comprehension of the report.

#### b) Description of Algorithm

We define the following symbols.

N: number of nodes in the graph (or subgraph) considered  
 II: the originating node  
 JJ: the destination node

$C(i,j)$ : cost factor for the link between  $i$  and  $j$  (cost factor can be distances, cost of operation, annual costs or any other relevant factor depending on the aims of the user)

$C(i,j) = \infty$  if no link exists between  $i$  and  $j$

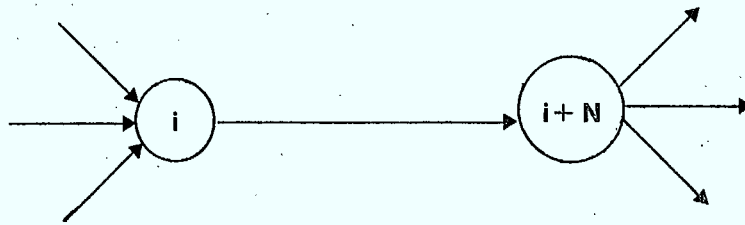
We create an "enlarged" network as follows. Each node is exploded into two nodes according to the following numbering system:

- i) Entry point of node is called  $i$  ( $i = 1, 2, \dots, N$ )
- ii) Exit point of node is called  $i + N$  ( $i = 1, 2, \dots, N$ )

In the enlarged cost matrix  $CC$  we assume

$$CC(i, i+N) = 0, CC(i+N, i) = \infty$$

The following figure shows briefly how this system appears in the graph



In the enlarged problem the cost matrix  $CC(i,j)$  has the following form

$\infty$	
MATRIX C	$\infty$

Each node of the enlarged problem is labelled with three identifiers  $P(i)$ ,  $T(i)$ ,  $PC(i)$  where  $P(i)$  is the name of the preceding node of an arc arriving in node  $i$ ;  $T(i)$  is the temporary minimum cost found to date for the chain from  $II$  to  $i$ ;  $PC(i)$  is the permanent minimum cost of the chain from  $II$  to  $i$ .

The entire algorithm is based on manipulation of these vectors, each of which has  $2N$  entries. The steps of the algorithm are as follows

#### Step 0 (Initialization)

Specify  $II$  and  $JJ$  (origin and destination) Set up the matrix  $CC$  as shown above.

#### Step 1

$P \doteq * [1, \dots, 11, 1]$  (2N entries)

$P \doteq [0, 0, \dots, 0, 1, 0, \dots, 0]$  (2N entries and a 1 in position  $II$ )

$T \doteq CC(II, \cdot)$  (that is the row  $II$  of the matrix  $CC$ )

#### Step 2

Let  $J = [j_1, j_2, \dots, j_k]$  be a vector of indices where  $j_h$  is an index for which  $T(j_h) < \infty$  and  $PC(j_h) = 0$

Let  $Min \doteq \min_{j_h \in J} \{T(j_h)\}$

If  $MIN \geq \infty$  go to step 7) Let  $KK$  be the index for which  $MIN = T(KK)$   
Let  $PC(KK) \doteq MIN$ , if  $KK = JJ$ , go to step 3); if not, let  
 $V = [i_1, i_2, \dots, i_k]$  where  $i_h$  is an index chosen so that  $CC(KK, i_h) < 0$ , and  $PC(i_h) = 0$ .

If the set  $V$  is empty go to step 7) Then let

$T(i_h) \doteq \min \{PC(KK) + CC(KK, i_h), T(i_h)\}$

$P(i_h) \doteq \begin{cases} KK & \text{if the new } T(i_h) \text{ is less than the old } T(i_h) \\ P(i_h) & \text{if not} \end{cases}$

Go to step 2

#### Step 3

Retrace using  $P$ , the shortest chain from  $II$  to  $JJ$  and store in vector  $IND$ . Examine all preceding chains to insure that no arc has been used in the opposite direction previously. If an arc has been reused go to step 5. If not go to step 4.

---

\* Note: the symbol " $\doteq$ " means replace the value of the left member by the value of the right member.

Step 4

Place the newly found chain in the list of chains. Go to step 6

Step 5

Label the two chains which use a common arc in opposite directions and disjoin these chains by attaching the first part of chain 1 to the last part of chain 2 and the first part of chain 2 to the last part of chain 1.

Step 6

Let  $IND = [II; II + N, S, \dots, S_X, JJ]$

Let  $CC(S_k, S_l) \doteq -CC(S_l, S_k)$  when  $S_k$  precedes  $S_l$  in  $IND$

Let  $CC(S_k, S_l) \doteq \infty$

Step 7

Write the chains obtained and stop or restart with a different pair of points.

The following remarks can be made about the algorithm and its efficiency.

- 1) The matrix  $CC$  is not stored fully in machine space since certain parts (the northwest and southeast corners) are never used in the calculations.
- 2) Computational efficiency can be improved if the algorithm is used on subgraphs which are decomposed from the original problem.

An example of a case where decomposition does not alter the nature of the problem is when a single link joins two sections of the network. In this case the points within each section can never be connected by disjoint paths which use the single link between sections.

## c) Use of Algorithm in Software

The software developed for this problem is intended to be used as a preempting device for the imposition of survivability constraints on the transmission network. This algorithm will therefore be applied as follows.

- 1) Considering the entire transmission network and demand from CSP to CSP, constrain the allocation of circuits to chains to use particular disjoint chains by assigning preemptively a certain fraction (say 10%) of the total demand to at least two disjoint chains.

- 2) Remove the capacity used in these chains from the various links of the transmission network and allocate the residual demand (say 80%) to the reduced transmission network. This preemptive application of survivability constraints will, of course, have some effect on the value of the objective obtained when the allocation is complete. The difference in the value of the objective function when survivability is imposed or not imposed represents the cost of imposing such a constraint.

An example of this procedure appears in section 2.1.2.3.

#### 2.1.4 Estimation of use of the transmission network, facilities by Message Streams

From the algorithm used to calculate the average hour by hour usage of the switching network (see description in section 2.2.2: Usage of the Switching Network, of the Interim Report on the Second Phase 15 September 1974) we are able to estimate for each link of the switching network, the total number of call seconds used per hour. Furthermore, for each link we are able to breakdown this total into the subtotal number of call seconds used by each of the message streams (O - D pairs or demand pairs) which have access to the link in question ("access" is of course determined by the switching hierarchy and the homing rules of the hierarchical network). Thus, if we define for each link a vector:  $W_i$  which has size  $N$  (where  $N$  is the total number of message streams) and which has entries  $w_{ij}$  as follows

1. If the link  $i$  does not carry traffic in the message stream  $j$  (i.e. if no message in the stream  $j$  uses the link  $i$ ) then  $w_{ij} = 0$
2. If the link  $i$  does carry traffic in the message stream  $j$  then  $w_{ij} =$  the amount of this traffic measured in call seconds.

Thus for each link  $i$  we have a vector  $W_i$  having positive or zero entries which represents the extent to which link  $i$  carries traffic in the various message streams.

The total traffic carried by the link  $i$  will be the sum of entries in the vector  $(\sum_{j=1}^N W_{ij})$

With the aid of the techniques (using linear programming and network algorithms) which are described in section 2.3: "Reseau Physique" of the Interim Report on the Second Phase 15 September 1974) we are now able to derive, using various management criteria (objective functions), the way in which the transmission network would be used to carry traffic from one central switching point to the next. This algorithm will give results of the following nature. For each link of the switching network, a single chain or collection of chains of physical facilities (not necessarily disjoint) over which the circuits required in the switching network are routed. Each of these chains is composed of a list of physical facilities.

The problem to be solved then, in this context, is to convert our information on the usage of links of the switching network to information regarding the usage of the physical facilities by the various message streams. Only if this information is available, will it be possible to estimate costs on a usage basis (usage being defined possibly as peak or average usage of the facility by a particular message stream).

The solution proposed for this problem, and included in the software, is based on the principle that if, for example, a particular link of the switching network carries the fraction  $p$  of all traffic of a particular message stream (say A-B) then a chain supporting the link and carrying the fraction  $q$  of all traffic of the link would carry the fraction  $p.q.$  of the total traffic of stream A-B. For example, consider a link identified as carrying 100ccs of which 30ccs are due to the message stream A-B, and which has two supporting chains each of which carry 50% of the total 100ccs. In this case the estimated traffic on each chain due to message stream A-B would be  $30/100 \times .5 = 15$  ccs (giving 30 ccs in total for the two chains).

This method can be expressed more precisely as follows.

Suppose the allocation of circuits to chains gives the allocation vector  $X^C = (X_1^C, X_2^C, \dots, X_m^C)$ , and that the relevant chains are identified as the matrix  $C = \{C_{kl}\}$  where  $C_{kl} = 0$  if chain  $K$  does not use the element  $l$  of the network

(elements  $l = 1, \dots, q$ ; chains  $K = 1, \dots, m$ ) and

$C_{kl} = 1$  if chain  $K$  does use the element  $l$  of the network.

Each of these chains, furthermore can be associated with one particular link  $X_i$  of the switching network.

The usage of an element of the transmission network is then a vector  $U_l$  containing entries for each possible message stream  $j$  ( $j = 1, M$ )

$$U_l = \sum_{k=1}^m C_{kl} X_k W_i$$

and where  $i$  is identified as the link of the switching network which chain  $k$  supports.

To clarify the procedures used in this estimation process we present the tableau 2.1.4A.

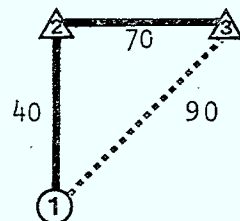
This tableau shows a typical (small) example of the layout of the algorithms involved. From the algorithm calculating usage of the switching network we obtain the information shown in item 1 namely the circuits required on each link of the switching network ( $V_1, V_2, V_3$ ) and for each link, a breakdown of the total demand on that link due to various message streams (OD pairs) giving the matrix  $V_{ij}$  of item 2. In item 3, the presentation of the circuit to facility allocation problem is shown and an hypothetical solution is given in the row labelled 'ALLOCATION'. Finally using this allocation we apply the principles described above to the element A of the facility network to obtain an estimate of responsibility for use as given in item 4.



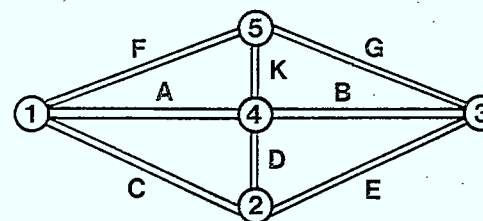
TABLEAU 2.1.4A

## LINEAR PROGRAMMING FORMULATION OF SAMPLE PROBLEM

SWITCHING NETWORK



TRANSMISSION NETWORK



- 1) TOTAL USAGE FROM SWITCHING NETWORK      2) USAGE BREAKDOWNS FROM SWITCHING NETWORK

$V_1 = 90$  circuits (link 1-3)  
 $V_2 = 40$  circuits (link 1-2)  
 $V_3 = 70$  circuits (link 2-3)

O-D Pairs	link 1-3	link 1-2	link 2-3
①-③	100%	30%	40%
①-②	0	70%	0
②-③	0	0	60%

that is

$V_{11}$	$V_{21}$	$V_{31}$
$V_{12}$	$V_{22}$	$V_{32}$
$V_{13}$	$V_{23}$	$V_{33}$

- 3) TRANSMISSION NETWORK

ALLOCATION		C <sub>1</sub>	C <sub>2</sub>	C <sub>3</sub>	C <sub>4</sub>	C <sub>5</sub>	C <sub>6</sub>	C <sub>7</sub>	C <sub>8</sub>	C <sub>9</sub>	C <sub>10</sub>	C <sub>11</sub>	RHS	SLACK
		30	40	0	0	20	30	10	0	50	10	10		
	A	1	1					1					$\leq U_1 = 90$	$S_1 = 10$
	B	1			1						1		$\leq U_2 = 60$	$S_2 = 10$
	C			1	1		1						$\leq U_3 = 30$	$S_3 = 0$
	D		1		1			1	1		1	1	$\leq U_4 = 70$	$S_4 = 0$
	E		1	1						1			$\leq U_5 = 100$	$S_5 = 10$
	F					1			1				$\leq U_6 = 25$	$S_6 = 5$
	G					1						1	$\leq U_7 = 20$	$S_7 = 0$
	K								1			1	$\leq U_8 = 50$	$S_8 = 40$
link 1-3		1	1	1	1	1							$\geq V_1 = 90$	
link 1-2							1	1	1				$\geq V_2 = 40$	
link 2-3										1	1	1	$\geq V_3 = 70$	

- 4) CALCULATION OF USAGE (AVERAGING OR PRORATING SYSTEM)

EXAMPLE

ELEMENT A

$$\begin{pmatrix} U_{11} \\ U_{12} \\ U_{13} \end{pmatrix} = 30 \begin{pmatrix} V_{11} \\ V_{12} \\ V_{13} \end{pmatrix} + 40 \begin{pmatrix} V_{21} \\ V_{22} \\ V_{23} \end{pmatrix} + 10 \begin{pmatrix} V_{31} \\ V_{32} \\ V_{33} \end{pmatrix} = 30 \begin{pmatrix} 1.00 \\ 0 \\ 0 \end{pmatrix} + 40 \begin{pmatrix} 1.00 \\ 0 \\ 0 \end{pmatrix} + 10 \begin{pmatrix} .3 \\ .7 \\ 0 \end{pmatrix} = \begin{pmatrix} 73 \\ 7 \\ 0 \end{pmatrix}$$

## 2.2 Costing Block

### 2.2.1 Asset Valuation Functions

This section outlines some of the technical information which must be specified before costing can take place and the techniques which we have developed to compute the asset costs of physical facilities. Some of this material represents an important advance in our knowledge of this area; those interested in the evolution of our approach to the problem of asset may wish to read the corresponding parts of the IRA-I Final Report and the IRA-II Interim Report in conjunction with this section.

We begin with a description of our conception of the technical relationships in the physical facilities network. We are concerned primarily with the TCTS long-haul, terrestrial microwave network. We also include the CN/CP microwave network but the description that follows is primarily oriented toward TCTS facilities.

The TCTS microwave system, which has been in operation for almost 20 years, operates on the 4 GHz band, that is, 3.7 to 4.2 GHz (this has recently been extended to include the 3.5 to 3.7 GHz range). The maximum capacity of this band is 16 full-duplex radio channels, a maximum of 14 of which are for operational use and 2 of which are used as standby, protection channels. With the development of high power transceivers (using TWT's) it has been possible to increase the number of voice circuits per R.F. channel from 600 to 960 to 1200. Sometime in the near future this is expected to increase to 1500 voice circuits per channel. The CN/CP system operates on the 6 GHz band with 8 full-duplex radio channels, 2 of which are standby, and with a maximum of 1800 voice circuits per channel.

The TCTS microwave system consists of a network of microwave repeaters or towers. Using the resources of the Network Development group of the National Telecommunications Branch we have been able to identify the location of all the repeaters in the TCTS 4 GHz long-haul network. In particular, the program ROUTE has been useful in locating, identifying and mapping towers in both the TCTS and CN/CP systems. We have also been able to identify the function or type of each repeater principally as a result of the support and assistance of the Domestic Long-Distance Communications Network Study group of the CRC. In particular a report issued by this group\*, has provided asset cost functions for each type of repeater which appear to be the most accurate available to date.

---

\* Bowen, R.R., Baser, R.V., Walker, E.A., and Hutchinson, R.L., "A Simplified Model of the Canadian Terrestrial Trunk Communications Network"; Communications Research Center, Dept. of Communications, Ottawa, 1974.

We will now examine the nature of each type of repeater recognized in our model of physical facilities and how their respective asset cost functions have been constructed. All repeaters in the TCTS system have been classified, according to their function, into 3 categories.

The first and simplest type is the regular repeater. At a regular repeater the microwave signal is simply received, amplified and retransmitted; no branching to several links or multiplexing takes place. These repeaters occur, on average, every 25 or 30 miles throughout the network and as a result are extremely numerous. Rather than including each as a separate node in the network, we calculate the number of regular repeaters on a given link in the network by dividing the length of the link by an average hop between repeaters. (It should be noted that the actual length of a link is, on average, 1.4 times the airline mileage).

The second type of repeater in the model is a branching or junction repeater. At this type of repeater long-haul signals are received and then retransmitted in several directions.

The third type of repeater is a terminal repeater. At terminals all signals are demodulated (or modulated for outgoing signals) and demultiplexed to the voice band. This type of installation is normally located in a large metropolitan node and is usually linked to a long-haul (toll) switching office.

The CRC study referred to above determines the components that are required to construct an installation of each type, provides the capital costs of the components and thereby constructs an asset cost function for each type of repeater. The asset cost varies, of course, with the size or capacity of the installation. Tableaux 2.2.1 A and 2.2.1 B (from the CRC study) showing the capital costs of the components of radio repeaters is attached. This data is the basis of the cost functions which have been formulated for each repeater type. A chart summarizing the major steps in the cost function for each repeater type, has also been attached (taken from the CRC study). This chart makes it possible to determine an asset cost for a given type of repeater as a function of its capacity in RF channels. As an example, we have constructed such a function in graphical form for regular repeaters (see tableau 2.2.1C). It should be noted that it is possible to modify the site cost and initial power cost components of these functions to account for higher costs caused by difficult terrain in certain parts of Canada.

Another type of equipment included in the physical facilities network is toll switching equipment. It is difficult to obtain the information necessary to derive asset cost functions which are characteristic of such equipment. Nevertheless, a formula to compute asset cost as a function of switching capacity has been derived. Work is continuing in this area to improve our knowledge and capability with respect to switching costs.

TABLEAU 2.2.1. A

## Capital Costs of Components of Radio Repeaters

Item	Factory Cost	Installation Mark-up	Installed Cost
Site, including land, buildings, road, prime power, and tower	75,000	1.0*	75,000
Antenna, horn	8,000	1.4	11,200
Antenna, paraboloid	6,000	1.4	8,400
Waveguide, circular/ft	33	1.4	46
Waveguide, rectangular/ft	10	1.4	14
Mode Filter	1,500	1.4	2,100
R.F. Branching Eqpt.	2,000	1.4	2,800
Alarm and order wire eqpt.			
-in regular repeaters	9,500	1.87	17,750
-in terminals	13,500	1.87	25,520
Protection Switching Eqpt./Site			
-initial cost	7,500	1.87	14,025
-incremental cost/channel	1,500	1.87	2,805
-increase to double protection	7,000	1.87	13,090
Power System			
-Initial in reg. repeater	50,000	1.0*	60,000
-Initial in drop repeater	67,320	1.0*	67,320
and in terminals			
-To introduce 6th channel	9,350	1.0*	9,350
-To introduce 11th channel	9,350	1.0*	9,350
F.M. Modem	5,500	1.87	10,285
Transceiver	14,000	1.87	26,180

\*The mark-up is 1.5 in remote sites.

Source: Bowen et Al.

TABLEAU 2.2.1. B

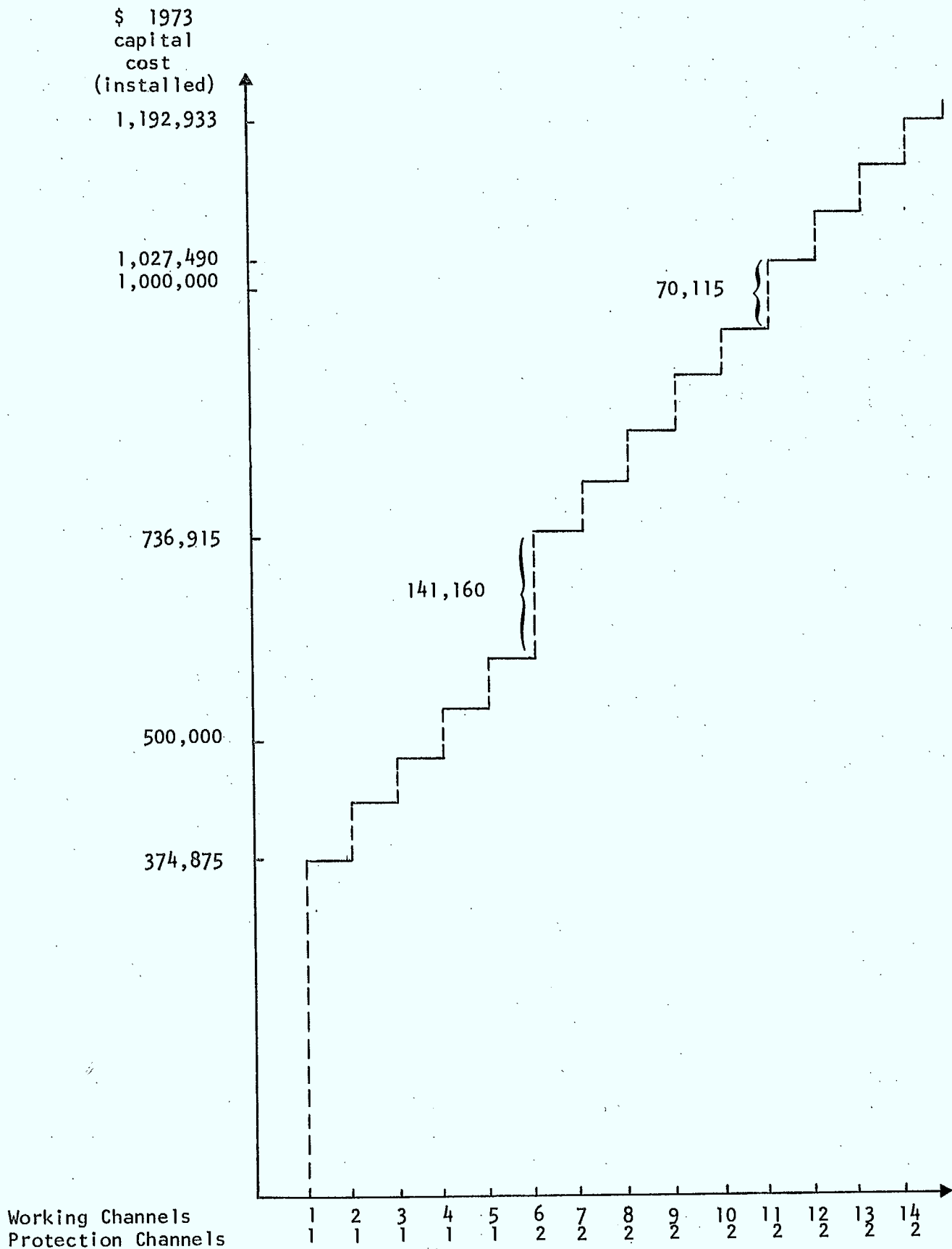
Summary of Installed Capital Costs of Different Repeater Types, 1973 Dollars

Repeater Type	Site Cost	Initial Power Cost	Other First-Channel Costs	Costs to Introduce 6th Channel	Costs to Introduce 11th Channel	Costs to Introduce Other Channels
Regular	75,000	60,000	239,895	141,160	70,115	55,165
Drop	75,000	67,320	295,465	196,730	70,115	55,165
Junction	75,000	67,320	151,940	101,170	36,460	28,985
Junction-Terminal	75,000	67,320	134,440	83,670	36,460	28,985
Terminal	75,000	67,320	163,990	102,370	51,420	39,270

Source: Bowen et Al.

TABLEAU 2.2.1. C  
Regular Repeater Cost Function

sorēs inc



It should be noted that the costing procedure that we have adopted is applied to the physical facilities on an element basis; each element, that is, each tower or switching machine is costed. This is in contrast to costing by links in which the asset costs of equipment are imputed to various links of the network. This also differs from techniques which employ cost functions on a per-mile basis.

Finally, the problems encountered in metropolitan nodes can be briefly described. Most major metropolitan nodes consist of more than one element, that is, a variety of types of physical equipment. Normally this includes one or two toll switching machines, several branching repeaters and several terminal repeaters. By reference to the DOC Microwave Facilities Catalogue and additional information provided in the CRC study mentioned above, we can identify most of the physical facilities and the function of each installation in any metropolitan nodes. Once this information has been obtained, one can apply the appropriate cost function to each facility, sum these and thus obtain an asset cost for the metropolitan node.

#### 2.2.2 Growth Module

Throughout the Costing Block (C.B.) the growth rate of gross additions is required. It is needed in the Aging and Depreciation Modules for the various calculations. In addition, it is used in the Deferred Tax Module which inputs in the Cost of Capital Module. These and other relationships are shown in the Costing Flowchart below.

The annual growth rate is represented by R where R is calculated for all Plant Types for each carrier. It is assumed that growth is compounded from year to year. This exponential nature can be modelled by equation (1) as follows:

$$GA(x) = GA(o) * R^x \dots\dots (1)$$

where GA (x) is the gross addition in year x

GA (o) is the gross addition in year o

i.e. initial base year GA

R is the growth rate =  $(1.00 + r)$  where r is annual rate of change

e.g.  $1.10 = (1.00 + .10)$

and  $R^x$  is the compounded growth for x years.

Given values of GA (x) for several consecutive years the problem is to solve for R.



```

graph TD
    VM[ASSET VALUATION MODULE] --> RA[REPRODUCTION ASSET VALUES]
    RA --> J1((X))
    GM[GROWTH MODULE] --> GR[GROWTH RATE]
    GR --> J1
    MI[MISCELLANEOUS INPUTS  
- SURVIVOR CURVE  
- INDEX FACTOR  
- AVERAGE LIFE  
- MAXIMUM LIFE  
- SALVAGE RATE] --> J1
    J1 --> AIM[AGING & INDEXING MODULE]
    AIM --> HGA[HISTORICAL GROSS ASSET BASE]
    HGA --> DM[DEPRECIATION MODULE]
    DM --> J2(( ))
    J2 -- NET ASSET BASED --> AD[ACCUMULATED DEPRECIATION]
    J2 -- GROSS ASSET BASED --> AR[ANNUAL DEPRECIATION RATE]
    AR --> NITR[NON-INCOME TAX RATE]
    NITR --> OC[OPERATING COSTS]
    OC --> IC[INCURRED COSTS]
    AD --> J3(( ))
    J3 --> CCR[COST OF CAPITAL RATE]
    CCR --> CC[CAPITAL COST]
    J3 --> NRB[NET RATE BASE]
    NRB --> CC
    CC --> IC
    DM --> DTM[DEFERRED TAX MODULE]
    DTM --> CC
    DM --> CCM[COST OF CAPITAL MODULE]
    CCM --> CC
  
```

The flowchart illustrates the process of calculating incurred costs, starting with the **ASSET VALUATION MODULE** which leads to **REPRODUCTION ASSET VALUES**. This value is then combined with inputs from the **GROWTH MODULE** (via **GROWTH RATE**) and **MISCELLANEOUS INPUTS** (including survivor curve, index factor, average life, maximum life, and salvage rate) at a junction point. The result flows into the **AGING & INDEXING MODULE**, then to the **HISTORICAL GROSS ASSET BASE**, and finally to the **DEPRECIATION MODULE**.

The **DEPRECIATION MODULE** branches into two paths: **NET ASSET BASED** and **GROSS ASSET BASED**. The **NET ASSET BASED** path leads to **ACCUMULATED DEPRECIATION**, which then feeds into the **COST OF CAPITAL** calculation. The **GROSS ASSET BASED** path leads to **ANNUAL DEPRECIATION RATE**, which then feeds into the **NON-INCOME TAX RATE** and **OPERATING COSTS**.

The **COST OF CAPITAL** calculation also receives input from the **DEFERRED TAX MODULE** and the **COST OF CAPITAL MODULE**. The **OPERATING COSTS** are then combined with the **CAPITAL COST** (derived from the **COST OF CAPITAL**) to determine the final **INCURRED COSTS**.

The approach taken is to apply the logarithm (log) of equation (1) which yields:

$$\log GA (x) = \log GA (o) + x \text{ Log} R \dots\dots (2)$$

This new equation represents a linear regression of log GA (x) on x. Log GA (x) is the dependent variable and x the independent one.

The log R can now be estimated by using the maximum likelihood estimate of the regression coefficient in this equation, and then the antilogarithm of this estimate yields an estimate of R. The intercept estimate of log GA (x) can be used to estimate the initial base year gross addition, GA (o).

Applying this algorithm to actual Bell Canada Construction Expenditures (1967-1971) yielded the following:

<u>Plant Category</u>	<u>R</u>
Switch by switch	1.014
Gross Bar	1.126
Circuit	1.096
Radio	1.226
Electronic	0.956

### 2.2.3 Aging and Indexing Module

As shown in the Costing Block Flowchart the Asset Valuation Module, which provides reproduction asset values, applies current cost functions and current technology to assets calculated in the Operating Block. However, these reproduction asset values must be transformed into historical asset values which are the ones which appear on companys' books. It is these historical values, which reflect original purchase prices which are one used in the calculation of gross assets at cost, are depreciated and which represent surviving plant values.

The Aging Module which applies various depreciation methods, survival characteristics and growth rates now also takes in account indexing which transforms reproduction values into historical values, along with the regular function of aging the asset i.e. distributing the historical values. The indexing factors are:

- 1) pricing which reflects dollar inflation over time; and
- 2) changes in technology which reflects differing real costs for the same capacity due to varying states of technology.

The same methodology developed in the Interim Report, September 1974, has been modified as follows.

The reproduction value of surviving plant installed x years ago now represented by

$$GA * R^{T-X} * SRV (100*X/L)*P^X$$

where P is an annual index factor (e.g. 1.02) which is compounded to the power X. At present P incorporates inflation only.

The old equation (1) (page 3-12 Iterim Report) is now revised to

$$GA = GTPRIM / \sum_{X=0}^T R^{T-X} * SRV(100*X/L)*P^X \dots\dots (1)$$

where GTPRIM is the reproduction gross telephone plant from the Asset Valuation Module.

Once equation (1) solves for GA all remaining aging, depreciation and other asset-based computations are done using historical values. The output from the Aging Module is historical gross asset base (GTP).

#### 2.2.4 Depreciation Module

The aging and depreciation modules have been programmed so that conversational time-sharing can be done for users to test this module with various simulation parameters. These are:

1. Survival Curve - 22 curves of varying skewness are available as shown in Survival Curve List
2. Depreciation Method - ASL or ELG
3. Average life, L.
4. Maximum Life, T. However  $(100*T/L)$  must not exceed the largest percentage of surviving plant for the selected survival curve.
5. Salvage rate
6. Inflation index
7. Growth rate
8. Reproduction value

In order to simulate a change in depreciation methods such as Bell Canada's conversion from ASL to ELG in 1971, the following would have to be done.

## SURVIVAL CURVE LIST

NUMBER	SURVIVAL CURVE	MAXIMUM PERCENTAGE AVERAGE LIFE
1	L0	358
2	L0.5	350
3	L1	291
4	L1.5	287
5	L2	262
6	L3	226
7	L4	196
8	L5	174
9	S-.5	200
10	S0	200
11	S0.5	200
12	S1	198
13	S1.5	197
14	S2	191
15	S3	180
16	S4	161
17	S5	143
18	S6	129
19	R1.5	200
20	R2.5	183
21	R3	166
22	R4	149

- Assume maximum life is T year old  
year of conversion is XC years ago

Therefore from T to (XC + 1) years ago use depreciation method 1:

XC to present use depreciation method 2.

Then the problem is to find accumulated depreciation for years T to (XC + 1) for surviving plant, and to also find accumulated depreciation for XC years ago to present and current annual depreciation rate based on depreciation methods.

Note that the depreciation method does not change value of surviving plant i.e. GTP which is a function of survival curve. But annual and accumulated depreciation will change.

- Computation of the Depreciation Reserve Ratios

Values of the accumulated depreciation by plant category for all carriers under consideration in the IRA model have been computed based on test run results on the theoretical depreciation reserve ratios (to the historical cost of the gross telephone plants by category at the end of year) through the conversational time-sharing Aging and Depreciation Algorithm (ADA).

The theoretical depreciation reserve ratios by category for all the carriers were computed by using estimate for some parameters and using the Iowa survivor curve L00 which is guide representative for mass property telephone plants.

The estimates of such theoretical reserve ratios by category of plants are:

	<u>Theoretical</u>	<u>Adjusted</u>
Building	18.57%	25.9%
Switching	19.51%	27.2%
Transmission	20.88%	29.1%
Outside Plant	19.03%	26.5%
Station Equipment	22.86%	35.9%
General Equipment	25.23%	35.1%

These theoretical ratios invariably differ from the historical ratios, necessitating adjustments on them, which are shown in the last column of the above table.

## 2.2.5

## Calculation of Incurred Costs

The cost valuation module provides beginning of year values for the five accounting types of plant. These are:

1. Switching (SW)
2. Transmission (T)
3. General equipment (G.E.)
4. Buildings (Bldg.)
5. Land (Land)

For each of these plant types a depreciation rate is provided and similarly a depreciation reserve ratio, a growth rate, and a retirement rate is available. All these rates use a beginning of year base. An end of year/beginning of year ratio is calculated as follows:

$$\frac{\text{end of year}}{\text{beginning of year}} = 1 + \text{growth rate} - \text{retirement rate}$$

A mid year rate is also calculated as the average of beginning and end of year rates.

The net rate base used for the cost of capital calculation is a mid year base. The net rate base, NRB is expressed by

$$\text{NRB} = (1 - \text{Acc. Dep}) (1 + \% \text{ W.C.})$$

Hence the NRB is calculated as shown here but scaled up by the ratio of mid year/beginning of year since all rates are beginning of year based.

The cost of capital CC is calculated using the following equation:

$$\text{CC} = \frac{1}{1 - t} * \text{RORE} * (1 - \text{DCR}) + i * \text{DCR}$$

where  $t$  = tax rate

RORE = rate of return on equity

DCR = debt/capitalization

$i$  = average interest rate on debt

The incurred cost rate is now the sum of the following:

- depreciation
- † cost of capital
- † operating costs rate (excluding depreciation)
- ✱ other tax rate

This has been done for all 5 plant types and 8 telephone carriers with the results shown in Incurred Cost Rate Table.

Incurred Cost Rate Table

Carrier	Plant type				
	SW.	Tr.	G.E.	Bldg.	Land
B.C. Tel.	.317	.337	.353	.307	.323
Alta. Tel.	.198	.214	.244	.186	.169
Sask. Tel.	.259	.276	.296	.247	.253
Man. Tel.	.189	.204	.236	.177	.156
Bell Can.	.308	.328	.344	.299	.315
N.B. Tel.	.245	.263	.287	.234	.233
M.T. & T.	.246	.264	.297	.235	.234
Nfld. Tel.	.305	.325	.343	.295	.306

The monthly ratios of Trans Canada Assigned Expenses/Trans Canada Plant Investment are available for the 1971 TCTS revenue sharing. These have been computed to a per annum basis and yield the following:

Expense Rate Table

Carrier	Annual expense/plant investment under TC assignment	Ratio to TCTS (.233)
B.C. Tel.	.250	1.12
Alta. Tel.	.188	.84
Sask. Tel.	.150	.67
Man. Tel.	.158	.71
Bell Can.	.258	1.16
N.B. Tel.	.210	.94
M.T. & T.	.213	.96
Nfld. Tel.	.116	.52
TCTS	.223	1.00

A comparative analysis can be made between these carrier rates and those of the Incurred Cost Rate Table. For the Incurred Cost the overall composite rate would be a value based on the weightings or distribution of the plant types. With this in mind the following observation can be made:

1. The Incurred Cost Rates are generally higher than Expense Rates
2. The relative values of the Incurred Cost Rates vs Expense Rates shows
  - a) B.C. Tel. and Bell Canada Incurred Costs are among the highest which concurs the Expense Rates;
  - b) Nfld. Tel. is relatively on opposite extreme of these tables;
  - c). Government-owned prairie carriers are in a reasonable relative position but within these Saskatchewan has the lowest expense rate but highest incurred cost rate

Possible explanations for these results are:

1. All depreciation rates are the same which is for Bell Canada. Similarly, for net rate base calculations of all carriers. For



example, differing growth rates will reflect more accurately the real situation;

2. Operating cost rates are realistic since they are calculated for each carrier based on 1971 Company Totals;
3. Cost of capital calculations for carriers' incurred cost rate may vary for TCTS assigned expense rates. For example, rates of return on capitalization components, interest rates and effective tax rates;
4. TCTS assignment for expenses may differ from IRA incurred cost calculation, in whole or in part.

### 2.3 Sharing Block

Relative to the work done in IRA-1 few concepts have changed in this block. The following section explains some detail the methods used for the estimation of revenue in the present version of the

#### 2.3.1 Calculation of Collected Revenue

In IRA-II the calculation of collected revenues has been improved through the incorporation of 24-hour traffic profiles, for average business day, Saturday and Sunday. The availability of these profiles permits the application of tariffs according to the hour at the call-originating point and the day of the week. Furthermore, although telephone traffic is not estimated statistically in three component parts (namely Direct-Distance Dialing, Station to Station, Operator Handled and Person to Person), it is nonetheless possible to apply certain average breakdowns of total traffic into these categories according to the hour of the day and the day of the week.

Data used in this version of IRA was obtained from a Bell Canada response to a question from the Province of Quebec at the recent CTC hearings (P (Q) 14 December 73B-71 P5). This data is presented below.

TABLEAU 2.3.1. A

ESTIMATED NUMBER OF INTERURBAIN  
CALLS BY PERIOD AND BY  
CALL CATEGORY

Weekdays	Direct Distance Dialing (Millions)	Station to Station Operator Handled (Millions)	Person to Person (Millions)	Total Calls (Millions)
08h to 18h	197.1 (79%)	40.2 (16%)	11.6 (5%)	248.9
18h to 23h	66.8 (77%)	17.6 (20%)	2.0 (3%)	86.4
23h to 8h	13.4 (74%)	4.1 (23%)	0.5 (3%)	18.0
Sundays				
08h to 18h	24.0 (84%)	4.1 (14%)	0.4 (2%)	28.5
18h to 23h	9.4 (79%)	2.3 (19%)	0.2 (2%)	11.9
23h to 08h	1.6 (76%)	0.5 (23%)	0.1 (1%)	2.1

Combining the information with the traffic profiles, we are able to estimate total revenue on a city to city basis as follows:

- 1) Revenue for the average business day (ABD) is

$$\sum_{\text{hour} = 1}^{24} \frac{\text{weighted average tariff}}{\text{CCS}} \cdot \text{CCS/hour}$$

In this calculation we use offered traffic; we recognize that the calculation of revenue should be based on carried traffic and not on offered traffic. This change will be made as soon as the final tuning of the software is complete.

Here the weighted tariff is obtained as follows. The average tariff per CCS is obtained from the TCTS average tariff matrix (published in the Final Report on the First Phase of the Project (IRA -1) March 31 1974 pages 19 and 20) for the relevant day and hour for each of the three categories. A weighted average of these three tariffs is obtained using the weightings of Tableau 2.3.1 A for the relevant day and hour.

- 2) The Saturday and Sunday revenues are calculated in a similar fashion using the relevant traffic profile. In this context we do not introduce the Saturday and Sunday profiles and derive profiles for time zone differences but rather use an hour by hour ratio "Sunday Traffic/ABD traffic" and "Saturday Traffic/ABD traffic" which is used to estimate traffic on these days.

- 3) Total revenue for a typical week is estimated as the sum of revenues for 7 days (5ABD + Saturday + Sunday). Annual revenue is 52.14 (weekly revenue).

It is recognized that these methods involve approximations and hence it is one of the aims of section 3.3.1 to calculate the total estimated revenues and attempt to compare these with published estimates.

2.4 Accounting Block

## 2.4.1 Simultaneous Equation System

In order to complete the financial statements the following variables are solved simultaneously:

<u>VARIABLE (X)</u>	<u>DESCRIPTION</u>
1. NETINC	net income
2. NEWDEB	new debt
3. NIA	net income available to common shareholders
4. EQ	equity, year-end (stock only)
5. DELEQ	change in equity
6. RE	balance of retained earnings, year-end
7. PR	preferred stock, year end
8. DELPR	change in preferred stock
9. L	debt, year-end
10. DIVI	common dividends

These variables and others are displayed in the Items of Financial Statements below. Below are the 10 simultaneous equations used to solve for these variables:

## 1) Funds Statement (Sources = Uses)

$$\begin{aligned}
 & \text{NETINC} - (\text{RHO} * \text{PRO} + \text{RHON} * \text{DELPR}/2) - \text{DIVI} \\
 & (1-\text{ALPHA}) + \text{CURDTX} + \text{PRDTX} + \text{DCRO} + \text{DEPN} + \text{DPRTVE} \\
 & + \text{NEWDEB} - \text{REPL} + \text{DELPR} + \text{DELEQ} + \\
 & = \text{GCE} - \text{DPRTVC} + \text{PLAMPS} + \text{DELINV} \\
 & + (\text{DCO} - \text{DCOO} - (\text{DCRO} - \text{DCROO}) + \text{CP} * \text{DELPR} \\
 & + \text{CE} * \text{DELEQ} + \text{CL} * \text{NEWDEB} - \text{NSV} \\
 & + (\text{CTIO} + \text{DELCTI} + \text{OCA} - \text{CL}) - (\text{CTIO} + \text{OCA} - \text{CLO}) \\
 & - (1-(1-\text{ALPHA}) * \text{T} * \text{DEPDIF}
 \end{aligned}$$

where the variables are described in the variable list below.

## 2) Net Income Equation

$$\begin{aligned} \text{NETINC} = & (1-T) * (\text{OPRV} - \text{OPXP} - \text{DEPN} + \text{TOI}) \\ & + (1-T) ((\text{DEBTO} - \text{REPL}/2) * \text{IO} + \text{NEWDEB} * \text{IN}/2) \\ & + \text{NOI} + \text{IDC} \\ & + \text{ALPHA} * \text{CURDTX} - (1 - (1 - \text{ALPHA}) * \text{I}) * \text{DEPDIF} \end{aligned}$$

$$3) \quad \text{NIA} = \text{NETINC} - (\text{RHO} + \text{PRO} + \text{PHON} * \text{DELPR}/2)$$

$$4) \quad \text{DIVI} = \text{DPR} * \text{NIA}$$

$$5) \quad \text{EQ} = \text{EQO} + \text{DELEQ}$$

$$\begin{aligned} 6) \quad \text{RE} = & \text{REO} + \text{NIA} - (\text{CE} + \text{DELEQ} + \text{CP} + \text{DELPR} + \text{CCL} * \text{NEWDEB}) \\ & - \text{DIVI} + \text{OTHADJ} \end{aligned}$$

$$7) \quad \text{PR} = \text{PRO} + \text{DELPR}$$

$$8) \quad \text{L} = \text{LO} + \text{NEWDEB} - \text{REPL}$$

$$9) \quad \text{DCR} (\text{EQ} + \text{RE} + \text{PR} + \text{L}) - \text{L} = \text{LO} - \text{DCR} (\text{EQO} + \text{REO} + \text{PRO} + \text{LOO})$$

$$10) \quad \text{PCR} (\text{EQ} + \text{RE} + \text{PR} + \text{L}) - \text{PR} = \text{PRO} - \text{PCR} (\text{EQO} + \text{REO} + \text{PRO} + \text{LO})$$

These equations can be solved simultaneously based on the following model:

$$\text{Ax} = \text{B}$$

where A is a coefficient matrix as shown in the Coefficient Tableau below.

COEFFICIENT TABLEAU

NETINC	DELPR	PR	DELEQ	EQ	RE	NEWDEB	L	NIA	DIVI	
1	-RHON/2.0 +1 - CP		1-CE			1-CCL			-1	EQ1
1						(1-T)* IN/2				EQ2
-1	RHON/2.0							1		EQ3
								-DPR	1	EQ4
			-1	1						EQ5
	CP		CE		1	CCL		-1	1	EQ6
	-1	1								EQ7
						-1	1			EQ8
		DCR		DCR	DCR		DCR-1			EQ9
		PCR-1		PCR	PCR		PCR			EQ10

X is the vector of 10 variables described above and B is the right hand side vector as follows:

$$\begin{aligned} B(1) = & RHO * PRO - (1 - ALPHA) * CURDTX + PRDTX - DCRO - DEPN - DPRTVE \\ & + REPL + GCE - DPRTVC - (-PLAMPS - DELINV - DCO + DCOO \\ & + (DCRO - DCROO) + NSV) + (CTI + OCA - CL) - (CTIO + OCAO - CLO) \\ & + OTHADS \end{aligned}$$

$$\begin{aligned} B(2) = & (1 - T) * (OPRV - OPXP - DEPN + TOI) - (1 - T) * ((LO - REPL / 2) * IO) \\ & + NOI + IDC + ALPHA * CURDTX - (1 - (1 - ALPHA) * T * DEPDIF) \end{aligned}$$

$$B(3) = -RHO * PRO$$

$$B(4) = 0$$

$$B(5) = EQO$$

$$B(6) = REO + OTHADJ$$

$$B(7) = PRO$$

$$B(8) = LO - REPL$$

$$B(9) = LO - DCR * (EQO + REO + PRO + LO)$$

$$B(10) = +PRO - PCR * (EQO + REO + PRO + LO)$$

## VARIABLE LIST

<u>NAME</u>	<u>DESCRIPTION</u>
AD	accumulated depreciation, year-end
ADO	accumulated depreciation, beginning of year
ALPHA	flow through coefficient of deferred taxes
BETA	% of taxable other income"total other income
CCL	issue expense rate debt
CE	issue expense rate, common shares
CL	current liabilities, year-end
CLO	current liabilities, beginning of year
CP	issue expense rate, preferred shares
CTI	cash & temporary investments, year-end
CTIO	cash & temporary investments, beginning of year
CURDTX	current deferred taxes
DCD	deferred charges-debt, year-end
DCDO	deferred charges-bebt, beginning of year
DCH	deferred charges - total, year-end
DCHO	deferred charges - total, beginning year
DCO	deferred charges - other, year-end
DCOO	deferred charges - other, beginning year



DCR	debt/capitalization ratio
DCRO	deferred credit - other, year-end
DCROO	deferred credit - other, beginning of year
DELEQ	change in equity
DELCTI	change in CTI
DELINV	change in investments
DELPR	change in preferred stock
DEPDIF	difference in depreciation due to change in depreciation method
DFPN	depreciation
DETAXO	deferred taxes, beginning of year
DIVI	company dividends
DPR	divident, payout ratio
DPRTVC	dep'n, tools & vehicles, capitalized
DPRTVE	dep'n, tools & vehicles, expenses
EQ	common equity (stock only), year-end
EQO	common equity (stock only), beginning of year
GAMMA	ratio of plant under construction (total plant)
GCE	gross construction expenditures
GTP	telephone plant at cost-acc. dep'n year-end
GTPAC	gross telephone plant at cost, year-end
GDPACO	gross telephone plant at cost, beginning of year
GTPO	telephone plant at cost-acc. dep'n beginning of year
IC	interest on construction (PUC)
IDC	interest during construction
IN	new interest rate
INV	investments, year-end
INVO	investments, beginning of year
IO	old (embedded) interest rate
IT	income taxes
ITC	interest times coverage

L	debt, year-end
LO	debt, beginning of year
NETINC	net income
NEWDEB	new debt
NIA	net income available to common shareholders
NOI	non taxable other income
NSV	net salvage value
OCA	other current assets, year-end
OCAO	other current assets, beginning of year
OPRV	operating revenue (gross
OPXP	operating expenses other than depreciation
OTHADJ	other adjustment (net)
OTHINC	other income - Total
PCR	preferred capital ratio
PLAMPS	plant acquired - plant sold
PR	preferred equity, year-end
PRO	preferred equity, beginning of year
PRDTX	prior deferred taxes
PVC	plant under construction, year end
PVCO	plant under construction, beginning of year
RE	retained earning, year-end
REO	retained earning, beginning of year
REPL	repayments of debt
RHON	new dividend rate on preferred shares
RHO	old dividend rate on preferred shares
ZETA	ratio of common stock dividends to transfer to government owners.

## 2.4.2 Mechanization of accounting block inputs

This block is now developed so that a minimum of data is required exogenously. The remaining input is calculated endogenously in the other blocks of the IRA model or is calculated automatically. This mechanization is described herein.

Step 1

The Company Total financial statement items are inputted in the same format as show in Appendix G, IRA 1, Final Report. These are complete for all financial statements except in that subtotals and totals are not provided and a breakdown of revenues and assets are not provided.

Step 2

Based on accounting principles the following values are directly supplied or calculated for completion of the company total financial statement and the simultaneous equations when used.

i) Interest rates

IN Exogenous

$$IO = (DSC - IN * NEW DEB/2) / (LO - REPL/2)$$

ii) Tax rate

$$T = IT / (OPRV - OPXP - DEPN + TOI - DSC)$$

iii) Issue expenses rates

$$CE = CP = SIE / (DELPR + DELEQ) \text{ and } CL = 0.$$

i.e. SIE provided is distributed between new common equity and new preferred equity and this expense rate is assumed equal for both. As for the debt issue rate it is included with the repayment of debt and is not calculated separately.

iv) Dividend payout ratios

$$RHON = RHO = PRDIVI / (PRO + DELPR/2)$$

i.e. preferred dividend is distributed between a midyear new preferred equity and old preferred equity and this ratio is assumed equal for both.

$$DPR = DIVI/NIA$$

Common dividend payout ratio is dividends net income available after preferred dividends are distributed.

v) Split of other income

$OTHINC = \text{Total other income} - IDC$

$\text{Taxable, TOI} = BETA * OTHINC$

$\text{Non-taxable, NOI} = (1-BETA) * OTHINC$

Where Beta is exogenous.

vi) Depreciation on tools and vehicles

$\text{Expensed, DPRTVE} = ONCC + IDC$

Where ONCC (other non cash changes)

$= \text{total non cash charges} - \text{depreciation}$   
and IDC is supplied

$\text{Capitalized, DPRTVC} = CNRF - IDC$

Where CNRF (changes not requiring funds) is supplied directly.

vii) Net salvage value

retirements,  $RET = GCE - GTPAC + GTPACO$  and

$NSV = AD - ADO - DEPN - DPRTVC - DPRTVE + RET$

viii) Plants acquired minus plant sold

Plants assumed = 0

ix Miscellaneous

$MISC = (DCH-DCHO) - DCRO - DCR00 + SIE-NSV - OTHADJ + PLAMPS.$

Step 3

The IR financial statements are generated using;

- 1) these same accounting inputs based on Company Totals as shown in Step 2
- 2) directly provided from costing and Sharing Blocks
- 3) based on proportionate distribution e.g. all beginning of year IR liabilities are in same proportion of Company Totals

These calculations are shown in Items of Inter-Regional Financial Statements below.

# ITEMS OF INTER-REGIONAL FINANCIAL STATEMENTS

## INCOME STATEMENT

VARIABLE NAME	DESCRIPTION	SOURCE OF CALCULATION
NETINC =	net income	S.E.
OPRV	OPRV operating revenue	from Sharing Block
-OPXP	operating expenses other than DEPN	Costing Block (C.B.)
-DEPN	depreciation	C.B.
+TOI	taxable other income (other than IDC)	BETA*OTHINC where OTHINC is exogenous
+NOI	non taxable other income	- (1-BETA)*OTHINC
+IDC	interest during const.	exogenous
-DSC	debt service charge = (DEBTO-REPL/2)*10 -NEWDEB*IN/2	IN, new interest rate exogenous DEBTO, from balance sheet REPL = $\frac{\text{Co.Total REPL*IR Assets}}{\text{Co.Total Assets}}$ 10 calculated from <u>Company Total</u> Total DSC as follows: $\text{DSCB} = \text{DSC} - \text{IN} * \text{NEWDEB} / 2$ $10 = \text{DSCT} / (\text{DEBTO} - \text{REPL} / 2)$ NEWDEB from Simultaneous Equations (S.E.)
-IT	income taxes	= t (OPRV - OPXP - DEPN + TOI-DSC) + NOI + IDC + FUNCTION where FUNCTION = ALPHA*CURDTX - ((1-(1-ALPHA) <sup>T</sup> )DEPDIF

# ASSETS BEGINNING OF YEAR

1) GTPACO = SWO +TRO +STATO +GENO +BLDGO +LANDO +PUCO	G.T.P. at cost   plant under construction	C.B. sum = SUMX   $= \frac{\text{SUMX (I.R.)} * \text{PUC (Co. Total)}}{\text{SUMX (Co. Total)}}$
2) ADO	Accumulated Dep'n	Sum of acc. dep'n over all depreciable plant types
3) GTPO	GTP at cost - acc. dep'n	= GTPACO - ADO
4) INVO	investments	$= \frac{\text{GTPACO (I.R.)} * \text{INVO (Co. Total)}}{\text{GTPACO (Co. Total)}}$
5) CAO =CTIO  +OCAO	current assets cash and temp. inv.  other current assets	$= \frac{\text{GTPACO (I.R.)} * \text{CTIO (Co. Total)}}{\text{GTPACO (Co. Total)}}$ $= \frac{\text{GTPACO (I.R.)} * \text{OCAO (Co. Total)}}{\text{GTPACO (Co. Total)}}$
6) DCHO =DCDO +DCOO	deferred charges deferred charges - debt " " - other	$= \text{DCDO (I.R.)} * \text{DCDO (C.T.)} / \text{GTPACO (C.T.)}$ $= \text{DCOO (I.R.)} * \text{DCOO (C.T.)} / \text{GTPACO (C.T.)}$
TOTAL ASSETS = 3)+4)+5)+6)		

# ASSETS END OF YEAR

1) GTPAC =SW +TR +stat +GEN +BLDG +LAND +PUC	<p>from C.B.</p> <p>e.g. <math>SW = SW_0 + GA(SW) - RET(SW)</math>  <math>sum = SUMY</math>  <math>= SUMY(I.R.) * PUC(C.T.) / SUMY(C.T.)</math></p>
2) AD	$= ADO + DEPN - RET + NSV$ Summed over all depreciable plant types
3) GTP	$= GTPAC - AD$ $= GTPACO + GA - RET +$ $- (ADO + DEPN - RET + NSV)$ $= GTPACO + GA + - ADO - DEPN - NSV$ $+ DPRTVE + DPRTVC$ $= GTPAC(I.R.) * INV(C.T.) / GTPAC(C.T.)$
4) INV	
5) CA = CTI  +OCA	$= CTIO + DELCTI$ where DELCTI = 0 or determined after S.E. $= GTPACO(I.R.) * OCA(C.T.) / GTPACO(C.T.)$
6) DCH = DCD + DCO	$= GTPACO(I.R.) * DCD(C.T.) / GTPACO(C.T.)$ $= GTPACO(I.R.) * DCO(C.T.) / GTPACO(C.T.)$

## LIABILITIES

BEG. OF YEAR		
1) TOTAL LIABILITIES = TOTAL ASSETS		
2) All components of I.R. Total Liabilities are in same proportion as company totals.		
END OF YEAR		
PR	preferred stock (including convertible)	= $PRO + DELPR$ where PRO is initial PR DELPR from S.E.
EQ	common stock	= $EQO \pm DELEQ$ where EQO is initial EQ DELEQ from S.E.
RE	retained earnings	from R.E. Statement or simultaneous Equation
SC = PR + E + RE	Shareholders capital	
L	debt - total components divided in same proportion as Co. Total i.e. - advances from gov't. - L.T.D. - notes payable	= $LO + NEWDEB - REPL$ where LO is initial debt NEWDEB from S.E. REPL (see D.S.C.)
TC = PR + E + RE + L	Total Capitalization	



# LIABILITIES - END OF YEAR (CONT'D)

CL	current liabilities	$= \frac{CA * CLO}{CAO} \text{ where } \begin{array}{l} CA - \text{year end} \\ CLO - \text{beg. of year} \\ CAO - \text{beg. of year} \end{array}$
DFTAX	deferred taxes	$= DFTAXO + CURDTX + PRDTX$ where DFTAXO is initial CURDTX is current, calculated in C.B. PRDTX is prior, exogenous
DCRO	deferred credits - other	$= \frac{DCRO (C.T.) * \text{Total Liab. (I.R.)}}{\text{Total Liab. (C.T.)}}$

# RETAINED EARNINGS

NETINC	net income	S.E.
- PRDIVI	preferred dividends	see FUNDS statement
= NIA	net income available	
- DIVI	common dividends	S.E.
= 1) CSD		split between 1) common share dividends
+2) TGO		2) transfers to govt. owners in same ratio as Co. Total
- S.I.E.	share issue expense	= CE*DELEQ + CP*DELPR (S.E.)
+ OTHADJ	other adjustments	exogenous
= DELRE	current retained earning	S.E.
+ REO	balance, beginning of year	balance sheet
= RE	balance, end of year	S.E.

# FUNDS STATEMENT

2-68

1) SOURCES =

NETINC	net income	SIMULTANEOUS EQUATIONS (S.E.)
- (PRDIVI + DIVI)	Dividend = preferred + common + transfers ordinary	= $RH00*PRO + RH0N*DELPR/2$ DIVI S.E.
+ CURDTX + PRDTX + DDCRO	Current deferred taxes prior deferred taxes delta deferred credits - other	deferred tax module exogenous = $DCRO - DCR00$
+ DEPN + ONCC ( = DPRTVE - IDC)	depreciation (total other non cash charges = dep'n charged (tools & vehicles) - int. during const.	income statement dep'n = $(1-PSI)* SIGMA*DEPN$ = $IC*GAMMA*(PUC0+PUC)/2$
+ NEWDEB - REPL	new debt repayments	S.E. = $(Co. Total REPL)*IR Assets/(Co.Total Assets)$
+ DELPR + DELEQ	delta preferred delta Equity ( common stock)	S.E. S.E.

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2) USES =

GCE -CNRF (= IDC +DPRTVC) +PLAMPS	Gross Const. Expenditures charges not requiring funds  Dep'n capitalized (tools & vehicles) plant acquired - plant sold	Sum of GA's  = $IC * GAMMA * (PUC. + PUC) / 2$ = $PSI * SIGMA * DEPN$ exogenous
+DELINV	delta investment	INV - INVO
+MISC	miscellaneous	= delta deferred charges - other DCO-DCOO - delta deferred credits - other DCR0-DCR00 + S.I.E. (= $CP * DELPR + CE * DELEQ$ + $CL * NEWDEB$ )  - NSV from change in Assets
+DELWC	delta working capital	= $(CTI + OCA - CL) - (CTIO + OCAO - CLO)$

- 4) solved values from simultaneous equations

All these various sources or calculations are shown in the Table Items of Inter-Regional Financial Statements" below.

2.4.3 The generation of the benchmark financial statements

- i) It is contemplated that the benchmark financial statements for the Regional and Inter-regional sectors and for the company totals will be generated and reported in the final report of the Phase 2, IRA. Here the benchmark financial statements refer to the reported or historical financial statements.

In the case of company totals such benchmark statements are readily available, and the missing input data is minimal. The establishment of benchmark statements is necessary to evaluate the impacts of introducing the various revenue sharing schemes and of other simulations.

- ii) The procedure for generating the Regional benchmarks will be as follows: first, estimate the Regional OPRV by subtracting the IR benchmark OPRV from the CO's total benchmark OPRV, where the IR benchmark OPRV is assumed to equal the sum of the TCTS and Adjacent Member OPRV of the company under consideration.

The Regional OPXP and GTP are similarly calculated as the residuals after subtracting the IR benchmark values from the CO's historical totals of such items. All the other items in the R sector could be computed either by employing the proportionality rule, or by applying CST and ACC block logic and algorithms.

The proportionality rule may be defined as just deriving a ratio of a certain item in a certain total company financial statement to a total value in that statement; then the ratio is applied corresponding value of the similar R statement, yielding the value of the financial item for the R sector. The rule could be used only if the total value for the R sector is known beforehand.

- iii) The IR benchmarks can be generated by applying the subtraction procedure if the R counterparts are already calculated. Otherwise, employ the similar procedures as described in (ii) above to the IR sector. Of course there are a number of algorithms available to complete the generation of such financial statements. The elaboration of such algorithms here would be redundant.

In any event, we need to prepare only two sets of the benchmark statements, one for the CO's totals and the other either for the IR or R sector. The third one could be computed as residual as long as there is positive residual income available for the sector.

- iv) The generation of the financial statements base on the sharing block inputs.

The variations of OPRV of the IR sector due to the employment of the various revenue sharing schemes will cause a chain of changes in the values of other financial items of the IR sector and hence of the company's (co) total revenue.

The procedures for regenerating the financial statements for such variations will be similar as in the case of benchmarks generation. The recalculation of the CO's total financial statements for such cases will be the sums of new IR values and the unchanged R benchmarks. Of course the accounting block logic is available to generate the financial statements.

## 2.4.4 Accounting block simultaneous equation system

$$\begin{aligned} \text{Eq. 1)} \quad & \text{GCE} + \text{CONST}_5 + \text{CONST}_{5A} \cdot \Delta \text{PR} + \text{DIVI} + \Delta \text{CTI} + \text{CONST}_{1A} \\ & = \text{NETINC} + \text{DEPR} + \Delta E (1 - C_e) + \Delta \text{PR} (1 - C_p) + \text{NEWDEBT} \\ & (1 - C_e) + 1 - (1 - \alpha) \cdot t \cdot \text{DEPDIF} + (1 - \alpha) \Delta \text{DEFTX} \end{aligned}$$

$$\begin{aligned} \text{Eq. 2)} \quad & \text{NETINC} = (1 - t) \text{OPRV} - \text{OPXP} - \text{DEPR} - \text{CONST}_2 - \text{CONST}_{2A} \cdot \\ & \text{NEWDEBT} + \beta \cdot \text{OTHERINC} + (1 - \beta) \text{OTHERINC} + 1c \cdot \gamma \cdot \text{GCE} \\ & + \alpha \cdot \Delta \text{DEFTX} - [1 - (1 - \alpha) \cdot t] \cdot \text{DEPDIF} \end{aligned}$$

$$\text{Eq. 3)} \quad \text{NIA} = \text{NETINC} - \text{CONST}_5 - \text{CONST}_{5A} \cdot \Delta \text{PR}$$

$$\text{Eq. 4)} \quad E = E_o + \Delta E$$

$$\text{Eq. 5)} \quad R = R_o + \text{NIA} - C_e \Delta E - C_p \cdot \Delta \text{PR} - \text{DIVI} + \text{CONST}_3$$

$$\text{Eq. 6)} \quad \text{PR} = \text{PR}_o + \Delta \text{PR}$$

$$\text{Eq. 7)} \quad L = L_o + \text{NEWDEBT} - \text{CONST}_4$$

$$\text{Eq. 8)} \quad \text{DCR} = \frac{L_o + L}{E_o + E + R_o + R + \text{PR}_o + \text{PR} + L_o + L}$$

$$\text{Eq. 9)} \quad \text{PCR} = \frac{\text{PR} + \text{PR}_o}{E_o + E + R_o + R + \text{PR}_o + \text{PR} + L_o + L}$$

$$\text{Eq. 10)} \quad \text{ITCE} = \frac{\text{NETINC} + \text{DSC}}{\text{DSC}}$$

$$\text{Eq. 11)} \quad \text{ITCI} = \frac{\text{NETINC} + \text{DSC} + \text{INCTX}}{\text{DSC}}$$

$$\text{Eq. 12)} \quad \text{ROREC} = \frac{\text{NIA}}{1/2 [E_o + E + R_o + R]}$$

$$\text{Eq. 13) } \text{RORC} = \frac{\text{NETINC} + \text{CONST}_2 + \text{CONST}_{2A} \cdot \text{NEWDEBT}}{1/2 [E_o + E + R_o + R + \text{PR}_o + \text{PR} + L_o + L]}$$

$$\text{Eq. 14) } \text{DPR} = \frac{\text{DIVI}}{\text{NIA}}$$

$$\text{Eq. 15) } \text{RORBI} = \frac{\text{NETINC} + \text{DSC} - \text{OTHERINC}(\text{excl. INTCON})}{1/2 [\text{GTP} - \text{GTP}_o + \text{ACCDEPR} - \text{ACCDEPR}_o] - \text{DEFTX}/2 + \frac{\Omega \cdot \text{OPXP}}{\Omega \cdot \text{OPXP} - \text{PNC}}}$$

$$\text{Eq. 16) } \text{RORBE} = \frac{\text{NETINC} - \text{DSC} - \text{OTHERINC}(\text{excl. INTCON}) - \text{INTCON}}{1/2 [\text{GTP} - \text{GTP}_o + \text{ACCDEPR} - \text{ACCDEPR}_o] - \text{DEFTX}/2 + \frac{\Omega \cdot \text{OPXP} - \text{PNC}}{\Omega \cdot \text{OPXP} - \text{PNC}}}$$

$$\text{CONST}_{1A} = \left[ -\Delta \text{DCRO} - \Delta \text{CL} - \text{NSALV} + \Delta \text{OCA} + \Delta \text{I} + \Delta \text{DCO} - \text{AMORT} + \text{CONST}_4 - \text{CONST}_3 \right]$$

$$\text{CONST}_2 = [L_o - 1/2 \text{RL}] * i_o$$

$$\text{CONST}_{2A} = 1/2 * i_n$$

$$\text{CONST}_3 = \text{OTHER ADJ NET}$$

$$\text{CONST}_4 = \text{RL}$$

$$\text{CONST}_5 = \text{PR}_o * \text{P}_o$$

$$\text{CONST}_{5A} = 1/2 \text{Pn}$$

$$\alpha = \text{Flowthrough coefficient}$$

$$\text{B} = \text{Ratio of INCTX on OTHERRINC to OTHERINC}$$

$$\gamma = \text{Ratio of avg PUC to avg ACE}$$



$\Omega$  = coefficient to be applied to OPXP in the RORBI&RORBE

ITCE = Interest Times coverage excluding INCTX

ITCI = Interest Time coverage including INCTX

### 3. TEST RUNS

In this section the aim will be to provide a series of test runs with the component parts of the model as constructed to date in order to:

- 1) Validate the computed results by comparison with published statistics whenever possible.
- 2) Examine the sensitivity of the parts of the model to changes in certain parameters (changes in single parameters while holding all others constant).
- 3) Through the two preceeding steps, obtain a benchmark run using acceptable and reasonable input data and producing valid results.

The sections following are organized in sequence around the 4 principal blocks of the model - namely the operating block, the costing block, the sharing block and the accounting block.

#### 3.1 The Operating Block

Test runs performed within this block will represent component parts of the block taken one at a time. The parts of the block to be examined are:

- i) computations dealing with the estimation of peak and total traffic.
- ii) computation and algorithms dealing with the estimation of load on switching network link.
- iii) examination of the transmission network in terms of survivability (understood in the limited sense used in this report)
- iv) algorithms dealing with the allocation of switching circuit requirements to supporting facility chains in the transmission network.
- v) tests to examine the cost of survivability

Finally one section is devoted to the overall results of the block and the formulation of a benchmark run.

##### 3.1.1 Traffic Parameters

The process for the estimation of traffic is described in section 2.1.1 of this report. In summary, in that section it is noted that we estimate:

- 1) point to point peak traffic (messages)
- 2) point to point traffic for each hour in a 24 hour day, and
- 3) estimation of point to point traffic in call-seconds

The data used in these calculations is briefly:

- 1) For each city or metropolitan area, its population and its geographical location (longitude and latitude)
- 2) data on average holding time per message according to various mileage bands.
- 3) Standard East to West and West to East Traffic profiles for different relative time differentials.

The derivation of these calling profiles is described in the section 2.1.1.

The first series of validation tests to be made involve the variation of the set of cities included in this subblock.

In order to estimate the differences in the amounts of traffic in a network we propose to vary the set of cities in the data specification. The criteria of comparison will be the total number of messages and the total ccs on a company to company basis.

The tests are organized as follows:

- 1) Using standard profiles and standard average holding times estimate numbers of messages and total call seconds on a company to company basis for the data base of 24 cities (base used in IRA I) and their respective city population.
- 2) The same test using the 60 mode (city) switching network data base proposed for use in IRA - II.

The results shown in the tables 3.1.1A and 3.1.1B are obtained through the use of these estimation procedures.

For these two tables we observe that the 24 city sample includes a total population of 5,148,410 persons and the 60 city sample covers a population of 5,915,796 persons. In principle to cover the entire country we should include cities and metropolitan areas in the sample until a population of 22,000,000 persons (Canada's total population) is approached. In practice, however, most of the additional population is located in smaller centres of up to 30,000 persons and these centres generate proportionately less inter-regional traffic than the larger centers (that is that two cities of 30,000 population each will generate less total traffic than one city of 60,000 population). This claim can be verified with reference to the estimation model used. What this means is that, in so far as the major metropolitan centres are included in the model we do not have to base on estimations on samples covering the entire population of the country.

The point to be made however, is that

- 1) the regression estimates were obtained by reference to a data base using city core to city core traffic
- 2) The traffic estimates are there obtained using this regression equation on city core populations.

In the near future these traffic estimates will be redone, using the same regression coefficients but metropolitan area populations since we may assume that the traffic demand follows a pattern between metropolitan areas which is similar to the pattern of demand between city cores.

TABLEAU 3.1.1 A

InterCompany peak traffic ( 24 city data base )

TO	B.C. TEL.	AGT.	SASK. TEL.	MANIT. TEL.	BELL CANADA	N.B. TEL.	MT. & TEL.	NFLD. TEL.	ALL COMPANIES	POPULATION
B.C. TEL.		327.8 466.9	28.4 40.4	17.4 24.8	106.8 152.2	3.0 4.2	0.2 0.3	0.9 1.3	484.5 690.1	473,824
AGT.	324.8 462.8		177.0 252.1	65.1 92.7	282.3 402.2	7.4 10.5	0.5 0.7	2.2 3.1	859.3 1224.2	841,471
SASK. TEL.	29.2 41.6	188.3 261.1		42.1 60.0	89.5 127.5	2.1 3.0	0.1 0.2	0.6 0.9	346.9 494.3	265,918
MANIT. TEL.	17.6 25.1	66.1 94.2	41.4 59.0		172.8 246.3	3.3 4.7	0.2 0.3	0.9 1.2	302.3 430.8	246,246
BELL CANADA	105.1 149.7	278.9 397.3	85.5 121.8	168.6 240.2		326.1 464.6	10.3 14.7	36.0 51.3	1010.5 1439.6	2,994,520
N.B. TEL.	3.1 4.4	7.7 11.0	2.1 3.0	3.4 4.9	346.2 493.3		1.3 1.9	3.9 5.5	367.7 524.0	89,039
MT. & TEL.	0.2 0.3	0.5 0.8	0.1 0.2	0.2 0.3	11.4 16.2	1.3 1.9		1.2 1.7	14.9 21.5	123,035
NFLD. TEL.	1.0 1.3	2.3 3.3	0.6 0.9	0.9 1.3	38.4 54.6	3.9 5.5	1.1 1.6		48.2 68.5	114,411
ALL COMPANIES	481. 685.3	866.6 1234.7	335.1 477.4	297.7 424.2	1047.4 1492.3	347.1 494.5	13.7 19.6	45.7 65.0		5,148,410

Messages  
ccs

TABLEAU 3.1.1 B

InterCompany peak traffic ( 60 city data base )

TO	B.C. TEL.	AGT.	SASK. TEL.	MANIT. TEL.	BELL CANADA	N.B. TEL.	MT. & TEL.	NFLD. TEL.	ALL COMPANIES	POPULATION
B.C. TEL.		343.7 489.7	33.0 47.0	19.5 27.8	117.7 167.7	4.5 6.5	0.5 0.8	1.4 2.0	520.3 741.5	487,998
AGT.	341.1 486.0		215.5 306.9	76.4 108.8	321.7 458.3	11.7 16.7	1.4 2.0	3.3 4.7	971.1 1383.4	925,976
SASK. TEL.	34.2 48.7	223.9 319.0		57.1 81.3	112.7 160.5	3.7 5.3	0.4 0.6	0.8 1.2	432.8 616.6	339,315
MANIT. TEL.	19.8 28.3	77.8 110.8	56.1 79.9		207.4 295.5	5.5 7.9	0.6 0.9	1.0 1.5	188.2 274.7	278,229
BELL CANADA	115.9 165.1	317.3 452.1	107.1 152.6	202.4 288.3		520.0 740.9	44.5 63.4	39.2 55.8	1346.4 1918.2	3,315,016
N.B. TEL.	4.8 6.8	12.4 17.6	3.7 5.3	5.7 8.1	560.0 797.8		30.8 43.8	6.4 9.1	623.8 888.5	217,561
MT. & TEL.	0.6 0.9	1.5 2.2	0.5 0.7	0.7 1.0	49.9 71.1	32.2 45.9		1.7 2.5	87.1 124.2	234,776
NFLD. TEL.	1.5 2.1	3.5 5.0	0.9 1.2	1.1 1.5	41.8 59.5	6.3 9.0	1.7 2.3		56.8 80.7	116,921
ALL COMPANIES	517.9 737.9	980.1 1396.4	416.8 593.6	362.9 516.8	1411.2 2010.4	583.9 832.2	79.9 113.8	53.8 76.8		5,915,795

Messages  
ccs

### 3.1.2 Utilization of the Switching Network

The test runs made on this subblock of the program are intended to fulfill the following goals.

1. to examine the validity of the traffic data (hourly traffic matrices) developed in the preceeding subblock in relation to the size of the network specified;
2. to produce, assuming data is valid, sample profiles of usage of switching links which show the message streams using each switching link;

This second goal is intended more specifically to show the relatively high powers of the software developed for producing results which may be used in cost analysis.

In order to verify the validity of the results (goal 1), we must calculate a number of indices related to switching networks, the most important of which is overall blocking probability. We can expect that the probability of blocking for a particular message stream should not exceed an average of  $x\%$  since this is the true quality of service parameter.

If such is the case on any particular message stream, we must assume that either traffic data, or circuits in place data are incorrect, and hence overall blocking probability is the principal tool for validation of the algorithms and data.

The types of results produced for costing analysis include, as mentioned above, the total traffic profiles for any particular switching link. On the same graph, for example, it will be possible to plot total offered traffic to the link (for 24 hours), total carried traffic for the link (for 24 hours), the component breakdown of total carried traffic into its component message streams. It is also possible to plot the probability of loss (or overflow) on a separate graph and compare these two results. Subsequently, an analysis of responsibility for peak usage can be made, comparing the different users of one link at the peak and the off peak period.

The benchmark data used for this run includes:

1. a series of 24 hourly traffic matrices derived in the traffic subblock;
2. a specification of this initial state of a switching network. These tests are based on a simplified 24 node version of the Canadian network which is used for tests because it is more manageable from the software point of view. The initial state specification includes a list of nodes and a list of point to point trunk groups including the number of trunks in the group and its type (H.U. group, Full group, Final group).

The results of these initial runs, and the benchmark data will be presented with analysis in an appendix to this report.

### 3.1.3 Survivability

As explained in the section 2.1.3, an algorithm to enumerate the node disjoint chains between any two points in a network has been developed. The test runs made on this algorithm alone (i.e. without integrating with the other parts of the software) were principally to enumerate the node disjoint chains in the western section of the base transmission network used. The purpose here is twofold:

1. to make validation and performance tests on the software;
2. to examine the general availability of 2 or more node disjoint chains in the transmission network in order to show if imposing survivability constraints on public message traffic is feasible or not;

The results of validation tests on this algorithm show that the software is operational but that in larger problems the computing time necessary to find disjoint chains is somewhat high. As a consequence, a first priority for the coming phase will be to improve the efficiency of the software in this area.

The results of tests on the full network will be shown in an appendix to this report.

### 3.1.4 Allocation of circuits on the transmission network with different objective functions

The test runs performed on this section of the model are based on the following data:

1. the initial state of the transmission network links in terms of capacity (these are capacities in the allocation program);
2. the initial state of switching network links (these are circuit demands in the allocation program);
3. the number of private line circuits required from point to point;
4. the selection of an objective function for the allocation of circuits.

The functions used in these tests were a) minimize the number of circuit miles in use for a particular allocation b) maximize the number of circuits used per dollar of annual holding cost;

The results of these tests will be used to analyse

1. the total "cost" of the allocation in terms of the objective function used;

2. the comparison of the two allocations to discover whether the objective functions used have a significant effect on the result. Normally, the effect should be significant but in the case of a relatively unmeshed network the resulting allocation may vary only marginally because of a lack of choice of chains to use;
3. the "natural" or resulting survivability of the solution to determine what degree of survivability is obtained without imposing these constraints;

The software developed for this problem is operational but requires a large relative amount of time in calculations and hence some additional effort is required to reduce computational time.

The results of these test runs will be presented and analyzed in an appendix to this report.

#### 3.1.5 Cost of Survivability

Two additional test runs are provided herein to examine the difference in "cost" (measured by the objective function) when all feasible survivability constraints are imposed before hand. These runs use the same objective functions as 3.1.4 and the analysis resulting from tests in 3.1.3 to establish feasibility of survivability constraints. The result of these tests, again, will be presented and analyzed in an appendix.



### 3.2 Costing Block

In the following sections, the main emphasis will be to produce benchmark results in order to validate our costing processes.

#### 3.2.1 Reproduction Cost Valuation of Assets

The data on the transmission network contains a representation of the Canadian long-haul microwave system and includes in total 60 CSPA, about 200 links and 125 transmission nodes. In this data base the companies owning these facilities include all the TCTS carriers, (except Newfoundland Telephone Company), Quebec Telephone and CN/CP. (There are of course others companies owning microwave facilities - but these facilities are relatively few and have been omitted from this data base. In the case of Newfoundland Telephone Company, the long-haul microwave facilities used for Trans-provincial and Blanc Sablon (Quebec)/Sydney (Nova Scotia) communications are leased from CN/CP. The coaxial cable facilities of Newfoundland Telephone Company have not been included since appropriate valuation functions have not yet been developed for this type of equipment. Furthermore, the CN/CP longhaul network is omitted, again for the reason that appropriate valuation functions are not yet developed.

Accordingly the table 3.2.1A shows the reproduction cost valuation for all TCTS carriers and one additional company, Quebec Telephone, and provides a numbers of statistics on the data base.

Statistics provided include node and link counts as well as measures of investment per circuit mile. The various categories of assets are regrouped into the 6 accounting categories for which asset values are provided and a corresponding percentage breakdown of assets is computed. Referring to the table we note that the percentage breakdown of assets into accounting categories is on the average 30 to 35% switching, 60 - 65% transmission, 4 to 8% general equipment, 1% buildings, and 1% land. These are percentage breakdowns obtained not by imposition before hand but by a general analysis of the types of equipment evaluated, assigning each to the most reasonable accounting category. This assignment of categories was done according to the following plan:

i) antennas	transmission
ii) Waveguide equipment	transmission
iii) Mode Filters	transmission
iv) R.F. Branching Equipment	transmission
v) Alarm and order Wire	
Equipment	general

vi) Protection switching	
Equipment	transmission
vii) Power system	transmission
viii) F.M. Moderns	transmission
ix) Transceiver	transmission
x) Site costs	transmission, general, land and building

We note also that percentage breakdowns vary some what from the general pattern for New Brunswick Telephone Company and Maritime Telephone and Telegraph where the proportions of switching and transmission are 50-55 and 35 to 40 respectively. This difference can be attributed to the relatively large number of CSP's in a smaller territory. Long distance transmission lines span relatively shorter distances in these areas.

The total asset valuations obtained in the model compare favorably with TCTS assigned plant investment. For example, a total of 57.5 million estimated for B.C. Telephone compares to 90 - 100 million TCTS assigned plant investment. But 50 - 60% of this latter figure is local and terminating equipment, the residual being interregional assets which is roughly comparable to our estimate.

Finally, we note that the total value of switching equipment is estimated only as an approximation since in an isolated test run not preceded by runs of the operating block we are unable to estimate the number of switched trunks at each switching machine and hence the switching assets are calculated on the assumption that each machine has fewer than 300 switched trunks. This estimation is therefore a minimum and will be increased somewhat in the benchmark runs when the size of switching machines is taken into consideration.

TABLEAU 3.2.1A

## STATISTICS BY COMPANY

	BCT	AGT	SASK	MANT	BQAN	QUET	NBT	MTT
<b>NODES</b>								
TOTAL NUMBER	13	24	20	9	34	7	15	15
SWITCHING NODES	5	6	6	3	11	3	8	8
BRANCHING NODES	5	7	4	4	6	0	4	4
REGULAR NODES	3	11	10	2	17	4	3	3
<b>LINKS</b>								
TOTAL NUMBER	17	32	24	15	45	8	17	20
TOTAL DISTANCE	1226	2125	1026	198	2961	704	631	801
CIRCUIT-MILES	870960	1738080	865740	626280	2600160	414180	351960	602520
INVEST. PER CIRCUIT-MILE	39	28	28	31	36	34	44	31
<b>QUANTITIES OF FACILITIES</b>								
SWITCHING MACHINES	6	7	6	3	17	3	5	8
TERMINAL REPEATERS	6	7	6	3	17	3	8	8
BRANCHING REPEATERS	1	2	2	3	9	2	1	2
REGULAR REPEATERS	52	87	32	33	131	30	19	31
<b>VALUE OF ASSETS</b>								
SWITCHING MACHINES	18000000	21000000	18000000	9000000	51000000	9000000	24000000	24000000
TERMINAL REPEATERS	2775330	2615410	2269830	1076010	7998470	1076010	3102630	2803910
BRANCHING REPEATERS	5694594	7832336	4956094	5569433	12979495	1299398	4545650	4390649
REGULAR REPEATERS	31037488	49135840	21703376	17038032	37894800	15228855	10950775	15449514
<b>ACCOUNTING BREAKDOWN</b>								
SWITCHING	18000000	21000000	18000000	9000000	51000000	9000000	24000000	24000000
TRANSMISSION	34012656	50273952	24477808	19965664	94117248	14427517	15688880	19158544
GENERAL EQUIP.	3819722	6459917	3101472	2592804	10255470	2201844	2055370	2735511
LAND	837500	1424999	675000	562500	2250000	487500	437500	599999
BUILDINGS	837500	1424999	675000	562500	2250000	487500	437500	599999
STATION	0	0	0	0	0	0	0	0
<b>PERCENTAGE BREAKDOWN</b>								
SWITCHING	31	26	38	27	31	33	56	50
TRANSMISSION	59	62	52	61	58	54	36	40
GENERAL EQUIP.	6	8	6	7	6	8	4	5
LAND	1	1	1	1	1	1	1	1
BUILDINGS	1	1	1	1	1	1	1	1
STATION	0	0	0	0	0	0	0	0
<b>TOTAL ASSETS</b>								
	57507378	80583867	46929280	32683468	159872718	26604461	42599050	47144053

STOP  
\*END  
\*GO

0

### 3.2.2 An Evaluation of the Results of Test Runs of the Aging and Depreciation Algorithm (ADA)

One of the most rigorously developed algorithms of the IRA project during the present phase is the AGING & DEPRECIATION ALGORITHM (ADA). The ADA is a combination of the five different algorithms, namely, an algorithm for determining ages or vintages, an algorithm for conversion of reproduction costs into historical costs, and algorithms for computing dollar values of the depreciation reserves and their ratios to the GTP for both ASL and ELG methods and finally an algorithm to compute the annual depreciation rates for the ELG and ASL depreciation methods. There has been a minor revision of the ADA concerning the feature of the conversion of reproduction costs into historical costs since the publication of the Interim Report of the IRA Phase 2. The revised AGING ALGORITHM is:

$$(1) \text{ --- } GTP = \sum_{x=0}^T Vx,$$

$$(2) \text{ --- } Vx = GA_i * R^{(T-X)} * SRV$$

$$(3) \text{ --- } GA_i = \frac{GTP'}{R^{T-X} * P^X * SRV}$$

where

- GTP : gross telephone plant at historical cost by categories of plant.
- GTP' : gross telephone plant in reproduction cost by plant categories.
- Vx : dollar value of vintage or age X (placed at X years ago) at historical cost.
- Vx' : dollar value of vintage X in reproduction cost.
- GAi : historical cost of the initial, oldest vintage of a mature plant.
- R : compound growth factor of the gross additions at historical cost.
- P : compound growth factor of reproduction costs.
- T : maximum life in calendar years which corresponds to GAI.
- L : average service life.
- X : age in calendar years.
- SRV : % of the survivors of vintages according to a certain type of survivor curve.

So far about three hundred test runs have been conducted with the ADA. The purposes of such test runs are primarily to ensure the operationality of the computer program and to check the validity of the logic of the program. Another purpose was to conduct sensitivity analyses of the parameters involved in the ADA. The sensitivity analysis refers to the determination of the directions and degrees of the differential impacts of variations of the parameters to the dependent variables. The parameters considered here and ranges of their variations are listed below:

- i) Types of survivor curves: Loo, L.5, L.1, L5, S-.5, So.o, S6, R1.5, R2.5, R4, out of the twenty three lowa-type SRV's.
- ii) Types of depreciation methods: ASL and ELG
- iii) Average service life: 30 years and 20 years
- iv) Maximum life: 50 years, 30 years and 20 years
- v) Salvage values: uniformly assumed zero
- vi) P: uniformly assumed unity
- vii) R: 1.15, 1.10, 1.05
- viii) GTP': uniformly assumed unity

The four dependent variables elected here are:

- ix) GAI: dollar value of the initial gross addition currently surviving at historical cost
- x) GAc: dollar value of the latest gross addition currently surviving at historical cost
- xi) DRR: depreciation reserve ratios (over the GTP at the end of the year)
- xii) ADR: annual depreciation rates

Since the variations of these parameters have to be done in a combinational manner and can easily become too complex, we were forced to proceed in a fashion of partial differentiation, by changing at a time the values of one or two parameters while keeping the values of the remaining parameters unchanged.

Table 1 represents the results of the first six test runs based on SRV = Loo, ASL, R = 1.1 and a range of values for average life and maximum life. From column 1, we can see the dollar value of the initial installation fifty years ago which still survives, GAI, is one mill and the survivor of the latest gross addition, GAc, 10.8 cents for the given dollar value of one dollar for GTP. The dollar value of depreciation reserve is 22.72 cents and its ratio 22.72%. Annual depreciation rate is 3.33%, which is equivalent to the inverse of the value of average service life 30. The remaining columns can be similarly read. For these tests the ASL method was used. A graphical presentation of Table 1 is given in Diagram 1.

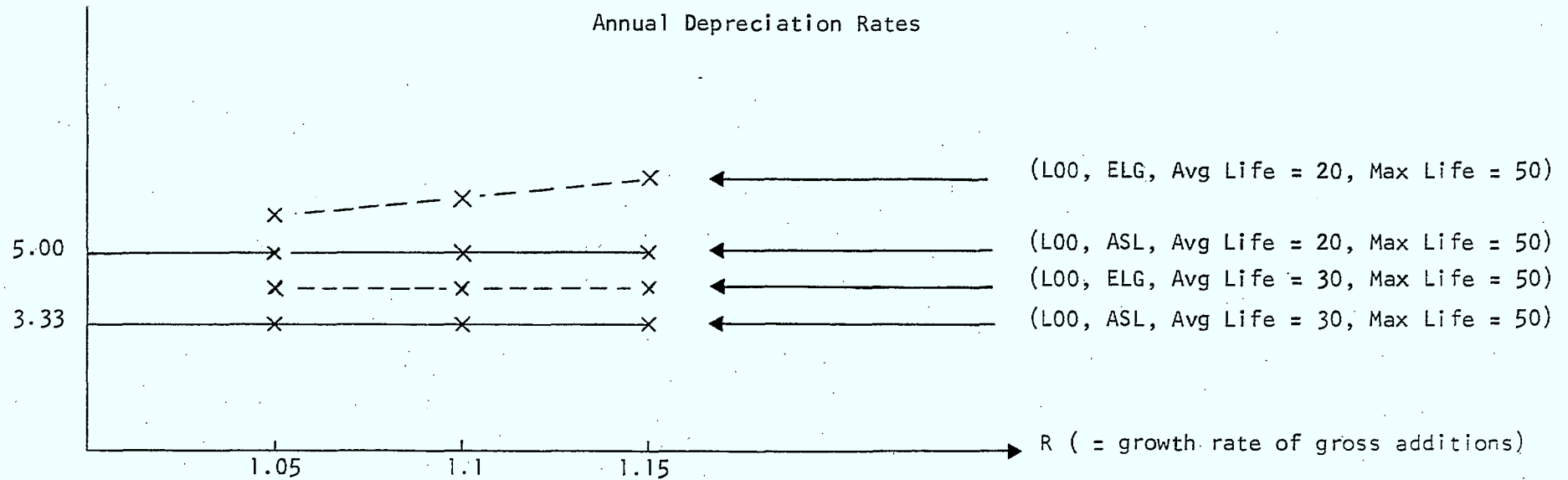
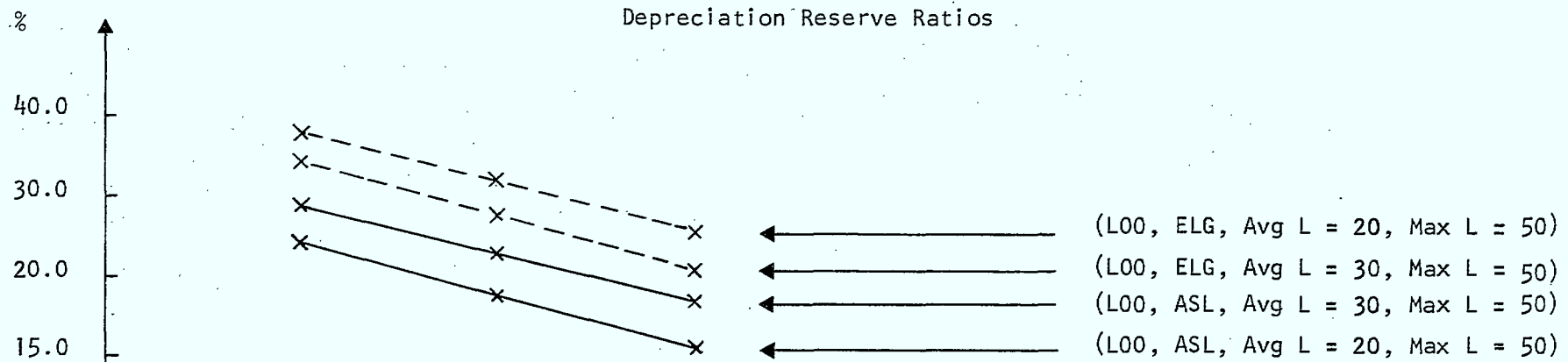
TABLE 1

Sensitivity Analysis  
of the Aging & Depr Algorithm

(Predetermination of parameters: P = 1,  
GTP' = GTP = 1, SALV = e)

	Col. 1	Col.2	Col.3	Col.4	Col.5	Col.6
Type of SRV	Loo	Loo	Loo	Loo	Loo	Loo
Method of Depr	ASL	ASL	ASL	ASL	ASL	ASL
AVG LIFE	30	30	30	20	20	20
MAX LIFE	50	30	20	50	30	20
R	1.1	1.1	1.1	1.1	1.1	1.1
GAI	.001	.006	.017	.001	.007	.019
GAc	.108	.110	.117	.120	.121	.126
Depr Res Ratio %	22.72	43.12	60.48	19.36	29.50	46.19
Annual Depr Rate %	3.33	3.33	3.33	5.00	5.00	5.00

DIAGRAM 1



The first three columns tell us that the depreciation reserves are highly sensitive to the variations of maximum life for a given average service life, while the annual depreciation rates are oblivious to these variations in this ASL regime. More specifically, Columns 1 and 2 reveal that given thirty year value for the average service life, the decrease of maximum life of 20 years from 50 years to 30 years causes to almost double the value of depreciation reserve ratios from 22.72% to 43.12%, a 90% increase. (See Diagram 2).

Let's look into impacts of the variations of the average service life, by comparing Col. 4 and 5. Col. 4 is identical to Col. 1 except a different average service life, i.e. 20 years instead of 30 years. The impact of this difference of average service lives are evident: GAI remains same but GAc rises to 12 cents, a 2 cents increase, and the depreciation reserve ratio drops to 19.36% from 22.72%, while the annual depreciation rate climbs to 5.00%.

From the above observations, we may conclude that the reserve ratios are more sensitive to the variations of maximum life than to variations of average service life, again in the regime of ASL depreciation method.

Another noticeable feature is that in this particular combination the smaller the average service life, the smaller the increase of the depreciation reserve ratios. For instance, for the case of Cols. 4 and 5, the reduction of max life from 50 to 30 with the same 20 years of average service life causes an increase of the reserve ratio from 19.36% to 29.50%. It is 50% increase in contrast to 90% increase (from 22.72 to 43.12) in the case of 30 year average service life. This phenomenon can be viewed from another angle as that the greater the depreciation rate, the lesser the impact and the reserve ratio of the variation of max life. This inference holds true to the case of ELG method too, as can be through the Table 2 and Diagram 2.

Table 2 exhibits the results of runs for the ADA. Table 2 contents are identical as Table 1 except for the depreciation method in the ELG. By comparing these two tables we may observe that firstly, the switching of the depreciation methods did not affect values of the GAI and GAc. In other words, the age distribution or values of vintages is obvious to the depreciation methods being used. Secondly, the annual depreciation rates in the case of ELG method vary with both variations of the average service life and max life, while in the case of ASL method they are influenced by variation of the average life but not by that of max life. Thirdly, the relationships between the reserve ratios and max life are opposite for these two depreciation methods; while the relationship in the case of the ASL Method is a strong inverse one, it is a weak proportional one in the case of ELG method. (See Diagram 2)



DIAGRAM 2

Variations of the Depreciation Reserve  
Ratios due to the changes of Maximum Life

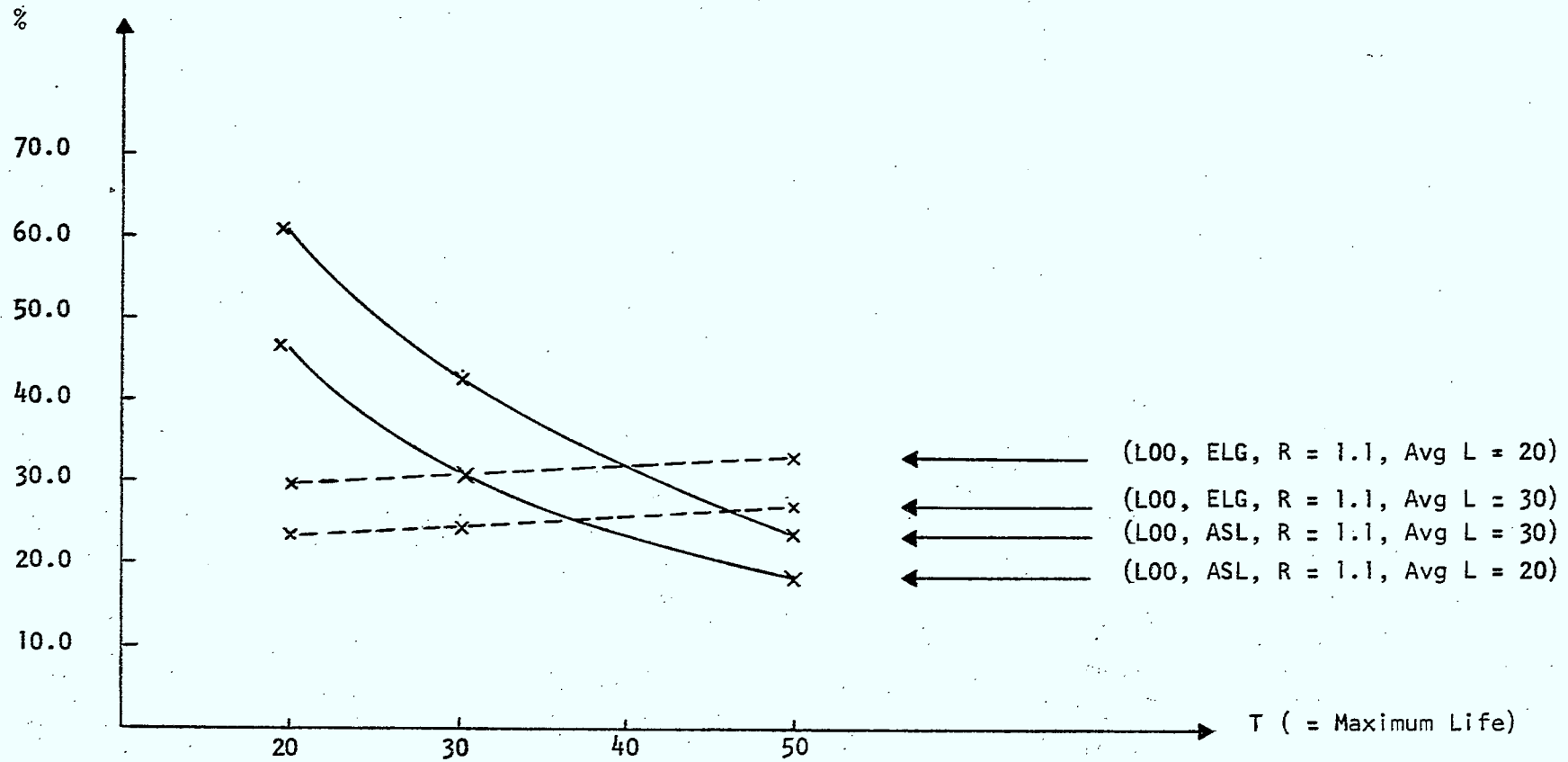


TABLE 2

SENSITIVITY ANALYSIS  
OF THE AGING & DEPR ALGORITHM

(Predetermination of parameters: P=1, GTP'=GTP=1, SALV= 0)

	Col 1	Col 2	Col 3	Col 4	Col 5	Col 6
Type of SRV	Loo	Loo	Loo	Loo	Loo	Loo
Method of Depr	ELG	ELG	ELG	ELG	ELG	ELG
AVG LIFE	30	30	30	20	20	20
MAX LIFE	50	30	20	50	30	20
R	1.1	1.1	1.1	1.1	1.1	1.1
GAI	.001	.006	.017	.001	.007	.019
GAc	.108	.110	.117	.120	.121	.126
Depr Res Ratio %	26.68	25.86	23.75	31.42	31.07	29.46
Annual Depr Rate %	4.63	4.68	4.82	6.53	6.57	6.72

The results of the test runs related to other lowa-type survivor curves reveal that they are essentially similar in their behaviours though there are a considerable variations in the values. Such a similarity is shown in the Diagram 3 below, which concerns the depreciation reserve ratios of a number of curves according in the function of growth rates of gross additions with the assumptions of average life of twenty years, max life of thirty years and on the basis of ASL depreciation methods. Here a general observation is made that the higher the numbers attached to a type of curves, such as L5, the greater are the values of depreciation reserve ratios. Also the higher the number of a SRV curve in a given type, the more sensitive the reserve ratios to the growth rates of gross additions, R's. These behaviours are fundamentally related to the shapes of the SRV's. (see Diagram 4 and 5).

A final comment: the ADA is a flexible conversational time-sharing program capable of handling any types of life and depreciation properties. Presently the ADA program contains the series of the lowa-type survivor curves which are most suitable for dealing with the mass-type properties. However the majority of telephone plants in switching and transmission is not mass-type of property but integrated types. In the next phase of the project the integrates type survivor curves will be incorporated in the ADA program.

DIAGRAM 3

Variations of the Depreciation Reserve Ratios  
due to the changes of SRV and R  
(Cases of ASL, Avg L = 20, Max L = 30)

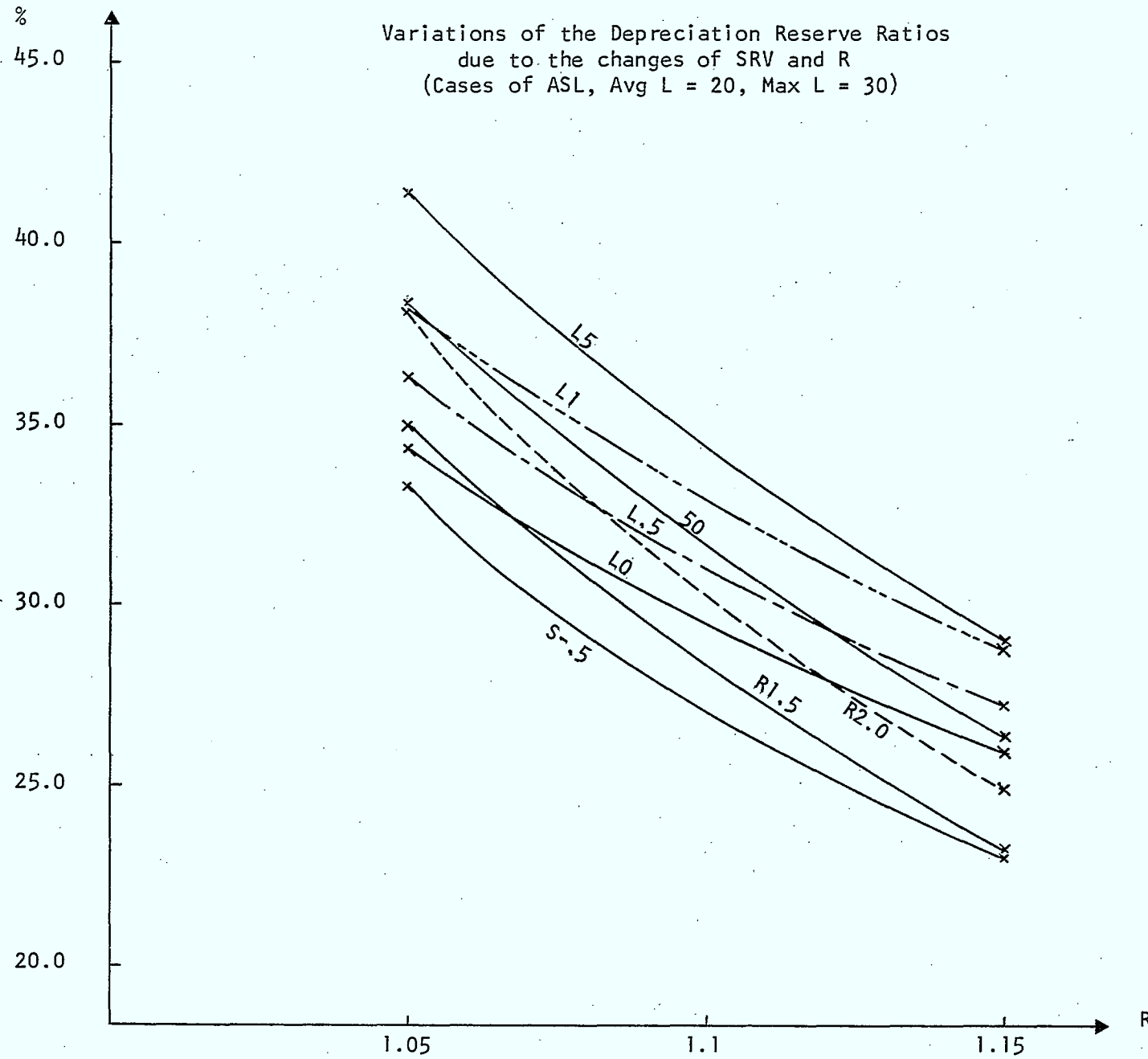


DIAGRAM 4

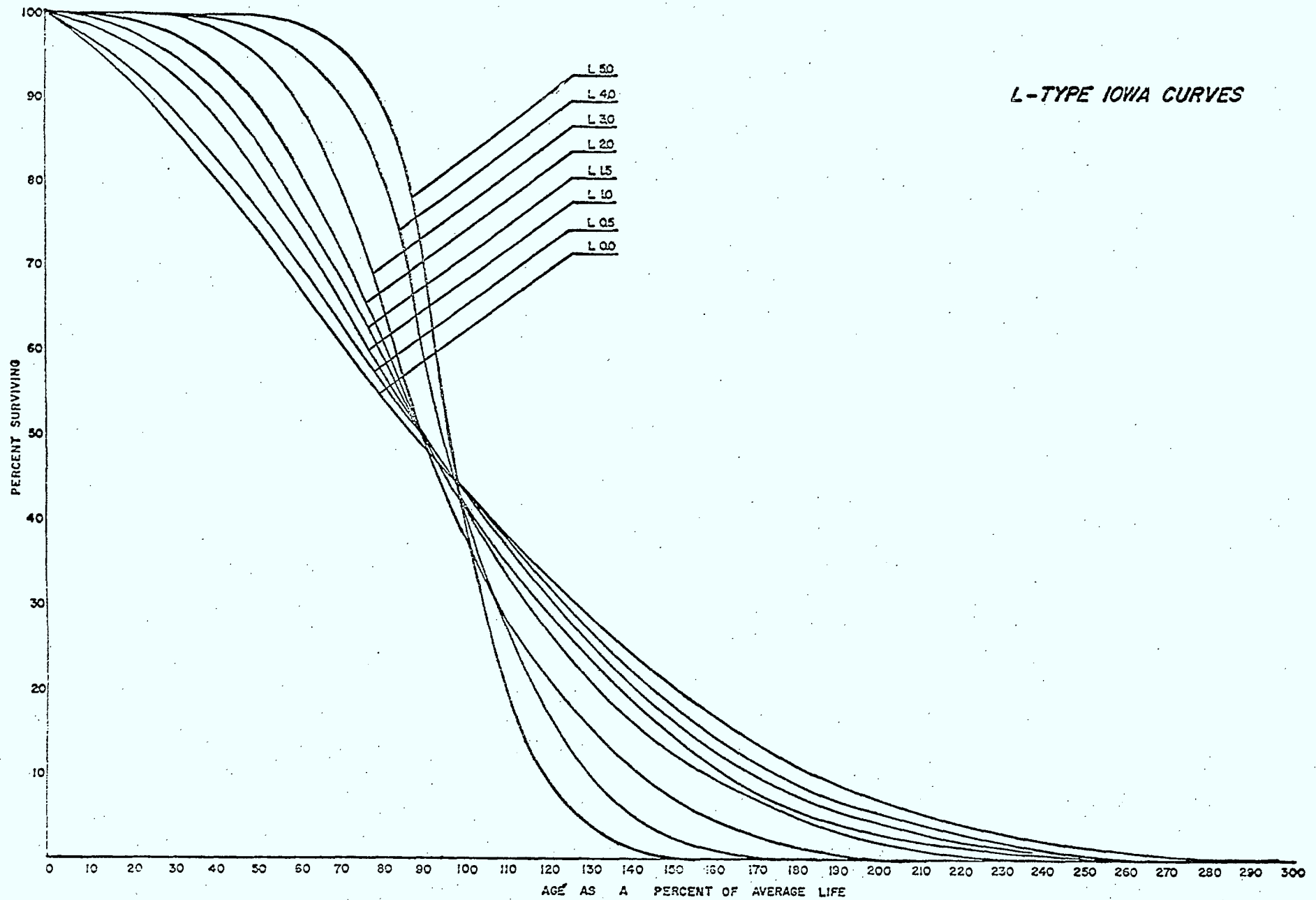
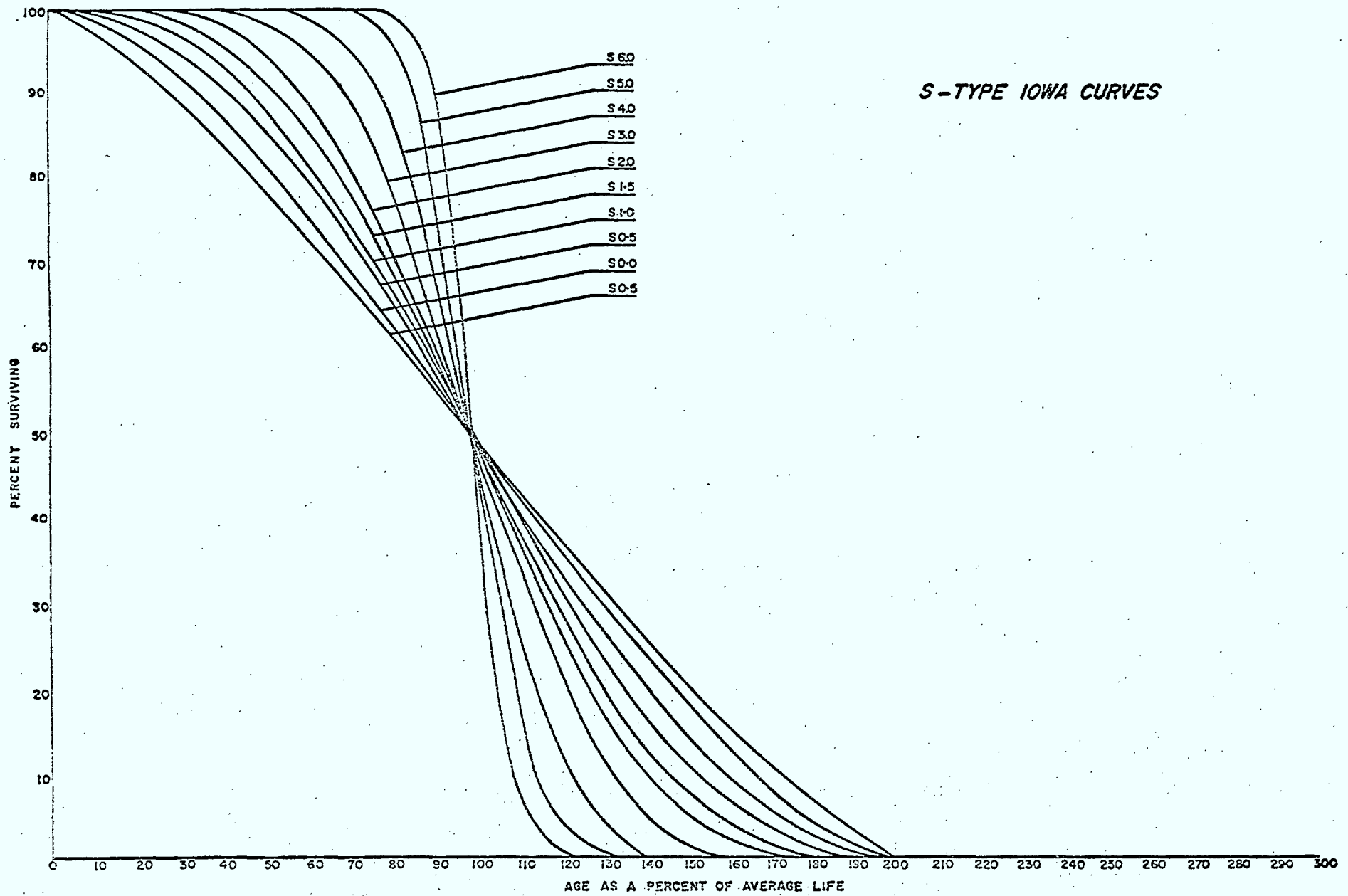


DIAGRAM 5



## 3.2.3 Bench mark - Incurred Cost

By applying the Incurred Cost Rate Table<sup>\*</sup> to Tableau 3.2.1A. The Accounting Breakdown, the Tableau 3.2.3A below is provided. The incurred cost rates have been multiplied against reproduction values. Ideally historical values will be calculated and used instead of reproduction values. Note as well that no incurred cost rates were available for Quebec Telephone assets and no asset values are presently assigned for Newfoundland Telephone so these two carrier are missing from the tableau below.

It is the Total data over all plant types that would be used in a TCTS full division of revenue (accounting for terminating costs not presently in the model) and Old Commonwealth sharing scheme.

TABLEAU 3.2.3A

Incurred Costs per Carrier by Plant Type (in \$1000/S)

	SW	TR	Gen.	Land	Bldg
B.C. Tel.	5706	11,462	1348	270	257
Alta. Tel.	4158	10,758	1576	241	268
Sask. Tel.	4662	6,756	918	194	189
Man. Tel.	1701	4,073	612	88	100
Bell Canada	15,708	30,870	3,528	709	673
N.B. Tel.	5,880	4,121	590	102	102
MT & Tel.	5,904	5,058	955	184	177

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\* See page 2-48.

### 3.3 Sharing Block

#### 3.3.1 Revenue Tests

The methods used in the calculation of revenue have been applied to a 24 city data base and a 57 city data base in order to

- 1) observe the variation in revenues when a more complete switching data base is incorporated
- 2) obtain a benchmark of revenues used in the model for comparison with published statistics where these are available.

Accordingly the results shown in tableaux 3.3.1A and 3.3.1B have been obtained for interregional revenues from the public switched network on a company to company basis.

Based on an analysis of these two tableaux we obtain the tableau 3.3.1c which gives the percentage increase in revenue relative to the increase in total population when a 57 city data base is substituted for a 24 city data base.



TABLEAU 3.3.1A

Interregional collected revenues  
Public Message  
Company to Company

(\$1,000'S)  
(24 CITY DATA BASE)

F R O M	TO	BC TEL.	AGT	SASKT	MANIT	BELL CAN.	NB TEL	MTT	NFLD T.	ALL COMPANIES	POPULATION
	BC Tel		10282.	1058.	737.	5484	152	10	46.	17770.	473,824
	AGT	9684.	-	5441.	2431	13942	382	26	113.	32019.	241,471
	SASK. Tel	964.	5343.	-	1237	4085	108	7	30.	11775.	265,918
	MTS MANIT. T.	660.	2345.	1215.	-	7066	158	10	43.	11497.	246,246
	BELL CAN.	4546.	12193.	3703.	6532	-	10404	398.	1543.	39318.	2,994,220
	NB Tel.	127.	336.	96.	143	10471.	-	39	135.	11348.	89,039
	MT & T	9.	23.	7.	9.	416.	41.	-	31.	537.	123,035
	NFLD T.	37.	91.	25.	37.	1455.	129.	29	-	1804.	114,411
	ALL COMPANIES	16027.	30614.	11544.	11128.	42921.	11374.	518	1942	126068.	5,148,410

TABLEAU 3.3.1b

Interregional collected revenues  
(Public Message)  
Company to Company

(\$1,000's)  
(57 CITY DATA BASE)

F R O M	To	BC TEL	AGT	SASKT	NTS	BELL CAN.	NB TEL	MTT	NFLD T.	ALL COMPANIES	POPULATION
	BC Tel	-	10783.	1231	825	6041	233	28	66.	19208 (12%)	487,998
	AGT	10171	-	6570	2831	15878	604	72	157	36283 (22%)	925,976
	SASK T.	1130	6476	-	1636	5123	187	22	41	14622 ( 9%)	339,315
	MAN. T.	742	2735	1607	-	8432	259	30	53	13858 ( 9%)	278,229
	BELL. CAN.	5013	13870	4620	7787	-	16678	1530	1692	51189 (32%)	3315,016
	NB. TEL.	197	538	169	239	17022	-	527	218	18909 (12%)	217,561
	MT T.	25	67	21	29	1625	551	-	50	2367 ( 1%)	234,776
	NFLD T.	55	129	35	45	1597	205	45	-	2111 ( 1%)	116,921
	ALL COMPANIES	17333	34598	14253	13392	55718	18717	2254	2277	158547 (100%)	5,915,792

TABLEAU 3.3.1c

Analysis of changes in revenue  
relative to changes in the number of cities  
in data base

Company	Total Population for			Revenues (\$1,000's)				
	24 cities	57 cities	% Change	total collected 24 cities	57 cities	% Change	#of CSP	
B.C. Tel.	473,824	487,998	3.0%	17770	19208	8.1%	4	6
A & T	841,471	925,976	10.0%	32019	36289	13.3%	2	6
SASK. Tel.	265,918	339,315	27.6%	11775	14622	24.2%	2	6
MANIT. Tel.	246,246	278,229	13.0%	11497	13858	20.5%	1	3
Bell Canada	2,994,520	3,315,016	10.7%	39318	51189	30.0%	10	13
NB Tel	89,039	217,561	144.3%	11348	18909	66.6%	2	9
MT & Tel	123,035	234,776	90.8%	537	2367	340.7%	1	10
NFLD Tel.	114,411	116,921	2.2%	1804	2111	16.9%	2	4
Totals	5,148,464	5,915,792	14.9%	126068	158547	25.8%	24	57

We note that the increase in revenues is greater proportionately than the increase in population for all cases except Saskatchewan and New Brunswick. This reflects the fact that additional cities in the 57 city sample in these provinces are relatively small and relatively isolated from other large centres and hence do not generate a proportionate increase in revenue. On the other hand it is apparent, by using a 24 city data base, the resulting revenue estimates will be too low and that inclusion of the 57 mode network is a significant improvement at this stage.

TABLEAU 3.3.1d

TCTS Member Toll Revenue Collected Revenues  
(after settlement) by Class of Service (\$1000, 1971)

Member	Trans- Canada	Adjacent Members	Total
BC Tel	30,255 (74%)	10,708 (26%)	40,963 (15%)
AGT	22,772 (65%)	12,184 (35%)	34,956 (13%)
Sask Tel	13,367 (71%)	5,377 (29%)	18,744 ( 7%)
MTS	14,216 (60%)	9,625 (40%)	23,841 ( 9%)
Bell Canada	100,205 (89%)	12,677 (11%)	112,882 (43%)
NB Tel	8,848 (55%)	7,252 (45%)	16,100 ( 6%)
MT & T	8,962 (75%)	3,066 (25%)	12,028 ( 5%)
NFLD Tel	3,860 (83%)	770 (17%)	4,630 ( 2%)
Total -All Members	202,485 (77%)	61,659 (23%)	264,144 (100%)

Several comparative analyses can be made between tableau 3.3.1b Interregional collected revenues (public message) company to company and tableau 3.3.1d TCTS Member Toll Revenue - Collected Revenues by Class of Service. (after settlement)

i) Total Collected Revenue:

The All Companies Total Collected revenue of Tableau 3.3.1B is \$158,547,000. Although Tableau 3.3.1D represent post-Settlement collected revenues i.e. those collected by carriers after settlements and negotiations the total of these \$264,144,000 is of comparative value. This latter total of \$264 million for post-settlement revenue is equal to pre-settlement revenue since this same sum is distributed in any given Settlement Scheme.

The difference between these two totals of collected revenues shows the IRA figure \$105 million lower. This difference is attributable to:

a) exclusion of private line and program transmission revenue.

Tableau 3.3.1B is solely for public message revenue. These other revenue sources are approximately 10% of total.

b) exclusion of Canada - U.S. revenue which is approximately \$100 million.

Incorporation of a) and b) would bring these totals in close accord and this fact shows the IRA revenue calculation to be realistic.

ii) Distribution of collected revenue among members: Even though the total collected revenue is realistic, as just mentioned, the distribution of this total does not correspond between Tableau 3.3.1B and D. For example, B.C. Tel, A.G.T. and Bell Canada are 12%, 22% and 32% in Tableau 3.3.1B respectively compared to 15%, 13% and 43% in Tableau 3.3.1D. \* Once again it must be pointed out that Tableau 3.3.1B is pre-settlement and Tableau 3.3.1D is post-settlement. When benchmark Sharing schemes are applied to distribute pre-settlement collected revenue these results will be of comparative interest. Also, with the inclusion of Canada - U.S. collected revenue these percentages might compare more favourably, especially since Bell Canada collects the majority of Canada - U.S. revenue and perhaps B.C. Tel to a lesser extent. This would simultaneously lower, other companies' percentages such as that of A.G.T.

## iii) Trans-Canada/Adjacent Members split

Tableau 3.3.1b could be analyzed to compute a Trans-Canada (3 or more members)/Adjacent Members split with a given set of rules defining adjacency. B.C. Telephone for instance is only adjacent to A.G.T. hence splits are as follows:

	BC Tel Trans-Canada	BC Tel Adjacent
Tableau 3.3.1b	56%	44%
Tableau 3.3.1d	74%	26%

Similarly for A.G.T., adjacency being BC Tel. and Sask. Tel.

	Trans-Canada	Adjacent
Tableau 3.3.1b	46%	54%
Tableau 3.3.1d	65%	35%

Discrepancies between these Tableaus are due to:

- Tableau 3.3.1B is pre-settlement and Tableau 3.3.1D is post-settlement (soon to be rectified)
- Canada - U.S. traffic
- special rate structure between B.C. Tel and A.G.T.
- revised holding times, per OD pair which is available and traffic volumes based on regression equation this area could be further developed and possible simulate the reality.

### 3.4 Accounting Block

#### 3.4.1 Accounting Parameters

The interrelationships of accounting variables, which must obey certain certain financial statement identities, are expressed by the Simultaneous Equation System. Therefore a change in any input variable or parameter can be tested by generating new financial statements according to the accounting identities and relationships.

The following is a partial list a values one might want to experiment with:

Ratios:	PCR	preferred equity/capital
	DCR	debt/capital
	ITC	Interest times coverage
Rates:	T	tax rate
	DPR	dividend payment (common)
	ALPHA	flowthrough coefficient
	IN	new interest rate
VARIABLES:	OPRV	operating revenue
	DEPDIF	difference in depreciation
	DELCTI	change in CTI

#### 3.4.2 Benchmark and Test Runs

##### 3.4.2.1 Benchmark

The benchmark used was Bell Canada Company Total 1971. The inputs for the Simultaneous Equations are shown in Tableau 3.4.2a. This tableau shows the list of variables generated automatically (see sub section 2.4.2) as well as those read in directly.

By applying the Simultaneous equation System to this data input the results are shown in Tableau 3.4.2b - Benchmark. The results shown in these financial statements serve as a validation simulation in that the financial variables reflect the figures shown on the company books. Discrepancies in balancing of Balance Sheet and Funds Statement and deviations from book figures of solved variables are in the insignificant order of the last of 7 significant digits and only by 1 to 4 units. These discrepancies are due to computer precision and should not detract from the financial statements.



### 3.4.2.2 Test Plan

The test run plan for the Accounting Block is illustrated by the following tableaux:

Tableau 3.4.2c	:	ALPHA	=	1.0
Tableau 3.4.2d	:	OPRV	=	1069726 (+5%)
Tableau 3.4.2e	:	DCR	=	.4409 (-5%)
Tableau 3.4.2f	:	DPR	=	.7752 (+10%)
Tableau 3.4.2g	:	ITC	=	3.0

Once again the data base is Bell Canada, Company Total, 1971. These 5 changes are made independently of each other i.e. only one change is made for any particular run.

### 3.4.2.3 Test runs and Simulation results in relation to the Financial Statements and Significant Ratios

The generation of the four financial statements, i.e., income statement, balance sheet, retained earning statement and fund statement for the Interregional sector represents a culmination of activities of the four individual blocks, in which numerous algorithms are involved.

The end results of the Interregional sector and Regional benchmarks merely have to be summed together item by item to obtain the company statements. Tableau 3.4.2B is an example of such company statements for Bell Canada 1971.

Several Accounting Block simulations concerning the generation of financial statements have been conducted, one in relation to impacts of the introduction of full flow-through of current deferred taxes, keeping the rest of items in the statements unchanged. A regeneration of financial statements reflecting the full flow-through is shown in Tableau 3.4.2C. Here the term "full flow-through" means the transferral of the change during a period of the current deferred taxes, in full amount, from the balance sheet, where it is a deferred credit item, to the income statement where it is treated as an income source.

In this Bell Canada example, the value of full flow-through amounts to \$47 M which gives rise to a chain of changes in other items in the financial statements. Net income increased by \$47 M or 32% from the book value. PRDIVI climbed slightly, the reason for the increase of the latter is that PCR (preferred share-total capitalization ratio) kept constant and the value of total capitalization increased. ROREC (rate of return on average common equity capital) is naturally boosted to 11.73% from 8.83%. Likewise, RORC (rate of return on average total capitalization) crept to 8.79% from 7.36%. DCR (average debt capitalization ratio) and DPRC (dividend payant ratio) were kept unchanged as constraints. ITCE (interest times coverage excluding income taxes) improved from 2.69 to 3.20.

When we view these impacts comparatively we can observe that the full flow-through in the case of Bell Canada 1971 has the strongest impact in relative term on ROREC, followed by RORC and ITCE. Put another way, ROREC, RORC and ITCE are sensitive to the different treatments of ALPHA (flowthrough coefficient). It is worthwhile to mention that various financial variables in the financial statements are affected by the introduction of positive values of ALPHA. (See Tableau 3.4.2C.)

TABLEAU 3.4.2a

## List of Variables

NETINC= 147290.000  
 DELPR = 104000.000  
 PR = 197997.000  
 DELEG = 4475.0000  
 EQ = 1293832.00  
 RE = 287842.000  
 NEWDEB= 155000.000  
 L = 1551504.00  
 NIA = 137940.000  
 DIVI = 97202.0000  
 RHO = 0.0640  
 PRO = 93997.0000  
 RHON = 0.0640  
 DCHO = 2669.0000  
 DEPN = 198438.000  
 DPRTVE= 8137.0000  
 REPL = 8250.0000  
 GCE = 471633.000  
 DPRTVC= 2339.0000  
 PLAMPS= 0.0  
 DELINV= 0.0  
 DCO = 0.0  
 DCOO = 0.0  
 DCHO0 = 2262.0000  
 NSV = -2942.0000  
 CTI = 80867.0000  
 OCA = 183936.000  
 CL = 164786.000  
 CTIO = 28145.0000  
 OCAO = 164264.000  
 CLO = 157927.000  
 CE = 0.0320  
 T = 0.5047  
 OPRV = 1018787.00  
 OPXP = 493525.000  
 ALPHA = 0.0  
 DEPDIF= 0.0  
 EQO = 1289357.00  
 OTHADJ= 0.0  
 DCR = 0.4641  
 LO = 1404754.00  
 REO = 250573.000  
 PCR = 0.0458  
 DPR = 0.7047  
 CP = 0.0320  
 BETA = 0.1500  
 OTHINC= 15784.0000  
 IC = 0.0  
 GAMMA = 0.0  
 PUCO = 122269.000  
 PUC = 171926.000  
 IN = 0.0790  
 IO = 0.0579  
 CURDTX= 47045.0000  
 PRDTX = 9641.0000  
 GTPD = 0.0  
 GTP = 0.0  
 INVO = 295919.000  
 INV = 309391.000  
 ZETA = 1.0000  
 DCDX = 0.0  
 DCD = 0.0  
 DCHO = 16068.0000  
 DCH = 20987.0000  
 DFTAXO= 170871.000  
 CCL = 0.0  
 AD = 1152567.00  
 ADO = 1039284.00  
 OUPACO= 3904629.00  
 OUPAC = 4283573.00  
 DELCTI= 52722.0000  
 ITC = 0.0

TABLEAU 3.4.2b

## Benchmark BELL CANADA COMPANY TOTAL 1971 (\$'000)

## INCOME STATEMENT

OPERATING REVENUE	1018787.
OPERATING EXPENSES	( 493525.)
DEPRECIATION	( 198438.)
TAXABLE OTHER INCOME	2368.
	329192.
NON-TAXABLE OTHER INCOME	13416.
INTEREST DURING CONSTRUCTION	14002.
	356610.
INCOME TAXES	( 122126.)
	234484.
DEBT SERVICE CHARGES	( 87194.)
NET INC	147289.

## RETAINED EARNINGS STATEMENT

NET INCOME	147289.
PREFERRED DIVIDENDS	( 9350.)
NET INCOME AVAILABLE	137939.
COMMON SHARE DIVIDENDS	97201.
TRANSFER TO GOV'T OWNERS	0.
	( 97201.)
INCOME RETAINED	40738.
SHARE ISSUE EXPENSE	( 3469.)
OTHER ADJUSTMENTS	0.
CURRENT RETAINED EARNINGS	37269.
BALANCE BEGINNING OF YEAR	250573.
BALANCE END OF YEAR	287842.

## BALANCE SHEET

ASSETS		LIABILITIES	
BEGINNING OF YEAR	END OF YEAR	BEGINNING OF YEAR	END OF YEAR
TEL. PL. # COST	3904629.	PREF. SHARES	93997.
ACC. DEP. #N	(1039284.)	COMMON SHARES	1289357.
TEL. PLANT	2865345.	RETAINED EARNING	250573.
INVESTMENTS	295919.	SHAREHOLD. CAP.	1633927.
CASH & TEMP. INV.	28145.	DEBT	1404754.
OTH. CUR. ASSETS	164264.	TOTAL CAP'N	3038681.
CURRENT ASSETS	192409.	CURRENT LIA.	157927.
DEF. CH. DEBT		DEFERRED TAXES	170871.
DEF. CH. OTHER		DEF. CREDITS OTH.	2262.
DEF. CHARGES	10068.	DEF. CREDITS	173133.
TOTAL ASSETS	3369741.	TOTAL LIA.	3369741.

## FUNDS STATEMENT

SOURCES:	USES:
NET INCOME	147289.
PREFERRED DIVIDENDS	( 9350.)
COMMON DIVIDENDS	( 97201.)
INCOME RETAINED	40738.
DEFERRED TAXES CURRENT	47045.
DEFERRED TAXES PRIOR	9641.
DEFERRED TAXES	56686.
DEPRECIATION	198438.
DEPRECIATION (TOOLS) EXP'D	8137.
INTEREST DURING CONSTRUCTION	( 14002.)
OTHER NON-CASH CHARGES	-5865.
NON-CASH CHARGES	192573.
ADDITIONS TO L.T.D.	154999.
LESS REPAYMENTS	( 8250.)
NET CHANGE IN DEBT	146749.
PREFERRED ISSUED	104000.
COMMON ISSUED	4476.
TOTAL SOURCES	545221.

TABLEAU 3.4.2C Alpha = 1.0

## BELL CANADA TOTAL 1971 (\$'000)

## INCOME STATEMENT

OPERATING REVENUE	1018787.
OPERATING EXPENSES	( 493525.)
DEPRECIATION	( 198438.)
TAXABLE OTHER INCOME	2368.
	<u>329192.</u>
NON-TAXABLE OTHER INCOME	13416.
INTEREST DURING CONSTRUCTION	14002.
	<u>356610.</u>
INCOME TAXES	( 74646.)
	<u>281964.</u>
DEBT SERVICE CHARGES	( 88050.)
NET INC	<u>193907.</u>

## RETAINED EARNINGS STATEMENT

NET INCOME	193907.
PREFERRED DIVIDENDS	( 9419.)
NET INCOME AVAILABLE	184488.
COMMON SHARE DIVIDENDS	130003.
TRANSFER TO GOV'T OWNERS	0.
	<u>( 130003.)</u>
INCOME RETAINED	54485.
SHARE ISSUE EXPENSE	( 3848.)
OTHER ADJUSTMENTS	0.
CURRENT RETAINED EARNINGS	50637.
BALANCE BEGINNING OF YEAR	250573.
BALANCE END OF YEAR	<u>301210.</u>

## BALANCE SHEET

ASSETS			LIABILITIES		
	BEGINNING OF YEAR	END OF YEAR		BEGINNING OF YEAR	END OF YEAR
TEL. PL. & COST	3904629.	4283573.	PREF. SHARES	93997.	200153.
ACC. DEPR'N	(1039284.)	(1152567.)	COMMON SHARES	1289357.	1303518.
TEL. PLANT	2865345.	3131006.	RETAINED EARNING	250573.	301210.
INVESTMENTS	295919.	309391.	SHAREHOLD. CAP.	1633927.	1804881.
CASH & TEMP. INV.	28145.	80867.	DEBT	1404754.	1573336.
OTH. CUR. ASSETS	164264.	183936.	TOTAL CAP'N	3038681.	3378217.
CURRENT ASSETS	192409.	264803.	CURRENT LIA.	157927.	164780.
DEF. CH. DEBT			DEFERRED TAXES	170871.	180512.
DEF. CH. OTHER			DEF. CREDITS OTH.	2262.	2669.
DEF. CHARGES	16068.	20987.	DEF. CREDITS	173133.	183181.
TOTAL ASSETS	<u>3369741.</u>	<u>3726187.</u>	TOTAL LIA.	<u>3369741.</u>	<u>3726184.</u>

## FUNDS STATEMENT

SOURCES		USES	
NET INCOME	193907.	GROSS CONSTRUCTION	471633.
PREFERRED DIVIDENDS	( 9419.)	INTEREST DURING CONSTRUCTION	14002.
COMMON DIVIDENDS	( 130003.)	DEPRECIATION (TOOLS) CAP'D	2339.
INCOME RETAINED	54485.	CHARGES NOT REQUIRING FUNDS	( 16341.)
DEFERRED TAXES CURRENT	0.	NET CONSTRUCTION EXPENDITURES	455292.
DEFERRED TAXES PRIOR	9641.	INVESTMENTS	13472.
DEFERRED TAXES	9641.	MISCELLANEOUS	11302.
DEPRECIATION	198438.	CHANGE IN WORKING CAPITAL	65535.
DEPRECIATION (TOOLS) EXP'D	8137.		
INTEREST DURING CONSTRUCTION	( 14002.)	TOTAL USES	<u>545601.</u>
OTHER NON-CASH CHANGES	-5865.		
NON-CASH CHARGES	192573.		
ADDITIONS TO L.T.D.	176832.		
LESS REPAYMENTS	( 9250.)		
NET CHANGE IN DEBT	160582.		
PREFERRED ISSUED	106156.		
COMMON ISSUED	14102.		
TOTAL SOURCES	<u>545600.</u>		

DO YOU WANT ANOTHER SIMULATION?

Y

NO

STOP  
46ND

0

## TABLEAU 3.4.2d

OPRV 1069726.  
?  
END

BELL CANADA COMPANY TOTAL 1971 (\$'000)  
OPRV (+5%)

## INCOME STATEMENT

OPERATING REVENUE	1069726.
OPERATING EXPENSES	( 493525.)
DEPRECIATION	( 198438.)
TAXABLE OTHER INCOME	2368.
	<u>380131.</u>
NON-TAXABLE OTHER INCOME	13416.
INTEREST DURING CONSTRUCTION	14002.
	<u>407549.</u>
INCOME TAXES	( 147833.)
	<u>259716.</u>
DEBT SERVICE CHARGES	( 87194.)
	<u>172521.</u>
NETING	

## RETAINED EARNINGS STATEMENT

NET INCOME	172521.
PREFERRED DIVIDENDS	( 9350.)
NET INCOME AVAILABLE	<u>163171.</u>
COMMON SHARE DIVIDENDS	114982.
TRANSFER TO GOV'T OWNERS	0.
	<u>( 114982.)</u>
INCOME RETAINED	48190.
SHARE ISSUE EXPENSE	( 3223.)
OTHER ADJUSTMENTS	0.
CURRENT RETAINED EARNINGS	<u>44967.</u>
BALANCE-BEGINNING OF YEAR	250573.
BALANCE-END OF YEAR	<u>295540.</u>

## BALANCE SHEET

		ASSETS		LIABILITIES	
		BEGINNING OF YEAR	END OF YEAR	BEGINNING OF YEAR	END OF YEAR
TEL.PL.# COST	3904629.		4283573.	PREF.SHARES	93997.
ACC.DEPRN	(1039284.)		(1152567.)	COMMON SHARES	1289357.
TEL. PLANT		2865345.	3131006.	RETAINED EARNING	250573.
INVESTMENTS		295919.	309391.	SHAREHOLD. CAP.	1633927.
CASH&TEMP. INV.	28145.		80867.	DEBT	1404754.
OTH.CUR.ASSETS	164264.		183936.	TOTAL CAP'N	3038681.
CURRENT ASSETS		192409.	264803.	CURRENT LIA.	157927.
DEF.CH.DEBT				DEFERRED TAXES	170871.
DEF.CH.OTHER				DEF.CREDITS OTH.	2262.
DEF.CHARGES		16068.	20987.	DEF.CREDITS	173133.
TOTAL ASSETS		<u>3369741.</u>	<u>3726187.</u>	TOTAL LIA.	<u>3369741.</u>

## FUNDS STATEMENT

## SOURCES:

NET INCOME	172521.
PREFERRED DIVIDENDS	( 9350.)
COMMON DIVIDENDS	( 114982.)
INCOME RETAINED	48190.
DEFERRED TAXES CURRENT	47045.
DEFERRED TAXES PRIOR	9641.
DEFERRED TAXES	56686.
DEPRECIATION	198438.
DEPRECIATION (TOOLS)EXP'D	8137.
INTEREST DURING CONSTRUCTION	( 14002.)
OTHER NON-CASH CHARGES	-5865.
NON-CASH CHARGES	192573.
ADDITIONS TO L.T.D.	154999.
LESS REPAYMENTS	( 8250.)
NET CHANGE IN DEBT	146749.
PREFERRED ISSUED	104000.
COMMON ISSUED	-3222.
TOTAL SOURCES	<u>544975.</u>

## USES:

GROSS CONSTRUCTION	471633.
INTEREST DURING CONSTRUCTION	14002.
DEPRECIATION (TOOLS)CAP'D	2339.
CHARGES NOT REQUIRING FUNDS	( 16341.)
NET CONSTRUCTION EXPENDITURES	455292.
INVESTMENTS	13472.
MISCELLANEOUS	10677.
CHANGE IN WORKING CAPITAL	65535.
TOTAL USES	<u>544976.</u>

## TABLEAU 3.4.2e

BELL CANADA COMPANY TOTAL 1971 (\$'000)  
DCR (-5%)

?  
DCR .4409  
?  
END

## INCOME STATEMENT

OPERATING REVENUE	1018787.
OPERATING EXPENSES	( 493525.)
DEPRECIATION	( 198438.)
TAXABLE OTHER INCOME	2368.
	<u>329192.</u>
NON-TAXABLE OTHER INCOME	13416.
INTEREST DURING CONSTRUCTION	14002.
	<u>356610.</u>
INCOME TAXES	( 125072.)
	<u>231538.</u>
DEBT SERVICE CHARGES	( 81356.)
NETINC	<u>150181.</u>

## RETAINED EARNINGS STATEMENT

NET INCOME	150181.
PREFERRED DIVIDENDS	( 9350.)
NET INCOME AVAILABLE	140831.
COMMON SHARE DIVIDENDS	99239.
TRANSFER TO GOV'T OWNERS	0.
	<u>( 99239.)</u>
INCOME RETAINED	41592.
SHARE ISSUE EXPENSE	( 8323.)
OTHER ADJUSTMENTS	0.
CURRENT RETAINED EARNINGS	33269.
BALANCE BEGINNING OF YEAR	250573.
BALANCE END OF YEAR	<u>283842.</u>

## BALANCE SHEET

	ASSETS		LIABILITIES	
	BEGINNING OF YEAR	END OF YEAR	BEGINNING OF YEAR	END OF YEAR
TEL. PL. & COST	3904629.	4283573.	PREF. SHARES	93997.
ACC. DEPN	(1039284.)	(1152567.)	COMMON SHARES	1289357.
TEL. PLANT	2865345.	3131006.	RETAINED EARNING	250573.
INVESTMENTS	295919.	309391.	SHAREHOLD. CAP.	1633927.
CASH & TEMP. INV.	28145.	80867.	DEBT	1404754.
OTH. CUR. ASSETS	164264.	183936.	TOTAL CAP'N	3038681.
CURRENT ASSETS	192409.	264803.	CURRENT LIA.	157927.
DEF. CH. DEBT			DEFERRED TAXES	170871.
DEF. CH. OTHER			DEF. CREDITS OTH.	2262.
DEF. CHARGES	16068.	20987.	DEF. CREDITS	173133.
TOTAL ASSETS	<u>3369741.</u>	<u>3726187.</u>	TOTAL LIA.	<u>3369741.</u>

## FUNDS STATEMENT

SOURCES:	USES:
NET INCOME	150181.
PREFERRED DIVIDENDS	( 9350.)
COMMON DIVIDENDS	( 99239.)
INCOME RETAINED	41592.
DEFERRED TAXES CURRENT	47045.
DEFERRED TAXES PRIOR	9641.
DEFERRED TAXES	56686.
DEPRECIATION	198438.
DEPRECIATION (TOOLS) EXP'D	8137.
INTEREST DURING CONSTRUCTION	( 14002.)
OTHER NON-CASH CHARGES	-5865.
NON-CASH CHARGES	192573.
ADDITIONS TO L.T.D.	7211.
LESS REPAYMENTS	( 8250.)
NET CHANGE IN DEBT	-1039.
PREFERRED ISSUED	104000.
COMMON ISSUED	156204.
TOTAL SOURCES	<u>550075.</u>

TABLEAU 3.4.2f

?  
DPR. .7752  
?  
END

BELL CANADA COMPANY TOTAL 1971 (\$'000)  
DPR (+10%)

## INCOME STATEMENT

OPERATING REVENUE	1018787.
OPERATING EXPENSES	( 493525.)
DEPRECIATION	( 198438.)
TAXABLE OTHER INCOME	2368.
	329192.
NON-TAXABLE OTHER INCOME	13416.
INTEREST DURING CONSTRUCTION	14002.
	356610.
INCOME TAXES	( 122126.)
	234484.
DEBT SERVICE CHARGES	( 87194.)
NETING	147289.

## RETAINED EARNINGS STATEMENT

NET INCOME	147289.
PREFERRED DIVIDENDS	( 9350.)
NET INCOME AVAILABLE	137939.
COMMON SHARE DIVIDENDS	106930.
TRANSFER TO GOV'T OWNERS	0.
	( 106930.)
INCOME RETAINED	31009.
SHARE ISSUE EXPENSE	( 3791.)
OTHER ADJUSTMENTS	0.
CURRENT RETAINED EARNINGS	27218.
BALANCE BEGINNING OF YEAR	250573.
BALANCE END OF YEAR	277791.

## BALANCE SHEET

		ASSETS		LIABILITIES	
		BEGINNING OF YEAR	END OF YEAR	BEGINNING OF YEAR	END OF YEAR
TEL. PL. & COST	3904629.	4283573.		PREF. SHARES	93997.
ACC. DEP'N	(1039284.)	(1152567.)		COMMON SHARES	1289357.
TEL. PLANT	2865345.	3131006.		RETAINED EARNING	250573.
INVESTMENTS	295919.	309391.		SHAREHOLD. CAP.	1633927.
CASH & TEMP. INV.	28145.	80867.		DEBT	1404754.
OTH. CUR. ASSETS	164264.	183936.		TOTAL CAP'N	3038681.
CURRENT ASSETS	192409.	264803.		CURRENT LIA.	157927.
DEF. CH. DEBT				DEFERRED TAXES	170871.
DEF. CH. OTHER				DEF. CREDITS OTH.	2262.
DEF. CHARGES	16068.	20987.		DEF. CREDITS	173133.
TOTAL ASSETS	3369741.	3726187.		TOTAL LIA.	3369741.

## FUNDS STATEMENT

SOURCES:	USES:
NET INCOME	147289.
PREFERRED DIVIDENDS	( 9350.)
COMMON DIVIDENDS	( 106930.)
INCOME RETAINED	31009.
DEFERRED TAXES CURRENT	47045.
DEFERRED TAXES PRIOR	9641.
DEFERRED TAXES	56686.
DEPRECIATION	198438.
DEPRECIATION (TOOLS) EXP'D	8137.
INTEREST DURING CONSTRUCTION	( 14002.)
OTHER NON-CASH CHARGES	-5865.
NON-CASH CHARGES	192573.
ADDITIONS TO L.T.D.	154991.
LESS REPAYMENTS	( 8250.)
NET CHANGE IN DEBT	146741.
PREFERRED ISSUED	104000.
COMMON ISSUED	14534.
TOTAL SOURCES	545543.

DO YOU WANT ANOTHER SIMULATION  
?  
NO



TABLEAU 3.4.2g

?  
ITC 3.0  
?  
END

BELL CANADA COMPANY TOTAL 1971 (\$'000)  
ITC = 3.0

## INCOME STATEMENT

OPERATING REVENUE	1018787.
OPERATING EXPENSES	( 493525.)
DEPRECIATION	( 198438.)
TAXABLE OTHER INCOME	2368.
	<u>329192.</u>
NON-TAXABLE OTHER INCOME	13416.
INTEREST DURING CONSTRUCTION	14002.
	<u>356610.</u>
INCOME TAXES	( 127607.)
	<u>229003.</u>
DEBT SERVICE CHARGES	( 76334.)
	<u>152668.</u>
NETING	

## RETAINED EARNINGS STATEMENT

NET INCOME	152668.
PREFERRED DIVIDENDS	( 9350.)
NET INCOME AVAILABLE	143318.
COMMON SHARE DIVIDENDS	100992.
TRANSFER TO GOV'T OWNERS	0.
	<u>( 100992.)</u>
INCOME RETAINED	42326.
SHARE ISSUE EXPENSE	( 12499.)
OTHER ADJUSTMENTS	0.
CURRENT RETAINED EARNINGS	29827.
BALANCE-BEGINNING OF YEAR	250573.
BALANCE-END OF YEAR	280400.

## BALANCE SHEET

ASSETS		LIABILITIES	
BEGINNING OF YEAR	END OF YEAR	BEGINNING OF YEAR	END OF YEAR
TEL. PL. & COST	3904629.	PRÉF. SHARES	93997.
ACC. DEPR'N	(1039284.)	COMMON SHARES	1289357.
TEL. PLANT	2865345.	RETAINED EARNING	250573.
INVESTMENTS	295919.	SHAREHOLD. CAP.	1633927.
CASH & TEMP. INV.	28145.	DEBT	1404754.
OTH. CUR. ASSETS	164264.	TOTAL CAP'N	3038681.
CURRENT ASSETS	192409.	CURRENT LIA.	157927.
DEF. CH. DEBT		DEFERRED TAXES	170871.
DEF. CH. OTHER		DEF. CREDITS OTH.	2262.
DEF. CHARGES	16068.	DEF. CREDITS	173139.
TOTAL ASSETS	3369741.	TOTAL LIA.	3369741.

## FUNDS STATEMENT

SOURCES:	USES:
NET INCOME	152668.
PREFERRED DIVIDENDS	( 9350.)
COMMON DIVIDENDS	( 100992.)
INCOME RETAINED	42326.
DEFERRED TAXES CURRENT	47045.
DEFERRED TAXES PRIOR	9641.
DEFERRED TAXES	56686.
DEPRECIATION	198438.
DEPRECIATION (TOOLS) EXP'D	8137.
INTEREST DURING CONSTRUCTION	( 14002.)
OTHER NON-CASH CHARGES	-5865.
NON-CASH CHARGES	192573.
ADDITIONS TO L.T.D.	-119932.
LESS REPAYMENTS	( 8250.)
NET CHANGE IN DEBT	-128182.
PREFERRED ISSUED	104000.
COMMON ISSUED	286848.
TOTAL SOURCES	554251.
DO YOU WANT ANOTHER SIMULATION	
?	
NO	

#### 4. SIMULATIONS

##### 4.1 General Considerations on the Concept of Simulation

##### 4.1.1 The diverse meanings of the term "simulation"

The terms "simulations" and "sensitivity studies" are often used without discrimination. It appears preferable to distinguish four broad categories according to the specific objective that is pursued.

- 1) Validating the logic of the model and obtaining some measures on the performance of the various algorithms and interfacing devices the model contains.

These simulations will start as a series of tests at a fine level of detail so that the source of any detected malfunctioning could be easily identified; as time goes on larger chunks of the model will be subjected to simultaneous testing once their individual components have been validated.

- 2) Testing the quality of the semi-realistic data by the model contained.

At first sight this might seem a somewhat futile exercise since at some future (and preferably near) stage these data are destined to be replaced by real data; however, experience suggests that the best approach to obtain real data, especially if they are to come to a very large extent from outside sources, is to develop gradually high quality semi-realistic data: a framework for the organization and integration of the real data, indications as to the orders of magnitude, identification of significant gaps in the information available at any given time; at the same time tests with semi-realistic data provide additional information concerning the quality of the logic of the model: the main objective of the preceding group of exercises.

- 3) Sensitivity testing with real and semi-realistic data for benchmarks and calibration.

The sensitivity tests will be run with the semi-realistic and whenever possible, real data; once again the tests will start as separate exercises at a fine level of detail; however, it will be essential to move fairly rapidly to working with large chunks of the model, and at the limit with the model as a whole; apart from serving as additional validations of both the logic and the data of the model these sensitivity tests will serve the following two groups of purposes

- provide benchmarks against which policy simulations proper will be measured
- calibrate the model, that is indicate the significant ranges of variation of the various parameters, suggest the rates at which or the steps by which it is of interest to change these parameters and finally to identify the configurations of parameter values which appear relevant for policy simulation purposes.

#### 4) Policy simulations proper

These simulations are the ultimate target, "la raison d'être" of the model. The carriers, the consumers, the regulation authorities have interrelated behaviors. The model is a formalized representation of parts of the relationships of the "true" system. A policy question should be able to receive some answer in order to help the regulating bodies to take decisions. This is why it is suggested that considerable effort be devoted to the planning of the simulation of that type. Some analytical examples will be presented in the Section 4.2 Policy simulations.

##### 4.1.2 Policy Simulation areas

The whole model is a set of algorithms more or less interfacing in the present state of development. Each algorithm can be conceived as 1. an input vector, 2. a logic block transforming the input vector, 3. an output vector resulting from the transformation. With certain qualifications, any input vector can be imposed exogenously by the analyst and an output vector computed and evaluated from the point of view of a policy question. But certain questions require the use of a sequence of algorithms, a part or the whole output of a preceding algorithm being the input of a following one. A more complex situation arises when a loop is envisaged between two algorithms. Finally all kinds of linkings could be conceived in a fully automatic monitoring policy block. In the present state of development it is necessary to stick to simple combinations of algorithms. Suffice here to say that planning policy simulations with a complex model is a major combinatorial problem where it is easy to get bogged down in a mass of hypotheses and results whose interrelationships it is impossible to establish and a plethora of unidentifiable indicators. A general remark is worth while at this stage: if the model is a detailed representation of the "true" system, the specification of any question is bound to need a lot of detailed inputs. The simplest way out is to provide typical input vectors, that is combinations of parameters "ready to serve" which may be modified by a scaling factor.

Whatever the complexity of asking precise questions from the model, the areas of concern for policy purposes seem to be the following:

##### 1. efficiency in resource allocation with problems like

- the right hierarchy and homing arrangement,
- the quality of service level,
- the interconnection potential,
- the private lines versus the public lines,
- the capital deployment strategy with respect to time, place and technology,

- the routing of circuits on the Transmission Network,
  - the survivability level
2. Competition among the carriers
  3. Financial viability in regulation proposals
  4. Cross-subsidization between
    - carriers,
    - services,
    - regions.

Adding to the potentiality of the model, last but not least, scores of simulation capabilities are available in the Accounting methods and financial matters.

4.1.3 The need for pursuing policy simulation planning in parallel with other simulations,

It is freely admitted that to start simulations and sensitivity tests planning at a stage where the model is only just beginning to be operational and when it is still quite some time off before it becomes really efficient and before it is put on a realistic and tested data base involves certain not inconsiderable risks. On the other hand, however, in spite of the likelihood that some duplication of effort will take place and some blind alleys will be entered into, proceeding in a sequential manner rather than by pursuing a number of parallel activities would have stretched the time needed to develop the model to an intolerable length. The activities in question are:

- the first three groups of simulations listed in 4.1.1 plus the planning and first runs of the simulations of the fourth groups, started in the sequence in which they are listed but with considerable overlaps.
- review and refinement of the conceptualization,
- development of the data base,
- last but not least, the development of efficient software geared to the requirements of simulation exercises.

Against the disadvantages of the simultaneous approach one has to set of course the opportunity it offers to coordinate the various activities involved.

## 4.2 Policy Simulation Examples

### 4.2.1 Examples taken from the efficiency question set.

Examples can be given at different levels of detail from general questions to operationally defined ones, the following examples throwing light on the simulation potentialities of the model:

1. Suppose the switching network hierarchy and/or the arrangement are changed, what is the resulting loads on the links of the switching network and what is the quality of service obtained for each stream?
2. Same question as 1. but with H.U. and/or full group creations.
3. Are automatic changes in routing arrangements worthwhile to envisage? These automatic changes although technically feasible and indeed in use as precautionary devices against emergencies might call for considerable investment if they are to become a general feature of the switching process; it will also be noted that these automatic changes would lose much of their interest if they did not cover changes (well over a dozen time a week) of certain groups of circuits from private lines to public switched traffic use and vice versa and also the creation and then the disconnecting of H.U. and full groups. These matters involve engineering considerations and advice from D.O.C. ought to be sought at the earliest possible date.
4. Although, this may sound far fetched, one might consider to make use of East-West connection during parts of the day to make the most of the existing capacity in peak periods in given time zones.

The peculiar geography of Canada would prevent such an arrangement to bring about any dramatic improvement, but it is becoming more and more urgent to economize on investment expenditures with interest rates at what they are and with the astronomical needs of other sectors of the economy which might justify considering such an arrangement.

5. The modified linear programming algorithm used to identify facility chains for pairs of Origin-Destination points; the modification consisting of the replacement of the usual simplex entry criterion by the shortest path procedure of the graph theory lends itself readily to sensitivity studies which in fact are simulations in the broad sense of the term. The simplest ones of such exercises at the present stage of the development of the model, at any rate, are:
  - exercises involving the modifications of the network of the types mentioned in the preceding point;

- modifying linear inequality or linear equation simultaneous constraints on subsets of the elements of the network;
- modifying linear inequality or linear equation simultaneous constraints on groups of Origin-Destination pairs;

Examples of the first group of exercises mentioned above would make it possible to evaluate the consequences of either making marginal additions to the network or, to the contrary, the hypothetical consequences of not having made some marginal additions in the past which in fact have been made. Setting these valuations against the capacity addition and or expansion costs involved would provide a first step in the working out of the trade-offs between the "physical" and the "financial" variables.

An example of the second group of simulation exercises mentioned above and involving linear equality or inequality constraints could be the imposition of requirements that the links belonging to a given carrier (or located in a given region) carry at least a certain given load. In order to do so, in the first table of Subsection 2.1.2 we must explicitly introduce the spare capacity since we cannot enumerate the chains involved in the problem. A new constraint will be added of the following form: the negative sum of the spare circuits allocated to a given set of links must be greater than or equal to the required lower bound minus the sum of the capacity of the same set of links.

The two groups of additional constraints would, of course, make worse or (at best) leave unchanged the value of the objective function with which the algorithm is run.

The third group of exercises involves aggregating constraints. It will thus lead to an improvement (or at worst leave unchanged) any objective function with which the algorithm is run. One could choose two (or more) geographically close origins or two (or more) geographically close destination and treat them as one origin or one destination respectively. In the first table of the Sub-Section 2.1.2 one would simply add up the two (or more) corresponding constraints and leave the algorithm otherwise unchanged. The resulting (if any) improvement in the objective function set against the cost of inter-connecting the origins or the destinations involved would then help to decide whether such a development might be considered advantageous.



Since the algorithm does not employ the usual simplex entry criterion but submits activities corresponding to facility assignment chains by using a shortest chain algorithm of the graph theory, it is awkward to impose constraints on the activities concerned. In any case constraints involving linear equations or inequalities on groups of these activities seem to be out of question, at any rate, at the present stage of the development of the model. It is however possible to have "exclusion" constraints on these activities whereby certain types of chains are ruled in advance as inadmissible. Or one would even envisage that a priori a number of chains of a given group cannot be exceeded in the final solution.

Since the shortest chain algorithm submits the candidate chains one by one, a simple test could check their admissibility and when chain fails the test, then the second shortest chain is proposed and so on. Obviously this can only make the value of the objective function worse or leave it unchanged. For instance one can exclude chains that cross more than once a given interprovincial boundary. This modification might also turn out to be providing certain formulations of the survivability requirements. It will be noted that in this case the solution will in general depend on the sequences of calculations.

Such simulation or sensitivity exercises would in most cases be limited to the completion of the algorithm in question, with its various alternative objective functions but in some instances would be carried right through the model to the stage where the financial statements of individual carriers are calculated; simulations of this type would be of particular interest in the formulations where the objective function to maximize is the length of time before additional capacity is required.

6. Is the lack of interconnection in the transmission network a great source of inefficiencies?

Interconnection involves delicate engineering problems but some simple simulations can be planned.

For a given pair of nodes with circuit requirement, a restricted connected transmission network is permissible. An allocation of circuits is done with that restricted network as constraints.

A new allocation is computed with a more connected network.

The value of the objective-function is computed in the two cases and the difference in values is an evaluation of the cost.

If the objective-function is expressed in physical units, the circuit allocation algorithm is sufficient. If some monetary costs are involved, outputs from the Costing Blocks are necessary.

7. What are the system survivability characteristics?

Survivability in networks because of the combinatorial nature of the problem to which it gives rise is notoriously difficult to handle. A relatively promising approach appears to be to work with the actual network, to identify the cases where the structure of the network already assures the required survivability and to plan a restricted number of simulation exercises destined to investigate the possibilities of improving survivability at a relatively low cost. Obviously the results will not be rigorously optimal. Several types of questions can be formulated:

- a) Given an origin-destination pair of nodes, of the transmission network, how many nodewise disjoint chains exist between the nodes in this transmission network?
- b) Or given a circuit requirement between any adjacent nodes of the switching network or any origin-destination of the private line service:
  - 1. before allocating the circuits on the transmission network, how many nodewise disjoint chains exist between the two given nodes?
  - 2. after allocating the circuits, had the circuits been allocated along at least two nodewise disjoint chains?

Since only the number of disjoint chains is looked for, the algorithms for the search of a minimal disconnecting set is enough for a) and b) 1.

For b) 2., part of the output of the circuit allocation algorithm must be fed into the minimal disconnecting set algorithm.

8. If the transmission network has not enough survivability capability, how much does it cost to improve it?

As already said, an optimal solution to that problem is really difficult to find. Some suboptimal solution is needed. The notion of cost is ambiguous and must be specified.

For given origin-destination pair with a given circuit requirement whose allocated transmission support is not comprised of nodewise disjoint chains:

- a) identify, if any, such chains on the transmission network before the circuit allocation. If there is no such chain, survivability as defined is not possible and some facility expansion should be needed,



- b) if at least two node disjoint chains exist, choose the two preferred ones and allocate a priori a given fraction of the circuit requirement on each of them.
- c) re-run the circuit allocation algorithm. Whatever the objective function, compare the value obtained before the a priori allocation to the value after allocation (it goes without to say that the objective function must be the same before and after); the difference in value is the "cost" of survivability.

This is an example of a loop between two algorithms: the circuit allocation algorithm and the node disjoint chain builder.

9. Some interactions between the switching and the transmission network.

Among the objectives to simulate some policy questions, there was the following general purpose: to try to obtain an efficient allocation of resources in the domain of telecommunications. This objective is of course very large: in the present example we will propose some simulations in view of determining an optimal way of reallocating the circuits on the transmission network between the different periods. In particular, we will show that as the Operating Block now stands, this reallocation has to be done for each period. But, in practice we know that this is not quite true, as this general allocation plan is made for one (or two) years with minor changes during the abnormal peak periods. The feedback on the quality of service will also be considered as well as the pertinence of the present matter on the study of cross-subsidization among services.

In order to appreciate the problem to be discussed, it is worth to summarize very briefly how the allocation of circuits on the Transmission Network is done for a given day or year. The precise duration of the preceding terms is not too much important, in so far as they include more than one sub-period. Now, for each sub-period, we proceed as follows: we determine the usage of the Switching Network or, in other terms we determine the requirements of each link on the switching Network. Taking these requirements and the initial capacity of each link into account and using a certain objective function we are now able to obtain an optimal solution, which will be the allocation of circuits on the Transmission Network. Now, if we go to another sub-period, we proceed in the same fashion as before. Of course, as the demand between each O-D pairs will probably be different, the requirements will also be different, which will have as a consequence that the allocation of circuits on the transmission network will be modified.

In the preceding point, we have seen how the allocation of circuits for each period will be done. But, this procedure has at least two weaknesses: the first is that it is sub-optimal (in a time perspective) in the sense that, in no criteria in so far used, we explicitly take the costs of reallocating circuits

between the periods into account, the second point being that, in practice, this allocation is done for more than one sub-period. For the time being, we don't know how the information concerning the costs of this reallocation can be obtained, neither if it is possible from a technical viewpoint. The consequence of the second point is that we have to find a "super" criteria which will take the difference of the requirements among the periods into account and which permits a certain optimal allocation of the resources. Of course, the problem here treated concerns the manner by which the network can be managed.

We now explicit our concern about the necessity of finding a "super" criteria and formulate some possible scenario of simulations. For the following discussion, we will assume that the criteria of maximizing the spare capacity that is equivalent to minimizing the cost when the cost per circuit is the same for each transmission link, is the criteria used to allocate the circuits on the Transmission Network. In the Interim Report, two possible solutions were proposed for this time allocation, so to speak. The first one was to take, for each component of the requirements, the maximum over all the sub-periods and make the allocation with this new vector of requirements. However, in so doing, we are not sure that a feasible solution will exist; that is to say one allocation which will not exceed the capacity already installed and which will satisfy these requirements, and if a feasible solution exists, there may be "very large" spare capacity for each sub-period. The second possibility suggested was to take the allocation which will minimize the maximum among all the sub-periods, of the spare capacity. In other words, as we can compute the value of the objective function, at the optimal solution and for each sub-period, we take the allocation corresponding to the sub-period where the value of the objective function, or the spare capacity, is at its minimum. But, in so doing, we have no reason a priori to suppose that the allocation obtained in this manner will satisfy all the requirements for all the sub-periods. However, it will be interesting to see if, in practice, the vectors of requirements are "substantially" different. Now, if we exclude the first proposal, every other suggestion will have the weakness of the second one. So, if we take the second proposal, or any compromise between the two suggested, we will be certain that some requirements will not be respected; what will be, as a feedback, the consequences on the quality of service?

We would like to make a general comment on the quality of service. If we knew all the information on the demand matrice we need, the grade-of-service can be given as a parameter. In this case, it will be possible to see the requirements on the Switching Network as function of this grade-of-service. And, since these requirements are used for allocating circuits on the Transmission Network, we

can see these allocated circuits as a function of the given quality of service (or more precisely a correspondance in the sense that for a given quality-of-service there are, in general, more than one best allocation). Now, in view of the third paragraph above, and of what have been said previously in the present point, whatever criteria is retained for allocating circuits for all the sub-periods, it could be possible, at least theoretically, to evaluate the impact on the quality of service of every proposed allocation. In this sense, we can propose the following scenario. Let the traffic load on the Switching Network be given or, in other words, let the requirements to be submitted to the Transmission Network be given. We can formulate a series of "super" criteria (as defined previously) and calculate a corresponding allocation and then obtain the quality of service by an inverse function. These simulations will then give use some "feelings" about the determination of an appropriate "super" criteria.

This problem of the reallocation of circuits between the sub-periods is, of course, crucial for many purposes and, in particular in evaluating the possible cross-subsidizing among the services because we have to consider this aspect in a time perspective. In effect, as the computed cost of each service depends on the allocation of the circuits, if we modify this allocation for every sub-period, we then vary the costs of the services. In other words, it can be perfectly possible that for a given period certain service does recover its cost and does not in another period. On the other hand, if we assume that the network is managed for a given year, the costs of the services will be strongly dependent on the "super" criteria which will be used.

#### 4.2.2 An attempt to measure the cross-subsidization among services

- 1) In the following, we propose a certain scenario using more than one block. Its main objective is, of course, to show the potentialities of the model, the integration (at least at a certain level) of different blocks, and to illustrate the idea of how technical and financial variables can be combined in a given simulation. We think that the proposed simulation can be easily modified to permit more flexibility and, of course, the introduction of loops between blocks. We also try to enumerate some weaknesses of the model or, more precisely, the aspects which can be modified with little effort.
- 2) To give certain policy aspects to the proposed simulation, we will show how the model, as it now stands, can answer the question of interfinancing among services. The question has to be formulated in a more precise manner and, in particular we have to define the criterion which will permit to say that there is, or is not, cross-subsidization among services. In the following, for many obvious reasons, we will use the criterion of total revenue versus total (investment or capital) cost per service.
- 3) First of all, we will assume that the data are available for the following three services: public message, private line and television. The first step is to go to the Operating Block and use the two algorithms which are already there. The first algorithm, concerning the usage of the Switching Network, serves two purposes. The first is to determine the requirements on each link in this network for the public message, these requirements being used as constraints for allocating circuits in the transmission network. The second purpose is to allocate these requirements between each Origin-Destination. Knowing this information, it is possible, by aggregation, to subdivide the public message traffic into the regional and inter-regional components and then also to analyse the interfinancing between regional, inter-regional, private line and television services.

The purpose of the second algorithm in the Operating Block is to allocate circuits in the transmission network taking the existing capacity of each link and the demand requirements into account and, eventually, some survivability constraints. As the model now stands, the television circuits are routed manually and subtracted for the original capacity on each link. At this level, no explicit objective function is used. However this is not the case for the routing of other circuits. We propose for the present simulation, to use the objective which consists of maximizing the number of circuits per dollar or, in other terms, the number of circuits divided by the annual capital cost per element (see sub-section 2.1.2.) This criterion necessitates the use of the Costing Block.

- 4) We have seen in the previous point that to formulate the proposed criterion we have to use the Costing Block. However, depending on which unit costing method we are interested in, some or all of the algorithms contained in this block can be used. For example, starting with the asset cost function per element, we can apply the algorithms of aging, indexing, etc. to arrive at a certain annual cost per physical element. It is then also possible, by taking a finite variation, to calculate the marginal (prospective as well as embedded) cost per physical element. Of course, our objective is to arrive at a cost per service: we will discuss this point in the next paragraph. In conclusion, having chosen a certain objective function we are now able, using the proper algorithm, to allocate the circuits on the physical network for the switched services. We then have the usage of each link in this network: since we also have the annual cost per element, dividing one by the other we can obtain a unit annual cost per element. We can notice that in so doing, we require the users of the Network to pay for the spare capacity.
- 5) We now propose an approach to arrive at a cost per service. First of all, having the unit cost per element it can be possible to compute the cost for each chain and, by addition to arrive at a usage cost for each link of the Switching Network. Now, as we said previously, using the algorithm for dimensioning the Switching Network, we also obtain the vector of the "true" Origin-Destination pairs. Then, for a given link, we can cost these O-D pairs using the particular link considered. Of course, we can do it for all links of the switching network. Finally, summing these different vectors we obtain a stream cost per "true" Origin-Destination pair. This procedure permits one to obtain directly a cost for the regional and inter-regional services, since at the beginning the data on the O-D pairs are classified between these two services. Then, summing the elements of the "cost vectors" obtained previously, pertaining to each of these services, we obtain a cost for the regional and a cost for the inter-regional services.
- 6) In point 5, we have only spoken about the costs of the switched services. But, we also consider the private lines and television services. The determination of the costs of these services causes no problem since we know first the unit cost per physical element and second the routing of these services. So, we have only to make the proper multiplication to obtain the desired results.
- 7) The preceeding points have showed how we can arrive at a certain cost evaluation per stream and by type of service, at a national level (i.e. without identifying the carriers). If we now look on the revenue side, it is very easy to compute the revenues per service, since we know the tariffs and also the demand for each

0-D pair and for each service. If we compare the cost versus revenue per service, we must be consistent in the sense that if the cost was obtained in a prospective sense, the revenue has to be computed on the same basis, if our objective is to compare them. In conclusion, then our approach allows to compare the net receipts of each service and then to check if there is an interfinancing among services.

- 8) Suppose now that the results of the simulation as described previously show that for a certain service the cost is greater than its revenue or, in other words, the other services finance this service. Suppose also that this situation is not admissible or, in other words, each service has to make profit (or at least no loss). Then, which parameters have to be modified in order for the loss to disappear? Of course this question leaves room for making other simulations for example, the commission can raise the tariffs for this service or lower the quality of service, or a mixture of both, and checks if the cost are now low enough to be equal to the revenues. Then, it is in the context like the present one that one can try to determine some trade-off between financial and technical variables.



#### 4.3 Local Simulations

##### 4.3.1 Simulations: multi-period simultaneous equations system

In the Interim Report of September 15, 1974, it was proposed to use a simultaneous system of equations consisting of seven equations and seven ratios. A revised version of this system appears in Section 2.4.4 of the present report. A multi-period extension of the system has also been studied. For more details, the reader is referred to the appropriate sections and sub-sections in reports already mentioned. However, the APL Language program constructed at this time, for the multi-period extension, was not perfectly adapted to the final version of the proposed system. Since that time, this adaptation has been done and some simulations with it have been performed. The reader will find in Appendix A of the present report a description of the program used and in Appendix B the specification of the simulations done with Bell's financial data as well as the results of these simulations.

When a simulation in a multi-period context has to be specified, at least three considerations have to be taken into account. The first one is that a technique of forecasting the future values of the truly exogeneous variables has to be incorporated in the program. By truly exogeneous variables, we mean those exogeneous variables for the next period which are not endogeneously obtainable for the present one. Of course, both of them can be present in the same model and in fact both are present in the suggested model. Then, a forecast of the truly exogeneous variables is needed if one wants to specify a simulation in a multi-period context. However, many techniques can be used, the sophisticated technique of regression as well as some kind of averaging or a subjective value. All of these possibilities have been incorporated in the program.

The second consideration is the following: for each simulation we have to specify five predetermined variables. The list of these five predetermined variables may be changed from one period to another. Thirdly, and strongly related to the preceeding point, which forecasted values of the predetermined variables should be used? In so far as the model is built, a message of error will be given each time an inconsistency is produced by these forecasted variables. For example, one equation in the system reads as follows: the equity at the beginning of the period is equal to the equity at the end of the period minus the variation in equity. The last two variables are endogeneous in the system and the value of the first is known. So, if one made a forecast of these two variables, he has to be consistent with the known initial value of equity.

#### 4.3.2 Goal Programming Approach

In our previous reports on the IRA Project, we introduced the technique of goal programming for evaluating the impact, on some goals, of the introduction of inequality constraints on different financial variables. The methodology is now completely operational. Moreover, we have illustrated it by reporting the results of a certain number of simulations performed with this approach. In consequence, we have judged that the presentation of results of the same kind in the present Report would add very little. In consequence, the reader who is interested in knowing more about the technique, as well as looking at some results and at their interpretation, has to consult our previous reports.



## 5. FUTURE DEVELOPMENT

### 5.1 Expansion of the Size of the Network

At present, the software is developed to handle problem sizes as follows:

- 1) Switching network algorithm
  - 24 to 30 nodes maximum
- 2) Transmission network algorithm
  - 600 constraints and any number of variables

The number of constraints in this latter problem is related to the network sizes as follows:

Number of constraints = number of transmission links +  
number of switching links + number of private demands

The following ratios and computations do not take into account the number of private demands. The maximum size of 600 relates therefore to a maximum total number of links. Since the transmission network is not highly meshed (few links for each node) the number of links exceeds the number of nodes by a factor of 1.2 to 1.5. In the switching network, which contrary to the transmission network is highly meshed (many links attached to each node) the number of links is related to the number of nodes by a factor of 2 to 3 (average for problems of 24 to 60 nodes).

Finally the number of transmission nodes exceeds the number of switching nodes by a factor of 3 to 5 for problems of 24 to 60 switching nodes. That is, to obtain a reasonable representation of transmission facilities, we estimate that for an  $n$ -node switching network  $3n$  to  $5n$  transmission nodes are required. The limit of 600 constraints translates therefore into a problem size having  $n$  less than  $600/10$  or roughly 60 nodes. Furthermore, the 600 constraint limit is not a difficult one to change. The software uses random access disk storage and therefore the size limit is not difficult to change.

The time of computation, however, becomes a decisive factor in large problems.

The restriction of 24 to 30 nodes in the switching network algorithm is more serious however since this limit is determined by software considerations. Furthermore, various test runs performed for this report have indicated that a 24 node network is too small to permit accuracy in computations of assets, traffic and other indicators. As a first priority for the coming phase, we plan therefore to convert the software to handle at least 60 nodes and if possible more. The method, of course, to perform this conversion involves the use

of virtual memory type techniques or sequential offline storage of data matrices. Without having studied the software problem in detail, we estimate that the conversion involved is a short term project and can be completed within the next few months.

For the other blocks of the program, the size problem does not arise since these operations deal with more manageable sized matrices and data bases.

## 5.2 Preparation of inputs

The simulation power of a model like this one is really increased if we have the capacity of rapidly and efficiently changing the specifications of the initial state namely with regard to the transmission and switching networks and also with regard to the point-to-point offered traffic. With the inclusion in the present software of the operation block of an estimated gravity model, we now have the capacity of generating point-to-point unidirectional traffic for any set of points within a matter of few seconds. At the present time, the only inputs required for this set of calculations are the population and geographical coordinates (longitude and latitude) of each point. This is an improvement from the previous approach, based on the use of Erlang and Poisson formulae given the number of existing point-to-point trunks and the blocking and overflow probabilities, which was tedious and time consuming.

Along the same lines, in a view of increasing the flexibility of the entire model the computerisation of the interface between Route and the Operating Block should be envisaged. This would allow the rapid specification of the microwave transmission network for different sets of points. So far the Route program has been extensively used in the definition of the existing microwave transmission network but the interface with the Operating Block was done manually. If such an interface is to be computerized, one might also consider at the same time modifying Route slightly to permit a search of the existing microwave routes for an entire set of points instead of a pair of points. The plotting capability of Route can also be very useful in the Operating Block.

However, there remains the problem of specifying and defining automatically the switching network for any given set of switching points. There exists at the present time in data base, like the one on microwave systems for example, which contains an up-to-date description of the switching network. The only data available at DDC on this subject are from the interregional study for the year 1970-71 and only includes the exchanges of class 1, 2 and 3. It then appears that the automatic updating of this information on the switching network can only be done at the present time through a dimensioning exercise, assuming that we can forecast the point-to-point traffic. This is where the linkage with Hermes III should be very seriously considered in due time.

### 5.3 Capacity Expansion

With growing demands and a large stock of physical capital, the planning of new and renewed switching and transmission equipments is of paramount importance in any regulatory situation.

Except for a few trials in the Operating Block with the allocation of circuits for growing circuit requirements and in the Accounting Block with a multi-period array, the available set of models do not permit simulation in time. There is no doubt that some physical expansion models are needed in the near future.

Delaying the conception of such models was based on the following reasons:

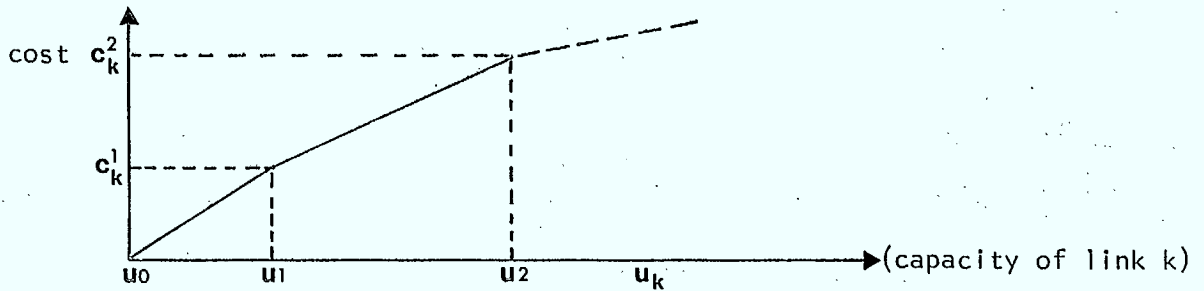
- a first attempt for a small expansion with step cost functions, for a network of around 60 nodes, was already made in the Hermes project;
- the "curse of dimensionality" which strikes many dynamic attempts in this domain was present in our mind;
- some building blocks for multi-period models were being constructed in the present phase of the IRA model and we are waiting for performance measures like running time and admissible size of inputs, in order to orient our model building endeavors;
- However, given the asset cost functions and the indexing procedures, we could devise some expansion model for a one period marginal capacity expansion. By marginal expansion it is meant an increase of the link capacity on the transmission network which can be approximately described by piece-wise linear functions. If big fixed costs are to be incurred in the expansion, a mixed integer mathematical programming method must be used and an algorithm of resolution must be conceived in order to include the "non chain enumeration" feature of the existing allocation algorithm since no known commercially available software exists for such a problem. Of course, the HERMES Models constitute, at the present time, the most appropriate basis from which any such dynamic capacity expansion model could be developed. On the contrary, if a certain set of links is ready for a piece-wise linear expansion cost, the following formulation can be used:

For a given period suppose the following:

- the expansion cost for the capacited link k is

$$C_k = \lambda_k^1 C_k^1 + \lambda_k^2 C_k^2 + \dots + \lambda_k^{n_k} C_k^{n_k}$$

Where  $0 \leq \lambda_i$  and  $\sum \lambda_i = 1$ , and the marginal cost function are as below



- the required circuits for demand constraint i is  $V_i$
- the  $D(i)$  is the set of transmission chains admissible for demand i
- $\delta_{jk} = 1$  if chain j belonging to  $D(i)$  uses the capacited link k, and zero otherwise
- $X_{ji}$  is the number of circuits allocated or chain j for demand i

Then, the linear programming expansion model is

$$\left[ \begin{array}{l} \text{MIN} \\ \lambda_k^t \end{array} \right] z = \sum_k \sum_{t=1}^{n_k} \lambda_k^t C_k^t$$

subject to

1° the capacity constraints, for all k

$$\sum_i \sum_{j \in D(i)} \delta_{jk} X_{ji} \leq \lambda_k^0 u_0 + \lambda_k^1 u_1 + \dots + \lambda_k^{n_k} u_{n_k}$$

2° the circuit requirement constraints, for all i

$$\sum_{j \in D(i)} X_{ji} \geq V_i$$

3° normalization constraints, for all k

$$\sum_{t=1}^{n_k} \lambda_k^t = 1$$

40 non negativity constraints, for all

$$\lambda_k^t \geq 0 \quad \text{and} \quad x_{ji} \geq 0$$

This is a separable programming which can be solved with a mixture of our "no chain enumeration" feature and the separable simplex method which guarantees that only pairs of contiguous  $\lambda_k^t$  enter the basis.

#### 5.4 Valuation and Depreciation

The production costs of the assets are converted into historical cost terms through the Aging and Indexing algorithms. It will be necessary to expand the algorithms to be capable of different annual growth rates and reproduction cost indices, i.e., by vintage. This approach is to replace the present uniform rates for all vintages which represent a good first approximation.

The effort indicated above may require the introduction of linear programming or some statistical method since the problem is mathematically indeterminate. This approach will enable both the depreciation and valuation (also costs) computations to become more accurate and flexible.

Considering the depreciation computation it will be necessary to introduce the integrated or "complex structure" type categories and survivor curves. These curves may be composed of survivor curves for mass type properties or of  $Y = mX$  lines cut off at the imposed age of final retirements.

### 5.5 Unit Costing Methods

There does not exist such a thing as a unique cost of service. In fact, each one depends on which cost definition one works with. Of course, the kind of cost one is interested in is strongly dependent on the objectives pursued. For these reasons, in the Interim Report of September 15, 1974, we proposed several costing methods. Among other things, these are being proposed for testing question of cross-subsidization among services and among the different periods. On the other hand, the different costs that can be measured are derived from some asset valuation function and from certain operations like aging, indexing, depreciation performed on the valuation. But, in fact, some of these operations and the kind of costing we like to arrive at are the two sides of the same piece. For example, to be able to compute the prospective incremental costs, it is necessary to index the asset cost function. These relationships were not fully developed to date. In consequence, we plan in the near future to develop this aspect of the problem in order to be able, at least, to formulate the related algorithms.

### 5.6 Multiperiod Goal Programming

In proceeding sections of the present Report, we show some results of simulations performed with the simultaneous equation system and also with the static goal programming technique. We motivate in previous reports the usefulness of these approaches and their respective advantages. In particular, the first one permits the linkage the different periods while the second one allows the introduction of constraints in inequality form. As a future improvement, we intend to combine these two advantages in order to give more flexibilities to those techniques. So, we will propose a multiperiod goal programming approach.

### 5.7 Future Development of Asset Cost Functions

In this phase of the project, some important advances have been made in the improvement of the asset valuation procedure. Investigations are continuing on several fronts which hopefully will extend and improve the evaluation procedures in the model.

There are plans to improve the accuracy of the asset cost functions applied to the various types of transmission equipment. In particular, comparisons will be made with the results of various investigations currently being carried out by other groups in the DOC.

The area which requires the most effort is undoubtedly the cost functions applied to switching machines. The present level of knowledge of switching costs is quite limited. Significant efforts will be made to advance in this area.

At the moment, the satellite facilities of Telsat are not included in the network. These will be included in the next phase. Thus, asset cost functions will have to be divided for earth stations and

the satellite link itself.

Similarly, the CN/CP network is not presently included in the model. When this network is included, asset cost functions for these facilities will have to be derived.

The approach taken in the current phase has been to consult with other groups in DOC which are familiar with the nature of the physical facilities of the network and their asset costs. This approach has been highly productive and will be continued in the future.

## 5.8

### Secondary Objective : First Steps Towards Behavior Studies

A number of the contemplated policy simulations as well as the simulations aiming at the adequation and the calibration of the model and also at the testing of the data base and refining its format have an important secondary objectives namely to prepare the ground for analyses and simulations having to do with the users' behaviors such as a future demand module. An example of this is the suggestion made earlier in this Report to consider decomposing (in a multiplicative element by corresponding element) the traffic matrices by Origin and Destination into the probability of call matrices and the average holding time matrices, of course by the time of the day and the day of the week. Another example related to some extent to the preceding one that might be worthwhile would be to pursue at the moment with hypothetical users' responses, the suggestion formulated in the Final Report on the Development of the HERMES III Model, p. 55, of having two kinds of long distance calls, the "immediate" and the "delayed" with hypothetical differences in the corresponding rates.



## APPENDIX A

Le programme SIMEQ2 de résolution du système d'équations simultanées

Le système comporte 12 équations en 41 variables exogènes (dont certaines peuvent être agrégées en constantes pour simplifier la notation) et 17 variables endogènes. Le programme, lorsqu'appelé, énumère ces variables en les numérotant.

1ère étape: Estimation des variables exogènes pour la période de prévision

Le programme demande les numéros des variables exogènes à estimer (de 1 à 41).

Pour chacune de ces variables, il demande ensuite:

- 1) le nombre d'observations et le nombre de régresseurs
- 2) d'entrer les observations sur chacun des régresseurs (si le nombre d'observations fourni pour un régresseur ne correspond pas à celui donné en 1), un message d'erreur est tapé sur la console et on doit recommencer à donner les observations sur ce régresseur à partir de la première)

- 3) d'entrer les observations sur la variable exogène à prévoir (si le nombre d'observations fourni ne correspond pas à celui donné en 1), un message d'erreur est tapé et il faut recommencer à partir de la première observation sur cette variable)
- 4) de fournir les valeurs des régresseurs pour la période de prévision. Les 3 étapes précédentes ont permis le calcul des coefficients de régression; on veut maintenant obtenir une prévision à partir de cette régression.

2ème étape: Résolution du système

Lorsque toutes les variables exogènes spécifiées ont été estimées, le programme demande les valeurs des autres variables exogènes (après les avoir énumérées). Si on a estimé toutes les variables exogènes au moyen de régressions, il passe directement à l'étape suivante. Un message d'erreur vérifie l'équation  $\Delta \text{DEFTX} = \text{DEFTX} - \text{DEFTX}_0$ , lors du calcul ou de l'introduction des valeurs de ces variables, pour la période de prévision.

Le système comporte 12 équations en 17 variables endogènes. Il faut donc choisir 5 de ces variables comme prédéterminées.

Le programme demande donc les numéros (1 à 17) de ces 5 variables. Puis il demande d'entrer (dans le même ordre) les valeurs de ces variables. Si l'on donne plus ou moins de 5 numéros de variables, un message d'erreur est tapé et il faut recommencer à donner ces numéros; de même, si l'on ne donne pas exactement 5 valeurs pour ces variables.

Le programme résout alors le système d'équations et fournit les résultats sous la forme d'une énumération des valeurs de toutes les variables. La dernière version du programme ne présente plus les résul-

tats sous la forme d'un Etat des Revenus et Dépenses et d'un Bilan. Certains problèmes méthodologiques nous empêchent pour le moment de rendre compatible cette méthode d'écriture des résultats avec l'approche comptable généralement acceptée. Trois nouvelles variables "pseudo-endogènes", RORBI, RORBE et POIVI, ainsi qu'une variable exogène supplémentaire PUC, sont calculées en fin de programme à partir des valeurs des autres variables. Lors de l'écriture des résultats, les variables endogènes prédéterminées et les variables exogènes estimées à l'aide de régressions sont marquées d'un astérisque.

N.B. Il est possible que les valeurs choisies pour les variables prédéterminées ne permettent pas de résoudre le système. Par exemple, une des équations est:

$$\Delta E = E - E_0 \quad \text{où } E \text{ et } \Delta E \text{ sont endogènes et } E_0 \text{ exogène.}$$

Si l'on choisit  $E$  et  $\Delta E$  comme variables prédéterminées, il faut se garder de provoquer une contradiction. Si cela se produit, le programme tape le message. "Les variables prédéterminées choisies ne permettent pas de résoudre le système". Pour recommencer, il suffit de répondre oui à la question "Désirez-vous effectuer d'autres simulations?"

Les variables du système sont:

Variables exogènes

1. GCE	Gross Construction expenditures
2. DEPR	Depreciation
3. OPXP	Operating Expenses
4. OINC	Other Income
5. INTCO	Interest on Construction
6. $E_o$	Ordinary Stock (beginning of period)
7. $R_o$	Retained earnings (beginning)
8. $L_o$	Long Term Debt (beginning)
9. $PR_o$	Preferred Stock (beginning)
10. T	Tax rate
11. $\alpha$	Coefficient of flowthrough
12. $\Delta$ CRT	$\Delta$ Deferred Credit Taxes
13. $\Delta$ CRO	$\Delta$ Deferred Credit (others)
14. $\Delta$ CL	$\Delta$ Current Liabilities
15. NSALV	Net salvage value
16. $\Delta$ OCA	$\Delta$ Other current assets
17. $\Delta$ I	$\Delta$ Investments
18. $\Delta$ DCO	$\Delta$ Other deferred charges
19. ISSO	Issue expenses of new ordinary shares
20. ISSL	Issue expenses of new debt
21. ISSP	Issue expenses of new preferred shares
22. AMORT	Amortization of debt issue expenses
23. RL	Debt retired
24. OADJ	Other adjustments net
25. $i_o$	Interest rate on debt existing at the beginning
26. $i_n$	Interest rate on new debt
27. $p_o$	Dividend rate on preferred existing at the beginning
28. $p_n$	Dividend rate on preferred issued during the period

## APPENDIX B

Simulation with the Multiperiod Simultaneous  
Equation System for Bell Canada

Below, we reproduce the results of a simulation made with the two-period simultaneous equation system as described in the case of the present report. For both periods we keep unchanged the list of the predetermined variables. Those are the following with the values pulled on (in '000) for the first period:  $DE = 0$ ,  $NEWDEBT = 127,188$ ,  $OPRV = 1,420,690$ ,  $DPR = 50,000$ . For the second period, the same values were fixed for all but the  $OPRV$  which was fixed at 1,435,690. Of course, many other simulations have been performed with other kinds of specification = so, the results reproduced here are mainly for illustrative purposes.

LA SOLUTION DE CE SYSTEME EST: Pour la première période.

## VARIABLES EXOGENES

1	GCE	721300.00000
2	DEPR	296607.00000
3	OPXP	690407.00000
4	QINC	28737.00000
5	INTCO	0.00000
6	EO	1303396.00000
7	RO	402415.00000
8	LO	1772238.00000
9	PRO	248988.00000
10	T	0.50400
11	$\alpha$	0.00000
12	ADCR	0.00000
13	ADCR0	0.00000
14	ACL	0.00000
15	NSALV	0.00000
16	ACCA	0.00000
17	AT	0.00000
18	APCC	0.00000
19	ISSC	0.00000
20	ISSL	0.00000
21	ISSP	113949.00000
22	AMORT	0.00000
23	RL	25000.00000
24	OADJ	0.00000
25	TO	0.06840
26	IN	0.08570
27	PO	0.06600
28	PN	0.06050
29	CF	0.02370
30	CL	0.01190
31	CP	0.02370
32	R	0.15000
33	G	0.26780
34	IC	0.07000
35	APCD	0.00000
36	CONST7	0.00000

## VARIABLES ENDOGENES

1	NETIN	190174.29332
2	NIA	172228.58532
3	F	1303396.00000
4	ACTI	5136.58532
5	NEWDEBT	250000.00000
6	DIVI	127188.00000
7	OPRV	1420690.00000
8	PR	298988.00000
9	DCF	0.48495
10	PCR	0.07050
11	ROBT	0.09962
12	ROPC	0.08266
13	POR	0.73848
14	AP	0.00000
15	APR	50000.00000
16	R	446270.58532
17	L	1997238.00000

LA SOLUTION DE CE SYSTEME EST: Pour la deuxième période.

VARIABLES EXOGENES		VARIABLES ENDOGENES			
1	GCE	721300.00000	1	NETIN	189980.85332
2	DEPR	296607.00000	2	NIA	168735.14532
3	OPXP	690407.00000	3	E	1303396.00000
4	QINC	28737.00000	4	ACTI	1643.14532
5	INTCO	0.00000	5	NDEBT	250000.00000
6	EO	1303396.00000	6	DIVI	127188.00000
7	RO	446270.58532	7	OPRV	1435690.00000
8	LO	1997238.00000	8	PR	348988.00000
9	PRO	298988.00000	9	DCR	0.50189
10	T	0.50400	10	PCR	0.07707
11	$\alpha$	0.00000	11	RORF	0.09534
12	ADCRT	0.00000	12	RORC	0.08004
13	ADGRO	0.00000	13	POR	0.75377
14	ACL	0.00000	14	AE	0.00000
15	NSALV	0.00000	15	APR	50000.00000
16	AOCA	0.00000	16	R	486632.73063
17	AI	0.00000	17	L	2222238.00000
18	ADGO	0.00000			
19	ISSO	0.00000			
20	ISSL	0.00000			
21	ISSP	-113949.00000			
22	AMORT	0.00000			
23	RL	25000.00000			
24	OADJ	0.00000			
25	IO	0.06840			
26	IN	0.08570			
27	PO	0.06600			
28	PN	0.06050			
29	CE	0.02370			
30	CL	0.01190			
31	CP	0.02370			
32	R	0.15000			
33	G	0.26780			
34	IC	0.07000			
35	APCD	0.00000			
36	CONST7	0.00000			

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