Canadian Manufacturing Technology


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I REVIEW OF LITERATURE AND CONGEPTS

1. Introduction

In the early $1970^{\prime}$ s a notable contribution was made in empirical production function analysis with the introduction of the "flexible form" specification for production technology. The new functional forms, such as the translog ${ }^{1}$ or the generalized Leontief ${ }^{2}$, prove to be particularly useful when considering more than two factor inputs to the production process.

The standard neo-classical theory of production posits certain well known properties for mathematical functions which are designed to describe the process of production. The flow of services from factors such as labour, materials, energy and capital, can be combined in various proportions to produce the flows of produced outputs, however only a restricted set of possible combinations are economically viable in the sense of not being wasteful of resources. This fact plus assumptions concerning divisibility of inputs and outputs gave rise to the characteristic properties of convex isoquants. Further assumptions yielded the homothetic property and the neo-classical production function can therefore be characterized as a "strictly quasi-concave homothetic function". While that description gave considerable scope, the major examples of production functions used in empirical analysis, the well known Cobb-Douglas ${ }^{3}$ and the CES ${ }^{4}$ forms, are nevertheless unduly restrictive for purposes of hypothesis testing when more than two factors of production are involved.

1 L.R. Christensen, D.W. Jorgenson, and L.J. Lau, (1971), "Conjugate Duality and the Transcendental Logarithmic Production Function", Econometrica, Vol. 39, No. 4, July 1971, pp. 255-256.
2 W. Erwin Diewert (1971), "An Application of the Shephard Duality Theorem: A Generalized Leontief Production Function", Journal of Political Economy, Vol. 79, May/June 1971, pp. 481-507.
3 C. Cobb and P.H. Douglas (1928), "A Theory of Production," American Economic Review, Supplement to Vol. 18, 1928, pp. 139-165.
K.J. Arrow, H.B. Chenery, B. Minhas, and R.M. Solow (1961), "Capital-Labor Substitution and Economic Efficiency, "Review of Economics and Statistics, Vól. 43, , August 1961, pp. 225-250.

Both of. these types of functional forms have built in the assumption that the elasticity of substitution between two factors is constant, and in the Cobb-Douglas case, that the constant is equal to unity. When only two factoss are present, the CES form, unlike the Cobb-Douglas, does allow one to estimate the single elasticity of substitution but with three or more factors there are several partial elasticities to consider and to assume them all to be equal is to miss some important economic phenomena. For example, there is evidence to show that non-production workers are complementary to capital while production workers are substitutable with both capital and other workers ${ }^{5}$. The CES form lacks the flexibility to allow the investigator to perceive the sign or size differences between the three partial elasticities examined in the above study, and other more suitable functionals are needed for such a task.

The issue can best be expressed by reviewing the relationship between substitution properties, the production function and its separability. As will become evident subsequently our focus of attention will rest chiefly on the separability properties of the estimated production function and this concern will determine our choice of available functional forms.

[^0]
## 2. Separability

Consider a production function with $n$ inputs $y=F\left(x_{1}, \ldots, x_{n}\right)$.
Partition the set of integers $N=\{1, \ldots, n\} \quad$ into $p$ mutually exclusive and exhaustive subsets $\left[N_{1}, \ldots, N p\right]$ to be called the partition $P$. The production function $F(x)$ is said to be weakly separable ${ }^{6}$ with respect to the partition $P$ if the marginal rate of substitution (MRS) between any two inputs $i$ and $j$ from any subset $N_{s}, s=1, \ldots, P$, is independent of the quantities of inputs outside of $N_{S}$, i.e.

$$
\frac{\partial}{\partial x_{k}} \frac{F_{i}}{F_{j}}=0 \text { for all } i, j \varepsilon \mathbb{N}_{S} \text { and } k \varepsilon \mathbb{N}_{S}
$$

where $F_{i}$ denotes the first order partial derivative $\partial F(x) / \not / x_{i}$. Weak separability with respect to the partition $P$ is equivalent to the production function $F(x)$ being of the form $F\left(v_{1}, \ldots, v_{p}\right)$ where $v_{s}$ is a function of the elements of $N_{s}$ only. For example, suppose $X_{1}$ stands for input of labour services, $x_{2}$ for machinery and equipment, and $x_{3}$ for structures. Then the weak separability between labour and machinery and equipment on the one hand and structures on the other can be expressed functionally as $F\left(v\left(x_{1}, x_{2}\right), x_{3}\right)$. Note that the sub-function $v$ is independent of the amount of services issuing from structures:

Clearly a stronger form of functional separability is available to us and this also has economic meaning. Define strong separability with respect to the partition $P$ if the MRS between any two inputs from different subsets $N_{s}$ and $N_{t}$ does not depend on the quantities of inputs outside of $N_{s}$ and $N_{t}$, i.e.

[^1]$$
\frac{\partial}{\partial \bar{x} k k_{j}} \frac{F_{i}}{F_{j}}=0 \text { for all i } \varepsilon N_{S}, j \varepsilon N_{t}, s \neq t \text {, and } k \notin N_{S} U N_{t}
$$

The corresponding functional form is $F\left(v_{1}+\ldots+v_{p}\right)$.
Both types of separability conditions can be written as
$F_{i k} F_{j}-F_{j k} F_{i}=0$ where $F_{i j}$ is the second partial derivative of $F$. This expression can be rewritten in terms of constraints on the estimated values of the parameters of the production function.

The Allen partial, elasticity of substitution (AES) between inputs $x_{i}$ and $x_{j}$ can be expressed in terms of the price elasticities of derived demand
as follows:

$$
\sigma i j=E_{i j} / w j
$$

where

$$
E_{i j} \doteq \frac{\partial X_{i}}{\partial p j} \frac{P_{j}}{X_{i}}
$$

and $w_{j}$ is the share of the $j$ th input in total cost, equal to $p_{j} x_{j} / \Sigma p_{i} x_{i}$, The AES correspond to conventional comparative statistics analysis; they measure: the response of derived demand to an input price change, holding output and all other input prices fixed. The separation of input 3 from inputs 1 and 2 can equivalently be expressed as: ${ }^{7}$
(i) functional separability: $F\left(x_{1} x_{2} \dot{x}_{3}\right)=H\left[J\left(x_{1} x_{2}\right), x_{3}\right]$
(ii) equality of the AES $\sigma_{13}=\sigma_{23}$

7 The second criterion is especially suitable whennestimating cost function.

## 3. Production and Cost Functions

For profit maximizing establishments who take market prices as given there is a duality theorem which shows a one-to-one correspondence between production and costs. To any production function whichobeysfertain regularity conditions (conditions that give the properties mentioned in Section 1) there corresponds a cost function which has the right properties, and conversely. Notice that a production function is defined on the space of input quantities while the (linear homogeneous) unit cost function is defined on the space of input prices. The Cobb-Douglas production function, for example, corresponds to a unit cost function of the same form in the space of input prices, this being an example of a function which is self-dual.

Using the first order efficiency conditions for profit maximization from the production function we are able to derive a system of factor demand equations, one equation for each factor, expressed in terms of input quantities. Analogously, using Roy's lemma', from the cost function we can obtain a system of factor demand equations, this time expressed in terms of input prices.

The duality between cost and production functions widens our choice of production structures to be tested. A cost function which is not self-dual corresponds to a production structure which in part is unknown. However, duality preserves separability properties ${ }^{9}$ so that the separability structure in the cost function is the same as that in the production function. Thus the same functional form, for example translog, can be eemployed either as a production function or as a cost function, giving us two distinct technology specifications through which the hypothesis of separability may be tested, using the available data in different modes.

[^2]
## 4. The Translog Function ${ }^{10}$

This is a quadratic function which relates the $\log$ of output
to the log of inputs and a technology index $A$ :

$$
\begin{aligned}
& \ln y=\ln \alpha_{0}+\alpha_{A} \ln A+\sum_{i}^{n} \alpha_{i} \ln x_{i}+\frac{1}{2} \gamma A A(\ln A)^{2} \\
& +\frac{1}{2} \sum_{i=1}^{n} \sum_{j=1}^{n} \gamma i j \ln x_{i} \ln x_{j}+\sum_{i=1}^{n} \gamma_{i A} \ln x_{i} \ln A
\end{aligned}
$$

where $\gamma_{i j}:=\gamma_{j i}$.
Constant returns to scale (CRTS) imply the following restrictions on the parameters of the translog production function:

$$
\Sigma_{i} \alpha_{i}=1, \Sigma_{i} \gamma_{i j}=0, \Sigma_{j} \gamma_{i j}=0, \sum_{i, j} \gamma_{i j}=0 \quad \Sigma_{i} \gamma_{i A}=0
$$

If production possibilities are characterized by both CRTS and Hicksneutral technical change ${ }^{11}$, the further restrictions on the translog parameters are implied as follows:

$$
\alpha_{A}=1, \gamma_{A A}=0, \gamma_{i A}=0
$$

Under these conditions the translog production function can be written as:

$$
\ln y=\ln A+\ln \alpha_{o}+\sum_{i} \alpha_{i} \ln x_{i}+\frac{1}{2} \sum_{i, j} \gamma_{i j} \ln x_{i} \ln x j
$$

10 This section is taken from Berndt E.R., and L. R. Christensen, The Translog Function and the Substitution of Equipment, Structures, and Labour in U.S. Manufacturing 1929-68.

11 If CRTS holds, factor augmenting technical change with equal rates of augmentation is equivalent to Hicks-neutral technical change. This shows why the Hicks-neutral type of change is readily handled and also shows why it is not a particularly plausible case for a study of information and non-information activities. Technical change seems to have been labour-saving over the period in question but this issue requires a more careful attention and will be looked at in a further study.

Define $F$ to be the output net of technical change, i.e. y =AF. Note that the logarithmic marginal productivity conditions on $y$ and $F$ are identical and independent of $A$ :

$$
\frac{\partial \ln y}{\partial \ln x_{i}}=\frac{\partial \ln F}{\partial \ln x_{i}}=\alpha_{i}+\sum_{j} \gamma_{i j} \ln x_{j}
$$

Since this is the relation to be used under the above conditions, there is no loss in generality in limiting our attention to the aggregate input function $F$.

The first order efficiency conditions for profit maximizing
behaviour is $\frac{\partial y}{\partial x_{i}}=\frac{P i}{P}$
where $P_{i}$ is the price of input $i$ and $p$ is the output price. In log form these conditions become $\cdot \frac{\partial \ln y_{i}}{\partial \ln x_{i}}=\frac{\mathrm{p}_{\mathbf{i}} x_{i}}{\mathrm{Px}}$
which gives the value share in total output value of the ith input. Thus we have the factor demand equations

$$
c_{i}=\alpha_{i}+\sum_{j} \gamma_{i j} \ln x j \quad i=1, \ldots, n
$$

where, under CRTS, $c_{i}$ are the cost share of the ith input in total cost. Note that the condition $\Sigma_{\mathbf{i}_{i}} \mathrm{c}_{\mathbf{i}}=1$ is satisfied by the set of conditions $\Sigma_{i} \alpha_{i}=1$ and $\sum_{i j} \gamma_{i j}=0$ which are implied by CRTS.

The translog production function, owing to its quadratic nature, is an ill-behaved production function globally. When at least one $\mathbb{V}_{i j} \neq 0$ there exists configurations of inputs such that neither monotonicity nor convexity is satisfied. We have to check that the estimated production function is well-behaved for each data point. This involves checking that
the estimated expression for $c_{i}$ are all positive and that the bordered Hessian matrix of first and second partial derivatives of $F$ is negative definite. ${ }^{12}$

Without going into the same detail, it can be shown that 13 anear homogeneous cost structure can be estimated from the system of demand equations.

$$
c_{i}=\alpha_{i}+\sum \gamma_{i j} \ln p_{j} \quad i=1, \ldots, n
$$

subject to $\Sigma \operatorname{ER}_{i}=1, \gamma_{i j}=\gamma_{j i}, \sum_{j} \gamma_{i j}=0$, where $c_{i}$ is the cost share as before and $P_{j}$ is the price of the $j$ th factor. 14 We can aply the separability tests to either the cost or production functions.

For the production function $F$ we noted that the separability conditions can be expressed in terms of the partial derivatives

$$
F_{i k} F_{j}-F_{j k} F_{i}=0
$$

where the ith and $j$ th factors are separated from factors $k \neq i, j$. From this we see that the necessary and sufficient condition for the translog function to be separable is: $c_{i} \gamma_{j k}-c_{j} \gamma_{i k}=0$

Monotonicity requires $c_{i}>0$, all $i$, so that if separability holds and if $\gamma_{j k}=0$, then $\gamma_{i k}=0$. Suppose however, that $\gamma_{j k} \neq \theta, \gamma_{i k} \neq / 0$, then the above condition can be rewritten as

$$
\alpha_{i} \gamma_{i k}-\alpha_{j} \gamma_{i k}+\sum_{m}\left(\gamma_{i m} \gamma_{j k}-\gamma_{j m} \gamma_{i k}\right) \ln _{m}=0
$$

12 Theorem 4.5 in Róckafellar R.T., Convex Analysis, Princeton University Press, 1970.

13 Berndt E.R. and R. Wood, WTechnology, Prices and the Derived Demand for Energy", Rev. Econ. Statist. Aug. 1975, 57, 259-68

For an expression of a non-homothetic cost structure see Fuss, M.A. "Demand for Energy in Canadian Manufacturing" Journal of Economics 1977, 5, 89-116.
and the global conditions (holding for all values of the $\mathrm{x}_{\mathrm{i}}$ ) for inputs $i$ and $j$ to be separated from $k$ becomes

$$
\begin{aligned}
& \alpha_{i} \gamma_{j k}-\alpha_{j} \cdot \gamma_{i k}=0 \\
& \gamma_{i m} \gamma_{j k}-\gamma_{j m} \gamma_{i k}=0 \quad m=1, \ldots, n
\end{aligned}
$$

Alternatively, we can write (for $\gamma_{i k}, \gamma_{j m}$ nonzero)

$$
\frac{\alpha_{i}}{\alpha_{j}}=\frac{\gamma_{i k}}{\gamma_{j k}}=\frac{\gamma_{i m}}{\gamma_{j m}} \quad m=1, \ldots, n
$$

Substitution possibilities among inputs can be measured by calculating the Allen partial elasticity of substitution. Allen (1938) has defined the partial elasticicity of substitution (AES) between inputsi and j as

$$
\begin{equation*}
\sigma_{i j}=\sum_{h=j}^{n} \quad F_{h} X_{h}\left|\bar{F}_{i j}\right| X_{i} X_{j}|\bar{F}| \tag{1}
\end{equation*}
$$

where $|\bar{F}|$ is the determinant of the bordered Hessian $\bar{F}$, and $\left|\vec{F}_{i j}\right|$ is the cofactor of $\mathrm{F}_{\mathrm{ij}}$ in $\overline{\mathrm{F}}$. Substituting the three-factor translog bordered Hessian in (1) yields

$$
\sigma_{i j}=\left|G_{i j}\right| /|G|
$$

where $|G|$ is the determinant of

$$
G=\left[\begin{array}{ccc}
0 & C_{1} & c_{2} \\
c_{1} r_{11}+c_{1}-c_{1} & \gamma_{12}+c_{1} c_{2} & r_{13}+c_{1} c_{3} \\
c_{2} r_{12}+c_{1} c_{2} & r_{22}+c_{2}^{2}-c_{2} r_{23}+c_{2} c_{3} \\
c_{3} r_{13}+c_{1} c_{3} & r_{23}+c_{2} c_{3} r_{33}+c_{3}^{2}-c_{3}
\end{array}\right]
$$

and $\left|G_{i j}\right|$ is the cofactor $G_{i j h}$ in $G$.

## 5. Estimation Technique

At this stage of the research we are estimating a four-factor translog prỏduction function using time series data, 1948-1973 on total Canadian manufacturing. The four factors are information labour I, non-information labour $N$ machinery and equipment E; and structures S.: The estimated coefficients can yield tests of various possible combinations of separability, but our main interest at this juncture is to test for separability between structures and the other inputs. To do this the derived demand relations are estimated by the Three Stage Least. Square (3SLS) technique and the estimated parameters can be subjected to the separability test.

With four input factors we obtain a system of four demand relations. By symmetry and homogeneity conditions the parameters for any one equation can be derived from the parameter estimates for the remaining three. Hence we estimate only the three equation system: corresponding to the two types of labour and the $M$ factor, from which can be constructed the demand for structures. Moreover, not all parameters in the remaining equations need be estimated. To see this, denote information labour, non-information labour, machinery and equipment, and structures by $I, N, E$ and $S$ respectively. Then we have the three equations ( $i==I, N, E)$ :

$$
c_{i}=\alpha_{i}+\gamma_{i I} \ln x_{I}+\gamma_{i N} \ln x_{N}+\gamma_{i E} \ln x_{\mathrm{E}}+\gamma_{i s} \ln x_{S}
$$

where $\gamma_{i . j}=\gamma_{j i}$.

However by the conditions $\gamma_{i I^{+}} \gamma_{i N}+\gamma_{i M^{+}} \gamma_{i E}=0, i=I, N, E$, we can write the system of equations as:

$$
\begin{gathered}
c_{I}=\alpha_{I}+\gamma_{I I} \ln \left(x_{I} / x_{S}\right)+\gamma_{I N} \ln \left(x_{N} / x_{S}\right)+\gamma_{I E} \ln \left(x_{E} / x_{S}\right) \\
c_{N}=\alpha_{N}+\gamma_{I N} \ln \left(x_{I} / x_{S}\right)+\gamma_{N N} \ln \left(x_{N} / x_{S}\right)+\gamma_{N E} \ln \left(x_{E} / x_{S}\right) \\
c_{M}=\alpha_{M}+\gamma_{I M} \ln \left(x_{I} / x_{S}\right)+\gamma_{N E} \ln \left(x_{N} / x_{S}\right)+\gamma_{E E} \ln \left(x_{E} / x_{S}\right)
\end{gathered}
$$

Tó each equation we add additive error terms ${ }^{15}$.

Applying the method of least squaressto this system we can obtain estimates for the parameters $\alpha_{i}, i=I, N, E$ and $\gamma_{i j}, i<j, j=I, N, E$. The variables on the right hand side cannot, however, be treated as exogenousi since the decisions of the firm involve : a simultaneous choice of all inputs. In order to avoid simultaneous equation bias produced by non-zero covariances among the disturbances, we identify a set of variables cexogenous to the system and use these in the three stage least squares estimation. In practice we use the LSQ section of the TSP programme package. This gives us a 3SLS facility by a non-1inear regression method which nonethetess is suitable for our linear system of equations.

15
The most appropriate method of making the equation system stochastic requires some thought. Also our estimation techniquesshould properly take into account the error term in the last, deleted equation.
II. CONSTRUCTION OF DATA ON INFORMATION/NON-INFORMATION LABOUR SERVICES

As described in the first interim report, as a first approximation, series of information/non-information labour services were derived for 1947-1974 from an aggregated man-hours series. It was intended that these data would be discarded when more accurate data became available.

More detailed data, described hereafter, have now been compiled and keypunched. The data consisted of man-hours worked by employees in each major manufacturing industry for the years 1961 to 1974 , disaggregated into two groups: "non-information" workers consisting of production workers in manufacturing operations, employees in new construction, outside piece workers; and other production and related workers;"and "information" workès consisting of administrative and office employees, sales and distribution workers, and employees at other locations.

For the purposes of time series estimation, these data presented two major problems: i) the disaggregated series began only in 1961 whereas we required data from 1947 onwards; ii) the methodology whereby the series were compiled changed in 1969 causing discontinuities in the data at that point. We first repaired the data by mending the discontinuity, then used the revised series to extrapolate for the period 1947-1960.

The Productivity Division of Statistics Canada describes the derivation. of the man-hours worked series as follows:
"In manufacturing, the Annual Census of Manufactures is the basic source of man-hours data, but it is supplemented by two other surveys; Earnings and Hours of Work in Manufacturing and Survey of Labour Costs. Man-hours
worked for production workers are taken from the Census. For the rest of the employees, man-hours paid are obtained by payroll deflation of Census salaries. These man-hours paid are then converted to man-hours worked by extrapolating the 1968 labour cost survey relationship between man hours paid and worked for the non-production workers. (The extrapolation was based on the movements of the same ratio for production workers). The average hours worked by other than paid workers, however, was applied to the number of working owners and partners to obtain an estimated man-hours worked.

The reason for the discontinuity in the series in 1969 was due to the cancellation of the labour cost survey in 1968, which was one of the primary data sources. After 1969, man-hours had to be estimated on the basis of extrapolation of the 1968 relationships. The effect of this extrapolation was to increase the number of man-hours worked of information workers as shown in Table 1. It did not however significantly affect the man-hours of non-information workers.

Our adjustment procedure consisted of multiplying the 1970 to 1974 figures by the ratio 1969 (old)/1969 (new). The original and revised man-hours data are shown below in Table 1.

## TABLE 1

## Estimates of Information Man-Hours Worked

Total Canadian Manufacturing, 1961-1974

| Year | Original | Revised |
| :--- | :--- | :--- |
| I96I | $774,401,306$ | $774,701,306$ |
| 1962 | $783,174,658$ | $783,174,658$ |
| 1963 | $796,825,394$ | $796,825,394$ |
| 1964 | $826,670,659$ | $826,670,659$ |
| 1965 | $872,590,484$ | $872,590,484$ |
| 1966 | $890,650,379$ | $890,650,379$ |
| 1967 | $905,104,692$ | $905,104,692$ |
| 1968 | $916,574,130$ | $916,976,098$ |
| $1969(1)$ | $918,574,130$ | $918,574,130$ |
| $1969(2)$ | $1,042,761,000$ |  |
| 1970 | $1,016,028,000$ | $895,024,877$ |
| 1971 | $990,656,000$ | $872,674,537$ |
| 1972 | $985,838,000$ | $868,430,334$ |
| 1973 |  | $1,025,581,000$ |

To extend the series of information and non-information man-hours back to 1947 a number of regressions were run relating man-hours worked (both original and revised) in each group to time and to the number of workers in the group.

The two following equations provided the best fit:

LIT $=7.01251+1.04101$ LIWKRS
(12.0136) (23.2451)
t - values in brackets
$R^{2}=.9846$

LIT $=$ the natural logarithm of the number of man-hours worked of information workers, using the revised series

LIWKRS $=$ published number of information workers in Canadian manufacturing

LNT $=9.37796+.872480$ LNWKRS
(20.8796) (27.0775)
t - values in brackets
$R^{2}=.9868$
LNT $=$ the natural logarithm of the number of man-hours worked of non-information workers (no adjustments required).

LNWKRS $=$ published number of non-information workers in Canadian manufacturing.

These equations were then used as a basis for estimating
information/non-information houvs worked for the period 1947-1960.
III. ESTIMATION OF FACTOR DEMAND EQUATIONS

Once the data were assembled, we applied the translog function to the problem of estimating substitution possibilities among machinery and equipment (E), structures (S), information labour (I) and non-information labour (N) in Canadian manufacturing 1948-1973.

## Summary of Data

The data required for this study are the prices and quantitiesof the inputs. Capital stocks and service prices were derived as discussed in the first interim report. Labour input data came from two sources: The quantity of $I$ and $N$ were derived as described in the preceding section. Corresponding prices were derived by dividing wages andfalaries data by the man-hours series. The quantities of all inputs were scaled to unity (zero logarithms) in 1948, the starting point of our 1948-1973 sample. Table 2 listis lists the series of quantity indexes for each of the four factor inputs.

The input prices and quantities are viewed as endogenous variables. The following variables are considered exogenous to the Canadian manufacturing sector and are used in our two and three stage least squares regressions to purge the quantities of $E, S, I$, and $N$ of correlation with the additive disturbances: (1) Canadian population, (2) Canảdianipopulation ófiworking age, (3) Total immigration to Canada, (4) immigrants destined for the labour force, (5) government expenditure on goods and services, (6) total exports, (7) longterm bond yield, (8) U.S. G.N.P., aand (9) total imports.

The dependent variables in the set of equations are the cost shares of the input factors. These are tabulated in Table 3.

TABLE 2
Quantity Indexes of Inputs in Canadian Manufactiring
1948-1973


Looking over the columns of Table 2, we note that over the 1948-1973 time period, the quantities of $I$, $S$, and $M$ rose more rapidly than the quantity of $N$, and the quantity of $M$ especially rapidly. This would appear to indicate that technological change has not been equally factoraugmenting, as we hypothesize, but rather has had the effect of substituting machinery and equipment for non-information workers. Whether this hypothesis holds will be further investigated when we examine the estimated parameters of the four-factor production function.

TABLE 3
Cost Shares dif Labour and Capital Inputs
Canadian Manufacturing
1948-1973


"Y
END OF OUTPUT FOR LABOUR
EXECUTION TIME $=2.200$ SECONOS


The cost shares tabulated in Table 3 are also depicted graphically in Figure 1. In the period 1948 to 1961 , the value shares of non-information labour and structures were decreasing, whereas the value shares of information labour and machinery and equipment were on the increase. However, in the period 1962 to 1973 the trends reversed themselves to some extent. The cost share of Information labour fell off steadily in the 1970-1973 period, and structures increased steadily.

The appearance of the factor shares tend to indicate a possible break in the pattern of technological change. To examine the question further, we should test whether the assumption of equal factor-augmenting technological change is supported by the data, and we should also test the hypothesis of parameter equality in the $1948-1961$ and $1962-1973$ period, using the Chow ${ }^{16}$ test. Couromaprobleminconductingesuch tests lies in-the paucity of degrees of freedom.

16
G.C. Chow: (1960), "Tests of Equality Between Sets of Coefficients in Two Linear Regressions," Econometrica, Vo1. 28, No. 3, July 1960, pp. 591-605.

## Empirical Results - Three-Factor Production Function

First we estimated a three-factor system with the quantity and price of capital services, $K$, derived as a Divisia index of equipmettend structures.

In Table 4 we present estimates of the unconstrained trans log parameters based on the OLS regressions, using instrumental variable (2SLS), and on ordinary least square regressions adjusted for first-order auto-correlation (CORC). The third procedure was tried because of the low Durbin-Watson statistics in the OLS regressions. (1.04 for the I equation, 1.38 for $N$ and 1.31 for $K$.)

## TABLE 4

Parameter Estimates of Unconstrained Translog Parameters Three-Factor Case

| Parameter | OLS | 2SLS | CORC |
| :---: | :---: | :---: | :---: |
| ${ }^{\alpha}$ I | . 0993 | . 1002 | . 0932 |
| ${ }^{\alpha}{ }_{N}$ | .3743 | .3760 | .3706 |
| ${ }^{\alpha}{ }_{K}$ | . 5264 | . 5238 | . 5349 |
| $\gamma_{\text {II }}$ | . 1025 | . 0936 | .1149 |
| $\gamma_{\text {IN }}$ | -. 1718 | -. 1703 | -. 1979 |
| $\gamma_{\text {IK }}$ | . 0190 | . 0253 * | . 0185 |
| $\gamma_{\text {NI }}$ | -. 0913 | -. 1191 | -. 0802 |
| $\gamma_{\text {NN }}$ | . 0806 | . $0571 \%$ | . $0595 \%$ |
| $\gamma_{\text {NK }}$ | . 0187 \% | . $0436 \%$ | . 0162 |
| $\gamma_{\mathrm{KI}}$ | -. 0112 * | . 0255 \% | $-.0334 \%$ |
| $\gamma_{\mathrm{KN}}$ | . 0911 | . 1132 | .1337 |
| $\gamma_{\mathrm{KIK}}$ | $-.0377$ | -. 0688 | $-.0339 \%$ |

* denotes not significantly different from 0 at $\alpha=.05$.

| Equation | OLS | 2SLS | CORC |
| :---: | :---: | :---: | :---: |
| I | .98 | .98 | .99 |
| N | .88 | .86 | .87 |
| K | .64 | .59 | .71 |

It is also of interest to compare the results obtained to those estimated for U.S. manufacturing $1929-68$ by Berndt. ${ }^{17}$ While the parameter estimates differ, of course, the signs (using 2SLS) of ${ }^{\gamma}{ }_{\text {IN, }}{ }^{\gamma}{ }^{\text {IK, }}{ }^{\gamma}{ }^{\gamma}$ NI are the same. However, the signs of $\gamma_{\mathrm{KK}}$ (to which further attention is devoted below), and $\gamma_{\text {KN }}$ which we obtain are opposite to those of Berndt.

Our estimated $\gamma_{\text {KK }}$ is less than zero, implying that if the quantity of capital is increased, then the value share of capital will decrease, labour factors being held constant, which is a clear contradiction. A tentative conclusion to be drawn from this result is that there is no consistent aggregate index of equipment and structures. Further study is required to ascertain whether this is so. Such a study would be a sub-component of a study of a four-factor production function, with equipment and structures treated separately. It is to this four-factor production function which we now turn.

## Empirical Results - Four-Factor Production Function

In Table 5 we present estimates of the unconstrained translog parameters for the four-年actor production function. As in the three-factor case, the parameters were estimated using three techniques: ordinary least-squares (OLS), ordinary least squares corrected for first-order auto-correlation (CORC), and two-stage least squares (2SLS).

[^3]| Parameter | OLS | CORC | 2SLS |
| :---: | :---: | :---: | :---: |
| ${ }^{\prime}$ I | . 1054 | . 1011 | . 1051 |
| ${ }^{\alpha}{ }_{N}$ | . 3768 | . 3705 | . 3767 |
| $\alpha_{\text {E }}$ | . 2375 | . 4609 | . 2351 |
| $\alpha_{S}$ | . 2802 | . 3376 | . 2830 |
| $\gamma_{\text {II }}$ | . 1124 | . 1151 | . 1080 |
| IN | -. 1874 | -. 1934 | -. 1890 |
| $\gamma_{\text {IE }}$ | -. 0412 | -. 0245 * | -. 0351 * |
| $Y_{\text {IS }}$ | . 0746 | . 0518 * | . 0703 |
| ${ }^{\gamma}{ }_{\text {NI }}$ | -. 0871 | -. 0801 | -. 1160 |
| $\gamma_{\text {INA }}$ | . 0744 | . 0598 * | . 0554 * |
| $\gamma_{\text {NE }}$ | -.0116** | . 0085 * | . 0134 * |
| $\gamma_{\text {NS }}$ | . 0364 * | . 0077 * | . 0317 * |
| $\gamma_{\text {EI }}$ | . 0564 | -. 0085 * | . 1458 |
| $\gamma_{\text {EN }}$ | -.0437 * | . 1001 * | . 0125 * |
| $\gamma_{\text {EE }}$ | . 1094 | -. 2433 * | . 0494 * |
| $\gamma_{\text {ES }}$ | -. 1606 | . 1727 * | -. 1699 * |
| $\gamma_{\text {SI }}$ | -. 0817 | -. 0461 | -. 1378 |
| $\gamma_{\text {SN }}$ | . 1567 | . 1442 | . 1210 |
| $\gamma^{\text {SE }}$ | -. 0567 | -. 2918 | -. 0277 * |
| $\gamma_{\text {SS }}$ | $\begin{aligned} & .0497 \% \\ & \mathrm{R}^{2} \text { Values } \\ & \hline \end{aligned}$ | . 3118 | . 0679 * |
|  | OLS | CORC | 2SLS |
| I | . 987 | . 987 | . 987 |
| N | . 8852 | . 872 | . 862 |
| E | . 6890 | . 748 | . 465 |
| S | . 9532 | . 978 | . 911 |

Denotes not significantly different from 0 at $\alpha=.05$

Next we imposed the assumed constraints of constant returns to scale, and symmetry $\left(\gamma_{i j}=\gamma_{j i}\right)$.

To estimate the parameters subject to these constraints requires dropping one equation. We first dropped the $S$ equation and used the TSP iterative three-stage least squares program to estimate the remaining free parameters (9). However the program did not converge successfully.

Next we estimated the system of the $I, N$ and $S$ equations simultaneously.

The equation system estimated was:
$C_{I}=\alpha_{I}+\gamma_{I T}(\ln I / E)+\gamma_{I N}(\ln N / E)+\gamma_{I S}(\ln S / E)$
$C_{N}=\alpha_{N}+\gamma_{\text {IN }}(\ln I / E)+\gamma_{N N}(\ln N / E)+\gamma_{N S}(\ln S / E)$
$C_{S}={ }^{\alpha} S+\gamma_{I S}(\ln I / E)+\gamma_{N S}(\ln N / E)+\gamma_{S S}(\ln S / E)$

In this case convergence was achieved after four iterations. The I3SLS estimated of the parameters from this system are presented in Table 6 below. The conventional $\mathrm{R}^{2}$ figures for the three estimated equations are 193 for the I equation, .88 (N) and .93 (S). The Durbin - Watson statistics are $.60(\mathrm{I}), 1.27(\mathrm{~N})$ and $.67(\mathrm{~S})$.

The starting values for the estimation were the 2 SLS estimates of the parameters, as shown in Table 5.

## TABLE 6

I3SLS Parameter Estimates of Translog Production Function

| Parameter | Estimate | Parameter | Estimate |
| :---: | :---: | :---: | :---: |
| $\alpha_{I}$ | .0901 | $\gamma_{\text {IS }}$ | -.0937 |
| $\alpha_{N}$ | .3788 | $\gamma_{\text {NN }}$ | .0496 |
| $\alpha_{E}$ | .2477 | $\gamma_{\text {NE }}$ | -.0299 |
| $\alpha_{S}$ | .2834 | $\gamma_{\text {NS }}$ | .0651 |
| $\gamma_{\text {II }}$ | .1154 | $\gamma_{\text {EE }}$ | .0303 |
| $\gamma_{\text {IN }}$ | -.0848 | $\gamma_{\text {ES }}$ | -.0635 |
| $\gamma_{\text {IE }}$ | .0631 | $\gamma_{S S}$ | .1021 |

A11 estimated parameters were significantly different from zero, at the $\alpha=.05$ level of significance.

Examination of the results shows that the fitted cost shares are positive for all observations. In addition $\gamma_{j j}(j=I, N, M, S)$ are all positive, as required.

We note that since the $\gamma_{i j} \neq 0$, i $\neq j$, this implies that our translog specification does not reduce to the simpler Cobb-Douglas case, and hence, that the conditions for complete global separability are not satisfied. It may. be, however, that some type of weak separability cannot be rejected.

## 4. Conclusion

In this interim report we have documented the recent econometric 1iterature relevant to our research, and have explained in some detail the methodology we are adopting.

We next described the data construction for the estimation of a production function for Canadian manufacturing, 1948-1973, and presented the empirical results of our estimation.

Completion of our work on the time series data involves a few more steps.

First, we should test whether the production function is convex at each point by determining whether the bordered Hessian matrix of partial derivatives is negative definite.

Next we wish to measure factor substitution possibilities by computing the estimated Allen partial elasticities of substitution, and price elasticities.

We also wish to test whether structures are weakly separable from the other three inputs, as a basis for our upcoming work with pooled crosssectional and time series data, for the twenty major manufacturing industries.

Finally, the exercise is to be repeated using the cost function rather than the production function.


[^0]:    5 E.R. Bernt and L:R. Christensen (1974), "Testing for the Existence of a Consistent Aggregate Index of Labour Inputs"; American Economic Review (June 197.4), 391-404.

[^1]:    6. This section closely follows the analysis of Berndt, E.R. and L. R. Christensen, "The Internal Structure of Functional Relationships": Separability, Substitution, and Aggregation," Review of Economic Studies (July 1973a), 403-410.
[^2]:    8 Diewert W.E., "Applications of Duality Theory," in Frontiers off Quantitative Economics. Vol II edited by M.D. Intritigator and D. A. Kendrick. Amsterdam: North Holland Pub. Co. 1974.
    9
    Lau, Ji. "Duality and the Structure of Utility Functions" Journai of
    Económic Theory, 1, 1970.

[^3]:    17 E. Berndt, The Economic Theory of Separability, Substitution and Aggregation with an Application to U.S. Manufacturing 1929-1968, Phd. Thesis, University of Wisconsin, 1972.

