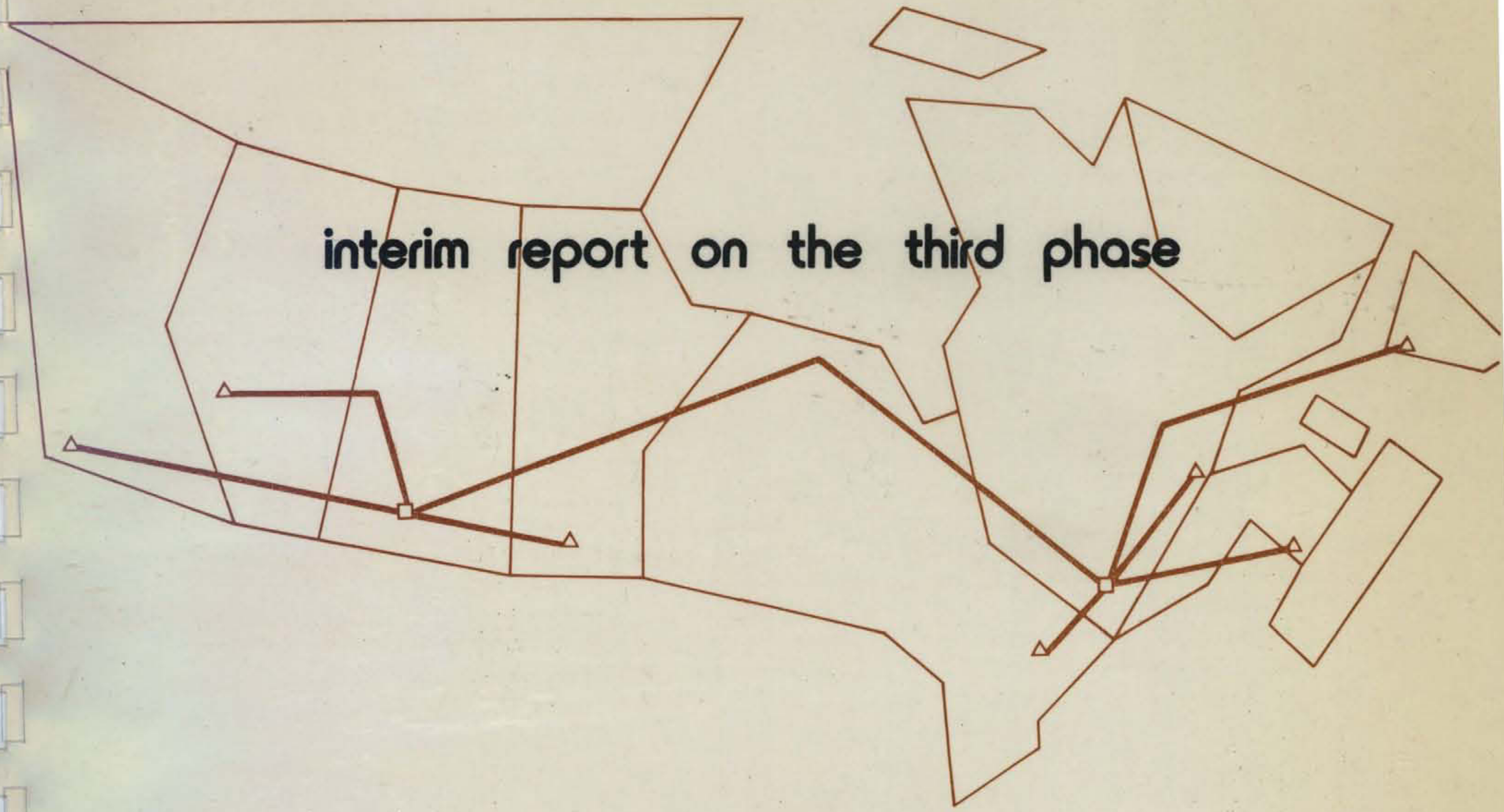
NP&PS PROJECT



interim report on the third phase

PREPARED FOR AND IN COLLABORATION WITH THE

NATIONAL TELECOMMUNICATIONS BRANCH

DEPARTMENT OF COMMUNICATIONS

BY

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NPPS PROJECT

NATIONAL POLICY AND PLANNING SIMULATION MODEL

Phase III

(formerly known as the Inter-Regional Accounting Project)

INTERIM REPORT ON PHASE III

prepared for and in collaboration with the

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DEPARTMENT OF COMMUNICATIONS

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Montréal

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FOREWORD

The National Policy and Planning Simulation Model (NPPS Model) aims at developing tools which respond to policy questions in the domain of telecommunications. Typical of these questions are the evaluation of costs per services, their relationships with tolls and consequently the identification and the measurement of the nature and the extent of cross-subsidization among the services. To this end, the model suggests costing separation procedures for operational implementation of these conceptual developments. It is to be noted that the NPPS model is a direct extension of, and an improvement upon, the previous Inter-regional Accounting Model.

ABSTRACT

During the first part of this Third Phase of the NPPS Project, previously called the IRA Project, the emphasis was put on the following four points: increase the operationality of the different algorithms, mainly by using a new computer system, strengthen the interrelationships among the blocks and especially between the Operating and the Costing ones in order to measure empirically some costing concepts, mechanize one step further the Accounting Block, and finally undertake a costing and accounting experiment focused on the Maritimes. The main goal in concentrating on these points is to develop costing separation procedures, to relate costs to tolls, in order that the nature and the extent of cross-subsidization among services might be identified and measured. The present Report describes, in detail, the content of each of these four points. It also suggests some new developments, for the short as well as the long term.

The report is divided into four chapters of which the first, "Introduction", summarizes the objectives both of this report and of the overall project, and presents a description of the structure of model and the computer system. Chapter 2, "Technical Status and Proposed Developments, presents a summary of the work undertaken to date and of the proposed development plan during the remainder of Phase III. On account of the importance of system conversion, an extensive description is incorporated of the strategy and the execution of the conversion of the NPPS System from the McGill University Computing Centre facilities (MUSIC system) to Computer Science Canada facilities (CSTS system). It may be useful to remind the reader that this conversion was mainly undertaken because problems of the size now required in the NPPS system could not be handled under the MUSIC system. At the time of writing the present Report, most of the conversion problems are now resolved.

The third chapter can be seen as the core of the Report: all the new conceptual developments since the Supplementary Report on the Second Phase of the IRA Project, March 31, 1975 are contained and explained in this chapter. The main efforts are devoted to the description of a procedure to implement empirically the marginal costing of each service, both at the transmission as well as the switching levels. The proposed approach integrates in a very intimate manner the Operating and the Costing Blocks. The Aging-Indexing-Depreciation Algorithms are now more fully mechanized and the manner by which this is done is also described, as well as the work to be done in order to integrate these algorithms with the proposed costing procedures. In regard to the Accounting Block, the vectors of BEADS and NON-BEADS are now integrated in the APL program, and some multi-period simulations are proposed. Appendices appearing at the end of the Report contain the more technical points on these different subjects and present the results of some of the accounting simulations that have been performed.

The fourth chapter contains the description of a costing experiment focused on the Maritimes, undertaken in order to better understand and improve the cost characteristics of the NPPS and HERMES Models, as well as to assist in the incremental costing developments of the NPPS Project. The chapter is structured as follows: objectives of the experiment and data sources, description of the switching and transmission networks. The details of the runs performed and presentation and evaluation of the obtained results will be incorporated into an appendix to be produced following completion of the entire experiment.

RESUME

Durant la première partie de cette troisième phase du projet NPPS, auparavant dénommé projet IRA, l'accent a été mis sur les points suivants: accroître l'opérationnalité des divers algorithmes, principalement par l'utilisation d'un nouvel ordinateur, renforcer l'interrelation entre les "blocs" et surtout entre le bloc d'opérations et le bloc des coûts, en vue de mesurer empiriquement quelques concepts de coûts, automatiser davantage le bloc comptable, et finalement expérimenter avec les Maritimes les concepts comptables et de coûts. Le principal but en se concentrant sur les points ci-dessus est de finaliser le développement de quelques procédures de séparation des coûts en vue de relier ces derniers aux tarifs, et donc d'identifier et de mesurer la nature et le degré d'inter-financement entre les services. Le présent rapport décrit en détail chacun des points ci-dessus en plus de suggérer des développements futurs, aussi bien pour le court terme que pour le long terme.

Le rapport se divise en quatre chapitres; le premier intitulé "Introduction", résume les objectifs de ce rapport et l'ensemble du projet NPPS, en plus de présenter une description de la structure du modèle, ainsi que du système d'ordinateur. Le deuxième chapitre, "Statut technique et développements proposés", résume les travaux entrepris depuis le début de la Phase III, et présente le plan de développement proposé à l'échéance de ladite phase actuelle. Dû à l'importance de la conversion du système NPPS, on explique dans le cadre du chapitre, la stratégie et l'exécution de la conversion du système NPPS, en le transférant à "Computer Science Canada" (système CSTS) alors qu'auparavant il était localisé au centre de calcul de l'Université McGill (système MUSIC). Il est peut-être utile de rappeler au lecteur que cette conversion fut entreprise surtout à cause de la taille requise par notre modèle actuel, taille que le système MUSIC ne pouvait plus suffire à contenir. Au stade actuel la presque totalité ou même la totalité de la conversion est complétée.

Le troisième chapitre peut être considéré comme le coeur du rapport: tous les nouveaux développements conceptuels y sont inclus et expliqués depuis le rapport supplémentaire sur la deuxième phase du projet IRA' du 31 mars 1975. Les principaux efforts ont porté sur la description d'une procédure visant à mesurer empiriquement les coûts marginaux de chaque service, aussi bien au niveau de la transmission que de la commutation. Pour évaluer le coût marginal par service au niveau de la transmission, l'approche suggérée nécessite l'intégration assez étroite des blocs d'opérations et de coûts. De plus, les algorithmes AIDIGO (Aging-Indexing-Depreciation) sont maintenant complètement automatisés et les détails pertinents y sont décrits, de plus, des travaux devront être entrepris afin d'intégrer ces derniers algorithmes avec les procédés de calcul des coûts. En ce qui concerne le bloc comptable, les vecteurs Beads et

NON-BEADS sont maintenant intégrés au programme APL, et quelques simulations multi-périodes sont proposées. Les annexes à la fin du rapport donnent plus de détails techniques sur ces sujets, de même que des résultats de ces simulations.

Le quatrième chapitre décrit les résultats d'un essai du modèle avec les Maritimes entrepris afin d'améliorer et de comparer l'aspect coûts des modèles HERMES et NPPS, aussi bien que pour développer les procédés de calcul des coûts d'accroissements (incremental) du projet NPPS. On y retrouve les sections suivantes: objectifs d'essais et sources de données, description du réseau de commutation et du réseau physique. Les détails sur les calculs effectués ainsi qu'une présentation et une évaluation des résultats obtenus seront incorporés en une annexe qui sera produite à la fin des essais, actuellement en cours.

1. INTRODUCTION

1.1 Objectives of the Project

The Government's objectives regarding Canadian telecommunications have been identified in the Green Paper on communications policy for Canada. As a summary statement these objectives may best be reflected by quoting from the last paragraph on page 35 of the Green Paper:

> "Economic, efficient, and adequate communications, making the best use of all available modes, are essential to the sovereognty, integrity, and defence of Canada, and for the political freedoms, social well-being, cultural development, economic prosperity, and safety of all Canadians."

It is the responsibility of the Minister of Communications to pursue national objectives. The Minister also needs to be advised and kept informed in the course of discharging his responsibilities.

The purpose of the "national policy and planning simulation model" developed by the Department of Communications is to provide for the Minister, at least in part, the necessary tools and methods for evaluating various policy options and alternatives as required for the Minister in the course of discharging his responsibilities.

One important way of evaluating the various policy options and alternatives is by examining the quantitative impacts of the scenarios under consideration through the utilization of techniques of simulation. Accordingly, the model has been designed for simulation purposes with a capability of dealing with alternatives and/or issues with respect both to the real operations of the telecommunications system and to the financial consequences for the carriers and for the system.

1.2 Context of this Report

1.2.1 Objective of this report

This report is an interim report on the progress of the Phase III of the NPPS Project, reporting on the progress of the work in relation to the planning as expressed in the Memorandum of Understanding, issued jointly as May 15th, 1975.

1.2.2 Objectives of Phase III

It is the purpose of the NPPS project to use the model operationally by applying it to the issues facing the Department in the process of making policy decisions and proposing changes in legislation, and to perfect the model to the end of executing such applications effectively and efficiently. For this purpose the following are the main objectives of the work to be performed in the current phase (Phase III).

a) to formulate, process and evaluate a number of simulation scenarios or studies concerning the Canadian Telecommunications Carrier Industry with respect to:

   i) the operation and expansion of the Canadian intercity network, e.g. routing of traffic, allocation of resources, quality of service, survivability, capacity utilization and expansion;

   ii) the embedded and prospective incremental and average costs associable with the different services;

   iii) the tolls to costs relationships of the different services;

   iv) the impact of various tariff policies and structures for the different services;

   v) the cross subsidies as between the different services and carriers;

   vi) the impact of different revenue settlement schemes among the carriers;

   vii) inter-carrier competition;

   viii) different rules for costing and accounting;

   ix) different taxation policies;

   x) the different methods of valuation of the physical assets of the carriers, e.g. reproduction costs, indexed (trended) original cost;

   xi) the impact of different corporate and financial structures in the carrier industry.

1.2.3 Participants in the Project

The present Report represents the results of a combined effort by three groups whose formal responsabilities were spelled out in the various official documents. In practice detailed sharing of the tasks was handled by more or less informal exchanges. The tripartite team consisted of the following organizations given here with the names of the specialists involved.

The National Telecommunications Branch, Communications Canada:

Mr. J.A. Guérin
Mr. G.G. Henter
Mr. C. Lee
Mr. P. Rogers

Sorès Inc., Montreal

Mr. J. Cluchey
Mr. A. Djenandji
Miss C. King
Mr. R. Lecompte
Mr. E. Manis
Mr. J.-P. Schaack
Mr. B. Webber

Laboratoire d'économétrie de l'Université Laval

Prof. C. Autin
Prof. G. LeBlanc

As members of le Laboratoire d'économétrie, the following research assistants have contributed to the Project:

Mr. R. Huppé
Mr. G. St-Cyr

Finally, Dr. I. Young from York University, in the capacity of consultant to the Department of Communications, has contributed in the area of multi-period accounting analysis in the simultaneous equation systems.

1.3 General Description of the National Policy and Planning Simulation Model

1.3.1 Introduction

In line with the objectives set out in the introduction, the model has been designed in a multi-faceted fashion to be capable of tying together a number of main activities. These are:

1) the structure and operations of the national telecommunications network, and the types of traffic it carries;

2) the costing processes which permit the association of various cost categories with the physical activities;

3) various alternative sharing schemes for the division of revenues generated by the services jointly provided by two or more Canadian carriers;

4) the financial and capital sourcing considerations, and the expression of the results of these activities in financial terms.

For each of these sets of activities there is a block, or module, within which the necessary computations are performed and outputs generated.

The general structuring of the model, the various blocks and their inter-relations are indicated by the Flowchart, figure 1. This flowchart shows the four blocks just described:

- Operating block
- Costing block
- Sharing block
- Accounting block

It also contains an indication of the simulation capability, which, as can be seen, may be applied at any point in the system. In summary, then, the model is designed to process information, which is at a highly disaggregated level, from block to block, for simulation of the operating, costing, sharing and accounting activities of the national communications network. The model's "building block" structure allows simulations to be run using any one block for the examination of a particular problem, or using all blocks together as an integrated unit. This structure allows great flexibility in the variety of scenarios which may be tested and evaluated.

The following brief notes describe what the blocks contain and what they do in a rather superficial manner, to provide a preliminary overview.

1.3.2 Operating block

The operating block deals with the physical activities. It contains the data and algorithms pertaining to the national network, both the switching network and the transmission facilities network, the traffic processed through the network, the routing, and the tariffs applicable to the various services. There are two main outputs from the operating block:

1) The usage of facilities by elements, by service and by stream, providing an input for the costing block;

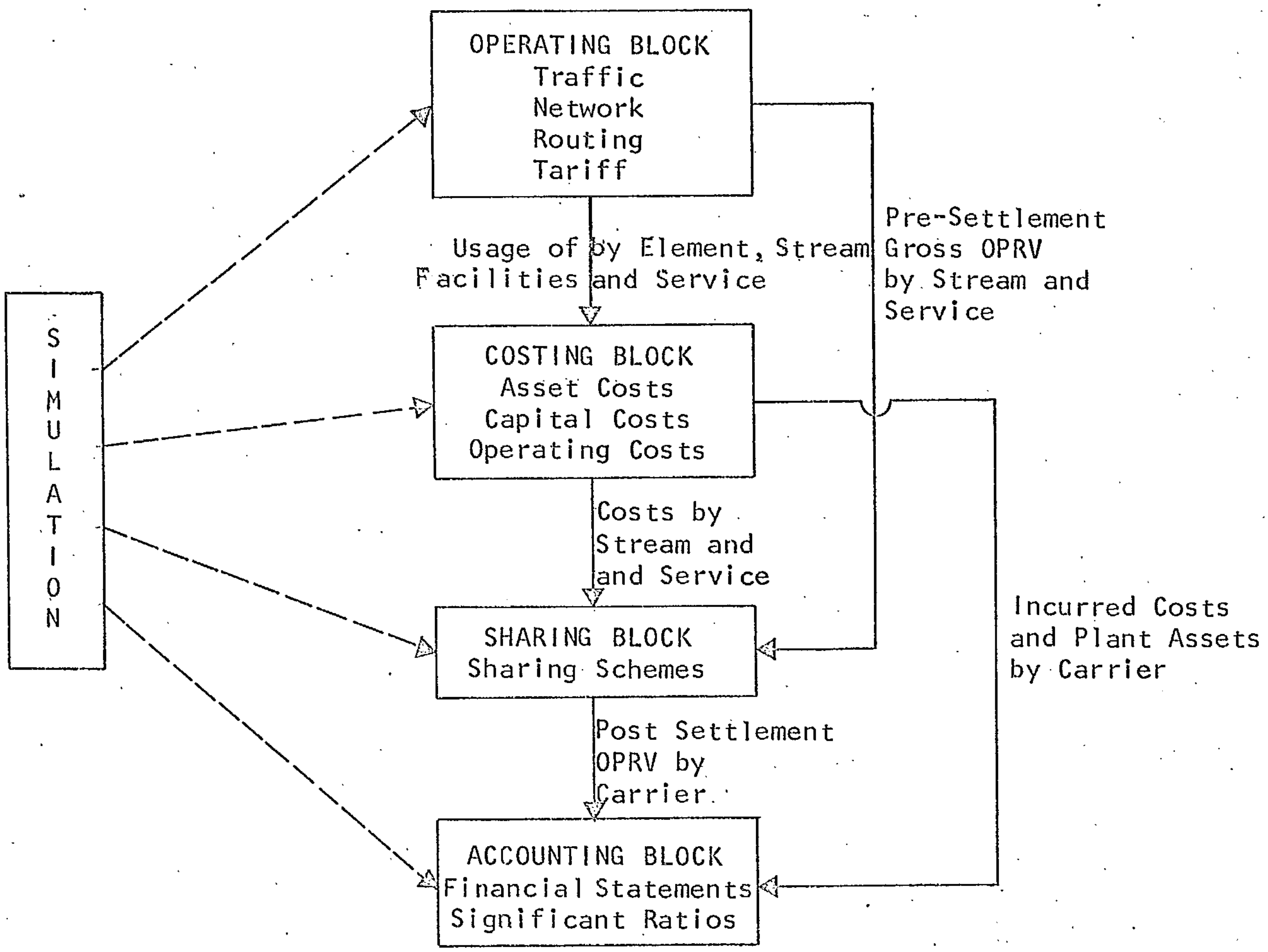
2) Presettlement gross operating revenues by stream and service, providing one component input for the revenue division process dealt with in the sharing block.

The term "element" means a switching node or a transmission link. The term "stream" refers to the traffic between a pair of origin and destination points. Service (in the present state of the model) is a category identified in the following division for intra-company or inter-company relation:

FIGURE 1

NPPS Project

Conceptual Structure



- public message toll
- TWX
- WATS
- private lines
- program transmission

It is worth noting that in addition to the specific outputs described above there are a number of other outputs available from the operating block. These relate to the various simulation alternatives and optimization procedures, with respect to the physical activities, which the operating block is capable of performing. These additional outputs may be treated as self-contained reports, evaluating different scenarios, or alternatively may be further processed through the other blocks for ultimate output from the accounting block reflecting the financial results or impacts of the particular alternatives considered.

It is also worth reminding the reader at this stage that this capability of producing intermediate outputs or optionally processing the information through the entire model, as indicated earlier, is similarly available in the other blocks.

1.3.3 Costing Block

The main function of the costing block is to associate costs, defined in various ways for various purposes, with the physical components (elements) of the network, and to allocate costs to various services and streams on the basis of usage. For this purpose a number of costing concepts and methodologies are employed in the costing block, e.g., fully distributed, incremental, reproduction, historical, etc. The main categories of costs are:

- asset costs: gross asset costs and net asset costs;
- capital costs: including depreciation, cost of capital and income taxes; and
- operating costs; such as maintenance, traffic etc.

The main outputs of the costing block are:

1. Costs by stream and service as the result of associating costs with the usage of facilities by elements, by service and by stream produced by the operating block. This output is fed into the Sharing Block.

2. The incurred costs such as depreciation and maintenance, and the plant assets by categories of plant by carrier. The plant assets by categories of plant are produced in the terms of both gross investment and net investment i.e. gross investment less accumulated depreciation. This output is fed into the accounting block, comprising the key components to be considered in the income statement (expenses) and the balance sheet (telephone plant assets) respectively.

1.3.4 Sharing Block

The main purpose of the sharing block is to consider the presettlement gross operating revenues, and the costs by stream and service in the context of the particular revenue sharing scheme which is applicable in the particular run. Out of the number of sharing schemes which may be considered not every one of the schemes require revenue and cost information by stream and service. The fine breakdown of these inputs, however, is required for some schemes as well as for other simulation purposes. The output of the sharing block is the post-settlement operating revenue by carrier. This provides an input for the accounting block (income statement) representing the results of employing a particular revenue division scheme.

1.3.5 Accounting Block

The last block in the model is the accounting block. It contains the accounting algorithms required to produce the financial statements, for each carrier, as the result of particular operating and/or costing and/or sharing scenarios, and as the result of the particular constraints and/or objective functions employed in the particular case. In addition to the financial statements a number of other ratios are also produced, e.g. rates of returns on various bases, debt and equity capital ratios, operating ratios etc. The outputs of the accounting block reflect the financial results and/or impacts consequential in a particular simulation.

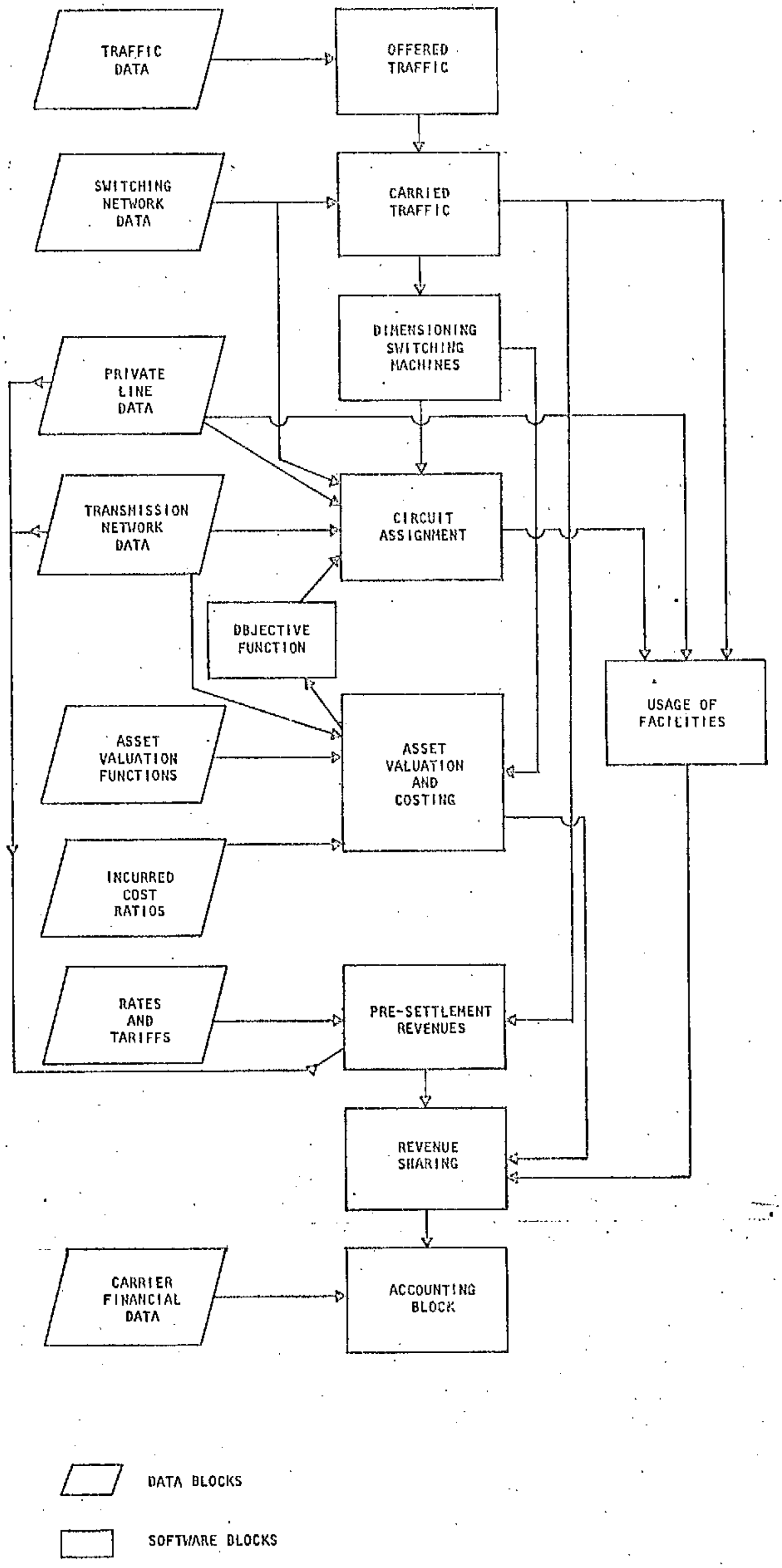
1.3.6 Simulation

From the foregoing it may be observed that the model has been structured to maintain and consider information, both exogenous and endogenous, at a level of fine disaggregation. These information elements are considered as "building blocks" which provide the capability of constructing complex modes of operating, costing, sharing, and accounting activities. By virtue of the "building block" concept and the structuring of the model in four main blocks, a very great flexibility in simulation capability has been provided. As indicated by the dotted line on the flowchart, (figure 1.3), simulation scenarios may be constructed through accessing and changing information elements, constraints, objective functions, etc. in any one, in more than one or in all of the four main blocks. These

FIGURE 2

NPPS MODEL

SCHEMATIC DIAGRAM
OF LOGIC FLOW



scenarios and their impacts then may be evaluated by running the model end-to-end, or for intermediate output from any one of the blocks. This structuring of the model also provides the capability of "local optimization", i.e. in a particular block only, through employing optimization algorithms which have been built into the operating and accounting blocks.

It is important that although the model has been designed to deal with the national network in entirety, it is capable of dealing with scenarios pertaining to individual carriers or regions treating them as subsets.

1.4 Structure and Functioning of the NPPS Software System

1.4.1 Conversational Character

The series of programs which constitute the NPPS model are incorporated into a conversational computer software system designed to permit a user with little knowledge of computer systems to rapidly perform a series of analyses incorporating complex and sophisticated programming and software techniques, with minimal prior instruction.

Use of the system is made via "teletype" style computer terminals, connected by telephone to the computer facility (currently a Univac 1108). To perform a particular function the user "signs-on", and then enters, on his terminal console the appropriate "keywords" for the function desired. After the keyword is entered, the system will respond with a certain number of messages and instructions for the running of the program. When a decision is required, the user will be given a question, a list of possible answers and will be prompted to answer by a question mark appearing on the terminal. The system waits for a response to this question before allowing processing to continue.

At the end of each simulation run or updating function the user is asked if he wants to terminate or repeat with revised inputs. The user then has the option of repeating, proceeding to a new block by entering another keyword, or of signing off.

1.4.2 Structure of the System

a) Logic Flow

The two key components of the NPPS model are data blocks, and software blocks. The interaction of these blocks, and the logic flow of the model are illustrated schematically in Figure 2.

In reality, the data blocks and the software blocks are each made up of a number of components, as presented in Figure 3 and discussed below.

b) Data Blocks

The "data blocks" of Figure 2, consist of a permanent data base component, connected to a temporary data base component via "intervention programs" shown as inverted triangles in Figure 3.

The permanent data base components (identified as parallelograms) consist of base data items used in the NPPS system, such as description of the initial state of the switching network, rate information, accounting and cost ratios. The role of the permanent data base is to provide a benchmark for users of the system.

The temporary data base components (shown as hexagons) serve as intermediate "scratch" storage space in which the user can incorporate modifications to the permanent data, for purposes of a particular simulation scenario.

The conversational intervention programs (shown as inverted triangle) represent for the user the main control points on the system. The use of these programs permits the user to define a particular simulation scenario by modifying elements of the permanent data base and transferring these modifications to the temporary data base.

c) Software blocks

The software blocks of Figure 2 consist of the main processing programs, the auxiliary processing programs and endogenously generated data base components.

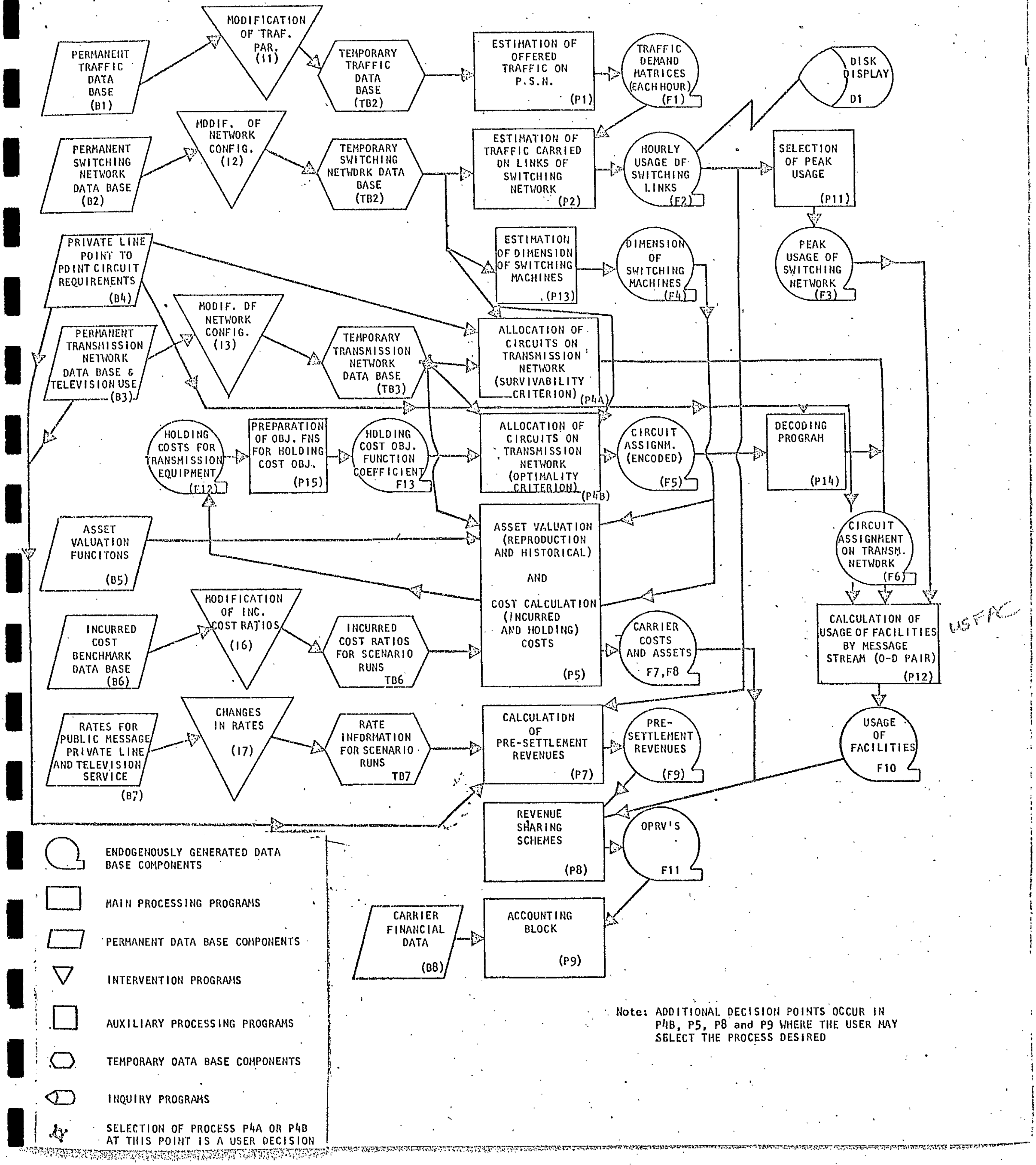
The main processing programs (Figure 3: rectangle) perform the principle calculations of the model and are usually organized around one particular algorithm or process. Due to data handling and computer memory constraints, the processing programs do not necessarily conform to the compartmentalization "Operating block" "Costing Block" etc, used to describe the model elsewhere in this paper.

The auxiliary processing programs (Figure 3: Square) serve as "housekeeping" links between main programs, generally transforming endogenously generated data into an appropriate form for subsequent computations.

The endogenously generated data base components (Figure 3 circles with tail) contain results of calculations of one processing program which must be stored for use by subsequent programs. In certain cases the contents of these endogenous

FIGURE 3

NPPS MODEL
SYSTEM FLOWCHART
(REVISED MARCH 1975)



Note: ADDITIONAL DECISION POINTS OCCUR IN P4B, P5, P8 and P9 WHERE THE USER MAY SELECT THE PROCESS DESIRED components can be printed on the users' terminal for examination and analysis before proceeding to subsequent calculations.

1.4.3 Using the System

a) Simulation Capabilities

The system is modular in nature and thus simulations can be run on one, or several or all of the blocks, as long as the flow logic is respected.

In addition to having control over the system through intervention programs, a number of main processing programs have option or control points built in. Thus the user can for example choose different bases for depreciation, in the Asset Valuation, Program "P.5" and can choose objective functions in the circuit assignment block "P. 4B".

b) An example of use

The user wishing to see the effects of a change in long distance telephone rates would refer automatically to the rate intervention program (17). By typing the appropriate keyword the user will be informed by the system of the present state of the permanent data base and will be allowed to copy this data base into a temporary work area, make changes in the data base and set up a scenario for the running of the main revenue processor (P.7).

The revenue program (P. 7) will perform a series of calculations, report upon them via the users terminal device and transmit the relevant information to the endogenously generated data base component (F 9). If the user wishes to pursue the effects of the changes made into the sharing scheme he may do so by typing the appropriate keyword on his console. To follow through the effects of changes in the system the connecting arrows are the only guide necessary. The user assumes that in running "P. 7" other entering information such as "F 2" does not change, but is available automatically from previous runs without requiring any action on the part of the user.

The "earlier" in the system that changes are made, the more extensive are the effects throughout the system. A change in data in "B1" via "I1" has an impact on almost every other result in the system.

2. STATUS OF THE MODELS AND PROPOSED DEVELOPMENTS

2.1 Introduction

The status of the NPPS model at the start of Phase III, and the developments to be undertaken during the phase was discussed at length in the Memorandum of Understanding* issued on 15 May 1975. The purpose of this chapter, therefore, is to review the progress of the work in the intervening period and to summarize the developments and simulations to be carried out in the remainder of the phase. Reference should be made specifically to section 5 (pp 14-15) of the Memorandum of Understanding.

This review and summary is expanded upon in greater detail in the appropriate sections of this report. Due to its importance in the first part of this Phase results of the System Conversion activity are discussed in the first section of this chapter.

2.2 System Conversion

2.2.1 Background

Conversion of the NPPS System from the McGill University Computing Centre facilities (Operating System - MUSIC) to Computer Sciences Canada facilities (Operating System - CSTS) was undertaken mainly because of size limitations imposed on NPPS by the use of the MUSIC System. Problems to be submitted to NPPS were of a size exceeding the capabilities of MUSIC. The choice of C.S.C. was made after a comparison with facilities available at Société de Mathématiques Appliquées (S.M.A.). C.S.C. was judged equivalent or slightly inferior on facilities, competitive in cost terms, but much more desirable for the immediate needs of rapid conversion. The design philosophy of CSTS is similar to that of MUSIC (mainly an interactive computing system) which was not the case for S.M.A. (mainly a batch processing oriented system).

Comparison of CSTS with MUSIC indicated a significant difference in costs. Initial estimates placed CSTS at 40% to 50% more expensive than MUSIC for day to day operations. More emphasis must be placed on system design and effectiveness in CSTS to avoid spending computer budgets unproductively; but if sufficient effort is placed on efficient use of CSTS, costs need not be significantly higher than MUSIC. Notably, because of the case of simulated conversational batch processing on CSTS many operations could be handled through batch

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* NPPS Project Memorandum of Understanding regarding the activities and scope of Phase III, May 15th, 1975.

processing (66% cheaper than conversational processing) without modifying the external appearance of the NPPS system to users. During the conversion phase, due to a multiplicity of other problems, advantage has not been taken of this possibility. However, our intent is to do so as soon as possible.

2.2.2 Conversion Strategy

Given the relative similarity of MUSIC and CSTS we were able to establish the following conversion plan.

STEP 1 Identify within NPPS on MUSIC 4 Types of Files as follows

i) FORTRAN

ii) BENCHMARK DATA

iii) SYSTEM CONTROL

IV) INTERMEDIATE DATA

Of these the Benchmark data is transferable without change; the Fortran is transferable with some relatively minor changes; the control files are not transferable and must be rewritten; and the intermediate data files need not be transfered since they can be regenerated on the new system.

The identification or these files is relatively simple since most are already identified by type in the USERS MANUAL

STEP 2 Physical Transfer

Files of type i) and ii) are physically transferred via punched cards from MUSIC to CSTS.

STEP 3 Rewrite the Control System: This step involves setting up the NPPS control system permitting the compilation of Fortran programs, linkages with data execution of programs, handling of intermediate data files, maintenance of libraries and handling of the interactions between major components of the NPPS system.

At the end of step 3 the control files generated would be a precise copy of the control system on MUSIC, as reflected in the USERS MANUAL.

STEP 4 Run the programs of the system to test linkages and regenerate all intermediate data files of the system. At this point a debug and quality control check is made. All intermediate data files are checked against their predecessors on MUSIC to ensure compatible formats and identical contents where possible. Printed and stored results are compared on CSTS and MUSIC to confirm that the conversion has not introduced any errors in the system.

Completion of step 4 represents completion of the conversion of the system. At this point all files can be dropped from MUSIC and full operation on CSTS can proceed.

STEP 5 Within limits of time and resources available, improve the control system to take advantage of special features of CSTS which will allow the reduction of operating costs, the improvement in clarity or the enhancement of the possibilities of NPPS on CSTS.

2.2.3 Conversion Record

Proceeding as planned, steps 1 and 2 were undertaken and completed by 20 June 1975. Step 3 was then undertaken, in parallell with Step 4 wherever possible. Step 3 was complete on July 4, 1975. Step 4 was undertaken and will be complete as of July 30, 1975, (As of July 11, 80% of Step 4 was complete, notably including the Operating, Costing and Accounting Blocks. Only the Revenue Sharing Block is Outstanding).

Parallel to the testing of the System, runs were made with a "regionalized data base for the so-called Maritimes Experiment. This is reported upon elsewhere.

2.2.4 Documentation

The following plan was adopted for the production of documentation (user's manual) concerning the NPPS system on CSTS.

1) As an interim measure produce a revised version of the Users Manual dated April 1st 1975.

2) Work closely with the users on runs of the system to improve the autodocumentation of NPPS through conversational input of data and prompting.

3) Produce for the end of the present phase, (oct. 75) a comprehensive definitive document on the NPPS Model, including an updated version of the Users Manual

Work has been completed, and in underway on point 2.

2.2.5 Results of Conversion

It will be recalled that the main reason for undertaking the conversion from MUSIC to CSTS was lack of space for the allocation of circuits on the transmission network. A comparison of problem size capabilities is presented in Table 2.1.

Within the Music System we had been able to solve problems related to 24 switching nodes but had no space for larger problems. Actually implemented tests on the newly converted system have demonstrated that we are now able to solve problems related to 60 switching nodes.

TABLE 2.1

Problem Size Comparison

	(1) MUSIC	(2) CSTS (actually implemented)	(3) CSTS Approximate max.
Transmission Nodes	200	200	
Switching Demands	110	190	
Private Line Demands	50	50	
Total Demand Constr.	150	215	250
Total Capacity Const.	90*	180**	250
Total Constraints	240	395	450-500

* with telescoping
** without telescoping

(1) problem solved on MUSIC after use of "OVERLAY" Techniques
(2) problem solved on CSTS without use of "OVERLAY"
(3) approximate size possible on CSTS with use of "OVERLAY"

2.2.6 Current Status

The implementation of NPPS on CSTS can be considered an operational success. (We are now able to and have solved problems using a 60 node switching network and a 40 constraint allocation problem). The costs are somewhat high but can be reduced significantly with additional systems work as discussed below.

A number of minor programs, are not yet operational (USEFAC, CMAXDE and SYSKOS) but this is due to linkage and format compatibility rather than to conversion problems per se. The formatting will be standardized and hence the linkages made operational prior to the completion of Maritime Regional Simulations.

2.2.7 Future Development

In moving from MUSIC to CSTS, for the same problem we anticipated a 30 to 50% increase in costs of runs (which proved to be the case) but total costs increased beyond 50% since the problem size increased. Certain developments are under investigation for the saving of operating costs including the use of simulated conversational batch processing (which could cut overall costs by 50%).

In the course of conversion, the various programs and linkages have been considerably cleaned up and this process of refinement, aimed at making the runs more economical, will continue. More specifically the software improvements will be geared to the results of cost benefit evaluation of improvements, reflecting the trade off between operating costs, frequency of use and cost of system improvement. The specific work to be carried out will include:

- Clean up all programs: with the objective of minimizing the memory space utilized by these programs.
- Standardize and improve upon the conversational comments used in the programs in order to improve user comprehension. This will require input from all users.
- Produce an executable version of the operating programs (as opposed to those in the development stage) in order to diminish execution time. At the same time a Fortran version of the program will be stored on tape in order to be able to readily make changes in the future.
- Increasing the power of the NPPS operating system by allowing users to construct private libraries of data for scenarios in use.
- Reducing the operating costs of the system through the use of simulated conversational batch processing.

2.3 Operating Block

2.3.1 General

Aside from system conversion the main developments in the operating block have been in regard to the regionalization of the model, conceptual developments, and the extension of the transmission network capability to 60 nodes. These are reported upon in Chapter 3.

Future developments in the operating block will be aimed at the modelling capability to run the regional and inter-regional simulation scenarios. Thus, assuming that no further conceptual development is undertaken, emphasis will be placed, in terms of software development, on automating linkages, use of different costing concepts and the quality of report generation. This would include capability to produce reports comparing runs, as would be needed to determine marginal service costs, for example.

2.3.2 Transmission Network

The transmission network program will be improved to increase its power and reduce cost, in line with conceptual developments aimed at improving efficiency. The model will also be expanded to include the CN/CP facilities and Telesat. Expansion over time capability will be investigated, in terms of software, and a decision reached on whether or not to proceed with this. The incorporation of survivability in the allocation program will be explored, as well the value of developing a program to evaluate survivability after allocation.

Thus it is expected that an acceptable and practical treatment of survivability will be operationally incorporated during the present phase.

2.3.3 Switching Network

The switching network programs are operational, and further developments will be restricted to the dimensioning capability, using the "Economic CCS" method. This is discussed in the section "Marginal Costing - Switching Network".

2.3.4 Demand Model

After exploring possible alternatives for further improvement of the Demand Model, it has been decided to retain the original version as the most practical approach at this time.

2.3.5 Data

Up to the time of the present report priority was given to data generation required for the Maritime Experiment. Further updating of the switching network is dependent on the development of Hermes III or alternatively obtaining the information from the carriers as a more direct route.

Once we have completed the present experiments the planned update of the data as set out in the Memorandum of the Understanding will be carried out.

2.4 Costing Block

2.4.1 General

The costing block contains three main elements,

- Asset valuation and Estimate of Annual Costs
- Aging, Indexing and Depreciation
- Service costing

The status of each and the work to be carried in the coming months are discussed below:

2.4.2 Asset Valuation Function

The program SYSCOS is not yet operational due to linkage and formatting incompatibilities, but will be made so at an early date. The revisions of asset cost functions, and switching costs and other circuit costs, as discussed in the technical memorandum remain to be finalized. Considerable improvements were already made regarding the transmission facilities as described in chapter 3.

2.4.3 Aging, Indexing and Depreciation

The basic AID programs, AGING I and AGING II, are now developed and operational, giving capability to perform various depreciation calculations, the former program using the Iowa survivor curves and the latter using Integrated Property survivor curves. Additionally certain rigidities existing in earlier versions with respect to the P, R, type of survivor CURVES, ASL, and ELG methods have been removed rendering the programs more flexible.

Developments in this phase will be primarily focused on the following aspects:

- an algorithm for the ASL depreciation method for integrated properties in AGING II;
- a subroutine for the reconciliation of theoretical depreciation rates and reserve ratios with the corresponding historical ones, at the level of disaggregation of assets as contained in the Asset Schedule (see the Supplementary Report of NPPS II);

- CCA algorithm;
- incorporation of the AGING I and II and the subroutine of reconciliation into the Costing Block;
- collection of relevant data of the carriers under consideration and computer storage of collected data.

2.4.4 Service Costing

Two models are available for service costing: the HERMES III model and the NPPS Model. HERMES has size limits much smaller than NPPS but HERMES is well adapted to the step cost functions used for costing; it can also receive upper bounds constraints to achieve some diversification in circuit allocation and it can obtain some trade-off between transmission cost and switching cost simultaneously. On the other hand the sub-model TRANCHE which is the allocation expansion sub-model of HERMES needs a chain enumeration to set up the mixed integer linear programming.

NPPS is more ambitious than HERMES, it has several functions: Costing Accounting, Sharing, Operating. The allocation-expansion linear programming sub-model CIRRES does not make the trade-off between switching and transmission cost; as a matter of fact, it receives some of its inputs on the circuits requirements from a switching sub-model which is not yet quite equipped for dimensioning. For service costing purpose, the cost of switching and the cost of transmitting have to be treated sequentially and trade-off are obtained by simulation; on the transmission side all cost functions have to be linearized in some way since we use a linear programming model. However, for incremental costing, for requirement variations not too big and total requirement not too near the ultimate capacity, it should be an acceptable approximation. The results of exercises with several type of slopes derived from the information on the cost functions will give some guidance in the choices of slopes, which could then be incorporated into the model.

Before the end of phase III, cost concepts other than reproduction cost will be made operational; particularly the embedded cost concepts with the help of the Aging, Indexing and Depreciation algorithms. The prospective cost concepts which require costs and demands structured by time periods will also be envisaged if the one period computation costs are not too high; maybe some "pedestrian" branch and bound methods could be used.

2.5 Sharing Block

The sharing block programs are not yet operational, due also to formating and linkage problems, connected in these case to the USEFAC program. Thus a priority in the early stages of the coming phase will be to make this program operational with the sharing schemes outlined in the previous reports.

2.6 Accounting Block

Since the Supplementary Report on the Second Phase, March 31, 1975, the main developments in the Accounting Block have been: the incorporation of the vectors of BEADS and NON-BEADS in the APL program, the introduction of the multi-period capability taking some new conceptual developments into account and finally the reworking of the reporting of results so that they conform with the published financial statements. The last two improvements are still to be completed. Apart from minor changes, the simultaneous equations system remains the same, and is presented in Appendix C.

During the remainder of the present phase we intend to complete the APL program at two levels: present the results of each simulation in a format similar to the published financial statements and, in the case of the multiperiod model, program in a more efficient manner the different NON-BEADS systems which depend on the retained optional BEADS.

Moreover, we propose to conduct a number of test scenarios, both at the static as well as the multiperiod level. Finally, we plan to reintroduce the multiperiod goal programming in the Accounting Block; but since the formal model is now available, put the emphasis on the determination of some scenarios and on the financial and economic interpretation of the results.

In a future phase we will have to integrate more fully this block with the other ones (Operating, Costing and Sharing Blocks) in order to determine the economic significance of some variables, like the GCE or PUC.

2.7 Simulations

The simulations to be conducted will involve two fronts, the continuation of the examination of regional questions initially using the Maritime regional example, and the use of the models to address inter-regional issues.

Given the advanced state of development of the models, these simulations will be aimed at extracting the maximum of knowledge from the models, rather than acting as a motivating force for further conceptual developments. The main scope of the simulations will be the development of various service costs in given operating scenarios in addition to the usual outputs already achieved and/or described in the previous reports.

3. CONCEPTUAL DEVELOPMENTS

3.1 Allocation on the Transmission Network

3.1.1 Survivability

a) La notion de survie dans un réseau de communication

Aucun réseau de communication n'est satisfaisant s'il n'assure des possibilités de communication lorsque des éléments physiques du réseau sont mis hors d'usage. Cette survie du réseau est une question de degré et les méthodes pour l'obtenir sont diverses. Pour un survol des habitudes canadiennes dans ce domaine on pourra consulter le texte "Survivability of the Canadian Telecommunication Network, T.C.T.S." (in Proceedings of the N.A.T.O. Conference, Ile de Bendor, France, June 17-21, 1968).

Les possibilités de restauration du réseau peuvent être assurées par les moyens suivants:

- par un routage diversifié des circuits pour la plupart des paires de points à mettre en communication;
- par une exploitation permettant la commutation et les débordements;
- par un contrôle centralisé permanent et des plans de restauration d'urgence.

Dans cette note, nous traiterons du routage diversifié des circuits, l'idée étant que plus la dispersion des lignes d'acheminement des messages est grande, moins grande est la vulnérabilité du réseau. Il semble qu'au Canada on se contentera d'ici 1980, d'au plus 3 routes pour une origine destination donnée, la charge étant également répartie entre ces dernières.

b) Historique

Dans la série des travaux dans le cadre des projets HERMES et IRA, on mentionne plusieurs fois le problème de la survie du réseau. Par ordre chronologique on pourra lire:

- HERMES Project, Report on the Second Phase, March 1972, Appendix C: Survivability Requirements.

  Cet annexe traite de façon combinatoire la survie pour l'expansion à coût minimal devant satisfaire 2, 3, 4 ou 5 canaux supplémentaires; les calculs étant de plus en plus lourds lorsque la demande augmente.

- HERMES Project, Interim Report on the Development of the HERMES III Model, November 15, 1972.

  On mentionne, sans expliciter, que la survie pourrait être obtenue, jusqu'à un certain degré, en incorporant de nouvelles contraintes dans le problème d'expansion-affectation (p. 18).

- IRA Project, Final Report on the Second Phase, December 31, 1974.

  Une méthode pour identifier des chaînes disjointes entre deux points de demande est exposée (p. 2-28 à 2-32) et quelques résultats sont rapportés (p. 3-6) en mentionnant une certaine lourdeur dans les calculs. De plus, dans une perspective de simulation sur la survie, quelques exercices sont proposés (p. 4-7 et 4-8).

- IRA Project, Supplementary Report on the Second Phase, March 31, 1975

  Une méthode d'affectation des circuits pour garantir a priori avant l'affectation optimale, un certain degré de survie est exposé dans ce dernier rapport (p. 2-8 et 2-9).

En résumé quelques tentatives ont été faites concernant la survie du réseau, mais ce problème ne fut jamais vu commu une priorité.

c) Position du problème

Nous abordons le problème de la survie dans un réseau physique déjà en place, les arêtes ayant des capacités données.

La demande est exprimée en nombre de circuits exigés entre deux points. Dans le modèle d'affectation CIRRES il s'agira de deux noeuds du réseau de commutation ou d'une paire de noeuds liés par des lignes privées. Puisqu'il s'agit de diversifier le routage des circuits, le besoin d'un circuit seulement laisse le problème sans solution, à moins d'augmenter la demande artificiellement. Supposons donc des demandes d'au moins deux circuits.

Il y a en fait deux problèmes:

- Un routage des circuits étant donné, quel degré de survie a-t-on obtenu?
- Comment trouver un routage qui garantisse un certain degré de survie?

La notion de degré de survie est liée à la dispersion des circuits. Voyons le premier problème

d) Survie d'une affectation de circuits donnée

Supposons que le routage soit donné sous forme du tableau 1.

		Noeuds					Nombre de circuits
		1	2	3	...	N	
	1	1	0	1	...		n_1
	2	1	1	0	...		n_2
Chaînes							
	.						.
	.				...		.
	.						.
	M						n_M
							=n (demande)

La somme n des circuits exigés est répartie selon M chaînes, sans cycle, allant d'une origine à une destination donnée. Supposons qu'il y a N noeuds dans le réseau, une chaîne est décrite par un vecteur de 1 ou de 0 suivant que le noeud fait partie de la chaîne ou non. Le produit scalaire de 2 vecteurs représentant deux chaînes i et j donne le nombre de noeuds en commun pour ces deux chaînes. En soustrayant 2, on obtient x_{ij} le nombre de noeuds intermédiaires en commun, au maximum N-2. On peut concevoir que la survie est d'autant plus petite que x_{ij} est grand et que le nombre de circuits $n_i + n_j$ sur les deux chaînes est grand d'où, en prenant des fractions, un indicateur de survie (ou plutôt de vulnérabilité):

$$V = \sum_{\substack{i=1 \\ i<j}}^{M} \frac{x_{ij}}{N-2} \cdot \frac{n_i + n_j}{n} \cdot \frac{1}{M(M-1)/2}$$

qui prend la valeur zéro si toutes les chaînes sont disjointes ($x_{ij} = 0$ poru tout i j) et qui prend la valeur 1 au maximum; pour ce dernier cas (où l'indicateur de vulnérabilité prend la valeur 1) cela signifie qu'en fait il n'y a qu'une seule chaîne acheminant tout le trafic pour une paire Origine-Destination donné, et donc l'indicateur doit être ajusté pour ce cas particulier où M = 1 afin d'éviter la division par zéro! Cette formule n'est qu'une ébauche du genre d'indicateur qu'on pourrait étudier dans le cadre de ce problème. Il resterait à caractériser la vulnérabilité du réseau dans son entier plutôt que par rapport à une seule paire origine-destination.

e) Recherche d'une affectation peu vulnérable

Quelque soit la méthode envisagée, il semble que l'utilisateur du modèle devrait d'abord ranger les origines-destinations selon un ordre d'importance du point de vue de la survie; il devrait aussi avoir à sa disposition les demandes rangées suivant le nombre de circuits demandés et connaître la fraction minimale de ces circuits dont on veut garantir la survie. Ces renseignements peuvent aider énormément à la recherche d'une solution.

Jusqu'à maintenant le sous-modèle d'affectation CIRRES (P. 4B) ne tient aucunement compte de la survie. On peut envisager de considérer la survie:

1) Après le passage par CIRRES;

2) Avant le passage par CIRRES;

3) simultanément avec l'affectation dans CIRRES.

1) Après CIRRES: Dans ce cas, ce qui a été dit au paragraphe 4 est valable. Si le degré de vulnérabilité est trop élevé pour certaines origines-destinations, une modification manuelle des affectations peut parfois, sans trop d'effort, restaurer un minimum de survie et ceci d'autant plus qu'il y aura des capacités en excès.

2) Avant CIRRES: Il s'agit d'imposer à une certaine fraction des circuits demandés des routes séparées, puis d'affecter le reste à l'aide de CIRRES. Dans un réseau physique donné peu "maillé" il se peut qu'il n'existe pas même deux routes séparées. Dans le rapport: IRA, décembre 1974, p. 2-28 à 2-32, un algorithme est décrit qui permet de compter et d'identifier les chaînes (routes) d'un ensemble maximal de chaînes disjointes au sens des noeuds ou des arêtes. Cet algorithme pourrait être utilisé de la façon suivante:

   a) En utilisant les renseignements sur le nombre de circuits demandés et sur la fraction dont il faut garantir la survie, calculer le nombre de circuits garantis.

   b) Choisir l'une des options suivantes:

      - prendre les classes des origines-destinations par ordre d'importance et à l'intérieur de chaque classe prendre les origines-destinations par ordre du nombre de circuits garantis (ordre croissant);

  - prendre les origines-destinations par ordre du nombre de circuits garantis seulement (ordre croissant).

c) Utiliser l'algorithme pour trouver un ensemble de chaînes disjointes pour une origine-destination de l'ordre déterminé en b).

d) Choisir l'une des options suivantes:

  - répartir uniformément les circuits garantis sur les chaînes disjointes (ou sur la seule chaîne existante s'il n'y en a pas au moins deux disjointes).
  - répartir uniformément sur pas plus de trois chaînes les circuits garantis.

e) Soustraire des capacités des arêtes les circuits affectés.

f) S'il reste une origine-destination non traitée, aller à c) sinon arrêter.

g) Si c) ne donne aucune chaîne à capacité 0 cf e) revenir en a) en diminuant certaines ou la totalité des fractions.

Cette procédure n'est qu'une suggestion qui demande à être discutée et certainement améliorée. S'il y a beaucoup de capacités disponibles on peut même concevoir d'augmenter les fractions garanties et arriver à une affectation complète des circuits demandés, CIRRES devenant inutile; on remarquera de toute façon qu'un critère économique a été utilisé dans la recherche de l'ensemble maximal des chaînes disjointes pour lever l'indétermination car cet ensemble n'est en général pas unique.

3) <u>Dans CIRRES:</u> Puisque CIRRES optimise l'affectation des circuits sur le réseau de transmission sans chercher nécessairement une certaine dispersion, il est tentant d'introduire de nouvelles contraintes pour imposer cette dispersion:

a) Dispersion obtenue par des bornes supérieures sur le nombre de circuits pouvant être affecté aux chaînes. Autrement dit chaque variable associée à une chaîne reliant une origine-destination donnée se voit imposer de ne pas dépasser une certaine fraction de la demande. Plus cette fraction est petite, plus on s'attend à trouver une solution "dispersée". Mais, d'autre part, l'existence de chaînes possédant en commun des noeuds ou des arêtes ne garantit pas que l'affectation se fera sur des chaînes disjointes.

Du point de vue programmerie, l'imposition de bornes supérieures sur les variables n'exige pas l'introduction de nouvelles contraintes explicites; des algorithmes spéciaux sont disponibles qui conservent au problème sa taille primitive (voir M. Simonnard, Programmation linéaire, ch. 10, édition 1962, Dunod).

b) Dispersion obtenue par des bornes supérieures sur le nombre de circuits d'une origine-destination donnée pouvant passer par une arête (ou un noeud) donnée. Formellement, cette méthode est la meilleure, surtout lorsqu'elle s'applique aux noeuds; mais elle a l'inconvénient d'augmenter considérablement la taille des matrices à inverser à moins d'avoir recours à des méthodes de décompositions (voir M. Simonnard, ch. 10) ou bien, et peut-être conjointement avec la décomposition, l'utilisation d'algorithmes généralisés de variables bornées (voir M. Simonnard, vol.2 Extension, ch. 9, édition 1973, Dunord) permettrait de faire disparaître explicitement toutes les contraintes de demandes. Le gain considérable en espace disponible pourrait être alors utilisé pour imposer des contraintes de dispersion.

En résumé, l'obtention de la dispersion des circuits durant la recherche du routage "optimal" exigerait que le programme CIRRES soit revu à la lumière des propositions précédentes.

f) Autre point de vue

Un modèle d'affectation comme HERMES qui travaille avec un ensemble de chaînes déjà énumérées pourrait facilement utiliser des bornes supérieures sur chaque chaîne, les programmes commerciaux traitent ce type de problème; par contre la généralisation mentionnée en b) ne semble pas disponible.

Finalement, dans ce même cas d'énumération des chaînes, on peut penser à trier ces chaînes pour chaque origine-destination donnée, suivant les ensembles disjoints - mais peut-être vides pour certains - ci-dessous:

- chaînes n'ayant aucun noeud intermédiaire en commun,
- chaînes n'ayant que le noeud i intermédiaire en commun et pas d'autres noeuds en commun avec les autres chaînes, pour tout i,
- chaînes n'ayant que la paire i, j intermédiaire en commun et pas d'autres noeuds en commun avec les autres chaînes, pour toute paire i, j.
- etc...

Ce triage peut être continué jusqu'à l'obtention d'un nombre minimal de chaînes qui seront utilisées dans le problème d'affectation optimale sans autres contraintes ou bien en imposant des contraintes de séquence dans l'affectation. Il faut avouer que cette approche n'a été qu'effleurée et qu'il faudrait déterminer un ordre de préférence si possible complet sur les ensembles précédents; d'autre part, si le nombre de chaînes est grand le triage peut être long et même prohibitif.

3.1.2 Generalized Upper Bound Algorithm

The current program "CIRRES", does the allocation of circuits on the physical network by means of the revised simplex technique in linear programming. The only constraints are firstly the satisfaction of traffic requirements, and secondly not to exceed the capacities of the physical links. If we want to impose some survivability constraints it must be done by pre-emption of a specified traffic portion on specified links.

A text is presented in Annex A that explains an algorithm which manages the survivability constraints <u>during</u> the CIRRES program. The underlying idea is quite simple: we impose upper bounds on the number of circuits that a chain can carry for a given O-D (1) pair. That won't assure disjoint chains but will diversify the traffic; the complete disjointness can be pursued by other means, more demanding in terms of computer space (2), but UB seems a good device for partial survivability.

We match the UB algorithm, which is quite simple, with a less simple one, the GUB algorithm, which is a very interesting feature since it will permit to cope with a much larger number of O-D pairs and a less than proportional additional space of computer memory. The only serious boundary would then be the number of links. For example, in the old CIRRES, if we jump from 60 to 200 O-D pairs it increases the number of constraints by 140 (i.e. 200-60), thus increasing the basis in CIRRES, and therefore the need for a non negligible computer space. The GUB has the advantage that it works with a reduced basis and the space required by the basis tableau in the linear programming technique would not be augmented at all; moreover the space required by CIRRES is reduced because the 60 first O-D pairs would not appear as explicit constraints in GUB.

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(1) O-D : Origin-Destination
(2) C. Autin, G. St-Cyr, Laboratoire d'économétrie, le 31 mai 1975, "Survie de routage des circuits", note technique.

The text in Annex A is divided in three parts: the first one introduces the theory and general methodology of UB ang GUB applied to the specific structure of our problem (mainly the non-enumeration of chains). The second part gives a precise and technical algorithm, and the last part applies the algorithm on a simple example, and compares the results with the case when there is no UB and also when there is a pre-emption before the allocation.

On account of the complex nature of this subject, the interested reader is referred to Annex A for further details.

FIGURE 3.13a

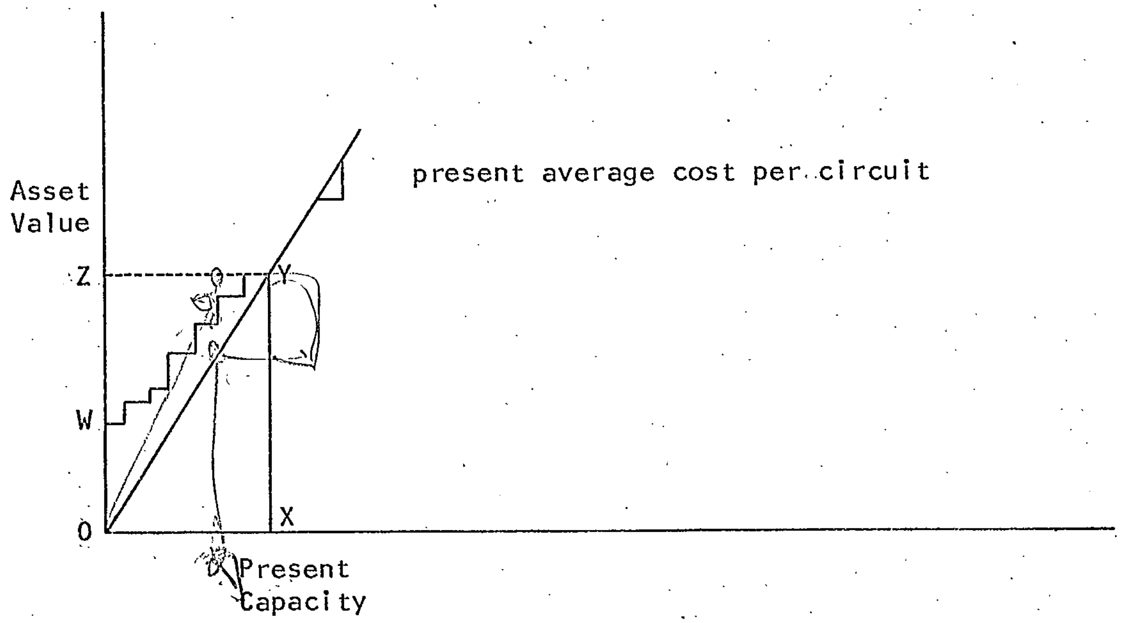
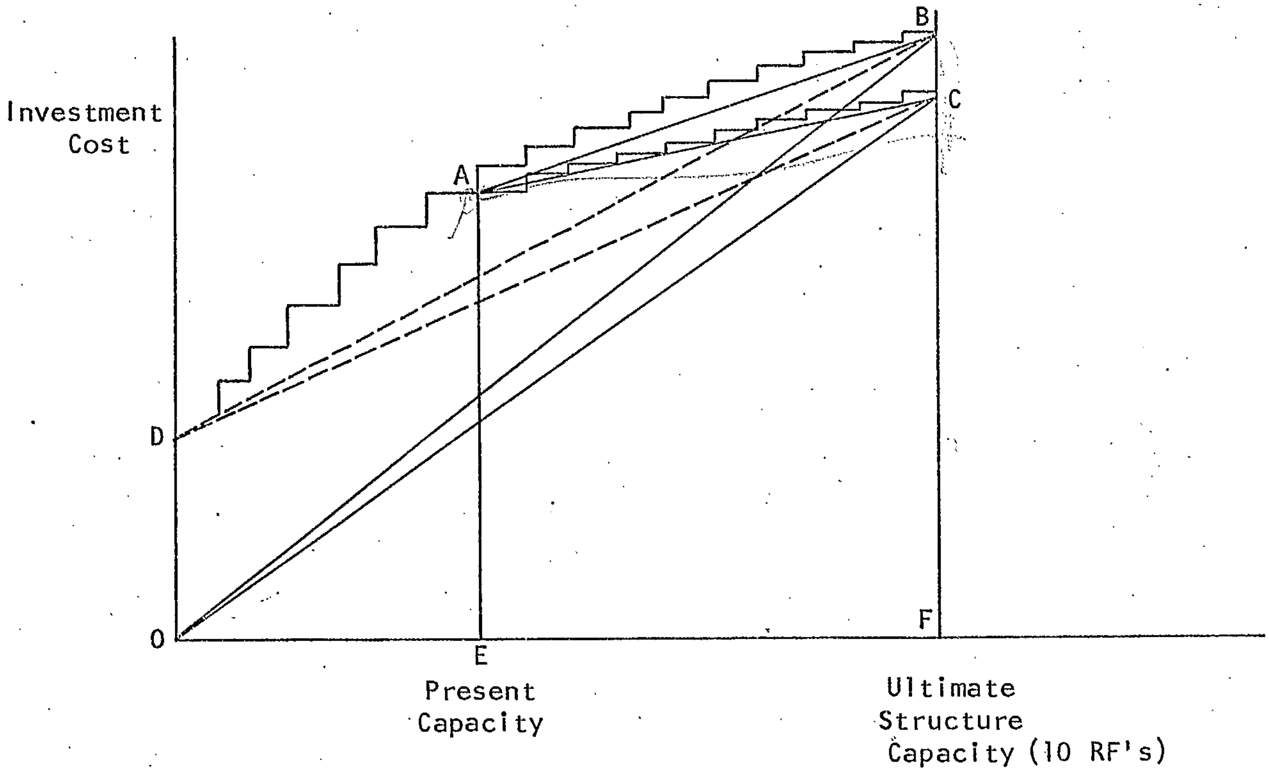


FIGURE 3.13b



3.1.3 Capacity, Valuation and Incremental Costing

a) Introduction

This section outlines the agreements we have reached and the problems still facing us regarding three related problems: determination of the circuit capacity of transmission facilities, the valuation (for accounting purposes) of these facilities, and the determination of a per circuit cost for incremental costing purposes. The same issue is discussed at greater length in terms of implications to the mathematical programming model CIRRES, in section 3.1.4

b) Effective Capacity

The first problem is to determine the effective capacity of a link measured in voice circuits. This can be done by examining the number of RF channels and the multiplexing plan of each channel. Since multiplexing plans used vary widely throughout the country, it has been agreed that rather than specifying every multiplexing plan on every channel on every link, we will make some assumptions about the nature of the multiplexing plans. In other words we will assume certain multiplexing plans to be "standard" or "typical" according to the type of traffic carried on a channel. There will be a "tag" system to identify what type of traffic a channel is primarily used for. Those channels which are used primarily for inter-regional, long-haul traffic will be assumed to have a configuration of master groups, super groups and groups installed so as to result in about 90 - 100% of the nominal circuit capacity (as given in the microwave catalogue) being effectively available. Those RF channels which are tagged as carrying mainly regional traffic with many drop-off points (multi pick-up, multi delivery) will be assumed to have a configuration of master groups, super groups, and groups installed which results in about 1/2 to 2/3 of the nominal circuit capacity being effectively available. By this scheme of identifying the channels according to the nature of the traffic carried we can translate multiplexing plans into an effective circuit capacity. Channels used for protection or TV transmission pose no problem since they are not multiplexed.

c) Valuation

The asset valuation procedure follows directly from the specification of the facilities in place, including the multiplexing plan as outlined above. Cost functions for the site, tower, power and RF equipment have already been defined and are presented in previous NPPS reports. The new procedure for identifying the appropriate multiplexing plan will determine for each channel the number of groups, supergroups and mastergroups installed. The costs applicable to each of these levels of multiplexing are outlined in Table 3.1. The other circuit costs (4 wire - 2 wire converter, and signalling) and echo suppression (for circuits greater than 1800 miles in length) can be simply multiplied by the number of circuits, once the multiplexing plan has determined the effective circuit capacity. Thus, asset valuation can be computed by a simple application of the cost factors for the structure, RF equipment, multiplexing equipment, channelizing equipment, and other circuit equipment. The cost factors for these types of equipment are applied according to the configuration of the facilities as specified in the data base.

There have been no advances in the construction of switching cost functions and their application remains as indicated in previous reports.

d) Per circuit incremental costs

The determination of a per circuit cost for incremental costing is the last of the three problems discussed here and the one involving the most judgement and speculation. From the discussion above, referring to Figure 3.1.3 a) it is clear that we are capable of determining the present effective capacity, OX, and computing its asset value, XY or OZ. The exact path or locus of the function between O and Y would depend on the order or sequence in which equipment was installed. This is not known (and thus the path WY is just a representational example) but this does not matter since we are sure about the starting point W and the present value XY. The simplist way to get a per cirucit cost for incremental costing studies is to take the present average cost per circuit as shown in Figure 3.1.3 a. This is easy to obtain (and can be done with the present software) but would probably be a poor estimate of long-term incremental or long-term variable cost.

In Figure 3.1.3 b) we focus on the ultimate structure capacity and its associated costs to define various measures of long-term incremental cost. As in Figure 3.1.3 a), we determine the present capacity OE and its present asset value EA and we can construct the cost curve DA (through the exact path of the function from D to A is dependent on the sequence of installations). Since it has been agreed that the long-term incremental cost must be based on the ultimate structure capacity we have projected the cost function up to the level it would reach at ultimate structure capacity (F in the diagram). The value or height of the cost function at F is somewhat ambiguous. It is clear that the ultimate capacity (of the 4GHz band) is 10 channels but it is not clear how the new channels E to F would be multiplexed. These additional channels could be multiplexed fully resulting in a high level of asset cost, FB (and as a result of full multiplexing, the maximum capacity measured in circuits); or, the new channels from E to F could be multiplexed only partially resulting in a lower level of asset cost, FC (and as a result of a lower level of multiplexing, a lower effective circuit capacity). The exact paths of the functions AB and AC, of course, depend on the sequence of the installations to expand capacity. The cost FB would be the value associated with the largest circuit and channel capacity the structure could physically support. The value FC would be that associated with all channels installed but not all channels fully multiplexed (the pattern which has existed up to the present time). FB would be the highest possible asset value or cost; FC would be a level more likely to be reached.

Once a decision between these two has been made, the determination of the long-term incremental cost comes down to choice among clearly specified, calculable alternatives. If we choose function DAB, we can calculate incremental costs from various slopes as follows:

Incremental cost from initial to ultimate capacity	Slope AB
Long-term ~~incremental~~ average cost	Slope OB
Long-term variable cost	Slope DB

(note: these are investment costs only, so operating costs)

The significance of these various alternatives is beyond the scope of the present discussion; but it is clear that once the choices outlined above are made these calculations can be made using the cost data discussed earlier.

TABLE 3.1

COST DATA FOR CONSTRUCTION OF FACILITY COST FUNCTIONS

Multiplexing	
Cost per group (12 cics)	~~$100 - 200~~
Cost per supergroup (60 circs)	~~100 - 200~~
Cost per mastergroup (600 circs)	~~100 - 200~~
Channelizing	
Cost per circuit terminated	$1000
Other circuit costs....	
4 wire - 2 wire converter	$ 50/circ
Signalling equipment	$300/circ
Echo suppressors......	
Cost per circuit	$2000 (?)

$100 - 200 for all three together

Not in now

$1040 - 1140 1200

3.1.4 Incremental Costing on the Transmission Network

a) Introduction

The "Maritime Experiment" and the full development of NPPS require operational definitions for the different meanings of the term "incremental cost". This paper will give some thought on the use of the Allocation Model (CIRRES) in that experiment.

b) The incremental costing (variational costing) concepts

Rather than using terms like short run incremental cost or long run incremental cost, it is much less ambiguous to establish what is considered as fixed in the system and what is subject to variations. Moreover, since only variations are significant, the initial state of the system must be carefully defined and the cost coefficients must be computed with an idea about the range and sign of the required variations. In a non linear system it is important to know whether the circuits requirements for example vary from 10 to 11 or 10 to 50. In the Maritime Experiment for instance, the suppression of the private line service could decrease the circuits requirements from 1/3 for most of the demand pairs, consequently the cost slope around the initial state must be chosen accordingly; on the other hand, the problem of costing one more circuit for a given private demand pair could lead to another cost slope.

Finally, the cost function which is relevant, depends upon the time dimension of the cost concept: embedded, reproduction, prospective cost.

These problems have been discussed previously in the following reports:

- IRA Project, Interim Report on the Second Phase, Sept. 15, 1974, 3.5 Approaches to Costing.
- IRA Project, Supplementary Report on the Second Phase, March 31, 1975, 2.2.1 d) Unit Costing Methods.

For the Maritime experiment we will restraint the choice of costs to

- the reproduction cost as for as the time is concerned
- infinitesimal (marginal) cost for each requirement
- avoidable and incremental (finite) costs for any combination of several requirement variations.

c) The allocation model as a costing model

The matrix formulation of CIRRES is the following:

$$\max z_o = cs$$

$$(1) \quad \text{subject to} \quad Ax + s = u_o \quad , \quad x \geqslant 0$$

$$Hx = v_o \qquad s \geqslant 0$$

where: c is a vector of weights (possibly costs) corresponding to the vector S of spare capacities on the links,

- u_o is a vector of capacities for the links*,
- v_o is a circuit requirement vector,
- A is associated to a set of "chains" on which the model allocates the circuits through the help of vector x,
- H agregates the circuits to satisfy the requirements.

The "incremental" costing problem, formally consists of varying v, $v_1 = v_o + \Delta v$, $\Delta v \gtreqless 0$

computing $z_1 = z_o + \Delta z$ and obtaining Δz the "incremental cost"**.

In a slightly different formulation, an expansion model is:

$$\min z = c \Delta u$$

$$(2) \quad \text{subject} \quad Ax - \Delta u + s = u_o \quad , \quad 0 \quad u$$

$$Hx = v_o + \quad v \quad ,$$

$$\Delta v \text{ given and} \quad x \geqslant 0$$

or for a "reduction" model ***:

$$\max \quad z = c \Delta u$$

---

* But node capacities could be envisaged

** The infinitesimal "marginal cost" can be computed through the dual variables of that system.

*** Model (1) with $S = S^+ - S^-$ and u_o interpreted as goal vector can replace (2) and (3).

$$\text{(3) subject to } Ax + \Delta u = u_o \quad , \quad 0 \leq \Delta u$$

$$Hx \qquad v_o - \Delta v \quad ,$$

$$\Delta v \text{ given and } x \geqslant 0$$

Whatever the model, the known data describing the real transmission network must be used to quantify the model parameters; particularly the u_o and c must be derived respectively from the equipment capacities and from some equipment cost functions.

d) The hierarchy of capacities in the transmission system

A physical system is always designed with an idea about the ultimate demand and quality requirement for that system. This is true also for the subsystems, and ultimately for the machines composing the physical support of those systems. For simplicity the following hierarchy of capacities will be defined:

i) installed voice circuits per channel

Whatever the type of channel equipment, the multiplexing equipment accompanying the channel machine has certains organs which are not installed before groups of circuits become required. Therefore, the number installed voice circuits is a first level of available capacity on each channel.

ii) type of channel capacity

Among the types of channel machines available are channels with a possibility of either 300, 600, 960, 1200, 1800 voice circuits. These nominal capacities could be used in our model or a coefficient could be used to reduce these capacities to an "efficient capacity" for good quality of transmission. In any event a channel capacity number for each channel is another level of capacity.

iii) transmission station capacity

For us, a transmission station is a site, a power generator, a tower, a set of antennae, a set of channel machines with multiplexing equipment in the junction and terminal repeaters or without multiplexing equipment in the regular repeater. A station has its own capacity in terms of the maximum number of channels which can be installed in one direction. We will suppose that it is 10 channels for any station.

iv) directional node capacity

Formally, the (1), (2), (3) models can accept any capacity constraint at the nodes, but the computing cost, for the moment, limits the number of such constraints. However, the concept of metropolitan node allows the grouping of switching machines, terminal and function repeaters every time it is impossible to relate a specific equipment to the end point of a link (a direction). Since the connecting arrangement within the metropolitan node is not known, the node capacity in any direction in also unknown. We will suppose that there is no node capacity but only directional capacities which are identical to the link capacities in the corresponding direction. Note this does not mean that the node cost will not be used in the costing exercise.

v) link capacity ("pipe" capacity)

What corresponds to a link in the model is, as a matter of fact, a series of regular repeaters or, if one prefers, a series of sections each of them with a set of capacities as described in a) to c). In order to alleviate the computing cost some section telescoping is necessary. Since the regular repeaters are not demand nodes, a sequence of sections with an unequal number of channels must be very rare. In these cases separate links will be established or the uniformity will be assumed. A link capacity is either of the type: "installed circuits" or of the type "installed channels"or of the type"ultimate number of channels" (see a) to c) above).

e) The cost functions

i) The cost functions we dispose of are reproduction cost functions for 1973 as mentionned and partially described in the following reports:

- IRA Project, Interim report on the Second Phase, September 15, 1974, 3.2.1 Asset Cost Functions
- IRA Project, Final Report on the Second Phase, December 31, 1974, 2.2.1 Asset Valuation Function.
- IRA Project, Supplementary Report on the Second Phase, March 31, 1975, 2.2.1 a) Asset Valuation Function.

The repeater cost functions are rather station (or "element") cost functions including site, power, repeater proper, etc. The arguments are in channel unit and the function shapes are step functions.

The multiplexing cost functions have the voice circuit as unit and the function shapes are linear functions passing through the origin; moreover, there is one particular function per type of channel (300, 600 voices, etc.). The multiplexing equipment are associated with terminal repeaters and junction repeaters.

The switching cost functions are not yet well defined; but since with CIRRES only the transmission network is relevant, there will be no further use of this concept in this paper.

ii) The problem is to aggregate the diverse equipment cost functions or rather to use the latter to derive unit incremental cost (slope) for each link. Whatever the capacity, it will be expressed in voice circuits.

Different assumptions will lead to different cost coefficients.

1) Positive variation of configuration of service without increasing the number of channels on each link. In that case the unit incremental cost for a link is computed as:

$$\frac{A + B}{C}$$

where

A = the total multiplexing cost in the adjacent metropolitan nodes prorated to the number of installed circuits of the incident links.

B = the total multiplexing cost on the link.

C = the number of installed circuits along the link.

2) Positive variation of configuration of services with a possibility of increasing the number of channels but without going beyond the ultimate link capacity. In that case, the unit incremental cost for a link is computed as $\frac{A + B}{C}$

where

A = the ultimate minus the actual multiplexing and antenna cost in each adjacent metropolitan node prorated to the ultimate minus the installed number of circuits of the incident links.

B = the ultimate minus the actual multiplexing and antenna cost on the link.

C = The ultimate minus the installed number of circuits along the link.

3) Negative variation of configuration of services will lead to a unit avoidable cost for a link computed as:

$$\frac{A \pm B}{C}$$

where

A = the total cost in the node in the adjacent metropolitan nodes prorated to the number of installed circuits of the incident links.

B = the total cost on the link

C = the number of installed circuits along the link.

The difference of treatment between positive and negative variation is due to the fact that the site, power and tower cost have possibly been "avoided" on a link.

3.2 Marginal Costing: Switching Network

3.2.1 General Introduction

Having recognized the importance of obtaining an estimate of the marginal cost of inclusion or exclusion of certain services, connections or routes in a switching network, it was decided to examine the problem of dimensioning of a switching network more closely. Dimensioning in this context simply refers to the process of estimating the necessary minimum size of switching facilities (in terms of switched trunks) to handle a given level of traffic respecting a minimum quality of service constraint at peaking conditions. Dimensioning is therefore a fundamental tool in the estimation of marginal costs through the following procedure. Given two traffic demand configurations, T1 and T2 calculate the dimension of the switching network for each D1 and D2. Evaluate the assets required for dimension D1 and D2 and compare to obtain the change in investment required to move from one traffic configuration to another.

3.2.2 The Economic C.C.S. rule

In the Hermes Project, a method of solving the dimensioning problem optimally under certain conditions was developed, programmed and tested. One limiting condition was the size of network (number of switching points) for which the method could be used. Hermes could not calculate the optimal dimension of a 60 node switching network. Not withstanding the existence of this dimensioning procedure in Hermes, it was therefore decided to attack the same problem, on a more conventional basis, that of the Economic C.C.S. rule (used extensively in the litterature and industry).

The economic CCS rule simply states that numbers of circuits to handle a given volume of traffic should be allocated between a direct route A from points i to j, and an alternate route B so that

$$\frac{MC_A}{MC_B} = \frac{MV_A}{MV}$$

where MC_A = marginal investment required to add 1 circuit to the route A

MC_B = marginal investment required to add 1 circuit to the route B

MV_A = marginal volume of traffic handled on the last circuit added to route A

MV_B = marginal volume of traffic handled on the last circuit added to route B

Alternatively, whenever the volume of traffic is such that

$$\frac{MC_A}{MC_B} \times MV_B < MV_A$$

Additional circuits should be added to route A (since MV_A is a declining function of C, the number of circuits).

Or whenever the volume of traffic is such that

$$\frac{MC_A}{MC_B} \times MV_B > MV_A ,$$

additional circuits should be added to route B.

In principle, all terms MC_A, MC_B, MV_A, MV_B are functions of the number of circuits already in place on each route C_A, C_B. In practice, however, it is usually assumed that the value of MV_B is 28 ccs regardless of the level of C_B, in so far as C_B exceeds a certain lower bound. This simply means that in large circuit groups, the additional traffic which can be handled by each added circuit is about 28 ccs (theoretical capacity of 1 circuit = 36 ccs = 1 erlang).

Furthermore, studies have shown that the so called cost ratio $\frac{MC_A}{MC_B}$

hovers around 1/1.3 to 1/1.8 and that the actual value derived for C_A using MV_A is relatively insensitive to slight errors in the calculation of the cost ratio (this remains to be verified in our case).

Solution for the number of circuits to be allocated to A and B (C_A and C_B) then becomes a 2 step process:

1. Determine the value of C_A so that

$$MV_A \text{ (for of } C_A) = MV_B * \frac{MC_A}{MC_B}$$

$$= 28 * \frac{1}{\text{Cost Ratio}}$$

Assuming the cost ratio is known.

2. Determine the value of C_B as the number of circuits required to handle overflow traffic from A respecting the minimum allowable grade of service (blocking probability).

3.2.3 Cost ratios

We may entertain a brief discussion regarding cost ratio. The asset valuation function used in NPPS is presented in Table 3.2.

On a direct switching route, switching investment is made at the originating and destinating point for the number of trunks in the group. According to our functions, the marginal switching investment for additional circuits in a direct route would range from $2,666./circuit to $10,000./circuit per switching machine. (That is $5,332./circuit to $20,000/circuit). For alternate routes, any number of additional switching points may be used yp to and inlcuding 5. However, the actual average number used is nearer to 1 or 2 due to the high frequency of occurence of H.U. groups, in our network at least.

Assuming relatively high development of alternate routes (over 300 trunks per group) and lesser development of direct routes (fewer than 300 trunks per group), the following sample cost ratios can be developed:

TABLE 3.3

Cost ratios

		Costs			
ALTERNATE ROUTE		2 terminals	1 interupt	TOTAL cost	COST RATIO
Assume 20,000/cir.	(3 csp's	20,000	2,666	22,666	1/1.13
	(4 "	20,000	5,322	25,332	1/1.27
or direct route	(5 "	20,000	7,998	27,998	1./1.39

Hence, it can be seen that the NPPS cost functions provide cost ratios in the same order of magnitude as those mentioned in the literature.

3.2.4 Application in NPPS

It is intended to proceed with addition research and develop the software necessary to handle these techniques in the context of NPPS.

TABLE 3.2

Switching Network

Valuation Functions

Switched Trunks	Investment (Rep. Cost)	Overall Average/Trunk Maximum	Overall Average/Trunk Minimum	Range Average/Trunk
1 - 300	3,000,000	--	10,000	10,000
301 - 600	3,800,000	12,624	6,333	2,666
601 - 900	4,600,000	7,653	5,111	2,666
901 - 1200	5,400,000	5,993	4,500	
1201 - 1500	6,200,000	5,162	4,133	
1501 - 1800	7,000,000	4,663	3,888	
1801 - 2100	7,800,000	4,330	3,714	
2101 - 2400	8,600,000	4,093	3,583	
2401 - 2700	9,400,000	3,915	3,481	
2701 - 3000	10,200,000	3,776	3,400	2,666
3001 - 3300	12,000,000	3,998	3,636	6,000
3301 - 3600	12,800,000			2,666

In order to accurately calculate the cost ratio, it would be necessary to proceed by solution of the problem for triangular sections of the switching network, and in all cases identify the direct and the alternate route. This computation is made in the NPPS switching network usage algorithm but only as necessity is setting up the problem. During the usage calculations only overflow links are computationally "remembered" for each direct connection (see the description of this algorithm in the Interim Report of the Second Phase: IRA Project, August 1974). Hence, precise calculation of the cost ratio in this context would be difficult. The alternative is to use an average, but fixed cost ratio of say 1/1.4 or /1.5 in all calculations as a parameter. Adopting this approach, dimensioning of the switching network could be done in the context of our present algorithm by a simple reversal of the "dimensioning formulae".

At present, we compute load carried as a function of circuits in place and load offered. To dimension, we may compute circuits required as a function of load offered, the cost ratio and the economic c.c.s.

Furthermore, this technique could probably be used either:

i) to dimension a switching network from the "ground" up. (i.e. assuming no starting dimension)

or

ii) To dimension a network for the addition of some demand assuming a previous network is in place.

The "dimensioning" being of a heuristic nature, the true optimal dimension will not be found (because no consideration is given to the alteration of the adjacency structure of the switching network). However, the method has much promise in terms of estimation of what takes place in actual fact since the industry has and does use the economic c.c.s. rule for day to day operations, and furthermore, can be expected to give near-optimal results for small relative changes in demand and offered traffic.

Given this tool, evaluation of marginal or incremental costs will be simple since the cost block is already able to produce the valuation of switching assets for any given network.

3.3 Aging, Indexing and Depreciation Algorithm (AID ALGORITHM)

The AID algorithm and its computer software, "AGING I", have been modified and expanded to incorporate those operational flexibilities as expounded in the Supplementary Report of the NPPS Phase II.

3.3.1 Conceptual Consideration

a) Status of the Algorithm

The revised version of "AGING I" program now can perform the following operations:

- individual, non-uniform growth rates of gross additions (R)
- individual, non-uniform indices of reproduction costs (P)
- conversion of gross telephone plant at book cost (GTP) into gross telephone plant at reproduction cost (GTP) and vice versa
- estimation of vintage distribution of GTP or GTP'
- mixture of different depreciation methods (i.e., ASL and ELG methods) and different types of survivor curves within a given run of AGING I program.
- distinction between vintage designators, X's and ages, (X - 1/2), reflecting that age is based on a mid-year concept whereas vintages merely refer to calendar years.
- dollar values of depreciation accruals and depreciation reserves and corresponding rates and ratios, respectively.

b) Uses of the programs

AGING I and II (see below) represent powerful tools for delving into the engineering aspects of depreciation and also for studying the interrelationships between the accounting, engineering and economic aspects of depreciation. We have already conducted an in-depth study on the differential impacts on GA, ADRAT and DEPRAT of variations of the types of survivor curves, depreciation methods, average life, maximum life and growth rate of gross additions. (See IRA II, page 3-11 to page 3-21).

We are naturally aware that there is much room for similar studies at rather theoretical levels, dealing, for instance, with various mixtures of different depreciation methods and comparative impact studies of Iowa-type survivor curves versus Interim-type survivor curves.

The more practical study areas include the comparative determination of the magnitudes of tax deferrals generated by depreciation accruals by various sets of assumptions, or scenarios, regarding arguments involved in the AID algorithm.

The latter task bears significance to the regulatory process as well as to the corporate financial management, as the expansion and modernization programs in the telecommunication companies are financed to a considerable degree by depreciation accruals.

The added flexibilities in the new AGING I program, and AGING II for integrated properties, give the users much more room for inputting combinations of factual information and estimated elements. This is a welcome addition in view of the fact that a depreciation calculation cannot be a scientifically exact process involving as it does a large element of judgement regarding future developments. The AID algorithms for integrated properties have undergone a number of modifications and the corresponding computer program, AGING II, will be based on these modifications.

AGING II comprises three main components. The first component, termed "AGING II: TYPE I," is for the single-vintage situations, i.e., for the case where there is one gross addition only and the age characteristics of such a gross addition, with respect to the depreciation accruals and depreciation reserves, are examined and finally the depreciation rates and reserve ratios are estimated.

The other two components, called "AGING II : TYPE II", concern the multi-vintage cases with different characteristics regarding the length of service life.

The first subprogram relating to the single-vintage situations indicate the fundamental characteristics of depreciation logic for the integrated properties in a static way.

The second subprogram dealing with the multi-vintage case, having a constant length of service life for each vintage in a dynamic set, may be regarded as depicting a compromise of the logic for the mass properties and that for the integrated properties. The last subprogram, also dealing with the multi-vintage case, represents a logic proper for the integrated properties in a dynamic situation, i.e., covering a number of periods and variations of engineering, accounting and economic characteristics and assumptions of plants under consideration. In this subprogram all vintages retire at the same time i.e., the total life span of each vintage is different.

3.3.2 Software developments

a) "AGING II : TYPE I"

This refers to the AGING II computer program dealing with the single-vintage case, which has the capability of computing dollar values of accumulated depreciation (AD), depreciation reserve ratios (ADRAT), dollar values of depreciation accruals (DEPN) and depreciation rates (DEPRAT) of each and every year or age of the plant under consideration. The modified formulae for these are:

$$(1)\quad AD\ (X) = GA(T) \left\langle M\ (X - 1/2)\ LN\ \left[(N - 1/2) \div (X - 1/2)\right] + \left[(X - 1/2) \div (N - 1/2)\right] \left[1 - M\ (N - 1/2)\right] \right\rangle$$

Notations:

AD (X) = accumulated depreciation in dollars attributable to the vintage X at the end of reference-year, i.e., at the year X = 1.

X = age designation (not vintage designation)

T = year of installation
(e.g. T = 30 installed 30 years ago)

N = total life span of the plant = T + S

S = remaining life (years) in the future

GA(T) = dollar value of the installation of T years ago

LN = natural logarithm

$$\text{(2)}\quad ADRAT(X) = \frac{AD(X)}{GA(T)\left[1 - M\,(X - 1/2)\right]}$$

Here the denominator yields the dollar value of the survivor at the end of the reference-year (i.e., X = 1)

Depreciation accruals attributable to age X, DEPN (X), are defined as the differences between the total accumulated recoupments (TAR) at age X and those of the preceeding year, X-1, for all the cases where $X \neq 1$. For the year X = 1 depreciation accruals are equal to the total accumulated recoupments; DEPN (X = 1) = TAR (X = 1). The formula for TAR is:

$$\text{(3)}\quad TAR(X) = GA(T)\left\langle M\,(X - 1/2)\left[1 + LN\left[(N - 1/2) \div (X - 1/2)\right]\right] + \left[(X - 1/2) \div (N - 1/2)\right]\left[1 - M\,(N - 1/2)\right]\right\rangle$$

Depreciation rates are defined as the ratios of DEPN(X) to the average survivors:

$$\text{(4)}\quad DEPRAT(X \neq 1) = \frac{DEPN\ (X)}{GA(T)\left[1 - M\,(X-1)\right]}$$

$$\text{(5)}\quad DEPRAT\ (X = 1) = \frac{DEPN\ (X = 1)}{GA(T)\left[1 - M \div 4\right]}$$

<u>Example 1:</u>

Using these five formulae and with the assumed values of GA(T) = \$100, annual retirement rate against the GA(T) of 10% and the life span of 7 years and the remaining life S being zero, the following statistics are obtained. (table 3.4)

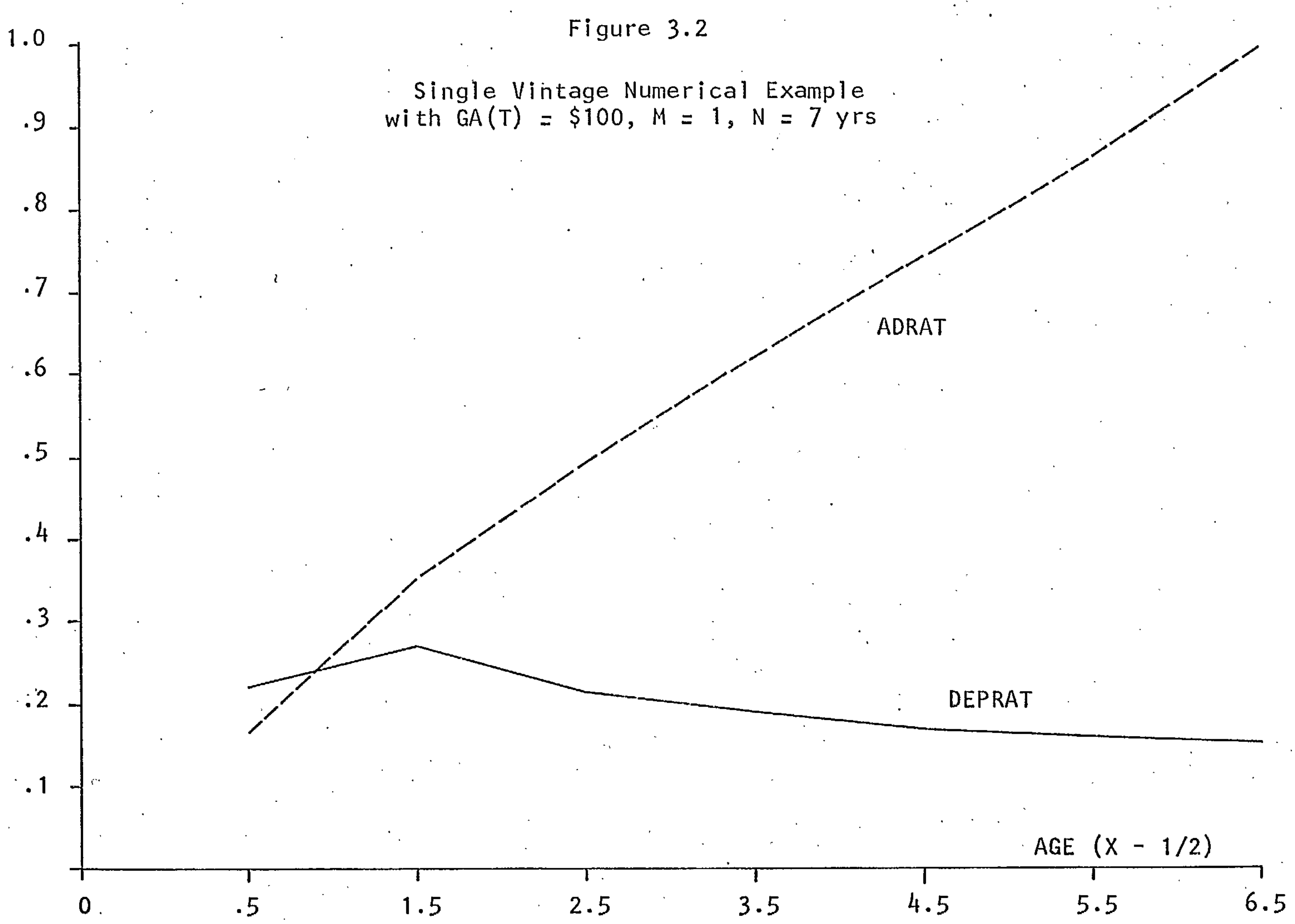


Figure 3.2

Single Vintage Numerical Example
with GA(T) = $100, M = 1, N = 7 yrs

Figure 3.2 demonstrates that depreciation reserve ratios in this simple static situation is a positive function of the ages and the depreciation rates are greater in the early ages than the later ages, reflecting the fact that the above single-vintage formulae are for the E.L.G. method.

TABLE 3.4

Single-Vintage Numerical Example
with GA(T) = $100. M = .1, N = 7 yrs

X	1	2	3	4	5	6	7	Total
AD ($)	15.81	30.06	37.34	40.49	40.74	38.79	34.97	
SRV ($)	95.00	85.00	75.00	65.00	55.00	45.00	35.00	
ADRAT (Number)	.1664	.3536	.4979	.6229	.7407	.8620	.9991	
TAR ($)	20.49	45.13	62.36	75.52	85.75	93.81	100.00	
DEPN ($)	20.49	24.64	17.23	13.16	10.23	8.06	6.19	100.00
AVG SRV ($)	95.00	90.00	80.00	70.00	60.00	50.00	40.00	
DEPRAT (%)	21.9	27.0	21.5	18.8	17.1	16.1	15.5	

b) "AGING II : TYPE II"

This is a computer program for the AID algorithm dealing with the integrated properties on the basis of E.L.G. method. It is capable of handling both cases of fixed life span and variable life span of gross additions. The formulae incorporated in it are of the following forms:

$$(6)\quad AD(Y) = \sum_{X=1}^{T} GA(X) \left\{ M\,(X - 1/2)\ LN \left[(N - 1/2) \div (X - 1/2)\right] + \left[(X - 1/2) \div (N - 1/2)\right] \left[1 - M\,(N - 1/2)\right] \right\}$$

Notations:

Y = reference year under consideration
(e.g. 1971)

X = age and vintage designation

N = T + S (fixed case)
or X + S (variable case)

S = remaining life in future years

T = year of the first installation

The formulae for ADRAT(Y), TAR(Y) DEPN(Y) and DEPRAT(Y) have also been modified as follows:

$$(7)\quad \mathrm{ADRAT}(Y) = \frac{\mathrm{AD}(Y)}{\sum_{X=1}^{T} \mathrm{GA}(X) \left[1 - M\,(X - 1/2)\right]}$$

$$(8)\quad \mathrm{TAR}(X) = \mathrm{GA}(X) \left\{ M(X - 1/2) \left[1 + \mathrm{LN} \left[(N - 1/2) \div (X - 1/2)\right]\right] + \left[(X - 1/2) \div (N - 1/2)\right] \left[1 - M\,(N - 1/2)\right] \right\}$$

$$(9)\quad \mathrm{TAR}(Y) = \sum_{X=1}^{T} \mathrm{TAR}(X)$$

$$(10)\quad \mathrm{DEPN}(Y) = \mathrm{TAR}(Y) - \mathrm{TAR}(Y-1)$$

$$(11)\quad \mathrm{DEPN}\,(Y=1) = \mathrm{TAR}\,(Y=1)$$

$$(12)\quad \mathrm{DEPRAT}(Y) = \frac{\mathrm{DEPN}(Y)}{\mathrm{GA}(X=1)\,\left(1 - \frac{M}{4}\right) + \sum_{X=2}^{T} \mathrm{GA}(X) \left[1 - M(X-1)\right]}$$

Example 2:

One could conceive a number of ways in experimenting with the AGING II algorithm. The following numerical example appears somewhat out of convention but may serve to demonstrate the versatility of the program. In it we assume that each vintage has a constant life span of seven years but their future lives vary from zero to maximum span. The purpose of this numerical exercise is to see the degree of sensivity of the arguments (e.g., AD, DEPN, etc) with respect to the variation of S (remaining life). The results are presented in the Table 3.4 and figure 3.3.

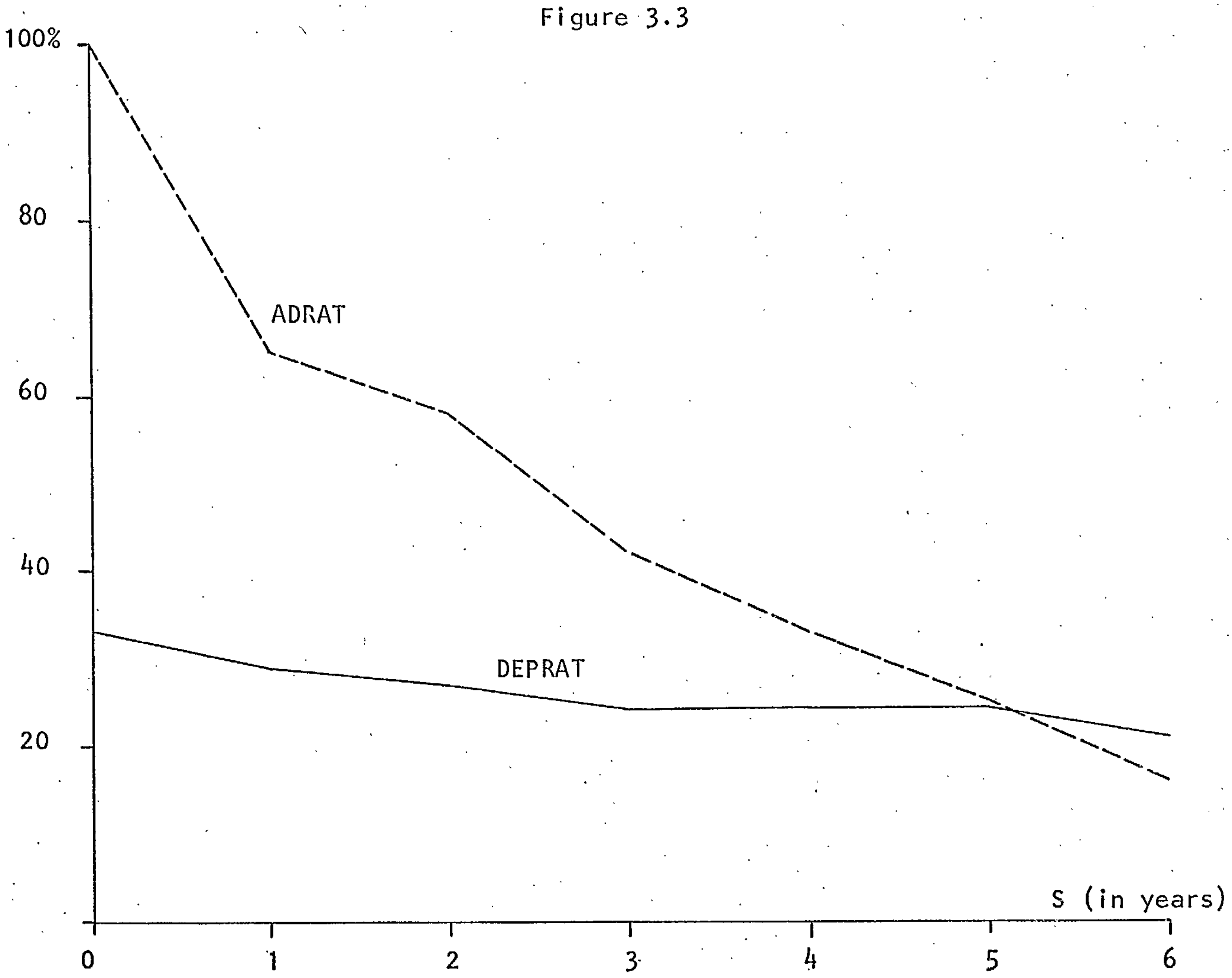
TABLE 3.4

Comparison of Cases of N=S+X = 7yr
M=.1, Maximum Life = 7 years; GA(T) = $100

S	T	AD	GTP	ADRAT	DEPN	AUG GTP	DEPRAT
0	7	455.00	455	1.000	180.	485	.372
1	6	274.48	420	.654	130.42	445	.293
2	5	219.20	375	.585	106.07	395	.269
3	4	134.52	320	.420	81.81	335	.244
4	3	86.22	255	.338	66.15	265	.246
5	2	46.15	180	.256	45.63	185	.247
6	1	15.52	95	.163	20.52	95	.216

The operational procedures are presented in Appendix B.

Figure 3.3



3.4 Accounting Block

3.4.1 Overview

Our intent in the present section is to expand on each of these developments and particularly on the multi-period accounting model. Numerical examples, illustrating for Bell 1971 and 1972 the content of the following paragraphs, are presented in Appendix C.

3.4.2 The APL Accounting Program

The main purpose of the APL program is to evaluate the impact on the financial statements of the carriers of various simulation scenarios for example if we assume an increase of 10% in OPRV, what will be the impact on some other financial variables and on some ratios like DCR. This impact is measured by resolving a simultaneous linear equation system. To do so, we have to construct a matrix of coefficients and partition the vector of variables among the exogenous and endogenous variables.

The usefulness of the vectors BEADS and NON-BEADS in the APL program is to mechanize the process of obtaining some numerical figures for the variables which appear in the simultaneous equations system, as well as to permit the construction of the matrix of the coefficients and to serve as bench-mark data. It is worth reminding the reader that there are 74 BEADS variables, 43 NON-BEADS ones and only 52 variables appearing in the equations system. The explanation for this is that some variables are aggregated in order to be used in the equation system. However, the program is so written that this aggregation is done automatically. This is the first main step.

The second step concerns the construction of the matrix of coefficients. At this level, the operator has two possibilities: to fix them manually or to take the coefficients automatically provided by the program once the NON-BEADS vector is calculated. Once these two steps are preformed the decision maker has to specify a certain simulation or, in other words, has to determine which variable will be considered as exogenous (and fix the values of them) and those which will be the result of the system (the endogenous variables). The program is so constructed that there is no constraint on the choice of the variables which will appear as exogenous.

Finally, the result of the simulation will be printed in a format similar to the published financial statements. As we said previously, this step has still to be improved in the APL program. At the end of these processes, the operator has two possibilities: to go back and perform a new simulation (with the unchanged coefficients) or to go to the multi-period simulation.

3.4.3 The Financial Reporting Program

As currently structured a problem arises once the result of a particular simulation has to be presented in a format similar to the published financial statements. It was previously agreed that the results will be presented in a format similar to the one which appears in the Supplementary Report on the Second Phase, March 31, 1975, page 3-39. However, some items do not appear as such in the simultaneous equation system, but can be obtained by aggregating some variables which are in this system: this operation has still to be made in order that our presentation be in accordance with the proposed one.

3.4.4 Multi-period simulations

In this section we discuss the multi-period simulation which can also be provided by the APL program. The objective for this kind of simulation can be the following:

> Suppose at the period t the tariffs are increased in such a way as to increase the OPRV by a certain percentage, and suppose also at the period t + 1, the tax rate (or the rate of depreciation) is modified, then what will be the impact, at the end of t + 1, on the other financial variables, of these modifications?

To perform these simulations, we proceed as follows: the values of the variables at the end of the first period become the values at the beginning of the next period. At this point, these variables are classified in several categories (the definition and the content of each class appears in Appendix C) in order to forecast the values of the exogenous variables. At the present time, there are three forecasting techniques which can be used: the regression technique, the averaging process and finally the operator can fix them manually.

Concerning the matrix of coefficients, the decision maker can take the same as in the period t or he has the choice to modify some or all coefficients. Finally, the financial statements appear as described previously.

Up to now, these conceptual developments are available and the software is almost completed.

4. THE MARITIME EXPERIMENT

4.1 Objectives of the Experiment

As a first step toward the implementation of the "regionalization" of the model proposed for this phase, we have undertaken a detailed examination and experiment on the Maritimes region. We plan to test the general cost characteristics and in particular the incremental costing capabilities of the model. We will run all of the costing, allocation and accounting modules with and without private line service in the Maritimes. This will allow us to operationally test some of the incremental costing concepts discussed elsewhere in this report (see sections II.1. c and d). At the same time, a similar experiment will be carried out with the same data and network using the Hermes model, and the incremental costs produced by the two models will be compared.

In addition to serving as a general test of the NPPS model, the experiment will guide our further work on incremental costing in the remainder of this phase. It will also demonstrate the model's capability of producing a magnified regional focus while still providing an analysis of telecommunications in a national context.

4.2 Data Sources

The switching network specified for the Maritimes includes all switching nodes down to level 4 in the hierarchy. This data on trunks and connectivity was obtained from sources provided by the Inter-Regional Study Group. The transmission network to support switching network was obtained from the latest DOC Microwave Catalogue.

Public message traffic was generated for all nodes by the standard gravity model developed previously. Private line data for the inter-regional sector was taken from the data used in previous phases (see IRA Report March 31, 1974). For the Maritime Region private line demand was assumed for any OD pair to equal ½ the number of public switched trunks. This procedure was followed since, according to a study done on Ontario private lines, this rule provides an excellent approximation of the actual private line demand. TV traffic for the Maritime region is the same as that provided for previous inter-regional runs of the model.

4.3 Comparison with HERMES III

4.3.1 General

The Hermes III model appears to be very well suited to studies of incremental cost per service. Hermes III is a network model which optimizes the capacity expansion of both transmission and switching networks simultaneously. However, for comparison purposes with the

NPPS model, since this latter model can only compute at the present time the incremental cost associated with the transmission network, Hermes III will be used as a transmission model only. The module "CHARGE" will not be used for the Maritimes experiment and it follows that all traffic, including public messages, will have to be expressed in number of point-to-point circuits.

4.3.2 Planned Simulations

The Hermes III model will be run for 2 services (i.e. public messages and T.V.) and 3 services (i.e., public message, T.V. and private lines) separately, with zero initial state in the transmission network in each case. The difference in the cost of the solution networks will represent the incremental cost associated with the provision of the private lines service.

4.3.3 Comparability of the Models

The comparison of the results obtained respectively with Hermes III and NPPS will only be meaningful to the extent that the input data are identical: i.e., the traffic and cost data and the initial network must be the same.

A problem may arise at the level of the traffic data since the public message traffic in the NPPS model must be specified in CCS where as the same traffic in the Hermes III model will be specified in number of point to point circuits requirements; this is so because we are bypassing CHARGE. Some reconciliation work will have to take place at this level before the experiment begins.

Concerning the cost data, Hermes III has already been used with updated transmission cost functions based on the number of regular repeaters instead of the number of miles. Those cost functions for a typical regular repeater were taken from NPPS and they represent quite an improvement over what we were using before. Nevertheless, the costing in Hermes III still differs slightly from what is done in NPPS, although on the average the results should be comparable, since Hermes III does not explicitly recognize the existence of different types of repeaters. There also remains the question of the inclusion of the multiplexing costs. We have not had a chance yet to test the incorporation of those costs into Hermes III but it should be feasible if we are proposed to accept a certain amount of average and short cuts.

Although we are focussing the incremental costing experiment on the Maritimes region, we have to take into account the traffic originating and terminating outside the Maritime network as it affects this latter network. Those traffic streams with the U.S. and other regions of Canada will be handled by the use of dummy nodes. Again we must ensure that the data are identical in both models and that the techniques employed to treat those traffic streams with the exterior are reasonably alike and compatible.

Incremental costing for switching network are scheduled to be implemented in the NPPS model before October 31st 1975 (see section).

At last, we also have to keep in mind that, the survivability constraint not being included in the present Hermes III model, the incremental cost of each service will be biased downward.

4.4 Network Data

The network data is presented in Figures 4.1 and 4.2

4.5 Planned Simulations

The plan of simulations outlined below is designed to evaluate the ability of the model to serve as a service costing tool and in addition, by performing a number of simulations with each of the major program the intermediate output capabilities can be fully tested and the nature of such information evaluated.

The planned program is as follows .

1. Switching Network - one pass of the 60 node, Maritime network in order to set up the traffic routing for use in the subsequent program modules.

   The results of the Switching Network run will be compared with those of earlier runs for the 60 node inter-regional network, and the 24 node inter-regional network.

2. Circuit Allocation on the Transmission Network - Allocation program will be run, with Private line data included, using three objective functions.

   Maximize surplus circuit miles and then maximizing surplus, average longrun cost and incremental cost.

   The results of these allocations will be compared in order to determine the behaviour of the allocation with different objectives functions.

   Finally, the allocation will again be run using the average cost, but excluding Private Lines. The difference between these results and those obtained using Private Lines will be the Incremental Cost of the Private Line-service.

3. Sharing block - Using the average cost allocation, the sharing block will be run for each of the three sharing schemes. This will provide a relative measure of the input of each upon the revenues of the individual carriers.

   The three sharing schemes are: TCTS, New Commonwealth and Old Commonwealth.

4. Accounting Block - Using the results obtained from the revenue sharing block with the New Commonwealth Scheme, the accounting block will be run in order to determine the impact upon the revenue of the companies.

   Additionally, local simulations using the accounting block will be conducted.

5. Costing Block - Using the results of circuit allocation on the transmission network, calculate incurred and holding costs, evaluate the assets on both reproduction and historical cost using the "aging, indexing and depreciation" algorithm.

4.6 Results

The results of the simulations are to be presented in a special annex to this report.

APPENDIX A

The usage of UB and GUB(1) techniques in the circuit allocation algorithm on the physical network.

The actual program, called CIRRES, does the allocation of circuits on the physical network by the means of the revised simplex technique in linear programming. The only constraints are firstly the satisfaction of traffic requirements, and secondly not to exceed the capacities of the physical links. If we want to impose some survivability constraints it must be done by pre-emption of a specified traffic portion on specified links.

The present text will explain an algorithm which manages the survivability constraints during the CIRRES program. The underlying idea is quite simple: we will impose upper bounds on the number of circuits that a chain can carry for a given O-D(2) pair. That won't assure disjoint chains but will diversify the traffic; the complete disjointness can be pursued by other means, more demanding in terms of computer space(3), but UB seems a good device for partial survivability.

We will match the UB algorithm, which is quite simple, with a less simple one, V.G. the GUB algorithm, which is a very interesting feature since it will permit to cope with a much larger number of O-D pairs and a less than proportional additional space of computer memory. The only serious boundary would then be the number of links. For example, in the old CIRRES, if we jump from 60 to 200 O-D pairs it in-

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(1) UB: upper bounds GUB: generalized upper bounds

(2) O-D: Origin-Destination

(3) C. Autin, G. St-Cyr, Laboratoire d'économétrie, le 31 mai 1975, "Survie du routage des circuits", note technique.

creases the number of constraints by 140 (i.e. 200-60), thus increasing the basis in CIRRES, and therefore the need for a non negligible computer space. The GUB has the advantage that it works with a reduced basis and the space required by the basis tableau in the linear programming technique would not be augmented at all; more over the old CIRRES required space is reduced because the 60 first O-D pairs would not appear as explicit constraints in GUB.

The text will be divided in three parts: the first one will introduce the theory and general methodology of UB and GUB applied to the specific structure of our problem (mainly the non-enumeration of chains). The second part will give a precise and technical algorithm, and the last part will apply the algorithm on a simple example, and compare the results with the case when there is no UB and also when there is a pre-emption before the allocation.

It is assumed for the remainder of the text that the reader is familiar with linear programming and with the mathematical formulation of the allocation problem (cf. I.R.A., Interim Report on the Second Phase, section 2.3).

1) General description of UB and GUB applied to our problem:

Notation:

S_i: spare capacity on the i-th link $(i=1,2,\ldots,m)$

x_{jk}: number of circuits carried on the j-th chain for the k-th O-D pair $(j=1,2,\ldots,n_k)$ $(k=1,2,\ldots,L)$

$x_o = \sum_{i=1}^{m} \alpha_i S_i$: to be maximized

α_i: weight for the i-th spare capacity

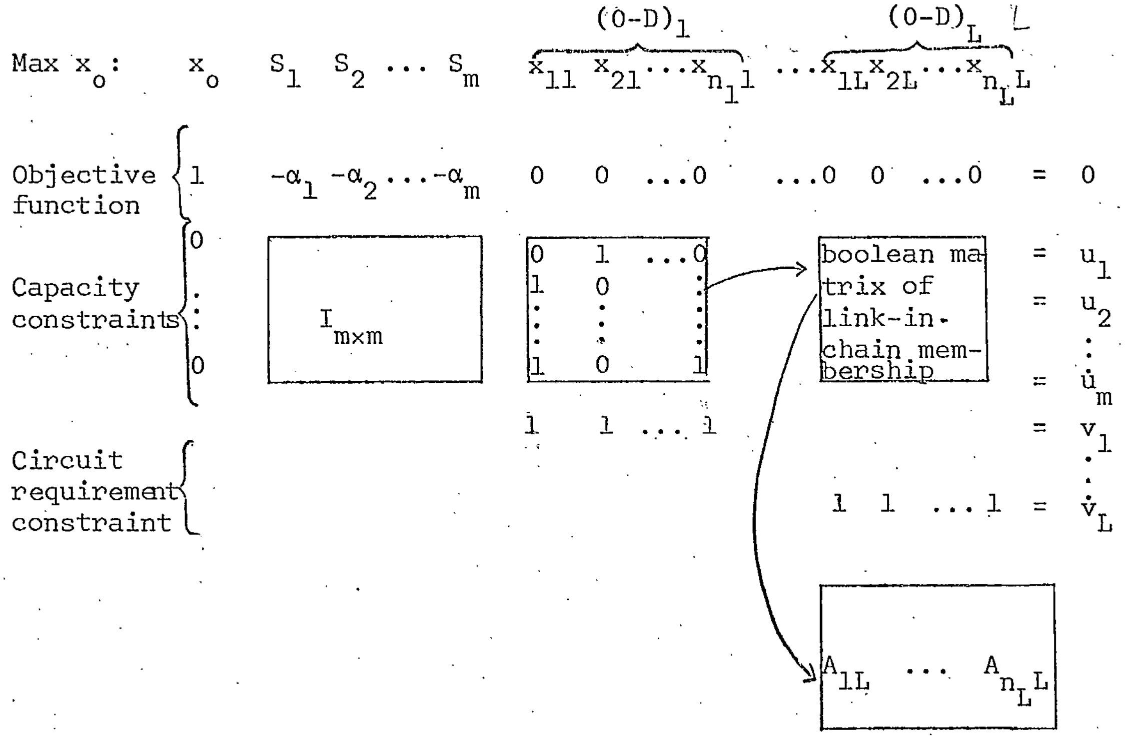
I_{mxm}: identity matrix of order m by m

u_i: capacity of the i-th link $(i=1,\ldots,m)$

v_k: circuit requirement of the k-th O-D pair $(k=1,\ldots,L)$

(The other symbols will be introduced when required).

The problem almost in extenso, the non-negativity and upper bounds constraints apart, has the following structure:



The traffic requirement constraints will be called <u>special</u> constraints in the remainder. The problem can be read in matrix formulation as follows:

Max x_o:

Subject to $x_o - \alpha S = 0$

$$S + \sum_{k=1}^{L} A^k X^k = u$$

$$1X^k = v_k \quad (k=1,\dots,L)$$

$$X^k \leqq h_k \quad \text{"}$$

$$X^k \geqq 0 \quad \text{"}$$

$$S \geqq 0$$

where $1 = [1\ 1\ldots 1]$, $X^k = \begin{bmatrix} x_{1k} \\ x_{2k} \\ \vdots \\ x_{n_k k} \end{bmatrix}$, $S = \begin{bmatrix} S_1 \\ S_2 \\ \vdots \\ S_m \end{bmatrix}$, A^k: boolean matrix of link-in-chain memberships.

$\alpha = [\alpha_1, \alpha_2, .., \alpha_m]$,

h_k is a vector for the upper bounds.

A) UB section:

The constraints $X^k \geqq 0$ in the simplex algorithm are taken into account not explicitly like the other constraints, but we make them interfere in the choice of the variable leaving the basis; the idea underlying UB is the same, except that it is a little more complicated. In the simplex algorithm, a variable not in the basis takes the value zero; but with UB a variable x_{jk} not in the basis can be at either of its bounds (the lower one being zero and the upper one being h_{jk}), so that it will interfere not only in the selection of the variable leaving the basis but also for the choice of the variable entering the basis; we will now see how it runs.

When we face a maximisation problem, the entry criterion is to choose the variable for which $Z_j - C_j$ is negative because this variable will take a value ≥ zero when it enters the basis. But here with the UB we must consider the case where the variable can improve the objective by making it <u>decrease</u> from its upper bound. For the variables at their upper bounds the entry criterion will then be $Z_j - C_j$ positive.

The algorithm will involve the calculations for the variables not in the basis according as they are at their lower or upper bounds. Since we do not enumerate chains we will have to retain the ones at their upper bounds, but the number of these cannot be large; for exemple if the upper bound is one third of v_k, then there cannot be more than three chains not in the basis and at their upper bound, for a given $(O\text{-}D)_k$. For the variables (chains) at their lower bound (i.e. zero) the enumeration is still unnecessary.

Thus it is quite simple for the entry criterion; for the exit criterion it is a little more complicated because we must make sure that no variable in the basis exceed its bounds by the entry of a new chain, and this new chain must itself stay inside of its bounds. Let x_{jk} be a chain not in the basis, but which has been selected by the criterion

$$Z_{jk} - C_{jk} < 0 \quad (\text{if } x_{jk} = 0)$$

$$Z_{jk} - C_{jk} > 0 \quad (\text{if } x_{jk} = h_{jk})$$

We then make x_{jk} increase (or decrease) up to the point where one variable reaches one of its bounds. If it is x_{jk} then the basis doesn't change and x_{jk} stays out of the basis; then we test the other $Z_j - C_j$ for another variable to enter the basis. If this variable reaches its bound before any other in the basis, we start again; maybe after a number of selection we will reach the optimum, i.e. for any x_j out of the basis we will find $Z_j - C_j > 0$ (for $x_j = 0$) and $Z_j - C_j < 0$ (for $x_j = h_j$).

But maybe we will meet another situation, i.e. a variable in the basis will reach its bound before the one out of the basis. Then the basic variable will leave the basis at its bound and the one previously out will get in (Note that if the basic variable leaves at zero we don't need to keep trace of it but we must do so if it leaves the basis at its upper bound).

Let's consider more precisely what happens. Let x_j be a basic variable, x_s the variable which enters the basis, and x_e the basic variable leaving at its bound. There are then two possibilities:

a) x_s decreases from its upper bound h_s :

Refering to the notation of section 2.3.3 of IRA's Interim Report on the Second Phase, the set of basic variables in terms of the others is given by: $x^B = B^{-1}b - B^{-1}Rx^R$.

When non-basic variables are all at their zero level we get $x^B = B^{-1}b$ as the values of basic variables; but here some non-basic variables are at their UB so that the values of the basic variables are given by: $\bar{x}^B = B^{-1}b - B^{-1}(\sum_{i\varepsilon I_1} a_i h_i)$ where I_1, is the set of columns a_i of R corresponding to non-basic variables at their upper bounds h_i.

When x_s enters the basis suppose it takes the value $\bar{x}_s$

(that we will derive below). Then the new values for basic variables are: $\bar{\bar{x}}^B = \bar{x}^B + B^{-1}a_s\ (h_s - \bar{x}_s)$

or $\bar{\bar{x}}_j = \bar{x}_j + (B^{-1}a_s)_j(h_s - \bar{x}_s)$ for the j-th basic variable.

We want this new solution to be feasible (i.e. to respect all constraints), so we must have: $0 \le \bar{\bar{x}}_j \le h_j$ and $0 \le \bar{x}_s$

i.e. $0 \le \bar{x}_j + y_{js}(h_s - \bar{x}_s) \le h_j$ and $0 \le \bar{x}_s$, where $y_{js} = (B^{-1}a_s)_j$, the j-th component of $B^{-1}a_s$.

We want these inequalities satisfied, so solving for $\bar{x}_s$ we get:

$$\begin{cases} 0 \le \bar{x}_s \text{ and } h_s - \dfrac{(h_j - \bar{x}_j)}{y_{js}} \le \bar{x}_s \le h_s + \dfrac{\bar{x}_j}{y_{js}} & \text{, for } y_{js} > 0 \text{ ,} \\ 0 \le \bar{x}_s \text{ and } h_s + \dfrac{\bar{x}_j}{y_{js}} \le \bar{x}_s \le h_s - \dfrac{(h_j - \bar{x}_j)}{y_{js}} & \text{, for } y_{js} < 0 \text{ ,} \end{cases}$$

The upper bounds on $\bar{x}_s$ above are always assured (since they are greater than h_s and since x_s is decreasing from its UB h_s), so that $\bar{x}_s$ must be given by:

$$\bar{x}_s = \max_j \left\{ 0 \ ; \ h_s - \frac{(h_j - \bar{x}_j)}{y_{js}} \text{ , for } y_{js} > 0 \ ; \ h_s + \frac{\bar{x}_j}{y_{js}} \text{ , for } y_{js} < 0 \right\}$$

Note that the maximum may not be unique; this is analogous to the case of degeneracy in ordinary linear programming (without UB constraints).

If $\bar{x}_s \doteq 0$ then x_s reaches its bound before any basic variable x_j whose value will there be

$$\bar{x}_j + y_{js}(h_s - 0) = \bar{x}_j + (B^{-1}a_s)_j h_s = (B^{-1}b)_j - \sum_{i \varepsilon I_2} (B^{-1}a_i)_j h_i$$

where $I_2 = I_1 - \{s\}$. So x_s stays out of the basis, its value passing from h_s to zero, and the basic variables are the same, their values being updated by the previous formula.

If $\bar{x}_s = h_s + \dfrac{\bar{x}_e}{y_{es}}$ (with $y_{es} < 0$) then x_s enters the basis with this value and the basic variable x_e will leave the basis at the zero level because:

$$\bar{\bar{x}}_e = \bar{x}_e + y_{es}(h_s - \bar{x}_s) = \bar{x}_e + y_{es}(h_s - h_s - \frac{\bar{x}_e}{y_{es}}) = 0$$

If $\bar{x}_s = h_s - \dfrac{(h_e - \bar{x}_e)}{y_{es}}$ (with $y_{es} > 0$) then x_s enters the basis with this value and the basic variable x_e leaves the basis at its upper bound; it must be kept in memory, with $e \varepsilon I_1$.

b) x_s increases from zero:

Suppose x_s takes the value $\bar{x}_s$ (that we will derive below). The new value of the j-th basic variable will be:

$$\bar{\bar{x}}_j = \bar{x}_j - (B^{-1}a_s)_j\bar{x}_s = \bar{x}_j - y_{js}\bar{x}_s$$

For feasability we want: $\bar{x}_s \leq h_s$ and $0 \leq \bar{x}_j - y_{js}\bar{x}_s \leq h_j$

Solving for $\bar{x}_s$ we get: $\begin{cases} \bar{x}_s \leq h_s \text{ and } \dfrac{(h_j - \bar{x}_j)}{-y_{js}} \leq \bar{x}_s \leq \dfrac{\bar{x}_j}{y_{js}}, \text{ for } y_{js} > 0, \\ \bar{x}_s \leq h_s \text{ and } \dfrac{\bar{x}_j}{y_{js}} \leq \bar{x}_s \leq \dfrac{(h_j - \bar{x}_j)}{-y_{js}}, \text{ for } y_{js} < 0. \end{cases}$

The lower bounds on $\bar{x}_s$ above are assured (since they are negative and x_s is increasing from zero) so that $\bar{x}_s$ must be given by:

$$\bar{x}_s = \min_j \left\{ h_s \ ; \ \frac{\bar{x}_j}{y_{js}} \text{ for } y_{js} > 0 \ ; \ \frac{(h_j - \bar{x}_j)}{-y_{js}} \text{ for } y_{js} < 0 \right\}$$

If $\bar{x}_s = h_s$ then x_s reaches its bound before any basic variable x_j whose value will then be $\bar{x}_j - y_{js}h_s$. x_s stays out of the basis but must be kept in memory, $s \varepsilon I_1$.

If $\bar{x}_s = \frac{\bar{x}_e}{y_{es}}$ (with $y_{es} > 0$) then x_s enters the basis with this value and x_e leaves at the zero level, for $\bar{x}_e - y_{es}\bar{x}_s = \bar{x}_e - y_{es}\frac{\bar{x}_e}{y_{es}} = 0$

If $\bar{x}_s = \frac{(h_e - \bar{x}_e)}{-y_{es}}$ (with $y_{es} < 0$) then x_s enters the basis with this value and x_e leaves the basis at its upper bound, for $\bar{x}_e - y_{es}\bar{x}_s = \bar{x}_e - y_{es}\frac{(h_e - \bar{x}_e)}{-y_{es}} = h_e$.

This is enough for UB section, which is not quite difficult.

B) GUB section:

The reader can find a good and concise description of GUB in reference (1). We will follow the same approach but with two qualifications: firstly we will give the peculiarities of our problem where they appear (mainly UB and the non-enumeration of chains), and secondly we will explicit the results where they are not too obvious in (1). We strongly suggest the reader to compare and contrast reference (1) with our text. As for UB section it is assumed that the reader is quite familiar with the revised simplex machinery.

Before going further it would be a good idea for the reader to review page 3 of this text. With this formulation in extenso of our problem, we will derive a reduced problem whose basis will be used to get the solution of the problem in extenso; we will also use the special constraints (i.e. circuit requirements constraints) because of the following theorem:

Theorem 1: Each basis of the problem in extenso (i.e. a set of m+L linearly independant columns) contains at least one chain from each $(O-D)_k$ $(k=1,\ldots L)$ (They will be called <u>key</u> chains).

Proof: If it were not so the set of columns would have a row of zeros, at least, and would not then be linearly independant.

So let us choose one of these chains in the basis from each $(O-D)_k$; without loss of generality let x_{1k} (for $k=1,2,\ldots,L$) denote these chains: we take the first chain from each set for $(O-D)_k$ to simplify the notation but it doesn't matter since we do not enumerate chains.

We will then formulate a "reduced" problem as follows: the k-th special constraint states that

$$\sum_{i=1}^{n_k} x_{ik} = v_k$$

or equivalently $x_{1k} = v_k - \sum_{i=2}^{n_k} x_{ik}$.

By replacing each key variable in terms of others, and without any explicit upper bounds constraint, an equivalent problem has then the following structure:

Maximize x_o

$$\begin{array}{lllllll}
 & \text{Variables:} & x_o & S_1 & S_2 \ldots S_m & \overbrace{x_{21} \quad \cdots \quad x_{n_1 1}}^{(O-D)_1} & \cdots \\
\text{subject to:} & \begin{matrix}\text{objective}\\ \text{function}\end{matrix}\Big\{ & 1 & -\alpha_1 & -\alpha_2 \ldots -\alpha_m & 0 \quad \cdots \quad 0 & \cdots \\
 & \begin{matrix}\text{Capacity}\\ \text{constraints}\end{matrix}\left\{\begin{matrix}\\ \\ \\ \end{matrix}\right. & \begin{matrix}0\\ \vdots \\ 0\end{matrix} & \multicolumn{2}{c}{\boxed{I_{m\times m}}} & \boxed{(A_{21}-A_{11})\ldots(A_{n_1 1}-A_{11})} & \cdots
\end{array}$$

$$\begin{array}{lcl} \cdots & \overbrace{x_{2L} \quad \cdots \quad x_{n_L L}}^{(O-D)_L} & \\ \cdots & 0 \qquad \cdots \qquad 0 & =0 \\ \cdots & \boxed{\begin{array}{c} (A_{2L}-A_{1L}) \ \cdots \ (A_{n_L L}-A_{1L}) \\ \\ \\ \end{array}} & \begin{array}{l} =d_1 \\ \vdots \\ =d_m \end{array} \end{array}$$

where $d_j = u_j - \sum_{k=1}^{L} \delta_{jk} v_k \quad (j=1,\dots,m)$

with $\delta_{jk} = \begin{cases} 1 & \text{if the key chain } k \text{ contains links } j \\ 0 & \text{if not.} \end{cases}$

Note that the δ_{jk}'s for $j=1,\dots,m$ form a column we noted A_{1k}

A basis for the reduced problem has $m+1$ columns, but since x_o is always in the basis we will usually speak of m variables (and associated columns) in the basis B, which we can call "working basis", since we will get the solution of the problem in extenso with its help.

Let's suppose for the moment that we got an extremal feasible solution for the problem in extenso, i.e. $m+L$ basic variables (of which there are L key chains by theorem 1) and the other variables at their bounds (lower or upper). By reordering the columns of the basis **B** for the problem in extenso (the boldface type will always refer to the problem in extenso), we can assume that the last L columns are for the key chains in order from 1 to L. Let us transform the first m columns of **B** as follows: if it is a column for a chain we subtract from it the corresponding key column of the key chain for the same O-D pair; if it is a slack variable we don't touch it. These m columns thus transformed will all have zeros for their last L components. By retaining the first m components of these m columns we get a matrix which is a basis for the reduced problem, since it is a subset of a linear combination of linearly independant columns. To make it more "visual" let us write:

$$\mathbb{B} = \begin{array}{c} \left[\begin{array}{c|c} A_{j_1} \;\cdots\; A_{j_m} & \begin{array}{c} A_{11} \;\cdots\; A_{1L} \\ \hline I_{L\times L} \end{array} \end{array}\right] \\ \text{m columns} \qquad \text{L columns} \end{array}$$

for the basis of the problem in extenso;

and by subtracting from A_{j_r} its key corresponding chain we get

$$\mathbb{B}^* = \left[\begin{array}{c|c} B_{m\times m} & A_{11} \;\cdots\; A_{1L} \\ \hline 0_{L\times m} & I_{L\times L} \end{array}\right]$$

where B is then a basis for the reduced problem.

Let $|A|$ stand for determinant of matrix A, then

$\mathbb{B}$ is a basis $\Rightarrow |\mathbb{B}| \neq 0$

$\Rightarrow |\mathbb{B}^*| \neq 0$ Since $\mathbb{B}^*$ is obtained by a linear combination of columns of $\mathbb{B}$ which are supposed to be linearly independant.

$\Rightarrow |B| \neq 0$ Since $|\mathbb{B}^*| = |B|$ by developing it from its south-east corner.

$\Rightarrow$ B Is regular, thus a basis.

a) Determining a set of prices:

For the sequel we will assume $\mathbb{B}$ and B "augmented", i.e. we will let the first column of these stand for the variable x_o which will always stay in the basis. Then the first row of B^{-1} gives the vector of shadow prices, $\hat{\Pi}$, for the reduced problem since by definition $\hat{\Pi} = C^B B^{-1}$ and $C^B = [1 \;\; 0 \ldots 0]$ (the one being for x_o, the remaining m zeros for other basic variables).

Knowing the key columns A_{1k} (for k=1,...,L) we can compute $\hat{u}_k = -\hat{\Pi} A_{1k}$ (for k=1,...,L). Let $\Pi = [\hat{\Pi}\; \hat{u}] = [\hat{\Pi}_o, \hat{\Pi}_1, \ldots, \hat{\Pi}_m, \hat{u}_1, \ldots, \hat{u}_L]$.

Theorem 2: Π is a vector of shadow prices for the problem in extenso.

Proof: We must show that $C^{\mathbb{B}}\mathbb{B}^{-1} = \Pi$ or $\Pi\mathbb{B} = C^{\mathbb{B}}$ where $C^{\mathbb{B}} = [1 \quad 0 \ \dots \ 0]$, with m+L zeros (we remind that $\mathbb{B}$ is augmented).

$$\Pi \mathbb{B} = [\hat{\Pi}_o, \Pi A_{j_1}, \dots, \Pi A_{j_m}, \ \Pi\begin{pmatrix} A_{11} \\ 1 \\ 0 \\ \vdots \\ 0 \end{pmatrix}, \dots, \ \Pi\begin{pmatrix} A_{1L} \\ 0 \\ \vdots \\ 0 \\ 1 \end{pmatrix}] \ ;$$

But $\Pi\begin{pmatrix} A_{11} \\ 1 \\ 0 \\ \vdots \\ 0 \end{pmatrix} = \hat{\Pi} A_{11} + \hat{u}_1 = \hat{\Pi} A_{11} - \hat{\Pi} A_{11} = 0$ and so on for $A_{1k} (k=2,\dots,L)$;

and $$\Pi A_{j_i} = \Pi\begin{pmatrix} B_{j_i} \\ 0 \\ \vdots \\ 0 \end{pmatrix} + \Pi\begin{pmatrix} A_{1k} \\ 0 \\ \vdots \\ 0 \\ 1 \\ 0 \\ \vdots \\ 0 \end{pmatrix} = \hat{\Pi} B_{j_i} + 0 = 0 + 0 = 0$$

$\leftarrow$ the one comes after (k-1) zeros

since we supposed that A_{j_i} was a chain for $(O\text{-}D)_k$ and as we know $B_{j_i} = A_{j_i} - A_{1k}$, and because we showed above that $\Pi\begin{pmatrix} A_{1k} \\ 0 \\ \vdots \\ 0 \\ 1 \\ 0 \\ \vdots \\ 0 \end{pmatrix} = 0$

finally, $\hat{\Pi}_o = 1$ because the fist column of $\mathbb{B}$ which is $\begin{pmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{pmatrix}$ implies the first column of $\mathbb{B}^{-1}$ is $\begin{pmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{pmatrix}$.

We thus have shown that $\Pi \mathbf{B} = [10\ldots0] = C^{\mathbf{B}}$ which gives the usual formula for prices $\Pi = C^{\mathbf{B}}\mathbf{B}^{-1}$.

b) Determining the variable entering the basis:

Having then a set of prices for the problem in extenso (prices obtained from the reduced problem) we already know from IRA's Interim Report on the Second Phase, sept. 15, 1974, section 2.3.3, how to obtain the variable entering the basis: without chain enumeration.

If one of the

$\hat{\Pi}_i$ is negative $(i=1,\ldots,m)$, we introduce the corresponding slack variable; if not, we search for a shortest chain (from the prices point of view); but as we explained in UB section, we must also calculate $Z_j - C_j$ for chains at their upper bounds, and choose the one for which Z_j in the most positive (since $C_j = 0$ for chains), or in other words a lengthiest positive chain (from the prices point of view) for these non-basic chains at their UB.

c) Variable leaving the basis:

Having then found the variable which enters the basis, we must determine the leaving one. The idea will still be to make use of the reduced problem.

Let $\mathbf{A}_s$ be the column of the variable entering the basis; the index s is used to simplify the notation and the bold-face type for $\mathbf{A}$ is to differentiate it from the column of the reduced problem. To put it differently $\mathbf{A}_s = \begin{pmatrix} A_s \\ 0 \\ \vdots \\ 0 \\ 1 \\ 0 \\ \vdots \\ 0 \end{pmatrix}$, where the one in the lower part is absent if we got a slack variable and present in the correct position otherwise.

Let $b = \begin{bmatrix} u \\ v \end{bmatrix}$, the so called RHS (right hand side). To determine the variable leaving the basis we need $\mathbf{B}^{-1}\mathbf{A}_s$ and $\mathbf{B}^{-1}b$ (we will get them without the knowledge of $\mathbf{B}^{-1}$), and we need also, because of UB constraints, the upper bounds of the basic variables, plus UB of the entering variable x_s. The notation here is almost that of reference (1); we will later make the correspondance with the notation of the section on UB.

Let's determine firstly how to obtain $\mathbf{B}^{-1}\mathbf{A}_s$. We can first easily obtain $\bar{D}_s = B^{-1}(A_s - A_{1k})$ where we suppose that s is a chain for $(O-D)_k$; if it were a slack variable, we would let $\bar{D}_s = B^{-1}A_s$. So $A_s - A_{1k} = B\bar{D}_s$ (or $A_s = B\bar{D}_s$ for a slack), i.e. $A_s = A_{1k} + B\bar{D}_s = A_{1k} + \sum_{i=0}^{m} \bar{D}_{is}B_i$ where B_i is the i-th column of B (the zero-th being for x_o). For the sequel we will forget the case for A_s being a slack but it is analogous.

By completing $\mathbf{A}_s$ we get

$$\mathbf{A}_s = \begin{array}{r} \\ \\ \\ \\ \text{k-th special constraint} \longrightarrow \\ \\ \\ \\ \end{array} \left(\begin{array}{c} A_s \\ \hline 0 \\ \vdots \\ 0 \\ 1 \\ 0 \\ \vdots \\ 0 \end{array}\right) = \mathbf{A}_{1k} + \left(\begin{array}{c} \sum_{i=0}^{m} \bar{D}_{is}B_i \\ \hline 0 \\ \vdots \\ 0 \\ 0 \\ \vdots \\ 0 \end{array}\right)$$

Let $\bar{\mathbf{A}}_s = \mathbf{B}^{-1}\mathbf{A}_s$, so that $\mathbf{A}_s = \mathbf{B}\bar{\mathbf{A}}_s = \sum_{j=0}^{m+L} \bar{A}_{js}\mathbf{B}_j$ where $\mathbf{B}_j$ is the j-th column of $\mathbf{B}$. By equating the two expressions for $\mathbf{A}_s$ we get:

$$\sum_{j=0}^{m+L} \bar{A}_{js}\mathbf{B}_j = \mathbf{A}_{1k} + \left(\begin{array}{c} \sum_{i\ 0}^{m} \bar{D}_{is}B_i \\ \hline 0 \\ \vdots \\ 0 \end{array}\right)$$

where $\mathbf{B}_j$ and B_i are as given on page 11(these matrix will be useful to understand what follows).

Since a vector is expressed in a unique way in terms of a basis, we have the following identifications: if $\mathbb{B}_j$ is the k-th key column, then $\bar{A}_{js} = 1 - \sum_t \bar{D}_{ts}$ where t is such that the t-th column of B is of the form $B_t = A_{j_t} - A_{1k}$, i.e. a variable for $(O-D)_k$;

if $\mathbb{B}_j$ is the r-th key column $(r \neq k)$ then $\bar{A}_{js} = -\sum_t \bar{D}_{ts}$ where t is the index for columns such that $B_t = A_{j_t} - A_{1r}$ $(r \neq k)$;

if $\mathbb{B}_j$ is a slack or a non-key column then $\bar{A}_{js} = \bar{D}_{is}$ where i and j are the corresponding indexes.

We now got $\mathbb{B}^{-1}\mathbb{A}_s$, but because of upper bounds we need also $\mathbb{B}^{-1}\mathbb{A}_i$ for $i \varepsilon I_1$ the set of non-basic variables at their upper bounds; the procedure is perfectly analogous, so we won't repeat it!

We now need $\mathbb{B}^{-1}b$ as we said before. Let $\bar{b} = \mathbb{B}^{-1}b$ so that $b = \mathbb{B}\,\bar{b} = \sum_{j=0}^{m+L} \bar{b}_j \mathbb{B}_j$.

On the other hand we know $\bar{d} = B^{-1}d$ where $d_j = u_j - \sum_{k=1}^{L} \delta_{jk} v_k$ as stated before. Let's write it with our present notation $d_j = u_j - \sum_{k=1}^{L} (A_{1k})_j v_k$ or in matrix form $d = u - \sum_{k=1}^{L} A_{1k} v_k$, so that $u = d + \sum_{k=1}^{L} A_{1k} v_k$. But $\bar{d} = B^{-1}d$ implies $d = B\,\bar{d} = \sum_{i=0}^{m} \bar{d}_i B_i$, so that $u = \sum_{i=0}^{m} \bar{d}_i B_i + \sum_{k=1}^{L} A_{1k} v_k$ and

$$b = \begin{bmatrix} u \\ v \end{bmatrix} = \left(\begin{array}{c} \sum_{i=0}^{m} \bar{d}_i B_i + \sum_{k=1}^{L} A_{1k} v_k \\ \hline \\ v \\ \\ \end{array} \right) = \left(\begin{array}{c} \sum_{i=0}^{m} \bar{d}_i B_i \\ \hline 0 \\ \vdots \\ 0 \end{array} \right) + \sum_{k=1}^{L} \mathbb{A}_{1k} v_k .$$

Comparing the two expressions for b we get

$$\sum_{j=0}^{m+L} \bar{b}_j \mathbb{B}_j = \left(\begin{array}{c} \sum_{i=0}^{m} \bar{d}_i B_i \\ \hline 0 \\ \vdots \\ 0 \end{array} \right) + \sum_{k=1}^{L} \mathbb{A}_{1k} v_k ;$$

and, again, since the coefficients are unique in terms of a given basis, we have the following identification:

if $\mathcal{B}_j$ is the r-th key column then $\bar{b}_j = v_r - \sum_t \bar{d}_t$ for t such that $B_t = A_{j_t} - A_{1r}$;

if $\mathcal{B}_j$ is a slack or non-key column then $\bar{b}_j = \bar{d}_i$ for the corresponding i and j .

To find the variable leaving the basis, we must know if x_s (the variable entering the basis) is increasing from zero or decreasing from its UB. If it is increasing from zero then we must find (as explaned in UB section)

$$\bar{x}_s = \min_j \{h_s \; ; \; \frac{\bar{x}_j}{y_{js}} \text{ for } y_{js} > 0 \; ; \; \frac{(h_j - \bar{x}_j)}{-y_{js}} \text{ for } y_{js} < 0 \}$$

where j = 1,...,L

$$\bar{x}_j = (\mathcal{B}^{-1}b)_j - \sum_{i \varepsilon I_1} (\mathcal{B}^{-1}A_i)_j \, h_i = \bar{b}_j - \sum_{i \varepsilon I_1} y_{ij} h_i$$

$$y_{js} = (\mathcal{B}^{-1}A_s)_j$$

For the case x_s decreases from its UB we would have to find

$$\bar{x}_s = \max_j \{ 0 \; ; \; h_s - \frac{(h_j - \bar{x}_j)}{y_{js}} \text{ for } y_{js} > 0 \; ; \; h_s + \frac{\bar{x}_j}{y_{js}} \text{ for } y_{js} < 0\}$$

Then the results of UB section are used to determine which variable stays out or goes out, depending on the value of $\bar{x}_s$.

d) Updating:

Now that the variables entering and leaving the basis are determined, we can go on the updating of B^{-1} and $\bar{d}$, our working aids of the reduced problem. Three main cases can occur:

a) Suppose the leaving variable was the only chain in the basis for a given $(O-D)_k$. Then by Theorem 1 we know that the entering variable must be a chain for the same $(O-D)_k$, and thus replaces it as the key variable for the traffic requirement constraint of $(O-D)_k$. The working basis needs no change since the key chain was supposed to be the only one in the basis; so B^{-1} needs no change either. But $\bar{d}$ needs to be replaced by $\bar{d} - B^{-1}(A_s - A_{1k})v_k$ where A_{1k} is the old key chain for $(O-D)_k$; we note that $B^{-1}(A_s - A_{1k})$ is what we noted $\bar{D}_s$ and was calculated in order to determine $\bar{A}_s$. So the updating for this case is very simple.

b) If the leaving variable is not a key chain, then B^{-1} and $\bar{d}$ are updated in the usual way of the revised simplex technique.

c) If the leaving variable is a key chain for a given $(O-D)_k$ and not the only chain for this $(O-D)_k$ in the basis, we must firstly choose another key chain (it may be the entering variable if it is a chain for $(O-D)_k$). We must then update B^{-1} by replacing the relevant columns in B with the new key chain and remove the old key chain (see reference (1) pages 218-219).

We will now order all the previous steps in a more rigorous fashion but without giving all the formulas (which are already available in the previous paragraphs).

Part II) Algorithm:

0) We suppose we got an extremal feasible solution, i.e. m+L basic variables (apart from x_0), from which L are key chains, and the other non-basic variables at their bounds (lower or upper). To get it, one can try to allocate the v_i's, starting from the lowest, as follows: you find a chain for $(O\text{-}D)_i$ such that every link's capacity is strictly positive, and you allocate circuits on it, up to the upper bound or to the residue (not allocated before). You then take off this number of circuits from every link of the chain and then start again for the residual circuits for this O-D pair or pass to another O-D pair. If at some stage you cannot allocate all the v_i circuits for $(O\text{-}D)_i$ you then assign an artificial variable to the i-th special constraint and pass to another O-D pair; the first objective function will be to minimize the sum of these artificial variables. Slack variables complete the starting basis B.

1) By the previous step we have B^{-1}, $\bar{d}$ and the set of L key chains.

Let $\hat{\Pi}$ be the fist row of B^{-1} : $\hat{\Pi} = [\hat{\Pi}_0 \hat{\Pi}_1 \ldots \hat{\Pi}_m]$

If for i = 1,...,m we have a $\hat{\Pi}_i < 0$ go to step 2)

If not, calculate $\hat{u}_k = - \hat{\Pi} A_{1k}$ for $k = 1,\ldots,L$ where A_{1k} is the key chain for $(O\text{-}D)_k$

Then form $\Pi = [\hat{\Pi},\hat{u}]$ the vector of prices, and find.

a) the shortest chain (from the prices point of view) for non-basic chains at their lower bound (i.e. zero). Remark that to increase the objective its "length" must be negative.

b) the lengthiest chain (from the prices point of view) for non-basic chains at their upper bound. Remark that to increase the objective its "length" must be positive.

If none from a) or b) improves the objective then go to step 3).

If not, choose the one which is improving the most x_o ; let's note this chain A_s (entering variable).

Go to step 4).

2) Let's note A_s the column for the entering slack variable corresponding to the most negative $\hat{\Pi}_i$.

Go to step 4)

3) Terminate ; we got an optimal solution.

4) Calculate $\bar{D}_s = B^{-1}(A_s - A_{1k_s})$ or $\bar{D}_s = B^{-1}A_s$ (for a slack) , where k_s is the index for the key chain associated to A_s.

Calculate $\bar{A}_s$, $\bar{A}_i$ (for $i \varepsilon I_1$, the set of non-basic variables at their UB), $\bar{b}$

a) if x_s increases from zero calculate $\bar{x}_s = \min\limits_{j=1,\ldots,} \{h_s L ; \frac{(h_j - \bar{x}_j)}{-y_{js}}$ for $y_{js} < 0$; $\frac{\bar{x}_j}{y_{js}}$ for $y_{js} > 0\}$.

a_1) if $\bar{x}_s = h_s$ then B^{-1} is unchanged but the values of basic variables change. Go to step 5).

The UB constraints on chains may, as we shall see, introduce some complications in the search for a shortest chain (from the prices point of view) because we do not enumerate chains (except the basic ones, the key ones, and those at their UB). We will first give a trivial case serving to identify the problem, and after we will give two means for solving it.

Let's suppose that in a given iteration we got the shortest chain among those at their zero level, and that the criterion

$$\min_{j} \left\{ h_s \; ; \; \frac{(h_j - \bar{x}_j)}{- y_{js}}, \text{ for } y_{js} < 0 \; ; \; \frac{\bar{x}_j}{y_{js}}, \text{ for } y_{js} > 0 \right\}$$

gives h_s (the UB) ; it means that the chain will stay <u>out</u> of the basis, but its level passing from zero to h_s. The basis will remain unchanged, and so will the shadow prices, implying that the search for a new shortest chain will give the same one we got previously.

A first temporary solution would be to search a shortest chain by eliminating the O-D pair from which we got the previous shortest chain, since it suffices to improve the objective function even if it is not in the direction of the steepest ascent.

This solution is temporary, because it is fairly probable that at a given time no other chain from the O-D pairs non-excluded could be found to improve the objective function. We would then have to search for a k-th shortest chain (k = 2,3,...) among the excluded O-D pairs, when for these O-D pairs the other chains (1,...,k-1) are out of the basis at their UB.

Searching a k-th shortest chain is an already solved problem; a review of some algorithms is given in reference [4] . Which one to choose is mainly a matter of how big k can be. For k = 2,3 a fairly simple algorithm exists, and since a realistic set of UB would be one half or one third of the traffic requirements, it implies that no more than one or two chains can be out of the basis at their UB.

Finally let's remark another case (other than the one mentionned above) for which the same problem can emerge. In the GUB algorithm (even without UB constraints) the entering chain in the basis may replaces a key one, without affecting B^{-1} of the reduced problem, and so let the door open for an identical set of shadow prices. The solutions given above still apply in this case.

(Addendum to the note on UB and GUB, to be inserted in 4)a_1) on page 19).

a_2) if $\bar{x}_s = \frac{h_e - \bar{x}_e}{-y_{es}}$ (for $y_{es} < 0$) then x_s enters the basis with this value and x_e leaves the basis at its UB ; for the updating go to step 6) and retain x_e in the set of non-basic variables at their UB.

a_3) if $\bar{x}_s = \frac{\bar{x}_e}{y_{es}}$ (for $y_{es} > 0$) then x_s enters the basis with this value and x_e leaves the basis at the zero level. For the updating go to step 6)

b) If x_s decreases from h_s calculate $\bar{x}_s = \max\limits_j \{0 ; h_s - \frac{(h_j - \bar{x}_j)}{y_{js}}$ for $y_{js} > 0$; $h_s + \frac{\bar{x}_j}{y_{js}}$ for $y_{js} < 0$ }

b_1) if $\bar{x}_s = 0$ then B^{-1} is unchanged but the values of basic variables change. Go to step 5).

b_2) if $\bar{x}_s = h_s - \frac{(h_e - \bar{x}_e)}{y_{es}}$ for $y_{es} > 0$ then x_s enters the basis with this value and x_e leaves the basis at its UB. Retain x_e in the set of non-basic variables at their UB. Go to step 6)

b_3) if $\bar{x}_s = h_s + \frac{\bar{x}_e}{y_{es}}$ for $y_{es} < 0$ then x_s enters the basis with this value and x_e leaves the basis at the zero level. Go to step 6).

5) Make the necessary changes in I_1, the set of non-basic variables at their UB.

Go back to step 1)a)
or 1)b)

6) If x_e (the leaving variable) was the only basic chain for a given $(O-D)_k$ then go to step 7).

If x_e (the leaving variable) was not a key chain then go to step 8).

If x_e (the leaving variable) was a key variable and not the only basic one for its (O-D) pair then go to step 9).

7) Since x_e was the only basic chain it was key, and, by theorem 1, x_s (the entering variable) must be a chain for the same O-D pair. Thus we replace x_e by x_s , the new key chain for this O-D pair, and B^{-1} remains unchanged. We update $\bar{d}$ as follows: let A_{er} and A_{s_r} be the old and new key columns,

$\bar{d}$ is replaced by

$$B^{-1}(u - \sum_{\substack{k=1 \\ k\neq r}}^{p} A_{1k}V_k - A_{er}v_r + A_{er}v_r - A_{sr}v_r) = B^{-1}(u - \sum_{\substack{k=1 \\ k\neq r}}^{p} A_{1k}v_k - A_{er}v_r) - B^{-1}(A_{sr} - A_{er})v$$

$= \bar{d} - \bar{D}_s v_r$ where $\bar{d}$ is the old one and where $\bar{D}_s$ has been computed in step 4). Go to step 1).

8) B^{-1} and $\bar{d}$ are updated in the usual way of the revised simplex technique. Go to step 1).

9) Suppose x_e is a chain for $(O-D)_r$. Firstly we must find another key (anyone other than x_e can fit) ; having it we must change B^{-1} because of the columns of B which are of the form $A_{j_i} - A_{1r}$ where A_{1r} is the old key. In reference (1) we find a method to do it but the method given in reference (3) seems more adapted to save computer space (it is based on the product-form of the inverse). $\bar{d}$ is updated in a similar way to B^{-1}.

REFERENCES

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(3) Simonnard, M., Programmation linéaire, Tome 2, Dunod, Paris, 1973, pages 14-21.

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APPENDIX B

AGING I and AGING II Operational Procedures

AGING I Program

a) Input

- Survival curve type
- Depreciation method
- Average life: L
- Maximum life: T
- Salvage rate: SAL
- Gross telephone plant at reproduction cost: GTPRIM
- Inflation index: P
- Growth rate: R

$\left.\begin{array}{l}\text{Inflation index: } P\\ \text{Growth rate: } R\end{array}\right\}$ for $x = 1, \ldots, T$.

b) Output

- Initial gross addition: GA (T)
- Current gross addition: GA (1)
- Gross telephone plant at historical cost: GTP
- Depreciation reserve (dollar value and percentage)
- Annual depreciation (dollar value and percentage)
- Age of vintage survivors
- Value of survivor of vintage X at book cost, at present.
- $\sum_{i=1}^{X} V(i)$

$\left.\begin{array}{l}\text{Value of survivor of vintage } X \text{ at book cost, at present.}\\ \sum_{i=1}^{X} V(i)\end{array}\right\}$ for $X = 1, \ldots, T$.

c) Remarks

1) After the user terminates typing in the input parameters the program confirms his receiving them by producing a list of these parameters as well as their value, and asks the user if he is willing to continue. If user's answer is positive, the

program will enter the computation phase and then the output phase. If user's answer is negative the program will re-enter the input phase.

The purpose of this feature is to allow the user to examin his input values, and possibly correct them, before entering the computation phase.

2) After the output is terminated the program asks the user if he wants to do another simulation. In the positive case the program enters the input phase and the user must type in the input parameters as requested by the program. In the negative case the program stops execution.

d) Sample terminal session

Let a simulation scenario consist of the following input:

- Survival curve of L0.0 type
- Average life: $L = 2$
- Maximum life: $T = 7$
- Salvage rate: $SAL = 0.1$ (10%)
- GTPRIM = 500.00
- Inflation index: $P = \begin{cases} 1.06\ (6\%) & \text{for } X = 1, \ldots, 3 \\ 1.02\ (2\%) & \text{for } X = 4, \ldots, 7 \end{cases}$

  $R = 1.1\ (10\%)$ for $X = 1, \ldots, 7$ (uniform)

The first simulation will use the A.S.L. method and the second will use the E.L.G. method.

```
!DO AGING1.VC
SRU'S:19.0
SRU'S:3.4
SURVIVAL CURVE NO. , OR "0" FOR HELP
0
NUMBER  SRV CURVE  MAX.PERC.AV.LIFE
   1      L0.0          358
   2      L0.5          350
   3      L1.0          291
   4      L1.5          287
   5      L2.0          262
   6      L3.0          226
   7      L4.0          196
   8      L5.0          174
   9      S-.5          200
  10      S0.0          200
  11      S0.5          200
  12      S1.0          198
  13      S1.5          197
  14      S2.0          191
  15      S3.0          180
  16      S4.0          161
  17      S5.0          143
  18      S6.0          129
  19      R1.5          200
  20      R2.5          183
  21      R3.5          166
  22      R4.5          149
```

The following relation must be satisfied:

$$\frac{\text{Max. life} \times 100}{\text{Av. Life}} \quad \text{Max. Perc. Av. Life}$$

```
1
TYPE DEPRECIATION METHOD : 1=ASL,2=ELG
1
TYPE IN AVG LIFE
3
MAX LIFE
7
SALVAGE RATE E.G. 0.10
0.1
GTP-PRIME E.G. 17536.
500.
P,INFLATION INDEX E.G. 1.02
RANGE
0,3
ERROR IN RANGE BOUNDS, TRY AGAIN
RANGE
1,3
VALUE
1.06
RANGE
4,7
VALUE
1.03
R,GROWTH RATE E.G. 1.10
RANGE
1,7
VALUE
1.1
```

The current year should be identified by 1, not 0

```
AVG LIFE= 3.000
MAX LIFE=  7
SALV RATE=   .10
GTP-PRIME   500.0
INFL. INDEX= 1.06   FOR PERIOD   1 TO  3
             1.03   FOR PERIOD   4 TO  7
```

An error has been noticed at this level: the average life should equal 2. The way to correct it, is to answer "no" to the next question.

```
DO YOU WISH TO CONTINUE: YES OR NO
NO
SURVIVAL CURVE NO. , OR "O" FOR HELP
1
TYPE DEPRECIATION METHOD : 1=ASL,2=ELG
1
TYPE IN AVG LIFE
2                                           Error is corrected
MAX LIFE
7
SALVAGE RATE E.G. 0.10
0.1
GTP-PRIME E.G. 17536.
500.
P,INFLATION INDEX E.G. 1.02
RANGE
1,3
VALUE
1.06
RANGE
4,7
VALUE
1.03
R,GROWTH RATE E.G. 1.10
RANGE
1,7
VALUE
1.1



AVG LIFE= 2.000
MAX LIFE=  7
SALV RATE=  .10
GTP-PRIME   500.0
INFL. INDEX= 1.06  FOR PERIOD   1 TO  3
             1.03  FOR PERIOD   4 TO  7
GROWTH RATE= 1.10  FOR PERIOD   1 TO  7
DO YOU WISH TO CONTINUE: YES OR NO
YES


GA INITIAL =      138.487
GA CURRENT =      328.876
GTP HISTORICAL =       450.257
DEPRECIATION RESERVE =      336.383 ( 74.71%)
ANNUAL DEPRECIATION =  50.00%
DO YOU WISH TO HAVE THE VINTAGES PRINTED
YES


 AGE      VALUE        SUM
  .5     220.26      220.26
 1.5     132.26      352.52
 2.5      64.99      417.51
 3.5      24.81      442.32
 4.5       6.74      449.06
 5.5       1.12      450.18
 6.5        .08      450.26
DO YOU WANT TO DO ANOTHER SIMULATION:YES OR NO
YES
```

```
SURVIVAL CURVE NO. , OR "O" FOR HELP
1
TYPE DEPRECIATION METHOD : 1=ASL,2=ELG
2
TYPE IN AVG LIFE
2
MAX LIFE
7
SALVAGE RATE E.G. 0.10
0.1
GTP-PRIME E.G. 17536.
500.
P,INFLATION INDEX E.G. 1.02
RANGE
1,3
VALUE
1.06
RANGE
4,7
VALUE
1.03
R,GROWTH RATE E.G. 1.10
RANGE
1,7
VALUE
1,1



AVG LIFE= 2.000
MAX LIFE=  7
SALV RATE=  .10
GTP-PRIME   500.0
INFL. INDEX= 1.06  FOR PERIOD   1 TO  3
             1.03  FOR PERIOD   4 TO  7
GROWTH RATE= 1.10  FOR PERIOD   1 TO  7
DO YOU WISH TO CONTINUE: YES OR NO
YES



GA INITIAL =     138.487
GA CURRENT =     328.876
GTP HISTORICAL =     450.257
DEPRECIATION RESERVE =     202.614 ( 45.00%)
ANNUAL DEPRECIATION =  40.71%
DO YOU WISH TO HAVE THE VINTAGES PRINTED
YES



 AGE       VALUE          SUM
  .5      220.26        220.26
 1.5      132.26        352.52
 2.5       64.99        417.51
 3.5       24.81        442.32
 4.5        6.74        449.06
 5.5        1.12        450.18
 6.5         .08        450.26
DO YOU WANT TO DO ANOTHER SIMULATION:YES OR NO
NO
 STOP
```

Second simulation uses ELG method

AGING II Program

a) Input:

- Simulation type: single vintage or multi vintage
- Year of first installation and eventually the current year. (cf. Remark -(1))
- Remaining life in future: S
- Growth rate: R(X) (multi vintage case only), for X=1, T
- Specify if Max. life is constant or variable (multi vintage case only).
- Initial gross addition: GA(T)
- Retirement rate: M

b) Output

- AGE
- Accumulated depreciation: AD(X), X = 1, ..., T
- Depreciation reserve ratioL ADRAT(X), X = 1, ..., T
- Total accumulated recoupments: TAR(X), X = 1, ..., T
- Depreciation accruals: DEPN(X), X = 1, ..., T
- Depreciation rate: DEPRAT(X), X = 1, ... T

c) Remarks

1) To input the year of first installation, the user has the choice of typing a specific calendar year or a number that refers to the year of first installation given that the present year is one.

In the former case the program will ask the user about the current year.

For instance, let the present eyar be 1959, and the year of first installation 1951, then the user can type in either 1951 or 9. In the former case the program will ask the user about the current year which is 1959.

2) In the multi vintage case, the maximum life could be calculated as follows:

   - constant case: $N = T + S$
   - variable case: $N = X + S$

d) Sample terminal session:

Let us try the following simulations:

Simulation 1:

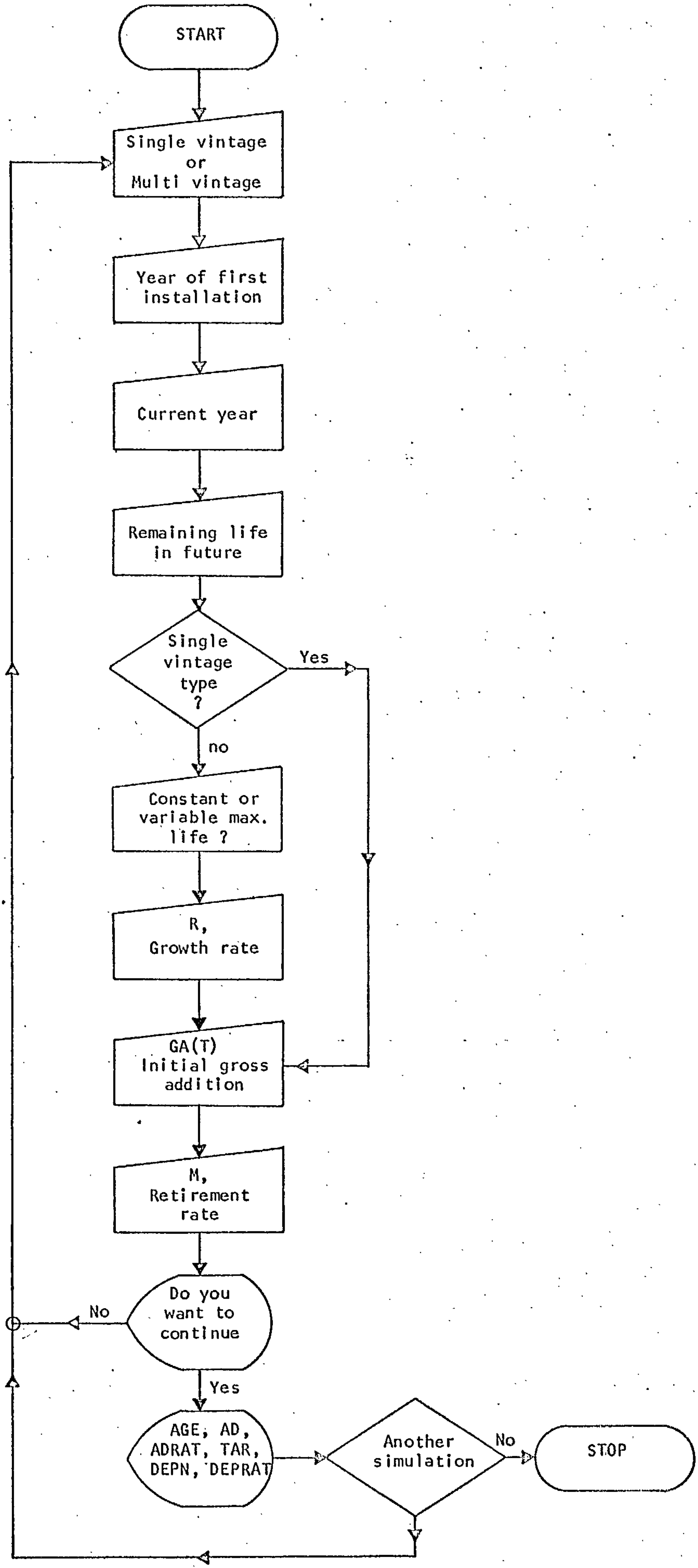
- Single vintage type
- GA (T) = \$100
- M = 10%
- T = 7
- S = 0

Simulation 2:

- Multi vintage type
- GA (T) = \$100
- $R(X) = 1. \quad , X = 1, \ldots, T$
- T = 7
- S = 0
- $N = S + X$ (variable Max. life)
- M = 10%

AGING II

LOGIC FLOWCHART



APPENDIX C

MULTIPERIOD ACCOUNTING MODEL

C-1 BEADS

1. EQO
2. REO
3. PRO
4. LO
5. PUCO
6. LANDO
7. GETVO
8. GTPO
9. ADO
10. DFTAXO
11. OCAO
12. CLO
13. INVO
14. RHO
15. IO
16. UCCO
17. CE
18. CP
19. CCL
20. ALPHA
21. BETA
22. OMEGA
23. IN
24. IC
25. RHN
26. T

27. IINV
28. REPL
29. DEPDIF
30. DELODCR
31. DELDCH
32. PRDTX
33. OTHADJ
34. ADJR
35. ADJP
36. ADJO
37. ADJD
38 ADJB
39. SPLIT
40. GAMMA

C-2 OPTIONAL BEADS

40. R
41. GCE
42. OPRV
43. ROREC
44. RORC
45. RORBI
46. RORBE
47. DELPR
48. PCR
49. NEWDEB

50	DCR
51	ITCE
52	ITCI
53	DELEQ
54	DELCTI
55	DIVI
56	DPR

C-3 REGRESSION BEADS

57	ARET
58	BRET
59	ADEPN
60	BDEPN
61	ANSV
62	BNSV
63	ALAND
64	BLAND
65	AGETV
66	BGETV
67	APUC
68	BPUC
69	ADPRTV
70	BDPRTV
71	ACCA
72	BCCA
73	AOPXP
74	BOPXP

75	ACL
76	BCL.
77	AOCA
78	BOCA
79	AAOCA
80	BBOCA
81	ROPRV
82	BOPRV
83	AINV
84	BINV
85	AOTHEXP
86	BOTHEXP

C-4 NON BEAD

The purpose of the NON-BEAD is to estimate certain items not available in BEAD. However, since some BEAD items are optional, the NON-BEAD changes somewhat depending on which options are exercised. At the moment, six NON-BEAD systems are distinguished.

NB-I

This system is appropriate if R and OPRV appear as BEAD

OPXP is NON-BEAD

1. GTP = (1+R)*GTPO
2. RET = ARET + BRET * GTPO
3. GCE = GTP + RET - GTPO -ADJP
4. AGTP = 1/2*(GTP + GTPO)
5. DEPN = ADEPN + BDEPN * AGTP
6. NSV = ANSV + BNSV * RET
7. LAND = ALAND + BLAND * GTP
8. PUC = APUC + BPUC * GCE
9. GETV = AGETV + BGETV * GTP
10. AGETV = 1/2*(GETV + GETVO)
11. DPRTV = ADPRTV + BDPRTV * AGETV
12. DPRTVE= SPLIT * DPRTV
13. DPRTVC = DPRTV - DPRTVE
14. IDC = 1/2* IC * (PUC + PUCO)
15. OTHEXP= ANOTHEXP + BOTHEXP * GCE
16. UCC = UCCO + GCE - IDC - DPRTVC - OTHEXP - ADJV

17. CCA = A CA + BCCA * UCC

18. OPXP = AOPXP + BOPXP * AGTP

19.

20. CL = ACL + BCL * OPXP

21. DELCL = CL - CLO

22. OCA = AOCA + BOCA * OPRV

23. DELOCA = OCA - OCAO

24. INV = AINV + BINV * GTP

25. AINV = 1/2* (INV + INVO)

26. OTHINC = IINV * AINV

27. DELINV = INV - INVO

NB-2

This system is appropriate if GCE and OPRV appear as BEAD

OPXP is NON-BEAD

The system is the same as NB-I except that

a) equation 1) is omitted

b) equation 3) is replaced by 3a) GTP = GTPO + GCE -RET +ADJP

NB-3

This system is appropriate if GCE is BEAD

OPRV and OPXP are NON-BEAD

The system is the same as NB-2 except that

a) equation 19) is added

19) OPRV = AOPRV + DOPRV * AGTP

NB - 4

This system is appropriate if R is BEAD

OPRV and OPXP are NON-BEAD

The system is the same as NB-I except that equation 19) is added

NB- 5

This system is appropriate if

GCE is BEAD

OPXP is NON-BEAD

OPRV is OUTPUT

The system is the same as NB-2 except that equation 22) is replaced by 22a)

22a) OCA = AAOCA + BBOCA * AGTP

NB- 6

This system is appropriate if

R is BEAD

OPXP is NON-BEAD

OPRV is OUTPUT

The system is the same as NB-I except that equation 22) is replaced by equation 22a)

C-5 UPDATING

Once the system has been run for one year, many of the BEAD variables must be updated. The following operations would be performed.

U1 $IO = \frac{(LO - REPL)*IO + IN * NEWDEB}{LO + NEWDEB - REPL}$

U2 $PDIVIO = \frac{RHO * PRO + RHN * DELPR}{PR + DELPR}$

U3 EO = E

U4 RO = R

U5 PRO = PR

U6 LO = L

U7 PUCO = PUC

U8 LANDO = LAND

U9 GETVO = GETV

U10 GTPO = GTP

U11 ADO = AD

U12 DFTAXO = DFTAXO + CURDTX + PRDTX

U13 UCCO = UCC - CCA

U14 OCAO = OCA

U15 CLO = CL

U16 INCO = INV

C-6 BEAD DATA

HB	FINANCIAL STATISTICS HANDBOOK 1973
SC	STATISTICS CANADA
CC	COMMUNICATIONS CANADA

1.	LOCAL	Local Revenue	HB-59
2.	TOLL	Toll revenue	HB-60
3.	MISOPRV	Miscellaneous operating revenue	HB-61
4.	UNCOLL	Uncollectible accounts	HB-62
5.	TOT OTHINC	Total other income	HB-66
6.	INCTAX	Income Taxes accrued	HB-68
7.	DSC	Debt Service Charges	HB-70
8.	MAINT	Maintenance expense	HB-72

9.	DEPN	Depreciation Expense	HB-73
10.	TRAF	Traffic expense	HB-74
11.	CAM	Commercial and Marketing Expense	HB-75
12.	OTHOPXP	Other Operating Expense	HB-76
13.	OTHTX	Other Taxes	HB-77

14. IDC Interest during construction

a) Bell
Notes to financial statements since 1972
SC reports

b) B.C. Tel
Notes to financial statements
SC reports

15. IN Interest rate on new debt

Find the par value of long term securities issued and the associated interest rate from the financial statements.

Calculate weighted average interest rate using as weights the relative size of the issue.
Assume that the rate on short term notes is the same as the average rate on long term debt.

16. RHON Dividend rate on new preferred shares

The rate on a new issue is found from the financial statements in one of two ways

a) If a dividend rate is quoted, simply take the rate

b) If a dividend amount is quoted, divide the dividend amount by the par (stated) value of the share

If more than one issue is made, calculate the weighted average rate.

17. ALPHA Flowthrough coefficient

a) Bell	0
b) B.C. Tele	0
c) NFLD	1

18. BETA Ratio of taxable other income to other income excluding IDC

This variable is designed to split other income into taxable and non-taxable components. Its size is at best an educated guess.
An estimate is provided by CC.

19. GTP Gross Telephone property HB-83

Note that B.C. Tel has been revising its value of GTP so that HB figures sometimes do not agree with financial statement figures

20.	AD	Accumulated depreciation	HB-84
21.	INV	Investments	HB-86
22	CTI	Cash and Temporary investments	HB-87
23.	ACCREV	Accounts receivable	HB-88
24	OTHCA	Other current assets	HB-89
25.	DCH	Deferred Charges	HB-91
26.	PUC	Plant under construction	

a) Bell SC

b) B.C. Tel SC

27. ADJP Adjustments to gross telephone property

This item can be calculated only if the details of the plant accounts are known

a) Bell

ADJP = Cost of plant acquired with traffic
+ plant acquired minus plant sold
+ increase in plant acquisition adjustment
- amortization of plant acquisition adjustment.

This can be calculated up to 1972

b) BC Tel - no information is available and ADJP is assumed to be zero.

28. ADJR adjustments to accumulated depreciation

This item can be calculated only if the details of the plant accounts are known

a) Bell

ADJR = accumulated depreciation on plant acquired with traffic

Can be calculated up to 1972

b) B.C. Tel

No information is available and ADJR is assumed to be zero.

29.	PR	Preferred stock	HB-93
30.	EQ	Common stock	HB-94
31	ADVGV	Advances by government	HB-97
32.	LTD	Long term debt	HB-98
33.	NOTES	Short term notes	HB-99
34.	CL	current liabilities	HB-102

35.	DFTAX	Deferred taxes	HB-103
36	PRDTX	prior year's deferred taxes	HB-120

a) Bell HB-120

b) B.C. Tel

Figure in handbook is zero, but PRDTX can be calculated as follows -
PRDTX = DFTAX-CURDTX-DFTAXO

where

CURDTX is figure taken from the "Statement of Sources of Funds"
For example, in 1972
PRDTX = 81852-15078-64253=2521

(Note: perhaps people preparing handbook should be informed of this).

37.	PDIVI	Dividends on preferred stock	HB-107
38.	DIVI	Dividends on common shares	HB-109
39.	TRANGV	Transfers to government	HB-110
40.	SIE	Share issue expenses	HB-112
41.	OTHADJ	Adjustments to retained earnings	HB-113

42. ADJD Adjustments to deferred taxes provided by CC

43. DCRO Other deferred credits HB-104

a) Bell - includes employees savings plan and Ontario tax credit.

44. DPNONC Depreciation and other non-cash charges HB-121

This item is included in BEAD because it can sometimes be used to force a value on DPRTVE See as a cross reference item 50-ADJA and equation 131 in section 2.

a) Bell

This item consists of

1) Depreciation
2) + DPRTVE
3) - IDC
4) + Amortization of bond issue expenses
5) + Amortization of plant acquisition adjustment
6) + Amortization of other deferred charges
7) - Amortization of other deferred credits (Ontario investment credit)
8) + Some other charges

ADJA = 4) + 5) +6) -7) +8)

Hence if ADJA can be roughly determined, we can deduce a value for DPRTVE

B) B C Tel

Much less is known, but the items are probably much the same.
However, I do not believe IDC is included in this item for BC Tel.
Therefore, in the BEAD data
DPNONC= figure in handbook
- IDC

In 1972
DPNONC = 44140
- 1666 = 42474

45 REPL Repayment of long-term debt and notes HB-124

46 GCE Gross construction expenditures HB-131

b) BC Tel. The handbook figure is not the same as the financial statement figure.
The handbook figure is equal to the financial statement figure plus interest during construction
I think this treatment by CC should be changed.

47 CNRF Charges to construction not requiring funds HB-132

a) Bell
It is <u>assumed</u> that

CNRF = Depreciation (T&V) capitalized
+ interest during construction
+ net salvage value

b) BC Tel
This figure was contrived for the handbook and is
CNRF = interest during construction
+ net salvage value

The figure in the handbook for CNRF is not ready for use as BEAD data. To use equation 132, CNRF must include DPRTVC. An independent estimate of DPRTVC must be obtained. This is done as follows:
From the accumulated depreciation account we find
DPRTVE + DPRTVC = AD + RET - DEPN - NSV - ADO
An estimate of DPRTVE is obtained from equation 131, and DPRTVC is a residual
For 1972
DPRTVE + DPRTVC = 219229 + 25561 - 43326
-2857 - 197190 = 1417
From equation 131, DPRTVE = 427 and this implies DPRTVC = 990
CNRF is then stated in BEAD AS
1) figure in bandbook = 4523
+ 2) DPRTVC + 990
5513

48 CCARAT Capital Cost allowance rate

Provided by CC

49 PHI

Provided by CC

50 ADJA Adjustments to calculate DPRTVE
See item 40 and section 2 equation 30

a) Bell
ADJA is defined in item 40, and

DPNONC = DEPN - IDC + DPRTVE + ADJA
If we can estimate ADJA, a figure for DPRTVE can be found.
ADJA can be difficult to derive, but we have done so up to 1972.

b) BC Tel
ADJA is assumed to be amortization of bond issue expenses only.

51 ADJU Adjustments to undepreciated capital cost provided by CC.

52 CCL Debt issue expenses as percentage of gross proceeds of issue

1) debt issue expenses can be found by taking
change in deferred charges debt (balance sheet)
\+ amortization of debt issue expenses (income statement)

2) gross proceeds of issue is found by taking change in long term debt (balance sheet).

Divide 1) by 2).

53 NSV Net salvage value

a) Bell

This figure is usually derived from the accumulated depreciation account.

AD = ADO + DEPN + DPRTVE + DPRTVC + ADJR - RET + NSV

If an independent estimate of DPRTVC can be made, NSV can be derived as a residual.
If this is not possible, NSV will have to be estimated directly.

B) BC Tel

This figure is found in the funds statement.

54 LAND Land

1) Bell - from SC

2) BC Tel from SC.

55 GETV General equipment tools & vehicles

1) Bell - From SC

a) BC Tel From SC

56 ADJB Regulatory adjustments to rate base.
Provided by CC.

57 OMEGA Ratio of regulatory working capital to operating expense.

Provided by CC.

58 DEPDIFF -

59. LO
60. GTPO
61. PRO
62. EQO
63. DFTAXO
64. REO
65. INVO
66. ADO
67. DCHO
68. ODCRO
69. OCAO
70. CLO
71. CTIO
72. LANDO
73. GETVO
74. PUCO

Items 59 to 74 are the values at the beginning of the year of certain BEAD variables or NON-BEAD variables.

C-7 NON-BEAD GENERATION SYSTEM

101 OPRV = LOCAL + TOLL + MISOPRV - UNCOLL

102 OPXP = MAINT + TRAF + CAM + OTHOPXP + OTHTX

103 OTHINC = TOTOTHINC - IDC

104 GAMMA = PUC ÷ GCE

105 IC = 2 * IDC ÷ (GAMMA * GCE + PUCO)

106 L = LTD + NOTES + ADVGV

107 NEWDEB = L - LO + REPL

108 IO = [DSC - 1/2 * IN * NEWDEB] ÷ [LO - 1/2 * REPL]

109 T = INCTAX ÷ [OPRV - OPXP - DEPN - DSC + BETA * OTHINC]

110 NETINC = OPRV - OPXP - DEPN + OTHINC + IDC - DSC - INCTAX

111 RET = GCE - GTP + GTPO + ADJP

112 NTP = GTP - AD

113 OCA = ACCREV + OTHCA

114 RE = NETINC - PDIVI - DIVI - TRANGV - SIE + OTHADJ + REO

115 SC = PR + EQ + RE

116 TC = SC + L

117 DELPR = PR - PRO

118 DELEQ = EQ - EQO

119 CE = SIE ÷ (DELEQ + DELPR)

120 CP = CE

121 CURDTX = DFTAX - DFTAXO - PRDTX

122 DELDTX = CURDT + PRDTX

123 DELCTI = CTI - CTIO

124 DELODCR = ODCR - ODCRO

125 DELCL = CL - CLO

126 DELOCA = OCA - OCAO

127 DELDCH = DCH - DCHO

128 DELINV = INV - INVO

129 DEPRAT = DEPN ÷ [GTPO + 1/2 [(1-GAMMA) *GCE - PUCO - RET - LAND - LANDO - GETVO - GETV + ADJP]]

130 RHO = [PDIVI - 1/2*RHN * DELPR] ÷ PRO

131 DPRTVE = IDC + DEPNONC - DEPN - ADJA

132 DPRTVC = CNRF - IDC - NSV

133 MIS = DELDCH - DELDCRO - NSV - OTHADJ + SIE + ADJP - ADJR

134 DELWK = DELCTI + DELOCA - DELCL

135 CCA = (1-PHI) (DURDTX ÷ T) + DEPN + DPRTVE

136 UCC = CCA ÷ CCARAT

137 OTHEXP = PHI * (CURDTX ÷ T) - IDC - DPRTVC

138 UCCO = UCC - GCE + ADJU + PHI (CURDTX ÷ T)

139 TOTASS = NTP + INV + CTI + OCA + DCH

140 TOTLIA = TC + CL + DFTAX + ODCR

141 TOTSOR = NETINC - PDIVI - DIVI - TRANGV + DELDTX + DEPN + DEPRTVE - IDC + NEWDEB - REPL + DELPR + DELEQ

142 TOTUSE = GCE - IDC - DPRTVC + MISC + DELWK + DELINV

143 NIA=NETINC - (RHO*PRO + 1/2*RHON*DELPR)