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Priority Assignment in a Network of Computers

by
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Terrestrial Planning Branch
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Introduction

This report treats the problem of allocation of priorities to the different types of message traffic that can flow in a computer-communication network, for eg. CANUNET [1]. The assumption made, is that messages flow in such a network between two host computers, that are interfaced to the network via Network Control Units (NCU) in the same way as they do, for example in the ARPA network [2].

Messages which flow between hosts are broken up into packets and a NCU can accept up to eight of these packets, each of which is routed to its destination NCU and corresponding over the same or different communication lines. Each time a NCU sends a packet to another NCU, the receiving NCU keeps a copy of it, until it receives an acknowledgement (ACK) from the transmitting NCU. If an ACK is not received within a given time period, the transmitting NCU repeats the transmission of the same packet. The absence of an ACK can be caused by detected error or by lack of buffer space in the receiving NCU. Giving priority to the ACK message traffic over the packet-message traffic empties the NCU buffers quickly and thus reduces traffic congestion and blocking.

Single Channel Queue - Two Priority Classes

Consider now two priority classes, (1 and 2). Messages of class 2 have priority over messages of class 1. A non-preemptive priority system is assumed, ie. if a message of class 2 arrives when a message of class 1 is being served, the class 1 message is not pre-empted.

For example, in a store-and-forward network such as ARPA or CANUNET, the acknowledgement messages would be considered as class 2 messages. Also within each priority class messages are served in a first in first out (FIFO) basis, by a single server.

Let λ_1 and λ_2 be the rates of class 1 and class 2 messages respectively, and $\lambda = \lambda_1 + \lambda_2$ be overall arrival rate. The service times for these messages are assumed to be an independently distributed, random variable; with distribution function $B_i(x)$, $i=1$ or 2 for class 1 or 2 messages respectively. The overall service time distribution $B(x)$ is therefore

$$B(x) = \frac{\lambda_1 B_1(x) + \lambda_2 B_2(x)}{\lambda}$$

Moments of Service Time

Denoting, the first and second moments of the distribution function $B_i(x)$ as

$$\frac{1}{\mu_i} = \int_{\infty}^{\infty} x \, dB_i(x)$$

$$\frac{1}{\eta_i} = \int_{\infty}^{\infty} x^2 dB_i(x)$$

The first and second moments of the distribution function B (x) are therefore;

$$\frac{1}{\mu} = \frac{\lambda_1/\mu_1 + \lambda_2/\mu_2}{\lambda}$$

$$\frac{1}{\eta} = \frac{\lambda_1/\eta_1 + \lambda_2/\eta_2}{\lambda}$$

and the average queuing time for class "i" messages has been found [3] to be

$$W_i = \frac{E[T_o]}{(1-\sigma_{i+1})\ (1-\sigma_i)} \quad , i = 1, 2 \qquad \ldots\ldots\ 1$$

where

$$\sigma_i = \sum_{i=1}^{2} \frac{\lambda_i}{\mu_i} = \sum_{i}^{2} \rho_i$$

therefore

$$\sigma_1 = \frac{\lambda_1}{\mu_1} + \frac{\lambda_2}{\mu_2} = \rho_1 + \rho_2 = \rho$$

$$\sigma_2 = \frac{\lambda_2}{\mu_2} = \rho_2 = \rho-\rho_1$$

The expected time to finish service on the message already in service when a new one comes in is $E(T_o)$ and is given by:

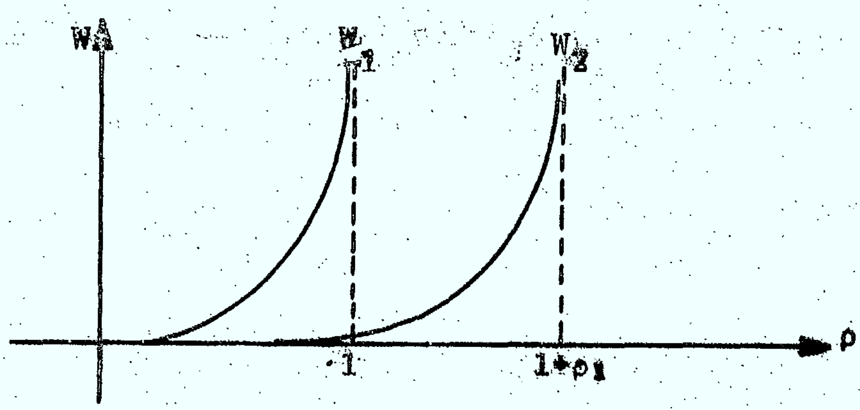
$$E[T_o] = \frac{\lambda}{2} \int_0^\infty x^2 dB(x) = \frac{\lambda}{2\eta}$$

This quantity represents the expected time to finish service on the message already in service when a new one comes in. By equation (1) the average queuing time for class 1 and class 2 messages, is;

$$W_1 = \frac{E[T_O]}{(1-\rho)\ (1-\rho+\rho_1)} \qquad \ldots\ldots\ldots\ldots\ 2$$

$$W_2 = \frac{E[T_O]}{(1-\rho+\rho_1)} \qquad \ldots\ldots\ldots\ldots\ 3$$

and using equations (2) and (3), the following curves are obtained;



The overall average queuing time is therefore given by;

$$W = \alpha W_1 + (1-\alpha)W_2 \qquad \ldots\ldots\ldots\ldots\ 4$$

where;

$$\alpha = \frac{\lambda_1}{\lambda}$$

The parameter α represents the percentage of the total number of messages that are of class 1. Using equations (2) and (3), equation (4) can be rewritten as:

$$W = E[T_O]\ \frac{1-\rho+\alpha\rho}{(1-\rho)\ (1-\rho+\rho_1)}$$

$$= E[T_o]\left(\frac{1}{1-\rho+\rho_1} + \frac{\alpha\rho}{(1-\rho)(1-\rho+\rho_1)}\right) \qquad \text{........ } 5$$

Queuing Time for Systems With and Without Priorities

It is interesting to compare the value of the average queuing time under priorities to the value of the average queuing time obtained assuming that messages are served under a first in first out basis (FIFO) with no priorities.

In a no priority situation, it is well known, that using the POLLACZEK KHINCHIN equation, the value of the average queuing time W for the FIFO case is

$$W = \frac{E[T_o]}{1-\rho} \qquad \text{........................... } 6$$

where

$$E[T_o] = \frac{\rho}{\mu} \quad ,$$

and since $\rho = \frac{\lambda}{\mu}$, (6) can be written as

$$E[T_o] = \frac{\lambda}{\mu^2}$$

Thus, for any particular value of α, the average queuing times as given by equations (5) and (6) are readily compared.

Optimum Assignment of Priorities

This section, now presents the solution (3) to the problem of finding the optimum division of a flow of messages in a network into two priority classes.

For simplicity, assume that the probability density function $b(x)$ of the service times is exponential,

$$b(x) = \mu e^{-\mu x}$$

Here $1/\mu$ is the average length of a message. This density function can be plotted as shown in Figure 2.

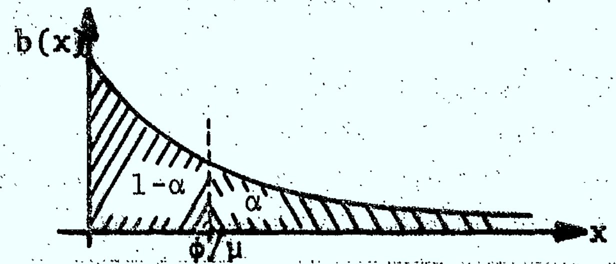


Figure 2

Where the parameter α denotes the percentage of total number of messages having a length greater than ϕ/μ.

For a Poisson arrival stream, the value of the overall average queuing time W as given by equation (4) is

$$W = \alpha W_1 + (1-\alpha) W_2$$

$$= E[T_o] \frac{1-\rho + \alpha\rho}{(1-\rho)\ (1-\rho+\rho_1)}$$

and the parameter ρ_1 can be related to ϕ and μ as follows

From Figure 2, it is obvious that the relationship between α and ϕ is

$$\int_{0}^{\phi/\mu} b(x)dx = 1-\alpha$$

$$= 1-e^{-\phi}$$

and therefore

$$\alpha = e^{-\phi}$$

Furthermore, since $\rho_1 = \lambda_1/\mu_1$, it follows that

$$\rho_1 = \lambda \int_{\phi/\mu}^{\infty} x dB(x)$$

$$= \lambda \int_{\phi/\mu}^{\infty} x\mu e^{-\mu x} dx$$

and integrating

$$\rho_1 = \rho\phi\alpha + \rho\alpha \qquad \text{.......... 7}$$

Substituting equation (7) into equation (5) the value of the overall average queuing time obtained as a function of α and ϕ is

$$W = \frac{E[T_0]\ (1-\rho + \alpha\rho)}{(1-\rho)\ (1-\rho + \rho\phi\alpha + \rho\alpha)} \qquad \text{.......... 8}$$

Now by drawing on the same graph, equations (6) and (8) the following curves are obtained for a given value of ρ.

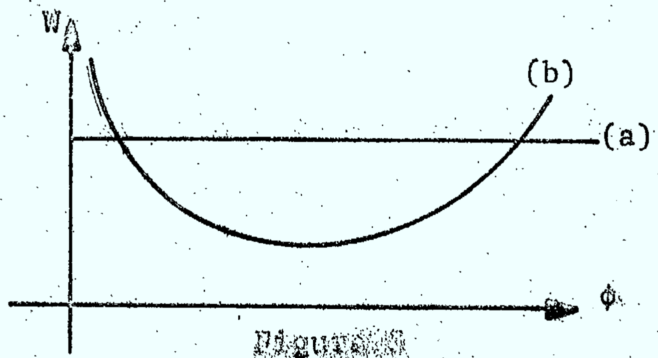


Figure 3

Curve (a) represents the average queuing time as given by equation (6) and curve (b) average queuing time as given by equation (8)

It is therefore clear from Figure 3, that an optimum value of ϕ can be found that minimizes the delay.

This value can be proved to be given by: [5]

$$\frac{1}{\rho} = 1 + \frac{e^{-\phi}}{\phi - 1} \qquad \text{.........} \quad 9$$

Thus it is possible by dynamic assignment of priorities to obtain an average queuing delay which is less than the average queuing delay with no priority.

In a typical problem one is given the value of λ and μ. With those values, ρ and ϕ are computed and priority is assigned to those messages whose length is less than ϕ/μ.

Equation (8) which was derived for the case of a single channel can be easily extended to the case of a network of channels.

If for the i^{th} channel, W as given by equation (8) is represented by W_i, the total average queuing time for a network of M channels is given by:

$$W_t = \sum_{i=1}^{M} \frac{\lambda_i}{\gamma} W_i \qquad \text{.........} \quad 10$$

where

λ_i = average number of messages on the i^{th} channel

γ = total input traffic to the network.

In a network of computers, the i^{th} node control unit (NCU) would compute the value of ρ_i at a given instant and find the value of ϕ which minimizes W_i. The optimum value of ϕ would be different for each channel. In this fashion the total average queuing time W_t would be much less than the total average queuing time obtained when priorities are not considered.

Example

The preceeding equations will now be applied to the case of the three node network shown in Figure 4. The speed of the lines and average message length are chosen to be respectively 50 kb/s and 400 bits. For simplicity, the traffic matrix was assumed to be symmetric in order to get exactly the same line utilization factors for the three branches.

For several levels of traffic the following matrices were computed;

a) Average queuing delay matrix with no priorities
b) " " " " " priorities
c) " " " " for class 1 messages
d) " " " " " " 2 "

Furthermore, the ratio of the total average queuing time under no priorities, over the total average queuing time with priorities, and the upper bound of the length of high priority messages, were calculated as a function of the line utilization factor.

The results obtained (Table 1) were used to plot several graphs (Figures 5,6,7,8).

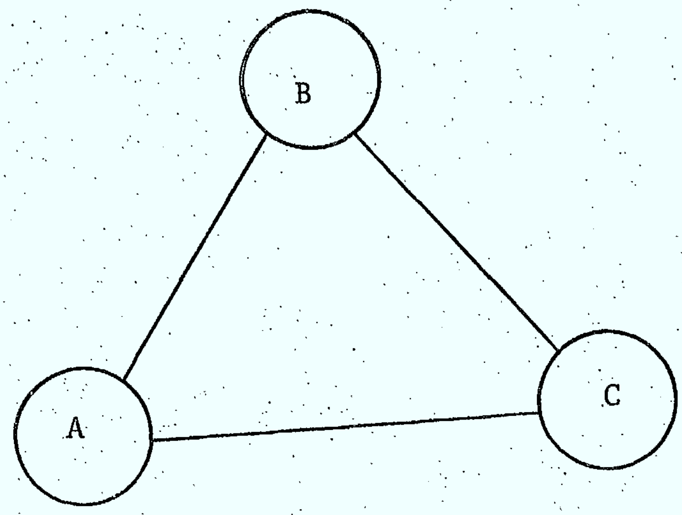


Figure 4

Three Node Network

Line Utilization	TAQ_{NP} (sec)	TAQ_p (sec)	$A.Q_1$ (sec)	$A.Q_2$ (sec)	$\frac{TAQ_p}{TAQ_{NP}}$	ϕ/μ (bits)
.994	1.2718	.3198	13.076	0.082	.2514	1600
.987	.6320	.1823	4.157	0.052	.2885	1380
.979	.3760	.1211	1.864	0.039	.3220	1240
.958	.1840	.0697	.637	.027	.3787	1060
.917	.0880	.0398	.215	.018	.4528	880
.875	.0560	.0284	.114	.014	.5068	780
.833	.0400	.0220	.072	.012	.5511	720
.750	.0240	.0150	.037	.009	.6236	640
.583	.0112	.0082	.015	.006	.7356	540
.417	.0057	.0047	.007	.004	.8248	480
.333	.0040	.0035	.004	.003	.8642	460
.208	.0021	.0019	.002	.002	.9186	440
.083	.0007	.0007	.001	.001	.9686	420

TABLE 1

TAQ_{NP} = Total Average Queueing Time (No Priorities)

TAQ_p = " " " " (Priorities)

$A.Q_1$ = Average Queueing Time for class 1 messages

$A.Q_2$ = " " " " class 2 messages

Figure 5

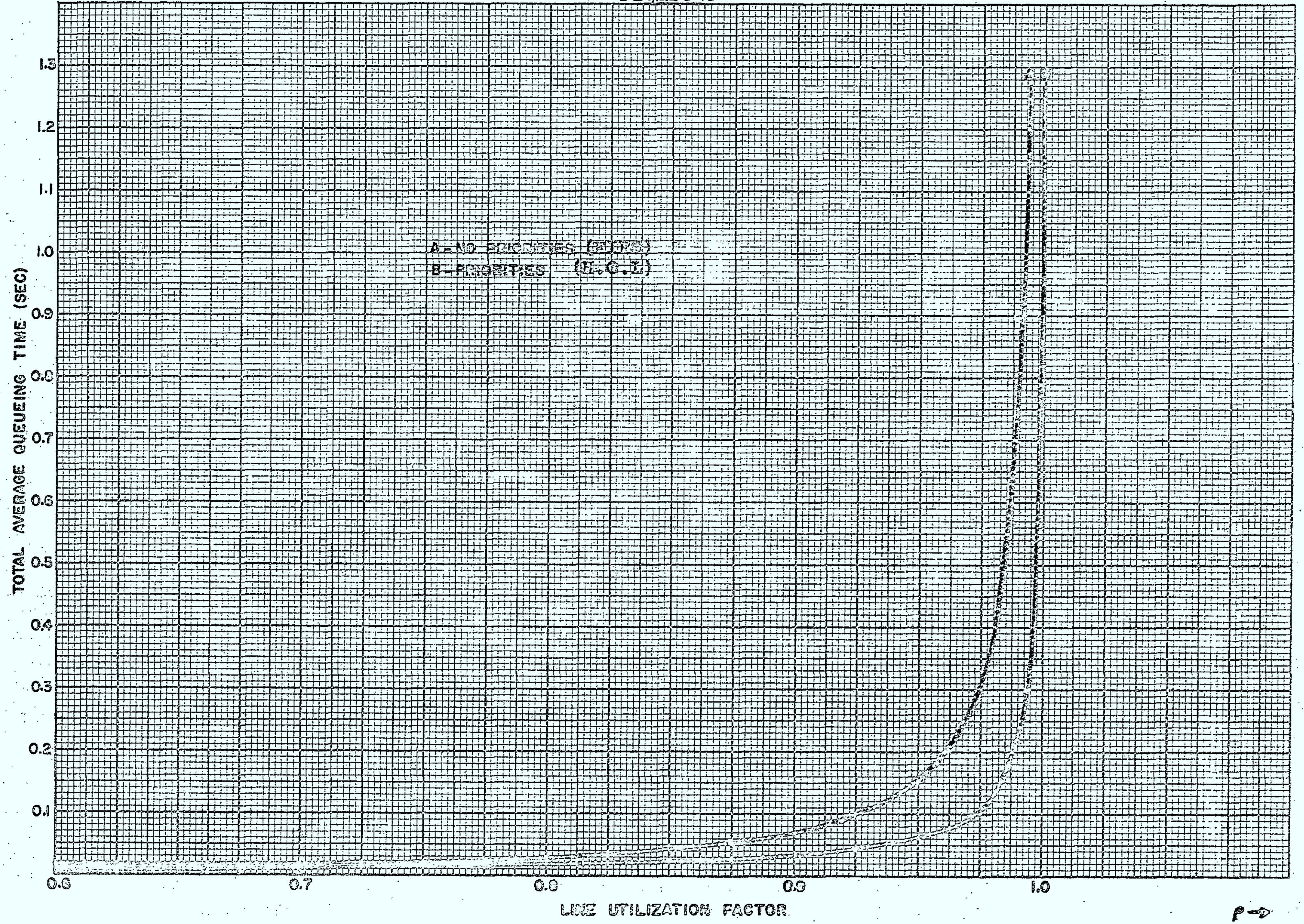

TOTAL AVERAGE QUEUEING TIME (SEC)
1.3
1.2
1.1
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
A-NO
B-PRIORITIES
0.6
0.7
0.8
0.9
1.0
LINE UTILIZATION FACTOR
ρ→

Figure 6

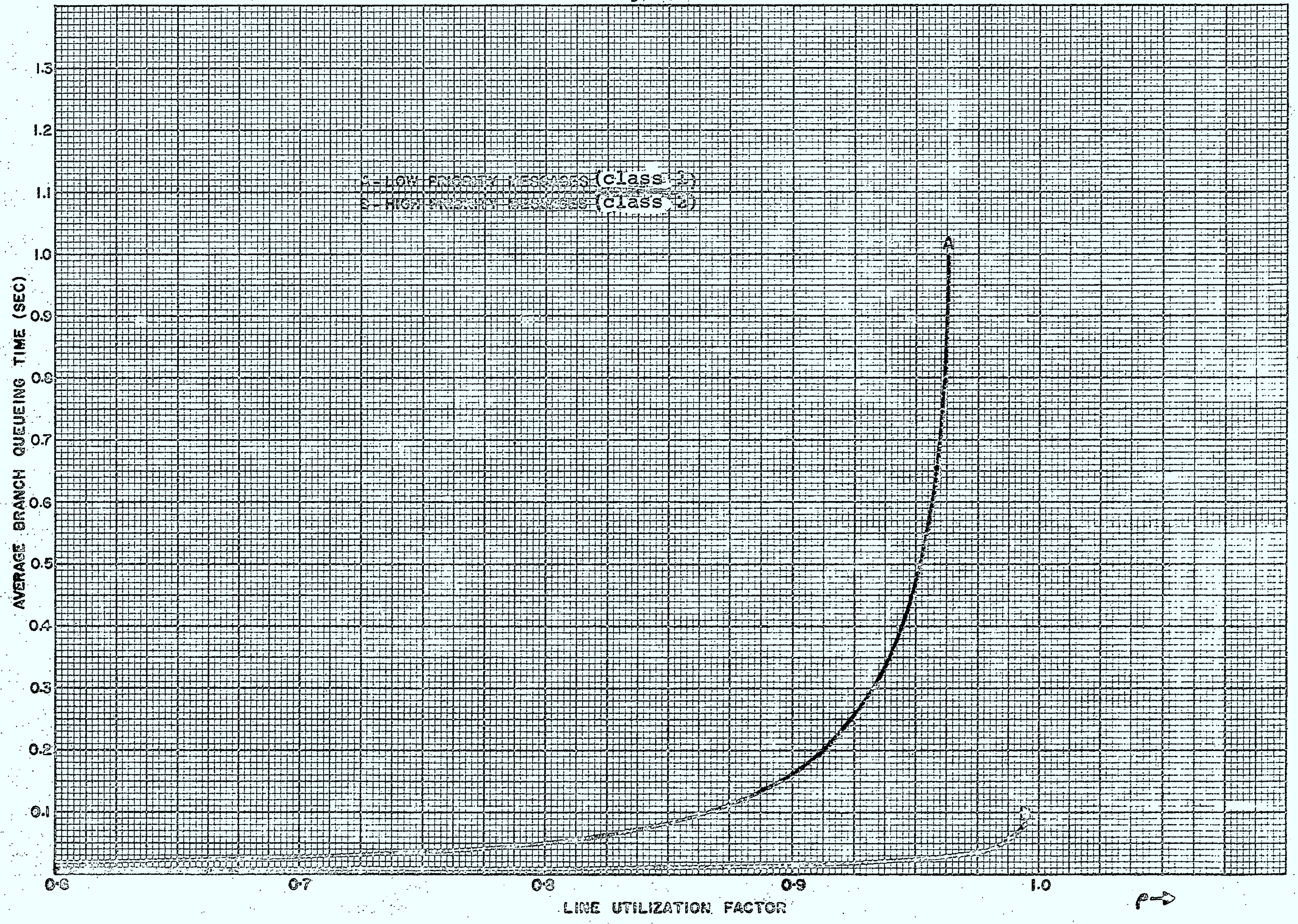
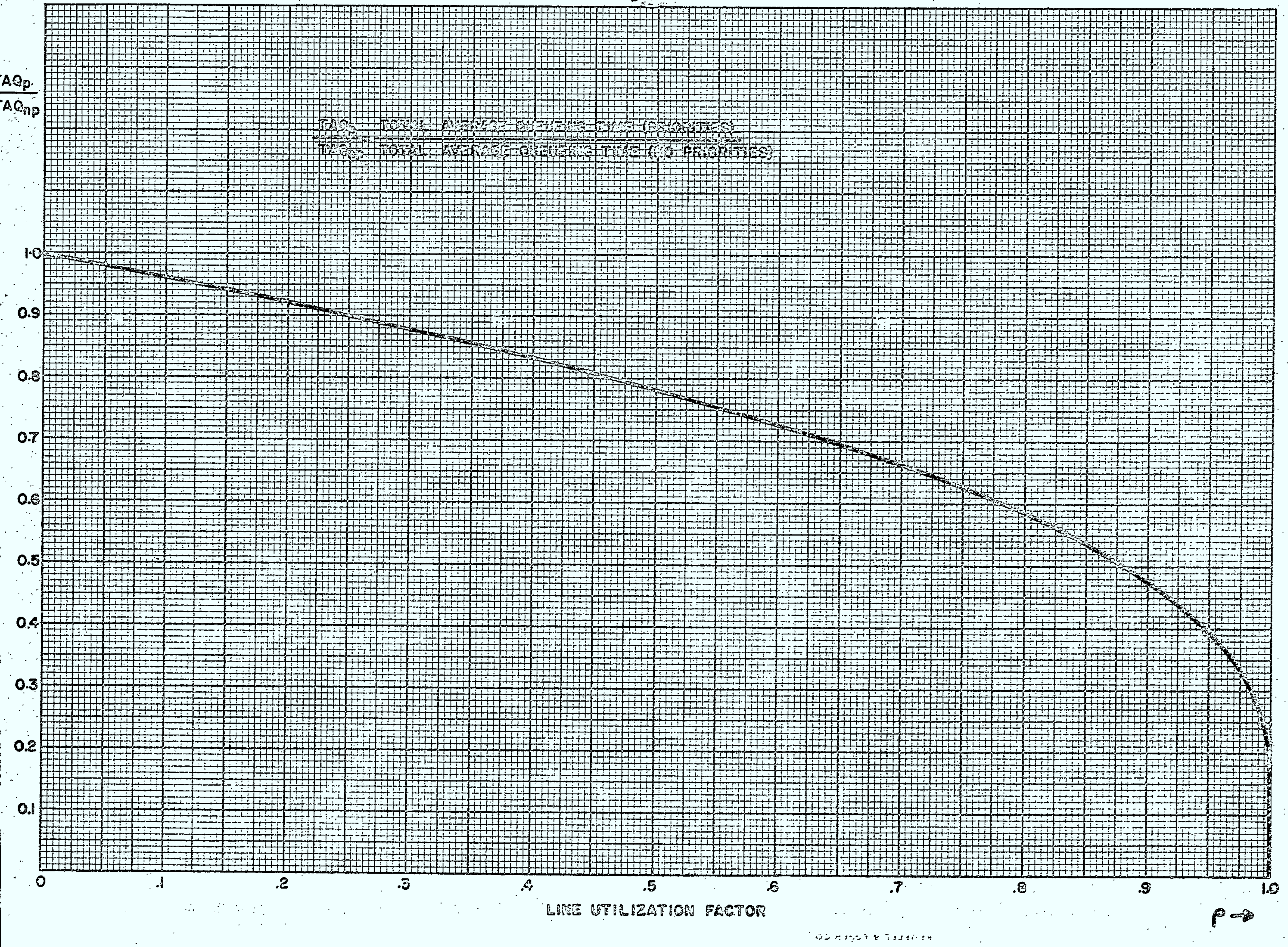
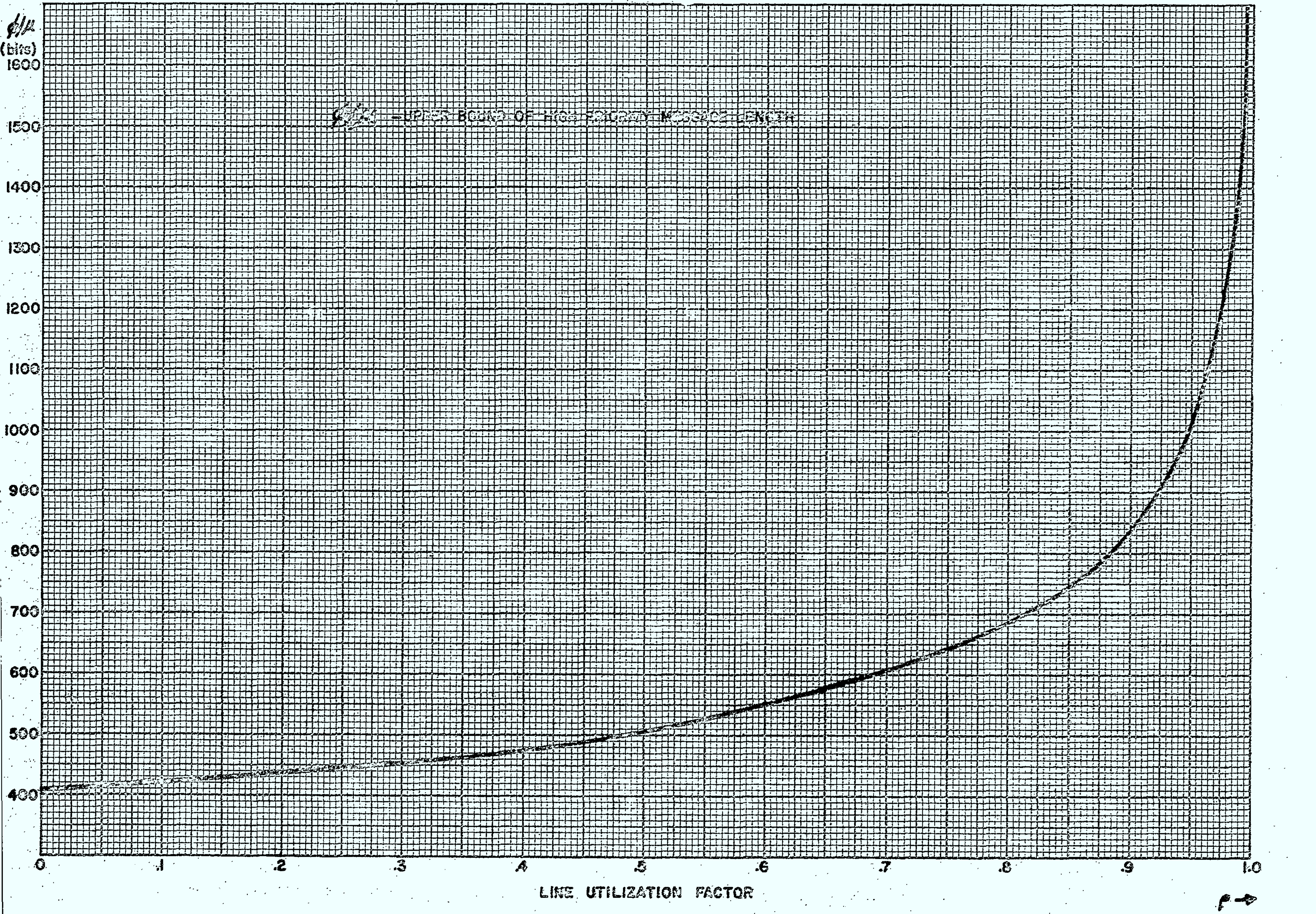

LOW (class
(class 2)
A
AVERAGE BRANCH QUEUEING TIME (SEC)
1.3
1.2
1.1
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.6
0.7
0.8
0.9
1.0
LINE UTILIZATION FACTOR
ρ

Figure 7



TAQp / TAQnp
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
.1
.2
.3
.4
.5
.6
.7
.8
.9
1.0
LINE UTILIZATION FACTOR
ρ →

Figure 8



(bits)
1600
1500
1400
1300
1200
1100
1000
900
800
700
600
500
400
—UPPER BOUND OF HIGH PRIORITY MESSAGE LENGTH
0
.1
.2
.3
.4
.5
.6
.7
.8
.9
1.0
LINE UTILIZATION FACTOR

Conclusion:

It has been shown and proven that the introduction of the type of priorities described in a network of computers is desirable in particular for line loadings exceeding 50%. Calculations made for the case of two priority classes, proved that the mean queuing time when using a non-preemptive head of the line priority scheme can be up to 25% less than the mean queuing time without priorities.

It can be shown that further gains are possible if more than two priority classes are introduced but the division of traffic into two priority classes seems reasonable for the type of traffic flowing in a store-and-forward computer network.

Finally, it should be noted that the software complexity of the node control units under an assignment of priorities could be increased substantially.

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