DYNAMIC ASSIGNMENT OF PRIORITIES IN A COMPUTER NETWORK

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J.A. SCHWARZ DASILVA

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Introduction

This paper treats the problem of assigning priorities to improve the traffic handling characteristics of store and forward computer networks, for example, of the CANUNET[1] type.

In such networks message traffic flows, between host computers that are connected to the network via node control units. Messages flowing between hosts are broken up into packets. These packets are routed by the node control units. Furthermore, the operation of the network is such that each time a node control unit sends a packet to another node control unit, it stores a copy. This stored copy is erased only after an acknowledgement has been received that the transmission is error free. In this paper, a scheme is developed for giving priority to the short messages in particular the acknowledgements. This results in a considerable improvement of the traffic handling characteristics of the network.

In section I, a review is given of the results for a single channel queue under a non-preemptive head-of-the line priority service discipline with two priority levels. The moments of the service time distribution and expressions for the average queueing times for messages in each priority level are presented. The expression for the overall average queueing time is shown to be a function of the percentage of the total number of messages that are of lower priority.

In section II, the solution to the problem of finding the optimum division of a flow of messages into two priority classes is presented. It is shown that the optimal assignment of priorities, that is, the assignment of priorities for which the average queueing delay is minimized, is a function of the traffic intensity.

In Section III, the results obtained in Section II, are applied to a network of channels. By way of an example of a fully connected network of channels, it is shown that a dynamic assignment of priorities offers a considerable advantage in terms of average message delay over the case of a first-in first-out service discipline, especially for high traffic intensities.

The application of a dynamic assignment of priorities to a non-fully connected network is then investigated. The major problem here is that messages travelling between any pair of nodes will on the average use more than one channel on their way to the destination node. In this case, the output of a channel will be the input of another channel and the property of Poisson arrivals which makes the analysis of complex networks easy could be destroyed. It is then shown that the property of Poisson arrivals is still preserved for any assignment of priorities.

In the appendix, the expression for the message interdeparture time distribution is derived and it is shown that the output of the queue is a Poisson process with mean equal to the input Poisson process.

I SINGLE CHANNEL QUEUE - TWO PRIORITY CLASSES

Consider two priority classes, (1 and 2), where messages of class 2 have priority over messages of class 1. The service is assumed to be non-preemptive priority, that is, whenever a message of class 2 arrives when a message of class 1 is being served, the class 1 message is not pre-empted.

In store-and forward networks of the ARPA or CANUNET[1] types, *ACKNOWLEDGEMENT* messages would be considered as class 2 messages. Also within each priority class messages are served in a first-in first-out (FIFO) basis, by a single server.

Let λ_1 and λ_2 be the rates of class 1 and class 2 messages respectively, and $\lambda = \lambda_1 + \lambda_2$ be overall arrival rate. The service time for these messages is assumed to be an independently distributed, random variable; with distribution function $B_i(x), i=1$ or 2 for class 1 or 2 messages respectively. The overall service time distribution $B(x)$ is therefore

$$B(x) = \frac{\lambda_1 B_1(x) + \lambda_2 B_2(x)}{\lambda}$$

MOMENTS OF SERVICE TIME

The moments of the service time are obtained from this distribution as

$$\frac{1}{\mu_i} = \int_0^\infty x \, dB_i(x)$$

$$\frac{1}{\eta_i} = \int_0^\infty x^2 dB_i(x)$$

it follows that the first and second moments of the distribution function B (x) are

$$\frac{1}{\mu} = \frac{\lambda_1/\mu_1 + \lambda_2/\mu_2}{\lambda}$$

$$\frac{1}{\eta} = \frac{\lambda_1/\eta_1 + \lambda_2/\eta_2}{\lambda}$$

and the average queueing time for class "i" messages in the case of Poisson arrivals is[3]

$$W_i = \frac{E[T_o]}{(1-\sigma_{i+1})\ (1-\sigma_i)} \quad , \ i = 1,\ 2 \qquad \ldots\ldots\ 1$$

where

$$\sigma_i = \sum_{j=i}^{2} \frac{\lambda_j}{\mu_j} = \sum_{j=i}^{2} \rho_j$$

therefore

$$\sigma_1 = \frac{\lambda_1}{\mu_1} + \frac{\lambda_2}{\mu_2} = \rho_1 + \rho_2 = \rho$$

$$\sigma_2 = \frac{\lambda_2}{\mu_2} = \rho_2 = \rho - \rho_1$$

$$\sigma_3 = 0$$

The expected time to complete service on a message already in service when a new one arrives is $E(T_o)$, where

$$E[T_o] = \frac{\lambda}{2} \int_o^\infty x^2 dB(x)$$

Using equation (1) the average queueing time for class 1 and class 2 messages, is found to be

$$W_1 = \frac{E[T_0]}{(1-\rho)\ (1-\rho+\rho_1)} \qquad 2$$

$$W_2 = \frac{E[T_0]}{(1-\rho+\rho_1)} \qquad 3$$

and the overall average queueing time per message is

$$W = \alpha W_1 + (1-\alpha)W_2 \qquad 4$$

where

$$\alpha = \frac{\lambda_1}{\lambda}$$

The parameter α represents the percentage of the total number of messages that are of class 1. Using equation (2) and (3), equation (4) can be rewritten as:

$$W = E[T_0]\ \frac{(1-\rho+\alpha\rho)}{(1-\rho)\ (1-\rho+\rho_1)}$$

Queuing Time for Systems with and without Priorities

Equation (5) is an expression for the overall average queuing delay encountered by messages arriving in a Poisson fashion. Here, α of these are low priority messages. It should be noted that no assumptions are made regarding the service time distribution. That is, the service time distribution is arbitrary and general. It is now possible to compare the value of the average queuing delay for a two priority class queuing system with the value of the average queueing time obtained for a system with no priority classes, that is, for systems where messages are served on a first-in first-out basis. For a general service time distribution, using the POLLACZEK-KHINCHIN equation, the value of the average queuing delay W[FIFO] for the first-in first-out (FIFO) case will be given by:

$$W|FIFO| = \frac{E[T_0]}{(1-\rho)} \qquad 6$$

dividing equation (5) by equation (6), it immediately follows that

$$W[PRIORITY] = W[FIFO] \quad \frac{(1-\rho + \alpha\rho)}{(1-\rho + \rho_1)} \qquad 7$$

It can be shown(7) that the factor multiplying the expected queueing delay under the FIFO discipline, on the right hand side of equation (7), is always less or equal than unity. This means that for any arbitrary selection of priorities the expected queueing delay under a priority service discipline is always less than the expected queueing delay under a FIFO service discipline. Again it should be noted that this result is valid for any service time distribution.

II OPTIMUM ASSIGNMENT OF PRIORITIES BASED ON MESSAGE LENGTHS

In this section, the solution is presented to the problem of optimally assigning priorities to a flow of incoming messages with exponentially distributed lengths. This optimum assignment of priorities is shown to be dependent on message lengths and involves minimizing the ratio of the expected queueing delays with and without priorities as given by the following equation:

$$\frac{W[\text{PRIORITIES}]}{W[\text{FIFO}]} = \frac{(1-\rho + \alpha\rho)}{(1-\rho + \rho_1)} \qquad 8$$

To minimize the ratio in equation (8), it is necessary to derive an expression for ρ_1 in terms of ρ and α. This expression is readily obtained as follows: letting the probability density functions of the average message lengths be $b(x)$ where

$$b(x) = \mu e^{-\mu x}, \quad x \geq 0$$

and $1/\mu$ is the average message length; then it is readily apparent that if priorities are to be assigned according to message lengths it is reasonable to give high priority to *short* messages and low priority to *long* messages. Therefore denoting the threshold level on message lengths below which messages are high priority messages by $\frac{\Theta}{\mu}$; recalling that α denotes the fraction of messages that are of *low* priority, that is, of class 1, then the following relationship is readily obtained between α and Θ; namely

$$1-\alpha = \int_0^{\Theta/\mu} b(x)dx = \int_0^{\Theta/\mu} \mu e^{-\mu x}dx =$$

and therefore

$$1-e^{-\Theta} = 1-\alpha$$

or

$$\alpha = e^{-\Theta}$$

Then the relationship between ρ_1, the traffic intensity of the low priority messages and ρ and α, is obtained as follows;

$$\rho_1 = \lambda \int_{\Theta/\mu}^{\infty} x\mu e^{-\mu x} dx$$

$$\rho_1 = \rho\Theta\alpha + \rho\alpha$$

that is

$$\rho_1 = \rho\alpha(\Theta+1) \qquad 9$$

and substituting the value of ρ_1 from equation (9) in equation (8), it follows that:

$$\frac{W[\text{PRIORITIES}]}{W[\text{FIFO}]} = \frac{[1-\rho + \alpha\rho]}{[1-\rho + \alpha\rho(\Theta+1)]} \qquad 10$$

To minimize equation (10) it is necessary to locate the maximum of $\rho\Theta\alpha$. This maximum will yield the optimum value of Θ. This value can be shown to be[2]:

$$\frac{1}{\rho} = 1 + \frac{e^{-\Theta}}{\Theta-1},$$

Therefore it is possible by dynamic assignment of priorities to obtain an average queueing delay which is less than the average queueing delay with no priority.

In a typical problem one is given the value of λ and μ. From these values, ρ and Θ are then computed and priorities are assigned to those messages whose length is less than Θ/μ.

III APPLICATION TO A NETWORK OF CHANNELS

In section II, an expression (equation (10)) for the expected queueing delay under an assignment of priorities based on message lengths was derived. This expression is valid in the case of a single channel system. In this section, these results are generalized and applied to a network of channels, under the assumption that *JACKSON's* theorem[5] and Kleinrock's independence result[6] are true. It is also proven in the appendix that these assumptions are in fact correct.

Consider now a network of M channels where the expected queuing delay W_i on the i^{th} channel is given by W[PRIORITIES] (equation 10). The total average queueing time W_t for the overall network will be

$$W_t = \sum_{i=1}^{M} \frac{\lambda_i}{\gamma} W_i \qquad 11$$

where λ_i represents the average number of messages on the i^{th} channel and γ, the total input traffic to the network from external sources.

To apply the results given in this paper and approve its traffic handling capabilities the networks i^{th} node control unit would compute the value of ρ_i at a given instant and find the corresponding value of θ that minimizes W_i. This optimum value of θ would of course be different for each channel. The networks total average message queueing time W_t (equation 11), using this approach, would be much less than the total average queueing time obtained under the normal first-come first-served service discipline.

EXAMPLE OF A FULLY CONNECTED NETWORK

These results will now be applied to the special case of a fully connected three node network shown in Figure 1. The routing scheme used is *that a message will always use the direct link connecting its origin to its destination.* The line speeds and average message lengths are chosen to be respectively 50 kb/sec. and 400 bits. For simplicity, the traffic matrix is assumed to be uniform and symmetric. This allows the same line utilization factors to be used for three channels.

The ratio of the total average queuing time under a priority service discipline over the total average queuing time under a FIFO service discipline and the upper bound of the length of high priority messages, were calculated as a function of the line utilization factor.

The results obtained (table 1) were used to plot several graphs (figures 2,3,4,5).

Several observations can be drawn from table 1 and figures 2,3,4,5,.

For low line utilizations the difference between the total average queueing time with and without priorities is negligeable and the average queueing times for high and low priority messages is almost the same. Also for low line utilizations the upper bound of high priority is close to the average message length which for this example is 400 bits.

As the line utilization increases it becomes more advantageous to consider a priority scheme. That is, for high line utilization the reduction in queueing time (see figure 4) obtained using a priority scheme is dramatic. For high line utilizations almost all messages are high priority messages (see figure 5), therefore the high average queueing times obtained for the low priority messages are acceptable.

CASE OF A DISTRIBUTED NETWORK OF CHANNELS

The preceding example of a fully connected network had the property that any message only used one channel of the network. Therefore the flow of arrivals at any node control unit would be described by a POISSON process. For a general distributed network this hypothesis is no longer necessarily valid, because the output of a channel constitutes the input of another channel. Therefore the intermessage departure time distribution for an M/M/1 queueing system with a priority service discipline has to be investigated. This work is the subject of another paper(4) and only some results are reproduced here (see Appendix), but the results obtained in the appendix clearly establish that the departure process of an M/M/1 queueing system with a priority service discipline of the type described in this paper, will be a POISSON process.

CONCLUSION

It has been shown that when average queueing delay is the main performance criteria of a computer network, the introduction of a non preemptive head-of-the-line priority service discipline, in particular for high traffic intensities, is desirable.

This study was undertaken considering only two priority classes but if more than two priority classes are introduced further gains will be possible at the expense of increased complexity of the node control units.

APPENDIX

THE OUTPUT OF THE M/M/1 QUEUE UNDER A PRIORITY SERVICE DISCIPLINE

In the appendix, an expression is derived for the interdeparture time distribution D(t) of an M/M/1 queue for the case where messages belong to two different classes or priority levels, and where the service discipline is of the type non preemptive head-of-the-line priority service discipline. This is accomplished by first deriving the stationary joint probability distribution of the number of messages of type 1 and type 2 left behind by the departure of the n^{th} message.

The model that is used here is shown in figure A-1. In this model, messages with exponentially distributed length (with mean $\frac{1}{\mu}$) arrive in a POISSON fashion with mean rate λ at the *decision* box. In this box, a decision, which is a function of the traffic intensity, is made whereby a particular message is sent to the queue #1 or the queue #2, depending on its length. A message joining queue #2 will be a high priority message. Messages in queue #1 will be low priority messages. Service at a given instant will be given to queue #2 if the number of messages in this queue is greater or equal than one, but a message of type 1 cannot be preempted from service if a message of type 2 arrives.

Let $\alpha_n^{(1)}$ and $\alpha_n^{(2)}$ denote the number of low (class 1) and high (class 2) priority messages left behind in the system by the n^{th} departing message. The total number of messages present in the system at time t=0 is assumed to be zero. Under this assumption the following stationary probabilities are obtained[4]:

$$\lim_{n\to\infty} \Pr\left[\alpha_n^{(1)} > 0,\ \alpha_n^{(2)} = 0\right] = \frac{\lambda_1}{\mu}$$

$$\lim_{n\to\infty} \Pr\left[\alpha_n^{(1)} \geq 0,\ \alpha_n^{(2)} > 0\right] = \frac{\lambda_2}{\mu}$$

$$\lim_{n\to\infty} \Pr\left[\alpha_n^{(1)} = 0,\ \alpha_n^{(2)} = 0\right] = 1-\rho$$

The Laplace transform of the interdeparture time distribution will therefore be given by:

$$D^*(s) = (1-\rho)\frac{\lambda}{\lambda+s}\ B^*(s) + \frac{\lambda_1}{\mu} B_1^*(s) + \frac{\lambda_2}{\mu} B_2^*(s) \quad \ldots\ldots\ldots \text{(A-1)}$$

where $B_1^*(s)$ and $B_2^*(s)$ are the Laplace transforms of the service time distributions of type 1 and type 2 messages. $B^*(s)$ is the Laplace transform of the overall service time distribution.

Equation (A-1) can be written in terms of the density function and the inverse transform of $D^*(s)$ is

$$d(t) = (1-\rho)\left[\lambda e^{-\lambda t} \otimes b(t)\right] + \frac{\lambda_2}{\mu} b_2(t) + \frac{\lambda_1}{\mu} b_1(t) \quad \ldots\ldots \text{(A-2)}$$

Performing the convolution product denoted by $\otimes$ and replacing all the density functions by their expressions gives

$$d(t) = \lambda\left[e^{-\lambda t} - e^{-\mu t}\right] + \frac{\lambda_2\mu_2}{\mu} e^{-\mu_2 t} + \frac{\lambda_1\mu_1}{\mu} e^{-\mu_1 t} \quad \ldots\ldots \text{(A-3)}$$

Using the expression of the overall service time distribution $B(x)$ in terms of the service time distributions of the two types of messages $B_1(x)$ and $B_2(x)$ it can easily be shown that equation A-3 can be written as:

$$d(t) = \lambda e^{-\lambda t} \qquad \text{..................(A-4)}$$

which means that the interdeparture time distribution is an exponential distribution with mean equal to the mean of the interarrival time distribution. This result also says that the output process will be a POISSON process regardless of the traffic intensity and the assignment of priorities to the two types of messages.

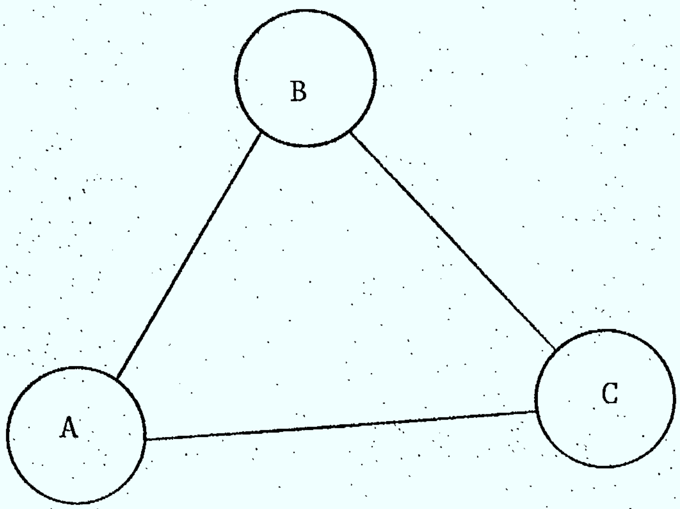


FIGURE 1

Three Node Network

Line Utilization	TAQ_{NP} (sec)	TAQ_p (sec)	$A.Q_1$ (sec)	$A.Q_2$ (sec)	$\frac{TAQ_p}{TAQ_{NP}}$	ϕ/μ (bits)
.994	1.2718	.3198	13.076	.082	.2514	1600
.987	.6320	.1823	4.157	.052	.2885	1380
.979	.3760	.1211	1.864	.039	.3220	1240
.958	.1840	.0697	.637	.027	.3787	1060
.917	.0880	.0398	.215	.018	.4528	880
.875	.0560	.0284	.114	.014	.5068	780
.833	.0400	.0220	.072	.012	.5511	720
.750	.0240	.0150	.037	.009	.6236	640
.583	.0112	.0082	.015	.006	.7356	540
.417	.0057	.0047	.007	.004	.8248	480
.333	.0040	.0035	.004	.003	.8642	460
.208	.0021	.0019	.002	.002	.9186	440
.083	.0007	.0007	.001	.001	.9686	420

TABLE 1

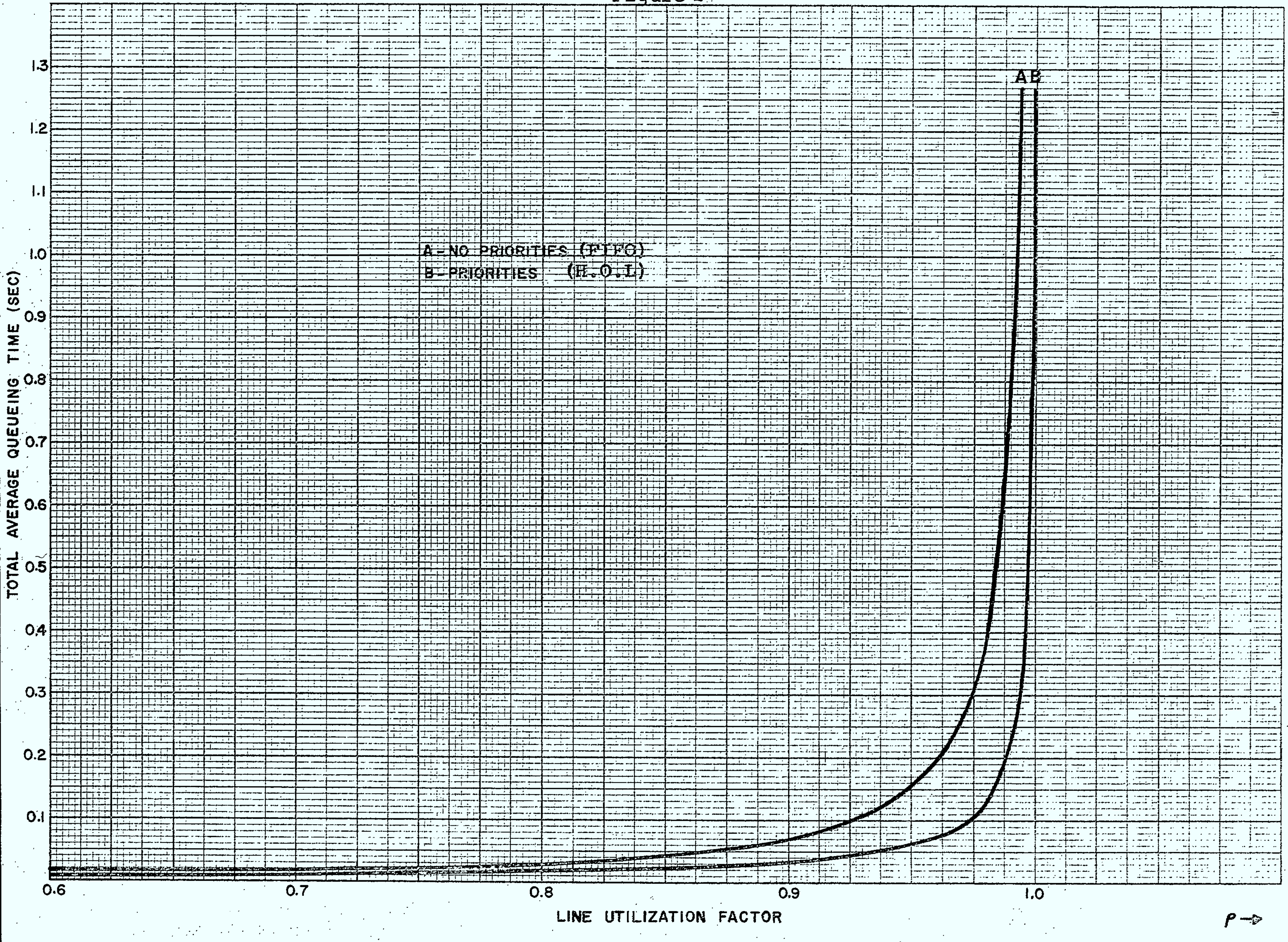
TAQ_{NP} = Total Average Queueing Time (No Priorities)

TAQ_p = " " " " (Priorities)

$A.Q_1$ = Average Queueing Time for class 1 messages

$A.Q_2$ = " " " " class 2 messages

Figure 2



TOTAL AVERAGE QUEUEING TIME (SEC)
LINE UTILIZATION FACTOR
A - NO PRIORITIES (FIFO)
B - PRIORITIES (H.O.L)
A B
1.3
1.2
1.1
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.6
0.7
0.8
0.9
1.0
$\rho \rightarrow$

Figure 3

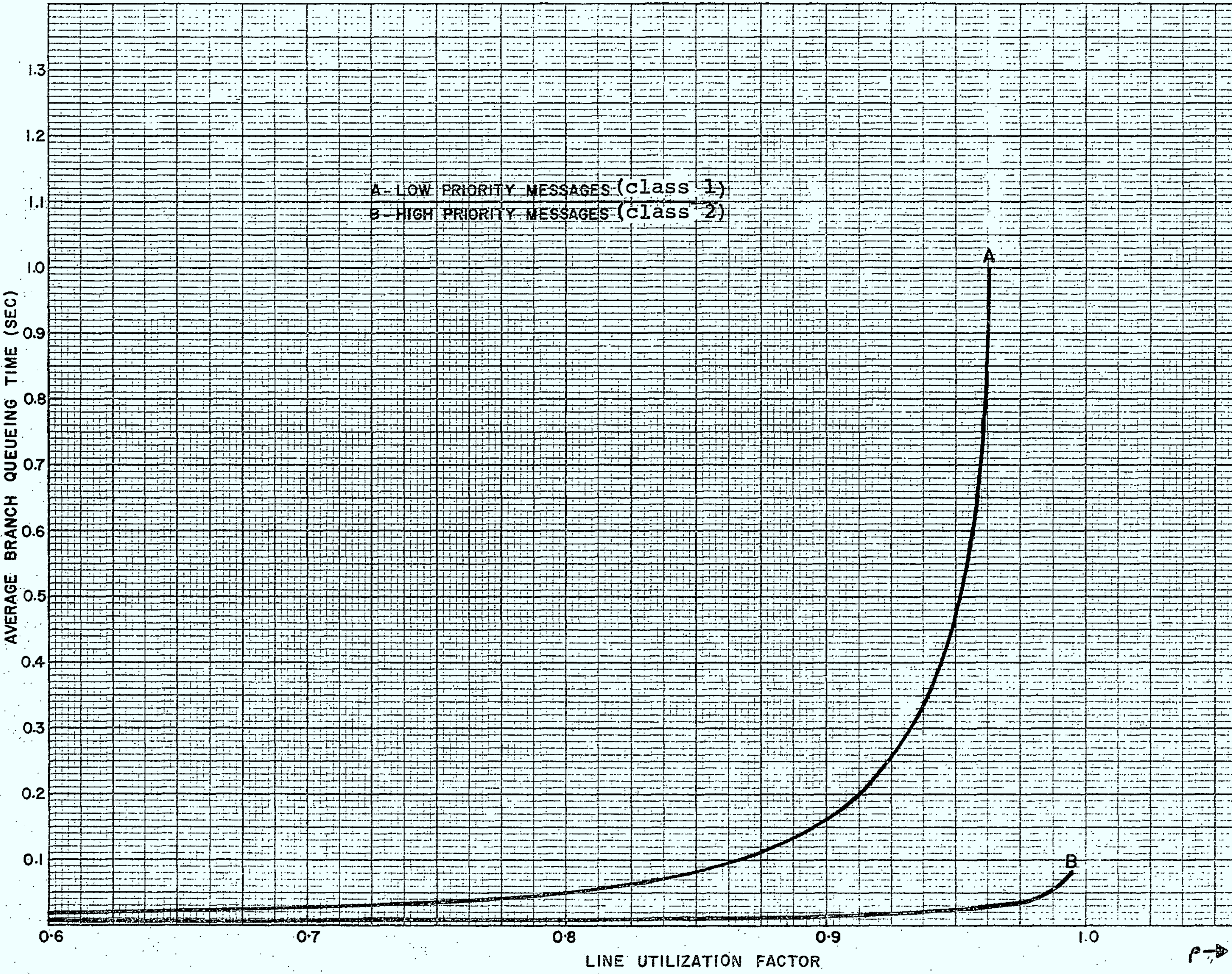
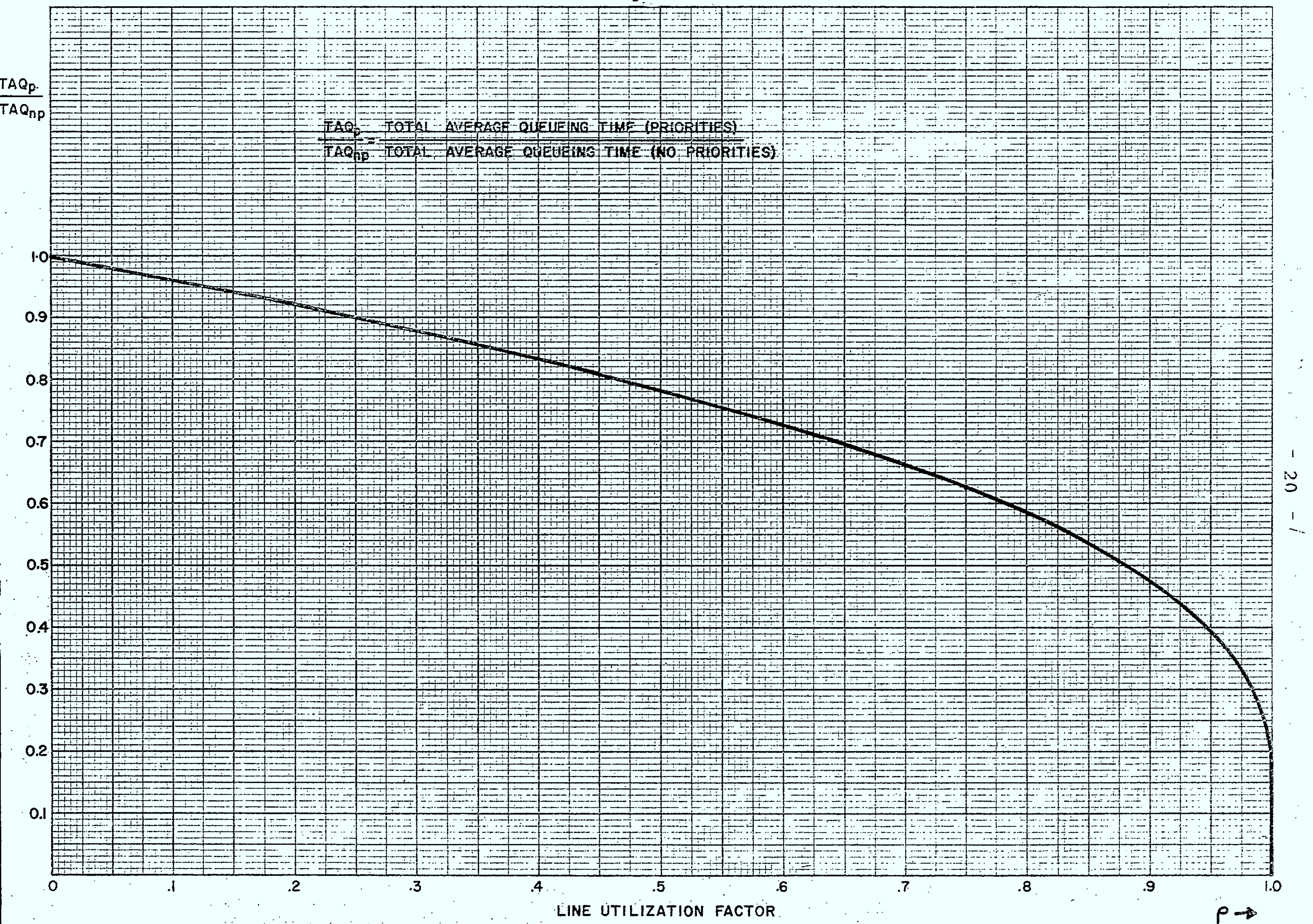
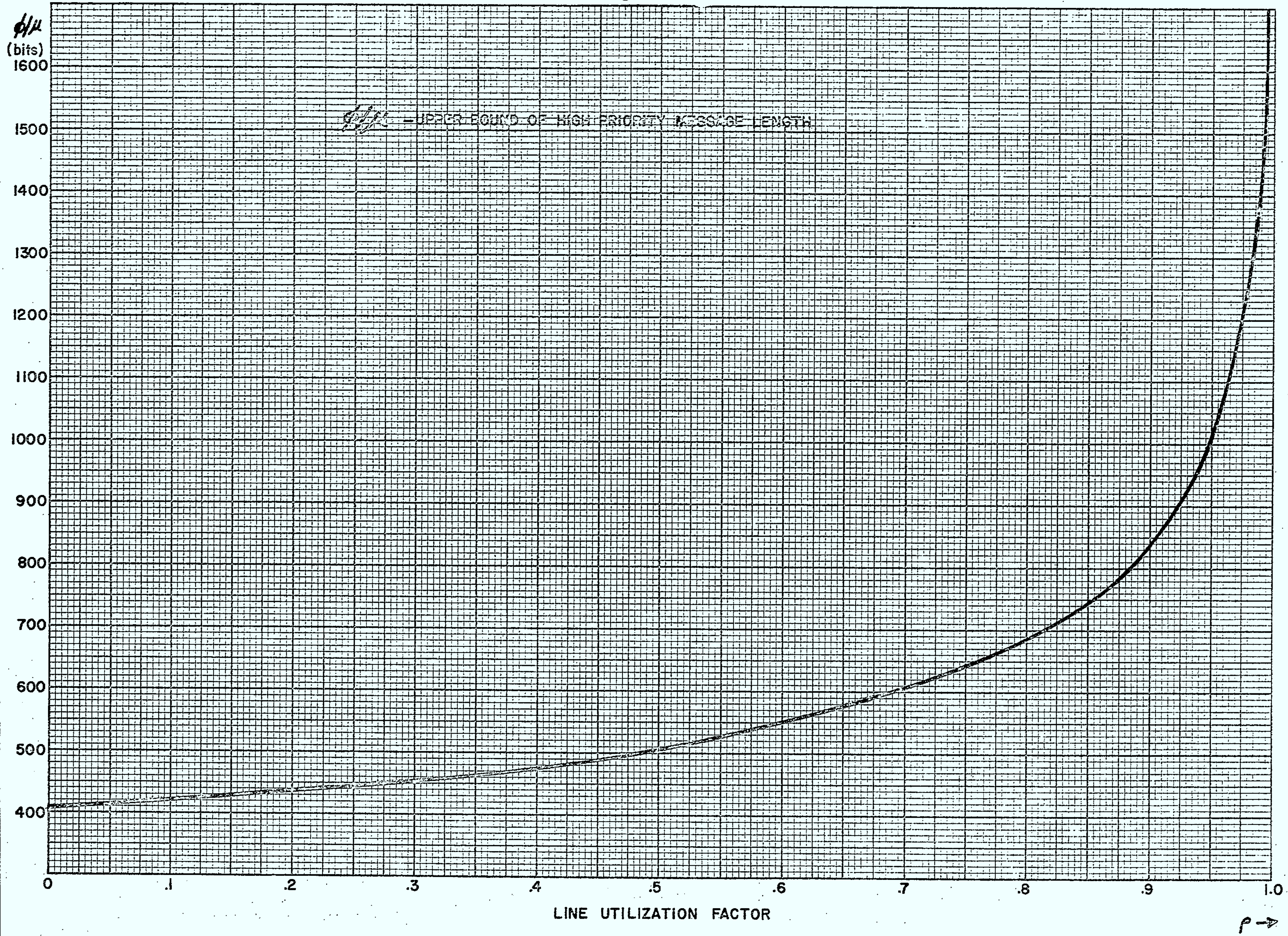

A- LOW PRIORITY MESSAGES (class 1)
B- HIGH PRIORITY MESSAGES (class 2)
A
B
AVERAGE BRANCH QUEUEING TIME (SEC)
1.3
1.2
1.1
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0·6
0·7
0·8
0·9
1.0
LINE UTILIZATION FACTOR
ρ →

Figure 4



TAQp
TAQnp
TAQp = TOTAL AVERAGE QUEUEING TIME (PRIORITIES)
TAQnp = TOTAL AVERAGE QUEUEING TIME (NO PRIORITIES)
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
.1
.2
.3
.4
.5
.6
.7
.8
.9
1.0
LINE UTILIZATION FACTOR
ρ →

Figure 5



φ/μ (bits)
1600
1500
1400
1300
1200
1100
1000
900
800
700
600
500
400
φ/μ — UPPER BOUND OF HIGH PRIORITY MESSAGE LENGTH
0
.1
.2
.3
.4
.5
.6
.7
.8
.9
1.0
LINE UTILIZATION FACTOR
ρ →

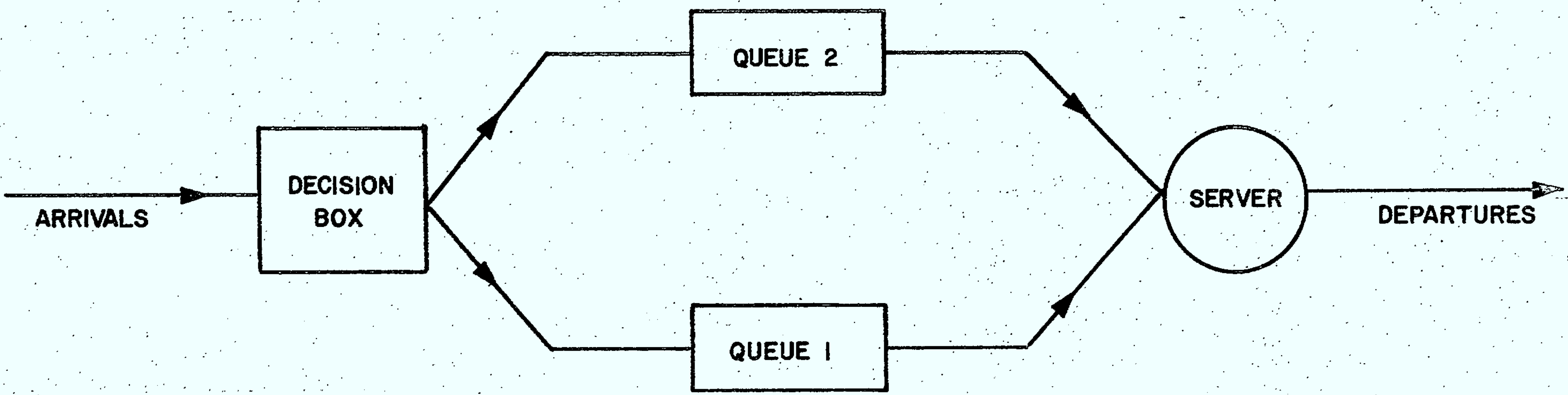


FIGURE A-1

Priority Queue Model

REFERENCES

(1) J. deMercado, R. Guindon, J. daSilva, M. Kadoch — "The Canadian Universities Computer Network: Topological Considerations"; First International Conference on Computer Communications, October 24-26, 1972. Washington, D.C.

(2) D.R. Cox, W.L. Smith — "Queues"; Methuen's monographs on Applied Probability and Statistics; Methuen and Co. London, 1961.

(3) R. Oliver, G. Pestalozzi — "On a Problem of Optimum Priority Classification"; Journal of Soc. Indust. Applied Mathematics; vol. 13, no. 3; pp. 890-901, September, 1965.

(4) J. daSilva, S. Mahmoud — "On the Output of a Priority Service Queue and Its Application to Computer Networks"; to be published.

(5) J.R. Jackson — "Networks of Waiting Lines"; Operations Research, vol. 5, 1957, pp. 518-521.

(6) L. Kleinrock — "Communication Nets: Stochastic Message Flow and Delay"; McGraw-Hill, New York, 1964.

(7) R. W. Conway, W.L. Maxwell, L.W. Miller — "Theory of Scheduling"; Addison-Wesley Publishing Company; Reading, Mass., 1967.