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DE MERCADO, JOHN.

Communications in Canada: mathematical studies of large communication-computer networks.

Notes for a Talk to be presented December 4th, 1970
at
The Centre for System Science -- Rochester University

COMMUNICATIONS IN CANADA

-- MATHEMATICAL STUDIES OF LARGE COMMUNICATION-COMPUTER NETWORKS --

by

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CAUTION

This is not a formal paper, it represents only a rough first draft of the gist of my talk here at Rochester. It undoubtedly contains many errors in content and style.

Please treat it as such.

ACKNOWLEDGEMENTS

I consider myself fortunate to work in a Government Department that is run by bonafide intellectuals and that one of these, my boss, Doug Parkhill, Assistant Deputy Minister of Planning, encourages me to pursue thoughts such as I will discuss today, in spite of our heavy day to day memorandum oriented and legal-political workload.

I wrote this paper on the eve of the formal creation of the Canadian Computer/Communications Task Force under Mr. Parkhill's guidance. I offer this piece of work as my first (hopefully useful) contribution to its work.

Most of the numerical examples were worked out by Mr. George Argatoff, a student in my course EE5654 Modern Communications and Computers at the University of Ottawa's Graduate School of Science and Engineering.

My secretary Judy Tremblay was kind enough to sacrifice a Grey Cup weekend, and type these notes from an almost unreadable handwritten manuscript. The many errors that remain are entirely my fault.

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PART I

First of all, let me thank you for inviting me to Rochester University to spend a few hours with you. I must confess that I was slightly concerned when Julian Keilson phoned to invite me here, and said "come and talk to us about real world communication problems and their solution using some of the latest and most sophisticated mathematical tools from the theory of stochastic processes", he went on to indicate that my audience would be experts in both these fields adding that forty-five minutes should suffice. I hope then that my presence here indicates that apart from being some sort of nut I am an optimistic enthusiast. Optimistic that sense can be made out of the subject and enthusiastic enough to think that I can say something new and interesting to you about it.

I propose to talk first of all very briefly about the present status of communication systems in Canada, and indicate some of the exciting new ideas that are being tossed around for sophisticated communication/computer networks. Next I would like to talk about two specialized topics, one from Operations Research and the other from theory of stochastic processes that can be effectively used for the analysis and synthesis of large scale communication-computer networks.

Let me begin by not boring you with lengthy details about statistics and types of communication systems, instead I will briefly outline what we have in the way of audio-video communication systems in Canada. These systems are sufficiently close in technical detail to those in the U.S., so that you can fill in the "gaps" easily.

In Canada we have essentially two types of telecommunication systems, these are

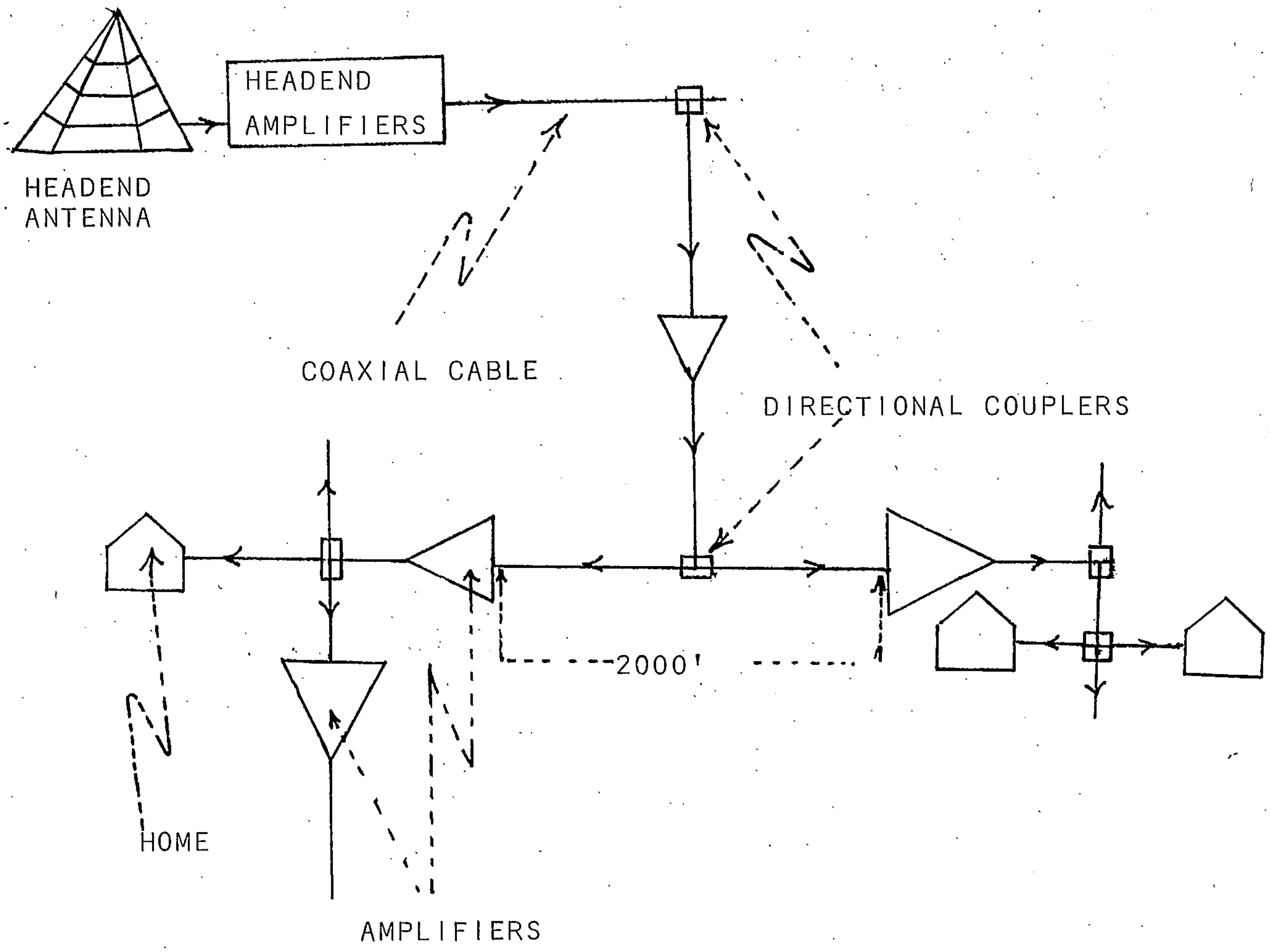
a) broadband (video - high speed) one way non-switched systems called CATV systems and

b) narrowband (audio - low speed) two way, switched systems which are of course the telephone systems.

<u>CATV Systems</u>

Cable television systems or Community Antenna Television (CATV) systems (figure 1) as they are usually called, receive television and FM signals off-the-air from broadcasting stations and distribute these signals via coaxial cables to their subscribers. These systems in effect provide a one-way broadband service. Usually, each subscriber pays an initial installment charge approximately $15.00 and a monthly service charge of about $5.00.

There are about 400 CATV systems in Canada serving over 1,000,000 households (over 25% of the urban households in Canada). These systems range in size from a few thousand to over 130,000 subscribers (the largest in the world).



CATV SYSTEMS

FIGURE I

The present CATV distribution systems employ no switching, are not readily adaptable to two-way transmission. They are not very exciting technically, but are an attractive proposition because they can distribute many (12 or more) television and FM broadcast stations on a single coaxial cable by the use of frequency division multiplexing techniques.

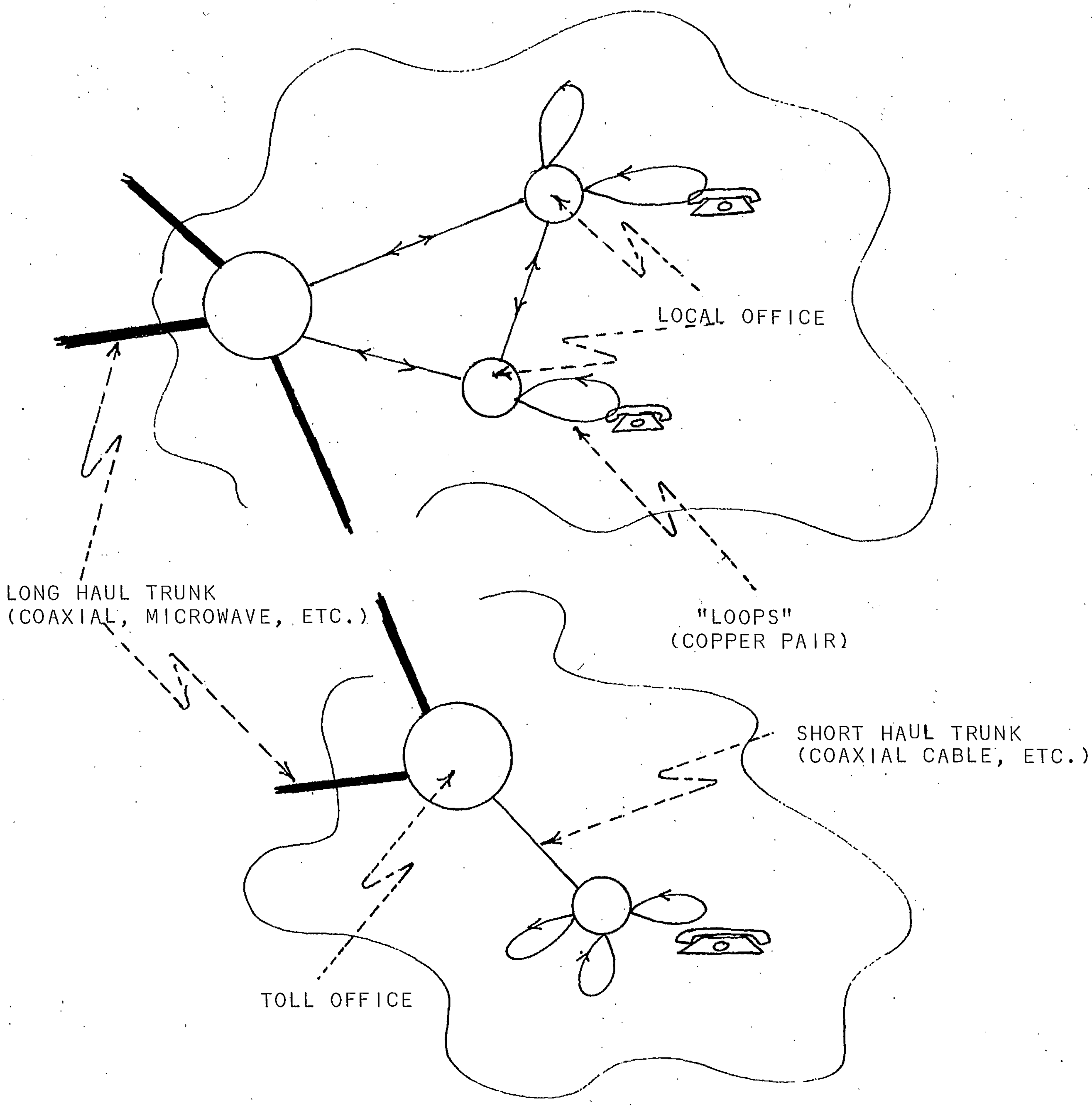
Most CATV systems provide 12 or less TV channels, even though they are capable of carrying 25 to 30 TV channels. There are a number of problems associated with the provision and utilization of this high channel capacity. First, the operators of CATV systems in Canada find that their reception is limited to 8 to 10 off-the-air channels. To provide additional channels on a closed circuit basis requires large expenditures for programming and costly studio equipment which cannot be supported by a small CATV operation. In addition the average (Canadian) home receiver is of VHF type and is capable of receiving only 12 channels. This receiver limitation can be overcome in various ways' nevertheless any proposed solution will have to pass the tests of economic viability and public acceptance.

Telephone Systems (See figure 2)

In Canada there are 16 major telephone companies, that own and operate approximately 2,000 intra-city telephone systems. In all, there are about 10 million telephones in Canada and the average Canadian makes some 1617 calls per year. Our telephone systems operate in much the same way as the U.S. systems. The total investment in these systems is around $7.5 billion dollars, and the industry is largely Canadian owned.

The present telephone systems although highly, developed and employing sophisticated switching techniques, utilize "pairs of copper wires" in their local distribution facilities that connect individual subscribersto switching centres. These wires have relatively small spectrum capabilities (less than 1 MHz of usable bandwidth)compared to coaxial cables (which have 300 MHz or more bandwidth),and are thus suitable for handling voice and low speed data signals only. That is, existing telephone systems are narrowband and are designed and optimized for voice traffic. Telephone systems also use analog signal processing techniques, and therefore, are not natural candidates for handling digital data.

In addition data traffic has different traffic statistics than voice traffic. Thus while the telephone system can accommodate a limited amount of data traffic it is likely that with heavy data loading of the telephone net, the high quality of the telephone service would deteriorate. One of the major problems facing the Department of Communications is to determine whether a separate dedicated network will be needed for computer applications in the coming ten or fifteen years.



TELEPHONE SYSTEMS

FIGURE 2

Nevertheless, there are many subscriber services that could be provided on existing telephone systems without seriously effecting the quality of service. There are however certain marketing unknowns, and venture capital seems to be tight at the moment. I should add however that certain telephone companies are considering dedicating either a pair of coaxial cables or three pairs of copper wires to each subscribers home, and then gradually introducing videophone services over the next ten to fifteen years. This use of six wires per subscriber would transform telephone systems into 1 MHz switched systems, capable of providing both videophone and a variety of other medium speed services. The use of coaxial cable in place of copper pairs would transform the system into a 300 MHz switched system capable of "total" telecommunication. A little later on in the talk, I will explain what I mean by the word "total".

I find it useful in comparing communication systems, to keep in mind two "state variable", namely information rate and direction of the information flow. Information rates can be low, medium or high, and the direction of information flow can be one-way (as in CATV) or two-way (as in telephony). Table I summarizes these characteristics and presents some typical examples.

TABLE 1

Directionality	Information Rate	Typical Example
One-Way	Low	Meter Reading
	Medium	Radio
	High	CATV
Two-Way	Low	Telephone Telex/TWX
		Computer to User,
	Medium	User to Computer, Videophone
	High	Computer-to Computer

The comparison is then completed when we say what rates are low, medium and high. Table 2 lists these values

TABLE 2

Information rate	Maximum bit	Typical Type of Service
Low	50K bits/sec	Telex/TWX voice
Medium	7M bits/sec	Videophone
High	50M bits/sec	Television

THE NEXT FIFTEEN YEARS

The telecommunication systems of the immediate future will be different from present systems in the following manner:

1. Multiplicity of services
2. Increased traffic handling capability
3. Increase in the number of medium and high information rate services
4. Increase in the number of two way services

The increase in the number of two way services is expected to be particularly dramatic in medium and high information rate types. This will affect significantly the traffic handling requirements of future telecommunication systems. In particular, the cost and complexity of switching facilities will increase considerably. Also as planners of future telecommunication systems, we must provide for versatility and adaptability to user demands for additional services as well as for the phasing out of services that have not received public acceptance. I usually think of future systems as total telecommunication systems, where "total" telecommunication system could provide: telephone, videophone, television, radio, computer, facsimile and utility metering services, to each home (figure 3). The various services provided on such a telecommunication system can be divided into three classes:

1. Broadcast

   Commercial and Instructional TV

   Commercial and Instructional Radio

2. Real Time Point to Point

   Telephone

   Videophone

   Telegraph and Teletype

   Certain Computer Services

3. Store and Forward

   Computer Services (time sharing and instruction)

   Facsimile (newsprint and magazines, library access)

   Financial Transactions (banking and remote purchasing)

   Interrogating (polling and meter reading)

   Mail

In order to achieve these capabilities for each urban home over the next fifteen to twenty years, it seems safe to say that a switched multiservice telecommunication system would be required. There is one convenient way to concieve of a "total" communication system. I talked about this conception under the name "Switched Broadband Systems" at a seminar on "Telecommunications and Participation" at the Universite de Montreal in April of this year. The non engineering types have had their imagination in high gear since then. My scientific friends have me up as suspect. Nevertheless, I am going to tell you about Switched Broadband Systems, because I find it easy to talk about them, and because I think the idea might prompt some of you

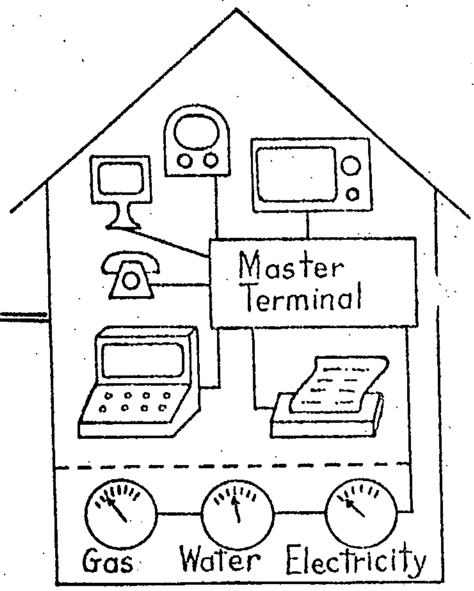


FIGURE 3

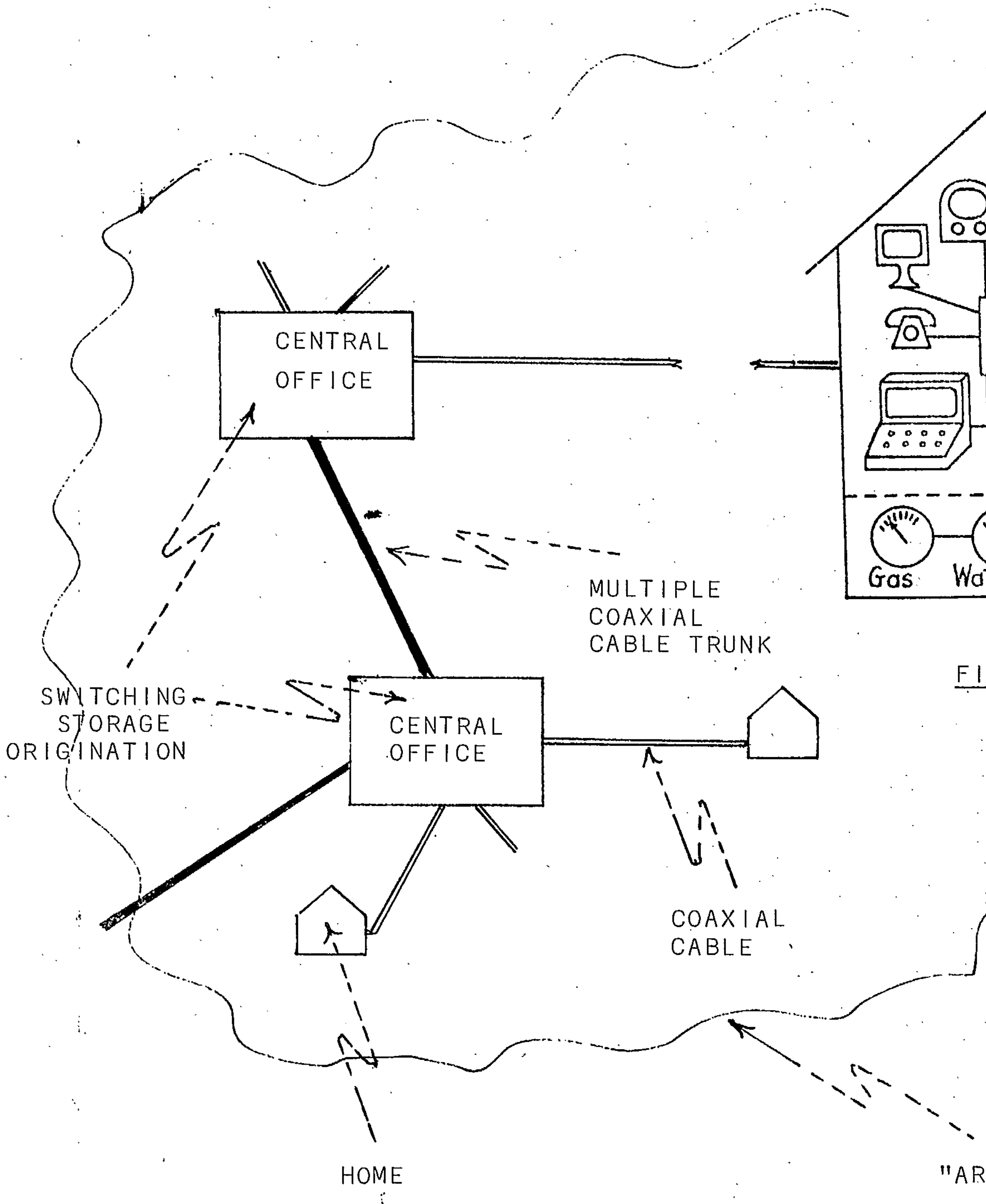


FIGURE 4

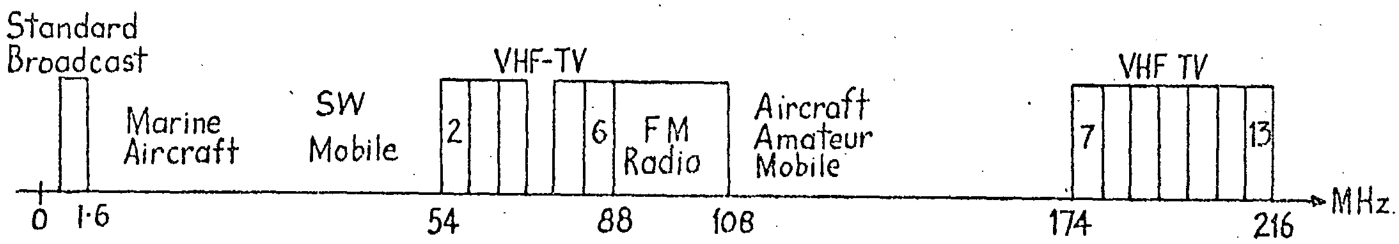
SWITCHED BROADBAND SYSTEMS

to try and model and simulate some particular aspects of them.

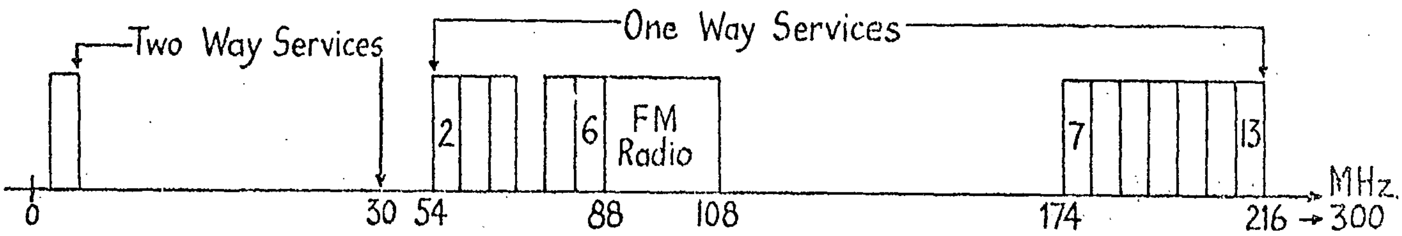
Switched Broadband Telecommunication Systems

The idea is simply this, suppose we were to increase the bandwidth of existing telephone system from 1 to 300 MHz, replacing their copper pairswith coaxial cables. The resulting system would be a switched broadband system (Figure 4). Such a system, with the use of suitable signal processing, would be capable of providing "total" telecommunication services to each home. These systems have the same philosophy of operation as existing telecommunication systems, but with the additional feature that they have usable bandwidths two orders of magnitude greater. Therefore, they can accommodate many more services. Some idea of the bandwidth gained by the use of coaxial cable can be obtained from Figure 5 which compares the spectrum (usable bandwidth) of a typical coaxial cable with the Department of Communications allocation of the Radio Spectrum and with the usable bandwidth of a typical copper pair.

If existing telecommunication systems were to evolve into switched coaxial cable systems, all Canadian cities would become "totally Wired Cities". It would be possible to build into such systems varying degrees of sophistication and complexity, depending on the type of signal processing used. The most sophisticated systems would be the ones in which all the signals are digital and time division multiplexing techniques are used. Furthermore, the use of computers for switching and for the performance of many other logical functions would make these systems very flexible.



D.O.C. Allocation of the Radio Spectrum



Possible Coaxial Cable Spectrum Allocation

Usable Spectrum of Typical Copper Pair

FIGURE 5

Before your imagination gets carried away, I would like to convey to you some rough appreciation for the large costs involved in the establishment of such total telecommunication systems. The system shown in figure(4)will be used as an example. If we wished to use such a system to provide the following capabilities to each home:

4 voice/data channels

12 TV channels, one-way-area selective

12 TV channels, one-way-subscriber selective

4 TV channels, two-way-area selective

an investment of the order $5,000 per subscriber would be required, assuming a density of 500 subscribers per square mile. This corresponds to ten times the cost of providing only telephone service. Another way of putting this is, instead of a $7 billion investment as in the existing telecommunication facilities an investment of the order of $70 billion would be required to provide such a system for the whole country.

Nevertheless between the bounds of what we have today, and the"total" telecommunication systems above, lie a whole range of systems having high probabilities of evolution. I must say that the technology at this time indicates several possibilities in terms of future types of multiservice telecommunication systems. These are:

Systems utilizing

1. Multiple paired (wires), each carrying single analog signals (as in the local distribution facilities of the present telephone system.

2. Sets of coaxial cables each carrying multiple analog signals. For this purpose the frequency division multiplex (FDM) technique would be used to combine signals.
3. Sets of coaxial cables each carrying multiple digital signals. For this purpose the time division multiplex (TDM) technique would be used to combine signals.
4. Sets of coaxial cables each carrying multiple digital and analog signals. The digital signals are used for low information rate services (such as telephone) and the analog signals are used for high information rate services (such as CATV).
5. Hybrid combinations of multiple paired wires and coaxial cables carrying analog and/or digital signals.

Digital systems offer significant advantages in versatility for low and medium information rate services (such as voice, data transmission, videophone, facsimile, etc.) The present high cost of digital systems will be reduced by at least one order of magnitude with the use of large scale integrated circuits. In addition a unified digital systemwould be the natural choice for the optimum realization of a nation-wide computer utility structure.

Analog systems, on the other hand, offer the most efficient use of the spectrum in the case of non-digital high information rate services.

Computer Utility Studies

In Canada, it recently became accepted that drastic steps had to be taken to ensure the optimal interaction of telecommunication and computer systems. The present sophistication of telecommunication systems is largely due to the use of computers as functional elements. The future potential of computers will be greatly enhanced by the utilization of advanced telecommunication networks in making computing power available on a widespread basis. The Canadian Government recognized the fundamental importance to Canada of the best Communication/Computer systems approved the creation within the Department of Communications of a Task Force, charged with ensuring the speedy and orderly development of "optimum" telecommunication/computer systems for Canada. This Task Force is Mr. Parkhill's** brainchild, and it is also his baby in that he is responsible for its work. The problems of the best networks in the legal-political economic and engineering sense are horribly entangled in semantics and personal prejudices;but in tackling the best networks in the engineering sense, there are acres of virgin territory. The brighter minds in the country are going to have a field year, and the fun has already begun in earnest for some of us. Those of you who are interested in some interesting reading see references (1) to (7).

** Parkhill is known to many both in Canada and the U.S. as the father of the computer utility, and his book "The Challenge of the Computer Utility" -- Addison Wesley 1965, is already a classic and still pertinent work.

PART II

Maximum Flow Techniques For Large Communication-Computer Networks

Leaving the H.G. Wells and 1984 stuff, I would like now in the second part of this talk to discuss two topics in integer programming that are rapidly gaining respectability in Operations Research but are not too well known to engineers.

I refer to the Ford-Fulkerson[8] max flow min cut theorem, and the Gomory-Hu[9,10] decomposition theory for finding maximum flows in networks. My main reason for discussing these two topics apart from the fact that I am familiar with them and have used them, is that they have for reaching applications in the analysis and synthesis of large computer-communication networks. Those of you who are familiar with these topics will recall that it is necessary to discuss the Ford-Fulkerson theory first as the Gomory-Hu theory is really based on it.

Luckily for me, as I need pictures to visualize things, both these two theories have a natural setting in the theory of finite graphs;this setting has an added attraction because the topological structure of a communication/computer network is naturally expressible as a graph. I will assume that you are familiar with the fundamental concepts[11,12,13,14] of graph theory, namely the oriented concepts of vertices(or nodes), arcs, paths, circuits lengths of paths, loops, symmetry as well as the non oriented concepts of edge, chain and cycle.

In general then with each communication/computer network, we can associate a graph (X,A,C), where X is a set of n nodes $\{x_1, --- x_a, - x_i, -- x_j -- x_b, -- x_n\}$; A, is the set of arcs $\{(x_i x_j)\}$ that is A contains all connections between nodes $x_i, x_j, \in X$. The set C is the set of integers, called arc (or channel) capacities denoted by

$c(x_i x_j) = c_{ij} \varepsilon C$.

For example, consider a switched communication network with three nodes denoting the switching centers (Figure 5).

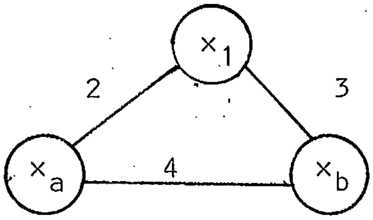


FIGURE 6

$X \equiv \{x_a, x_1, x_b\}$

$A \equiv \{(x_a x_b), (x_a x_1), (x_1 x_b)\}$

$\mathcal{B} \equiv \{4, 2, 3\}$

Between each pair of nodes (x_a, x_b) we can define an integer valued flow function ϕ, satisfying**, $0 \leq \phi (x_i x_j) \leq c(x_i x_j) \equiv c_{ij} \varepsilon C$

General Considerations

We encounter essentially two types of problems

I) Analysis problems

Given (X,A,C), find $\phi^*(x_a, x_b) \equiv \max \phi (x_a, x_b), \forall x_a, x_b \varepsilon X$

subject to constraint that for each $(x_i x_j) \varepsilon A$; $0 \leq \phi^*(x_i x_j) \leq c(x_i x_j) \varepsilon C$

II) Synthesis problems

I) Given X, A, ϕ^* find C

** Flow functions ϕ for networks that satisfy $b_{ij} \leq \phi(x_i x_j) \leq c(x_i x_j)$ offer further interesting possibilities in the studies of communication networks. I do not have the time to go into them here, but see Berge[12] if you are interested.

2) Given X, C and ϕ^* find A

3) Given C and ϕ^* find X and A

I might add that the interpretation of I) and II) in the context of communication/computer networks is obvious. In one variation of the analysis problem, we wish to find the maximum possible number ϕ^* circuits that could be utilized between all pairs of nodes in the network. In the synthesis problems, we usually are given the desired number ϕ^* circuits (flow function), and asked to find,

1) the channel capacities $c_{ij} \varepsilon C$ for a given network configuration X and A,

2) the channel capacities C, as well as the required connections A for a given set of nodes (switching centers) X and desired number of circuits ϕ^*,

3) the entire network X and A, given the desired number of circuits ϕ^* and available channels capacities C.

The way, I usually do synthesis is to first do analysis, and often synthesis problems are atleast partially defined in that we usually have some knowledge of X, A and C. The road to optimum synthesis however is littered with the bodies of umpteen analytical exercises. In addition, and this is something you should never forget namely that there are usually apriori unknown legal, political and economic constraints added to X, A and C, and therefore to ϕ^*, which will often negate your valid system studies.

Ford-Fulkerson Theory

Many communications-computer network maximum flow problems that you will meet, can be formulated as maximum flow problems for transport networks.

A transport network is a finite connected graph (X,A,C) without cycles, in which

1) X contains two nodes, a source x_a and sink x_b
2) For each arc $(x_i x_j) \epsilon A$, there exists an integer valued function ϕ with maximum value ϕ^* satisfying $o \leq \phi^* (x_i x_j) \leq c(x_i x_j) \epsilon C$
3) The flow $\phi (x_a, x_b)$ from the source x_a to sink x_b, is equal to the capacity $c(Y\overline{Y})$ of the cut $(Y\overline{Y})$ separating x_a and x_b, where $x_a \epsilon Y$, $x_b \epsilon \overline{Y}$.

The question, that Ford and Fulkerson [8] supplied the answer to, was -- "what is $\phi^* (x_a, x_b)$, the maximum flow, between source x_a and sink x_b". I quote the theorem below without giving its proof which is quite simple[14].

THEOREM I

In a transport network $\phi^* (x_a, x_b) = \max \phi(x_a, x_b)$ is equal to the cut of minimum capacity separating x_a and x_b. That is

$$\phi^*(x_a, x_b) = \min_{\substack{x_a \epsilon Y \\ x_b \epsilon \overline{Y}}} c(Y\overline{Y})$$

COMMENTS

1) You will recall, a cut $(Y\bar{Y})$ is therefore a set of arcs the removal of which would disconnect the network. The capacity of the cut, denoted by $c(Y\bar{Y})$ is the sum of the capabilities of all arcs incident into $\bar{Y}$ from Y. That is

$$c\,(Y\bar{Y}) = \sum_{\substack{x_i \in Y \\ x_j \in \bar{Y}}} c(x_i x_j)$$

In general, $c(Y\bar{Y}) \neq c(\bar{Y}Y)$. Networks in which $c(\bar{Y}Y) = c(Y\bar{Y})$ are called pseudo symmetric,[10] as distinct from symmetric,[9] in which $c(x_j x_i) = c(x_i x_j) \;\; \forall \; x_i, x_j \in X$.

2) It is well known, that although the maximum flow $\phi^*(x_a, x_b)$ is unique, there may be many cuts of minimum capacity that give this unique maximum flow, in fact it can be shown,[14] that if $(Y,\bar{Y})$ and $(Z\bar{Z})$ are minimum cuts, so are $(Y \cup Z \;\; \overline{Y \cup Z})$ and $(Y \cap Z \;\; \overline{Y \cap Z})$

There are therefore many different algorithms[11,12,14,15] for locating these minimum cuts. I assume you are familiar at least with one of them. The one I am familiar with in Berge [12], involves first finding a complete flow in the network, by saturating all <u>paths</u> leading from x_a to x_b, and then using a labelling procedure (until x_b cannot be labelled) on <u>chains</u> joining x_a and x_b.

For example 2, suppose we are given the following communication system, and want to find the maximum number of circuits $\phi^*(x_a, x_b)$, i.e., the maximum flow possible between nodes x_a and x_b, in figure(7).

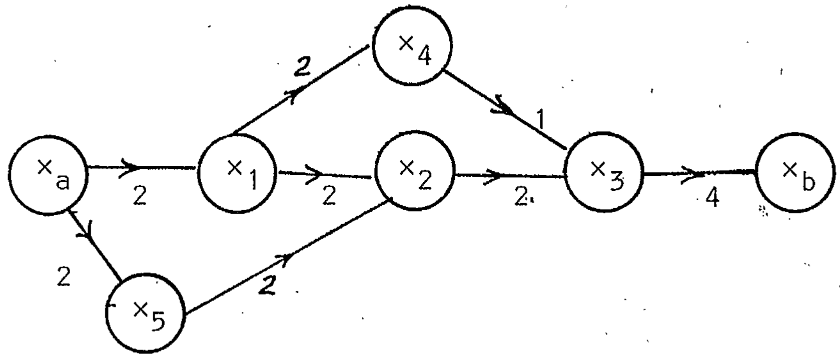


FIGURE 7

The channel capacities are shown in figure(7). Applying the algorithm in Berge,(12, pp. 74-76) it is easy to find that $\phi^*(x_a x_b) = 3$.

As a practical example of a real life communication- computer network problem that admits an integer programming formulation, I offer the following example 3.

Example 3 Consider a number of interconnected remote offices that each have small computers or terminals $x_1, ---, x_n$. At night messages from these small computers are forwarded to a central computer Y in a given interval of time, or if this is not possible, are stored back in a small computer. Let $c(x_i x_j) = c_{ij}$ be channel capacity between two of these small computers. Let t_{ij} be the average time it takes a message to go from x_i to x_j over the channel $(x_i x_j)$. Let $c_{ii} \equiv c(x_i x_i)$ be the number of messages that can be stored at x_i. The problem is this "how should the output of the small computers be scheduled, so as to have as many messages as possible arrive at Y in a given interval of time Θ." To draw the transport network for this problem, define the set of vertices X by the cartesian product $\otimes$

$$X \equiv \{x_1, x_i, x_n, Y\} \otimes \{0,1,2,-,t,-,--\theta\}$$

and at each vertix $(x_i, t) \equiv x_i(t)$, define two arcs, $(x_i(t), x_j(t+t_{ij}))$ of capacity $C_{ij})$ and $(x_i(t), x_i(t+1))$ of capacity c_{ii}. Denote by $\phi(x_i(t), x_j(t+t_{ij}) \equiv x_{ij}$, the flow, that is the number of messages leaving computer x_i at time t, to go to computer x_j, and by $\phi(x_i(t), x_i(t+1)) \equiv x_{ii}$ the number of messages that remain in x_i between t and t + 1. I will draw the transport network for the case of 3 small computers x_1 x_2 and x_3. To do this connect a source x_a to x_1, x_2 and x_3 with arcs of capacities $c(x_a, x_i) = a_i$, i = 1, 2, 3. Then connect Y(1), ---- Y(2), -- Y(t), Y(θ), to a sink x_b by arcs of infinite capacity. We obtain the required network shown in figure (8). (12, 13)

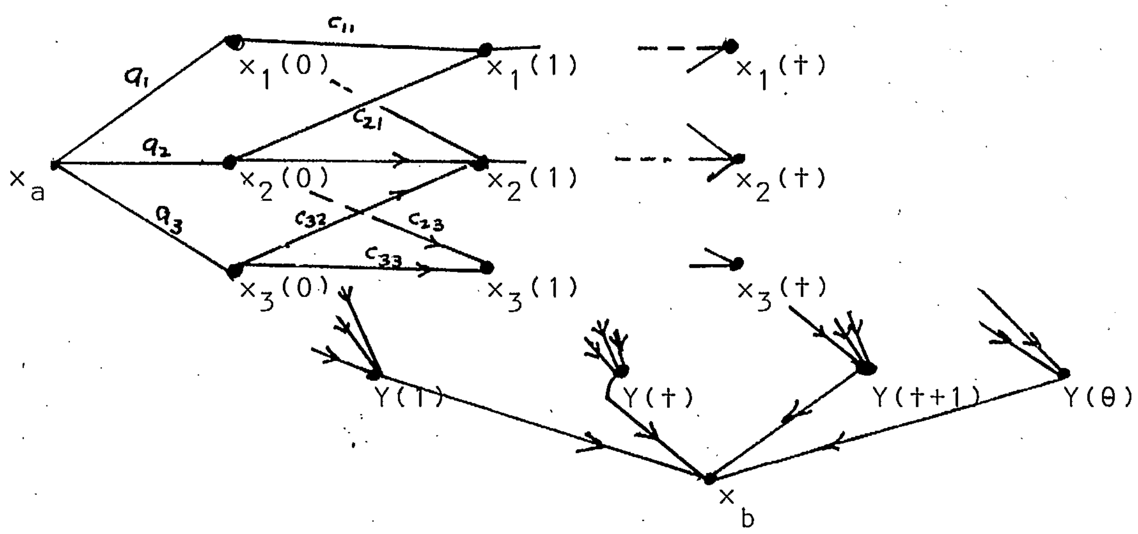


FIGURE 8

We can apply the labelling procedure as in example 2, and determine the maximum flow $\phi^*(x_a, x_b)$; once this is done, then the optimum schedule can be copied off of the graph for this optimum flow.

COMMENT

As most of you are aware, there is an intimate connection between transport network problems and linear programming problems. I will now give the linear programming formulation for this problem. It is as follows; defining the numbers

$$C_{ij} = \begin{cases} 0 & ;(x_i x_j) \notin A \\ c(x_i x_j) ; & (x_i x_j) \in A \end{cases}$$

$$x_{ij} = \phi(x_i x_j), \text{ where } 0 \leq x_{ij} \leq c_{ij}$$

Then the solution consists in finding numbers x_{ij}, so that the following expression for the flow is maximized namely

$$\max \sum_{i=0}^{n} x_{ib} \equiv \phi^*(x_a, x_b).$$

subject to the constraints

$$\underbrace{\sum_{k=0}^{n} x_{ki}}_{\text{Flow into } x_i \text{ at time } t.} - \underbrace{\sum_{J=1}^{n+1} x_{ij}}_{\text{Flow out of } x_i \text{ at time } t+1} = 0$$

This is a sort of classical programming problem and it could for instance be solved by the Simplex Method.

GOMORY-HU THEORY[9,10,13,14]

The Ford-Fulkerson method can be applied as a brute force resort n(n-1) times to find the maximum flows between all pairs of nodes in a n node network. However for large scale networks, i.e., where n is very large, the computation required tends to increase almost without bound. The Gomory-Hu Theory which I have used extensively is elegantly worked out in a recent paper by Gupta[10] and applies to pseudo symmetric networks and hence to a large class of communication network problems. The method was originally worked out for symmetric networks by Gomory and Hu[9] and extended to pseudo symmetric networks by Gupta.

The method essentially is a technique for transforming a given network $G \equiv (X,A,C)$ into a flow equivalent network $G_1 \equiv (X,A_1,C_1)$ which is a tree. Then $\phi^* (x_i, x_j)$, the maximum flow between any pair of nodes is the same for G and G_1. The nice thing about the Gomory-Hu method is that it involves only n-1 applications of the Ford Fulkerson labelling method instead of n(n-1) as would be required in the brute force technique.

You will recall the flow equivalent tree G_1, is a simple connected graph with n-1 arcs (that is why A_1 is different from A), and the maximum flows function ϕ^* can be read of G_1 by inspection. We usually tabulate ϕ^* as a (nxn) matrix, whose entries are the maximum flow between all pairs of nodes in the network. This is shown below in figure(9).

$\phi^* \equiv$

	x_1	x_j	x_n
x_1	$\phi^*(x_1x_1)$	$\phi^*(x_1x_j)$	$\phi^*(x_1x_n)$
x_i	$\phi^*(x_ix_1)$	$\phi^*(x_ix_j)$	$\phi^*(x_ix_n)$
x_n	$\phi^*(x_nx_1)$	$\phi^*(x_nx_j)$	$\phi^*(x_nx_n)$

FIGURE 9

The Gomory-Hu method is interesting because it rests on two basic but far reaching theorems from graph theory. These are

THEOREM 2 Consider a network (X,A,C) with a path

$\{(x_i,x_1), (x_1x_2), ---,(x_r,x_j)\}$ joining a source x_i and terminal x_j, then

$$\phi^*(x_ix_j) \geqslant \min\{\phi^*(x_ix_1), \quad \phi^*(x_1x_2), ---, \phi^*(x_rx_j)\} --- 1$$

Proof

Consider the case where the path joining x_i and x_j is $\{(x_ix_1), (x_1x_j)\}$. Then we have to prove

$$\phi^*(x_ix_j) \geq \min\{\phi^*(x_ix_1), \phi^*(x_1x_j)\} ---------- 1(a)$$

Applying the Ford Fulkerson Labelling procedure, we can find a minimum cut of capacity $c(Y\overline{Y})$ with $x_i \epsilon Y$ and $x_j \epsilon \overline{Y}$, so that $\phi^* (x_i x_j) = c(Y\overline{Y})$; now two possibilities a) and b) exist

a) $x_1 \epsilon X$ then $\phi^* (x_1 x_j) \leq c(Y\overline{Y})$

b) $x_1 \epsilon \overline{X}$ then $\phi^* (x_i, x_1) \leq c(Y\overline{Y})$

Substituting a) and b) in I(a) gives the desired result. the general result (1) follows by induction.

QED

Corollary: The function ϕ^* can have at most n-1 numerically distinct values.

The following theorem can be found in Gupta [10]

Theorem 3 Let $G \equiv (X,A,C)$ be a network (symmetric or pseudo symmetric). Let $(Y\overline{Y})$ be a minimum cut separating x_i and x_j,

i.e. $x_i \epsilon Y$, $x_j \epsilon \overline{Y}$ Let $G_1 \equiv (X, A_1, C_1)$ be the network obtained from G by condensing the set $\overline{Y}$ into a single node, and let x_a, x_b, be in Y.

Then $\phi^* (x_a, x_b)$ in G is equal to $\phi^* (x_a x_b)$ in G_1 that is for the purpose of computing maximum flows in Y, the networks G and G_1 are flow equivalent.

Proof

Let $(Z\bar{Z})$ be a min cut separating x_a and x_b. With reference to figure (10) define sets

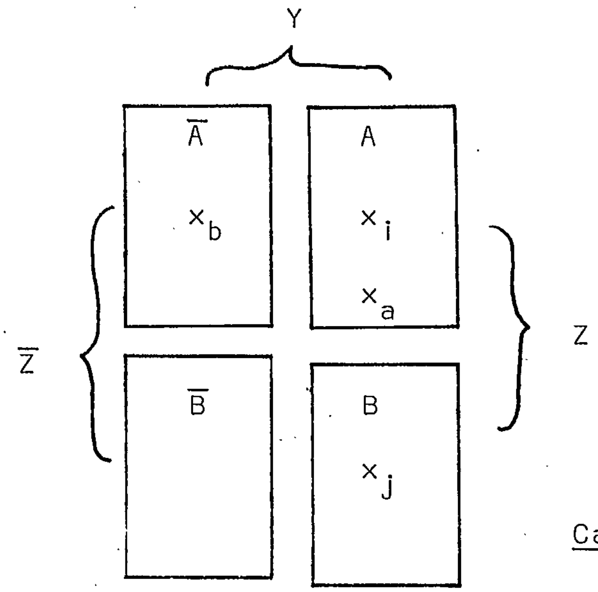
$Y \cap Z \equiv A$

$Y \cap \bar{Z} \equiv \bar{A}$

$\bar{Y} \cap Z \equiv B$

$\bar{Y} \cap \bar{Z} \equiv \bar{B}$

These $\bar{A}$ is the complement of A in Y. That is $A \cup \bar{A} = Y$. Likewise $\bar{B}$ is the complement of B in $\bar{Y}$. That is $B \cup \bar{B} = \bar{Y}$.



Case 1 shown.

FIGURE 10

Without loss of generality, we can assume that x_i, $x_a \in A$, and $x_b \in \bar{A}$. We have two cases to consider, namely $x_j \in B$ and $x_j \in \bar{B}$

Case 1 $x_j \in B$ from figure (10) we can immediately write

$$c(Z\bar{Z}) = c(A\bar{A}) + c(A\bar{B}) + c(B\bar{A}) + c(B\bar{B}) \quad \text{----2}$$

$$c(Y\bar{Y}) = c(AB) + c(A\bar{B}) + c(\bar{A}B) + c(\bar{A}\bar{B}) \quad \text{----3}$$

Now since $(Z\overline{Z})$ is a minimum cut separating x_a and x_b and $(AUBU\overline{B}\ \overline{A})$ is some other cut separating x_a and x_b, and since

$$c\ (AUBU\overline{B}\ \overline{A}) \quad = \quad c(A\overline{A}) + c(B\overline{A}) + c\ (\overline{B}\overline{A}) \quad ---4$$

subtracting (4) from (2) we find

$$c(Z\overline{Z}) - c(AUBU\overline{B}\ \overline{A}) = c(A\overline{B}) + c\ (B\overline{B}) \quad - c\ (\overline{B}\overline{A}) \leqslant 0 \ ---5$$

Again since $(Y\overline{Y})$ is a minimum cut separating x_i and x_j and since

$(AU\overline{A}U\overline{B}B)$ is some other cut separating x_i and x_j and since

$$c(AU\overline{A}U\overline{B}\ B) = c(AB) + c(\overline{A}B) + c(\overline{B}B) \quad ---6$$

subtracting (6) from (3) we get

$$c(Y\overline{Y}) - c(AU\overline{A}U\overline{B}\ B) = c(A\overline{B}) + c(\overline{AB}) - c(\overline{B}B) \leqslant 0 \ ---7$$

adding 7) and 5) we find using pseudo symmetry

$$c(Z\overline{Z}) - c(AUBU\overline{B}\ \overline{A}) \quad + \quad c(Y\overline{Y}) - c(AU\overline{A}U\overline{B}\ B)$$

$$= 2c(A\overline{B}) \leqslant 0$$

but since $c(A\overline{B}) \geqslant 0$, we must have that

$$c(Z\overline{Z}) - c(AUBU\overline{B}\ \overline{A}) \quad + \quad c(Y\overline{Y}) - c(AU\overline{A}U\overline{B}\ B) \quad = 0$$

Now the sum of two non negative numbers can be zero if both are zero, therefore

$$c(AUBU\overline{B},A) \quad = c(Z\overline{Z})$$

that is, $\quad c(AU\overline{Y},\overline{A}) \quad = c(Z\overline{Z})$

$(A \cup \overline{Y}, A)$ is therefore a minimum cut separating x_a and x_b.

Case 2 $x_b \varepsilon \overline{B}$

Proceeding as in Case 1, it is easy to show

$c(A, \overline{A} \cup \overline{Y}) = c(Z\overline{Z})$ which is again a minimum cut separating x_a and x_b.

What we have shown is that there is always a minimum cut separating x_a and x_b such that the set of nodes $\overline{Y}$ is always on one side of this cut. Consequently condensing $\overline{Y}$ into a single node does not affect the value of the maximum flow from x_a to x_b in the condensed network G_1 and thus the theorem is proven.

QED

COMMENTS

With a little reflection you will see this means that if we have a network (X,A,C) and use the Ford Fulkerson method to find $\phi^* (x_{i,} x_j)$ where $x_i \varepsilon Y$ and $x_j \varepsilon \overline{Y}$ and $Y \cup \overline{Y} \equiv X$;

then for the purposes of find $\phi^*(x_a x_b)$, where x_a and x_b are in Y and x_a or x_b could be x_i; we can condense all the nodes of $\overline{Y}$ into a single node. This implies that I should be able to find some algorithm which would permit me to find all n^2 flow functions ϕ^*, in only n-l applications of the Ford Fulkerson labelling method. This is precisely what Gomory and Hu recognized. Their theorem which I will soon give, exploits the condensation property of theorem 3 and the theorem 2 inequality (I) to the maximum.

I pass now to the proof of the Gomory-Hu theorem. Before giving the proof, I shall present one form of their algorithm[15] and discuss some examples.

The procedure for applying the Gomory-Hu theorem to find the maximum flows between all n nodes in a network, in only n-1 labelling operations, is as follows. Consider the network of figure (11), where $Y \cup \overline{Y} = X$, and A and C are given. That is we have $G \equiv (X,A,C)$.

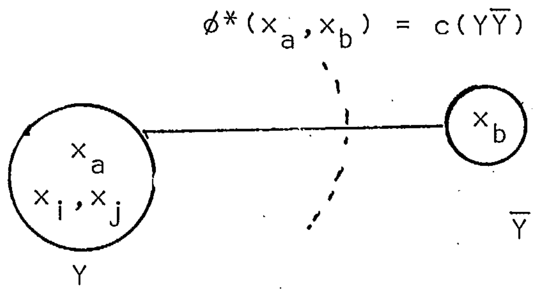


FIGURE 11

in figure (11), we have used a Ford-Fulkerson labelling and found $\phi^*(x_a,x_b)$; we next condense the nodes of $\overline{Y}$ into a single node (that is we join the nodes of $\overline{Y}$ by arcs of infinite capacities), and with reference to Y only, find $\phi^*(x_i,x_j)$. We obtain figure(12).

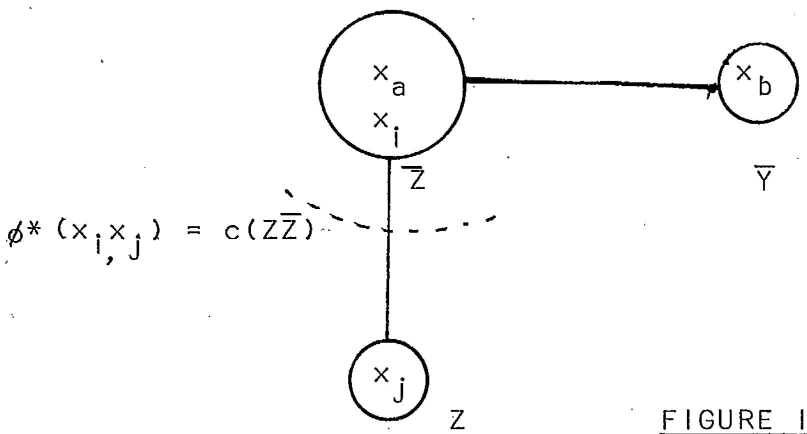


FIGURE 12

Continuing this procedure n-l times, we find a flow equivalent tree G_1 typically of a form shown in figure (13).

$G_1 \equiv$

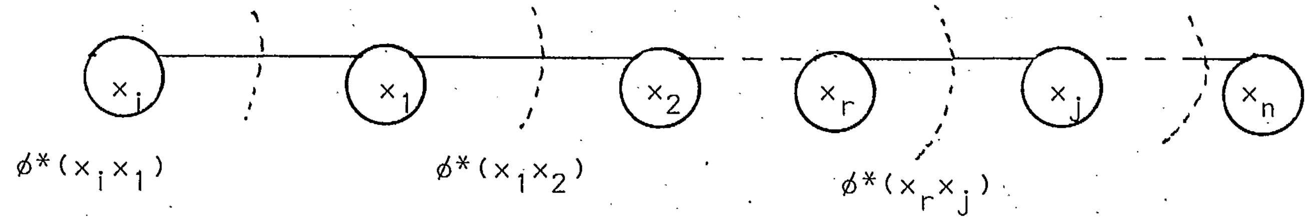


FIGURE 13

The Gomory Hu Theorem which comes next, tells us we can read off all the ϕ^* flow functions by inspection of the flow equivalent tree of the type shown in figure (13), once it has been found as described above.

THEOREM 4 (Gomory-Hu, Gupta) Given a network $G \equiv (X,A,C)$;

The maximum flow $\phi^*(x_i x_j)$ between any two nodes x_i and $x_j \in X$ is given as

$$\phi^*(x_i x_j) = \min \left\{\phi^*(x_i x_1),\ \phi^*(x_1,x_2)\ ---\phi^*(x_r x_j)\right\} \quad ---8$$

where $\phi^*(x_i,x_j)$, $j=1,---,$ r are the values of the maximum flows along the path $\left\{(x_i x_1),\ ---,(x_r x_j)\right\}$ joining x_i and x_j in the flow equivalent tree G_1 obtained by the n-l applications of the Ford Fulkerson method (See figures (11) to (13).

Proof [(10)] I will now give a brief outline of the proof. From the inequality (1) of theorem 2, with reference to figure (13), we have immediately,

$$\phi^*(x_i x_j) \geq \min \left\{\phi^*(x_i x_1),\ ---,\phi^*(x_r x_j)\right\} ---9$$

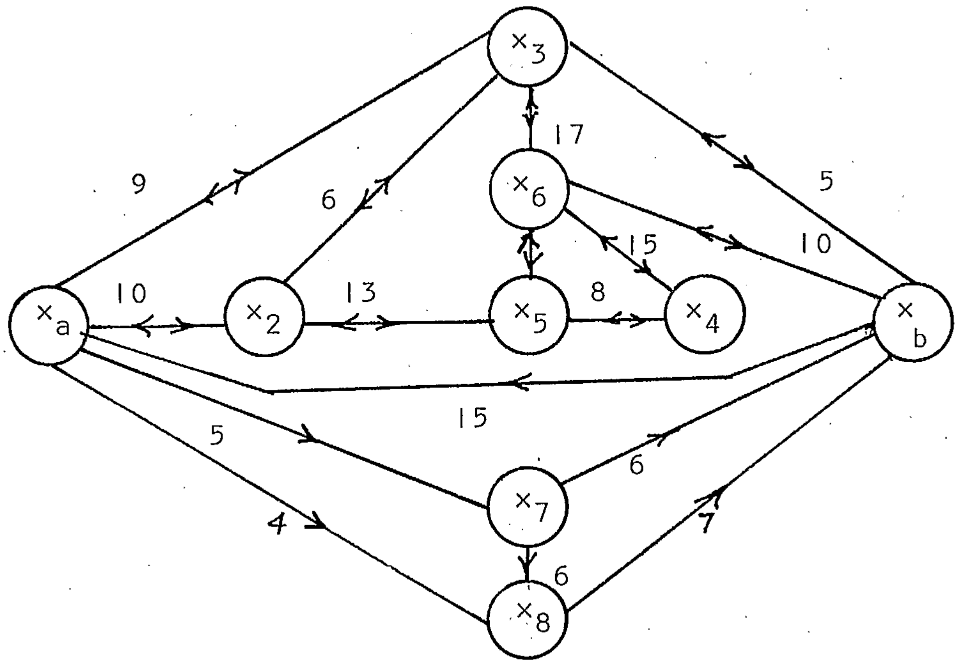
The reverse inequality to (9) follows trivally from inspection of the tree in figure (13). Obviously since minimum cut sets $(x_i x_1),(x_1,x_2),\ ---,(x_r x_j)$ are in the unique chain joining x_i and x_j we must have

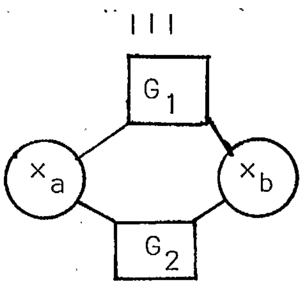
$$\phi^* (x_i x_j) \leq \min \left\{ \phi^*(x_i x_1), \ --- \ \phi^*(x_r, x_j) \right\} \qquad ---10$$

combining (9) and (10) completes the proof of the equality (8).

QED

As an indication now of how this theory fits into computer-communication problems. Consider the graph G of figure(14) which shows two centers x_a and x_b connected through switching centers over two distinctly different networks G_1 and G_2.





$G_1 = (X_1, A_1, C_1)$

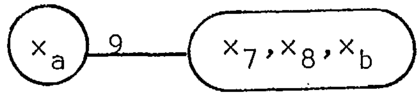
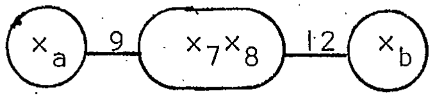
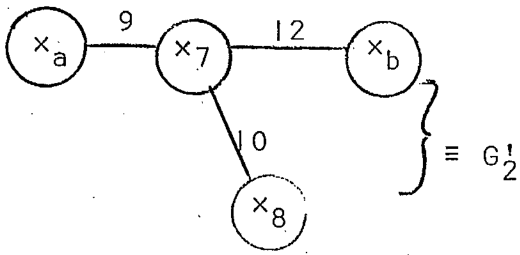
$X_1 = \{x_a, x_2, x_3, x_4, x_5, x_6, x_b\}$

$G_2 = (X_2, A_2, C_2)$

$X_2 = \{x_a, x_7, x_8, x_b\}$

FIGURE 14

The procedure for applying theorem 4, to find the flow equivalent trees G_1' and G_2' for networks G_1 and G_2 is illustrated below.[15] With reference to G_2 to obtain G_2' we proceed as follows, obtaining the flow equivalent tree G_2' on the 3rd iteration. (Compare G_2' with figure(13).

Iteration	x_i, x_j	Max. flow	Tree
1	x_a, x_b	9	
2	x_7, x_b	12	
3	x_7, x_8	10	

Now using equation (8) of Theorem 4, we can immediately on inspecting G_2', tabulate ϕ_2^*, for G_2. This tabulation is shown below.

$\phi_2^* =$

	x_a	x_7	x_8	x_b
x_a	/	9	9	9
x_7	9	/	10	12
x_8	9	10	/	10
x_b	9	12	10	/

from the above table for example $\phi_2^*(x_8x_b) = 10$, $\phi_2^*(x_ax_b) = 9$ etc. Similarily for G_1 we find the tree G_1' shown in figure (15).

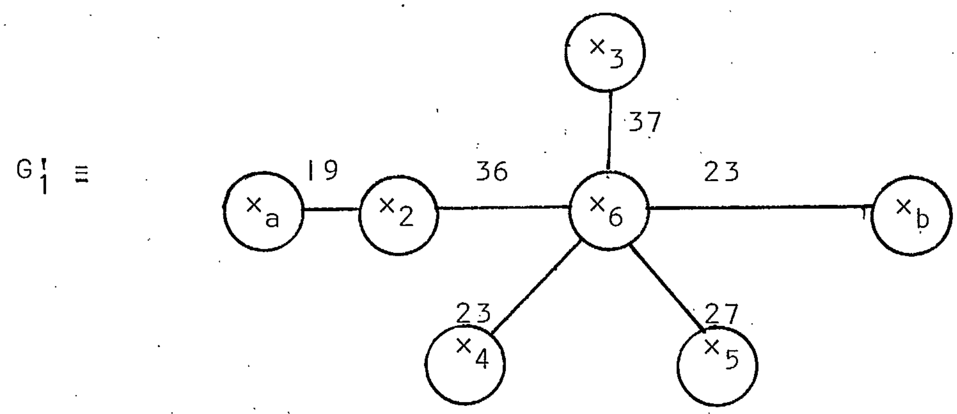


FIGURE 15

Then ϕ_1^* for G_1, can be read off from G_1' above, using theorem 4; for example $\phi^*_1(x_a, x_4) = 19$. Thus we have that the network of figure(13) for the purposes of obtaining ϕ_1^* and ϕ_2^* can be represented by the pair of trees, figures(14) and(15). In effect then we have figure(16).

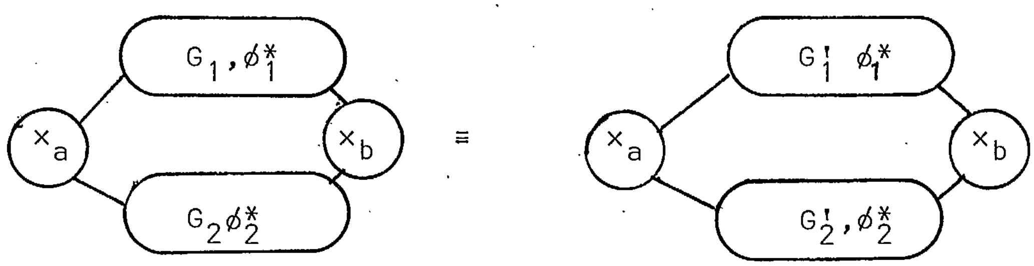
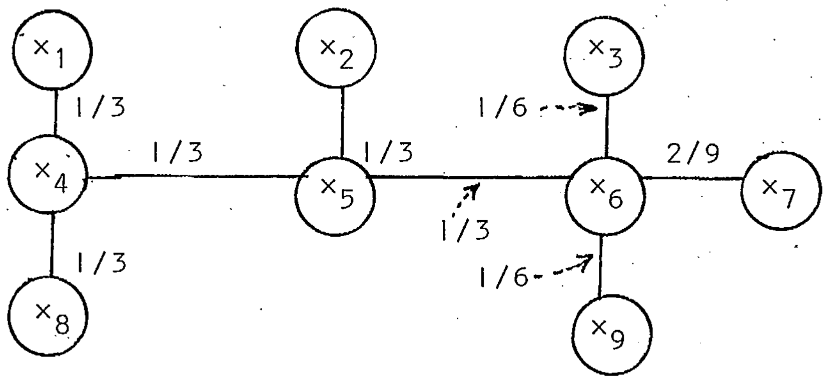


FIGURE 16

Figure (17) shows a flow equivalent tree for Avalon Telephone network in Newfoundland. The number in the numerator of the fraction indicates the maximum number of circuits under the present situation. The number in the denominator indicates the maximum possible number of circuits if "certain" changes were made.

Flow Equivalent Tree For Avalon Telephone Company

NEWFOUNDLAND



Identification of Centres (see any map of Newfoundland)

x_1 ≡ Freshwater

x_2 ≡ Harbour Main

x_3 ≡ Bay Roberts

x_4 ≡ Cabinet

x_5 ≡ Mile four Pond

x_6 ≡ Kenmount

x_7 ≡ St. John's

x_8 ≡ Point La Haye

x_9 ≡ Bell Island

FIGURE 17

PART III

RELIABILITY PREDICTION THEORY

Because I have spent many years worrying about reliability problems, I could not bring myself to leave Rochester without saying something about reliability prediction theory. Since the time is short, it is impossible to develop the theory (which is quite straight forward). What I would like to leave you with then, is some general techniques that George Glinski and I worked out[16,18] for handling large computer-communication networks and which have served as well.

These techniques can be used to obtain

I) Reliability Models; in particular to obtain time dependent state transition functions, as well as a time dependent reliability function for a given network (X,A,C)

II) Moments of Time to First Failure; in particular to obtain the moments of the first time it takes to go from specified acceptable states to specified failed states in a given network (X,A,C)

for the following class of networks, namely

Networks (X,A,C) that operate in a repair environment, and that have r,$(r \geq 1)$ satisfactory and m,$(m \geq 1)$ failed states. The assumptions are:

a) the failure and repair rates λ and μ of the subsystems X and A are known,

b) it is possible from an understanding of the operation of the network to specify to set $\mathcal{S}$ of satisfactory states and a set $\mathcal{F}$ of failed states.

I call this the reliability probem $(X,A,C,\lambda,\mu,\mathcal{S},\mathcal{F})$. The methods[16,18] that are given for reliability modelling of such networks, rest on the assumption that the state transition behaviour of the network is characterizable by a stationary Markov Chain with a finite dimensional state space, (namely $\mathcal{S} \cup \mathcal{F}$) and discrete time set.

COMMENT

The more general class of networks (X,A, C) which are degradation-prone, that is which can be described by a set of degradation-prone parameters is treated in our paper[20]. The assumption made there, is that the time function characterizing each degradation-prone parameter, is a sample function of a Feller-Markov process, that is a sample function of a continuous Diffusion process.[21,22] I do not have the time to go into this today, but those of you who have the courage to read Itô and Mackean[21,23] will find enough goodies for many years of original and rewarding research in Operations Research, Economics, stock market predictions, etc. as well as in optimum filtering and prediction theory[24] in electrical engineering, etc.

I will now state the results for the reliability problem $(X,A,C,\lambda,\mu,\mathcal{S},\mathcal{F})$, and then do an example. As you will see from what follows, the key to using these results, is realizing that the failure and repair rates multiplied by appropriate units of time, are the one step state transition probabilities of a stationary

Markov chain, that is, are the entries of some transition matrix $[M]$ which completely characterizes the behaviour of the chain.

Specifying the satisfactory states $\mathcal{S}$ and failed states $\mathcal{F}$ produces the following partitions of $[M]$, as shown below

$$[M] \equiv \begin{array}{r|c|c|l} \mathcal{S}\to\mathcal{S}\Big\{ & [S] & [B] & \Big\}\mathcal{S}\to\mathcal{F} \\ \hline \mathcal{F}\to\mathcal{S}\Big\{ & [0] & [I] & \Big\}\mathcal{F}\to\mathcal{F} \end{array}$$

where

$[S]$, is the (rxr) matrix of the one step transition probabilities between the satisfactory states in $\mathcal{S}$. The entries of $[S]$ are denoted by s_{ik}, etc.

$[B]$, is the (rxm) matrix of the one step transition probabilities from states in $\mathcal{S}$ to states in $\mathcal{F}$.

$[I]$, is the (mxm) unit matrix, which shows that transition between failed states are not allowed**

$[0]$, is the (mxr) null matrix, which shows that transitions cannot take place from failed states in $\mathcal{F}$ to satisfactory states in $\mathcal{S}$.

** The theory that I present here could be further generalized by replacing $[I]$ by some general matrix which allows transitions between the failed states of $\mathcal{F}$.

The following results were obtained[16,18] for the reliability problem $(X, A, C, \lambda, \mu, \mathcal{S}, \mathcal{F})$. They assume $[M]$ can be specified and partioned into $[S]$, $[B]$ and $[I]$ as shown above.

1) <u>Reliability Models</u>

Letting

a) $s_i(n) = \text{Prob}\left\{\text{network is in state } S_i \epsilon \mathcal{S} \text{ at time } n\right\}$

b) $P_{ij}(n) = \text{Prob}\left\{\text{network goes from state } S_i \epsilon \mathcal{S} \text{ to state } F_j \epsilon \mathcal{F} \text{ in time } n\right\}$

c) $p_{ij} = \lim_{n \to \infty} p_{ij}(n)$

It can be shown

a) $\overline{s}(n) = \overline{s(o)}\, [M]^n$: $\overline{s}(n) \equiv 1 \times (r \times m)$ row vector of $s_i(n)$

b) $[P(n)] = [S]^{n-1} [B]$: $[P(n)] \equiv (r \times m)$ matrix of $p_{ij}(n)$

c) $[P] = \left[[I] - [S]\right]^{-1} [B]$: $[P] \equiv (r \times m)$ matrix of p_{ij}

therefore we can define a reliability function R(n) as

d) $R(n) \equiv \text{Prob}\left\{\text{System is in state in } \mathcal{S} \text{ at time } n\right\}$

$$= \sum_{\mathcal{S}} s_i(n)$$

II) <u>Moments of the First Time to Failed State</u>

Letting

e) τ_{ij} be the random variable -- "time taken for first transition from successful state S_i to failed state F_j".

f) $\tau_{ij}(k)$ be the k^{th} moment of τ_{ij}

g) $t_{ij}(z)$ be the generating function for the moments $\tau_{ij}(k)$

It can be shown

h) $$\left[\tau(k)\right] = \frac{d^k}{dz^k}\left(\left[T(z)\right]\right)\Big|_{z=1}, \quad k=1,2,\text{--}$$: $\left[\tau(k)\right] \equiv$ (rxm) matrix of $\tau_{ij}(k)$.

g) $$\left[T(z)\right] = z\left[\left[I\right] - z\left[S\right]\right]^{-1} B$$: $\left[T(z)\right] \equiv$ (rxm) matrix of $t_{ij}(z)$

<u>COMMENTS</u>

(I) The nice thing about the above techniques, is that they are immediately usable by any engineer, who understands how the network (X,A,C) operates and can therefore specify his satisfactory states $\mathcal{S}$ and failed states $\mathcal{F}$, and thus determine the partition matrices $[S]$, $[B]$ and $[I]$ of $[M]$.

(2) In an important sense, we can also use these techniques to do synthesis. This is because by knowing the failure and repair rates $\{\lambda\}$ and $\{\mu\}$ which are the entries of $\{[M]\}$, for a number of possible systems $\{X\}$ and $\{A\}$ that could be used to build the network (X,A,C), we can do a computer simulation and select the subsystems $X \in \{X\}$ and $A \in \{A\}$ that give the "best" R(n) and $[\tau(k)]$ for the network (X,A,C). This is precisely what I meant when I talked about an analytical approach to synthesis at the beginning of this talk.

Example

To illustrate the method for applying the above results to progressively complicated networks, I would like to refer back to figure (14). Let us suppose we know failure and repair rates $\{\lambda_1\}$ and $\{\mu_1\}$ for subsystem X_1, and A_1, of G_1, and repair and failure rates $\{\mu_2\}$ and $\{\lambda_2\}$ for subsystems X_2 and A_2 of G_2. Then we could find a $[M]_1$ and $[M]_2$ and $R_1(n)$, $[T(z)]$ and $R_2(n)$ and $[T_2(z)]$ for G_1 and G_2.

To apply the reliability prediction theory above to the composite network G of figure(16), we have to define overall failure and repair rates for G_1 and G_2. Let these be λ_1, μ_1 and λ_2, μ_2 respectively. Where

$$\lambda_1 = \frac{1}{\text{smallest}\,\{\tau_{ij}(1)\}\ \text{for}\ G_1} \quad ; \quad \lambda_2 = \frac{1}{\text{smallest}\,\{\tau_{ij}(1)\}\ \text{for}\ G_2}.$$

$$\mu_1 = \frac{1}{\text{largest repair time in} \left\{ \frac{1}{\mu_1} \right\} \text{for } G_1}$$

$$\mu_2 = \frac{1}{\text{largest repair time in} \left\{ \frac{1}{\mu_2} \right\} \text{for } G_2}$$

Then for figure (16) we can draw figure 18 below

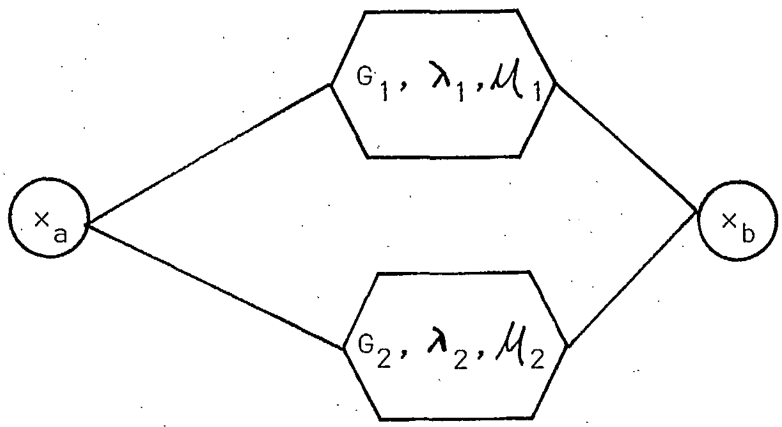


FIGURE 18

The** question now is how do we find $\mathcal{S}$, $\mathcal{F}$, and [M] with its partitions [S], [B] and [I]. In order to apply the modelling techniques we have to obtain [M] for the system of figure (18). To do this is easy if you first construct the state-word assignment to find $\mathcal{S}$ (satisfactory states) and $\mathcal{F}$ (failed states).

I usually do this by making a state-word assignment as shown in figure (19).

** The same considerations as these, apply to G_1 or G_2 individually.

STATE	WORD DESCRIPTION
S_1	Both G_1 and G_2 provide a path from x_a to x_b.
S_2	G_1 fails and the only path is provided by G_2. Repairs to G_1 are not yet started.
S_3	Repairs to G_1 start. G_2 is still providing the connections between x_a and x_b.
F_1	G_2 fails before repairs to G_1 have begun.
F_2	G_2 fails before repairs to G_1 are completed.

FIGURE 19

Let us assume that $\lambda_1 = \lambda_2 = .002/\text{hour}$, $\mu_1 = \mu_2 = .004/\text{hour}$ and that delay rate $\rho = \dfrac{1}{(\text{average time before repairs begin})} = .2/\text{hour}$.

Then from the word description of figure (19), we can draw the transition graph figure(20)below.

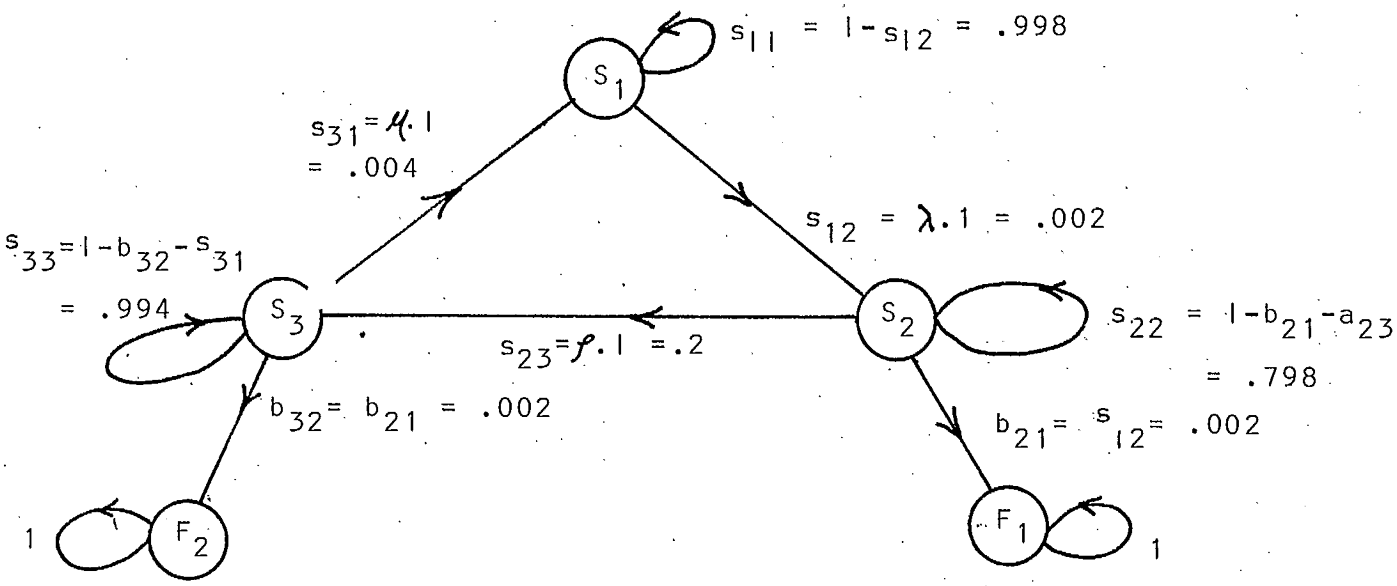


FIGURE 20

From figure (20) we can copy down [M] immediately and obtain figure (21).

[M] =

	S_1	S_2	S_3	F_1	F_2
S_1	.998	.002	0	0	0
S_2	0	.798	.2	.002	0
S_3	.004	0	.994	0	.002
F_1	0	0	0	1	0
F_2	0	0	0	0	1

$$= \left[\begin{array}{c|c} [S] & [B] \\ \hline [0] & [I] \end{array}\right]$$

FIGURE 21

We now have the matrices [S], [B] and [I] of [M] and can now do some numerical computation using a) to g).

For example <u>The reliability function R(n) gives the probability that a connection exists between x_a and x_b in figure (18) at time n</u>. We have that $s(n) = \bar{s}(o)\,[M]^n$; and $R(n) = \sum_{i=1}^{3} s_i(n)$.

Taking $\bar{s}(o) = [1,o,o]$ we find typically

$$R(3) = .999988, \quad R(4) = .999976$$

Similarily it can be shown that

$$\left[[I] - z\,[S]\right]^{-1} = \frac{1}{1-2.8z+2.6z^2-.8z^3}\begin{bmatrix} (1-.8z)\,(1-z), & .002z(1-z), & -.0004z^2 \\ .0008z^2, & (1-z)(1-z), & .2z(1-z) \\ .004z(1-.8z), & .000008z^2, & (1-z)\times(1-8z) \end{bmatrix}$$

Therefore we can easily find

$$[T(z)] = z\left[[I] - z\,[S]\right]^{-1}[B]$$

and, for example

$$[P] = \left[[I] - [S]\right]^{-1}[B]$$
$$= [T(z)]\Big|_{z=1}$$

and it is easily shown

$[P] =$

	F_1	F_2
S_1	.03	.97
S_2	.03	.97
S_3	.02	.98

We observe from $[P]$, that the small delay ($\rho = .2$/hr.) of average duration 5 hours, before repairs start to G_1 does not seriously affect the long run operating performance of the overall system. Specifically we see that failure will usually occur to F_2 rather than F_1.

Thank you for your attention.

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SLIDES

S L I D E I

PART I

Discussion of Present and Future Communication Systems In Canada

CATV Systems

Telephone Systems

The Next Fifteen Years

Switched Broadband Telecommunication Systems

Computer Utility Studies

PART II

Maximum Flow Techniques For Large Communication-Computer Networks

Analysis Problems

Synthesis Problems

Ford Fulkerson Theory

Practical Example of Flow Network Formulation of a Computer Network Problem

Gomory Hu Theory

Flow Equivalent Tree for Avalon Telephone Company Newfoundland

PART III

Reliability Prediction Theory

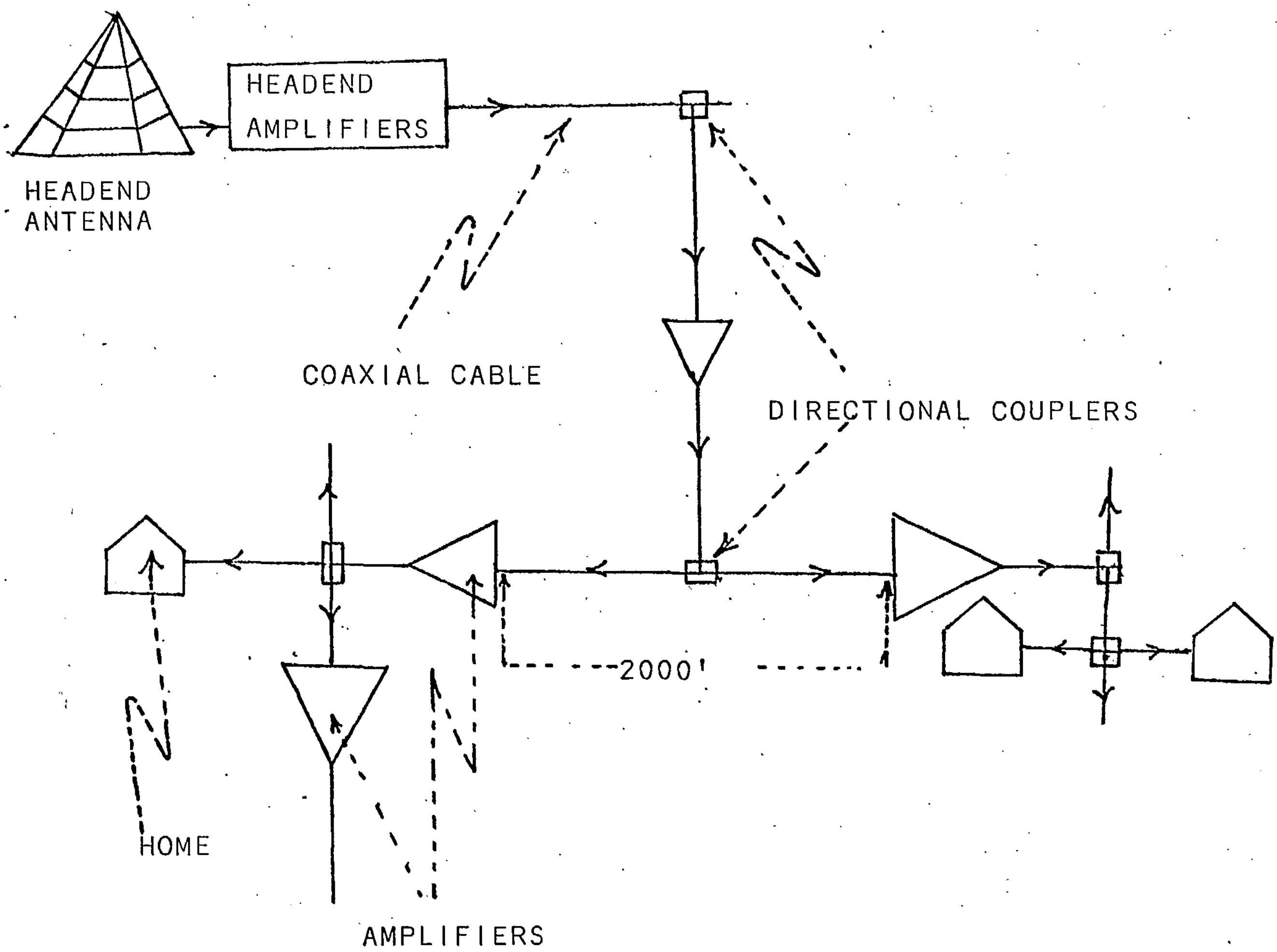
Reliability Models

Moments of Time to First Failed State

Example

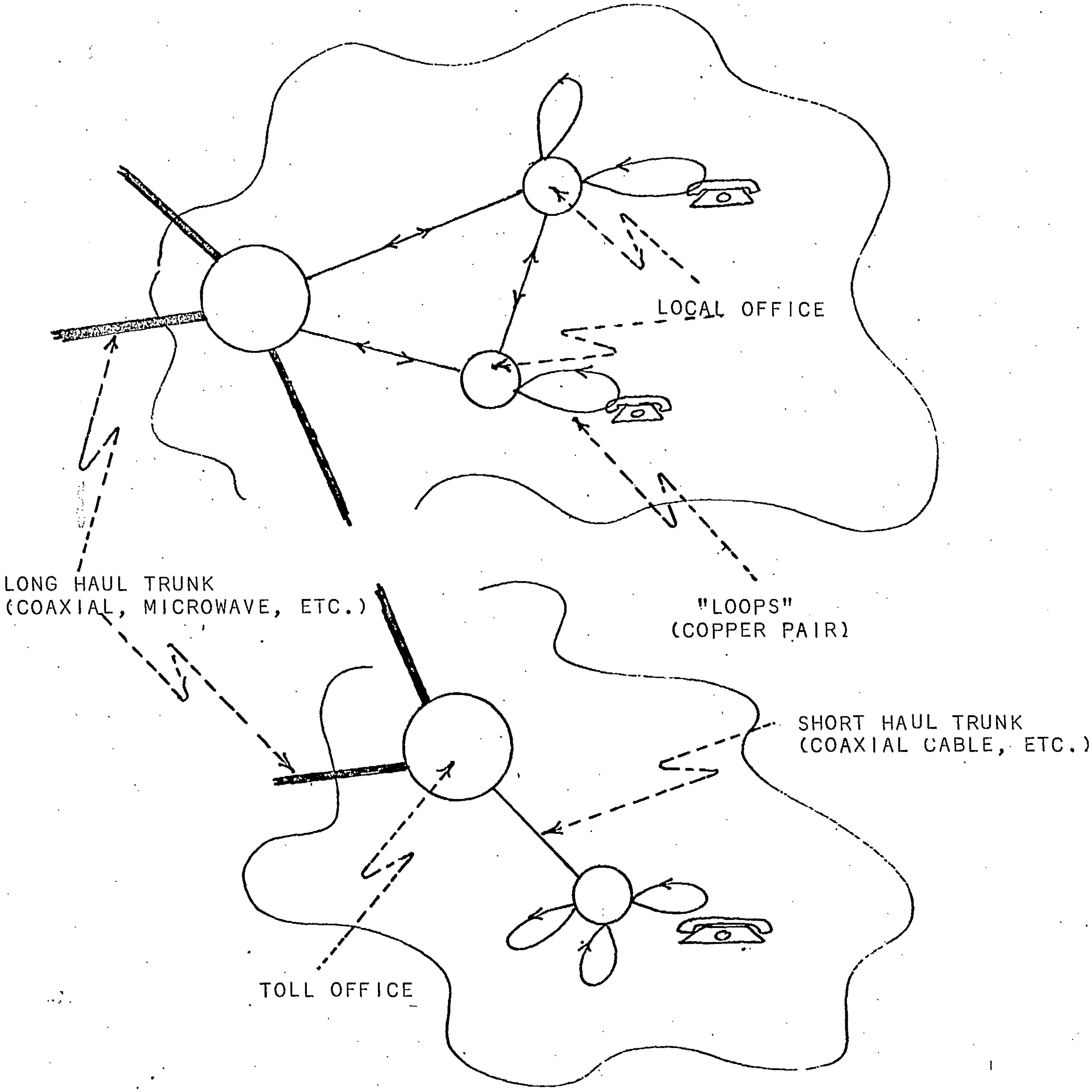
REFERENCES

SLIDE 2.



CATV SYSTEMS

SLIDE 3



TELEPHONE SYSTEMS

SLIDE 4

Directionality	Information Rate	Typical Example
One-Way	Low	Meter Reading
	Medium	Radio
	High	CATV
Two-Way	Low	Telephone Telex/TWX
		Computer to User,
	Medium	User to Computer, Videophone
	High	Computer-to Computer

Information rate	Maximum bit	Typical Type of Service
Low	50K bits/sec	Telex/TWX voice
Medium	7M bits/sec	Videophone
High	50M bits/sec	Television

S L I D E 5

THE NEXT FIFTEEN YEARS

1. MULTIPLICITY OF SERVICES

2. INCREASED TRAFFIC HANDLING CAPABILITY

3. INCREASE IN THE NUMBER OF MEDIUM AND HIGH INFORMATION RATE SERVICES

4. INCREASE IN THE NUMBER OF TWO WAY SERVICES

S L I D E 6

TOTAL COMMUNICATIONS

1. BROADCAST

   COMMERCIAL AND INSTRUCTIONAL TV

   COMMERCIAL AND INSTRUCTIONAL RADIO

2. REAL TIME POINT TO POINT

   TELEPHONE

   VIDEOPHONE

   TELEGRAPH AND TELETYPE

   CERTAIN COMPUTER SERVICES

3. STORE AND FORWARD

   COMPUTER SERVICES (time sharing and instruction)

   FACSIMILE, (newsprint and magazines, library access)

   FINANCIAL TRANSACTIONS (banking & remote purchasing)

   INTERROGATING (polling and meter reading)

   MAIL

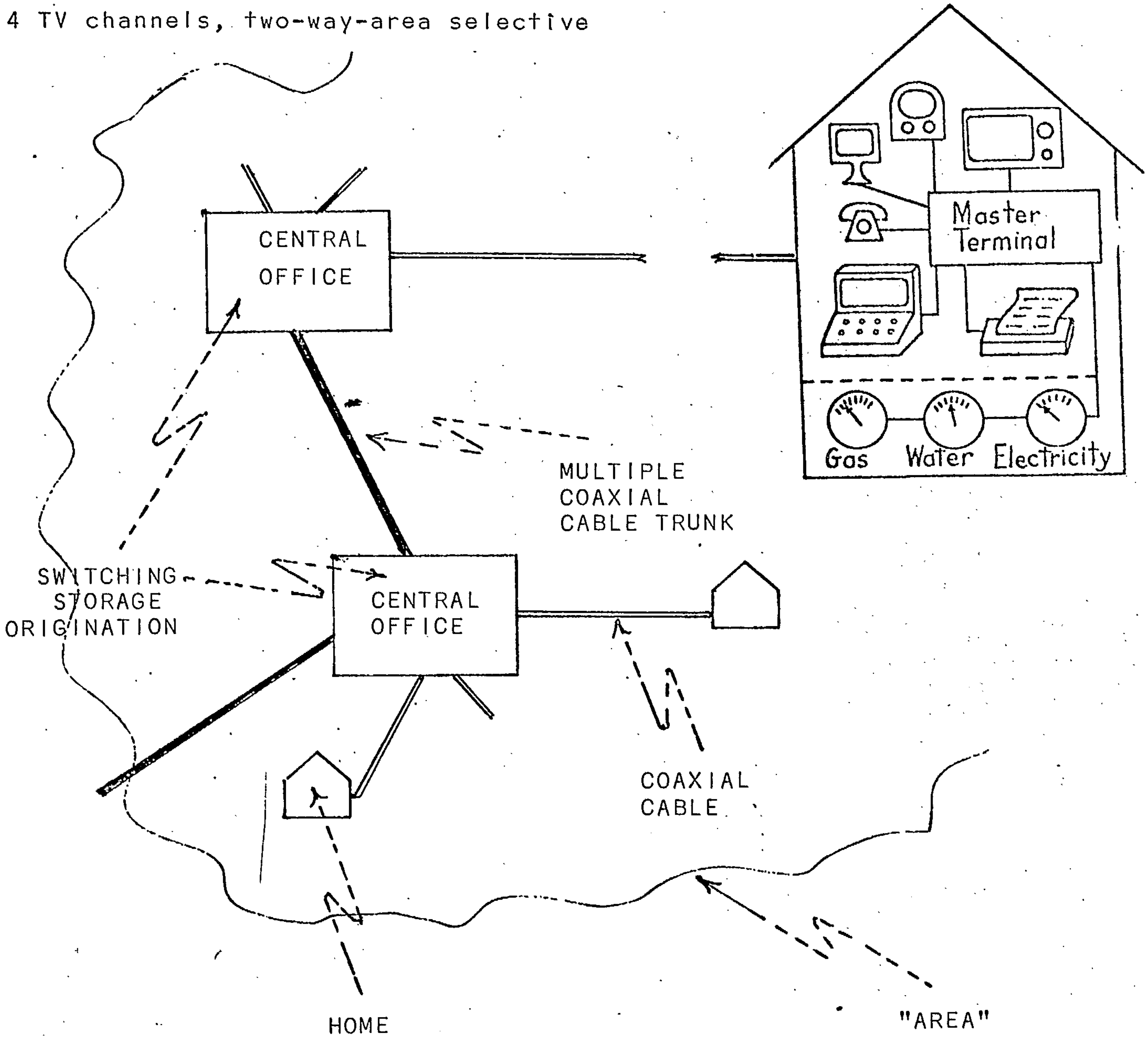
SLIDE 7

4 voice/data channels

12 TV channels, one-way-area selective

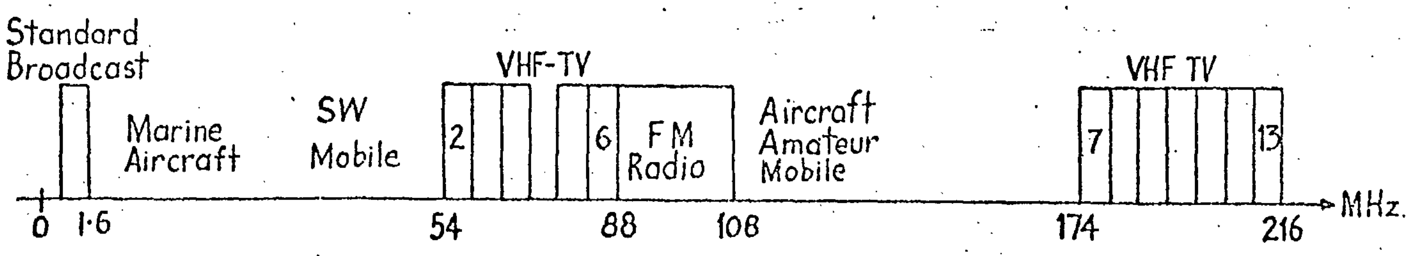
12 TV channels, one-way-subscriber selective

4 TV channels, two-way-area selective

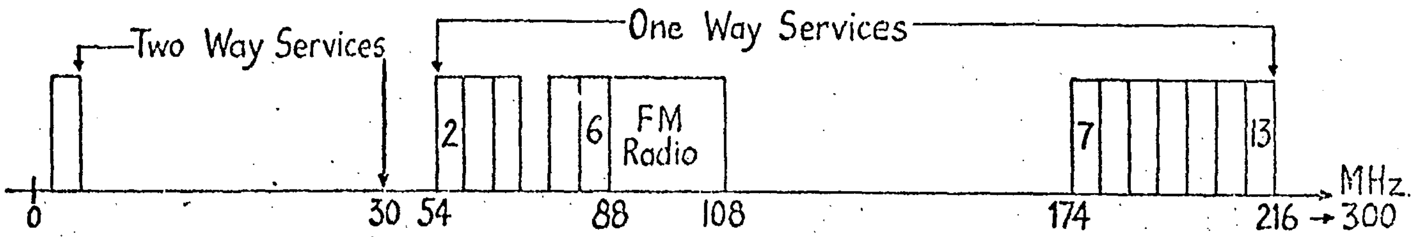


SWITCHED BROADBAND SYSTEMS

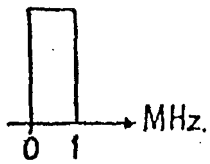
SLIDE 8



D.O.C. Allocation of the Radio Spectrum



Possible Coaxial Cable Spectrum Allocation



Usable Spectrum of Typical Copper Pair

SLIDE 9

POSSIBLE BROADBAND CABLE CONFIGURATIONS

I. MULTIPLE PAIRED (WIRES), EACH CARRYING SINGLE ANALOG SIGNALS.

II. SETS OF COAXIAL CABLES EACH CARRYING MULTIPLE ANALOG SIGNALS.

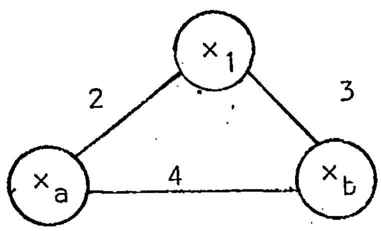
III. SETS OF COAXIAL CABLES EACH CARRYING MULTIPLE DIGITAL SIGNALS.

IV. SETS OF COAXIAL CABLES EACH CARRYING MULTIPLE DIGITAL AND ANALOG SIGNALS.

V. HYBRID COMBINATIONS OF MULTIPLE PAIRED WIRES AND COAXIAL CABLES.

SLIDE 10

NETWORKS AS GRAPHS



$$X \equiv \{x_a, x_1, x_b\}$$

$$A \equiv \{ (x_a x_b), (x_a x_1), (x_1 x_b) \}$$

$$B \equiv \{4, 2, 3\}$$

I) Analysis problems

Given (X,A,C), find $\phi^*(x_a, x_b)$

II) Synthesis problems

1) Given X,A,ϕ^* find C
2) Given X, C and ϕ^* find A
3) Given C and ϕ^* find X and A

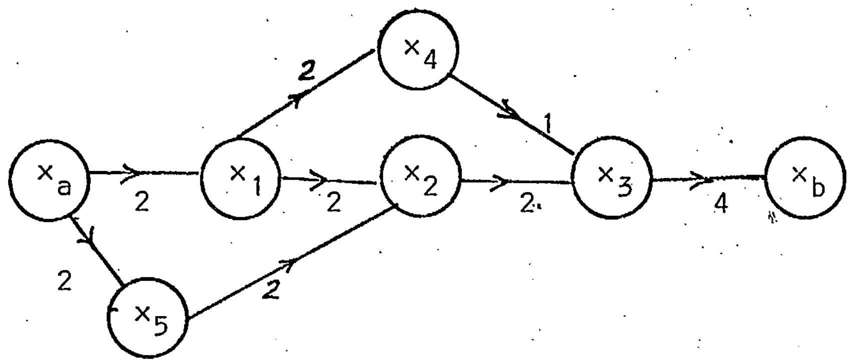
FORD-FULKERSON THEORY

THEOREM 1

In a transport network $\phi^*(x_a, x_b) \equiv \max \phi(x_a, x_b)$ is equal to the cut of minimum capacity separating x_a and x_b.

$$\phi^*(x_a, x_b) = \min_{\substack{x_a \in Y \\ x_b \in \overline{Y}}} c(Y\overline{Y})$$

EXAMPLE -- NETWORK OF STORE AND FORWARD SYSTEMS



$\emptyset^* (x_a, x_b) = 3$

SLIDE 13

EXAMPLE -- COMPUTER-COMMUNICATION NETWORK

CONSIDER A NUMBER OF INTERCONNECTED REMOTE OFFICES THAT EACH HAVE SMALL COMPUTERS OR TERMINALS $x_1, ---, x_n$. AT NIGHT MESSAGES FROM THESE SMALL COMPUTERS ARE FORWARDED TO A CENTRAL COMPUTER Y IN A GIVEN INTERVAL OF TIME, OR IF THIS IS NOT POSSIBLE, ARE STORED BACK IN A SMALL COMPUTER. LET $c(x_i x_j) = c_{ij}$ BE CHANNEL CAPACITY BETWEEN TWO OF THESE SMALL COMPUTERS. LET t_{ij} BE THE AVERAGE TIME IT TAKES A MESSAGE TO GO FROM x_i TO x_j OVER THE CHANNEL $(x_i x_j)$. Let $c_{ii} \equiv c(x_i x_i)$ BE THE NUMBER OF MESSAGES THAT CAN BE STORED AT x_i. THE PROBLEM IS NOW "HOW SHOULD THE OUTPUT OF THE SMALL COMPUTERS BE SCHEDULED, SO AS TO HAVE AS MANY MESSAGES AS POSSIBLE ARRIVE AT Y IN A GIVEN INTERVAL OF TIME θ".

GOMORY-HU METHOD

THE METHOD ESSENTIALLY IS A TECHNIQUE FOR TRANSFORMING IN n-1 APPLICATIONS OF THE FORD FULKERSON LABELLING METHOD NETWORK $G \equiv (X, A, C)$ INTO A FLOW EQUIVALENT NETWORK $G_1 \equiv (X, A_1, C_1)$ WHICH IS A TREE.

THEN $\phi^* (x_i, x_j)$, THE MAXIMUM FLOW BETWEEN ANY PAIR OF NODES IS THE SAME FOR G AND G_1.

$\phi^* \equiv$

	x_1	x_j	x_n
x_1	$\phi^*(x_1 x_1)$	$\phi^*(x_1 x_j)$	$\phi^*(x_1 x_n)$
x_i	$\phi^*(x_i x_1)$	$\phi^*(x_i x_j)$	$\phi^*(x_i x_n)$
x_n	$\phi^*(x_n x_1)$	$\phi^*(x_n x_j)$	$\phi^*(x_n x_n)$

BASIS FOR GOMORY-HU METHOD

THEOREM 2 Consider a network (X,A,C) with a path $\{(x_i,x_1), (x_1x_2), ---, (x_r,x_j)\}$ joining a source x_i and terminal x_j, then

$$\phi^*(x_ix_j) \geq \min\{\phi^*(x_ix_1), \quad \phi^*(x_1x_2), ---, \phi^*(x_rx_j)\} \text{ --- } 1$$

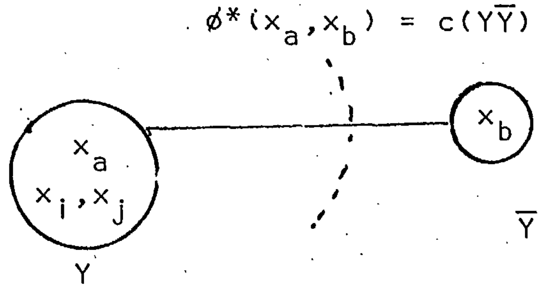
Theorem 3 Let $G \equiv (X,A,C)$ be a network (symmetric or pseudo symmetric). Let $(Y\bar{Y})$ be a minimum cut separating x_i and x_j,

i.e. $x_i \in Y$, $x_j \in \bar{Y}$ Let $G_1 \equiv (X,A_1,C_1)$ be the network obtained from G by condensing the set $\bar{Y}$ into a single node, and let x_a, x_b, be in Y.

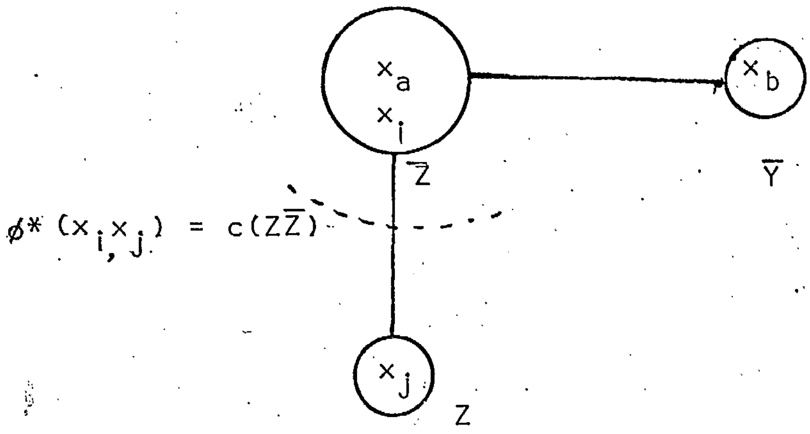
Then $\phi^*(x_a,x_b)$ in G is equal to $\phi^*(x_ax_b)$ in G_1 that is for the purpose of computing maximum flows in Y, the networks G and G_1 are flow equivalent.

GOMORY-HU METHOD

STEP 1

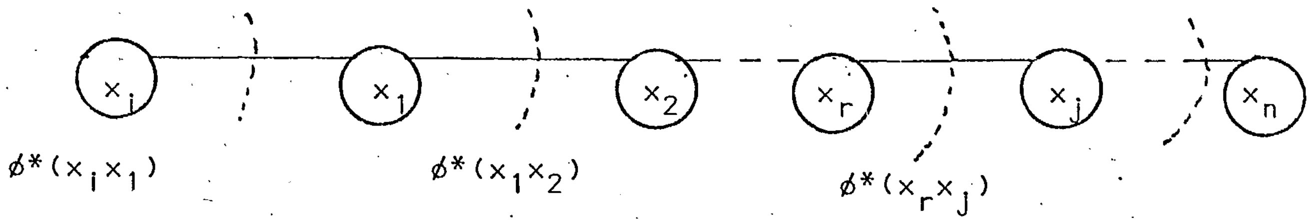


STEP 2



STEP n-1

$G_1 \equiv$



THEOREM 4 (Gomory-Hu, Gupta) Given a network $G \equiv (X, A, C)$;

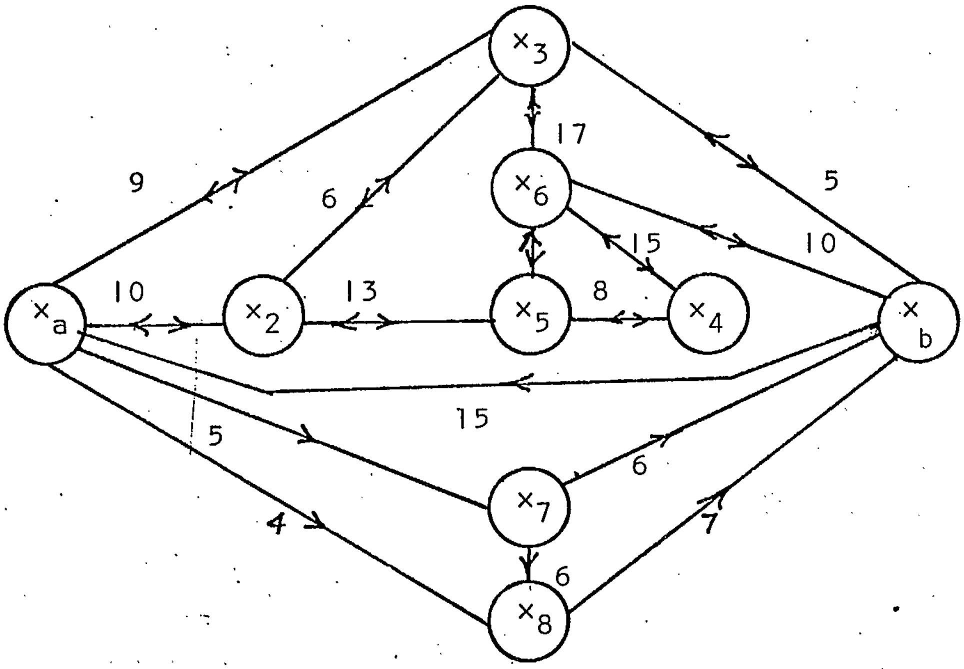
The maximum flow $\phi^*(x_i x_j)$ between any two nodes x_i and $x_j \in X$ is given as

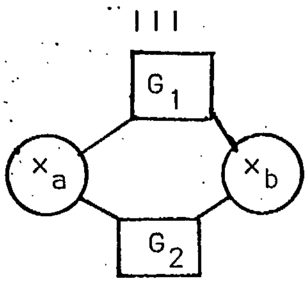
$$\phi^*(x_i x_j) = \min \left\{\phi^*(x_i x_1),\ \phi^*(x_1, x_2)\ \text{---}\ \phi^*(x_r x_j)\right\}$$

where $\phi^*(x_i, x_j)$, $j = 1, \text{---}, r$ are the values of the maximum flows along the path $(x_i x_1)$, --- $(x_r x_j)$ joining x_i and x_j in the flow equivalent tree G_1 obtained by the n-1 applications of the Ford Fulkerson method

SLIDE 17

EXAMPLE





$G_1 = (X_1, A_1, C_1)$

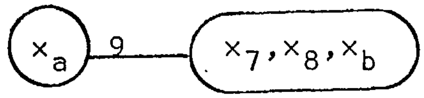
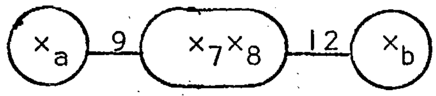
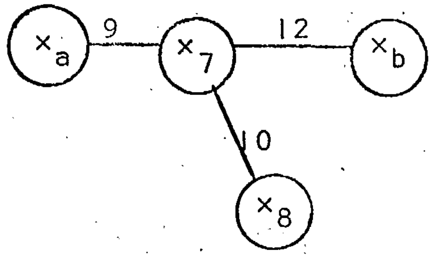
$X_1 = \{x_a, x_2, x_3, x_4, x_5, x_6, x_b\}$

$G_2 = (X_2, A_2, C_2)$

$X_2 = \{x_a, x_7, x_8, x_b\}$

EXAMPLE Cont'd

FLOW EQUIVALENT TREE FOR G_2

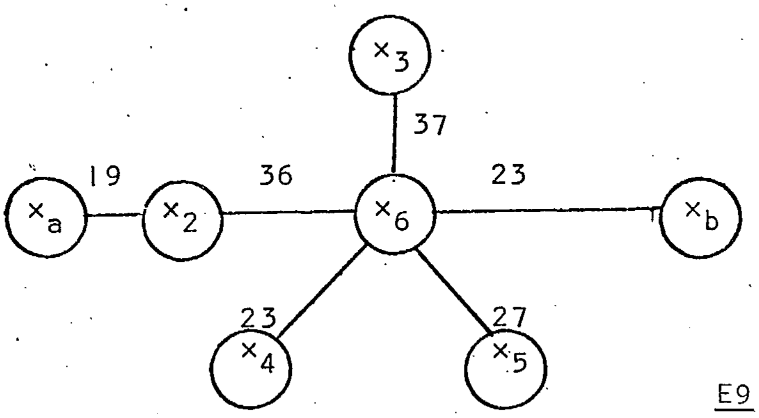
Iteration	x_i, x_j	Max. flow	Tree
1	x_a, x_b	9	
2	x_7, x_b	12	
3	x_7, x_8	10	 $\equiv G_2'$

$\phi_2^* \equiv$

	x_a	x_7	x_8	x_b
x_a	/	9	9	9
x_7	9	/	10	12
x_8	9	10	/	10
x_b	9	12	10	/

FLOW EQUIVALENT TREE FOR G_1

$G_1' \equiv$

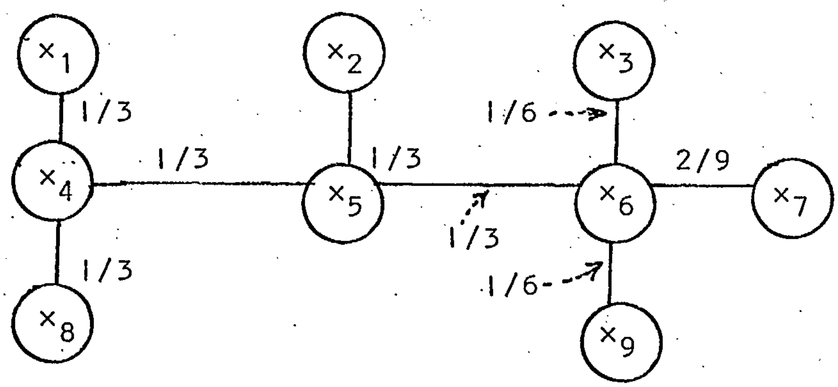


E9

$\phi^*(x_a x_5) = 19$

Flow Equivalent Tree For Avalon Telephone Company

NEWFOUNDLAND



Identification of Centres (see any map of Newfoundland)

$x_1 \equiv$ Freshwater

$x_2 \equiv$ Harbour Main

$x_3 \equiv$ Bay Roberts

$x_4 \equiv$ Cabinet

$x_5 \equiv$ Mile four Pond

$x_6 \equiv$ Kenmount

$x_7 \equiv$ St. John's

$x_8 \equiv$ Point La Haye

$x_9 \equiv$ Bell Island

RELIABILITY PREDICTION THEORY

NETWORKS (X,A,C) THAT OPERATE IN A REPAIR ENVIRONMENT, AND THAT HAVE $r (r \geq 1)$ SATISFACTORY AND $m, (m \geq 1)$ FAILED STATES.

I) RELIABILITY MODELS; IN PARTICULAR TO OBTAIN TIME DEPENDENT STATE TRANSITION FUNCTIONS, AS WELL AS A TIME DEPENDENT RELIABILITY FUNCTION.

II) MOMENTS OF TIME TO FIRST FAILURE; IN PARTICULAR TO OBTAIN THE MOMENTS OF THE FIRST TIME IT TAKES TO GO FROM SPECIFIED ACCEPTABLE STATES TO SPECIFIED FAILED STATES.

ASSUMPTIONS

a) THE FAILURE AND REPAIR RATES λ AND μ OF THE SUBSYSTEMS X AND A ARE KNOWN,

b) IT IS POSSIBLE FROM AN UNDERSTANDING OF THE OPERATION OF THE NETWORK TO SPECIFY A SET $\mathcal{S}$ OF SATISFACTORY STATES AND A SET $\mathcal{X}$ OF FAILED STATES.

THIS IS CALLED THE RELIABILITY PROBLEM

$$(X, A, C, \lambda, \mu, \mathcal{S}, \mathcal{X})$$

S L I D E 20

FOR $(X, A, C, \lambda, \mu, \mathcal{S}, \mathcal{F})$, THE FAILURE AND REPAIR RATES MULTIPLIED BY APPROPRIATE UNITS OF TIME, ARE THE ONE STEP STATE TRANSITION PROBABILITIES OF A STATIONARY MARKOV CHAIN WITH TRANSITION MATRIX [M]

$$[M] \equiv \begin{array}{r} \mathcal{S}\to\mathcal{S}\left\{\right. \\ \mathcal{F}\to\mathcal{S}\left\{\right. \end{array} \left|\begin{array}{c|c} [S] & [B] \\ \hline [0] & [I] \end{array}\right| \begin{array}{l} \left.\right\}\mathcal{S}\to\mathcal{F} \\ \left.\right\}\mathcal{F}\to\mathcal{F} \end{array}$$

[S] , is the (rxr) matrix of the one step transition probabilities between the satisfactory states in $\mathcal{S}$. The entries of [S] are denoted by $_{ik}$, etc.

[B] , is the (rxm) matrix of the one step transition probabilities from states in $\mathcal{S}$ to states in $\mathcal{F}$.

[I] , is the (mxm) unit matrix, which shows that transition between failed states are not allowed

[0] , is the (mxr) null matrix, which shows that transitions cannot take place from failed states in $\mathcal{F}$ to satisfactory states in $\mathcal{S}$.

1) <u>Reliability Models</u> For $(X, A, C, \lambda, \mathcal{M}, \mathcal{S}, \mathcal{F})$

Letting

a) $s_i(n)$ = Prob $\{$network is in state $S_i \epsilon \mathcal{S}$ at time n$\}$

b) $P_{ij}(n)$ = Prob $\{$network goes from state $S_i \epsilon \mathcal{S}$ to state $F_j \epsilon \mathcal{F}$ in time n$\}$

c) $p_{ij} = \lim_{n \to \infty} p_{ij}(n)$

$$M \equiv \begin{array}{c|c} S & B \\ \hline 0 & I \end{array}$$

It can be shown

a) $\overline{s}(n) = \overline{s(o)}\,[M]^n$: $\overline{s}(n) \equiv$ 1x(rxm) row vector of $s_i(n)$

b) $[P(n)] = [S]^{n-1}\,[B]$: $[P(n)] \equiv$ (rxm) matrix of $p_{ij}(n)$

c) $[P] = \left[[I] - [S]\right]^{-1}\,[B]$: $[P] \equiv$ (rxm) matrix of p_{ij}

therefore we can define a reliability function R(n) as

d) $R(n) \equiv$ Prob $\{$System is in state in $\mathcal{S}$ at time n$\}$

$$= \sum_{\mathcal{S}} s_i(n)$$

II) Moments of the First Time to Failed State

Letting

e) τ_{ij} be the random variable -- "time taken for first transition from successful state S_i to failed state F_j".

f) $\tau_{ij}(k)$ be the k^{th} moment of τ_{ij} .

g) $t_{ij}(z)$ be the generating function for the moments $\tau_{ij}(k)$.

It can be shown

h) $$[\tau(k)] = \frac{d^k}{dz^k} \left([T(z)] \right) \Big|_{z=1} , \quad k=1,2,\text{--}$$: $[\tau(k)] \equiv$ (rxm) matrix of $\tau_{ij}(k)$.

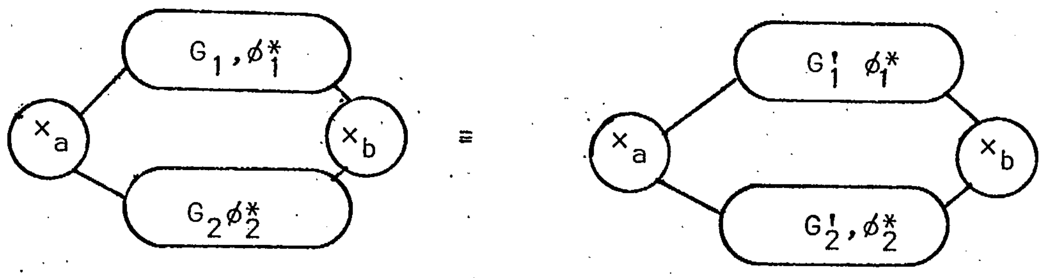
g) $$[T(z)] = z \left[[I] - z [S] \right]^{-1} B$$: $[T(z)] \equiv$ (rxm) matrix of $t_{ij}(z)$

COMMENTS

(1) THESE TECHNIQUES, ARE IMMEDIATELY USABLE BY ANY ENGINEER, WHO UNDERSTANDS HOW THE NETWORK (X,A,C) OPERATES AND CAN THEREFORE SPECIFY HIS SATISFACTORY STATES $\mathcal{S}$ AND FAILED STATES $\mathcal{I}$ AND THUS DETERMINE THE PARTITION MATRICES $[\bar{S}]$, $[B]$ AND $[I]$ OF $[\bar{M}]$.

(2) IN AN IMPORTANT SENSE, WE CAN ALSO USE THESE TECHNIQUES TO DO SYNTHESIS. THIS IS BECAUSE BY KNOWING THE FAILURE AND REPAIR RATES $\{\lambda\}$ AND $\{\mu\}$ WHICH ARE THE ENTRIES OF FOR A NUMBER OF POSSIBLE SYSTEMS $\{X\}$ AND $\{A\}$ THAT COULD BE USED TO BUILD THE NETWORK (X,A,C), WE CAN DO A COMPUTER SIMULATION AND SELECT THE SUBSYSTEMS $X \in \{X\}$ AND $A \in \{A\}$ THAT GIVE THE "BEST" R(n) AND $\{\tau(k)\}$ FOR THE NETWORK (X,A,C).

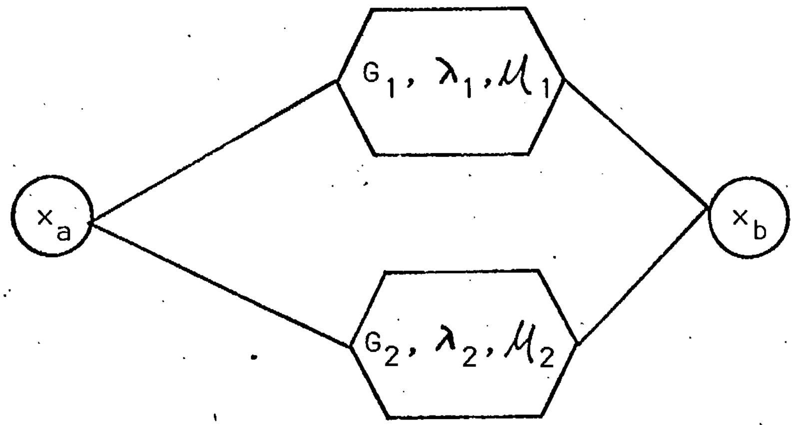
EXAMPLE



$$\lambda_1 = \frac{1}{\text{smallest}\ \{\tau_{ij}(1)\}\ \text{for}\ G_1}\ ;\ \lambda_2 = \frac{1}{\text{smallest}\ \{\tau_{ij}\ (1)\}\ \text{for}\ G_2}.$$

$$\mu_1 = \frac{1}{\text{largest repair time in}\ \left\{\frac{1}{\mu_1}\right\}\ \text{for}\ G_1}\quad \mu_2 = \frac{1}{\text{largest repair time in}\ \left\{\frac{1}{\mu_2}\right\}\ \text{for}\ G_2}$$

EXAMPLE Cont'd



STATE	WORD DESCRIPTION
S_1	Both G_1 and G_2 provide a path from x_a to x_b.
S_2	G_1 fails and the only path is provided by G_2. Repairs to G_1 are not yet started.
S_3	Repairs to G_1 start. G_2 is still providing the connections between x_a and x_b.
F_1	G_2 fails before repairs to G_1 have begun.
F_2	G_2 fails before repairs to G_1 are completed.

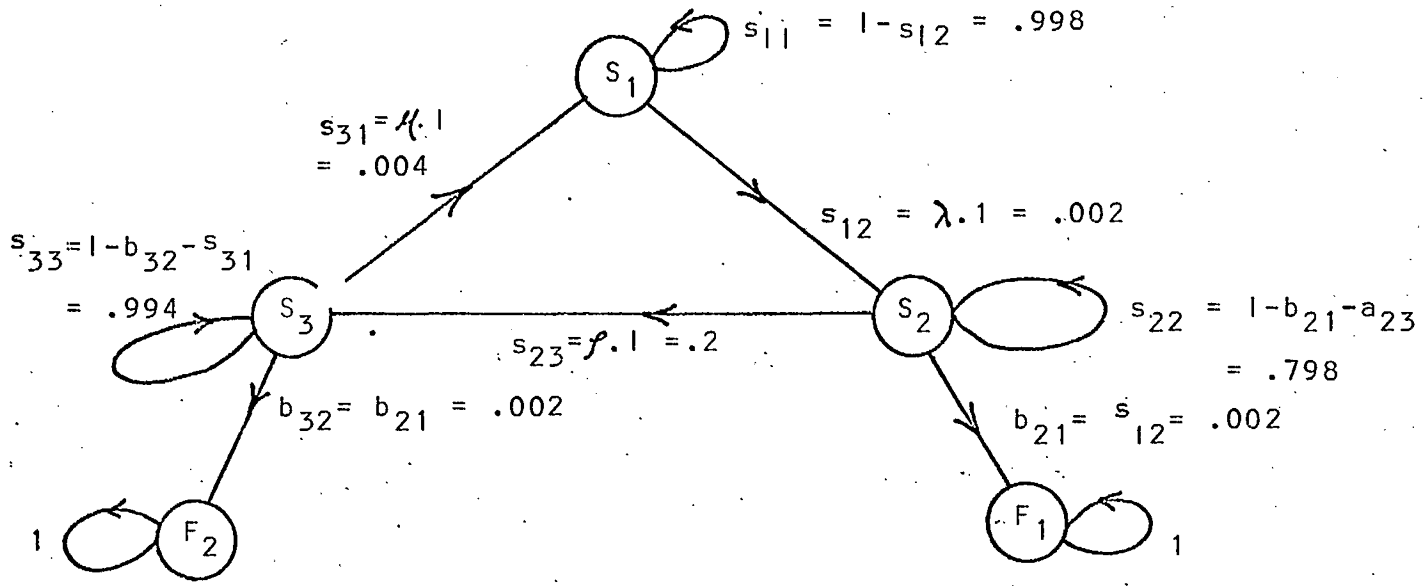
ASSUME

$\lambda_1 = \lambda_2 = .002/hr.$

$\mu_1 = \mu_2 = .004/hr.$

repair rate $\rho = .2/hr.$

EXAMPLE Cont'd



$[M] =$

	S_1	S_2	S_3	F_1	F_2
S_1	.998	.002	0	0	0
S_2	0	.798	.2	.002	0
S_3	.004	0	.994	0	.002
F_1	0	0	0	1	0
F_2	0	0	0	0	1

$$= \left[\begin{array}{c|c} [S] & [B] \\ \hline [0] & [I] \end{array}\right]$$

EXAMPLE Cont'd

THE STATE PROBABILITY VECTOR $\overline{s(n)}$ SATISFIES

$$\overline{s(n)} = [M]^n \quad ; \; \overline{s} \equiv (1 \times 5) \text{ row vector}$$

SINCE THERE ARE ONLY THREE SATISFACTORY STATES

$$R(n) = s_1(n) + s_2(n) + s_3(n)$$

AND WE FIND TYPICALLY

$$R(3) = .999988, \quad R(4) = .999976$$

etc.

R(n) IS THE PROBABILITY THAT A CONNECTION EXISTS BETWEEN x_a AND x_b AT TIME n.

SIMILARILY

$$[P] = [G(z)]\Big|_{z=1}$$

$[P] \equiv$

	F_1	F_2
S_1	.03	.97
S_2	.03	.97
S_3	.02	.98

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COMMENT

THE MORE GENERAL CLASS OF NETWORKS (X,A,C) WHICH ARE DEGRADATION-PRONE, THAT IS WHICH CAN BE DESCRIBED BY A SET OF DEGRADATION-PRONE PARAMETERS IS TREATED IN THE PAPER [20]. THE ASSUMPTION MADE THERE, IS THAT THE TIME FUNCTION CHARACTERIZING EACH DEGRADATION-PRONE PARAMETER, IS A SAMPLE FUNCTION OF A FELLER-MARKOV PROCESS, THAT IS A SAMPLE FUNCTION OF A CONTINUOUS DIFFUSION PROCESS.

DE MERCADO, JOHN,
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