Government of Canada
Department of Communications

Gouvernement du Canada
Ministère des Communications

CAPACITY DEGRADATION OF PACKET RADIO FADING CHANNELS

by

J. Schwarz daSilva

April 79

Tele
Reg

Service de la Réglementation
des Télécommunications

Government of Canada
Department of Communications

Gouvernement du Canada
Ministère des Communications

CAPACITY DEGRADATION OF PACKET RADIO FADING CHANNELS

by

J. Schwarz daSilva

April 79

Telecommunication
Regulatory Service

Service de la Réglementation
des Télécommunications

ABSTRACT

A mathematical model is developed to determine the probability of successful transmission of a packet over a fading land mobile data channel.

The model is further extended to packet radio channels and the throughput degradation of three random access schemes is analyzed. A relationship between throughput degradation and distance from the base station is finally derived and numerical results presented.

I. INTRODUCTION

Several authors [1], [2] have shown that packet radio, brought about by the proliferation of computer resources and the need to offer communication capabilities, in the form of mobile radio channels, leads to a considerably efficient spectrum use.

The problem of designing packet radio systems encompasses a number of system variables such as the network topology, channel configuration, access policy and error control and flow control techniques. A review of the available literature on packet radio shows however that the effect of channel errors is largely ignored. The assumption is generally made that packet retransmissions due to collisions outnumber the retransmissions due to channel errors. A previous study [3] by the same authors that addressed the question of finding the optimal packet size for a fading mobile radio channel indicated that the error characteristics of such channels should not be ignored.

Fading occurs when more than one signal from the same transmitter reaches the receiver and causes either a nulling or a reinforcement of the direct path signal. Fading phenomena are often characterized by a specific type of short term multipath signal reception whose amplitude can be approximated very closely by the Rayleigh distribution over short distances. Large area variations of the received signal envelope are found to be lognormally distributed [4].

In this paper, we examine the throughput degradation of three classical random access schemes, namely Pure ALOHA, Slotted ALOHA and Non Persistent CSMA, in the presence of Rayleigh fading.

A mathematical model is used to determine the probability of successful packet transmission assuming a single user. The model is then extended to three random access schemes and the relation between channel capacity and distance from the base station is derived.

We assume that transmission errors, in the absence of packet collisions, are only due to fast fading i.e errors due to ignition noise or slow fading are not considered. We further assume that the acknowledgement traffic is carried by a separate channel and arrives at the terminals reliably and at no cost and we ignore propagation delays.

II. Model

Consider the characteristics of a Rayleigh fading mobile radio channel where lognormal variations of the received mean power are not taken into account. The average level crossing rate, N_ρ, and average fade duration, τ_ρ, are given by [5]:

$$N_\rho = f_D \sqrt{2\pi\rho} \quad \exp(-\rho) \qquad \text{.......... (1)}$$

$$\tau_\rho = \frac{\exp(\rho) \quad -1}{f_D \sqrt{2\pi\rho}} \qquad \text{.......... (2)}$$

where $\rho = x/x_o$ is the ratio of the instantaneous signal power level to the R.M.S. power level and f_D the Doppler frequency is given by:

$$f_D = \frac{v}{\lambda_c} \qquad \text{.......... (3)}$$

where v is the vehicle speed and λ_c the carrier wavelength.

Since we are dealing with a Rayleigh fading channel we have:

$$\tau_\rho \, N_\rho = 1-\exp(-\rho) \qquad \text{.......... (4)}$$

Studies [6] of these Rayleigh fading channels at frequencies of 850 Mhz have indicated that the interfade interval distribution (or time between consecutive fade arrivals) and the fade width distribution are closely approximated by an exponential distribution.

Denoting by Z and X the random variables corresponding to the interfade duration and the fade duration we can generally write:

$$f_Z(t) = \lambda_1 \exp(-\lambda_1 t) \qquad \text{.......... (5)}$$

and

$$f_X(t) = \lambda_2 \exp(-\lambda_2 t) \qquad \text{.......... (6)}$$

Relating equation (1) and (2) with (5) and (6) we have:

$$N_\rho = \lambda_1 \qquad \text{.......... (7)}$$

$$\tau_\rho = \frac{1}{\lambda_2} \qquad \text{.......... (8)}$$

That is for a given value of ρ we obtain the corresponding values for N_ρ and τ_ρ and consider these to be the average fade arrival rate (of a Poisson process) and the average fade duration.

For a given power level we can then consider on a time axis a sequence of fades followed by no fade gaps as follows:

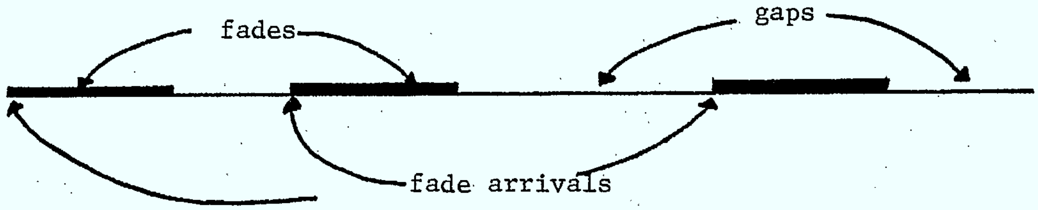


FIGURE 1

Assuming that the random variables X and Z are independent we can easily obtain the probability density function of the random variable Y representing the gap duration. Denoting by $\Psi_Y(s)$ the Laplace transform of the unknown density function of the random variable Y we obtain:

$$\Psi_Y(s) = \frac{\lambda_1}{\lambda_2} \left(\frac{s+\lambda_2}{s+\lambda_1} \right) \qquad \text{.......... (9)}$$

To obtain the Laplace transform of the cumulative distribution of Y we divide $\Psi_Y(s)$ by s:

$$\frac{\Psi_Y(s)}{s} = \frac{1}{s} - \left(1 - \frac{\lambda_1}{\lambda_2}\right) \left(\frac{1}{s+\lambda_1}\right) \qquad \text{.......... (10)}$$

Inverting the transform we have:

$$P[Y \leq t] = 1 - \left(1 - \frac{\lambda_1}{\lambda_2}\right) \exp(-\lambda_1 t) \qquad \text{.......... (11)}$$

Replacing λ_1 and λ_2 by the corresponding equations (7) and (8) we finally obtain:

$$P[Y \leq t] = 1 - (1-N_\rho \tau_\rho) \exp(-N_\rho t) \qquad \text{.......... (12)}$$

Using (4) we have:

$$P[Y \leq t] = 1 - \exp(-\rho-N_\rho t) \qquad \text{.......... (13)}$$

Assume now that a packet of data of length B bits is transmitted on the radio channel at a speed of C bits/sec. The packet transmission time will be given by:

$$T = \frac{B}{C} \qquad \text{.......... (14)}$$

This packet will be successfully transmitted (no errors) if it can be totally contained within a gap of length greater or equal than T. If we assume that the packet arrives at a time t_o from the beginning of a given gap, it will be successfully transmitted with probability:

$$P[Y-t_o>T / Y>t_o] = \frac{P[Y>T+t_o]}{P[Y>t_o]} \qquad \text{..........(15)}$$

After a few manipulations equation (15) will be written as:

$$P\left[Y-t_o \geq T \,/\, Y>t_o\right] = \exp(-N_\rho T) \qquad \ldots\ldots\ldots\ldots (16)$$

We note that the answer given by (16) is memoryless in the sense that it is not a function of t_o. However, we also note that the distribution function given by (13) has a singularity at the origin, hence the answer obtained above is valid if and only if $t_o>0$.

Considering the process, depicted in Figure 1, as a renewal process the probability of successful transmission of a packet is given by:

$$P_s = \frac{\bar{Y}}{\bar{Z}} \exp(-N_\rho T) \qquad \ldots\ldots\ldots\ldots (17)$$

The average value of Z is simply given by $1/N_\rho$ while the average value of Y can easily be obtained from (9) as follows:

$$\bar{Y} = -\frac{d\Psi_Y(s)}{ds} \Big/_{s=o} = \frac{\lambda_2-\lambda_1}{\lambda_2 \cdot \lambda_1} \qquad \ldots\ldots\ldots\ldots (18)$$

Replacing again by the appropriate expressions (7) and (8) we obtain:

$$\bar{Y} = \frac{1-N_\rho \tau_\rho}{N_\rho} \qquad \ldots\ldots\ldots\ldots (19)$$

Hence P_s will be given by:

$$P_s = \exp\left[- (\rho + N_\rho T)\right] \qquad \text{.......... (20)}$$

III. Random Access Channels

The expression obtained above for the probability of successful packet transmission assumed that only one terminal was using the radio channel. However, if we consider that this channel is a random access channel equation (20) should be modified to include the effects of the random access scheme that is used. In particular we can consider the following three random access schemes with their corresponding throughput-traffic characteristics:

Pure ALOHA

$$S = G \exp(-2G) \qquad \text{.......... (21)}$$

Slotted ALOHA

$$S = G \exp(-G) \qquad \text{.......... (22)}$$

Non Persistent CSMA

$$S = \frac{G \exp(-\beta G)}{G(1+2\beta) + \exp(-\beta G)} \qquad \text{.......... (23)}$$

where S is the channel input rate (average number of new packets generated per packet transmission interval T), G is the channel traffic rate (average number of new packets plus previously collided packets generated per packet transmission time) and β is the normalized propagation delay (ratio of propagation delay to packet transmission time)

In the available analysis of maximum channel utilization of these schemes the effect of channel errors (random noise, fading etc) is ignored. Hence only those packets that collided with other packets are part of the traffic rate G. However, as we have seen, even in the absence of transmission from other terminals there is a finite probability of packet error.*

By applying equation (20) to the three selected random access schemes we obtain:

Pure ALOHA

$$S = G \exp\left[-(2G+\rho+N_\rho T)\right] \qquad \text{.......... (24)}$$

Slotted ALOHA

$$S = G \exp\left[-(G+\rho+N_\rho T)\right] \qquad \text{.......... (25)}$$

Non Persistent CSMA

$$S \doteq \frac{G \exp\left[-(\beta G+\rho+N_\rho T)\right]}{G(1+2\beta) - \exp(-\beta G)} \qquad \text{.......... (26)}$$

We note, however, that G should now be interpreted as the traffic rate due to new packets, collisions and errors due to fading. It is also

* We note that for the Reservation ALOHA [7] system, packet collisions do not occur however packet errors do occur.

clear that equation (20) gives the probability of successful packet transmission as a function of ρ, which is clearly a function of the distance between transmitter and receiver. Hence the degradation in throughput increases as the mobile terminals move away from the center of the system.

Denoting by x_t the receiver threshold level and by x_o the average received power we can write:

$$\rho_t = \frac{x_t}{x_o} \qquad \text{.......... (27)}$$

If we express ρ_t in dB we have:

$$\rho_t(\text{dB}) = 10 \ \log\left(\frac{x_t}{x_o}\right) \qquad \text{.......... (28)}$$

Hence if our receiver is located at some distance D from the transmitter the average received power will be $x_o(D)$. So long as the receiver threshold level x_t is below the average received power there will be "reliable" communications. However, as x_t approaches and exceeds x_o there will be a loss of communication since the signal will be buried in the noise.

If we now consider equation (20) we remark that for a given receiver threshold level as the terminal moves away from the receiver there will be an increase in ρ as well as an increase in N_ρ. As shown by Jakes [5] the maximum value of N_ρ is attained for a value of ρ of about -3dB. Above this point there will be a decrease in N_ρ.

As an example assume a carrier frequency of 850 Mhz, a vehicle speed ranging from 30 to 60 km/hour, and a packet transmission time of 0.1 sec. For various values of ρ and the corresponding values of N_ρ we obtain the following results for P_s as given by equation (20):

ρ(dB)	V(km/h)	N(fades/sec)	P_s	ρ(dB)	V(km/h)	N(fades/sec)	P_s
-30	30	1.87	0.83	-30	60	3.74	0.68
-20	30	5.86	0.55	-20	60	11.72	0.30
-10	30	16.93	0.17	-10	60	23.86	0.08
0	30	21.7	0.04	0	60	43.4	0.004

Applying these results to equation (24) (25) and (26) we obtain the curves depicted in Figure 2. Note for each access mode, Figure 2 only consideres the degradation in channel capacity.

IV. PATH LOSS MODEL

The above results indicate that there is a degradation in throughput when errors due to fades are considered. It was furthermore shown that such errors tend to increase as the mobile terminal moves away from the central station. It is therefore important to relate the distance to the value of ρ. To do so we consider a path loss model proposed by Okumura [8].

The total path loss attenuation is then approximated by

$$P_L(\text{dB}) \approx 37 + 20 \log f + 20 \log D + A_{Fs} \qquad \text{.......... (29)}$$

where:

f = frequency in MHz

D = distance in miles

A_{Fs} = median attenuation relative to free space (dB)

Using average effective transmissions heights for the base station and terminal of 45 meters and 3 meters respectively, the median attenuation A_{Fs} can be approximated by:

$$A_{Fs} = 13.3 \log D + 36 \qquad \text{(for } D \lesssim 14 \text{ miles)} \qquad \text{.......... (30)}$$

Hence at a carrier frequency of 850 MHz the total path loss will be:

$$P_L(\text{dB}) \approx 131.6 + 33.3 \log D \qquad \text{(for } D \lesssim 14 \text{ miles)} \qquad \text{.......... (31)}$$

and the following table results:

D(mi)	1	2	5	8	10	14
P_L(dB)	131.6	141.6	154.9	161.7	164.9	169.7

If we now assume an effective radiated power of +40 dBm and a received threshold power of -130 dBm we obtain the following relationship between ρ and the distance:

D(miles)	1	2	5	8	10	14
ρ(dB)	-38.4	-28.4	-15.1	-8.3	-5.1	-0.3

Comparing these values with the curves of Figure 2 it becomes clear that if the average distance between the mobile terminals and the base station exceeds 2 miles the throughput degradation becomes substantial.

CONCLUSION

The above analysis has provided a means by which the throughput degradation of packet radio fading channels can be evaluated. We have shown the effects of system parameters such as vehicle speed and relative power level. Other variables such as the carrier frequency, the channel speed or the packet length were not explicitly considered since their effects can be easily evaluated. We note however that for low values of ρ the dominant factor in the probability of successful packet transmission is the ratio of packet transmission time to the average interfade duration.

Numerical results obtained for typical values of mobile transmit power and receiver threshold level indicate that the channel capacity degradation can attain one order of magnitude whenever the distance between the mobile station and the base station exceeds 5 miles.

REFERENCES

[1] N. Abramson, "The Throughput of Packet Broadcasting Channels" IEEE Trans. on Communications, Vol. COM-25, No. 1, pp 117-128, January 1977.

[2] R.E. Kahn, "The Organization of Computer Resources into a Packet Radio Network". IEEE Trans. on Communications, Vol. COM-25, No. 1, pp 169-178, January 1977.

[3] J.S. daSilva, H.M. Hafez, S. Mahmoud, "Optimal Packet Size for Fading Land Mobile Data Channels". Technical Report, Telecommunications Regulatory Service, Department of Communications, Ottawa, Canada, December 1978.

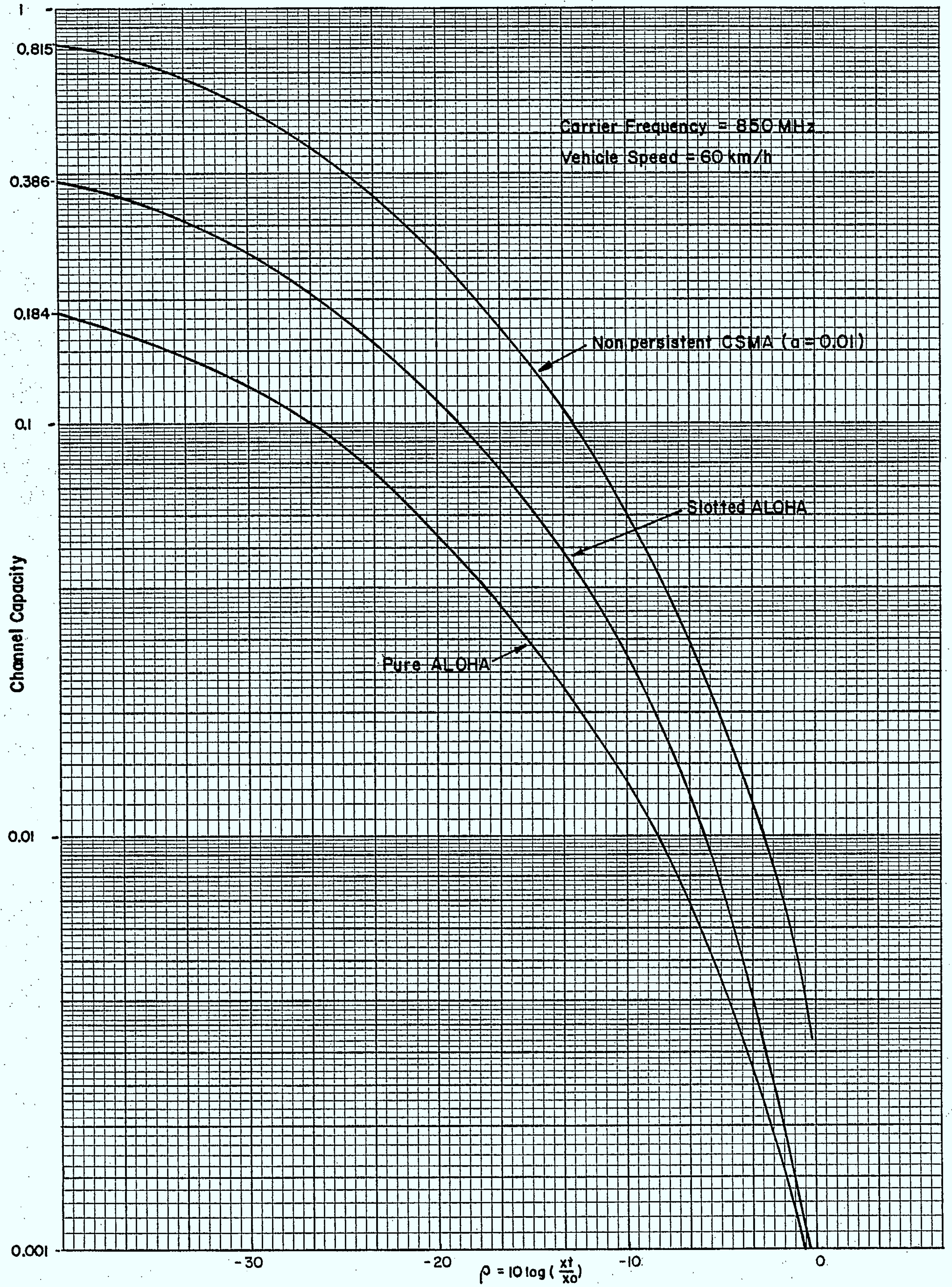
[4] F. Hansen, F.I. Meno "Mobile Fading - Rayleigh and Lognormal Superimposed" IEEE Trans. on Vehicular Technology, Vol. VT-26, No. 4, November 1977.

[5] W.J. Jakes "Microwave Mobile Communications" John Wiley, 1974.

[6] G. Arredondo, J. Smith "Voice and Data Transmission in a Mobile Radio Channel at 850 MHz". IEEE Trans. on Vehicular Technology, Vol. VT-26, No. 1, February 1977.

[7] L.G. Roberts "Dynamic Allocation of Satellite Capacity through Packet Reservation". National Computer Conference, Vol. 42, pp 711-716, June 1973.

[8] Y. Okumura "Field Strength and its Variability in VHF and UHF Land-Mobile Radio Service" Rev. Elec. Comm. Lab, Vol. 16, pp 825-878, September 1968.


Carrier Frequency = 850 MHz
Vehicle Speed = 60 km/h
Non persistent CSMA (a = 0.01)
Slotted ALOHA
Pure ALOHA
Channel Capacity
1
0.815
0.386
0.184
0.1
0.01
0.001
-30
-20
-10
0
ρ = 10 log (xt / xo)