RATE ADJUSTMENT GUIDELINES FOR REGULATED INDUSTRIES:

A MODEL FOR BELL CANADA

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"RATE ADJUSTMENT GUIDELINES FOR REGULATED INDUSTRIES: A MODEL FOR BELL CANADA"

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FORWARD

Sir George Williams University and L'Ecole des Hautes Etudes Commerciales affiliated to the Université de Montréal jointly established on June 2nd 1969 the International Institute of Quantitative Economics (I.I.Q.E.) to initiate original research and promote international scientific collaboration in the field of quantitative economics.

A major reorganization of the I.I.Q.E. took place in April 1976 resulting in the adopting of a new policy statement and set of objectives as well as the renaming of the I.I.Q.E. to the Institute of Applied Economic Research (I.A.E.R.). Consequently, the I.A.E.R. located at the Sir George Williams Campus, has been established as Concordia University's institute for programs of socio-economic research and training related to both the developing world and Canada.

Nations both rich and poor, individually and collectively share many common domestic and international problems, which contribute to the growing threat of global deterioration. Prominent among these problems are the need for economic development of less developed countries and the need for readjustments in the economic policies of industrialized societies. Recognition of the importance of these problems should lead institutions and interested individuals to apply existing socio-economic knowledge to their solution.

The I.A.E.R. believes that a major step towards finding acceptable solutions to the above problems is domestic and international cooperation. To this end, the I.A.E.R. utilizes the most modern methods of scientific analysis available, as well as the services of internationally recognized experts in the relevant fields in:

1) initiating, organizing and implementing major economic research projects, at both international and Canadian levels, occasionally in collaboration with other research institutes and interested specialists;

2) organizing seminars and conferences on specific economic issues of particular international and Canadian interest;

3) serving as a link between the Department of Economics, Sir George Williams Campus and other Departments of Concordia University and the Canadian private sector with the objective of increasing the latter's awareness of, participation in and support for applied economic research.

The I.A.E.R. believes that it has a necessary and useful role to play in both Canada and the developing world, particularly Latin America and Francophone Africa, given the accumulated experience and expertise of its research staff.

Professor V. Corbo
Director

INTRODUCTION

The purpose of this study is twofold. First to develop a testable model of the functioning of a regulated public utility, in our case Bell Canada. Second to use the said model to develop a formula for automatic rate adjustments applied to the same company.

In the development of a model to study the behaviour of this regulated public utility we assume that Bell Canada maximizes profits subject to a given production function and a rate of return constraint. We study in detail the specification of technology and we conclude that Bell Canada technology can be approximated by a Cobb-Douglas production function on Capital and Labour. Our model belongs to the family of models for regulated firms subject to a rate of return constraint. If the constraint is binding then the Averch-Johnson effect as developed by H. Averch and L. Johnson (1962), is present. This effect states that a regulated firm will choose a capital-labour ratio greater than that which would be needed to minimize cost at the level of output selected by the firm, and that the firm will have an incentive to serve competitive markets even if revenues fall below marginal costs in those markets, with the marginal loss more than compensated by increased net revenues permited through price increases in its monopoly services. Until now little empirical analysis has been done to study the importance of these distortions in the real world. Some tests have been done in the U.S. for electric utilities but none for telecommunication industries, although in their original paper Averch and Johnson had in mind the telephone and telegraph industry. Our model will allow for testing of this effect in the context of Bell Canada. After testing for the presence of the Averch-Johnson effect, the other related issue is to study how important is this effect in practice. For this, it is needed to quantify in terms of dollar values the over-capitalization of Bell Canada. These computations are done here.

It is well known that unregulated monopolies will create inefficiencies on the use of resources but also that regulation creates its own distortions; therefore, regulation has costs and benefits and both should be compared. Efficient use of resources (Pareto optimality) requires first efficient combination of inputs given the level of output, which implies cost minimization given social prices of inputs. In the measure of costs to be minimized we include capital cost measured at its economic (opportunity) cost which in the case of a regulated industry will usually be higher than the implicit capital cost used for managerial decisions. Second it requires efficient scale of production, which implies that price equals marginal cost of production, with cost satisfying the first rule.

In the case of a competitive firm, maximization of profits satisfies the two conditions for efficient use of resources; hence, regulation is not necessary in this case. However, in the case of an unregulated monopoly, only the first rule will be satisfied as the profit maximizing firm will set marginal revenue (instead of price) equal to the marginal cost; therefore it will produce less output than what is the optimum one from the point of view of society. Different regulatory devices have been proposed to eliminate this inefficiency. These regulatory rules can also have other effects on resource allocation as developed in the pioneering work of Averch and Johnson and discussed above. Therefore, before implementing a regulatory devise its cost and benefits should be studied more carefully.

In periods of stable prices a regulatory policy implies only infrequent rate adjustments, generally after long and arduous discussions on what is a "fair rate of return" and the cost of capital for a public utility. This requires acceptable measures of value of output and cost of inputs, taking into account changes in productivity. These problems magnify themselves in periods of inflation. When inflation is present, the utilities are forced to more frequently apply for obtaining rate increases from the regulatory authority. Two major forces work in this direction. First, as unit costs of inputs (excluding capital) increase, their realized rate of return tends to drop relative to the allowed rate of return. Second, utilities have to seek higher rates of return on investment to stay competitive in capital markets. In these circumstances the frequency of rate applications increases and the process of going from one rate case to another becomes extremely costly. In this situation, it becomes important to develop some formula which can be used as a guideline for short term rate adjustments. This formula should incorporate allowed increases in cost and a productivity target for the sector under consideration. For long term rate structure a full rate case should take place to take care of circumstances not taken into account in the design of an automatic rate adjustment formula.

This study is divided in three parts. In the first part we develop a model of Bell Canada for its choice of input-mix under regulation. In this part we also study the efficiency in the choice of inputs followed by Bell Canada and hence the side effects of regulation. In the second part we study the problems arising from the regulation of a public utility. Finally, in the last part of this study, we develop and evaluate a formula for short term automatic rate adjustments applied to Bell Canada. This formula takes into account the functioning of Bell Canada as described by the model developed in the first part of the study.

I. AN ECONOMETRIC MODEL OF BELL CANADA: OUTPUT CREATION AND FACTOR REQUIREMENTS

1.1 An Overview of Bell Canada Regulation Framework

Bell Canada is a federally chartered firm, and is subject to the regulation of the Canadian Transport Commission. In the period covered in our study (1952-1972), rate hearings have been held in 1952, 1958, 1969, 1970 and 1972. In 1950 and 1952 the regulatory board ruled that a 40% debt-equity ratio, as contended by Bell, was the highest possible to keep the company financially sound. An explicit rule for regulation was not developed until 1958 but in every rate case the major element taken into account was the rate of return of the firm. In 1958 the regulatory Board granted a rate increase that was supposed to allow Bell a level of earnings of $2.43 per share based on a 40% debt-equity ratio. As we can see in Appendix D, in that year 40% was also the actual debt-equity ratio. In 1966 for the first time an explicit rate of return regulation is introduced; in that year, after substantial deliberations, the board ruled that Bell was entitled to a rate of return at the range of 6.2 to 6.6%. In the 1969 rate case the allowed rate of return was increased to 7.3%. In 1972 the regulatory body (CTC) concluded that 8.2% was the maximum permissible rate of return on average invested capital. At that moment Bell had claimed that this was the minimum, while 9% was the maximum. In all these rate cases other elements were also discussed such as rate structure for toll revenue, relations between Bell and Northern Electric, etc. In this brief summary we only wanted to emphasize that in general a major consideration given in the rulings about the appropriate level of revenues to be allowed for Bell, was related to the rate of return. It was indeed the rate of return that played the key role in the period under examination.

1.2 General Characteristics and Properties of the Model

The main purpose of this part of the study is to derive the behaviour followed by Bell Canada on the production of its output and the hiring of its inputs. For the derivation of this behavioural relation we have to specify a simplified model of the working of Bell, from where we will estimate econometrically the functions involved.

The model that we are going to use is of the Averch-Johnson family. In this type of model three main assumptions are made:

(i) The firm seeks to maximize profits;

(ii) the decision process is constrained by the available technology and by the imposition of an upper limit on the rate of return that it can earn on its capital. This rate of return is the "allowed rate of return". It is further assumed that the "allowed rate of return" is greater than the firm's cost of capital but lower than the rate of return that the firm would achieve if it were an unregulated monopoly;

(iii) no regulatory lags exist.

If this model is an appropriate description of the behaviour of a regulated company then the following propositions follow:

(1) At the output selected by the regulated firm the capital-labour ratio chosen is greater than that which minimizes cost.

(2) The output of the regulated firm would not rise above that of the unregulated profit-maximizing firm, except in the unusual circumstance in which capital is an inferior input. That is, when an increase in output is accompanied by a decrease in capital used.

(3) The regulated firm has an incentive to expand to competitive markets even if marginal revenues fall below marginal costs in those markets, with the difference more than compensated by increased net revenues allowed through rate increases in the regulated market.

(4) The capital-labour ratio will increase rather than decrease as the allowable rate of return is lowered towards the market cost of capital.

(5) The overcapitalization discussed in (1) above does not, in general, imply "gold-plating" or purchase of plant solely to be held idle; it rather means that the firm seeks to obtain whatever additional revenue is obtainable through overcapitalization. As long as the objective of the firm is to maximize profits subject to a rate of return constraint, an "entirely productive use of capital is always preferable to one entailing waste".[1]

(6) The regulated firm will have an incentive to reduce the acquisition cost of capital goods, in this way the firm would be able to invest in new ventures and achieve higher levels of profits for a given regulatory constraint. This is an extension of (5) above.

Now we are going to develop a model of a regulated firm. The notation to be used is as follows:

Q = the firm's value added in real terms

K = the amount of physical capital employed by the firm

L = the amount of labour employed by the firm

P = the price of value added

w = the wage rate

P_K = the rental price of a unit of physical capital

s = the "allowed rate of return" on a unit of physical capital, $s > P_K$

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1 Johnson (1973, p. 90).

Π = profits

R = P.Q = total value added, at current prices

$\frac{\partial R}{\partial L}$ = marginal revenue product of labour, i.e. increase in revenue due to a (small) increase in the labour hired

$\frac{\partial R}{\partial K}$ = marginal revenue product of capital

The regulated firm then seeks to maximize profits

(1) $\Pi = PQ - wL - P_K K$

subject to the rate of return constraint

(2) $s \geq \frac{PQ - wL}{K}$

and to the production technology constraint

(3) $Q(L,K) \geq Q$

If the regulatory and production function constraints are assumed to be binding, that is if the inequalities (2) and (3) are satisfied with equalities, then this maximization problem implies the following conditions for choosing inputs and the level of output.[1]

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1 For the derivation of this relations, see Appendix A below.

(4) $\frac{\partial R}{\partial L} = w$ (5) $\frac{\partial R}{\partial K} = P_K - \frac{\lambda}{1-\lambda}(s-P_K)$

(6) $R(L,K) = wL + sK$

where the only new symbol introduced is λ which is a Lagrangian multiplier from the maximization problem. Furthermore, Baumol and Klevorick (1970) have shown that $1 > \lambda > 0$. Let us see the meaning of each of these equations required to be fulfilled for the firm to maximize profits subject to the regulatory constraint. Equation (4) states the condition that the marginal revenue product of labour has to be equal to the wage rate. This equation takes the same form as in the case of unregulated monopolies, although this does not mean that the regulated firm will choose the same level of labour input as the unregulated monopoly, because the level of factor inputs is obtained from the simultaneous solution of equations (4), (5) and (6) above. Now, for the case of an unregulated monopolist, $\lambda = 0$ and therefore in (5) above we will have $\frac{\partial R}{\partial K} = P_K$, the well known equality between the marginal revenue product of capital and the price of capital. While for the regulated monopolist, $P_K - \frac{\lambda}{1-\lambda}(s-P_K) = \frac{P_K-\lambda s}{1-\lambda} < P_K$ and from here the over capitalization for the chosen value of output follows.

In the next section we are going to estimate an econometric model for Bell Canada and then we are going to test if a model of this fashion is a good representation of the behaviour of Bell Canada. Specifically we are going to study econometrically if λ is significantly different from zero and furthermore if it is in the interval (0,1). If this hypothesis is accepted then we will conclude that Averch-Johnson inefficiency is present in the behaviour of Bell Canada. Then we are going to go on measuring the importance of this effect in terms of actual figures for capital-labor ratios.

1.3 An Econometric Model of Bell Canada and a Test of the Over-capitalization Hypothesis

1.3.1 The Production Function

In order to test the Averch-Johnson hypothesis we need to specify the production function relevant for Bell Canada. With respect to the first we will assume that it is such that the own price elasticity is constant. With respect to the production function we reviewed the existing production functions for Bell Canada, i.e. Dobell et al (1970), Millen (1974), and we concluded that there is no clear-cut evidence with respect to what is the appropriate specification of technology. An attractive avenue for further empirical research is to assume a more general form of production function which can be considered as a second order approximation to any production function around a point in which the logarithms of each of the inputs are made equal to zero. This form of production function is called the Transcendental Logarithmic Production Function (translog) due to Christensen et al (1971, 1973) and it has the advantage that it reduces to a Cobb-Douglas form as a special case.

We write the translog production function as:

$$(8) \quad \ell n\, Q_t = \alpha_0 + \alpha_D D_t + \alpha_1 \ell n\, L_t + \alpha_2 \ell n\, K_t + \tfrac{1}{2}\gamma_{11} (\ell n\, L_t)^2 + \tfrac{1}{2}\gamma_{22} (\ell n\, K_t)^2 + \gamma_{12} (\ell n\, L_t)(\ell n\, K_t) + \varepsilon_t$$

where the only new variables introduced are D_t which is an index of technology and ε_t which is the random error of the regression. We have further assumed here Hicks-neutral technical change.

Before moving on the testing of our model we will make a brief description of our data. A detailed description appears in Appendix C.

The following definitions were used for the different variables.

Q = Total value added (including uncollectibles) minus Indirect taxes and raw materials in millions of 1967 dollars re-escalated to make the average equal to one during the sampling period.

L = Weighted man-hours, were the weights are the relative hourly wage rate of the different labour categories, re-escalated as above.

K = Net capital stock in millions of 1967 dollars, re-escalated as above.

s = Actual rate of return defined as total revenue minus indirect taxes, plus uncollectibles, minus cost of materials, rent and supplies, minus the wage bill, all in current dollars divided by the value of the net capital stock in millions of 1967 dollars.

P_K = Price of capital services, computed using the Jorgenson formula.[1]

D_t = Percentage of calls direct distance dialled

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1 In this formula

$$P_K = [\delta * q_t + CC * q_{t-1} - (q_t - q_{t-1})] * \frac{(1-u*z)}{1-u}$$

where q = Price index of capital goods 1967 = 1.00

CC = Cost of capital

u = Income tax rate

δ = Rate of replacement

z = Present value of depreciation deductions on a dollar's investment in plant

The source of each of these elements appears in the Appendix C.

First we will estimate a general translog model with Hicks-neutral technical change. The results, after correcting for autocorrelation are the following (the figures in parentheses are the t-ratios):

$$\ell n\ Q_t = \underset{(-.162)}{-.020} \underset{(.760)}{+.172}\ \ell n\ L_t \underset{(4.077)}{+1.230}\ \ell n\ K_t \underset{(.882)}{+2.125}\ (\ell n\ L_t)^2$$

$$\underset{(2.999)}{+.559}\ (\ell n\ K_t)^2 \underset{(-1.639)}{-.929}\ \ell n\ L_t\ \ell n\ K_t \underset{(-.091)}{-.041}\ D_t$$

$$\hat{\rho} = \underset{(1.230)}{.265}\ ,\ SSR = .003789,\ D.W. = 1.89,\ \text{Years: } 1953\text{-}1972$$

$$R^2 = .9993$$

Now we will test for constant returns to scale in the above function. Constant returns to scale condition imposes the following restrictions on the parameters of a translog model (for details see Appendix B).

$$(9) \quad \alpha_1 + \alpha_2 = 1.0$$

$$\gamma_{11} + \gamma_{12} = 0$$

$$\gamma_{12} + \gamma_{22} = 0$$

Imposing these restrictions in the model given by equation (8) we obtain the following restricted model:

$$(10) \quad \ell n\ (Q_t/L_t) = \alpha_0 + \alpha_2\ \ell n\ (K_t/L_t) + \gamma_{12}\ [\ell n\ Lt \cdot \ell n\ K_t - \tfrac{1}{2}\ (\ell n\ L_t)^2 - \tfrac{1}{2}\ (\ell n\ K_t)^2] + \alpha_D\ D_t + \varepsilon_t$$

We can test for the set of restrictions given by (9) by considering the model (10) as a restricted case of model (8).

Thus we perform F test. When we estimate the restricted model (equation (10)), the results are the following:

$$\ell n\ (Q_t/L_t) = \underset{(-1.748)}{-.147} \underset{(3.976)}{+.905}\ \ell n\ (K_t/L_t) \underset{(-2.329)}{-.754} \left[\ell n\ L_t\ \ell n\ K_t - \tfrac{1}{2}\ (\ell n\ L_t)^2 - \tfrac{1}{2}\ (\ell n\ K_t)^2\right] \underset{(1.446)}{+.446}\ D_t$$

$\hat{\rho} = \underset{(2.066)}{.419}$, SSR = .005270, D.W. = 2.01, Years: 1953-1972

R^2 = .9988

The constant returns to scale hypothesis is tested using the statistic

$$F = \frac{\dfrac{SSR^{M10} - SSR^{M8}}{3}}{\dfrac{SSR^{M8}}{13}}$$

where SSR^{M10} = sum of squared residuals of the restricted model (equation 10)), and

SSR^{M8} = sum of squared residuals of the unrestricted model (equation (8)).

If the null hypothesis (constant returns to scale) is true then the above statistic is distributed as F- Snedecor with 3 and 13 degrees of freedom. The computed F statistic in our case is 1.69 and $F_{.01}$ (3, 13) = 5.74. Therefore we accept the null hypothesis of constant returns to scale.

Accepting constant returns to scale as the maintained hypothesis we want to test now for global separability in the two inputs of the translog model (for details of this test see Appendix B). Global separability requires in this case $\gamma_{12} = 0$. With this additional restriction, the translog function reduces to a Cobb-Douglas with constant returns to scale. The restricted model in this case is given by:

(11) $\ell n\ (Q_t/L_t) = \alpha_0 + \alpha_2\ \ell n\ (K_t/L_t) + \alpha_D D_t + \varepsilon_t$

When this model is estimated we obtain the following results:

$$\ell n\ (Q_t/L_t) = \underset{(-9.356)}{-.330} \ \underset{(6.486)}{+.391}\ \ell n\ (K_t/L_t) \ \underset{(11.826)}{+1.136}\ D_t$$

$$\hat{\rho} = \underset{(1.900)}{.391},\ SSR = .007050,\ D.W. = 1.83,\ \text{Years: } 1953\text{-}1972$$

$$R^2 = .9984$$

The null hypothesis that the production model is a Cobb-Douglas subject to constant returns to scale is tested using the statistic

$$F = \frac{\dfrac{SSR^{M11} - SSR^{M10}}{1}}{\dfrac{SSR^{M10}}{16}}$$

where

SSR^{M11} = sum of squared residuals of the restricted model (equation (11)).

If the null hypothesis (constant returns to scale Cobb-Douglas) is true then this statistic is distributed as F-Snedecor with 1 and 16 degrees of freedom. The computed F statistic in our case is 5.405. The 1% critical value for the F - statistic with 1 and 16 degrees of freedom is 8.53. Therefore we accept the null hypothesis that the production function is Cobb-Douglas with constant returns to scale.[1]

We should also mention that we estimated also the Cobb-Douglas function with gross production as the dependent variable and three factors of production (labor, capital and raw materials) but the coefficient of the raw materials variable was never statistically significant. Therefore the hypothesis of fixed coefficient for raw materials and a constant returns Cobb-Douglas function for value added is shown out by our data.[2]

One could claim that the poor showing of the general translog model is due to the strong collinearity among the regressors in equation (8) and that therefore the translog function should not be estimated directly but from side conditions for profit maximization.

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1 This test is equivalent to a t-test on γ_{12} of equation (10).

2 The measure of raw materials available is the one reported in R. Millen (1974) and it includes rents and other supplies besides raw materials and therefore part of its poor performance in the equation could be due to an error in the variables problem.

Thus as a further search into the technology of Bell Canada we will take the unrestricted translog as a general production function describing the technology of Bell and then we will use the side conditions for profit maximization subject to a rate of return constraint to identify the parameters of the production function.

Using the translog function as a description of technology we now move onto derive some side conditions for profit maximization to be used to estimate the parameters of the production function.

For the case of constant price elasticity of the demand for value added we obtain the following set of relations from the profit maximization subject to a rate of return constraint.[1]

(12) $M_L = \frac{Lw}{PQ} = \beta\alpha_1 + \beta\gamma_{11}\ \ell n\ L + \beta\gamma_{12}\ \ell n\ K + \varepsilon_1$

(13) $M_K = \frac{P_K K}{PQ} = \beta\alpha_2(1-\lambda) + \beta\gamma_{12}(1-\lambda)\ell nL + \beta\gamma_{22}(1-\lambda)\ell nK + \lambda Z + \varepsilon_2$

where M_L = is the share of labour payments in value added

M_K = is the share of capital payments in value added when capital services are paid at its cost to the company

Z = Share of capital and abnormal profits in value added

That is $Z = \frac{PQ - Lw}{PQ}$

ε_1 and ε_2 are random errors

Now if we assume that λ is fairly constant during the sampling period the above model can be used to obtain estimates

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1 For details of these derivations see Appendix A.

of the parameters involved. Also the Averch-Johnson hypothesis can be tested by estimating the regression (13) and running a test on the coefficient of the variable Z. If the coefficient of this variable is between zero and (P_K/s) and it is statistically significant then we conclude that an Averch-Johnson effect is present. More efficient estimates for the coefficients of equation (13) are obtained if we take into account that equations (12) and (13) have coefficients in common. When the translog model was estimated in this fashion, independent of the definition for the cost of capital (see Appendix C), the production function was not well-behaved. It was nor monotonic neither quasi-concave (see Appendix B for details of these concepts). Therefore the translog model cannot be considered as a proper description of the technology used by Bell-Canada. We also estimated a simultaneous model with a C.E.S. function but the elasticity of substitution was always negative. Therefore the C.E.S. model was also rejected by the data.

We conclude therefore that the technology of Bell Canada can be approximated by a Cobb-Douglas production function.

1.3.2 The Demand Equation

Consumer demand theory suggests that given the tastes of a consumer the quantity demanded of a commodity (Q_t) is a function of real income (y_t) and relative prices (p_t). Factor demand theory for a cost minimizing producer suggests the same type of relationship, but with production levels playing the role of real income. Therefore, a demand model can be expressed as follows:

$$Q_t = f\ (y_t\ ,\ p_t)$$

Now, economic theory does not restric the form of the function f. However, for estimation purposes we have to go further; we have to specify the form of the function f. We will assume that f is a linear in the logs function on all the variables. In this case, we have:

$$\ell n\ Q_t = a_1 + a_2\ \ell n\ y_t + a_3\ \ell n\ p_t$$

We could interpret the dependent variable of this last equation as the desired demand for telephone with the actual demand for telephone moving towards the desired one following a Cagan-Nerlove partial adjustment scheme.

That is the partial adjustment model is given by:

$$\ell n\ Q^*_t = a_1 + a_2\ \ell n\ y_t + a_3\ \ell n\ p_t$$

$$\ell n\ Q_t - \ell n\ Q_{t-1} = \theta\ (\ell n\ Q^*_t - \ell n\ Q_{t-1})$$

where Q^*_t is the "desired level" of telephone services.

The reduced form of this system of equations is given by

(14) $\ell n\ Q_t = a_1\theta + a_2\theta\ \ell n\ y_t + a_3\theta\ \ell n\ p_t + (1-\theta)\ \ell n\ Q_{t-1}$

In equation (14) a_3 is the long run price elasticity of demand.

When we estimated this last equation we obtained[1]:

$$\ell n\, Q_t = \underset{(-1.058)}{-.920} \; \underset{(1.223)}{+.154} \; \ell n\, y_t \; \underset{(-1.123)}{-.160} \; \ell n\, P_t \; \underset{(11.765)}{+.899} \; \ell n\, Q_{t-1}$$

$$\hat{\rho} = \underset{(-1.229)}{-.271}\;,\; R^2 = .999\;,\; D.W. = 1.98\;,\; \text{Years } 1954\text{-}1972$$

where the only new variable introduced is:

y_t = real gross domestic product in Ontario and Quebec

From this demand equation we compute a long-run price elasticity (η) of -1.58. This value is then used in the rest of the model.

1.3.3 Testing for an Averch-Johnson Effect

Now we will perform a test of the Averch-Johnson hypothesis by estimating λ from equation (5) in section 1.2. For easier reference we reproduce that equation below

$$(5) \quad \frac{\partial R}{\partial K} = \frac{P_K - \lambda s}{1 - \lambda}$$

But we also know that:

$$\frac{\partial R}{\partial K} = P\left(1+\frac{1}{\eta}\right) \cdot \frac{\partial Q}{\partial K}$$

where

η = price elasticity of demand

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1 We also used as an explanatory variable the log of P_t divided by a combined consumer price index of Toronto and Montreal but the results were inferior.

From our estimated Cobb-Douglas production function we obtain:

$$\frac{\partial Q}{\partial K} = .391 \frac{Q}{K}$$

From the demand equation we had $\eta = -1.58$. Therefore using equation (5) we obtain:

$$(.365)(.391) \quad \frac{PQ}{K} = \frac{P_K - \lambda s}{1 - \lambda}$$

Finally we obtain:

$$\frac{P_K K}{PQ} = .1428 \ (1-\lambda) + \lambda \frac{sK}{PQ}$$

That is:

$$M_K = .1428 \ (1-\lambda) + \lambda Z$$

This equation was estimated for the three different definitions of the cost of capital obtaining:

(15) $M_{K1} = .1428 \ (1-.715) \ +.715 \ Z$
$\qquad\qquad\qquad\qquad (7.510) \quad (7.510)$

$\hat{\rho} = .592$, $R^2 = .290$, D.W. = 1.77 , Years 1954-1972
$\qquad (3.189)$

(16) $M_{K2} = .1428 \ (1-.762) \ +.762 \ Z$
$\qquad\qquad\qquad\qquad (11.395) \quad (11.395)$

$\hat{\rho} = .487$, $R^2 = .108$, D.W. = 1.89 , Years 1954-1972
$\qquad (2.576)$

(17) $M_{K3} = .1428 \underset{(12.184)}{(1-.785)} + \underset{(12.184)}{.785} Z$

$\hat{\rho} = \underset{(2.479)}{.473}$, $R^2 = .097$, D.W. = 1.89 , Years 1954-1972

Therefore from the three different definitions of the cost of capital we conclude that λ is statistically significant and a number between zero and one as predicted by the theory of a regulated firm.

To get some idea of the effect of regulation on the capital-labor ratio we will compute this ratio under regulation and without regulation.

With regulation

$$\frac{K}{L} = \frac{\alpha_2}{1-\alpha_2} \frac{w}{\frac{P_K - \lambda s}{1 - \lambda}}$$

without regulation ($\lambda = 0$)

$$\frac{K}{L} = \frac{\alpha_2}{1-\alpha_2} \frac{w}{P_K}$$

In the first order condition λ is a variable, therefore in the above equations λ is a sort of an average value, therefore we will compute K/L for the case in which the right hand side variables take values equal to their average for the period 1952-1972, to obtain some measure of the over-capitalization in Bell Canada.

Effect of Regulation on the Capital-Labor Ratio

	Observed	Regulation Model	No-Regulation
PK1	33.44	34.4	16.9
PK2	33.44	30.5	16.1
PK3	33.44	28.8	15.8

From this part of the study we conclude that there is strong evidence of over-capitalization in Bell Canada as predicted by the theory of regulation.

II. PROBLEMS ARISING FROM THE REGULATION OF NATURAL MONOPOLIES

2.1 Introduction

As discussed in the previous section many public utilities in particular the telephone industry operate under conditions of natural monopoly. In order to prevent obvious abuses such as the making of monopoly profits, public authorities have to resort to various forms of regulation which in turn give rise to other difficulties. Certain problems arising from the imposition of fair rate of return regulation have been discussed in several articles under the general heading of the Averch-Johnson (A-J) effect. These problems and the methods by which regulations could overcome them are dealt with in this section.

The analytical intractability of real world situations precludes the use of a theoretically rigorous framework. However, some of the basic economic principles relevant to regulation which have been derived from a rigorous analysis of idealized models, and discussed in Baumol (1973), Leland (1974) and Bailey (1974), are summarised below in general terms.

One of the most obvious goals of regulation is the prevention of monopoly profits, while allowing the firm to earn a reasonable rate of return. At the same time pricing policies should be designed to minimize the misallocation of resources which are used to produce the output. A widely used method of imposing a ceiling on profits is the so called fair rate of return criterion applied to a suitable "rate base". There is a tendency for the regulated firm to raise the prices for those of its services having inelastic demand, until the allowed rate of return (which is higher than the cost of capital) becomes a binding constraint on further profits. Further increases in net earnings can be achieved only by increasing the rate base.

2.2 Some Consequences of Regulation

This section deals with some of the consequences of regulation which should be of concern to policy makers; major issues include:

(a) The regulated firm may lose much of its incentive for technological improvements and efficiency increases. If the firm can always earn profits up to the allowed ceiling by increasing prices for its demand inelastic services, then there is no financial reward for cost saving.

(b) There will be a tendency for the regulated monopoly to expand its rate base beyond the normally optimal level (i.e. over-interest). The effect of the binding regulatory constraint reduces the perceived cost of capital to the firm below the market rate, and therefore for any given level of output the mix of inputs becomes more capital intensive than if the firm were not regulated. In fact, as long as prices in inelastic demand markets can be raised, further investments will be permitted to earn the allowed rate of return.

(c) If there is some measure of vertical integration in the industry, and the rate base is computed in terms of replacement costs, then transfer pricing may be encouraged. The new capital acquired from its suppliers at inflated prices will not decrease the firm's allowed profits, while at the same time the replacement of older machines by more expensive ones will actually increase the rate base.

(d) Destructive price cutting by the monopoly may occur for those services which are supplied in competitive markets. Any losses which the firm may incur in such markets can be made good (up to the allowed rate of return) by price increases in its inelastic demand services, and once competitors have been driven out of the competitive market by predatory pricing, the monopoly can invest further in this area in order to expand its rate base.

2.3 Methods of Supplementing the Efficiency of Regulation

The prevention of monopoly profits requires that the regulatory authority establishes a rate base and a fair rate of return (r) which exceeds the cost of capital for the firm. Although usual economic practice based on resource allocation grounds dictates the use of the forward looking replacement cost to determine the capital base, a number of regulatory problems may be avoided by using a sunk cost approach.

Firstly, the use of historical costs (including of course depreciation and price level corrections for inflation) will protect the investor from losses in a situation where technological or other factors have resulted in substantially reducing the costs of replacement capital. Thus, it seems fair to protect investors from unforeseen losses, since corresponding windfall gains are precluded by the regulated ceiling on earnings. The temptation to inflate suppliers prices discussed in Section 2.2 also will not occur when sunk costs are used for the rate base.

The level and mechanism chosen for imposing the rate of return have an important hearing on the firms' viability and incentive for cost saving innovation. The rate of return r must exceed the effective cost of capital for the company's risk class, or the firm will not be able to attract external capital in the long-run. Therefore, r must reflect not only the interest rate on debt but also the cost of equity (which is usually substantially higher).

Baumol (1973) has suggested an interesting use of the usual regulatory lag mechanism to encourage efficiency. Thus if the rates are set every three years, then the value of r and the corresponding tariffs would be based on data for year 0. In the three year interval, there would be no restriction on any increased company earnings arising from cost saving, but no interim tariff increases would be allowed. This procedure would allow the firm to benefit, albeit temporarily, from increases in efficiency etc.

If the regulated monopoly is allowed considerable freedom to set price levels within the limits of the overall rate of return, then the possibility of unjustified price increases in inelastic demand markets arises, because the firm may attempt to transfer losses incurred from its other services to the inelastic demand service. One approach to the problem of protecting customers of the latter type of service, involves the so-called "test for compensatory services" which may be applied to see whether in fact an inefficiently managed service is being subsidized.

A service is defined as compensatory if the provision of this service does not adversily affect either the profits of the company or the customers of other company services, i.e. if the revenues from the service exceed the long-run costs imputable to it. Thus, if overall earnings are constrained by regulation, then the provision of a compensatory service will in fact encourage price reductions for inelastic demand services.

If a service is determined as being compensatory in nature then several regulatory problems may be avoided;

(i) as described above there will be no additional burden on the users of other company services;

(ii) investments undertaken in this area will be on a sound economic basis, rather than for the purposes of rate base padding;

(iii) predatory pricing to eliminate competitors will be ruled out. In fact, if the regulated firm were a more efficient supplier of the compensatory service and thereby dominated its rivals, this process would be consistent with the most economic use of resources to produce the given output.

Several problems arise in the practical application of the test of compensation such as the used to:

(i) accurately determine long-run revenue and especially cost streams relating to the service;

(ii) reapply the test often when market conditions or company operating procedures change (e.g. changes in prices, costs etc.);

(iii) apply the test not only in terms of replacement cost but also sunk or historical costs in order to have a consistent basis for comparison of the rate of return on the service, with the overall regulated rate of return which is also based on sunk costs.

The problem of destructive pricing competitive services is discussed next. Firms could be prevented from increasing tariffs once they have lowered them in a competitive market, unless they demonstrate compelling reasons for doing so such as unforeseen cost increases etc. Such a provision would help to prevent monopolies from adopting predatory price cutting tactics.

A related question arises from the need to determine prices for joint outputs. The "full-cost pricing" approach is unsatisfactory because certain joint costs have to be allocated arbitrarily, and the society's welfare will not be optimized because the output of decreasing cost industries is under-consumed etc. In another approach, the prices for elastic demand services are set to maximize net profits from these services, and the tariffs for inelastic demand services are set so as to achieve the regulated ceiling on earnings. Although the ideal of strict marginal cost pricing derived from welfare maximization principles is inapplicable in the real world, a second best approach provides us with some theoretical guidelines. A theorem derived originally by Mann (1952) and developed by others shows that if marginal cost pricing cannot be used because a firm is required to meet a certain profit constraint, the consumers' surplus in maximized if for any two of the firm's outputs, the ratio of the % deviation of price from the marginal cost is equal to the inverse ratio of demand elasticities. The sense of this

From this section we conclude that any Automatic Rate Adjustment Formula could at the most be considered as a guide for the complex process of regulating a natural monopoly that produces a differentiated set of products and satisfies a whole set of different consumers.

III. AUTOMATIC RATE ADJUSTMENT FORMULA

3.1 Introduction

In this section of the study we will use the model developed in the first part to derive a formula for long-term cost. The formula to be developed gives the cost as a function of the price of the inputs used by the firm and the level of production and should only be used as a long-term trend of the costs of a regulated firm. We will first develop the formula and then we will comment on its main components.

3.2 An Automatic Rate Adjustment Formula for Bell Canada

In the first part of this study we analysed in detail the technology in use by Bell Canada. In that part of the study we concluded that a constant returns to scale Cobb-Douglas function was a proper representation of the production function facing Bell Canada. We have also shown there that the regulatory constraint was binding and that therefore the cost of capital services facing Bell Canada is not equal to its market cost (P_K) but some transformation of it given by $\frac{P_K - \lambda s}{1 - \lambda}$. Now we will solve the model of the first part for the total Cost of Bell.

Total cost of Bell can be written as:

$C = wL + \tilde{P}_K K$ where

$\tilde{P}_K$ is the "shadow" or "economic cost" of capital services for Bell and it is given by $\frac{P_K - \lambda s}{1 - \lambda}$

To derive a cost function for Bell we minimize cost subject to the estimated Cobb-Douglas production function.

The minimization of costs for a given level of output subject to a constant returns to scale Cobb-Douglas production function yields the following Lagrangian form:

$$J = wL + \tilde{P}_K K - \theta \left(AL^{1-\alpha_2} K^{\alpha_2} e^{\alpha_D D} - Q\right)$$

and this leads to the first order conditions:

$$\frac{\partial J}{\partial L} = w + \theta\,(1-\alpha_2)\,\frac{Q}{L} = 0$$

$$\frac{\partial J}{\partial K} = \tilde{P}_K + \theta \alpha_2\,\frac{Q}{K} = 0$$

$$\frac{\partial J}{\partial \theta} = AL^{1-\alpha_2} K^{\alpha_2} e^{\alpha_D D} - Q = 0$$

From the first two equations we obtain:

$$\frac{w}{\tilde{P}_k} = \frac{(1-\alpha_2)}{\alpha_2}\,\frac{K}{L}$$

substituting in the production function we obtain:

$$L = \frac{Q}{A} \left(\frac{w\,\alpha_2}{\tilde{P}_K (1-\alpha_2)}\right)^{-\alpha_2} e^{-\alpha_D D}$$

$$K = \frac{Q}{A} \left(\frac{w\,\alpha_2}{\tilde{P}_K (1-\alpha_2)}\right)^{1-\alpha_2} e^{-\alpha_D D}$$

Replacing back in the definition of cost we obtain the final expression for the cost function

$$C = C_0\, w^{1-\alpha_2}\, \tilde{P}_K^{\alpha_2}\, Q e^{-\alpha_D D}$$

where $C_0 = \frac{(1-\alpha_2)^{\alpha_2 - 1} \alpha_2^{-\alpha_2}}{A}$

Substituting now here the value of the estimated parameters we obtain:

$$C = 4.75 \; w^{.609} \; \tilde{P}_K^{.391} \; Q \; e^{-1.136 \; D}$$

This function gives total cost as a function of the wage rate (w), the "economic cost" of capital ($\tilde{P}_K$), value added (Q) and a technology measure (D). Now to appreciate in more detail the relation between total cost and the different elements which enter in the "economic cost" of capital we will rewrite for easy reference $\tilde{P}_K$.

$$\tilde{P}_K = \frac{P_K - \lambda s}{1 - \lambda}$$

where detail expressions for P_K and s are derived in Appendix A and we reproduce them here for easy reference

$$P_K = q\left[i^B \alpha + \frac{i^E(1-\alpha)}{1-u} + \left(\frac{1-uz}{1-u}\right)\delta\right]$$

$$s = q\left[r + \left(\frac{1-uz}{1-u}\right)\delta - \frac{u i^B \alpha}{1-u}\right]$$

Therefore the evolution of costs for the regulated firm will also be a function of all the variables and parameters that enter into the measurement of P_K and s. This cost function, for a given evolution of Q, w, $\tilde{P}_K$ and D could be used to compute the Minimum Cost. This minimum cost in conjunction with the value of the existing capital stock and the demand function should be used then to determine the rate to be permitted to the carrier, i.e. the allowed rate of return. In this way we derive an automatic rate adjustment formula.

One of the main elements of this formula for normative costs is the price of capital goods for Bell Canada. The implementation of any formula for automatic rate adjustment requires the development by the regulatory authority of an independent measure for the price of capital goods. We think that the use of an Automatic Rate Adjustment Formula along the lines suggested here would facilitate tremendously the functioning of the regulatory process. As this formula shows there are some crucial variables and parameters that have to be measured and estimated by the regulatory authority before any formula is implemented. Any rate of adjustment formula will concentrate on providing enough revenue to achieve a given rate of return for the carrier but the regulation process also has to deal with structure of rates for which not only revenue elements should be consider but also the welfare of the consumers. Therefore we conclude that an Automatic Rate Adjustment Formula should be only one, though very important, element to be used for regulation of a carrier.

APPENDIX A

A MODEL OF A REGULATED FIRM

A.1 Introduction

In this appendix we will develop in detail the model used in the first part of this study.

In this model we assume that Bell Canada's objective is to maximize profits after tax subject to a rate of return constraint. It is further assumed that the "allowed rate of return" is greater than the firm's cost of capital but lower than that rate of return the firm would achieve if it were an unregulated monopoly. The development of this model in detail will allow us to have a precise definition of the different elements entering in the model presented in the first part of this report.

Let us define:

R = value added in current dollars

Q = value added in constant dollars

K = the amount of physical capital employed by the firm

L = the amount of labor employed by the firm

P = the price of value added

q = the price of a unit of physical capital

i^B = the interest rate on bonds

i^E = the cost of equity capital

α = the share of debt into total capital

u = the corporate income tax

δ = the economic depreciation rate

w = the wage rate

z = the present value of depreciation deductions totaling one dollar over the lifetime of the investment

r = the "allowed before tax rate of return" net of depreciation

A.2 The Objective Function

The regulated firm then seeks to maximize profits after taxes. Profits after tax are given by:

$$\text{(A.1)} \quad \Pi = R - wL - i^B \alpha qK - i^E (1-\alpha) qK - \delta qK - u[R - wL - i^B \alpha qK - \delta zqK]$$

where

$i^B \alpha qK$ is the cost of debt capital

$i^E (1-\alpha) qK$ is the cost of equity capital

δqK is the cost of economic depreciation

The corporate income tax is applied on taxable income defined as current value added (R) minus labor cost (wL), minus the cost of debt capital ($i^B \alpha qK$) and depreciation allowance.

A.3 The Constraints

Profits after tax are maximized subject to a rate of return constraint and a technology constraint. The rate of return constraint can be written as:

$$\text{(A.2)} \quad \frac{R - wL - \delta qK - u[R - wL - i^B \alpha qK - \delta zqK]}{qK} \leq (1-u) r$$

In this expression the numerator of the left hand side term is net income available for dividend payments and the denominator is the value of capital stock. In the right hand side we have the "allowed before tax rate of return" times, one minus the corporate income tax rate.

The technology constraint can be written as:

(A.3) $Q(L,K) \geq Q$

If the regulatory and production function constraints are assumed to be binding, the firm's decision problem can be written as maximizing (A.1) subject to (A.2) and (A.3) the last two taken as equalities.

The maximization of profits subject to the rate of return constraint yields the Lagrangian

$$H = R - wL - i^{B}\,\alpha qK - i^{E}\,(1-\alpha)\,qK - \delta qK$$
$$- u\left[R - wL - i^{B}\,\alpha qK - \delta zqK\right] - \lambda\{\,R - wL - \delta qK$$
$$- u\left[R - wL - i^{B}\,\alpha qK - \delta zqK\right] - (1-u)\,rqK\}$$

where $R = PQ = \Phi(Q).Q(L,K)$

$= \Phi\left[Q(L,K)\right].Q(L,K)$

$= R(L,K)$ and

$\Phi(Q)$ is the inverse demand function.

This leads to the following first-order conditions:

(A.4) $\frac{\partial R}{\partial L} = w$, (A5) $\frac{\partial R}{\partial K} = \frac{P_K - \lambda s}{1 - \lambda}$

(A.6) $R(L,K) = wL + sK$

where $P_K = q\left[i^B\alpha + \frac{i^E(1-\alpha)}{1-u} + \left(\frac{1-uz}{1-u}\right)\delta\right]$[1]

$$s = q\left[r + \left(\frac{1-uz}{1-u}\right)\delta - \frac{ui^B\alpha}{1-u}\right]$$

If capital gains are included as part of the revenue then a term equal to $\frac{\dot{q}/q}{1-u}$ has to be substracted within the parenthesis of the expression for P_K .

The system of equations (A.4), (A.5) and (A.6) can be solved in principle for equilibrium values of L, K and λ .

Baumol and Klevorick (1970) have shown that $0 < \lambda < 1$ and from here using (A.5) follows the Averch-Johnson result that the regulated firm would choose in equilibrium a level of capital such that the marginal value added product of capital $(\frac{\partial R}{\partial K})$ is less than the cost of capital (P_K) and therefore the production process is too capital intensive.

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1 This is the formula to be used in the continuous case. The formula actually used, fully described in Appendix C, deals with the discrete case.

APPENDIX B

THE TRANSLOG PRODUCTION FUNCTION

B.1 Introduction

In the estimation of production models the usual hypothesis is that the function is one of a restricted class which satisfies some a priori restrictions in technology. The production functions most frequently used are the Cobb-Douglas, CES, and Translog, with the last one of more recent development (Christensen, Jorgenson, and Lau, 1971). The Cobb-Douglas production function restricts all Allen partial elasticities of substitution to be equal to one. The CES function, restricts the above elasticities to be constant and equal for any pair of inputs and for all points in input space.

In addition, the Cobb-Douglas and the CES functions assume strong separability. On the other hand, the translog function does not assume strong separability and furthermore it can attain an arbitrary set of pairwise elasticities of substitution at any point in input space. The estimation of translog functions has become very popular lately for the flexibility that it provides (E. Berndt and L.R. Christensen, 1973; E. Berndt and L.R. Christensen, 1974; E. Berndt and D. Wood, 1975; D. Humphrey and J.R. Moroney, 1975). A translog function with three inputs has nine regressors besides the constant. To avoid multicollinearity problems in small samples, the usual estimation procedure has been to work with side conditions for profit maximization in competitive product and factor markets. In this procedure, the parameters of the associated translog function have been estimated from a system of semi-logarithmic equations with one equation for each input. Each of these equations gives the cost share of an input as a linear in the logs function of each of the inputs. The problem with this approach is that it is impossible to know if the parameters that one is estimating are the parameters of a translog function, or some other parameters resulting from the specification error introduced if any of the untested assumptions indicated above are not fullfilled.

In this Appendix we review the main characteristics and properties of a translog function. We study in detail the case of a three inputs function. The two inputs function used in the first part of this study is a special case of it.

B.2 The Translog Model

The translog function with symmetry imposed ($\gamma_{sk} = \gamma_{ks}$) can be written as:

(1)
$$\begin{aligned} \ell n\, Q_i = {} & \alpha_0 + \alpha_1\, \ell n\, L1_i + \alpha_2\, \ell n\, L2_i + \alpha_3\, \ell n\, K_i \\ & + \tfrac{1}{2}\gamma_{11}(\ell n\, L1_i)^2 + \gamma_{12}(\ell n\, L1_i)(\ell n\, L2_i) \\ & + \gamma_{13}(\ell n\, L1_i)(\ell n\, K_i) + \tfrac{1}{2}\gamma_{22}(\ell n\, L2_i)^2 \\ & + \gamma_{23}(\ell n\, L2_i)(\ell n\, K_i) + \tfrac{1}{2}\gamma_{33}(\ell n\, K_i)^2 \end{aligned}$$

where

Q = Value Added, with units defined in such a way that mean of Q_i equals to one

L1 = Blue Collar labor, with units defined in such a way that mean of $L1_i$ equals to one

L2 = White Collar labor, with units defined in such a way that mean $L2_i$ equals to one

K = Capital, with units defined in such a way that mean of K_i equals to one

i = time

The elasticity of substitution in a translog function is different at every data point. Therefore, the size of the observations will be affecting the substitution properties of the technology.

The hypothesis of constant returns to scale can be tested directly from (1). Constant returns to scale imply the following restrictions on the parameters of this function (E. Berndt and L. Christensen 1973, p. 84).

(i) $\sum_{k=1}^{3} \alpha_k = 1$

(ii) $\sum_{k=1}^{3} \gamma_{sk} = 0 \quad s=1,2,3$

(iii) $\sum_{s=1}^{3} \gamma_{sk} = 0 \quad k=1,2,3$

(iv) $\sum_{s=1}^{3} \sum_{k=1}^{3} \gamma_{sk} = 0$

With symmetry imposed a priori, restrictions (iii) and (iv) are not independent of (i) and (ii). Therefore, we test for constant returns to scale in model (1) by imposing constraints (i) and (ii) on the parameters.

A production function is considered to be well-behaved only if it has positive marginal products for each input (monotonicity) and if it is quasi-concave. The translog function does not satisfy these restrictions globally. On the other hand, we can have wide enough regions in input space, which include the observed input combinations, where these restrictions are satisfied, and therefore the translog function can be considered as a well-behaved function for relevant input combinations. In any case monotonicity and quasi-concavity of the estimated translog function should be checked for every data point in the sample. Monotonicity requires $\partial Q/\partial L1 > 0$, $\partial Q/\partial L2 > 0$ and $\partial Q/\partial K > 0$.

Differentiating the translog function we find:

$$F1_i \equiv \frac{\partial Q_i}{\partial L1_i} = \frac{Q_i}{L1_i} (\alpha_1 + \gamma_{11} \ln L1_i + \gamma_{12} \ln L2_i + \gamma_{13} \ln K_i)$$

$$F2_i \equiv \frac{\partial Q_i}{\partial L2_i} = \frac{Q_i}{L2_i} (\alpha_2 + \gamma_{12} \ln L1_i + \gamma_{22} \ln L2_i + \gamma_{23} \ln K_i)$$

$$F3_i \equiv \frac{\partial Q_i}{\partial K_i} = \frac{Q_i}{K_i} (\alpha_3 + \gamma_{13} \ln L1_i + \gamma_{23} \ln L2_i + \gamma_{33} \ln K_i)$$

From these expressions we can compute the relevant partial derivatives for a given set of parameters values and for a sample of input and output values.

The translog function is strictly quasi-concave (strictly convex isoquants) if the bordered Hessian matrix is negative definite. In the case of three inputs this requires that the bordered principal minors be positive and negative respectively (see Takayama 1974, p. 123).

Differentiating the partial derivatives computed above we obtain expressions of the following form:

$$F11_i \equiv \frac{\partial^2 Q_i}{\partial L1_i^2} = \frac{Q_i}{L1_i^2}\left[\gamma_{11} + F1_i \frac{L1_i}{Q_i}\left(F1_i \frac{L1_i}{Q_i} - 1\right)\right]$$

$$F13_i \equiv \frac{\partial^2 Q_i}{\partial L1_i \; \partial K_i} = \frac{Q_i}{L1_i \; K_i}\left[\gamma_{13} + F1_i \frac{L1_i}{Q_i} F3_i \frac{K_i}{Q_i}\right]$$

Similar expressions can be derived for the other inputs.

The bordered Hessian matrix is given by:

$$\bar{H}_i = \begin{bmatrix} 0 & F1_i & F2_i & F3_i \\ F1_i & F11_i & F12_i & F13_i \\ F2_i & F21_i & F22_i & F23_i \\ F3_i & F31_i & F32_i & F33_i \end{bmatrix}$$

The bordered principal minors of this matrix are computed for every data point, that is at every i.

One of the main characteristics of a technology is the elasticity of substitution. The Allen elasticity of substitution between $L1_{ij}$ and K_{ij} (Allen 1938, p. 504) is given by:

$$\sigma L1K_i = \frac{F1_i \; L1_i + F2_i \; L2_i + F3_i \; K_i}{L1_i \; K_i} \left(|R13_i| / |\bar{H}_i|\right)$$

Where $|R13_i|$ is the cofactor of $F13_i$ in $\bar{H}_i$. Analogous expression can be derived for $\sigma L1L2_i$ and $\sigma L2K_i$. These elasticities of substitution are computed again at every data point.

The translog function does not assume separability. Separability can be tested. In the case of three inputs three types of weak separability may exist: the weak separability of L1 and L2 from K (denoted L1L2-K), L1 and K from L2 (denoted L1K-L2), and L2 and K from L1 (denoted L2K-L1). In the case of the translog function of equation (1), these separability conditions are fullfilled globally if and only if (E. Berndt and L. Christensen (1973, p. 102))

(2) L1L2-K (i) $\alpha_1 \ \gamma_{23} - \alpha_2 \ \gamma_{13} = 0$

(ii) $\gamma_{11} \gamma_{23} - \gamma_{12} \gamma_{13} = 0$

(iii) $\gamma_{12} \gamma_{23} - \gamma_{22} \gamma_{13} = 0$

(3) L1K-L2 (i) $\alpha_1 \ \gamma_{23} - \alpha_3 \ \gamma_{12} = 0$

(ii) $\gamma_{11} \gamma_{23} - \gamma_{13} \gamma_{12} = 0$

(iii) $\gamma_{13} \gamma_{23} - \gamma_{33} \gamma_{12} = 0$

(4) L2K-L1 (i) $\alpha_2 \ \gamma_{13} - \alpha_3 \ \gamma_{12} = 0$

(ii) $\gamma_{22} \gamma_{13} - \gamma_{23} \gamma_{12} = 0$

(iii) $\gamma_{23} \gamma_{13} - \gamma_{33} \gamma_{12} = 0$

If we impose constant returns to scale (CRTS) then in each of the set of conditions ((2), (3) and (4)) only one between (ii) and (iii) is independent.

The linear restrictions $\gamma_{13} = \gamma_{23} = 0$ satisfy the conditions for global separability L1L2-K. In the same way $\gamma_{23} = \gamma_{12} = 0$ satisfy the set of restrictions (3) and $\gamma_{13} = \gamma_{12} = 0$ satisfy restrictions (4). All the global separability conditions are satisfied simultaneously if and only if $\gamma_{13} = \gamma_{12} = \gamma_{23} = 0$ and in the CRTS case the function is a Cobb-Douglas[1].

If we substitute the CRTS restrictions in (2) and (3) above then a set of nonlinear separability conditions can be derived (E. Berndt and L. Christensen 1973, p. 91). A summary of these conditions is reproduced below

Table 1

Parameter Restrictions for Global Functional Separability

Separability type	Linear separability restrictions CRTS & non CRTS	Non-linear separability	Non-linear separability restrictions (under CRTS)
L1L2 - K	$\gamma_{13} = \gamma_{23} = 0$	$\alpha_1\gamma_{23} - \alpha_2\gamma_{13} = 0$ $\gamma_{11}\gamma_{23} - \gamma_{12}\gamma_{13} = 0$ $\gamma_{12}\gamma_{23} - \gamma_{22}\gamma_{23} = 0$	$\gamma_{33} = \gamma_{23}^2/\gamma_{22}$ $\alpha_3 = 1 + (\alpha_2\ \gamma_{23}/\gamma_{22})$ $(\sigma_{13} = \sigma_{23} \neq 1)$
L1K - L2	$\gamma_{12} = \gamma_{23} = 0$	$\alpha_1\gamma_{23} - \alpha_3\gamma_{12} = 0$ $\gamma_{11}\gamma_{23} - \gamma_{12}\gamma_{13} = 0$ $\gamma_{11}\gamma_{23} - \gamma_{33}\gamma_{12} = 0$	$\gamma_{33} = \gamma_{23}^2/\gamma_{22}$ $\alpha_3 = (\alpha_2 - 1)\gamma_{23}/\gamma_{22}$ $\sigma_{12} = \sigma_{23} \neq 1$
L2K - L1	$\gamma_{13} = \gamma_{12} = 0$	$\alpha_2\gamma_{12} - \alpha_3\gamma_{12} = 0$ $\gamma_{22}\gamma_{13} - \gamma_{23}\gamma_{12} = 0$ $\gamma_{23}\gamma_{13} - \gamma_{33}\gamma_{12} = 0$	$\gamma_{33} = \gamma_{23}^2/\gamma_{22}$ $\alpha_3 = \alpha_2\ \gamma_{23}/\gamma_{22}$ $\sigma_{12} = \sigma_{13} \neq 1$

It can also be shown that if one set of non-linear separability restrictions holds, then neither of the other two sets can be satisfied.

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1 If we do not restrict the translog function to exhibit CRTS then the restricted translog function will also include terms with the square of the logs of each input and therefore will not be a Cobb-Douglas function.

One of the difficulties with the tests for weak separability in a translog function is that they require the aggregator function to be linear in the logs. Thus the tests presented above are a joint test of weak separability and a linear logarithmic aggregator. Under the translog specification of technology the joint character of the tests makes them unseparable. Therefore these tests are biased in favour of rejecting the hypothesis of weak separability (see Blackorby, Primont and Russell, 1976).

APPENDIX C

DATA USED IN THE STUDY

C.1 Sources of data

The major sources which provided us with the data necessary to carry out this study are: the "Memorandum on Productivity", Bell Canada Application, File 955.182.1, volumes I and II, Exhibit B-73-61 to B-73-67 by the Telecommunications Committee of the Canadian Transport Commission; the yearly financial statistics of Bell Canada (Income Statements and Balance Sheets) for the range of the sample period used in the study; and "Automatic Rate Adjustments and Short-Term Productivity Objectives for Bell Canada" by Ronald H. Millen, a doctoral thesis presented in the Department of Economics of Concordia University in September 1974. Occasionally, from now on the sources listed above will be referred to as first, second (all the statistics collectively) and third source respectively. All the deflated time series have 1967 as the base year.

C.2 Data

C.2.1 Gross Value Added

Ideally output should have been used measured in physical units as the inputs should refer to services, to be within the theoretical framework of production theory. In our case we did not consider raw materials for reasons referred to in the text thus we used gross value added instead at constant 1967 prices (GVAB). Thus from what appears in the income statements as Total Operating Revenues, by adding uncollectibles and substracting other than income taxes and raw materials (taken from the third source) we arrive to what we define as Gross value added at current prices. Then by dividing this latter by its price index we get what we define as Gross value added at constant prices. The price index used was calculated from data appearing in the first source using the formula PIGVA = (TORC-ITC-RMC)/(TORB-ITB-RMB) where TOR is total operating revenues, IT is indirect taxes and RM is raw materials at current (C) and basic (B) prices (all data appear in Appendix D).

C.2.2 Labor Input and Payments to Labor

As labor input in the production function we used total manhours worked (excluding hours used in construction) adjusted for differences in quality among different types of labor before aggregating them. A description of the procedure of adjustment can be found in the first source. We can refer in passing that the main assumption in it is that differences in wages reflect differences in quality. The year of reference for these weights was 1967. Then, after further adjusting for taking into account payments received which do not represent remuneration for working time and other elements, we end up with the Total Weighted Manhours (TWM). As payments to labor (CL) we consider the figures reported in the third source as Employee Expense. In that study there is also a description of how this is calculated. Furthermore, we can say that it includes, apart from salaries, most of the fringe benefits (except those considered as tax payments).

C.2.3 Capital Input

As capital input in the production process we consider the figures given in the first source as Net Capital Stock (NCS) at 1967 prices. A good description of the technique used for these estimates can be found in the third source. In passing it can be said that these figures include average plant under construction. Furthermore, vintage curves of the life expectancy of the different types of capital goods were considered to find the net capital stock and its economic depreciation (replacement) for the company.

C.2.4 Price of Capital Service

By this we mean what is paid back to capital for its services rendered to the production. For quick reference we reproduce here the specific form of the Jorgenson's formula used in our computation.

$$P_K = \left[\delta * q_t + CC_i^* q_{t-1} - (q_t - q_{t-1})\right] * \frac{(1-u*z)}{(1-u)}$$

where $i = 1,2,3$

then

q is the Total Plant Price Index for Bell Canada (TPPI) taken directly from the first source.

u is the Effective Corporate Tax Rate (ECTR) derived as the ratio of income taxes (INCT) over profits (PBT) before taxes, or more simply, income taxes over income taxes plus net income, all these taken directly from the income statements of the company.

δ is the rate of replacement defined as the ratio of depreciation (DEP) over net capital stock both at constant prices were taken from the third source.

$q_t - q_{t-1}$ is the price change of capital goods and it stands for potential capital forms (losses) accruing to the investors from appreciation (devaluation) of them.

z is the present value of depreciation deductions on a dollar's investment in plant. z for the case of straight line depreciation procedure, for the continuous case, is given by:

$$z = \left[\frac{1}{(1-U)\rho T}\right] \left[1 - e^{-(1-u)\rho T}\right]$$

or the discrete approximation of that

$$z = \left[\frac{1}{(1-u)\rho T}\right] \left[1 - \left(\frac{1}{1+(1-u)\rho}\right)^{T}\right]$$

where ρ stands for the before-tax rate of return and T is the lifetime of the asset. We obtained average T for Bell Canada by dividing the unit over the average composite depreciation rate for tax purposes 5.3% so T = 18.868 years. For u we used the average of the 21 years considered while for ρ we used 15% which is very close to the average s. So z was found to be equal to .54079.

C.2.5 Cost of Capital

C.2.5.1 Introduction

This is represented by the CC_i term in the formula for the price of capital services. It is a very important factor in determining P_K so it deserves special attention in its calculation.

By cost of capital we mean the cost to the firm of raising one dollar of funds. In calculating the cost of capital, Bell Canada's financing policies are taken as given. In other words, even though the cost of funds on the market may be sensitive to the particular mix of debt and equity capital employed, it is assumed that an optimal debt-equity ratio has been determined each year by management. This is a reasonable assumption in the case of Bell Canada since the debt-equity ratio has remained fairly stable and management has little discretion in exploring alternative financing policies. The principal question explored in this section is the determination of the cost of capital to Bell Canada, given that the capital structure in any given year is considered optimal.

The appropriate cost of capital for a firm is referred to the weighted marginal cost of capital, for it is a weighted average of the specific marginal costs of each individual sources of funds, where the weights are the proportions of the total value of the firm contributed by each source of funds. Since these weights can be observed directly and are considered optimal, the only remaining problem is the computation of the specific marginal cost of each source of funds, namely long-term debt, preferred stock, and equity capital.

The central concept involved in determining these specific marginal costs is the market's capitalization rate, that is, the investor's opportunity cost or simply the investor's required rate of return. In the absence of corporate taxes and underwriting costs, the specific marginal costs to the firm are identical with the market capitalization rates. For example, if the market capitalization rate for bonds or the rate of interest is 9%, the firm must undertake to offer investors in its bonds a rate of return of 9% if it is to be successful in placing its bonds. Similar considerations apply to preferred shares. In the case of common equity, however, the relationship is more subtle, for the firm makes no explicit promise as to the magnitude of the return it offers. Nevertheless, if the required return on equity is 12%, and the firm raises new equity on which it can earn only a return less than 12%, the result will be that investors will revise their valuation of the firm's shares downwards so that at the new price the shares will once more offer a prospective return of 12%. If the firm is to avoid this decline in its share prices resulting from its investments, it must offer a return on its equity capital at least equal to 12%, and this is therefore the required rate of return or specific marginal cost of equity capital. In the presence of taxes and issuing expenses, these market capitalization rates are adjusted to account for market imperfections.

Thus we procede as follows. The specific marginal cost of debt, preferred stock, and equity capital are first determined separately, and then combined to arrive at the total marginal cost of all the forms of capital raised by Bell Canada.

C.2.5.2 The Marginal Cost of Debt Capital

The dollar cost of debt capital is the contractual obligation to pay interest while the debt is outstanding and to pay the principal amount at maturity. An equally important cost of debt financing is the additional financial risk to the common shareholders which results from the introduction of senior securities. This latter cost is reflected in the equity component rather than in the debt component of the weighted cost of capital, and will be treated at that time.

The marginal cost of debt is the yield to maturity required by debt investors, i.e., their required rate of return, related to the net proceeds of the issue. To determine the marginal cost of debt in any given year, we considered two alternatives. First the long-term Government Bonds Yield (GBY) and second the average yield on bonds of ten industries (BUS) as it is calculated by Mc Leod, Yound and Weir. The assumption for using (GBY) is that Bell Canada's bonds may not be considered as risky assets thus the potential investors would be indifferent in buying Bell Canada's or Government bonds. While the rationale for using (BUS) is that Bell Canada's bonds are considered as equally risky as the ones of other industries. The (GBY) series was obtained from the Statistical Review, Statistics Canada and the (BUS) from the Bank of Canada Statistical Review.

C.2.5.3 The Marginal Cost of Preferred Stock

The dollar cost of straight preferred capital is simply the weighted average of the indicated dividend rate for each preferred stock issue divided by its net proceeds. As in the case of debt securities, there is also a risk cost to common stockholders which is reflected in the common equity component of the total cost.

Only in 1970 did Bell Canada resort to preferred stock financing. Two issues have been outstanding since then: the \$3.20 convertible Ser. A preferred, and the \$3.34 convertible Ser. B preferred. For each of these two issues, the indicated dividend rate was related to the average price during the year $\frac{(\text{High} + \text{Low})}{2}$ to obtain the preferred stockholder's required return. To obtain the cost of preferred capital, each issue was weighted in proportion to the amount outstanding in the capital structure (47% and 53% of total preferred stock financing respectively).

Strictly speaking, purchasers of convertible preferred shares as opposed to straight preferred shares make their investment in the expectation of obtaining more than the fixed income provided by the dividend. They also purchase the appreciation provided by the expected growth in the value of the common shares and/or growth in the dividend on the common shares that they would receive upon conversion. Instead of recognizing the additional cost associated with the conversion feature explicitly, it is assumed that the return on equity recognizes this need; in other words, it is assumed that the difference between the dividend requirement and the total cost of convertible preferred is adequately reflected in the estimate of the cost of common equity capital.

C.2.5.4 The Marginal Cost of Common Stock

While the determination of the cost of debt and preferred capital is relatively straightforward, the cost of equity capital requires judgment and introduces additional complications. Investors must anticipate some minimum degree of compensation to induce them to invest their capital in the equity of a particular company. The compensation which is expected may take the form of income or capital appreciation, or both, and when related to the market price of the security, is the investor's required rate of return.

Several methods are available in approaching the problem of the determination of the marginal cost of common stock. Here we follow the so-called "Discounted Cash Flow Method". In this method, the required return is defined as the discount rate which makes the present value of the dividends and capital appreciation expected by investors for the common shares equal to the current market value of the shares. Under certain assumptions as to these expectations, the required return can be expressed as the ratio of the dividend expected by investors at the end of the year and the market price at the beginning the year plus the expected future rate of growth in dividends, or some proxy. The DCF method is most commonly identified with the following equation:

$R = D_1/P_0 + g$

R = required rate of return where

D_1 = dividends expected at end of the year

P_0 = current market value of the stock at beginning of the year

g = estimated future growth in dividends, or some proxy

In applying this method, the average dividend yield was estimated each year from 1952 to 1972 by relating the indicated dividend rate on the common stock to the year's average market price $\frac{(\text{High} + \text{Low})}{2}$.

The principal problem in applying the DCF method arises from the fact that 'g' is an expectation which lies in the mind of the investor and cannot be measured directly. To arrive at a plausible estimate of 'g' for each year, the simple average of the log-linear least squares growth rates of earnings per share

(GER) and dividends per share (GDR) was computed for the previous ten years. The regression coefficients were significant for the period 1962-1972 for the dividends and 1965-1972 for the earnings and these were the only ones considered (for the other years g was taken as zero).

C.2.5.5 The Marginal Cost of Total Capital

The last step is to combine the specific costs of each component of the capital structure and to calculate the weighted cost of capital, where each component is weighted by its book value in the existing financial structure which is deemed optimal.

With (GBY) we consider two alternatives. First, that there is no difference in between the marginal cost on Bell Canada's debt and the return on equity capital except the fact that the first is tax-exempted. So we do not use the DCF method either. This we call case (1). Second, we use (GBY) to approximate the marginal cost of long-term debt and DCF method as described above for common equity return and the relate for preferred equity return. This we call case (2). Finally we consider case (3) similar to the second case however using (BUS) instead of (GBY) for the marginal cost of long-term debt.

Thus for the three cases we apply:

$CC_1 = (1-U*WD)*GBY$

$CC_2 = [(1-U)*WD*GBY+WP*RPS+WC*RCS]$

$CC_3 = [(1-U)*WD*BUS+WP*RPS+WC*RCS]$

where

WD is the percentage of long term debt (LTD) on total capital (TCAP) defined as the summation of long-term debt plus common (CS) and preferred stock (PS).

WP is the percentage of preferred stock on total capital

WC is the percentage of common stock on total capital

RPS is the rate of return on preferred stock defined above

RCS is the rate of return on common stock defined above

So finally:

PK1, PK2, PK3 are the three definitions of the price of capital services.

C.2.6 Other Variables

KL is the capital-labor ratio (stock of capital over total man-hours).

s is the allowed rate of return as it is defined in the text

PL is the price of labor defined as total employee expenses over total man-hours (taken from the third source).

APPENDIX D

DATA

In millions (when applicable)

YEAR	TORC	ITC	RMC
1952.000000	184.756000	6.597000	28.730000
1953.000000	202.349000	7.184000	30.770000
1954.000000	219.884000	7.734000	35.000000
1955.000000	245.457000	8.835000	40.330000
1956.000000	274.639000	9.838000	49.010000
1957.000000	303.891000	11.859000	50.410000
1958.000000	329.945000	12.902000	56.260000
1959.000000	377.965000	14.526000	60.380000
1960.000000	406.444000	16.692000	63.870000
1961.000000	435.321000	18.862000	66.910000
1962.000000	472.920000	20.160000	72.660000
1963.000000	505.229000	21.501000	78.000000
1964.000000	545.014000	23.121000	80.110000
1965.000000	595.762000	25.313000	90.210000
1966.000000	648.201000	29.905000	98.010000
1967.000000	705.556000	35.716000	98.710000
1968.000000	761.802000	38.796000	107.290000
1969.000000	846.150000	44.681000	133.440000
1970.000000	942.768000	45.479000	138.820000
1971.000000	1023.409000	52.226000	160.930000
1972.000000	1129.382000	53.880000	185.900000

YEAR	GVAC	GVAB	PIGVA
1952.000000	149.429000	148.659000	1.005180
1953.000000	164.395000	162.535000	1.011443
1954.000000	177.150000	175.680000	1.008367
1955.000000	196.292000	192.672000	1.018789
1956.000000	215.791000	212.351000	1.016200
1957.000000	241.622000	240.463000	1.004824
1958.000000	260.783000	257.943000	1.011010
1959.000000	303.059000	281.459000	1.076745
1960.000000	325.882000	301.652000	1.080325
1961.000000	349.549000	325.199000	1.074877
1962.000000	380.100000	361.580000	1.051220
1963.000000	405.728000	385.698000	1.051932
1964.000000	441.783000	423.463000	1.043263
1965.000000	480.239000	463.869000	1.035290
1966.000000	520.285000	513.575000	1.013065
1967.000000	571.130000	571.130000	1.000000
1968.000000	615.716000	621.716000	.990349
1969.000000	668.029000	678.789000	.984148
1970.000000	758.469000	753.089000	1.007144
1971.000000	810.253000	784.598000	1.032767
1972.000000	889.602000	849.810000	1.047426

YEAR	TWM	CL	NCS	DEP
1952.000000	44.926000	75.330000	615.560000	34.660000
1953.000000	46.059000	83.050000	677.510000	38.160000
1954.000000	48.188000	90.630000	752.230000	41.720000
1955.000000	51.889000	101.760000	856.650000	45.380000
1956.000000	55.661000	111.730000	973.810000	51.400000
1957.000000	57.798000	121.080000	1109.160000	64.510000
1958.000000	57.596000	127.290000	1259.980000	72.880000
1959.000000	56.529000	130.980000	1405.650000	83.950000
1960.000000	54.597000	134.470000	1554.700000	91.860000
1961.000000	52.442000	136.680000	1693.410000	99.900000
1962.000000	52.279000	142.320000	1827.350000	108.920000
1963.000000	53.518000	150.480000	1971.520000	121.050000
1964.000000	54.427000	157.030000	2110.690000	131.050000
1965.000000	55.799000	166.000000	2241.630000	142.320000
1966.000000	57.470000	181.220000	2384.180000	154.760000
1967.000000	56.578000	192.580000	2538.860000	165.900000
1968.000000	55.488000	204.790000	2684.910000	178.580000
1969.000000	56.598000	226.280000	2836.990000	194.960000
1970.000000	57.835000	255.780000	2984.120000	205.940000
1971.000000	58.125000	282.120000	3147.160000	218.000000
1972.000000	58.998000	314.180000	3312.240000	243.990000

YEAR	DDD	Y	INCT	PBT
1952.000000	0.000000	19946.858813	23.745000	46.315000
1953.000000	0.000000	21479.480924	22.715000	49.564000
1954.000000	0.000000	21210.786958	23.697000	52.246000
1955.000000	0.000000	23014.651592	24.617000	56.595000
1956.000000	.006000	25514.158354	26.686000	61.635000
1957.000000	.013000	26645.426102	27.871000	63.908000
1958.000000	.053000	26501.824702	29.118000	68.017000
1959.000000	.091000	27935.244981	44.556000	94.840000
1960.000000	.159000	28776.775821	48.039000	101.551000
1961.000000	.224000	29926.271453	54.621000	112.312000
1962.000000	.263000	31716.753138	61.441000	126.726000
1963.000000	.311000	33845.190053	63.332000	131.626000
1964.000000	.373000	36504.394454	72.916000	150.655000
1965.000000	.433000	39376.525984	80.788000	167.313000
1966.000000	.471000	42442.941991	84.527000	176.490000
1967.000000	.507000	44113.000000	94.848000	203.385000
1968.000000	.568000	46013.610462	102.319000	216.648000
1969.000000	.623000	48672.499045	103.835000	217.531000
1970.000000	.678000	51514.709200	126.531000	259.793000
1971.000000	.721000	55136.330075	122.126000	269.416000
1972.000000	.766000	58526.951248	126.808000	291.596000

YEAR	TPPI	ECTR	D
1952.000000	.864000	.512685	.056312
1953.000000	.848000	.458296	.056324
1954.000000	.839000	.453566	.055456
1955.000000	.838000	.434968	.052976
1956.000000	.851000	.432968	.052780
1957.000000	.856000	.436111	.058160
1958.000000	.861000	.428099	.057842
1959.000000	.862000	.469802	.059727
1960.000000	.866000	.473053	.059084
1961.000000	.863000	.486333	.058990
1962.000000	.872000	.484833	.059606
1963.000000	.881000	.481151	.061402
1964.000000	.879000	.483993	.062086
1965.000000	.894000	.482855	.063492
1966.000000	.937000	.478934	.064911
1967.000000	1.000000	.466347	.065344
1968.000000	1.049000	.472282	.066512
1969.000000	1.099000	.477334	.068720
1970.000000	1.178000	.487045	.069009
1971.000000	1.241000	.453299	.069269
1972.000000	1.312000	.434876	.073661

YEAR	LTD	CS	PS	TCAP
1952.000000	192.627000	306.428000	0.000000	499.055000
1953.000000	231.646000	377.106000	0.000000	608.752000
1954.000000	267.584000	387.612000	0.000000	655.196000
1955.000000	266.402000	461.902000	0.000000	728.304000
1956.000000	305.019000	536.006000	0.000000	841.025000
1957.000000	343.407000	622.315000	0.000000	965.722000
1958.000000	423.000000	631.362000	0.000000	1054.362000
1959.000000	453.000000	734.400000	0.000000	1187.400000
1960.000000	545.000000	751.245000	0.000000	1296.245000
1961.000000	570.000000	848.160000	0.000000	1418.160000
1962.000000	630.000000	956.839000	0.000000	1586.839000
1963.000000	710.000000	981.212000	0.000000	1691.212000
1964.000000	735.000000	1100.001000	0.000000	1835.001000
1965.000000	794.353000	1139.033000	0.000000	1933.386000
1966.000000	944.803000	1331.782000	0.000000	2276.585000
1967.000000	1070.228000	1380.241000	0.000000	2450.469000
1968.000000	1193.052000	1428.367000	0.000000	2621.419000
1969.000000	1262.504000	1480.331000	0.000000	2742.835000
1970.000000	1386.504000	1539.930000	93.997000	3020.431000
1971.000000	1541.504000	1581.674000	197.997000	3321.175000
1972.000000	1652.238000	1640.672000	197.991000	3490.901000

YEAR	WD	WC	WP
1952.000000	.385984	.614016	0.000000
1953.000000	.380526	.619474	0.000000
1954.000000	.408403	.591597	0.000000
1955.000000	.365784	.634216	0.000000
1956.000000	.362675	.637325	0.000000
1957.000000	.355596	.644404	0.000000
1958.000000	.401190	.598810	0.000000
1959.000000	.381506	.618494	0.000000
1960.000000	.420445	.579555	0.000000
1961.000000	.401929	.598071	0.000000
1962.000000	.397016	.602984	0.000000
1963.000000	.419817	.580183	0.000000
1964.000000	.400545	.599455	0.000000
1965.000000	.410861	.589139	0.000000
1966.000000	.415009	.584991	0.000000
1967.000000	.436744	.563256	0.000000
1968.000000	.455117	.544883	0.000000
1969.000000	.460292	.539708	0.000000
1970.000000	.459042	.509838	.031120
1971.000000	.464144	.476239	.059617
1972.000000	.473298	.469985	.056716

YEAR	GBY	BUS	GER	GDR
1952.000000	.035630	.043300	0.000000	0.000000
1953.000000	.037050	.044200	0.000000	0.000000
1954.000000	.031760	.039000	0.000000	0.000000
1955.000000	.031370	.037000	0.000000	0.000000
1956.000000	.036250	.043800	0.000000	0.000000
1957.000000	.041130	.052800	0.000000	0.000000
1958.000000	.041120	.049300	0.000000	0.000000
1959.000000	.050740	.057100	0.000000	0.000000
1960.000000	.051850	.057600	0.000000	0.000000
1961.000000	.050460	.055200	0.000000	0.000000
1962.000000	.051130	.055100	0.000000	.010040
1963.000000	.050880	.054600	0.000000	.012658
1964.000000	.051830	.055500	0.000000	.014112
1965.000000	.052080	.056700	.019892	.014394
1966.000000	.056900	.064100	.030273	.013504
1967.000000	.059370	.069200	.038552	.016948
1968.000000	.067460	.077700	.039375	.019575
1969.000000	.075840	.086500	.037387	.019448
1970.000000	.079130	.092300	.036287	.018242
1971.000000	.069480	.082800	.038481	.019382
1972.000000	.072320	.082800	.039655	.022187

YEAR	RPS	ADY	GR	RCS
1952.000000	0.000000	.054585	0.000000	.054585
1953.000000	0.000000	.052165	0.000000	.052165
1954.000000	0.000000	.046168	0.000000	.046168
1955.000000	0.000000	.041365	0.000000	.041365
1956.000000	0.000000	.041982	0.000000	.041982
1957.000000	0.000000	.048356	0.000000	.048356
1958.000000	0.000000	.048204	0.000000	.048204
1959.000000	0.000000	.049008	0.000000	.049008
1960.000000	0.000000	.048748	0.000000	.048748
1961.000000	0.000000	.041889	0.000000	.041889
1962.000000	0.000000	.041929	.005020	.046949
1963.000000	0.000000	.040095	.006329	.046424
1964.000000	0.000000	.039125	.007056	.046181
1965.000000	0.000000	.036691	.017143	.053834
1966.000000	0.000000	.047285	.021889	.069174
1967.000000	0.000000	.051824	.027750	.079574
1968.000000	0.000000	.056054	.029475	.085529
1969.000000	0.000000	.054837	.028417	.083254
1970.000000	.064000	.058630	.027265	.085895
1971.000000	.061950	.056721	.028932	.085653
1972.000000	.066430	.060200	.030921	.091121

YEAR	PK1	PK2	PK3
1952.000000	.160155	.175345	.177228
1953.000000	.125231	.136464	.138232
1954.000000	.107001	.116984	.118876
1955.000000	.091396	.098595	.099916
1956.000000	.077693	.081827	.083585
1957.000000	.100767	.106137	.108836
1958.000000	.099377	.104257	.106415
1959.000000	.121474	.120176	.121735
1960.000000	.117177	.114988	.116539
1961.000000	.127788	.121419	.122635
1962.000000	.112580	.109464	.110468
1963.000000	.114781	.111567	.112574
1964.000000	.133606	.129337	.130294
1965.000000	.112098	.113396	.114629
1966.000000	.083305	.092433	.094413
1967.000000	.065361	.080303	.083311
1968.000000	.104032	.117924	.121394
1969.000000	.124338	.130295	.134114
1970.000000	.100253	.104954	.109848
1971.000000	.120940	.132738	.138235
1972.000000	.131168	.145448	.150156

YEAR	KL	S	PL
1952.000000	13.701643	.120377	1.677000
1953.000000	14.709612	.120065	1.803000
1954.000000	15.610318	.115018	1.881000
1955.000000	16.509279	.110351	1.961000
1956.000000	17.495374	.106860	2.007000
1957.000000	19.190283	.108679	2.095000
1958.000000	21.876172	.105949	2.210000
1959.000000	24.865998	.122420	2.317000
1960.000000	28.475924	.123118	2.463000
1961.000000	32.291103	.125704	2.606000
1962.000000	34.953806	.130123	2.722000
1963.000000	36.838447	.129468	2.812000
1964.000000	38.780201	.134910	2.885000
1965.000000	40.173301	.140183	2.975000
1966.000000	41.485645	.142215	3.153000
1967.000000	44.873626	.149102	3.404000
1968.000000	48.387219	.153050	3.691000
1969.000000	50.125269	.155710	3.998000
1970.000000	51.597130	.168455	4.422000
1971.000000	54.144688	.167813	4.853000
1972.000000	56.141564	.173726	5.326000

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