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A MODEL OF MOBILE TO TELEVISION INTERFERENCE

by

G.J. van der Maas and J. Schwarz daSilva

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Abstract-One of the major problems created by the allocation of new frequencies to high capacity land mobile systems is that of their interference impact to UHF television receivers. This paper proposes a new interference model and provides an insight into the interference analysis. The results presented indicate that the extent of interference to television sets caused by mobile stations is negligible.

I. INTRODUCTION

The development of high capacity cellular systems for land mobile communications is currently proceeding in various countries. In both the U.S. and Japan technical and market trials were conducted in 1978 and it is expected that the demand for such a service will allow its rapid expansion to major metropolitan areas in the near future.

In the U.S. the upper portion of the UHF-TV band (806-890MHz) has been reallocated on a nation-wide primary basis to the high capacity land mobile service. In Canada the Department of Communications in a recent policy [1] announcement recommended that the same band be used to provide for the growth of mobile services. It is expected that high capacity cellular systems will be available in Toronto or Montreal by 1982.

One of the problems created by such frequency allocations is that of interference. In effect it has been argued that the density of mobile telephones will be such that they will interfere with consumer electronic equipment and in particular with UHF television sets. Several studies [2], [3] of M.T.¹, that is interference to television sets from mobile transceivers have been conducted in the past. Whether mathematical or experimental, these studies attempt to predict a radius of potential interference around a hypothetical TV receiver. A recent study [4] suggests that this interference radius is of the order of 1056 ft and the conclusion has been drawn that it is not wise to assign TV channels 58 through 61 for use in an area also served by a high capacity mobile system.

¹ We suggest the introduction of a standard notation for interference, where the first symbol is the interference source and the second symbol the interfered object. Hence the notations M.T. is interpreted as a mobile interfering with a television set.

In this paper a mathematical model based on the work of Chandrasekhar [5] is suggested to compute the probability of interference and the mean interference duration. The model is general enough to warrant its application to various interference problems. These include the interference from mobile transceivers to UHF TV receivers as well as the interference from CB transceivers to consumer electronic equipment.

II. INTERFERENCE MODEL

Our objective is to determine how various parameters affect the probability that a randomly selected TV receiver will suffer from interference caused by mobile transceivers in an area served by a high capacity cellular system.

The model is based on the assumption that several mobile transceivers are geographically scattered over a given interference area (Fig. 1). The radius of this circular area, denoted by R_1 , corresponds to the distance beyond which a mobile transceiver will not cause interference to the TV receiver located in the center of the area. If the geographical density of mobile stations is denoted by D , the average number of mobiles in a circular area, A , of radius R_1 will be given by:

$$\bar{n} = \pi R_1^2 D = AD \quad \dots\dots\dots (1)$$

We now assume that the number of mobile stations within this area is Poisson distributed with mean \bar{n} . Hence the probability of finding n mobiles in this area will be given by:

$$P(n) = \frac{\bar{n}^n e^{-\bar{n}}}{n!} \quad \dots\dots\dots (2)$$

If α denotes the probability that a mobile leaves the area A within a time τ , given that n mobiles were present in A at the beginning of this time interval, the probability $P_i^{(n)}$ that i mobiles will leave the area in time τ is given by:

$$P_i^{(n)} = \frac{n!}{i! (n-i)!} \alpha^i (1-\alpha)^{n-i} \dots\dots\dots (3)$$

The probability P_i that i mobiles leave the area A, regardless of the value of n , is then given by:

$$P_i = \sum_{n=i}^{\infty} P(n) P_i^{(n)} \dots\dots\dots (4)$$

Replacing in (4) the appropriate expressions of $P(n)$ and $P_i^{(n)}$ we obtain the following Poisson distribution with mean $\bar{n}\alpha$:

$$P_i = \frac{(\bar{n}\alpha)^i e^{-\bar{n}\alpha}}{i!} \dots\dots\dots (5)$$

Assuming stationarity we conclude that P_i is also the probability for i mobiles entering the area A in time τ .

Having obtained an expression for P_i we now determine the various transition probabilities corresponding to a change of system state. If at time t there are n mobiles in an area A, at time $t+\tau$ the probabilities

that for $k > 0$ there are $n+k$ or $n-k$ mobiles present, are given respectively by:

$$P [n:t;n+k:t+\tau] = \sum_{i=0}^n P_i^{(n)} P_{i+k} \dots\dots\dots (6)$$

and

$$P [n:t;n-k:t+\tau] = \sum_{i=k}^n P_i^{(n)} P_{i-k} \dots\dots\dots (7)$$

For mathematical convenience we now introduce the variables $x = n-i$ and $y = i-k$. Then we can rewrite equations (3) and (5) as follows:

$$W^{(n)}(x) = \frac{n!}{x!(n-x)!} (1-\alpha)^x (\alpha)^{n-x} \dots\dots\dots (8)$$

and

$$P_y = \frac{(\bar{n}\alpha)^y e^{-\bar{n}\alpha}}{y!} \dots\dots\dots (9)$$

A. State Behaviour

It is clear from (8) and (9) that the expected value of the random variables x and y is given by:

$$\bar{x} = n(1-\alpha) \dots\dots\dots (10)$$

$$\bar{y} = \bar{n}\alpha \dots\dots\dots (11)$$

Defining now the new random variable m as the sum of the independent random variables x and y , its expected value will be given by:

$$\bar{m} = \bar{x} + \bar{y} = n(1-\alpha) + \bar{n}\alpha \quad \dots\dots\dots (12)$$

In order to describe the time behaviour of the state of the system we let:

$$\bar{m} - n = (\bar{n} - n)\alpha = dn \quad \dots\dots\dots (13)$$

and

$$\alpha = \alpha_0 dt \quad \dots\dots\dots (14)$$

Where α_0 is the probability for entering or departing from the interference area per unit of time.

Thus we obtain:

$$\frac{dn}{dt} = (\bar{n} - n)\alpha_0 \quad \dots\dots\dots (15)$$

with the solution:

$$n(t) = \bar{n} + (n_0 - \bar{n})e^{-\alpha_0 t} \quad \dots\dots\dots (16)$$

Equation (16) describes how a state n_0 at time $t = 0$ will on the average "decay" to state \bar{n} when t goes to infinity.

B. Mean Life and Recurrence times

The interference model introduced in section 2 will now be used to determine the mean life of a state and the average state recurrence time, where the state of the system is simply given by the number of mobile stations within area A at time t.

The state transition probabilities given by equations (6) and (7) can be expressed in terms of Laguerre Polynomials, L_n , and their derivatives. It can be shown that the following expression for the transition probability from state n at time t to state n at time t+τ is obtained:

$$P [n:t;n:t+\tau] = e^{-\bar{n}\alpha} (1-\alpha)^n L_n \left(-\frac{\bar{n}\alpha^2}{1-\alpha} \right) \dots\dots\dots (17)$$

Using equation (14) and making $\tau = \Delta t$ we can replace equation (17) by the following approximation:

$$P [n:t;n:t+\Delta t] \approx 1 - \bar{n}\alpha_0 \Delta t - n\alpha_0 \Delta t \dots\dots\dots (18)$$

The probability that, having started in state n at time t the system remains in state n over the next k-1 occasions, Δt units of time apart and exits from state n at the kth occasion will be given by:

$$\phi_n [(k-1)\Delta T] = P^{k-1}_{(n,n)} [1-P(n,n)] \dots\dots\dots (19)$$

where for notational convenience we have denoted the probability given by equation (18), by P(n,n).

The probability density function $\phi_n(t)$ is obtained by rewriting equation (19) as follows:

$$\phi_n [(k-1)\Delta t] \approx \alpha_0(\bar{n}+n) e^{(k-1)\ln P(n,n) \Delta t}$$

or

$$\phi_n [(k-1)\Delta t] \approx \alpha_0(\bar{n}+n) e^{-(k-1)\alpha_0(\bar{n}+n)\Delta t}$$

Taking the limit $\Delta t \rightarrow 0$, $(k-1)\Delta t \rightarrow t$ we obtain:

$$\phi_n(t) = (n+\bar{n})\alpha_0 e^{-(n+\bar{n})\alpha_0 t} \dots\dots\dots (20)$$

The expected value of t , that is the mean life T_n of state n is therefore:

$$T_n = \int_0^{\infty} t \phi_n(t) dt \dots\dots\dots (21)$$

or:

$$T_n = \frac{1}{(n+\bar{n})\alpha_0} \dots\dots\dots (22)$$

Similarly we can find the mathematical expectation of the time required for the first return to state n . The mean recurrence time to state n , denoted by θ_n is given by:

$$\theta_n = T_n \frac{[1-P(n)]}{P(n)} \dots\dots\dots (23)$$

Replacing $P(n)$ by the appropriate expression given by equation (2) we find:

$$\theta_n = T_n \frac{\left[1 - \frac{\bar{n}^n}{n!} e^{-\bar{n}} \right]}{\frac{\bar{n}^n}{n!} e^{-\bar{n}}} \dots\dots\dots (24)$$

The "availability" of state n , that is the fraction of time spent in state n will be given by²:

$$A_n = \frac{T_n}{\theta_n} = \frac{(\bar{n}^n e^{-\bar{n}}) / n!}{(1 - \bar{n}^n e^{-\bar{n}}) / n!} \dots\dots\dots (25)$$

Since we are interested in the interference-free state, ($n=0$) we can conclude that an arbitrary TV receiver will be interference-free for a fraction of time A_0 given by:

$$A_0 = \frac{1}{e^{\bar{n}} - 1} \dots\dots\dots (26)$$

C. Interference Probability

A probably more meaningful measure of the availability of state n is given by:

$$\frac{T_n}{T_n + \theta_n} = \frac{\bar{n}^n e^{-\bar{n}}}{n!} \dots\dots\dots (27)$$

which is simply the probability $P(n)$ that n stations are present when their average number is \bar{n} .

² It should be realized that A_n as given by equation (25) is not normalized, i.e. $\sum_{n=0}^{\infty} A_n \neq 1$.

Hence the probability of occurrence of an interference-free state ($n=0$) is:

$$P(0) = e^{-\bar{n}} \dots\dots\dots (28)$$

Therefore it can be concluded that the probability of interference is given by:

$$P(n \geq 1) = 1 - e^{-\bar{n}} \dots\dots\dots (29)$$

Up to this point our model assumed that the mobile stations were continuously transmitting. However, it is known that the mobile traffic characteristics are bursty, in the sense that their peak to average load is extremely high. If we denote by ρ the average traffic load per mobile station, the probability of no interference when there are n stations in the interference area is given by:

$$P_{NI}(n) = P(n) (1-\rho)^n \dots\dots\dots (30)$$

Summing over all possible values of n we obtain:

$$P_{NI} = \sum_{n=0}^{\infty} P(n) (1-\rho)^n$$

$$P_{NI} = e^{-\bar{n}\rho} \dots\dots\dots (31)$$

Equation (31) above, plotted in Fig. 2, gives then the probability of no interference when there are on the average \bar{n} mobile stations in the interference area and when each station contributes to the traffic load by an amount ρ .

III. RESULTS

The interference model suggested in the previous sections allowed us to derive expressions for the mean life time, the recurrence time and the probability of interference. We will now discuss by way of an example the typical numerical results given by the model. Several authors indicate that a typical interference area is about 0.1 mi^2 and that an average mobile density of 10 mobiles per mi^2 is characteristic of a mature cellular system. We then conclude that \bar{n} , the average number of mobiles within an interference area, is of the order of 1. Lacking any experimental results we suggest that α_0 the probability density for entering or leaving the interference area, per unit of time is directly proportional to the average mobile speed, \bar{v} , and inversely proportional to the radius, R_1 , of the interference area. Hence, if we assume that the constant of proportionality is one, we obtain:

$$1/\alpha_0 = \frac{R_1}{\bar{v}} \dots\dots\dots (32)$$

Assuming an average vehicle speed of 10 mi/h and a radius R_1 corresponding to an interference area of 0.1 mi^2 we conclude that $1/\alpha_0$ is equal to 64.23 s.

Substituting the values of $\bar{n}=1$ and $1/\alpha_0=64.23 \text{ s.}$ in equations (22) and (23) we obtain:

$$T_n = \frac{64.23}{n+1} \text{ s}$$

and

$$\theta_n = \frac{64.23}{n+1} (en!-1) \quad s$$

The probability of no interference computed from equation (31) is:

$$P_{NI} = e^{-\rho}$$

It is interesting to note that for typical values of ρ between 0.01 and 0.02 erlangs/mobile the probability of no interference varies between 99% and 98%.

IV. CONCLUSION

One type of interference has been examined namely the interference caused by mobile stations in high capacity cellular systems to UHF television receivers. A model was presented that is useful in determining various interference related measures such as the mean life time and the mean recurrence time of the system state.

It has been shown that for the expected mobile densities and traffic loads the probability of interference is for all practical purposes negligible. It is therefore suggested that TV channels 58 through 61 could certainly be assigned for use in an area which is also served by a cellular mobile telephone system.

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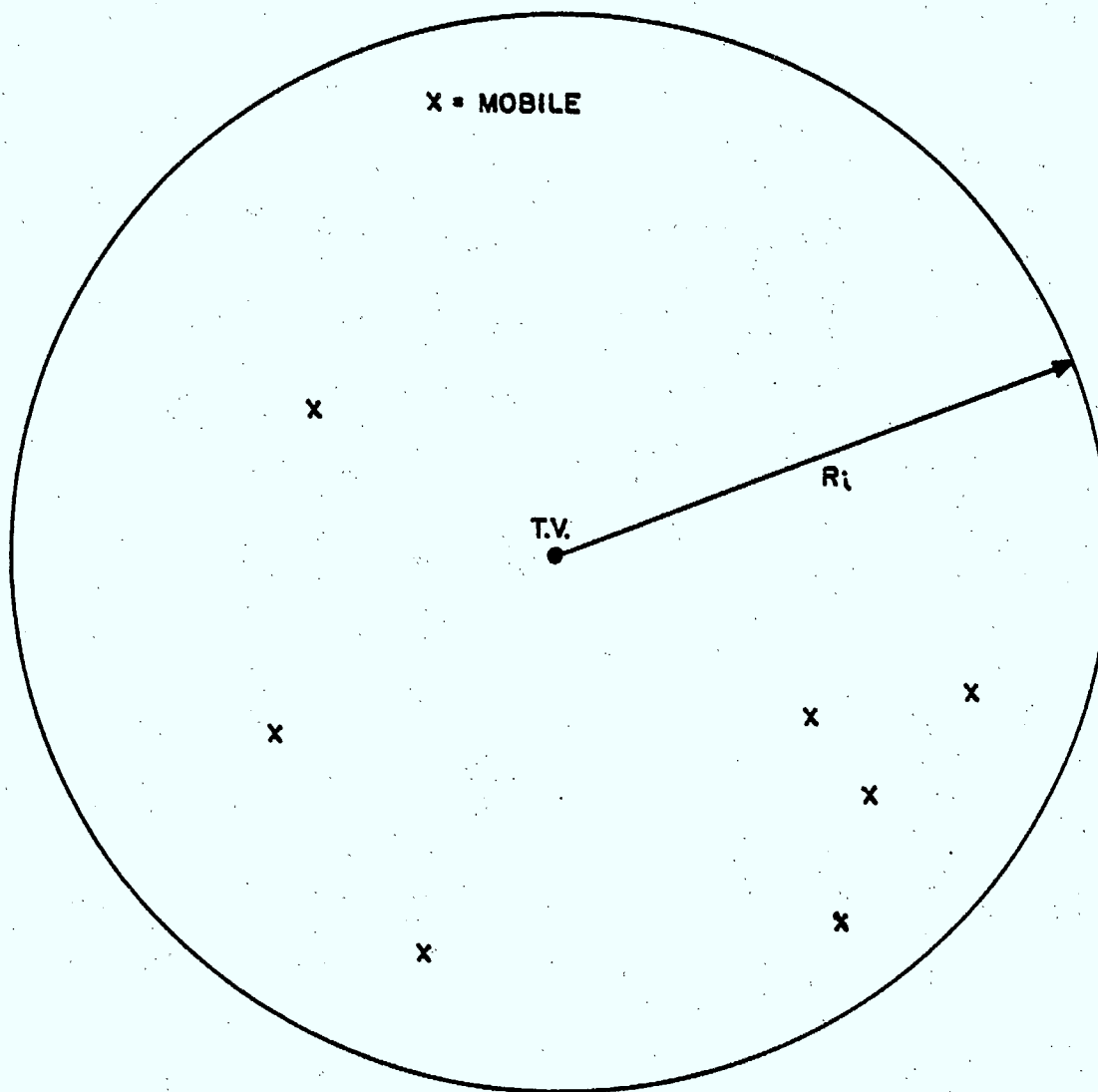


FIGURE 1. MODEL OF THE INTERFERENCE AREA

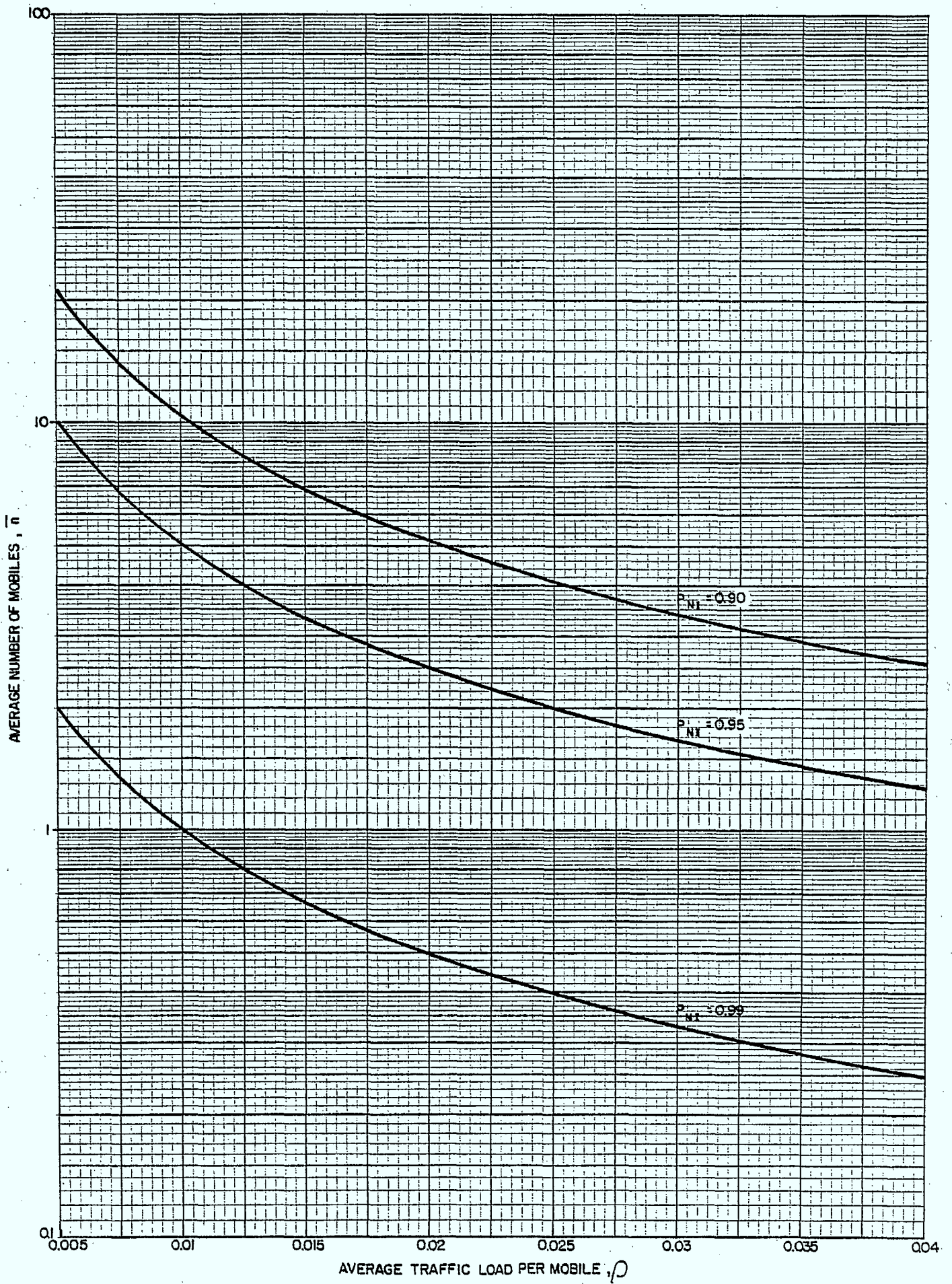


FIGURE 2. PROBABILITY OF NO INTERFERENCE

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A model of mobile to television interference.

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