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Intermodulation Study

UHF Multipurpose Satellite Transponder

DSS Contract PL 36100-4-2003

Serial OPL4-0184

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REPORT NO. 552-2

Study on Intermodulation Analysis

and Recommendations

for the

UHF Multipurpose Satellite Transponder

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Study on Intermodulation Analysis and Recommendations for the UHF Multipurpose Satellite Transponder

Under

DSS Contract PL 36100-4-2003

Serial OPL4-0184

July 15, 1975

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1.0 Introduction

1.1 Statement of Work

The report covers work carried out for the Department of Communications under DSS Contract PL 36100-4-2003, Serial OPL4-0184 to Work Plan Reference CS 4.6.

1.2 Background

System concepts provided by the Department of Communications for the proposed multi-purpose UHF communications satellite provide potential system configurations for UHF, UHF/SHF, and UHF/SHF/L Band transponders.* The final system configuration, not firmly established at the present time, will be dependent upon factors including traffic characteristics, RF Interference environment, and equipment availability. The UHF section of the system was stressed in the present investigation since it is common to all configurations, and presents the main problem areas in respect to passive component intermodulations interference.

* Department of Communications Report "Multi-purpose UHF Satellite Communications System Feasibility Study" December 1974.

The UHF band uplink and downlink frequency assignments within the mobile-satellite service bands 240-328.6 MHz and 335.4-399.9 MHz are not confirmed at the present time.

The following bands have been under consideration, with the latter 275-285 MHz downlink band producing the most suitable frequency plan from intermodulation requirements.

D/L Band (MHz)	U/L Band (MHz)	Lowest Intermod Order on UL Band
300 - 328.6	350 - 400	3
300 - 328.6	370 - 400 (370-385 SSMA) (385-400 FDMA)	5
321.5 - 328.5	370 - 400	13
275 - 285	370 - 400	19

Additional uplink UHF bands include 401 - 403 MHz (DRP) and 406 - 406.1 MHz (EPIRB).

Approximately eighty carriers at a nominal one watt average power level per carrier are to be considered, with an additional number of carriers (up to 20) ranging between $\pm$ 10dBw. Transmitter (HPA) output signal-to-intermodulation levels for a few carriers ranging between -10dB and +10dB from nominal levels vary from 12dB to 27dB respectively, to 19dB for nominal carriers.

The satellite receiver (LNA) thermal noise power in a 25KHz bandwidth is -128dBm. Transmitter spurious and intermodulation noise generated by all sources within the transponder and satellite occuring on the uplink bands must be reduced to 10dB below thermal noise (-138dBm) for a maximum c/n degradation of 0.5dB.

Assuming that the transmitter thermal noise, spurious, and intermods falling on the receive band are reduced to -138dBm by output filtering, and negligibly small receiver intermods due to incident transmit signals occur due to adequate input filtering, the principal intermodulation interference becomes that due to passive component generated intermodulation. The main sources are transmission components and structure, which share a common transmission path to transmit signals, where due either to location of the component in the transponder circuit or to re-radiation of antenna coupled signals, intermodulation reduction cannot be provided by isolation techniques.

The following hardware items present the most important passive component intermodulation sources in a diplexed antenna system:

1) Diplexer

(a) Output terminal and transmit-receive filters combining section.

(b) Transmitter output filter; principally the filter sections adjacent to the antenna terminal where attenuation to filter-generated intermodulation on the receive band approaches minimum.

(c) Receiver input filter; principally the filter sections adjacent to the antenna terminal where the attenuation to transmit signals is insufficinet to reduce the filter-sections generated intermod to the required receiver terminal level.

2) Antenna feed, including the UHF radiator, balun, transmission line, and SHF and L-band feeds coupled to UHF feed system currents.

3) Antenna reflector; principally the high current density surfaces.

4) Spacecraft structure and components coupled to the UHF antenna high intensity fields.

1.3 Summary

1) Performance data and pertinent information obtained in discussions with TRW, Hughes, and the Naval Research Laboratory pertaining to the Fleetsatcom and Marisat UHF band satellite transponders is included in Appendix III for reference.

High order (i.e., nineteenth) passive component intermodulation data for a large number of transmitting carriers such as is representative of the proposed Canadian multipurpose UHF satellite transponder was not available.

Of primary importance was confirmation of the range of practical third-order two-tone intermod specifications for spacecraft hardware, and provision of some high-order products roll-off rate and power back-off rate data, though the latter are reasonably unique for the particular transponder design.

2) TRW test data for Fleetsatcom was limited to third and fifth-order interfering intermodulation products. Third to eleventh-order transmitting channels products fall on the receive frequency band, with fifth-orders affecting the greatest number of receive channels, and seventh and higher orders sufficiently low so as to be of no problem.

A third-order intermod test specification for the spacecraft antenna and filter hardware of -100 dBM for two-tone 30 watt clean carriers was employed. The specification is moderately easy to accomplish for production model hardware and approximately 30 dB greater than state-of-the-art results for laboratory test models.

3) Hughes data for the Marisat transponder was limited to thirteenth to twenty-first order three-carrier products. Thirteenth-order antenna intermods were -150 dBM for three +46 dBM test signals, increasing to -117 dBM for the antenna and rotary-joint assembly mounted on the spacecraft. The diplexer comprising three transmit filters and a nine-section receive filter and including rotary-joint generated thirteenth-order intermods of about -125 dBM under similar test conditions. Strict attention was not given to intermod reduction techniques during Marisat transponder manufacturing, and it is estimated that equivalent third-order two-tone intermods for which measurement data was not available may be of the order of -50 dBM.

4) Intermodulation testing can be performed utilizing a number of basic approaches to test signal conditions, and quantity of test signals employed. Unmodulated test signals are typical for laboratory testing, yielding worst case amplitude-frequency spectrum results over the noise-modulated test signal case for which a dispersion of the intermodulation spectral distribution occurs.

The following test methods are representative:

a) Two-tone CW signal test, for which the product amplitude specification is designed for equivalence to the intermodulation result under the final system actual signal conditions.

b) Modelling of a test signal in which the number of and modulation of test signals suitably simulates the final system actual signal conditions. The method will employ the smallest number of test signals permissable, and is practical at the sub-system and final system test levels. Three-tone testing employing CW signals can be included as a variation of the approach and yields additional data for development analysis over two-tone data, particularly where frequency band allocations require measurement of the more dominant A-B+C product. In addition, the three-tone product amplitude distribution provides an improved approximation to the Gaussian distribution over two-tone data.

c) Intermodulation noise level measurement under actual signal conditions. The method is impractical at the component development level, and is principally employed at a system acceptance test level.

5) The two-tone CW test method is generally accepted as the most practical means for carrying out intermodulation development tests and diagnostics at the component and sub-system level due to the inherent simplicity and performance level attainable by the measurement circuit over a test situation involving larger numbers of carriers. Further advantages provided by the two-tone test are the relative ease of determination and detection of discrete product orders and frequencies, and in standardization of the testing method.

Correlation between the two-tone product test data and the satellite intermodulation noise level due to a number of noise loaded carriers is essential for justification of a two-tone test specification.

The Fleetsatcom transponder specification requires that the receiver sensitivity degradation due to twenty three (operating) noise loaded carriers be 0.5 dB, or intermod noise be approximately -10 dB from receiver thermal noise. The equivalent third-order two-tone product amplitude was established at -150 dBM (including a 20 dB margin for additional degradation due to the spacecraft).

6) A practical third-order intermod test specification for the proposed multipurpose UHF satellite which is applicable for spacecraft hardware at the component level, was established at -110 dBM for two-tone +45 dBM test signals. Transponder passive hardware developments should be carried out to 20 to 30 dB better than spacecraft hardware specifications for the purpose of increasing the margin for satellite system degradation to the greatest practical amount.

7) A summary of applicable transponder passive hardware intermodulation level achievements for state of the art fabrication techniques are given in Para. 4.1.

Third-order intermodulation levels to approximately -145 dBM for two +45 dBM carrier test, or -190 dBM from carrier average power have been obtained for UHF filters and diplexers under laboratory conditions. However, the inconsistency of the laboratory results and possible performance degradation due to environmental factors indicates such very low levels to be impractical for space hardware at the present time.

Helical transmitting antenna third-order intermodulation levels to -130 dBM have been obtained, degraded to -100 dBM for the final antenna-reflector assembly specification for spacecraft hardware. Very low levels of antenna intermodulation are usually difficult to evaluate due to re-radiation from nonlinearities in the antenna test environment. The above levels were obtained in an anechoic chamber where some degree of nonlinear behaviour is to be expected due to high intensity field regions and incident signal reflection from both the resistive material and the metal reflector interface.

Due to inherent difficulties in low level antenna intermodulation testing, special test facilities in the nature of either a free-space or anechoic high linearity test range will be required for the proposed satellite development program.

8) The power back-off rate for seventh and higher-order intermods is estimated to be 1 dB/dB. A roll-off rate of 5 dB per odd order for high orders is assumed for specifying purposes. Since the nonlinearity coefficients for the transponder passive components and complete system are not predictable, simulation testing coupled with computational methods are required to establish final intermodulation performance.

9) A diplexed (common) antenna UHF transponder appears feasible for frequency band allocations such that lowest intermod orders of about fifteenth to nineteenth order due to satellite transmitting signals fall on the receive channels.

The estimated achieveable system intermod specification as determined by a two-tone CW test is given in Figure 4-1 versus the receive band lowest interfering product order. The repeater receive band intermod noise performance can be accurately determined only on establishing the necessary correlation between the two-tone product amplitude at the receive band lowest order interfering product, and the noise-loaded carriers intermodulation noise falling in the receive channels bandwidth.

The conversion coefficients inter-relating the two cases can be predicted only after measurement of the transponder transfer characteristic nonlinearity coefficients, although computational accuracy becomes the major limitation at high order products. Alternatively, the correlation can be obtained through a direct measurement of receive channel noise when employing a modelled test signal.

10) Multipactor breakdown, better described as a predecessor to plasma, is a significant problem area in transponder hardware design. The effect occurs at a range of pressures of 10^{-2} to 10^{-9} torr with the principal parameters being frequency, conductor spacing, conductor work function, and field intensity. The effect has been obtained at 15 watt average power levels, and is inititated by microdischarge occuring at discontinuities in the transmit signal path.

1.4 Passive Component Intermodulation

Design Guidelines for the Multipurpose UHF Communications Satellite Transponder

1.4.1 NRL Guidelines for UHF Transponder Development

Difficult problem areas were encountered in the Fleetsatcom UHF transponder development due to passive component intermodulation in a diplexed antenna system, and multipactor breakdown. The transmit-receive frequency allocations for Fleetsatcom resulted in some third-order, but principally fifth-order intermodulation interference in the receive band. Since the frequency bands could not be altered for an improved compatability with the intermod spectra, separate transmit and receive UHF antennas providing 50dB of antenna isolation were employed in the final satellite configuration, rather than the originally proposed diplexed antenna system.

Several important guidelines toward a UHF satellite development program based upon the Fleetsatcom program experience were stated by NRL:

1) Carry out a transponder simulator development and testing phase to establish as many potential problem areas as possible. Simulation of the entire communications package (UHF-SHF-UHF) was recommended. The importance of the simulator test phase in relation to the Fleetsatcom program is such that simulator data provides the reason for proceeding with the spacecraft program.

2) Carry out a knowledgeable testing program on the simulator package to determine the system intermodulation noise and related problem areas to the greatest possible extent. It was stated that all operational intermodulation problems will still not have been completely solved after simulator testing.

The implications relate primarily to the Fleetsatcom program experience where the array of detrimental factors (Frequency plan, diplexed antenna, state of the art hardware requirements and techniques, and multipactor effects) produced initially unsolveable intermodulation problems.

A comprehensive test program involving unit specifications, subsequent relating to noise-loaded-carriers transponder specifications, through to performance specifications in a simulated space environment is envisaged for the transponder simulator test phase.

3) Design the spacecraft following the transponder design stage; The note again reflects Fleetsatcom experience and suggests the situation in which transponder design may result in extensive packaging and antenna system modifications.

Completion of the transponder simulator test phase, is probably a satisfactory event at which to commit the spacecraft design.

1.4.2 Canadian Multipurpose UHF Satellite Transponder Intermodulation Design Considerations

A major decision area for the Canadian Multipurpose UHF Satellite Transponder in regard to intermodulation interference concerns diplexed or isolated transmit and receive UHF antennas. The decision depends directly upon the UHF band frequency allocations and the intermodulation products minimum order falling in the receive band.

Intermods generated by satellite transmitting signals, which due to the transmit segment bandwidth and separation of the segment from the satellite receive band, which result in low order intermods (and up to approximately fifteenth-order) on the receive band, will impose a requirement for isolated transmit-receive antennas.

Isolated UHF antennas are to be preferred from the stand-points of:

1) Reducing the transmitting filter and antenna circuit generated intermods at the receiver terminal.
2) Reducing the receiver antenna circuit generated intermods.
3) Reducing the transmit filtering requirement for suppression of transmitter amplifier generated noise in the receive band.
4) Reducing the receive filtering required for carrier suppression at the receiver input.
5) Increasing the margin for intermod degradation.

6) Preference of frequency assignment for the transmit band; that is, added antenna circuit isolation will allow for decreasing orders of intermod products to appear on the receive band by simultaneously reducing the degradation margin.

A transmit band frequency allocation which results in nineteenth and higher intermod product orders on the receive band appears quite feasible for the diplexed (common) UHF antenna approach to transponder design. A transmit band frequency allocation which results in about fifteenth order products on the receive band appears to be borderline for diplexing. In this case additional costs and development efforts are envisaged to accommodate the reduced intermod specification required to meet the diplexed antenna design approach.

Alternatively, for a fifteenth order case, a simulator development approach incorporating isolated antennas is suggested, with filters and antenna system hardware development carried out to specifications suitable for the diplex approach.

A mechanically complex common-mast transmitting antenna assembly containing the UHF, SHF, and TT&C antennas presents potentially difficult intermodulation design problems. Physical separation decoupling of all antennas from the UHF transmitting antenna feed is preferred from intermodulation requirements.

The levels of intermod products falling on the receive band cannot be mathematically predicted at the onset of design, but must rather be established by a measurement-computation approach. A conservative initial estimate for a two-tone specification for the UHF transponder, which is based upon available practical data is given in Figure 4-1. A 10 dB roll-off rate per odd-order product is considered to be a good rule-of-thumb, and for this case is applied to the transponder-antenna system up to fifteenth-order products. The 5 dB rate for seventh-orders and up for component specifying is again by rule-of-thumb although little supporting data is available. The actual transponder intermodulation spectra will be reasonably unique. A 20 dB degradation factor decreasing with product order, is included in the system specification. The estimate will depend upon transmitting antenna illumination of spacecraft hardware (the solar panels and telemetry receiver diode front-end were noted to be prominent sources on Fleetsatcom).

Recent discussions with NRL confirmed that a 20 dB margin for solar panels and satellite structure contributions should be satisfactory.

No deliberate allowance for space deterioration of the intermodulation noise level has been included. At this time no data has become available with the exception of the Lincoln Labs LES-6 result which indicated an intermod malfunction improvement in space rather than deterioration. Confirming information within this potential problem area is required.

The transponder system specification given in Figure 4-1 includes an additional factor for (n) noise-loaded carrier noise level conversion from the two-tone +45 dBM carriers product amplitudes. For a total permissable intermodulation noise level of -138 dBM at the receiver terminal, approximately +6 dB is allowed for fifteenth-order product conversion, increasing to approximately +17 dB at nineteenth order.

1.5 Recommendations for Transponder Hardware Development Program

1.5.1 Program Plan

Suggestions toward a program plan up to the transponder simulator (RF package) testing stage are given in Figure 1-1. It is recommended that the UHF and VHF/SHF systems RF package testing be accomplished by the earliest possible date in the overall program. NRL have recommended that the complete communication package including power supplies and necessary spacecraft equipment be included in the overall simulation.

1.5.2 Notes Pertaining to Development Program Plan

<u>100-110</u> UHF Primary Feed:

An initial development of the UHF transmitting and receiving feed only employing a solid reflector (and possibly SHF feed mockup) is recommended to simplify the intermodulation design; then progressing to increasing complexity.

<u>120-130</u>

UHF feed intermod testing is performed on the feed, mast, balun, and solid reflector sub-assembly. Both two-tone and modelled test signal intermod testing is performed to determine the two-tone test specification for the sub-assembly.

<u>130-140</u>

Multipactor effect testing of the above assembly at this development stage is included to provide mechanical design guidance.

200-210 Transponder Test Signal Intermod Test Facility:

The facility as suggested will comprise a number of channelized low-power solid-state amplifiers with noise modulation capability.

Eight to ten couplers with common output, providing approximately 0.5% frequency separation channels at 70-80 dB isolation for amplifier intermodulation protection is envisaged for the test signal model.

220-230 Two-tone Intermod Test Facilities:

Several testing facilities, at least for antenna and diplexer development testing phases are suggested. The following upgrading of the present configuration of UHF Intermod Test Facility is proposed:

1) Addition of band-reject filters to the present band-pass three-channel assembly.
2) Addition of manual-tune frequency ganging to filters.
3) Improvement of spectrum analyzer-preamplifier detection capability to approximately -155 dBM noise level in a 10 Hz bandwidth.
4) Addition of frequency stabilized 100 watt amplifiers.

700-710 UHF Diplexer Development Model

A brass-board development to initial design specifications is recommended.

710-720

Two-tone and modelled test signal intermod testing is performed to determine the two-tone test specification for the sub-assembly

725-735

Finalization of the diplexer frequency response characteristics following amplifiers simulation signal testing is proposed.

720-730

Multipactor testing is included to provide mechanical design guidance.

900-910 UHF Simulator Testing

The test phase covers only the UHF section, employing preferably the UHF HP Amplifier (or the channelized test signal Intermod Test Facility).

920-930 UHF/SHF Simulator Testing

Components at the service test model stage are shown. Development stage UHF/SHF simulator testing is recommended as well.

2.0 Passive Component Intermodulation

2.1 General

Passive component intermodulation presents a significant problem in satellite transponder design in terms of establishing a suitable channel performance (S/N) within any given channel. Passive component intermodulation of major importance is that generated by linear passive components where, due to location and usage of the components in the transponder circuit, intermodulation elimination or reduction cannot be adequately facilitated by filtering and isolation techniques.

All transmission line components including antennas, filter configurations, various forms of transmission lines, and RF connections are typical and are treated as linear under normal voltage-current test conditions. However, under simultaneous excitation by two or more carriers, small non-linearities which exist in the dynamic transfer characteristic for a linear device will result in generation of spurious intermodulation products. In addition to the devices comprising the receive and transmit antenna packages, other obscure hardware items in their immediate surroundings may be significant intermodulation sources.

Experience with passive component intermodulation has shown that the most prominent sources occur at regions or circuit locations most strongly excited by currents at the local transmit signal frequencies. The prominent sources are also likely to be the most efficient radiators of intermodulation signals due to antenna-like characteristics.

The levels of the intermods produced cannot be predetermined accurately in the design stage, and can only be ascertained from measurement. Rough estimates of performance prior to actual component fabrication and measurement must be based upon previous measurements of devices incorporating similar hardware and fabrication techniques.

Lower odd-order products which fall in the receive band are the most significant intermodulation interferences in the UHF (225-400 MHz) band with interferences decreasing with increasing order of product. Low even order products do not cause interferences in UHF band systems.

Dependent upon the frequency allocations for the transmit band and the receive band, and upon product amplitide decay (roll-off) with product order, the high-order, even-order products can be potential interferences at the high end of the 225-400 MHz band.

As a rule of thumb, the highest orders of intermodulation products are generated by the most prominent third-order intermod sources, while all sources usually contribute to the lower order products. Multi-tone excitation (greater than two tones) produces the situation where higher order intermodulation signals are superimposed on the lower-order intermodulation frequencies, and the observed signal level is primarily due to the lowest order product. In comparison, two-tone excitation produces observed signal levels due to the summation of all similar order contributions at a given order frequency.

Passive component intermodulation signals generated by all sources produce interferences when the intermodulation spectra appearing in the receive band at the receiver terminal, is significant relative to the receiver thermal noise power level. The transponder specification requires that all interferences be reduced to 10dB below receiver thermal noise for maximum degradation in uplink C/N of approximately 0.5 dB.

2.2 Non-linearity Mechanisms

A complete quantitative characterization of the weak non-linearities in linear passive components is not available due to the complexity of the task. However, the dominant causes for nonlinearities can be categorized according to the following:

1) Junction or semiconductor effects.

2) Plasma effect (Multipacing and corona).

3) Resistive nonlinearity due to local high current density. (Thermal conduction modulation)

4) Low level surface effects (Nonlinear bulk materials, nonlinear absorbing materials, contaminants.)

Material surface finishes, bulk materials, and RF connections according to the following items tend to be major non-linearity contributions in passive hardware development:

1) Coaxial connectors, RF cables, flanges.

2) Metal-to-metal pressure contacts; screw covers; spring-finger, press-fit, and point contacts; tuning screws.

3) Dissimilar metal contacts; oxidized metal contacts (metal-oxide-metal tunneling).

4) Corrosion (solder flux, etc.)

5) Contacting surface texture (loose contacting junctions and microdischarge mechanisms); roughness, burrs, cracks, scratches, imbedded chips, filings, metallic filament conductors.

6) Contaminants; water vapour, solder flux, etc.

7) Magnetic materials; ferrite, stainless steel, etc.

Reduction of passive component nonlinearities requires designs which minimize high voltage stresses and high current densities. Manufacturing to state-of-art intermodulation levels requires precision machining, clean room facilities, and tight assembly control.

3.0 Intermodulation Amplitude Relationships

3.1 The transfer characteristic for a nonlinearity without memory (i.e., output voltage is an instantaneous function of the input voltage) can be described by a Taylor series expansion:

$$E_{out} = a_1 E_{in} + a_2 E_{in}^2 + \ldots + a_n E_{in}^n + \ldots \text{ where}$$

$a_1, \ldots a_n$ are constant coefficients of curvature, and E_{in} represents the input signal applied to the nonlinearity.

Two expansions are shown for reference (Appendix I); a two carrier input for expansion order $n \leq 5$, and an m carrier input each of equal amplitudes.[1]

The first case demonstrates the generation of output terms (d.c. components, fundamentals, harmonics, and intermods) up to fifth order, and the second case demonstrates contributions to third and fifth order products to fifteenth order curvature.

The following observations can be drawn for the amplitudes of lower order of products relative to harmonic amplitudes:

1) Two frequency third-order products, type $2 W_1 \pm W_2$ are 9.6 dB greater than third harmonic products for equal amplitude carriers.
2) Three frequency third-order products, type $W_1 \pm W_2 \pm W_3$ are 15.6 dB greater than third harmonic products, and 6 dB greater than $2 W_1 \pm W_2$ type.

3) Two frequency fifth-order products are 20 dB greater (i.e. $3 W_1 \pm 2 W_2$) and 14 dB greater ($4 W_1 \pm W_2$) than fifth harmonic products.

For a large number of simultaneously applied carriers, the important product of each order is $W_1 \pm W_2 \pm W_3 \pm \ldots$ type, as to both amplitude and number of products.

(1) Research into Non-Linear Characteristics of Antenna systems, D.I.R. Report 239-1, F.G. Buckles, Sinclair Radio Laboratories.

3.2 Characterization of Low-Order Nonlinearities

The nonlinearity can be characterized given the values of the a_n's of the voltage transfer characteristic. Direct measurement of the transfer characteristic to the accuracy needed to determine the Taylor series coefficients cannot be carried out, however the following methods can be employed for determination of the low order coefficients:

1) Direct measurement of harmonic amplitudes with single carrier input, and determination of the coefficients. The method becomes impractical at other than the low harmonic orders due to inadequate detection sensitivity.

2) Direct measurement of a two equal carrier product at a given input voltage, and determination of the coefficient, or

3) Direct measurement of a two equal-carrier product of order n for a range of input voltage (i.e. assumes constant coefficients) and solution of the set of coefficients (a_n, a_{n+2}, a_{n+4}, ...) derived from the Taylor series expansion for the measured product. The method is likely to be limited by computational errors when dealing with a large set of simultaneous equations or large n.

3.3 Determination of System Total Intermod Power

3.3.1 The following methods for prediction of multi-noise loaded carrier system intermod levels, for low orders of nonlinearity, are given in the literature.

1) Nonlinear Distortion in Transmission Systems, R.A. Brockbank and C.A.A. Wass, J.I.E.E., 1945, Pt. III. (Appendix II).

   The method obtains the distribution of total intermodulation power over the operating band for low orders of expansion (2 to 5 approx.) of the power series. Calculations are referenced to measured harmonic powers.

2) Transmission Systems for Communications, 4th Edition, Bell Telephone Labs. Inc.

The Intermodulation noise computation (at low orders) is obtained from the spectral density and autocorrelation functions, with the input signal to the nonlinearity represented by a band of noise.

3.3.2 A measurement of the total intermod by means of noise loading[2] or simulation signal input to the nonlinearity appears the most suitable approach for a wide band system in which high order intermods occur on the receive band.

(2) Communications System Design: Philip F. Panter, McGraw Hill Co.

As the probable number of superimposed products exceeds 20 or 30, they can be assumed to become indistinguishable from random noise, becoming increasingly random with the masking due to noise loaded inputs.

4.0 Intermodulation Data

4.1 Summary of Third-Order Intermod Level Achievements

The following is a summary of stated achievement levels for passive hardware third-order two-clean carrier intermodulation levels under laboratory conditions. The levels can be assumed to relate to hardware components which are designed and fabricated according to present state-of-art techniques.

1) UHF Diplexer (NRL), -140 dBM third order at +50 dBM test carrier levels.

   The level was not consistent. Precision machining and extreme care in assembly was exercised. A 20 to 30 dB satellite environment deterioration was expected.

2) UHF Diplexer (MIT Lincoln Labs), -125 dBM third-order at +45 dBM carrier levels.

   Levels were within a -125 to -135 dBM range with soldered 0.250 inch diam. semi-rigid cable connections, and within a -100 to -110 dBM range with 0.141 inch diam. semi-rigid cable and TNC connectors. The range of levels resulted from signal summing with line length change between the diplexer and load filter (used to isolate the load intermod from diplexer intermod).

3) UHF Diplexer (TRW), -130 dBM third order at +45 dBM carriers.

4) UHF Intermod Test Bed (SRL), -135 dBM third order at +50 dBM carriers. The result includes and is limited by 50 ohm high power load intermod. The level is stable and repeatable with routine maintenance of the output RF connectors. A consistent level without routine maintenance is approximately -110 dBM. Voltage stresses and current densities are generally reduced by hardware design, (conductor dimensions) over equivalent levels occuring in satellite diplexer designs.

5) UHF Helical Transmitting Antenna (TRW), -130 dBM third order. The test conditions were not defined.

6) UHF Biconial Dipole, including 200 foot feeder (Heliax FHJ-5-50) and four pairs of Type N connectors (SRL), -100 dBM. The result includes the coupled environment contribution. The RF circuit intermod sources are stable and repeatable with connector maintenance.

4.2 Practical Third-Order Intermod Specification for UHF Passive Hardware

An initial development goal for transponder passive hardware components to be used for simulator evaluations should be of the order of 20 to 30 dB better than final requirements.

For the purpose of increasing the margin of protection to total system intermods at the receiver to the greatest practical amount, component development to state-of-the-art intermodulation levels or typically -130 dBM for laboratory models at two-tone +45 dBM signals should be carried out. The figure applies for reasonably simple mechanical structures.

A practical specification for spacecraft hardware at the component level appears to be -110 dBM for similar test conditions.

Refer: 1) Fleetsat Antenna Systems
Specification: -100 dBM at +45 dMB test carriers.
(Achieved -110 dBM)

2) Fleetsat Transmitting Filters
Final Specification: -100 dBM at +45 dBM test carriers.
(Procurement Spec: -115 dBM)

3) SRA-503(V) UHF Four channel multicoupler: -105 dBM at +50 dBM test carriers, for production quality tunable filters and combiner.

4.3 Transponder Third-Order Intermod Margin

The margin included for transponder third-order intermod level degradation due to spacecraft coupled sources is 20 dB.

Reference: TRW Trip Report, Para. 3.1(b).

The significant coupled intermod sources in Fleetsatcom are the solar panels and common-mast antenna equipment.

Space deterioration of intermod levels may occur with time. No data is available in support of the concept, and additional margin for such deterioration is not included.

4.4 Power Back-Off Rate for Intermods

The theoretical power back-off rates for third order intermods is 3 dB/dB. Measured third order values generally fall within a range of 1.5 to 4 dB/dB. A 3 dB/dB rate has been measured (SRL) for a high linearity 50 ohm high-power UHF Intermod Test Bed termination. The assumed conservative rate for seventh and higher order products is 1 dB/dB.

4.5 Two-Tone Product Roll-off Rate

The roll-off rate for intermodulation products of increasing product order cannot be predetermined accurately, and must be established from measurements on the individual passive hardware components and complete transponder system.

An initial estimate for specifying the transponder components is given on Fig. 4-1. The 10 dB rate estimate for low odd order products is confirmed by Fleetsat data and SRL test experience. A 5 dB rate for product orders above seventh is an estimated average rate for initial purposes.

A 6 dB allowance is included for vector summing of all contributions to the transponder system result. The 20 dB margin for transponder system degradation due to the spacecraft is allowed to decrease with increasing product order (i.e. number of contributory sources decreases).

Hughes measurements (Marisat Transponder data) at thirteenth to twenty-first order products (Fig. 4-2) indicate a 5 dB per odd-order average rate at high orders, and (2.5 dB per odd-order minimum rate). The measured data has been extrapolated to a -50 dBM level estimated for two-tone third order products. A -50 dBM third-order two-tone level typifies an assembly produced with normal manufacturing control standards but without strict adherence to low-intermodulation manufacturing techniques.

Test data in support of the estimate is as follows:

1) VHF Intermod Test Bed (150 MHz band) with Noisy Termination; the RF Circuit is given on Fig. 4-3. Product roll-off rates (Fig. 4-4) are 30 dB at third to fifth orders, decreasing to about 5 dB at ninth to eleventh orders.
   Back-off rates with power are 1.8 dB/dB at third-order, and less than 1 dB/dB at eleventh order.

2) UHF Biconical Antenna (Preliminary design model) and feeder; (Fig. 4-5). Product roll-off rates are typically 10 dB at third to fifth orders. The third order power back-off rate is approximately 1.7 dB/dB. The plotted data indicates a variation in the third order intermod levels across the UHF band for the particular antenna (approx. 14 dB for three sets of two-tone measurements spread over the band.)

4.6 Transponder Total Intermods

The total intermod power as a function of a large number of varying amplitude noise-loaded carriers can be predicted only after measurement of the transponder transfer characteristic nonlinearity coefficients.

The determination of the total intermod power falling in a given receive channel is dependent upon the particular transfer characteristic and a certain minimum number of noise loaded signals for which the channel noise becomes independent of additional input signals.

A simulation measurement of the total noise on a given receive channel can be performed by applying unmodulated carriers two-at-a-time, then proceeding to three-at-a-time, etc. and noting the increase in channel intermod at a worst case product (i.e. $f_1 \pm f_2 \pm f_3 \pm \ldots$). The total average applied power (typically) increases with each added carrier. At a certain density of products, the channel noise increase flattens out with additionally applied carriers.

5.0 UHF Low Noise Amplifier

The UHF Transponder low noise amplifier is susceptible to self generated intermodulation due to transmitter signals direct coupled through the amplifier isolating network.

Under two tone conditions, the relationship between input power levels and self generated third order IM levels is predictable by means of the intercept point method.

Measured two tone intermodulation performance of a Bi-polar transistor input stage 3 dB NF amplifier (AUH-1018) is shown in Fig. 5-1. The measured third and fifth order products followed the predicted behaviour.

The predicted intermods for a UHF amplifier having an intercept of +10 dBM is shown in Fig. 5-2.

Transmitter signals at the amplifier input are required to be attenuated to -68 dBM in order to reduce third order intermods at the amplifier output to -96 dBM level (-138 dB +42 dB).

6.0 Ferrite Isolators

Isolators employed for output circuit impedance matching and for input impedance stabilization of cascaded amplifier stages under power level variation produce a high level broad spectrum intermodulation compared to antenna system passive hardware generated intermods. Isolators employed in the high-power amplifier circuit previous to the output filter will produce negligible contributions relative to the amplifier noise level.

Measured data for a Western Microwave Model I-201-LIB isolator is included as being representative of VHF/UHF isolator performance.

Isolator Specifications

Frequency Range - 148-174 MHz

Bandwidth - ± 5 MHz

Isolation (typical) - 25 dB

Insertion Loss - 0.5 dB

VSWR - Under 1.3

Max. Forward Power - 100 W

Max. Reverse Power - 20 W

The measured intermod spectra for two-tone test are shown in Figs. 6-1 to 6-5 and are summarized in Fig. 6-6.

Third order intermodulation levels of four manufacturers' units tested fall within ±10 dB of the values given.

The roll-off rate per odd order varies typically from 5 dB at +41 dBM inputs to 7 dB at +33 dBM inputs. Power Back-off rates varied from approximately 1 dB/dB for low order intermods to 2.5 dB/dB for higher orders.

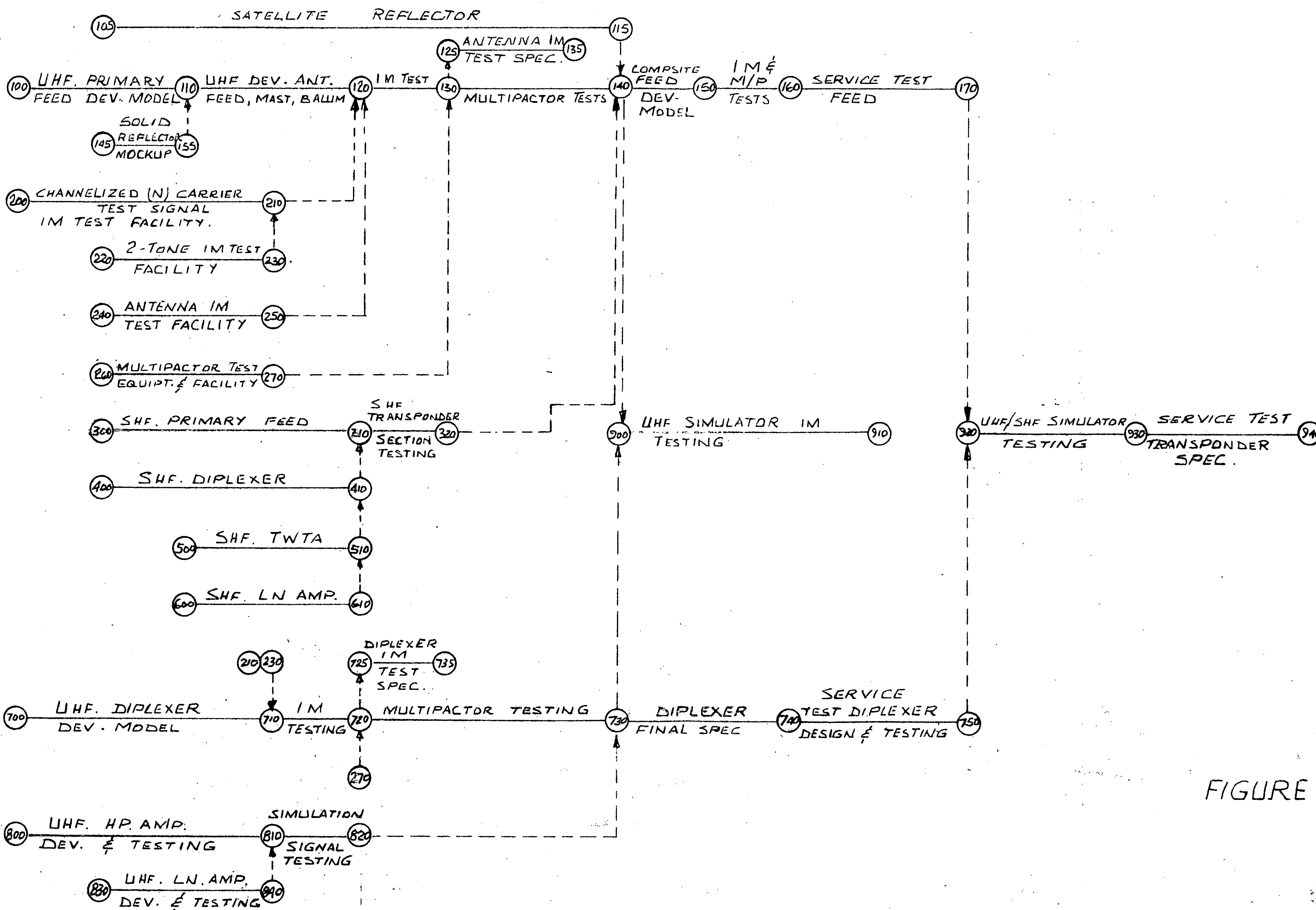


FIGURE 1-1

JUL 17 1975

SINCLAIR

G-11 SQUARE 10 X 10 TO THE HALF INCH

GRAPHIC CONTROLS CANADA LTD.
MADE IN CANADA

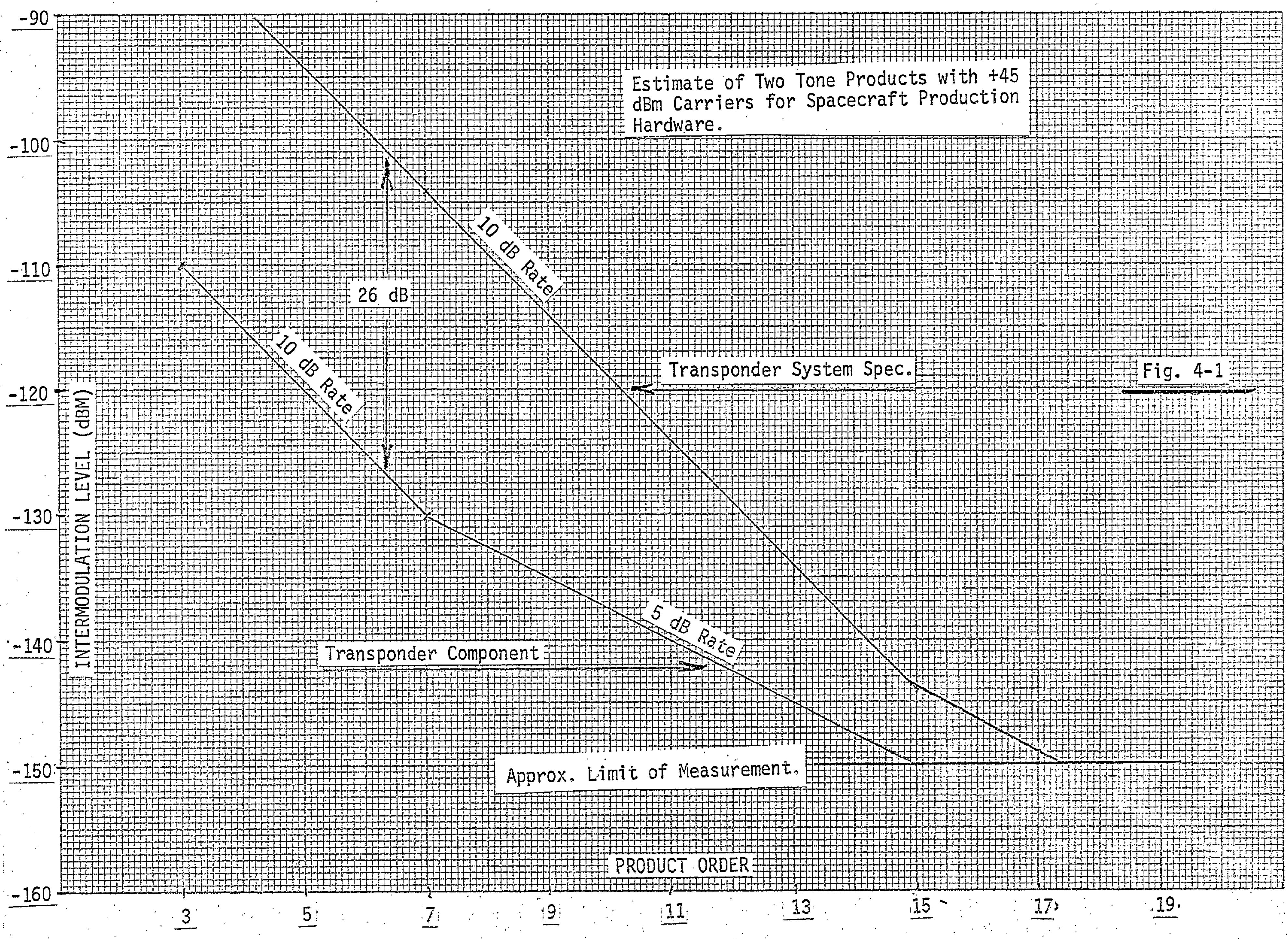


Fig. 4-1

G-11 SQUARE 10 X 10 TO THE HALF INCH
GRAPHIC CONTROLS CANADA LTD.
MADE IN CANADA

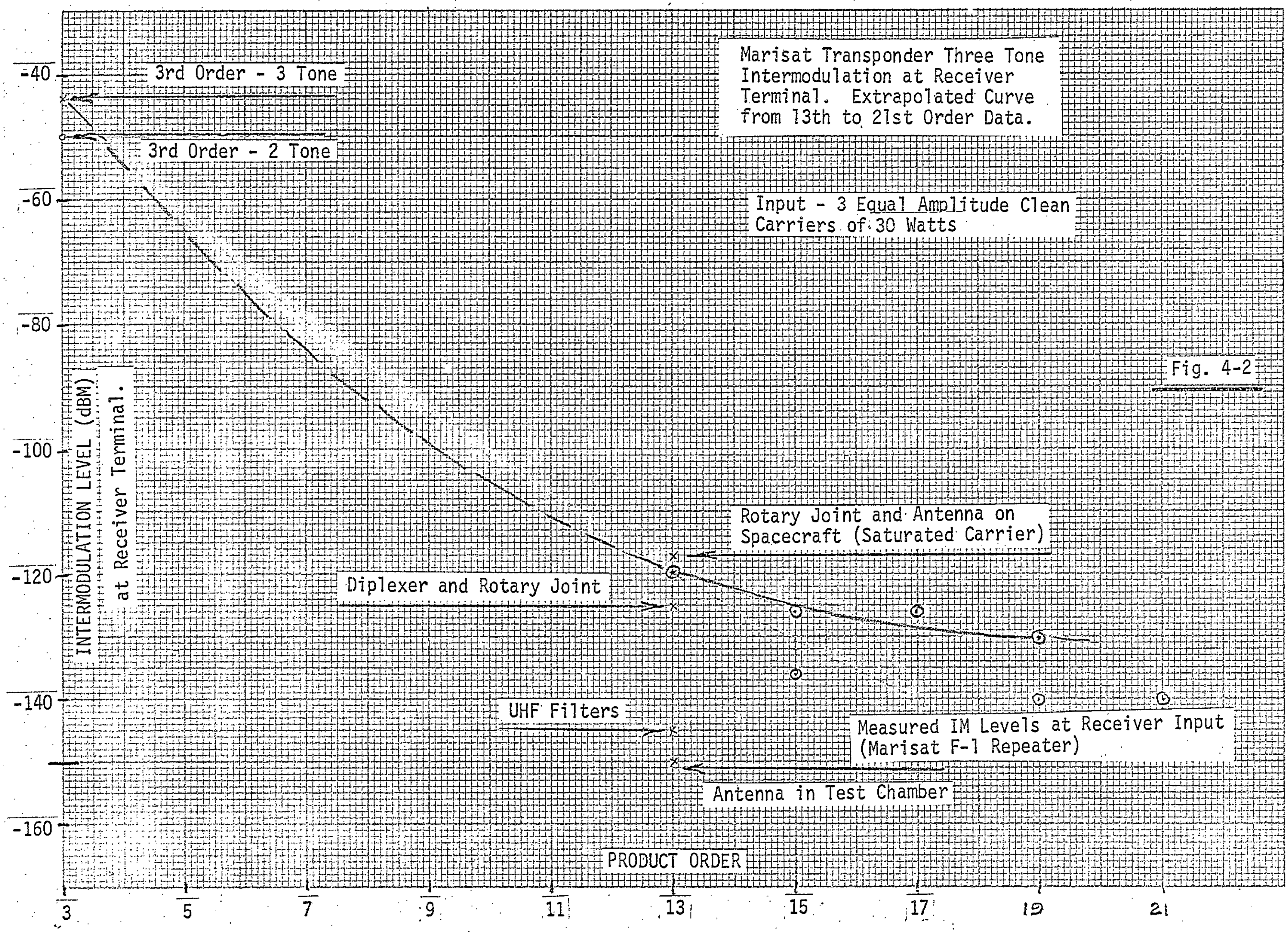


Fig. 4-2

COMPOSITE BANDPASS TRANSMITTING FILTERS & WIDE-BAND RECEIVING FILTER

(VHF INTERMODULATION TEST BED)

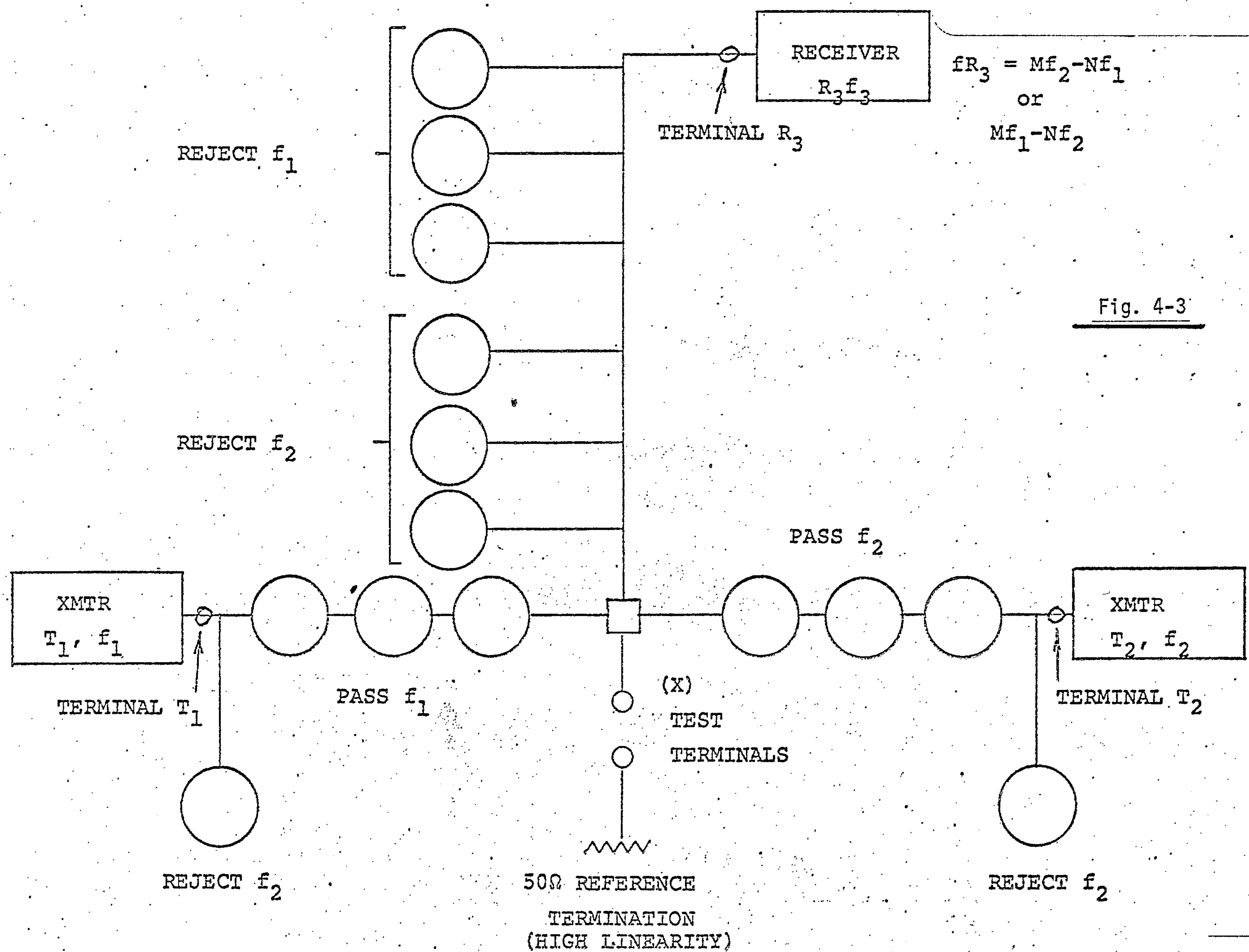
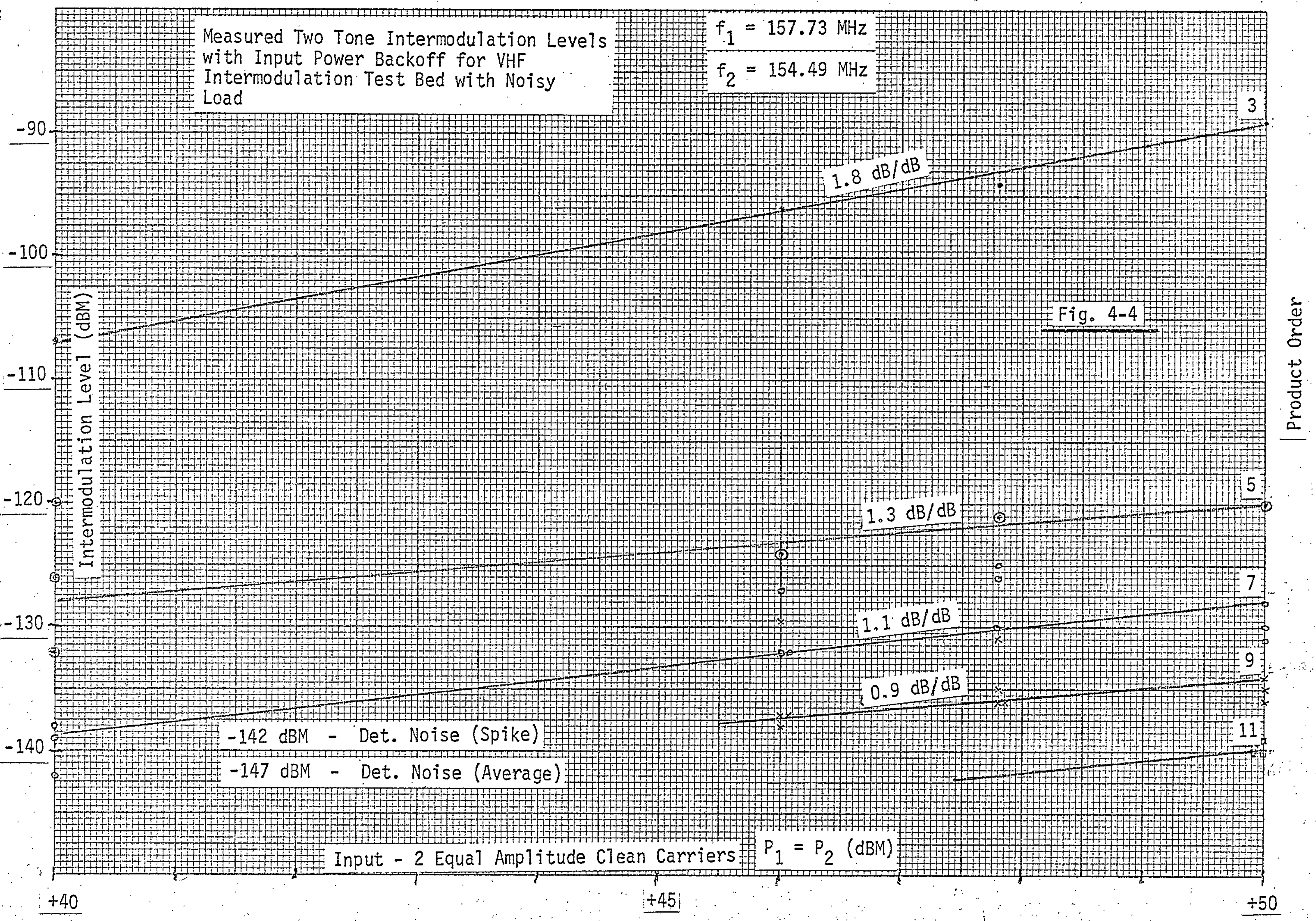
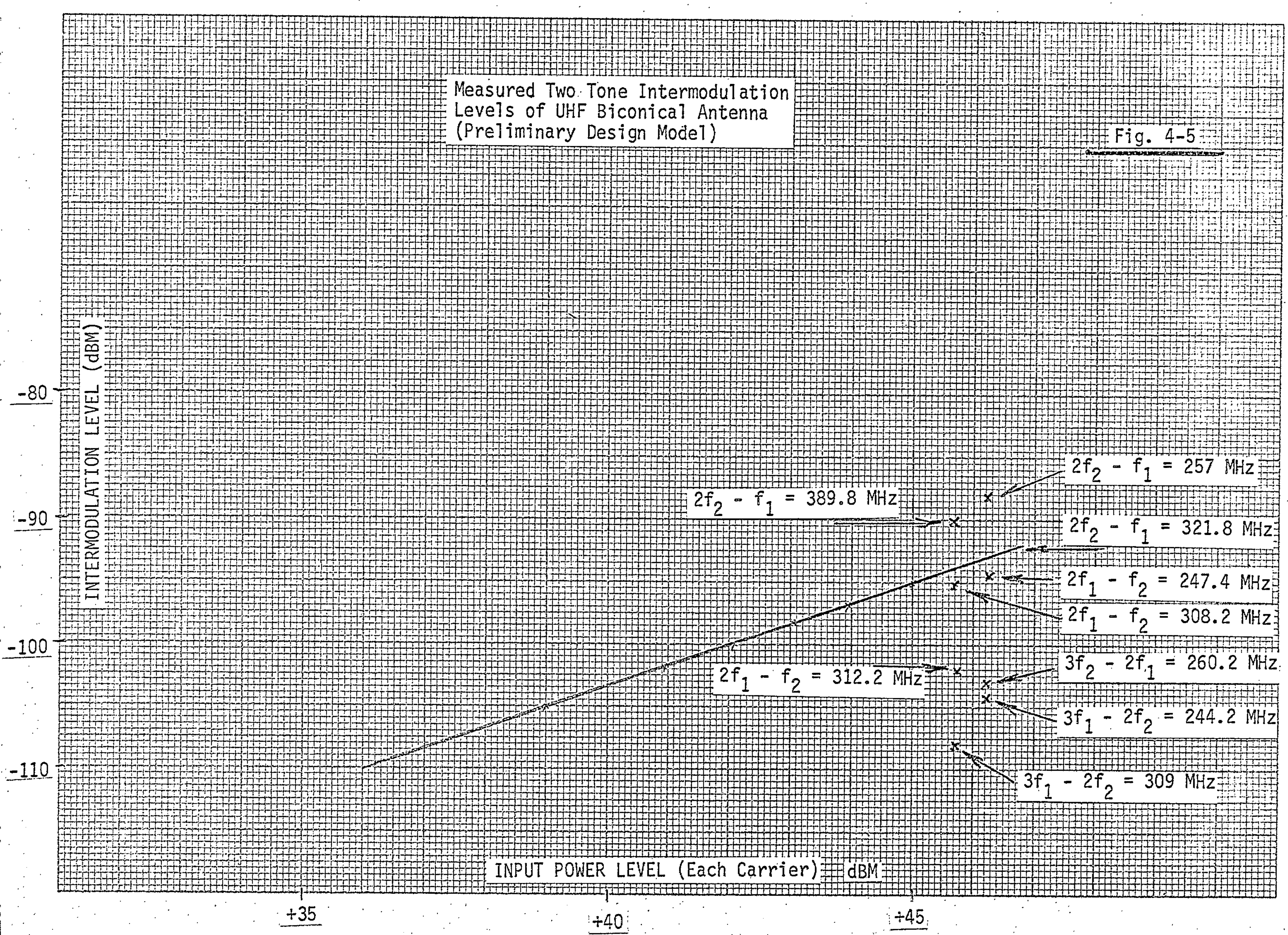


Fig. 4-3

G-11 SQUARE 10 X 10 TO THE HALF INCH
GRAPHIC CONTROLS CANADA LTD.
MADE IN CANADA




Measured Two Tone Intermodulation
Levels of UHF Biconical Antenna
(Preliminary Design Model)
Fig. 4-5
INTERMODULATION LEVEL (dBM)
-80
-90
-100
-110
INPUT POWER LEVEL (Each Carrier) dBM
+35
+40
+45
2f2 - f1 = 257 MHz
2f2 - f1 = 389.8 MHz
2f2 - f1 = 321.8 MHz
2f1 - f2 = 247.4 MHz
2f1 - f2 = 308.2 MHz
3f2 - 2f1 = 260.2 MHz
2f1 - f2 = 312.2 MHz
3f1 - 2f2 = 244.2 MHz
3f1 - 2f2 = 309 MHz

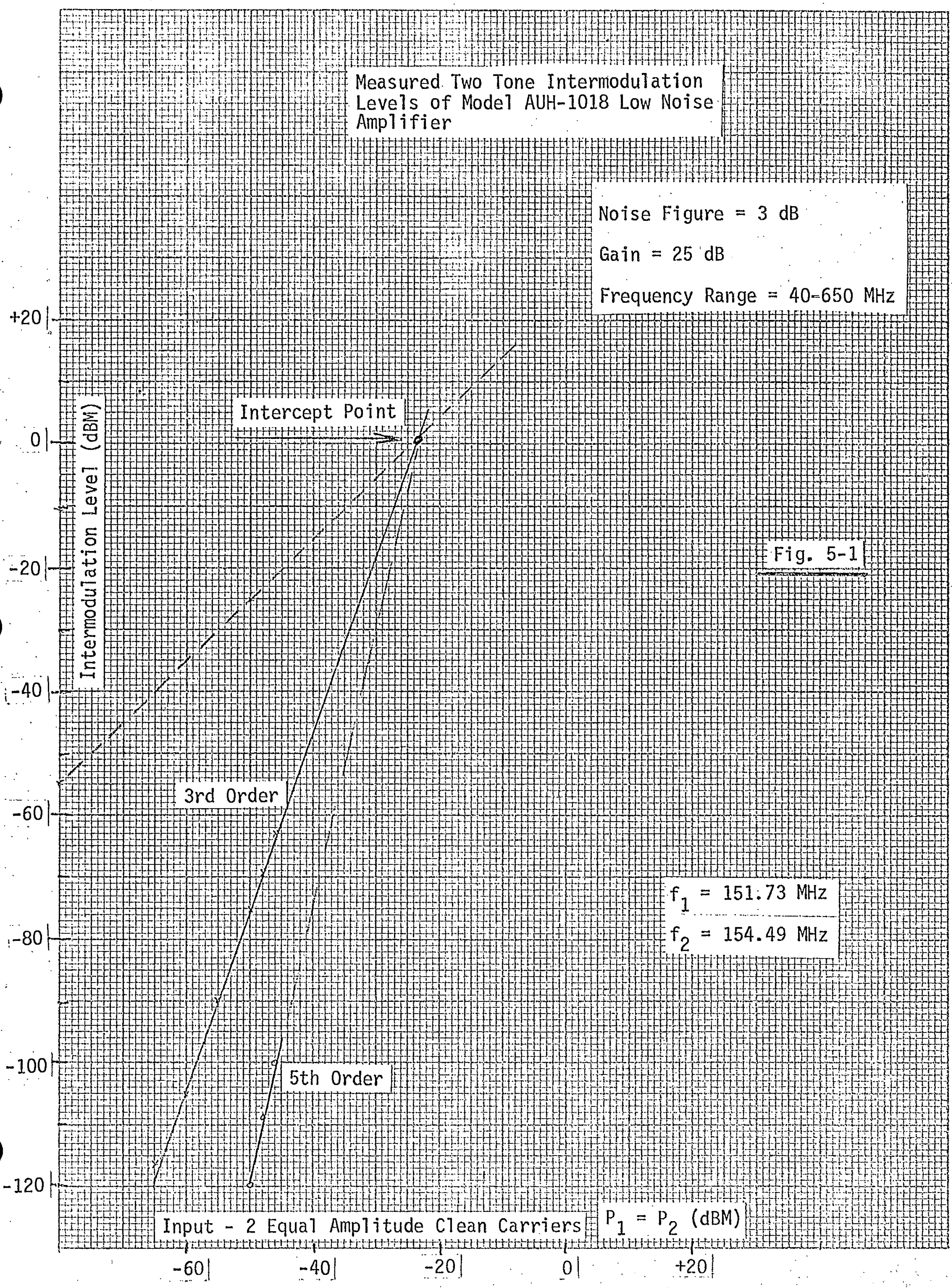


Fig. 5-1

GRAPHIC CONTROLS CANADA LTD. MADE IN CANADA
G-11 SQUARE 10 X 10 TO THE HALF INCH

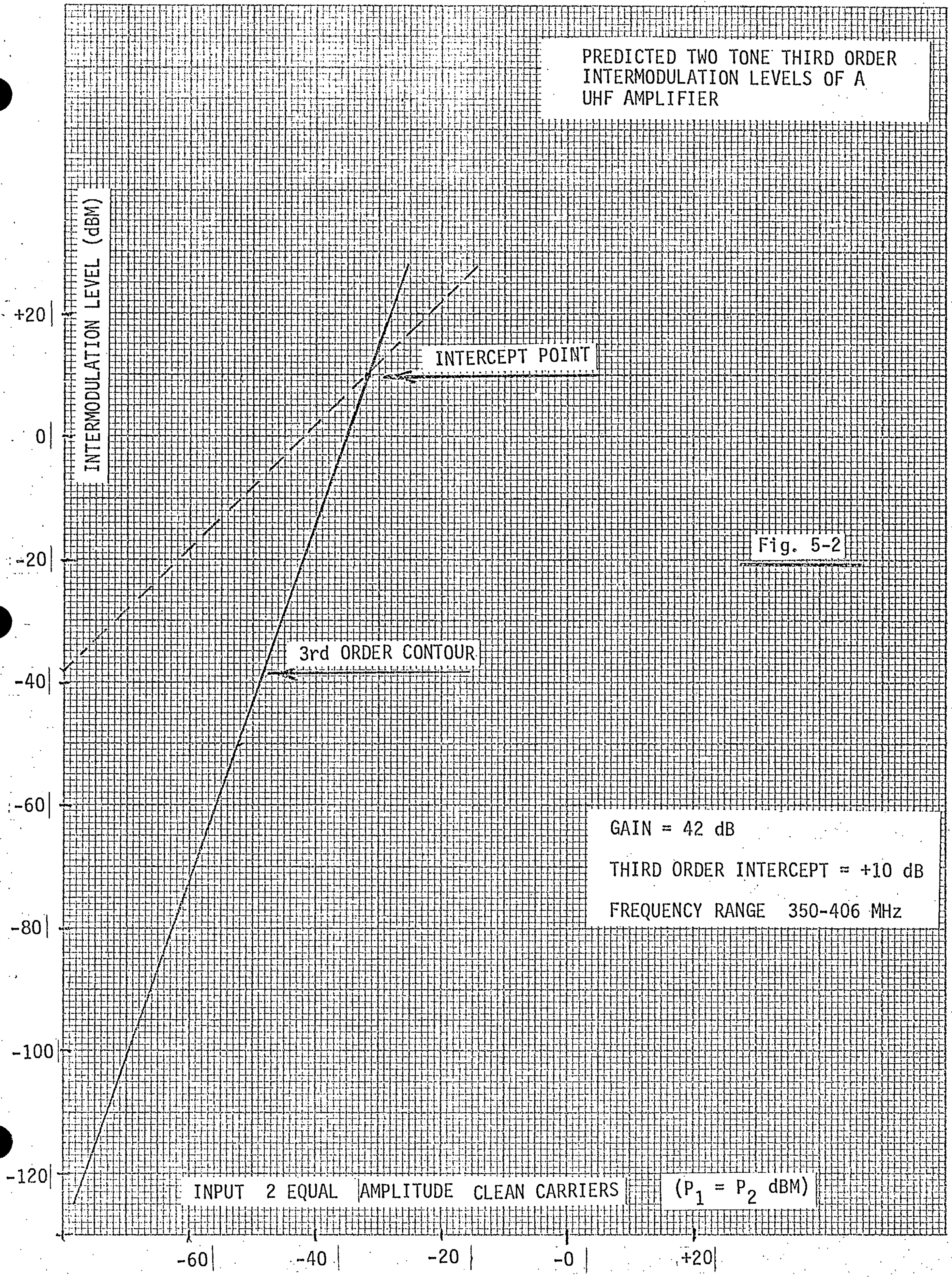
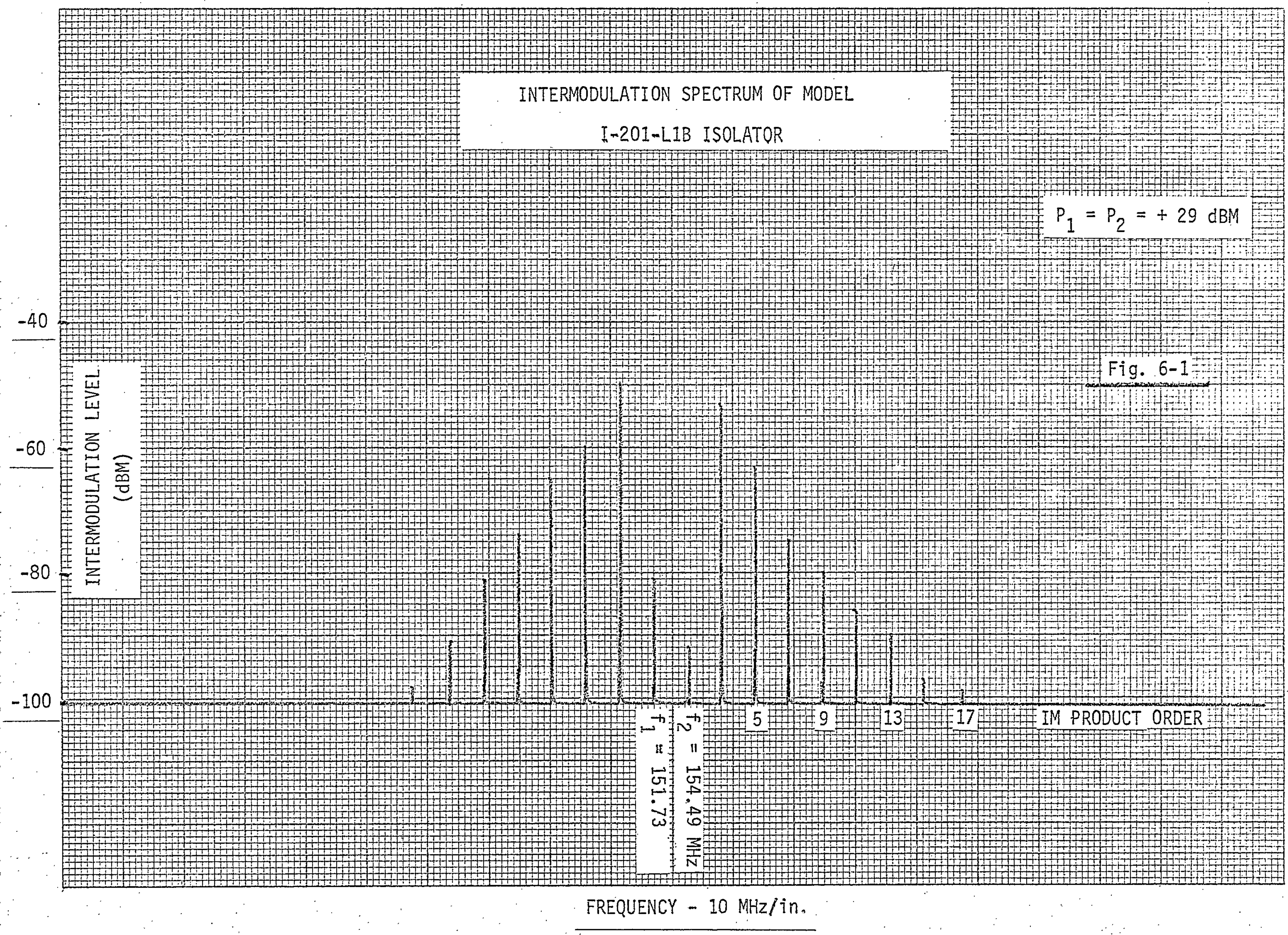


Fig. 5-2

GRAPHIC CONTROLS CANADA LTD. MADE IN CANADA
G-11 SQUARE 10 X 10 TO THE HALF INCH

G-11 SQUARE 10 X 10 TO THE HALF INCH
GRAPHIC CONTROLS CANADA LTD.
MADE IN CANADA



G-11 SQUARE 10 X 10 TO THE HALF INCH

GRAPHIC CONTROLS CANADA LTD.
MADE IN CANADA

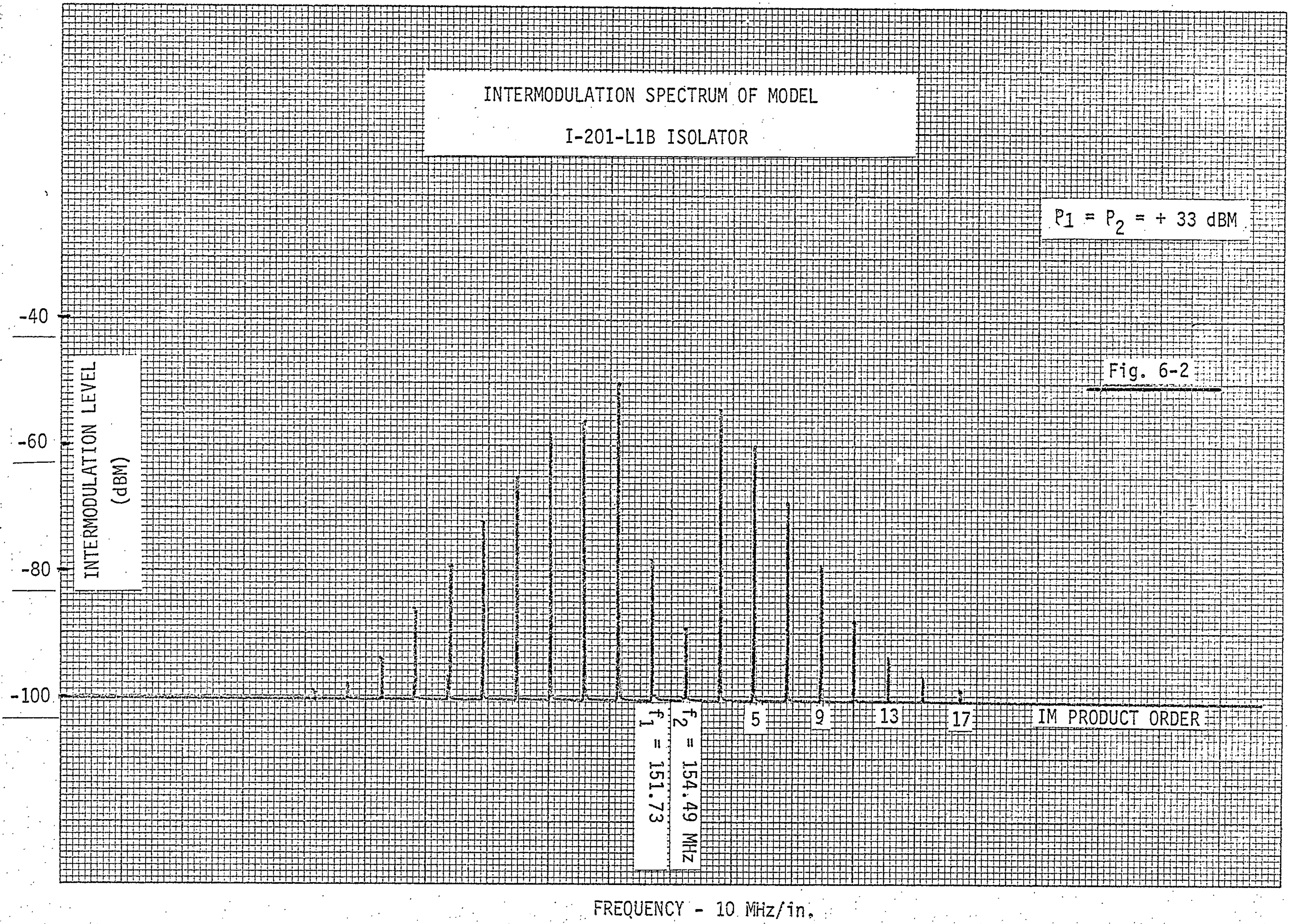


Fig. 6-2

G-11 SQUARE 10 X 10 TO THE HALF INCH
GRAPHIC CONTROLS CANADA LTD.
MADE IN CANADA

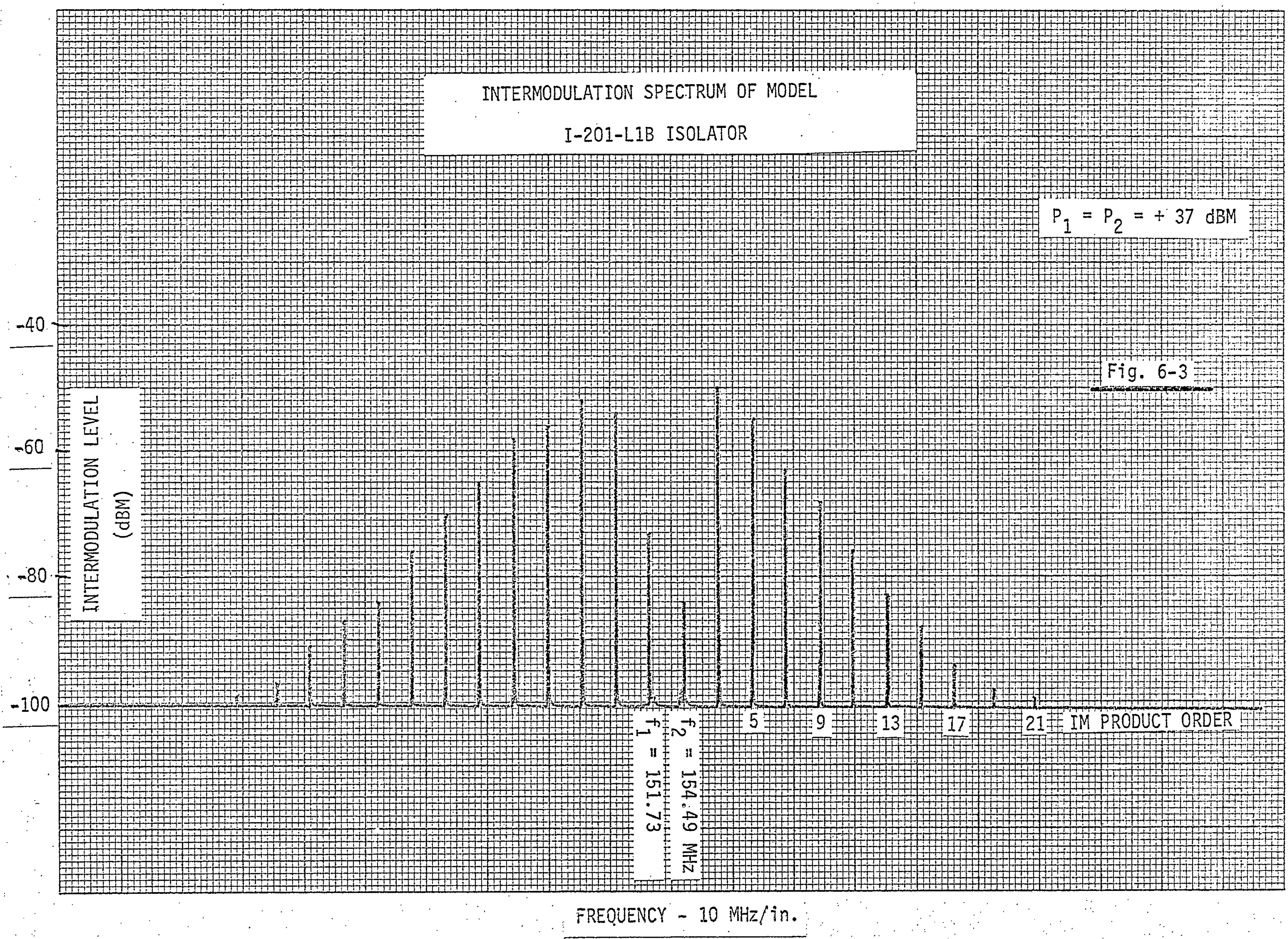
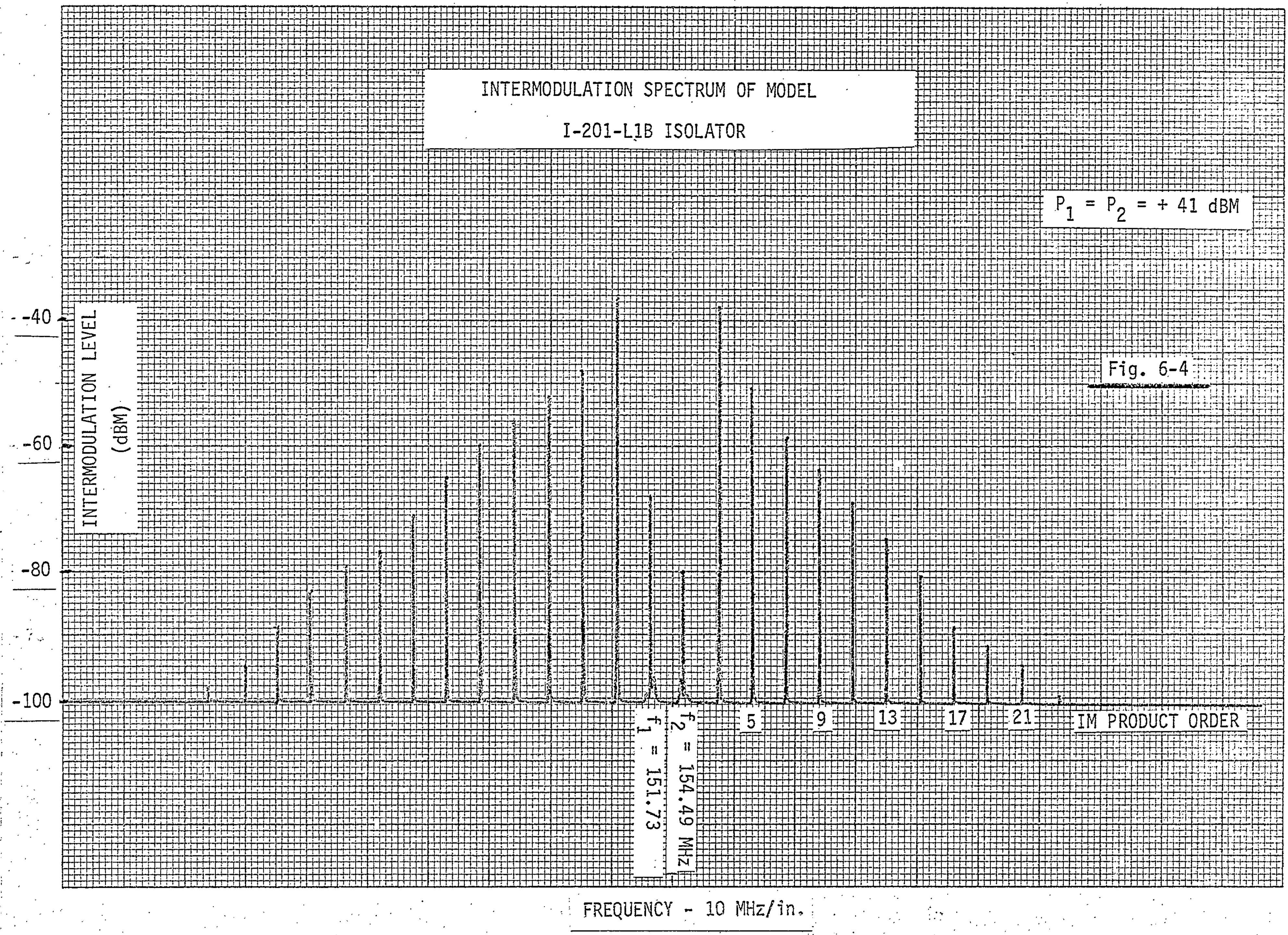


Fig. 6-3

G-11 SQUARE 10 X 10 TO THE HALF INCH

GRAPHIC CONTROLS CANADA LTD.
MADE IN CANADA



INTERMODULATION SPECTRUM OF MODEL
I-201-L1B ISOLATOR

Fig. 6-4

G-11 SQUARE 10 X 10 TO THE HALF INCH
GRAPHIC CONTROLS CANADA LTD.
MADE IN CANADA

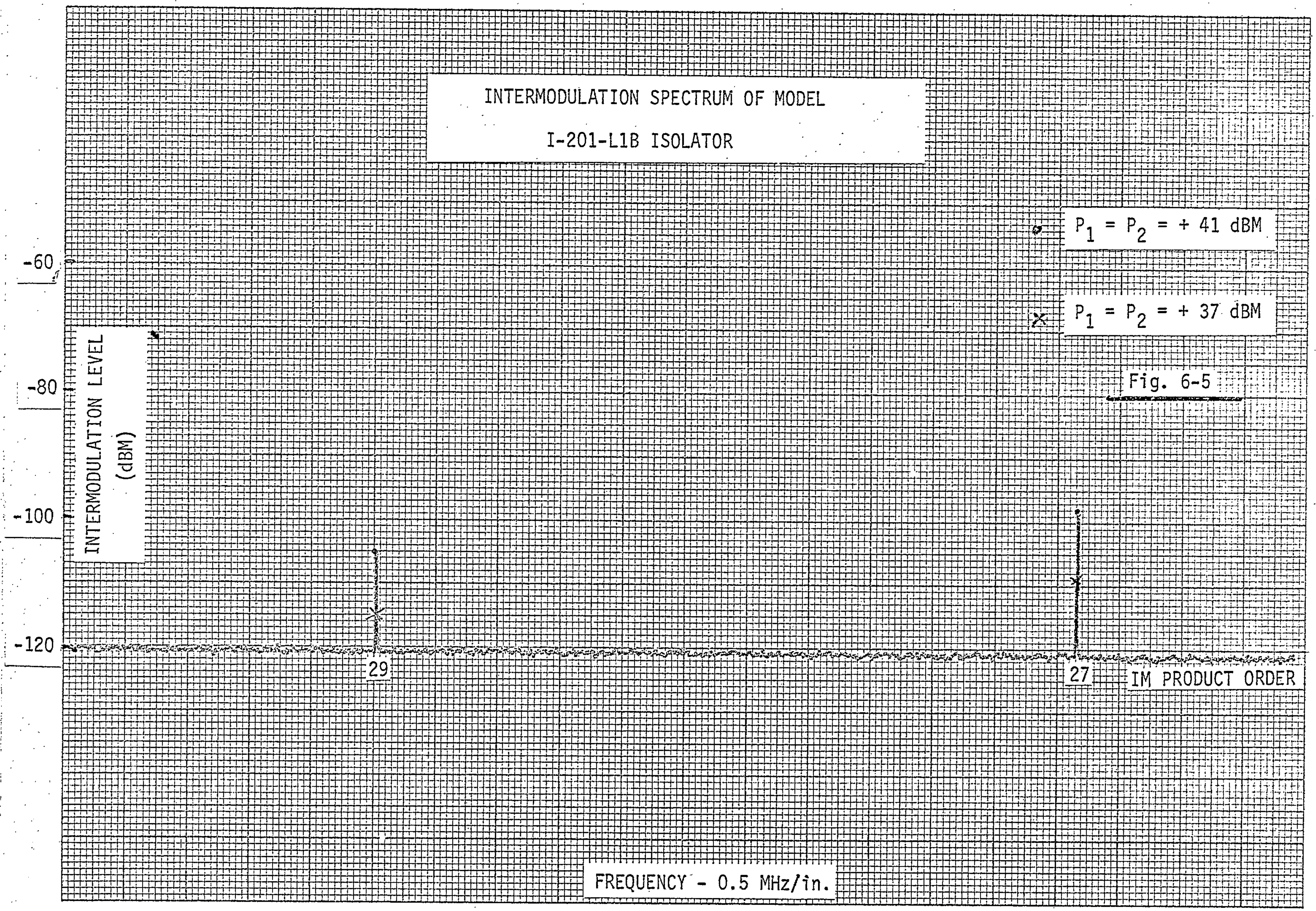


Fig. 6-5

G-11 SQUARE 10 X 10 TO THE HALF INCH
GRAPHIC CONTROLS CANADA LTD.
MADE IN CANADA

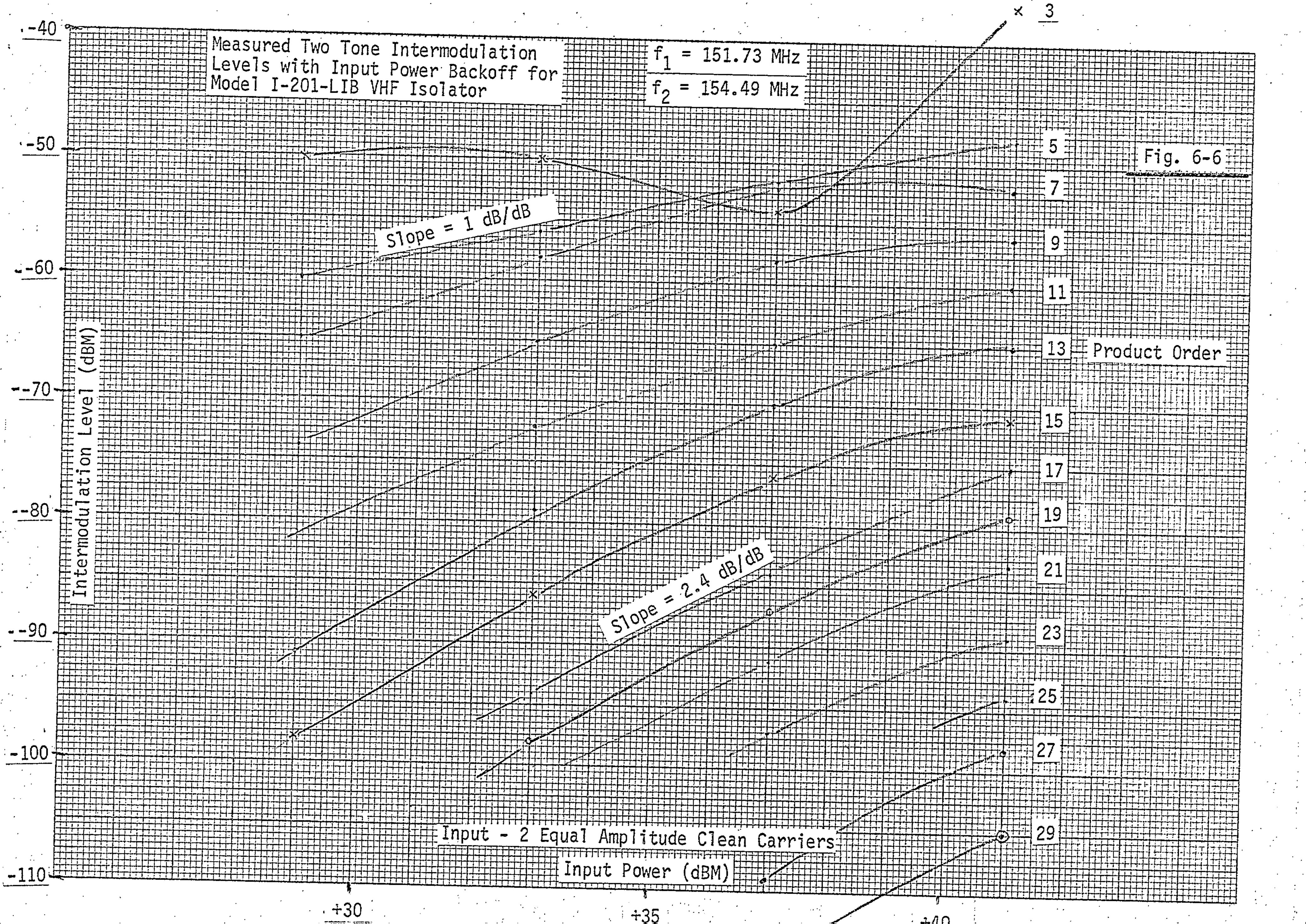


Fig. 6-6

APPENDIX I

Report No. 239-2

Progress Report
Defence Industrial Research Project
E168

Research into Non-Linear Characteristics of Antenna Systems

for

The Period Covering June 1, 1971

to Nov. 30, 1971

F.G. Buckles

Sinclair Radio Laboratories Limited
122 Rayette Road
Maple Ontario.

3.0 Power Series Representation of Transfer Characteristic

3.1 Introduction

The output spectrum produced by a nonlinear transfer characteristic whose steady-state output varies nonlinearly with a steady-state input, can be derived by representing the transfer characteristic as a power series of the input signal, for those cases where the characteristic is representable by a finite series.

Power series expansions based on Maclaurin's and Taylor's theorems have been used extensively in distortion analysis of nonlinear or variable systems, and the methods are applicable to the analysis of the behaviour of nonlinear elements in general.

The fundamental assumptions on which the analysis is based are the following:

1) A single valued input/output characteristic is assumed. In power series form, the $e_o(e_i)$ functional relation for a nonlinear element is,

$$e_o = a_o + a_1e_i + a_2e_i^2 + . . . + a_Ne^N + . . \qquad (1)$$

where e_o is the instantaneous output voltage and e_i is the instantaneous input voltage. The output is expressed as a power series function of the input which generally consists of a sum of cosine waves at different frequencies. The coefficients a_o, a_1, . . . a_N are constants for the polynomial which describes the transfer characteristic, and are independent of voltage over the range of the polynomic approximation. The coefficients are usually determined experimentally; the number of terms required depends on the rate of decrease of a_n as n increases,

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where e_o is the instantaneous output voltage and e_i is the instantaneous input voltage. The output is expressed as a power series function of the input which generally consists of a sum of cosine waves at different frequencies. The coefficients a_o, a_1, . . . a_N are constants for the polynomial which describes the transfer characteristic, and are independent of voltage over the range of the polynomic approximation. The coefficients are usually determined experimentally; the number of terms required depends on the rate of decrease of a_n as n increases, and on the accuracy required of the approximation.

2) The nonlinear characteristic of the element is assumed to be independent of frequency over the working frequency range.

3) For a simplification of the analysis which follows, a pure resistance load is assumed, and nonlinearity in the phase characteristic of the transfer function is neglected.

The transfer function is actually described by gain and phase shift ($\frac{\Theta_k}{\omega}$) characteristics, which produce nonlinear distortion due to variation either in input signal amplitude or frequency.

The utility of the power series has been limited by the relatively large number of terms that are often required to accurately represent a given function, and secondly by the unwieldy manipulation involved in the expansion and regrouping of common terms, particularly as the order of the expansion and as the number of terms contained within e_i increases.

Through the use of a general expression for the expansion of each term in the power series, and evaluation of the amplitude and corresponding frequency terms by digital computer, the power series approach becomes a reasonably direct method for evaluation of a non-linear element output spectrum. No particular restriction need be place on the number of fundamental terms contained in e_i.

The expansion of a power series function expressing a nonlinear transfer characteristic, for a single-frequency input voltage produces typically a d.c. term, the fundamental, and harmonically related frequency terms. All terms are related in amplitude by coefficient series which are a function of the input voltage amplitude, and of the constant co-efficients which specify a polynomial which suitably approximates the transfer characteristic. The method of harmonic analysis by successive difference based upon a Taylor Series expansion* is representative of the single-frequency input voltage case.

When the input voltage to the non-linear characteristic is comprised of two or more frequencies simultaneously, the output spectrum contains in general, a d.c. term, the fundamentals and harmonic frequencies of the fundamentals, and the intermodulation product frequencies formed by all possible additive and subtractive combinations of the fundamental frequencies and their harmonics. A grouping of the common frequency terms which appear in the power series expansion of the input voltage, results in a modified power series function which expresses the output voltage amplitude for each frequency component in the output spectrum as a function of the input voltage and the constant coefficients of the transfer characteristic approximating polynomial.

The output voltage given by a power series expansion of an input voltage comprising M cosine waves with frequency ω_k, amplitude E_k, and instantaneous phase θ_k is as follows;

Input voltage $e_i = E_1 \cos(\omega_1 t + \theta_1) + E_2 \cos(\omega_2 t + \theta_2)$

$$+ \ldots + E_M \cos(\omega_M t + \theta_M).$$

Substitution of e_i into (1) gives the

output voltage $e_o = a_o + a_1 \left\{E_1 \cos(\omega_1 t + \theta_1) + \ldots + E_M \cos(\omega_{Mt} + \theta_M)\right\}$

* Reference: Harmonic Analysis by the Method of Central Differences, D.C. Espley, Phil. Mag. 1939(28) P. 238

$$+ \ldots + a_N \left\{ E_1 \cos (\omega_1 t + \theta_1) + \ldots + E_M \cos (\omega_M t + \theta_M) \right\}^N$$

or in compact form,

$$e_o = \sum_{n=o}^{N} a_n \left\{ \sum_{m=1}^{M} E_m \cos \phi_m \right\}^n \qquad (2)$$

3.2 Two-Frequency Input Voltage Expansion

The output voltage given by the expansion of a two-frequency input voltage as a simple case, and considering only terms generated for up to the fifth-power of the input voltage, demonstrates the modified power series amplitude functions for the output frequency components.

Where $e_i = E_1 \cos \omega_1 t + E_2 \cos \omega_2 t$ and the instantaneous phase of the input is neglected,

$$\begin{aligned} e_o \quad &= a_o + \frac{1}{2} a_2 E_1^2 + \frac{1}{2} a_2 E_2^2 + \frac{3}{8} a_4 E_1^4 \\ &+ \frac{3}{8} a_4 E_2^4 + \frac{3}{2} a_4 E_1^2 E_2^2 + \cdots \\ &+ \cos \omega_1 t \left\{ a_1 E_1 + \frac{3}{4} a_3 E_1^3 + \frac{3}{2} a_3 E_1 E_2^2 \right. \\ &\left. + \frac{5}{8} a_5 E_1^5 + \frac{15}{4} a_5 E_1^3 E_2^2 + \frac{15}{8} a_5 E_1 E_2^4 + \cdots \right\} \\ &+ \cos \omega_2 t \left\{ a_1 E_2 + \frac{3}{2} a_3 E_1^2 E_2 + \frac{3}{4} a_3 E_2^3 \right. \\ &\left. + \frac{15}{8} a_5 E_1^4 E_2 + \frac{15}{4} a_5 E_1^2 E_2^3 + \frac{5}{8} a_5 E_2^5 + \cdots \right\} \end{aligned}$$

$$+ \cos 2\omega_1 t \left\{ \frac{1}{2} a_2 E_1^2 + \frac{1}{2} a_4 E_1^4 + \frac{3}{2} a_4 E_1^2 E_2^2 + \ldots \right\}$$

$$+ \cos 2\omega_2 t \left\{ \frac{1}{2} a_2 E_2^2 + \frac{3}{2} a_4 E_1^2 E_2^2 + \frac{1}{2} a_4 E_2^4 + \ldots \right\}$$

$$+ \cos 3\omega_1 t \left\{ \frac{1}{4} a_3 E_1^3 + \frac{5}{16} a_5 E_1^5 + \frac{5}{4} a_5 E_1^3 E_2^2 + \ldots \right\}$$

$$+ \cos 3\omega_2 t \left\{ \frac{1}{4} a_3 E_2^3 + \frac{5}{4} a_5 E_1^2 E_2^3 + \frac{5}{16} a_5 E_2^5 + \ldots \right\}$$

$$+ \cos 4\omega_1 t \left\{ \frac{1}{8} a_4 E_1^4 + \ldots \right.$$

$$+ \cos 4\omega_2 t \left\{ \frac{1}{8} a_4 E_2^4 + \ldots \right.$$

$$+ \cos 5\omega_1 t \left\{ \frac{1}{16} a_5 E_1^5 + \ldots \right.$$

$$+ \cos 5\omega_2 t \left\{ \frac{1}{16} a_5 E_2^5 + \ldots \right.$$

$$+ \cos (\omega_1 - \omega_2) t \left\{ a_2 E_1 E_2 + \frac{3}{2} a_4 E_1^3 E_2 + \frac{3}{2} a_4 E_1 E_2^3 + \ldots \right\}$$

$$+ \cos (\omega_1 + \omega_2) t \left\{ a_2 E_1 E_2 + \frac{3}{2} a_4 E_1^3 E_2 + \frac{3}{2} a_4 E_1 E_2^3 + \ldots \right\}$$

$$+ \cos (2\omega_1 - \omega_2) t \left\{ \frac{3}{4} a_3 E_1^2 E_2 + \frac{5}{4} a_5 E_1^4 E_2 + \frac{15}{8} a_5 E_1^2 E_2^3 + \ldots \right\}$$

$$+ \cos (\omega_1 - 2\omega_2) t \left\{ \frac{3}{4} a_3 E_1 E_2^2 + \frac{5}{4} a_5 E_1 E_2^4 + \frac{15}{8} a_5 E_1^3 E_2^2 + \ldots \right.$$

$$+ \cos (2\omega_1 + \omega_2) t \left\{ \frac{3}{4} a_3 E_1^2 E_2 + \frac{5}{4} a_5 E_1^4 E_2 + \frac{15}{8} a_5 E_1^2 E_2^3 + \ldots \right.$$

$$+ \cos (\omega_1 + 2\omega_2) t \left\{ \frac{3}{4} a_3 E_1 E_2^2 + \frac{5}{4} a_5 E_1 E_2^4 + \frac{15}{8} a_5 E_1^3 E_2^2 + \ldots \right.$$

$$+ \cos (2\omega_1 - 2\omega_2) t \left\{ \frac{3}{4} a_4 E_1^2 E_2^2 + \ldots \right.$$

$$+ \cos (2\omega_1 + 2\omega_2) t \left\{ \frac{3}{4} a_4 E_1^2 E_2^2 + \ldots \right.$$

$$+ \cos (3\omega_1 - \omega_2)t \left\{ \frac{1}{2} a_4 E_1^3 E_2 + \ldots \right.$$

$$+ \cos (\omega_1 - 3\omega_2)t \left\{ \frac{1}{2} a_4 E_1 E_2^3 + \ldots \right.$$

$$+ \cos (3\omega_1 + \omega_2)t \left\{ \frac{1}{2} a_4 E_1^3 E_2 + \ldots \right.$$

$$+ \cos (\omega_1 + 3\omega_2)t \left\{ \frac{1}{2} a_4 E_1 E_2^3 + \ldots \right.$$

$$+ \cos (3\omega_1 - 2\omega_2)t \left\{ \frac{5}{8} a_5 E_1^3 E_2^2 + \ldots \right.$$

$$+ \cos (2\omega_1 - 3\omega_2)t \left\{ \frac{5}{8} a_5 E_1^2 E_2^3 + \ldots \right.$$

$$+ \cos (3\omega_1 + 2\omega_2)t \left\{ \frac{5}{8} a_5 E_1^3 E_2^2 + \ldots \right.$$

$$+ \cos (2\omega_1 + 3\omega_2)t \left\{ \frac{5}{8} a_5 E_1^2 E_2^3 + \ldots \right.$$

$$+ \cos (4\omega_1 - \omega_2)t \left\{ \frac{5}{16} a_5 E_1^4 E_2 + \ldots \right.$$

$$+ \cos (\omega_1 - 4\omega_2)t \left\{ \frac{5}{16} a_5 E_1 E_2^4 + \ldots \right.$$

$$+ \cos (4\omega_1 + \omega_2)t \left\{ \frac{5}{16} a_5 E_1^4 E_2 + \ldots \right.$$

$$+ \cos (\omega_1 + 4\omega_2)t \left\{ \frac{5}{16} a_5 E_1 E_2^4 + \ldots \right.$$

$$+ \ldots \tag{3}$$

From the expansion given by (3), it is noted that each term of the power series contributes only to intermodulation products of order equal to or less than the index of the expansion. In general, the n^{th} term contributes only to intermodulation product orders of n, n-2, n-4, . . to 1 or 0 for n odd or even, respectively. Figure 3-1 displays the two frequency expansion case to order 7.

3.3 General Term

A general term* for the expansion of the n^{th} term in the power series of (1) is given from a multinomial expansion of

$$\left\{ \sum_{m=1}^{M} E_m \cos \phi_m \right\}^M$$

For an input voltage,

$$e_i = E_1 \cos \omega_1 t + E_2 \cos \omega_2 t + \ldots .$$

The general term is,

$$e_n = a_n \left(E_1 \cos \omega_1 t + E_2 \cos \omega_2 t + \ldots\right)^n$$

$$= a_n \frac{n!}{2^{n-1}} \frac{E_1^{\alpha} E_2^{\beta} \ldots\ldots}{(\alpha-\alpha_1)!\,(\beta-\beta_1)!\ldots\,\alpha_1!\,\beta_1!\cdots}$$

$$\cos\left\{(\alpha-2\alpha_1)\omega_1 t \pm (\beta-2\beta_1)\omega_2 t \pm \ldots\right\} \qquad (4)$$

Each of α, β, . . . are zero or positive integers such that $\alpha + \beta + \ldots = n$.

and,

α_1, β_1, . . . are zero or positive integers such that $\alpha_1 \leq \frac{\alpha}{2}$,

* <u>Reference</u> A Table of Intermodulation Products, C.A.A. Wass, Journal of Inst. of Elec. Engrs.(London)Part III P.31.Jan. 1948.

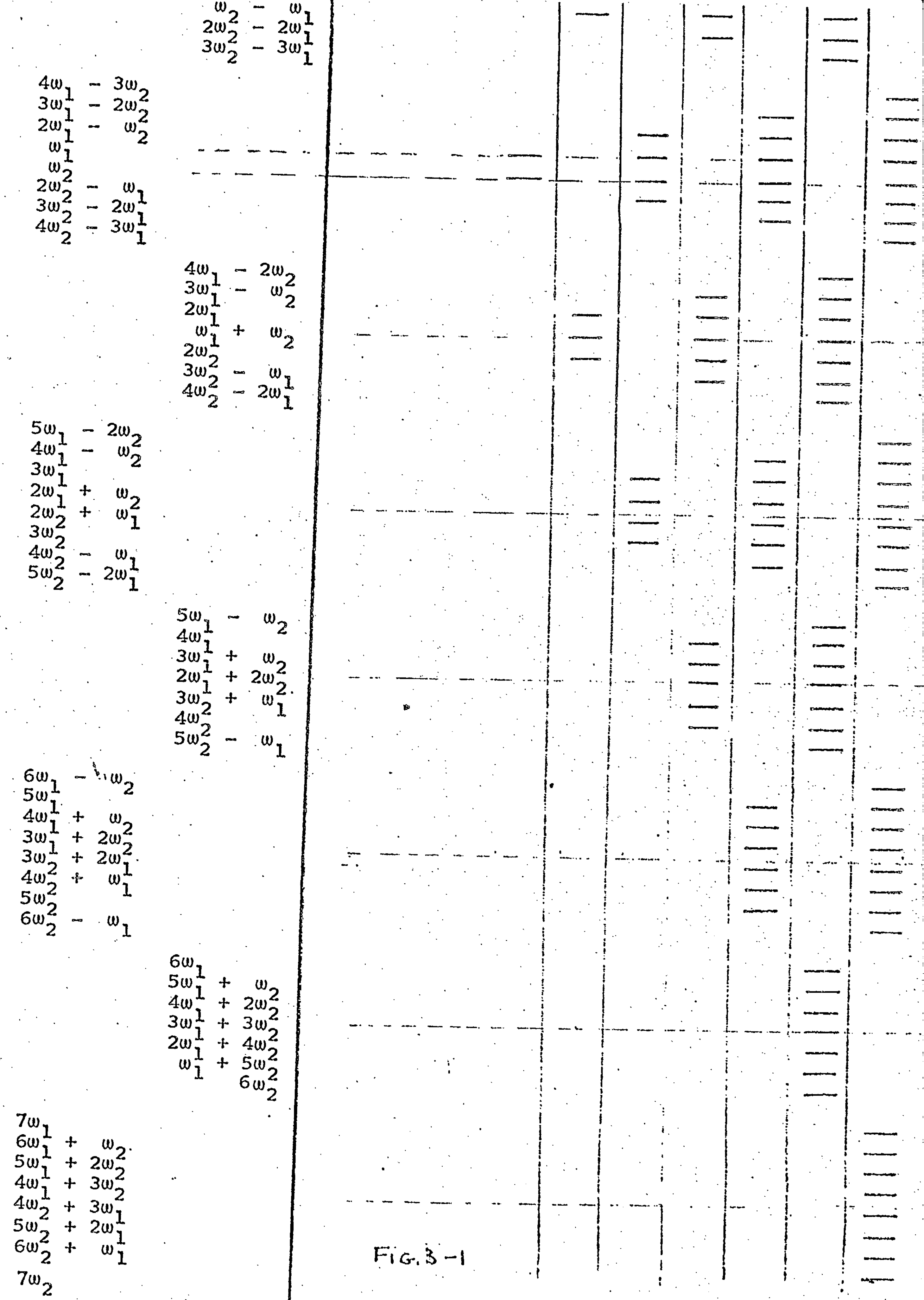
$\beta_1 \leq \frac{\beta}{2}$, . . . Equation (4) generates all terms which contribute to the output voltage from the n^{th} term in the power series by taking all values of α, β, . . ., α_1, β_1, . . . which satisfy the requirements. The exception noted to the rule is for cases where the cosine argument is zero, when the correct value is one-half the value given by Eqn. (5).

For example, when n = 2, the expansion of $(E_1 \cos \omega_1 t + E_2 \cos \omega_2 t)^2$ produces the following sets of α, β, α_1, β_1, and corresponding frequency terms for $\alpha + \beta = 2$, $\alpha_1 \leq \frac{\alpha}{2}$, and $\beta_1 \leq \frac{\beta}{2}$.

α	β	α_1	β_1	Frequency Term
0	2	0	0	$2\omega_2$
0	2	0	1	0
1	1	0	0	$\omega_1 \pm \omega_2$
2	0	0	0	$2\omega_1$
2	0	1	0	0

The complete expansion for this particular term in the power series is as an example,

$$a_2 \left\{ \frac{1}{2} E_1^2 + \frac{1}{2} E_2^2 + \frac{1}{2} E_1^2 \cos 2\omega_1 t + \frac{1}{2} E_2^2 \cos 2\omega_2 t + E_1 E_2 \cos (\omega_1 \pm \omega_2) t \right\}.$$



$\omega_2 - \omega_1$
$2\omega_2 - 2\omega_1$
$3\omega_2 - 3\omega_1$
$4\omega_1 - 3\omega_2$
$3\omega_1 - 2\omega_2$
$2\omega_1 - \omega_2$
ω_1
ω_2
$2\omega_2 - \omega_1$
$3\omega_2 - 2\omega_1$
$4\omega_2 - 3\omega_1$
$4\omega_1 - 2\omega_2$
$3\omega_1 - \omega_2$
$2\omega_1$
$\omega_1 + \omega_2$
$2\omega_2$
$3\omega_2 - \omega_1$
$4\omega_2 - 2\omega_1$
$5\omega_1 - 2\omega_2$
$4\omega_1 - \omega_2$
$3\omega_1$
$2\omega_1 + \omega_2$
$2\omega_2 + \omega_1$
$3\omega_2$
$4\omega_2 - \omega_1$
$5\omega_2 - 2\omega_1$
$5\omega_1 - \omega_2$
$4\omega_1$
$3\omega_1 + \omega_2$
$2\omega_1 + 2\omega_2$
$3\omega_2 + \omega_1$
$4\omega_2$
$5\omega_2 - \omega_1$
$6\omega_1 - \omega_2$
$5\omega_1$
$4\omega_1 + \omega_2$
$3\omega_1 + 2\omega_2$
$3\omega_2 + 2\omega_1$
$4\omega_2 + \omega_1$
$5\omega_2$
$6\omega_2 - \omega_1$
$6\omega_1$
$5\omega_1 + \omega_2$
$4\omega_1 + 2\omega_2$
$3\omega_1 + 3\omega_2$
$2\omega_1 + 4\omega_2$
$\omega_1 + 5\omega_2$
$6\omega_2$
$7\omega_1$
$6\omega_1 + \omega_2$
$5\omega_1 + 2\omega_2$
$4\omega_1 + 3\omega_2$
$4\omega_2 + 3\omega_1$
$5\omega_2 + 2\omega_1$
$6\omega_2 + \omega_1$
$7\omega_2$

FIG. 3-1

3.7 Amplitude of Carrier and Third and Fifth-Order Intermodulation Products Power Series for m-Carrier Frequency Input Signal

The amplitude power series generated by the general term(4), for the case in which m carrier frequencies, all of equal amplitude A, are applied to the non-linear characteristic is given in published literature * for n up to and including 15.

Reference

Calculation of Nonlinear Distortion Products,
R.G. Medhurst, R.A. Harris, Proc. IEE,
Vol. 115, No. 7, July 1968

The transfer characteristic was expressed by an odd-order power series function of the type,

$$e_o = a_1 e_i + a_3 e_i^3 + a_5 e_i^5 + \ldots$$

In the following tabulated series, $k_n = a_n A^n$.

The result is similar to that given herein for the expansion of Equation (1) since only odd values of index contribute to the frequency terms tabulated.

Carrier output

$$
\begin{aligned}
k_1 &+ \{0{\cdot}75 + 1{\cdot}5(m-1)\}k_3 + \{0{\cdot}62 + 5{\cdot}62(m-1) \\
&\quad + 3{\cdot}75(m-1)(m-2)\}k_5 \\
&+ \{0{\cdot}55 + 18{\cdot}59(m-1) + 39{\cdot}38(m-1)(m-2) \\
&\quad + 13{\cdot}12(m-1)(m-2)(m-3)\}k_7 \\
&+ \{0{\cdot}49 + 61{\cdot}52(m-1) + 319{\cdot}92(m-1)(m-2) \\
&\quad + 295{\cdot}31(m-1)(m-2)(m-3) \\
&\quad + 59{\cdot}06(m-1)(m-2)(m-3)(m-4)\}k_9 \\
&+ \{0{\cdot}45 + 207{\cdot}99(m-1) + 2436{\cdot}33(m-1)(m-2) \\
&\quad + 4737{\cdot}30(m-1)(m-2)(m-3) \\
&\quad + 2436{\cdot}33(m-1)(m-2)(m-3)(m-4) \\
&\quad + 324{\cdot}84(m-1)(m-2)(m-3)(m-4)(m-5)\}k_{11} \\
&+ \{0{\cdot}42 + 718{\cdot}49(m-1) + 18331{\cdot}79(m-1)(m-2) \\
&\quad + 67743{\cdot}46(m-1)(m-2)(m-3) \\
&\quad + 67743{\cdot}46(m-1)(m-2)(m-3)(m-4) \\
&\quad + 22170{\cdot}59(m-1)(m-2)(m-3)(m-4)(m-5) \\
&\quad + 2111{\cdot}48(m-1)(m-2)(m-3)(m-4)(m-5)(m-6)\}k_{13} \\
&+ \{0{\cdot}39 + 2527{\cdot}03(m-1) + 138720{\cdot}12(m-1)(m-2) \\
&\quad + 927315{\cdot}55(m-1)(m-2)(m-3) \\
&\quad + 1408755{\cdot}98(m-1)(m-2)(m-3)(m-4) \\
&\quad + 979200{\cdot}88(m-1)(m-2)(m-3)(m-4)(m-5) \\
&\quad + 221703{\cdot}88(m-1)(m-2)(m-3)(m-4)(m-5)(m-6) \\
&\quad + 15836{\cdot}13(m-1)(m-2)(m-3)(m-4) \\
&\qquad (m-5)(m-6)(m-7)\}k_{15}
\end{aligned}
$$

$f_1 \pm f_2 \pm f_3$ products

$$
\begin{aligned}
1{\cdot}5k_3 &+ \{11{\cdot}25 + 7{\cdot}5(m-3)\}k_5 + \{78{\cdot}75 \\
&\quad + 137{\cdot}81(m-3) + 39{\cdot}38(m-3)(m-4)\}k_7 \\
&+ \{561{\cdot}09 + 1988{\cdot}44(m-3) + 1417{\cdot}50(m-3)(m-4) \\
&\quad + 236{\cdot}25(m-3)(m-4)(m-5)\}k_9 \\
&+ \{4101{\cdot}15 + 26867{\cdot}28(m-3) + 36409{\cdot}57(m-3)(m-4) \\
&\quad + 14617{\cdot}97(m-3)(m-4)(m-5) \\
&\quad + 1624{\cdot}22(m-3)(m-4)(m-5)(m-6)\}k_{11} \\
&+ \{30669{\cdot}31 + 356682{\cdot}50(m-3) \\
&\quad + 828757{\cdot}62(m-3)(m-4) \\
&\quad + 607051{\cdot}76(m-3)(m-4)(m-5) \\
&\quad + 158361{\cdot}33(m-3)(m-4)(m-5)(m-6) \\
&\quad + 12668{\cdot}91(m-3)(m-4)(m-5)(m-6)(m-7)\}k_{13} \\
&+ \{233780{\cdot}91 + 4734035{\cdot}95(m-3) \\
&\quad + 17918298{\cdot}35(m-3)(m-4) \\
&\quad + 21477755{\cdot}13(m-3)(m-4)(m-5) \\
&\quad + 9930574{\cdot}95(m-3)(m-4)(m-5)(m-6) \\
&\quad + 1829073{\cdot}34(m-3)(m-4)(m-5)(m-6)m-7) \\
&\quad + 110852{\cdot}93(m-3)(m-4)(m-5)(m-6) \\
&\qquad (m-7)(m-8)\}k_{15}
\end{aligned}
$$

$2f_1 \pm f_2$ *products*

$$
\begin{aligned}
&0{\cdot}75k_3 + \{3{\cdot}12 + 3{\cdot}75(m-2)\}k_5 \\
&\quad + \{11{\cdot}48 + 42{\cdot}66(m-2) + 19{\cdot}69(m-2)(m-3)\}k_7 \\
&+ \{41{\cdot}34 + 374{\cdot}06(m-2) + 472{\cdot}50(m-2)(m-3) \\
&\quad + 118{\cdot}12(m-2)(m-3)(m-4)\}k_9 \\
&+ \{148{\cdot}89 + 3043{\cdot}15(m-2) + 8053{\cdot}42(m-2)(m-3) \\
&\quad + 5143{\cdot}36(m-2)(m-3)(m-4) \\
&\quad + 812{\cdot}11(m-2)(m-3)(m-4)(m-5)\}k_{11} \\
&+ \{535{\cdot}18 + 24202{\cdot}89(m-2) + 121586{\cdot}31(m-2)(m-3) \\
&\quad + 150443{\cdot}26(m-2)(m-3)(m-4) \\
&\quad + 58065{\cdot}82(m-2)(m-3)(m-4)(m-5) \\
&\quad + 6334{\cdot}45(m-2)(m-3)(m-4)(m-5)(m-6)\}k_{13} \\
&+ \{1965{\cdot}77 + 191776{\cdot}67(m-2) \\
&\quad + 1745625{\cdot}72(m-2)(m-3) \\
&\quad + 3759761{\cdot}86(m-2)(m-3)(m-4) \\
&\quad + 2678945{\cdot}80(m-2)(m-3)(m-4)(m-5) \\
&\quad + 692830{\cdot}81(m-2)(m-3)(m-4)(m-5)(m-6) \\
&\quad + 55426{\cdot}46(m-2)(m-3)(m-4)(m-5) \\
&\qquad (m-6)(m-7)\}k_{15}
\end{aligned}
$$

$3f_1 \pm 2f_2$ *products*

$$
\begin{aligned}
&0{\cdot}62k_5 + \{3{\cdot}83 + 6{\cdot}56(m-2)\}k_7 + \{17{\cdot}72 \\
&\quad + 98{\cdot}44(m-2) + 59{\cdot}06(m-2)(m-3)\}k_9 \\
&+ \{74{\cdot}44 + 1051{\cdot}23(m-2) + 1759{\cdot}57(m-2)(m-3) \\
&\quad + 541{\cdot}41(m-2)(m-3)(m-4)\}k_{11} \\
&+ \{299{\cdot}54 + 9926{\cdot}91(m-2) + 35455{\cdot}34(m-2)(m-3) \\
&\quad + 28153{\cdot}12(m-2)(m-3)(m-4) \\
&\quad + 5278{\cdot}71(m-2)(m-3)(m-4)(m-5)\}k_{13} \\
&+ \{1179{\cdot}46 + 88742{\cdot}83(m-2) \\
&\quad + 612385{\cdot}45(m-2)(m-3) \\
&\quad + 951487{\cdot}65(m-2)(m-3)(m-4) \\
&\quad + 438792{\cdot}85(m-2)(m-3)(m-4)(m-5) \\
&\quad + 55426{\cdot}46(m-2)(m-3)(m-4)(m-5)(m-6)\}k_{15}
\end{aligned}
$$

APPENDIX II

NON-LINEAR DISTORTION IN TRANSMISSION SYSTEMS*

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(The paper was first received 10th December, 1943, and in revised form 21st December, 1944.)

SUMMARY

The problem of the distortion noise generated by a signal in its transmission through one or more non-linear devices is analysed. This subject has become of increasing importance in connection with the development of the transmission of different types of signals on a multi-channel basis through a common network, e.g. modulator, amplifier or transmitter. The provision of such networks is now of frequent occurrence, and in order that economic and satisfactory designs may be obtained it is essential to be able to determine precisely the effect of non-linearity and also the limiting linearity requirements, preferably in a form which can be readily comprehended and applied by designers. The problem has been considered to be very complex, and existing literature is both scanty and involved. This paper presents a simple solution, which is capable, in general, of immediate application to any type of complex signal (e.g. speech, music, television, voice-frequency telegraphy or thermal noise) covering any frequency band or frequency allocation. The results are presented to the designer in the form of the limiting levels of the various harmonics of a single tone, which levels must not be exceeded if the noise power from the non-linear device in any specified frequency band is not to exceed a predetermined value. The differences in the linearity requirements of a single amplifier and of a repeatered system are demonstrated. Provided the input/output characteristic of the network can be expressed as a single-valued power series independent of the frequency in the working band, the results obtained should cover with adequate accuracy the design of all types of transmission systems.

(1) INTRODUCTION

Most of the fundamental requirements governing the design of networks for the transmission of electrical signals‡ with minimum distortion are now fairly well established. The design information, however, for calculating intermodulation and harmonic noise caused by interaction between the various signals, or between the various frequency components of a single signal traversing a network having a non-linear input/output characteristic, is very unsatisfactory. Previous contributions to the specialized case of multi-channel telephone transmission through wide-band amplifiers have been made by two authors,[2, 3] but even such results are very complex and are unsuited to general design applications.

The necessity for a more perfect understanding of the behaviour, limitations and requirements of non-linear systems has arisen from the modern development of the simultaneous amplification of a number of signal channels by one common amplifier, e.g. wide-band coaxial-cable telephone systems and high-power wide-band modulator amplifiers. The whole design and economics of a transmission system are based on the permissible noise output per channel at the receiving terminal. It is the contribution due to intermodulation and harmonic noise that has so far proved very difficult to assess since, unlike thermal noise, it is dependent on the speech volume in each channel, the frequency band, the actual frequencies, and the output levels. The magnitude of this noise can be progressively decreased by the application of negative feedback, but, although in theory it is possible to improve the non-linearity of an amplifier to an enormous extent by the application of feedback, it is usually uneconomical in practice to design for a higher degree of linearity than will ensure that the noise contribution due to intermodulation will not unduly increase the noise power due to other sources. This statement becomes of increasing importance as the highest working frequency increases.

It is therefore necessary in most high-frequency amplifiers to be able to fix, at an early stage in the design, the degree of linearity required. When this requirement is determined it is readily possible, from consideration of the output valve and output coupling, to arrive at the amount of feedback necessary to obtain this degree of linearity, and hence a satisfactory freedom from distortion. The application of feedback improves other characteristics of the repeater, e.g. flatness of gain/frequency characteristic and stability against supply voltage variations, but in practice it is usually found that the amount of feedback necessary to meet linearity requirements is more than sufficient to meet specification limits for these other characteristics. Similar design and economic problems arise in the design of high-power modulators, transmitters and receivers used in multi-channel radio circuits.

Also, in the last few years it has become increasingly necessary to obtain concise information on the linearity requirements of amplifiers, in order that a system composed of a chain of such amplifiers shall realize the predetermined noise limits in service without excessive and uneconomic factors of safety.

(2) BASIC THEORY

(2.1) List of Symbols

n = number of simultaneous tones on the system.
N = total number of channels in the system.
f_1, f_2 = lowest and highest frequencies of working band, c/s.
t_r = rth harmonic output power (mW) produced by that sinusoidal input which gives a fundamental output tone of 1 mW.
T_r = total rth-order distortion power (mW).
P = total fundamental power output (mW) from the non-linear network.
R = number of repeater stations in the system.

The unit of power adopted is immaterial, provided t_r, T_r and P are measured in the same units.

(2.2) Basic Assumptions

The fundamental assumptions on which the analysis has been based are as follows:—

(*a*) The treatment assumes a single-valued input/output characteristic such as is obtained with valves and rectifiers where the transit time is negligible. The relation can then be expressed by the following power series:—

$$V = av + bv^2 + cv^3 + \ldots \quad (1)$$

where V is the output voltage, v the input voltage, and a, b, c . . . are constants.

(*b*) The non-linear characteristic of the amplifier is independent of frequency over the working band.

* Radio Section paper.
† Post Office Engineering Radio Station.
‡ It is assumed here and throughout this analysis that the intelligence is conveyed by amplitude variation and not by frequency- or phase-modulated signals.

The distorting device is referred to as an amplifier throughout the analysis, but the treatment is applicable to any network which conforms substantially to these assumptions.

(2.3) Distortion Power Produced by n Tones of Equal Powers

If the input signal to the amplifier consists of n tones, such that the fundamental output powers are all equal, the output signal will contain, in addition, a number of harmonics and intermodulation products. The numbers and relative powers of the most important of these are calculated in Appendices 9.1 and 9.2 and are shown in Table 1. It is evident that the proportion of the distortion contributed by the harmonics is relatively small, even when n is 2, and becomes quite negligible when n is greater than, say 10. It is also shown that, provided n is not small, the only important type of product in each order is of the form $A \pm B \pm C \pm \ldots$

(2.4) Distortion Power Produced by n Tones of Uncontrolled Powers

(2.4.1) Second-Order Distortion.

If the input signal is such that the output powers are not equal, but have values $p_1, p_2, \ldots p_q, \ldots p_r, \ldots p_n$, then the distortion power in each $(A+B)$ and each $(A-B)$ product will be of the form $4t_2 p_q p_r$, and the total second-order distortion power will be

$$T_2 = \tfrac{1}{2} \cdot 8 \cdot t_2 \sum_{q=1}^{n} \sum_{r=1}^{n} p_q p_r \qquad (2)$$

The fraction $\frac{1}{2}$ is necessary since otherwise $p_q \cdot p_r$ and $p_r \cdot p_q$ would both be counted.

Expanding (2),

$$T_2 = 4t_2(p_1 + p_2 + p_3 + \ldots p_n)^2$$
$$= 4t_2 P^2 \qquad (3)$$

where P is the total output power.

This result is slightly in error, since products of the type p_x^2 have been treated as $p_x \cdot p_y$ products. Strictly,

$$4t_2P^2 = 4t_2\{n \text{ products of form } p_x^2 + \tfrac{1}{2}n(n-1) \text{ products of form } 2p_xp_y\}$$
$$= t_2\{n \text{ products of form } 4p_x^2 + n(n-1) \text{ products of form } 4p_xp_y\}$$

whereas the total distortion (see Appendix 9.1) is equal to

$$t_2\{n \text{ products of form } p_x^2 + n(n-1) \text{ products of form } 4p_xp_y\}$$

Thus (3) assumes the contribution due to the harmonic power to be four times too large.

Table 1

CHARACTERISTICS OF DISTORTION PRODUCTS

Order	Type of product	Number of products	Power per product relative to harmonic of same order	Total power per type of product when n is equal to—				
				2	3	10	30	100
2nd	$2A$	n	1	2	3	10	30	100
	$A+B$, $A-B$	$n(n-1)$	4	8	24	360	3 480	40 000
3rd	$3A$	n	1		3	10	30	100
	$2A+B$, $2A-B$, $A+2B$, $A-2B$	$2n(n-1)$	9		108	1 620	15 660	178 000
	$A+B+C$, $A+B-C$, $A-B+C$, $A-B-C$	$\frac{2}{3}n(n-1)(n-2)$	36		144	17 280	585 000	23×10^6
4th	$2A \pm B \pm C$	$2n(n-1)(n-2)$	144			207 500	7×10^6	
	$A \pm B \pm C \pm D$	$\frac{1}{3}n(n-1)(n-2)(n-3)$	576			968 000	126×10^6	
5th	$2A \pm B \pm C \pm D$	$\frac{4}{3}n(n-1)(n-2)(n-3)$	3 600			24×10^6	3×10^9	
	$A \pm B \pm C \pm D \pm E$	$\frac{2}{15}n(n-1)(n-2)(n-3)(n-4)$	14 400			58×10^6	33×10^9	

If the largest fundamental output tone has a power λ times that of the smallest, and if the total power output is P, it can be shown that the maximum error due to using (3) occurs when one of the fundamental tones has a power $\lambda P/(n + \lambda - 1)$ and the remaining $(n-1)$ tones each have a power $P/(n + \lambda - 1)$. The magnitude of the error for this most unfavourable distribution is shown in Table 2 for various values of n and λ. The last column of the Table shows the error for power levels distributed approximately according to a "normal" speech-volume probability.

Table 2

ERROR DUE TO ASSUMING THAT THE TOTAL SECOND-ORDER DISTORTION IS GIVEN BY $4t_2P^2$

Value of n	Maximum error for value of λ equal to				Error for "normal" distribution
	1	10	20	100	
	db	db	db	db	db
3	+1·2	+2·0	+4·2	+5·6	+2·0
10	+0·3	+1·1	+2·0	+4·3	+0·6
30	+0·1	+0·3	+0·6	+2·6	+0·2
100	+0·0	+0·0	+0·0	+0·9	+0·0

In all cases the true distortion is less than that calculated from $4t_2P^2$.

The somewhat surprising result is therefore obtained that the total 2nd-order distortion power produced by a number of tones n of total power P, provided n is reasonably large, is independent of n and of the relative value of the power of each tone and depends only on the square of the total power P.

(2.4.2) Third-Order Distortion.

Following the same method of analysis the total third-order distortion is given by

$$T_3 = \tfrac{1}{6} . 4 . 36 . t_3 \sum_{q=1}^{n} \sum_{r=1}^{n} \sum_{s=1}^{n} p_q p_r p_s$$

$$= 24t_3P^3 \qquad (4)$$

This result is also slightly in error because products of the form p_x^3 and $p_x^2 p_y$ have been treated like products of the form $p_x p_y p_z$. The maximum possible error, and the error for "normal" speech distribution of powers, are given in Table 3.

In all cases the true distortion is less than $24t_3P^3$.

The total 3rd-order distortion power is therefore dependent only on the cube of the total fundamental power P, provided the number n of component tones is reasonably large.

(2.4.3) Higher-Order Distortion.

Extending the method to orders higher than the third, it can be shown that the rth order distortion power is given by

$$T_r = 2^{r-1} r! \, t_r P^r \qquad (5)$$

Table 3

ERROR DUE TO ASSUMING THAT THE TOTAL THIRD-ORDER DISTRIBUTION IS GIVEN BY $24t_3P^3$

Value of n	Maximum error for value of λ equal to				Error for "normal" distribution
	1	10	100	1 000	
	db	db	db	db	db
3	+ 4·0	+ 8·1	+ 13·0	+ 14·0	+ 5·3
10	+ 1·0	+ 1·0	+ 10·0	+ 13·0	+ 1·7
30	+ 0·4	+ 0·6	+ 7·0	+ 12·0	+ 0·5
100	+ 0·1	+ 0·2	+ 2·2	+ 9·0	+ 0·1
500	+ 0·0	+ 0·0	+ 0·3	+ 4·0	+ 0·0

and the total distortion power due to all orders can thus be written:

$$T = T_2 + T_3 + T_4 + \ldots T_r$$

$$= 4t_2P^2 + 24t_3P^3 + 192t_4P^4 + \ldots 2^{r-1} r! \, t_r P^r \qquad (6)$$

(2.5) Distribution of Distortion Power

It is now necessary to determine what proportion of the total distortion power due to each order falls within the band, and how this distortion power is distributed over the working band. When n is reasonably large (Section 3.2) only the distortion powers produced by products of the type $A \pm B \pm C \pm \ldots$ are of importance. As explained later in Section 3.4, the products of this type are divided into two Groups—Group 1, consisting of

Table 4

DISTRIBUTION OF NOISE POWERS OF ALL ORDERS

Order r	Group 1 products		Group 2 products		
	Proportion y of T_r in working band	Distribution over band, i.e. $\frac{\text{max. power}}{\text{min. power}}$	Proportion y of T_r in working band if $f_2/f_1 >> 1$	No power over portion of band if $f_2/f_1 <$	No power in band if $f_2/f_1 <$
2	zero	—	0·75 (a)	3	2
3	0·5 (b)	1·50	0·16 (b)	5	3
4	zero	—	0·60 (c)	5/3	3/2
5	0·34 (d)	1·31	0·21 (d)	7/3	2
6	zero	—	0·51	7/5	4/3
7	0·26	1·22	0·22	9/5	5/3
r odd	(e)	(f)	(g)	$(r+2)/(r-2)$	$(r+3)/(r-1)$
r even	zero	—	(g)	$(r+1)/(r-1)$	$(r+2)/r$

References:—

$$T_r = 2^{r-1} r! \, t_r P^r$$

(a) to (d) See Figs. 1 to 4, respectively.

$$(e) = \frac{(r+1)^r - (r+1)(r-1)^r + \dfrac{(r+1)r(r-3)^r}{2!} - \dfrac{(r+1)r(r-1)(r-5)^r}{3!} + \ldots \text{ to } \dfrac{r+1}{2} \text{ terms}}{2^{(2r-1)}\left(\dfrac{r-1}{2}\right)!\left(\dfrac{r+1}{2}\right)!}$$

$$(f) = \frac{r^{r-1} - r(r-2)^{r-1} + \dfrac{r(r-1)(r-4)^{r-1}}{2!} - \ldots \text{ to } \dfrac{r+1}{2} \text{ terms}}{(r+1)^{r-1} - r(r-1)^{r-1} + \dfrac{r(r-1)(r-3)^{r-1}}{2!} - \ldots \text{ to } \dfrac{r+1}{2} \text{ terms}}$$

$$(g) = \frac{r+1}{r!\,2^{r-1}}\left[(1)^r + \frac{r}{2!}\left\{(2)^r - (r+1)(1)^r\right\} + \frac{r(r-1)}{3!}\left\{(3)^r - (r+1)(2)^r + \frac{(r+1)r(1)^r}{2!}\right\} + \ldots\right]$$

to $\frac{1}{2}(r-1)$ terms when r is odd
to $\frac{1}{2}r$ terms when r is even

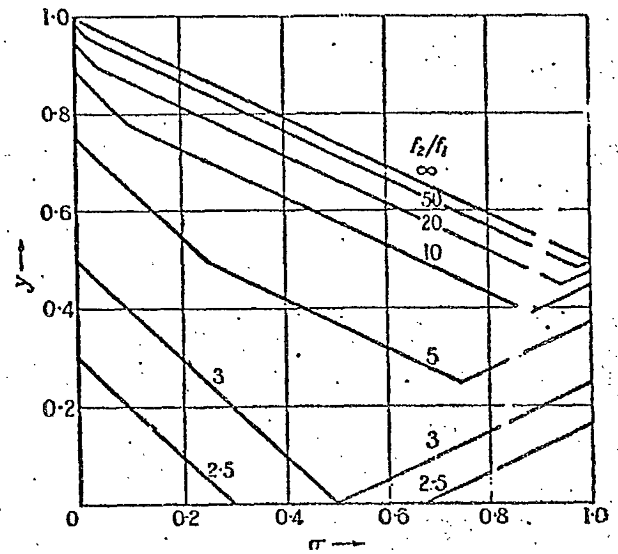

Fig. 1.—Distribution of 2nd-order distortion power in a system working between frequencies f_1 and f_2.

Second-order distortion power falling in a narrow band b c/s wide at mid-frequency f c/s

$$= \frac{b}{B} \cdot y \cdot (4t_2P^2)$$

where $B = (f_2 - f_1)$ c/s

$$\sigma = \frac{f - f_1}{f_2 - f_1}$$

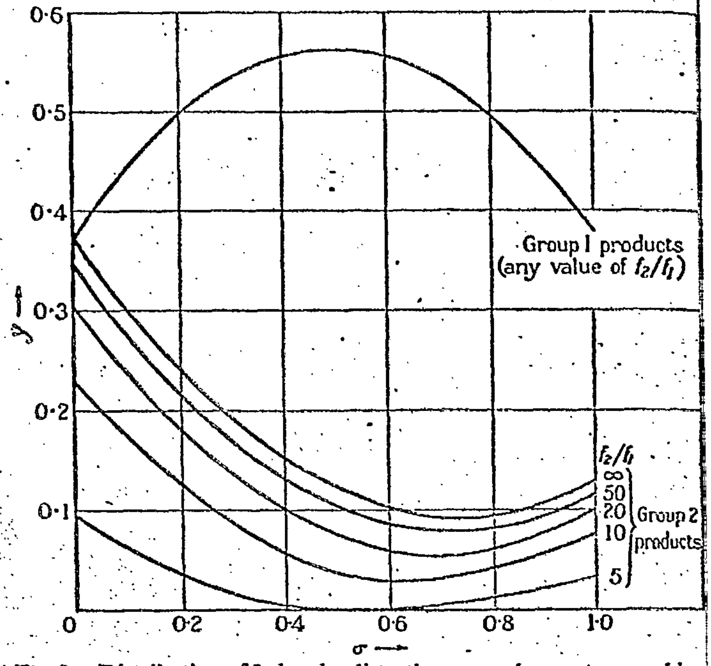

Fig. 2.—Distribution of 3rd-order distortion power in a system working between frequencies f_1 and f_2.

Third-order distortion power falling in a narrow band b c/s wide at mid-frequency f c/s

$$= \frac{b}{B} \cdot y \cdot (24t_3P^3)$$

where $B = (f_2 - f_1)$ c/s

$$\sigma = \frac{f - f_1}{f_2 - f_1}$$

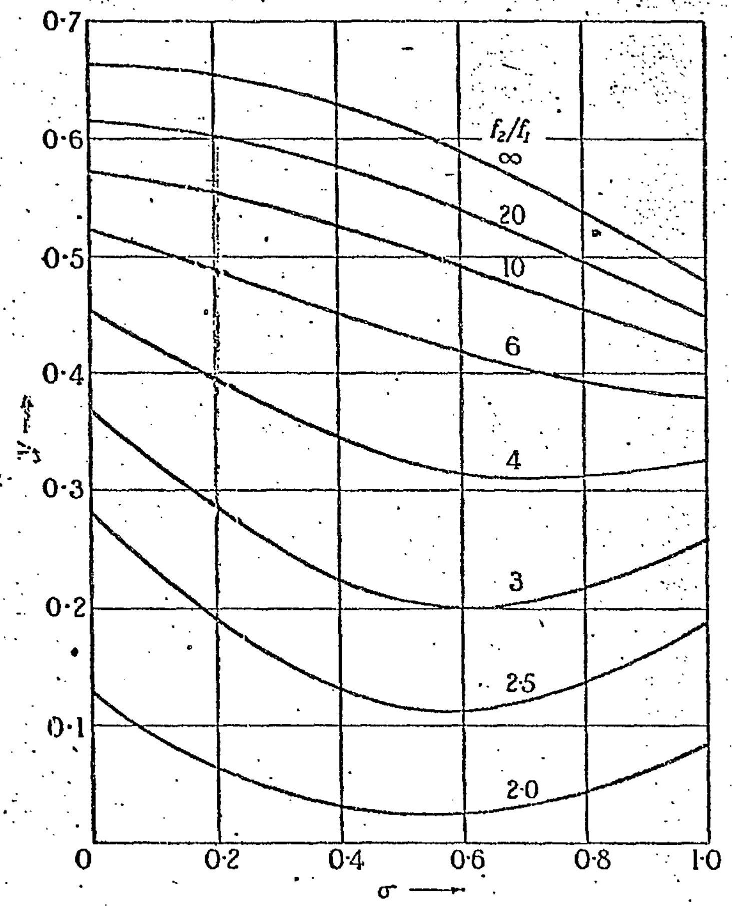

Fig. 3.—Distribution of 4th-order distortion power in a system working between frequencies f_1 and f_2.

Fourth-order distortion power falling in a narrow band b c/s wide at mid-frequency f c/s

$$= \frac{b}{B} \cdot y \cdot (192t_4P^4)$$

where $B = (f_2 - f_1)$ c/s

$$\sigma = \frac{f - f_1}{f_2 - f_1}$$

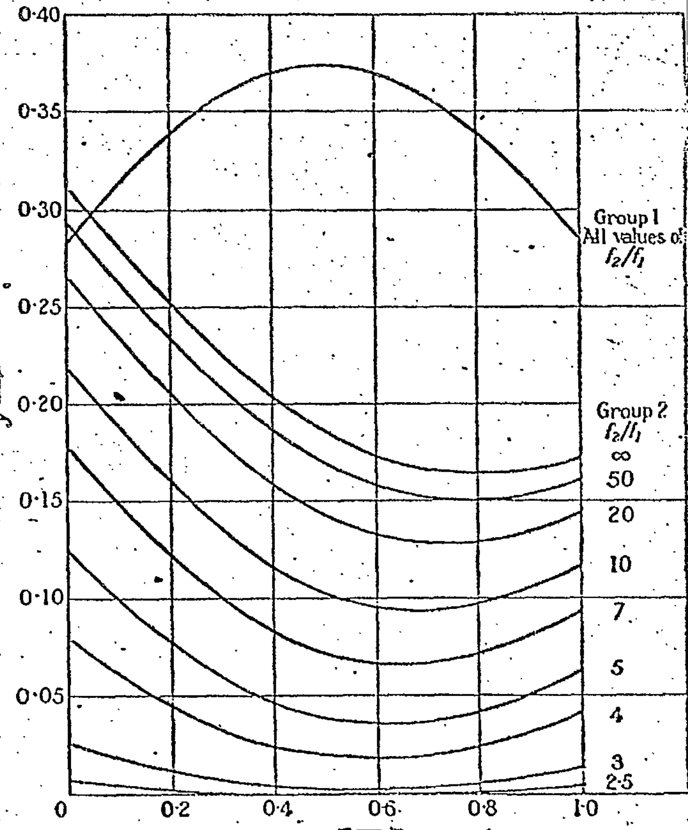

Fig. 4.—Distribution of 5th-order distortion power in a system working between frequencies f_1 and f_2.

Fifth-order distortion power falling in a narrow band b c/s wide at mid-frequency f c/s

$$= \frac{b}{B} \cdot y \cdot (1\,920t_5P^5)$$

where $B = (f_2 - f_1)$ c/s

$$\sigma = \frac{f - f_1}{f_2 - f_1}$$

distortion products which add on a voltage basis at successive repeaters, and Group 2 in which the addition is normally on a power basis. The method of determining the frequency distribution is indicated briefly in Appendix 9.4 for products of all orders. The results are collected together in Table 4 and Figs. 1–4. The most important conclusions can be summarized as follows:—

(a) The distributions are in all cases independent of the actual frequencies, but may depend on the value of the frequency ratio f_2/f_1.

(b) *Group* 1 *Products.*—The distribution is independent of the ratio f_2/f_1 and is fairly uniform over the working band for the third order, becoming still more uniform for higher orders (see Figs. 2 and 4).

(c) *Group* 2 *Products.*—The distribution depends on the ratio f_2/f_1. There is a value of f_2/f_1 for each order below which no Group 2 products appear in the band. Between this value and a higher value of f_2/f_1, Group 2 products appear in only a portion of the working band, and above this higher value of f_2/f_1, Group 2 products appear over the whole working band. The highest density of Group 2 products always occurs at the lowest frequency in the band, and it is greatest when f_2/f_1 is infinity.

Figs. 1–4 contain all the information necessary to determine the magnitude of the 2nd, 3rd, 4th and 5th-order distortion powers which will fall in any particular band of width b c/s located at a mid-frequency of f c/s, provided the fundamental components of the complex wave are uniformly distributed over the working band. For example, to determine the 3rd-order distortion power in a band 4 kc/s wide located at 1 Mc/s when the working band of the complex wave extends from 0·2 to 4 Mc/s we have, from Fig. 2, $B = (4 - 0{\cdot}2)10^6 = 3{\cdot}8 . 10^6$; $\sigma = \frac{1 - 0{\cdot}2}{4 - 0{\cdot}2} = 0{\cdot}21$, whence the proportion y due to Group 1 products is 0·50 and to Group 2 products 0·18. The total 3rd-order distortion power in this particular 4-kc/s band is therefore

$$b/B . y . 24t_3P^3 = \frac{4 . 10^3}{3{\cdot}8 . 10^6}(0{\cdot}50 + 0{\cdot}18) . 24t_3P^3 = 0{\cdot}017t_3P^3$$

For 6th and higher-order distortions the values of y can be obtained from Table 4. These values are an average over the working band, and the accuracy in assuming an average value is adequate for practical purposes.

(2.6) Non-Uniform Distribution of the Fundamental Tones

The distribution of the distortion power derived in Section 2.5 assumes that the fundamental wave consists of a large number of tones uniformly distributed over the working band, at least on a long-time basis. This condition may not always obtain in practice, so that it is desirable to determine the effect of a deviation from a uniform fundamental distribution.

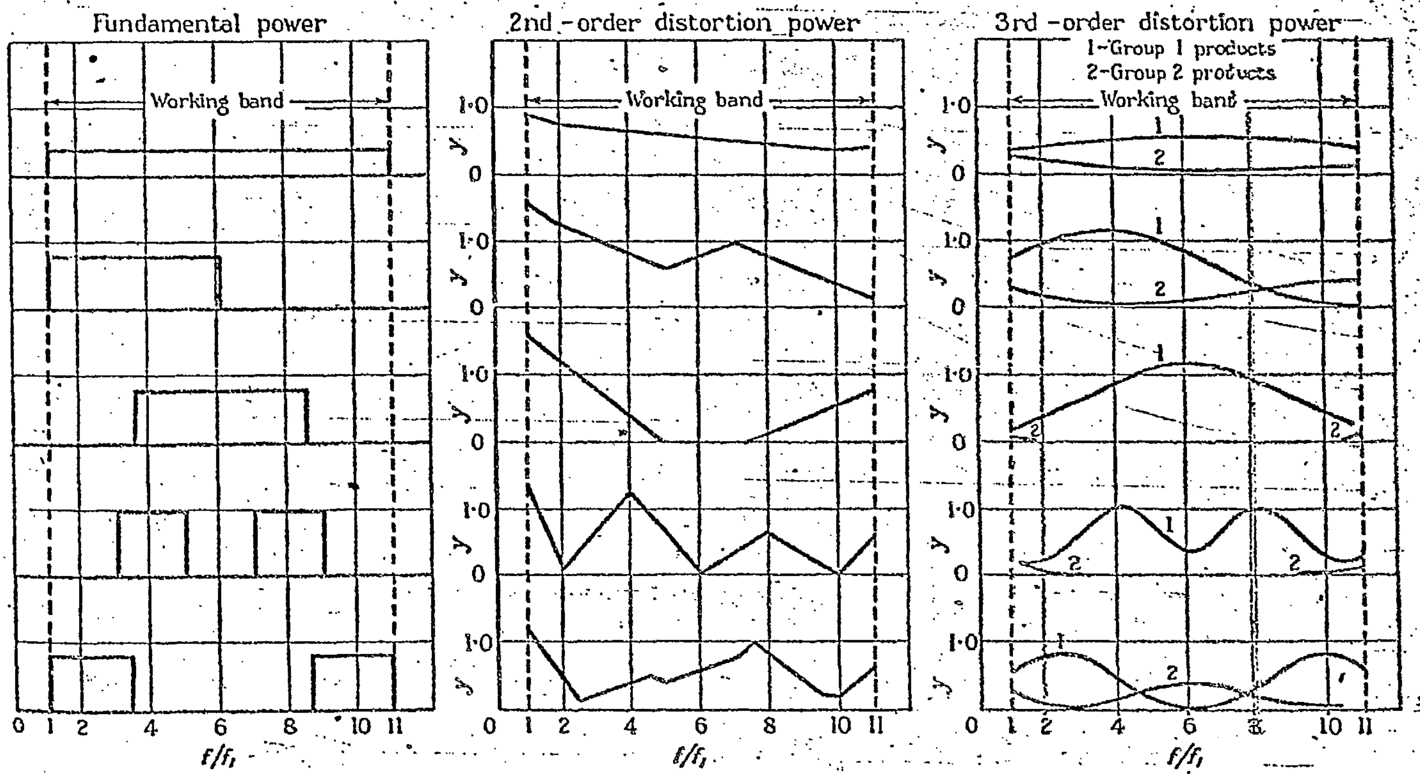

Fig. 5.—Distribution of 2nd- and 3rd-order distortion powers with non-uniform fundamental distributions. The total fundamental power is the same in each case.

In Fig. 5 is shown a comparison between a uniform distribution of the fundamental power over the working band and four non-uniform distributions each having the same total power as the uniform distribution. It will be observed that, although the non-uniform distributions are much more marked than would occur on, say, a wide-band speech system, the peak noise power in the band, for which the system has usually to be designed, does not vary greatly between the different distributions.

(3) PRACTICAL APPLICATION

(3.1) Power/Time Relation

The methods used in the previous Sections to calculate the distortion power assume implicitly that a steady-state condition exists for the fundamental tones. The measurement or calculation of the instantaneous fundamental power will give a value which cannot be correlated with the average or long-time power, and similarly the instantaneous distortion power will not be related to the average or long-time distortion power. It is the average power that is of concern in this analysis, and this cannot be obtained accurately unless the time period under considera tion is several times the periodic time of the lowest frequency in the working band; e.g. with an amplifier covering the frequency band 60–3 000 kc/s the average fundamental and distortion powers can be measured and calculated with accuracy only over a minimum time interval of about 50 microsec with steady-state conditions. For most practical purposes this time interval can be considered as instantaneous.

(3.2) Effect of Type of Signal

The intelligence that can be conveyed by a single-frequency tone is strictly limited, and in all accepted methods of telecommunication the effective band is used for some type of complex signal, e.g. speech, music, and voice-frequency telegraphy. It follows that n will be large even for a complex wave in a single channel, and extremely large for the resultant wave in any multichannel system. The one restriction to the use of equation (6), i.e. that n should be large, is thus completely removed, and this expression for the total distortion power can be employed without reserve for all types of systems.

(3.3) Determination of Value of P

The appropriate value of P to substitute in eqn. (6) will depend on the system requirements and the economical degree of safety. The following methods of determining P will cover most practical cases.

(*a*) Measure the peak value of the fundamental power at the output of the amplifier on a typical system. This is a comparatively simple method and quite safe since it will ensure that the average distortion noise is well below the calculated maximum noise. This method may, however, be unnecessarily severe and impose uneconomical conditions on the designer.

(*b*) Determine experimentally or theoretically the power/probability function for the signal. The value of P to give a predetermined peak or average noise power can then be evaluated.

(*c*) For speech, adopt the value of P taken from Fig. 6, curve 2. This refers to peak values and may be unnecessarily severe.

(*d*) For speech, adopt the value of P given in Fig. 6, curve 3. This will give a value of P for determining the average noise power during the busy hour.

The method of assessment of speech power given in (*b*), (*c*) and (*d*) above is referred to briefly in Appendix 9.5.

It must be noted that the values of P in Appendix 9.5 and in Fig. 6 relate to an amplifier with an output channel level equal to the sending switchboard level. If the output level from the amplifier is not equal to the switchboard level the value of P must be modified accordingly; for example, if the output level of each channel is − 13 db relative to the sending switchboard, the value of P will be 13 db below that determined from Fig. 6.

(3.4) Voltage or Power Summation

It is a peculiarity of a multi-repeater system that there is a difference in the behaviour and effect of the different products even if they are of the same type and order. In Appendix 9.6 the summation of distortion products in a multi-repeater system is analysed and it is shown that all distortion products fall into one of the two following groups:—

Group 1. A product is said to be a Group 1 product if the number of frequency components prefixed by a plus sign exceeds by + 1 or − 1 the number of frequency components prefixed by a minus sign, and the product frequency is positive in the former case and negative in the latter case.

Group 2. All other products are included in this group.

Examples: 60 + 300 − 200 = 160 is a Group 1 product
but 60 + 300 − 500 = − 140 is a Group 2 product

In an amplifier system with a linear phase/frequency characteristic over the working band, Group 1 products add at successive amplifiers on a voltage basis, i.e. their distortion waveforms coincide. Group 2 products can add on a voltage basis only if, in addition to the linear phase/frequency characteristic over the working band, the phase intercept of this characteristic at zero frequency is zero or a multiple of 2π radians.

(3.5) Effective Channel Noise

Since the effective channel working band must be less than the total channel spacing, the total output noise power per channel spacing must be multiplied by a factor K. For speech this factor will also take into account the relative aural importance of the noise powers throughout the channel, and it can be shown from the standard C.C.I.F.[5] psophometric weighting curve that the values of K are approximately 0·32, 0·40 and 0·50 for speech channels of 5, 4 and 3 kc/s spacing, respectively.

(3.6) Pilot and Ringing Tones

Telephone systems generally employ some type of voice-frequency signalling for setting-up calls, and pilot tones for supervisory purposes. It is usually satisfactory to increase P to cover the total speech power plus signalling power.

(3.7) Factor of Safety

The actual errors involved in the theoretical analysis are small compared with the discrepancies that will arise in practice, e.g. variation in valves, output levels and speech powers. The allocation of an adequate factor of safety will depend on the circuit design and the system conditions and requirements.

(3.8) Testing of Overall Systems

A problem which has existed hitherto has been the specification for an overall test that will adequately and accurately determine whether the linearity of a system is sufficient to carry the ultimate busy traffic conditions for which the system is designed. It is usually not a practical proposition to attempt to load a multi-channel system with the numbers of talkers and corresponding levels that would obtain in service. The preceding analysis indicates, however, how simple tests can be devised to determine the noise characteristics of a system. For example, to

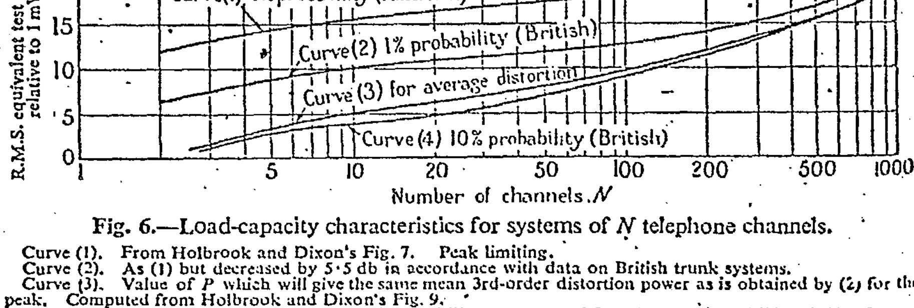

Fig. 6.—Load-capacity characteristics for systems of N telephone channels.

Curve (1). From Holbrook and Dixon's Fig. 7. Peak limiting.
Curve (2). As (1) but decreased by 5·5 db in accordance with data on British trunk systems.
Curve (3). Value of P which will give the same mean 3rd-order distortion power as is obtained by (2) for the peak. Computed from Holbrook and Dixon's Fig. 9.
Curve (4). Corresponds to (2) but for 10% probability. Computed from Holbrook and Dixon's Fig. 9.

determine the noise power due to 3rd-order Group 1 products, a source of thermal-agitation noise can be applied to any block of, say, three consecutive channels and the distortion noise voltage measured in an adjacent channel. If the total thermal noise power output be adjusted to equal the total output power which would be obtained on the complete system when in service, then if N is the total number of channels in the system the noise voltage per channel on the system when in service will be $\sqrt{(3/N)}$ times the noise voltage measured in the adjacent channel. To determine the noise power due to 2nd-order distortion the same test can be applied, but the measured power should now be the sum t_m of the individual noise powers occurring in the six or so channels located at about twice the frequency of the three fundamental channels. This measurement will give the total $(A + B)$ distortion power and is equal to $2t_2P^2$. The noise power per channel on the complete system will therefore be $b/B \, . \, y \, . \, 4t_2P^2 = b/B \, . \, y \, . \, 2t_m$, where y is obtained from Fig. 1.

(4) SUMMARIZED DESIGN

The general design problem is to determine the output distortion noise power produced in a given band-width or channel located at a particular point in the frequency spectrum of a wide-band system. The noise power components due to the Group 1 and Group 2 products of the different orders should be evaluated separately and added to give the total distortion noise at the output of the system. Each noise component can be evaluated individually from the following general formula, which can be used up to the highest order necessary:—

$$T_{rR} = \frac{b}{B} y T_r R^{\lambda} K 10^{-x/10} \qquad (7)$$

where

T_{rR} = noise power at a zero-level point in a band-width b c/s due to rth order distortion (Group 1 or Group 2) in a system of R networks in tandem,
b = channel band-width (c/s),
f = mid-frequency of channel band b (c/s),
B = total band-width of system carrying signals (c/s), $= f_2 - f_1$,
f_2 = upper frequency of wide band (c/s),
f_1 = lower frequency of wide band (c/s),
$\sigma = \dfrac{f - f_1}{f_2 - f_1}$ in Figs. 1–4,
y = proportion factor in Figs. 1–4 and Table 4, cols. 2 and 4,
t_r = rth harmonic output power produced by that sinusoidal input to the non-linear network which gives a fundamental output tone of unit power,
T_r = total noise power produced by rth order intermodulation in each network,
$= 4t_2P^2$ for 2nd order,
$24t_3P^3$ for 3rd order,
$2^{r-1}r!\,t_rP^r$ for rth order,
P = total fundamental output power from the non-linear network (for speech on trunk lines see Fig. 6, Curve 3),
R = number of non-linear networks in tandem in the system,
λ = 1 for Group 2 products; 2 for Group 1 products,
K = psophometric weighting factor,
$= 0{\cdot}5, 0{\cdot}4$ or $0{\cdot}32$ for speech channels with 3, 4 or 5 kc/s channel spacing, respectively,
x = output level of channel band b from each network, in decibels relative to a zero level point, e.g. sending switchboard level. x is negative for levels below zero.

The unit of power adopted throughout is immaterial, but the milliwatt is usually convenient.

The value of y can be obtained accurately from Figs. 1–4 for 2nd, 3rd, 4th and 5th-order distortion for any location of the channel b. For higher-order distortion the value of y can be taken with sufficient practical accuracy from cols. 2 and 4 of Table 4.

In certain cases the derivation of the harmonic limits from eqn. (7) can be further simplified, e.g. P is usually a function of B and may therefore be eliminated as a variable in the equation; also, for a given system many of the factors in the equation are independent of the harmonic order, so that it is often convenient to relate by simplified expressions all the harmonic limits t_r to, say, the 3rd-order limit t_3.

(5) EXAMPLES OF APPLICATION

Two examples of the application of this analysis are given below.

(5.1) Design for a Coaxial-Cable Telephone System

It is required to determine the harmonic limits for a wide-band repeater to operate under the following conditions:—

(a) The telephone channels are uniformly distributed at 4 kc/s spacing over the frequency band 60–2 788 kc/s.

(b) The average psophometric total distortion voltage per channel during the busy hour is not to exceed 0·6 mV into 600 ohms at a zero-level point on a 500-mile system.

(c) A 500-mile system consists of four 125-mile links, each consisting of 25 repeatered sections.

(d) The output level per channel from each repeater is − 13 db relative to the sending switchboard level.

The noise power in the four links will sum on a power basis, so that the noise allowance for each link will be $0{\cdot}6/\sqrt{4} = 0{\cdot}3$ mV.

From curve 3 in Fig. 6 the appropriate value of P is + 17 db relative to 1 mW at a zero-level point, i.e. sending switchboard level. The actual value of P at each repeater output will therefore be + 17 − 13 = + 4 db on 1 mW = 2·5 mW.

For 2nd-order distortion we have, in eqn. (7),

$$T_{rR} = \frac{0{\cdot}3^2 \, . \, 10^{-6} \, . \, 10^3}{600} \text{ mW} \qquad K = 0{\cdot}4$$
$$b = 4\,000 \qquad R = 25$$
$$f_1 = 60 \, . \, 10^3 \qquad \lambda = 1$$
$$f_2 = 2\,788 \, . \, 10^3 \qquad x = -13$$
$$B = 2\,728 \, . \, 10^3 \qquad P = 2{\cdot}5$$
$$f_2/f_1 \simeq 50 \qquad T_r = 4t_2P^2 = 4t_2 \, . \, 2{\cdot}5^2$$

The maximum distortion power will occur in the lowest-frequency channel, so that $\sigma = 0$ and $y = 0{\cdot}98$ (Fig. 1).

Solving equation (7) for t_2 it is found that

$$t_2 \simeq 2{\cdot}1 \, . \, 10^{-8} \text{ mW} = -77 \text{ db relative to 1 mW}$$

For 3rd-order intermodulation the maximum noise power will occur in the centre of the band, and for Group 1 terms in Fig. 2 $\sigma = 0{\cdot}5$ and $y = 0{\cdot}56$. Also, now, $\lambda = 2$ and $T_r = 24t_3P^3 = 24t_3 \, . \, 2{\cdot}5^3$. Substituting in eqn. (7) and solving for t_3, it is found that

$$t_3 = 10^{-10} \text{ mW} = -100 \text{ db relative to 1 mW}$$

The Group 2 products are quite negligible.

The 4th, 5th and higher orders can be calculated similarly, using values of y taken from Figs. 3 and 4 and Table 4, respectively.

Straight lines can then be drawn through these points with slopes appropriate to the harmonic order, as shown by the dotted lines in Fig. 7. Each repeater must therefore be so designed that its measured harmonic curve lies above the corresponding theoretical curve, if the distortion noise is to meet the specified requirements. Since the limiting values of t_2, t_3, etc., have

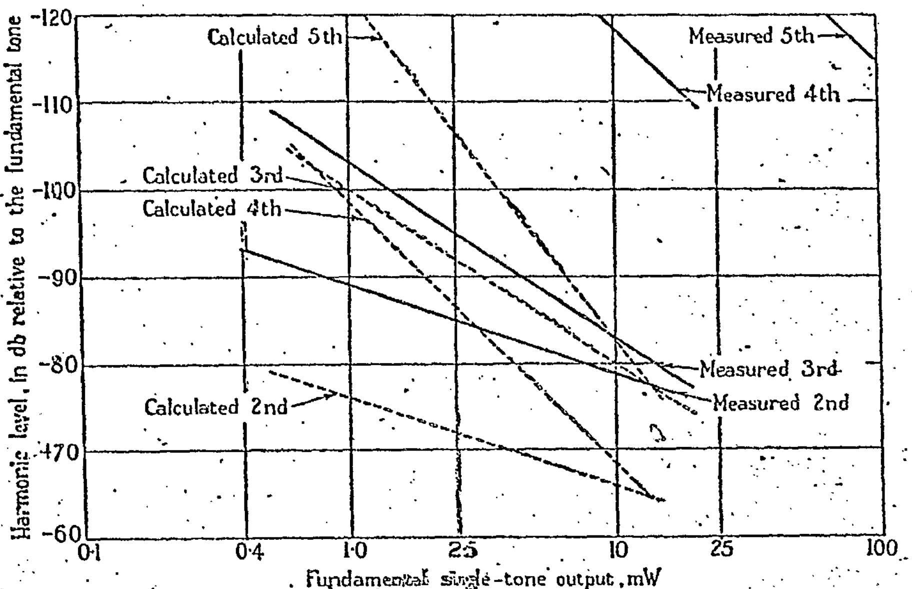

Fig. 7.—Harmonic characteristics of a wide-band repeater.

been calculated on the assumption that each alone produces the total permissible noise power, it will only be permissible for one measured harmonic curve to approach very closely to its theoretical limit, provided that at the same time all other harmonics lie well above (e.g. 6 db) their theoretical curves. In Fig. 7 the full lines show the measured characteristics of a three-valve negative-feedback repeater which meets the calculated limiting values.

(5.2) Output of Broadcast Receiver

If the output valve of a broadcast receiver is rated at 4 watts for 5% 3rd-harmonic voltage, i.e. a single-tone signal/noise ratio of 26 db, then the value of t_3 (in watts with a 1-watt tone output) is $(5/100)^2 . 4 . 1/64 = 156 . 10^{-6}$. With a complex wave of peak power 4 watts the peak distortion power will, from Table 4, be $(0\cdot5 + 0\cdot16)24t_3P^3 = 16 . 156 . 10^{-6} . 4^3 = 0\cdot16$ watt, i.e. a signal/noise ratio of $10 \log_{10} 4/0\cdot16 = 14$ db at peak output. This method of treating audio-frequency distortion appears to lead to interesting results.

(6) SUMMARIZED RESULTS

If the input/output characteristic of a non-linear network can be expressed as a single-valued function, e.g. $V = av + bv^2 + cv^3 + dv^4 + \ldots$, where V is the output voltage, v the input voltage and a, b, c, d, etc., are constants, then the distortion noise generated by a complex signal can be determined as follows.

(6.1) The total instantaneous distortion power T produced by a complex wave of "instantaneous" output power P is given by

$$T = 4t_2P^2 + 24t_3P^3 + \ldots 2^{r-1}r!\,t_rP^r$$

Here t_r is the rth harmonic power produced by that sinusoidal input which gives a fundamental output tone of unit power. T, P and t_r are measured in the same units. "Instantaneous" is defined in Section 3.1. The term "complex wave" includes all signals associated with speech, music, television, voice-frequency telegraphy, and random noise. If P is the maximum power of a complex wave during any period of operation, then the expression above will give the maximum distortion power produced during the same period.

(6.2) The proportion of the power T which occurs within the frequency range of the complex wave or in any portion of this band can be determined as follows, if the frequency components of the complex wave are distributed fairly uniformly over the working band.

Even-order products.

(*a*) No distortion power due to even order products of order r can be produced within the frequency band if the ratio of the upper frequency to the lower frequency, i.e. the frequency ratio, is less than $1 + 2/r$.

(*b*) As the frequency ratio increases, the proportion of the distortion power within the band increases to a limiting value. The distortion power in any band can be determined from Figs. 1 and 3 for 2nd and 4th orders respectively, and from Table 4 for higher orders of f_2/f_1 much greater than unity.

Odd-order products.

(*c*) Products of all odd orders are present for any value of the frequency ratio.

(*d*) The products can be divided into Group 1 products, which are voltage-adding, and Group 2 products, which are power-adding (Section 3.4).

(*e*) The distribution of Group 1 products is independent of the frequency ratio and is substantially uniform over the working band.

(*f*) The distribution of the Group 2 products depends on the frequency ratio.

(*g*) The distortion power in any band can be obtained from Figs. 2 and 4 for 3rd and 5th orders respectively, and from Table 4 for higher orders.

(*h*) The distortion power of the Group 2 products is usually much less than that of the Group 1 products. For most practical purposes Group 2 can be neglected, particularly when there are a number of non-linear networks in series.

(6.3) In a multi-repeater system* certain products, called Group 1 products, will tend to add on a voltage basis at successive repeaters. The criterion for such a product is that the number of frequency components prefixed by a plus sign must exceed by $+1$ or -1 the number of frequency components prefixed by a minus sign; in the former case the product frequency must be positive and in the latter case negative. It follows that Group 1 products can be obtained only from odd orders. Group 2 includes all remaining products. Group 1 products add on a voltage basis if the phase/frequency characteristic of each repeater section is linear over the working band. Group 2 products add on a power basis except in the case of a linear phase/frequency characteristic combined with a phase intercept at zero frequency of zero or a multiple of 2π radians. This condition is not readily obtained in practice.

(7) ACKNOWLEDGMENT

This study was carried out in the Radio Laboratories of the British Post Office, and the authors' thanks are due to Col. Sir A. Stanley Angwin, Engineer-in-Chief, for permission to publish this paper.

(8) REFERENCES

(1) HOLBROOK, B. D., and DIXON, J. T.: "Load Rating Theory for Multi-Channel Amplifiers," *Bell System Technical Journal*, 1939, **18**, p. 624.

(2) BENNETT, W. R.: "Cross Modulation in Multi-Channel Amplifiers below Overload," *ibid.*, 1940–41, **19**, p. 587.

(3) JACOBSEN, B. B.: "Effect of Non-Linear Distortion in Multi-Channel Amplifiers," *Electrical Communication*, 1940–41, 19, p. 29.
(4) Lord RAYLEIGH: "Theory of Sound" (Macmillan), vol. 1, chap. II.
(5) Comité Consultatif International Téléphonique: Directives (Oslo, 1938), chap. 15.

(9) APPENDICES

(9.1) Distortion Due to Second-Order Products

The second-order distortion can be examined by inserting in eqn. (1)

$$v = r \sin 2\pi At + s \sin 2\pi Bt \quad . \quad . \quad . \quad (8)$$

Neglecting the production of second-order products by dv^4 and higher even powers of v (Appendix 9.3), the bv^2 term only need be considered. The output voltage due to this term can be written

$$V_2 = bv^2 = b(r \sin 2\pi At + s \sin 2\pi Bt)^2$$

This can be expanded to

$$V_2 = b\left\{\begin{array}{l}\frac{1}{2}r^2 \cos 4\pi At + \frac{1}{2}s^2 \cos 4\pi Bt + rs \cos 2\pi t(A + B)\\ \quad - rs \cos 2\pi t(A - B) + \text{d.c. terms which can}\\ \quad \text{be neglected}\end{array}\right\} \quad (9)$$

If in equation (8) the number of tones is increased from 2 to n, it will be found that the distortion products produced are still only of the types exemplified by $2A$, $(A + B)$ and $(A - B)$, and no new types appear.

It follows that if a single tone of power 1 mW produces t_2 mW of second harmonic, n tones, each of power q mW, will produce n second harmonics each of power q^2t_2 mW, together with $\frac{1}{2}n(n - 1)$ products of the form $(A - B)$ and $\frac{1}{2}n(n - 1)$ products of the form $(A + B)$, each of power $4q^2t_2$ mW.

(9.2) Distortion Due to Third-Order Products

Proceeding on similar lines to Appendix 9.1, the substitution

$$v = r \sin 2\pi At + s \sin 2\pi Bt + t \sin 2\pi Ct \quad . \quad . \quad (10)$$

is made in equation (1). The output voltage due to cv^3 is therefore

$$V_3 = c(r \sin 2\pi At + s \sin 2\pi Bt + t \sin 2\pi Ct)^3$$

Expansion of this expression and reduction to compound angles gives

$$\frac{V_3}{c} = \frac{1}{4}\left[\begin{array}{l}3r^3 \sin 2\pi At - r^3 \sin 6\pi At + 6r^2s \sin 2\pi Bt\\ \qquad\qquad - 3r^2s \sin 2\pi t(2A + B)\\ + 3r^2s \sin 2\pi t(2A - B) + 6r^2t \sin 2\pi Ct\\ \qquad\qquad - 3r^2t \sin 2\pi t(2A + C)\\ + 3r^2t \sin 2\pi t(2A - C)\\ + 8 \text{ corresponding terms with amplitudes } s^3, s^2r, s^2t\\ + 8 \text{ corresponding terms with amplitudes } t^3, t^2r, t^2s\end{array}\right]$$
$$+ \frac{3}{2}rst\left[\begin{array}{l}\sin 2\pi t(A + B + C) - \sin 2\pi t(A + B - C) - \sin 2\pi t\\ \quad (C + A - B) - \sin 2\pi t(B + C - A)\end{array}\right] (11)$$

Again no new types of products are produced if the number of fundamental tones is increased to n.

Neglecting products of the types $3r^3 \sin 2\pi At$ and $6r^2s \sin 2B\pi t$, which only modify inappreciably the fundamental tones, there remain three types of distortion products, i.e. $3A$, $2A \pm B$ and $A \pm B \pm C$. It can readily be shown that if there are n fundamental tones each of power q mW the numbers of products of each type are n, $2n(n - 1)$ and $\frac{2}{3}n(n - 1)(n - 2)$, and the distortion power in each type is q^3t_3, $9q^3t_3$ and $36q^3t_3$ mW, respectively.

(9.3) Low-Order Products from High-Order Terms

It was assumed in Appendices 9.1 and 9.2 that second- and third-order products were produced only by the terms bv^2 and cv^3 respectively. Actually all the even powers of v in eqn. (1) will also produce lower-order even products; and the odd powers of v will also produce lower-order odd products. Thus, the term ev^5 will produce both fifth-order and third-order products, e.g. terms of the types $A \pm B \pm C$ in addition to terms of the types $A \pm B \pm C \pm D \pm E$. It can be shown that, although the power in each such lower-order product may exceed the power in each higher-order product, the number of products is so much lower when n is not small that the total distortion power in the higher-order products is considerably greater than the power in the lower-order products. If, therefore, the design limits are based on the distortion power of the high-order products, the effect of the lower-order products which are simultaneously generated by the high-order terms can be neglected.

(9.4) Frequency Distribution of Intermodulation Products

In the following treatment of the frequency distribution of intermodulation products it will be assumed that the signal passing through the non-linear system consists of a large number of sine waves, of equal amplitudes and random phases, uniformly spaced in frequency, the unit of frequency being so chosen that the frequencies of the waves are f_1, $(f_1 + 1)$, $(f_1 + 2) \ldots (f_2 - 1)$, f_2. Second harmonics will be regarded as products of the type $(A + B)$ for which $A = B$, but it is shown in Table 1 that the resultant error due to this assumption is small. Corresponding approximations will be made in dealing with higher-order products in which one fundamental appears more than once.

Considering only products of the type $(A + B)$, the wave of frequency f_1 will combine with all the waves, including itself, to give products of frequencies $2f_1$, $(2f_1 + 1)$, $(2f_1 + 2)$, ... $(f_1 + f_2)$. This set of products is represented by the lowest rectangle in Fig. 8(*a*). The wave of frequency $(f_1 + 1)$ will combine with all the waves to give a set of products of frequencies $(2f_1 + 1)$, $(2f_1 + 2)$, . . . $(f_1 + 1 + f_2)$, represented by the second rectangle in Fig. 8(*a*): and similarly each wave $(f_1 + 2)$, $(f_1 + 3)$, . . . f_2 will combine with all the waves to give sets of products which can be represented by rectangles displaced successively one unit to the right, the last one occupying the range $(f_2 + f_1)$ to $2f_2$. The number of products occurring at any particular frequency can be found by counting the number of rectangles which include that frequency and dividing by 2, since in the summation any product $(f_x + f_y)$ where $x \neq y$ will have been counted twice, i.e. as $(f_x + f_y)$ and as $(f_y + f_x)$. If this is done for all frequencies the result can be approximated very closely by an isosceles triangle [Fig. 8(*b*)] of base $2f_1$ to $2f_2$ and height $\frac{1}{2}(f_2 - f_1)$. Thus, the number of products in unit bandwidth at frequency f is given by

$$\left.\begin{array}{ll} m = \frac{1}{2}(f - 2f_1) & \text{when } 2f_1 < f < f_1 + f_2 \\ m = \frac{1}{2}(2f_2 - f) & \text{when } f_1 + f_2 < f < 2f_2 \end{array}\right\} \quad . \quad (12)$$

The two equations (12), together with the statements $m = 0$, $f < 2f_1$, and $m = 0$, $f > 2f_2$, can be combined in the single equation

$$m = \tfrac{1}{2}\{[f - 2f_1] - 2[f - f_1 - f_2] + [f - 2f_2]\} \quad . \quad (13)$$

(The device is adopted here and throughout the rest of Appendix 9.4 that quantities in square brackets[] are to be ignored if they are negative.) Further, since every pair of fundamentals produces one $(A + B)$ product and one $(A - B)$ product, the complete isosceles triangle represents half the total 2nd-order distortion power, and it is easy to show that the proportion of the total 2nd-order distortion power which falls as $(A + B)$-type products in a narrow band $(f - \frac{1}{2}b)$ to $(f + \frac{1}{2}b)$ is given by

$$y_2 = \frac{1}{2} \cdot \frac{b}{B}\left\{\frac{[f - 2f_1] - 2[f - f_1 - f_2] + [f - 2f_2]}{(f_2 - f_1)}\right\} \quad (14)$$

where $B = (f_2 - f_1)$. The frequencies in (14) can be expressed in any desired units, and the shape of the corresponding curve depends only on the ratio f_2/f_1, not on the absolute values of f_2 and f_1.

In cases where the fundamental powers are unequal it is possible to use the above method as a completely graphical method.

The distribution of the $(A - B)$-type products can be treated in the same manner as the $(A + B)$-type, but it is preferred to use a different method, which is better capable of application in the case of third and higher orders.

The frequencies of a set of $(A + B)$-type products having f_x as one component can be found by adding f_1, $(f_1 + 1)$, . . . f_2 in turn to f_x. Similarly, the frequencies of a set of $(A - B)$-type products having f_x as one component can be found by adding $-f_2$, $(-f_2 + 1)$, . . . $(-f_1 - 1)$, $-f_1$, in turn to f_x. The two sets can be represented thus:—

$(A + B)$:
(f_x+f_1), (f_x+f_1+1), (f_x+f_1+2), . . . (f_x+f_2-1), (f_x+f_2)
$(A - B)$:
(f_x-f_2), (f_x-f_2+1), (f_x-f_2+2), . . . (f_x-f_1-1), (f_x-f_1)

Corresponding to every frequency in the $(A + B)$ set there is in the $(A - B)$ set a frequency which is smaller by $(f_2 + f_1)$. This applies for all f_x for which $f_1 \leqslant f_x \leqslant f_2$, and since there are equal numbers of $(A + B)$ and $(A - B)$ products, all of equal power, it follows that the $(A - B)$ distribution curve can be obtained by sliding the $(A + B)$ curve bodily to the left a distance $(f_1 + f_2)$. The result is to put half the isosceles triangle in the negative frequency region, as shown by the broken lines in Fig. 8(*c*). The negative signs can be ignored, as they have no physical significance, so that the negative part of the curve can be transferred to the positive region by lateral inversion about the ordinate axis, and the final curve can be found by adding ordinates. Overlapping parts of the $(A + B)$ and $(A - B)$ curves can also be added. A typical result is shown by the heavy lines in Fig. 8(*c*).

A set of curves showing the distribution of 2nd-order distortion power in the working band for different values of f_2/f_1 is shown in Fig. 1.

The method used for the 2nd-order products can be extended to the third and higher orders, but it tends to become rather cumbersome. The same principle can, however, be used to establish a general expression which will give the frequency distribution of products of any order.

The frequency of any product of order r is formed by adding the frequencies of $(r - q)$ fundamentals and then subtracting the frequencies of q other fundamentals. It is convenient to deal with products in classes characterized by the number q of negative frequency components involved; thus $A + B - C - D$ and $A + B + C + D - E - F$ are both "$q = 2$" products although of different orders.

Considering the same set of fundamentals as before, and confining attention for the moment to products of the rth order for which $q = 0$, i.e. those for which all the r component frequencies are added, it can be shown that the number of products falling in a band of unit width at frequency f is given by

$$m = \frac{1}{r!(r-1)!}\Big\{[f - rf_1]^{(r-1)} - r[f - (r-1)f_1 - f_2]^{(r-1)} + \frac{r(r-1)}{2!}[f - (r-2)f_1 - 2f_2]^{(r-1)} - \frac{r(r-1)(r-2)}{3!}[f - (r-3)f_1 - 3f_2]^{(r-1)} + \ldots \text{ to } (r+1) \text{ terms}\Big\} \quad (15)$$

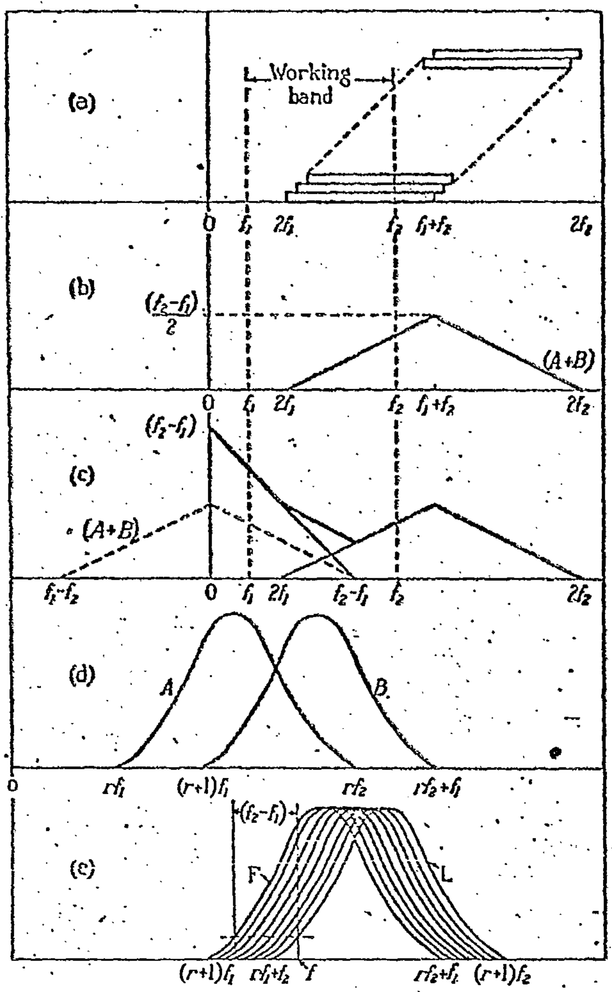

Fig. 8.—Derivation of distortion distribution curves.

This expression can be derived from elementary considerations, but an inductive proof is sufficient for the present purpose.

All the products included in (15) are represented by the curve *A* of Fig. 8(*d*). If f_1 is added to the frequency of each of these products the new set of products can be represented by curve B of Fig. 8(*d*), which has the same shape as curve A but is displaced f_1 to the right. This new curve represents all the $(r + 1)$th-order products having f_1 as one component frequency. Further curves can be drawn, each displaced successively one unit to the right, corresponding to the addition of $(f_1 + 1)$, $(f_1 + 2)$, . . . f_2, respectively, to the frequencies represented in eqn. (15). The complete set of curves, a few of which are indicated in Fig. 8(*e*), includes each $(r + 1)$th-order product $(r + 1)$ times, for a product of frequency $(f_p + f_q + f_r + \ldots$ to $r + 1$ terms) will appear under each of the curves, $(r + 1)$ in number, formed by adding f_p, f_q, f_r, . . . to the frequencies included under curve A. The complete distribution curve for the $(r + 1)$th-order $q = 0$ products can thus be found by adding the ordinates of all the curves in Fig. 8(*e*) and dividing each sum by $(r + 1)$.

From Fig. 8(*e*) it will be seen that, instead of adding the ordinates of all the curves at frequency f, the same result can be obtained by measuring the ordinates of curve F at unit intervals over the range $f - (f_2 - f_1)$ to f_1 and adding. Thus, the sum of the ordinates at f is equal to the area under curve F between

$(f - f_2 + f_1)$ and f. Now curve F is simply the curve of eqn. (15) displaced a distance of f_1 to the right, so that its equation is

$$F(f) = \frac{1}{r!(r-1)!}\Big\{[f-(r+1)f_1]^{(r-1)} - r[f - rf_1 - f_2]^{(r-1)} + \frac{r(r-1)}{2!}[f-(r-1)f_1 - 2f_2]^{(r-1)} - \dots\Big\}$$

and, if the frequency between successive fundamentals is now called df, the ordinate of the $(r+1)$th-order curve at frequency f will be given by

$$m = \frac{1}{r+1}\int_{f-f_2+f_1}^{f} F(f)df$$

$$= \frac{1}{(r+1)r!(r-1)!}\int_{f-f_2+f_1}^{f}\Big\{[f-(r+1)f_1]^{(r-1)} - r[f-rf_1-f_2]^{(r-1)} + \frac{r(r-1)}{2!}[f-(r-1)f_1-2f_2]^{(r-1)} - \dots\Big\}df$$

$$= \frac{1}{(r+1)!r!}\Big([f-(r+1)f_1]^r - r[f-rf_1-f_2]^r + \frac{r(r-1)}{2!}[f-(r-1)f_1-2f_2]^r - \dots\Big)_{f-f_2+f_1}^{f}$$

$$= \frac{1}{(r+1)!r!}\Big\{[f-(r+1)f_1]^r - (r+1)[f-rf_1-f_2]^r + \frac{(r+1)r}{2!}[f-(r-1)f_1-2f_2]^r - \dots \text{ to } (r+2) \text{ terms}\Big\}$$

which is equation (15) rewritten for order $(r+1)$ instead of r.

Equation (13) shows that (15) holds for the second order, and it therefore holds for all orders.

Considering now the products for which $q \neq 0$, it will be seen that while a set of r fundamentals will give only one $q = 0$ product it will give r products with $q = 1$, since the negative sign can be associated with any of the r fundamental frequencies, and each choice gives a different product-frequency. Similarly, the number of $q = 2$ products will be $r(r-1)/2!$ times the number of $q = 0$ products, and, generally, the ratio of the number of products with a given value of q to the number of products with $q = 0$, formed from the same set of fundamentals, is $r(r-1)(r-2)\dots$ to q terms$/q!$ This ratio holds for $q < \frac{1}{2}r$ only. If r is even and $q = \frac{1}{2}r$, the ratio must be halved; otherwise, in the case of sixth-order products, for example, $A+B+C-D-E-F$ and $-A-B-C+D+E+F$ will both be counted as separate products, whereas they are identical when the negative sign before one of the resultant frequencies is ignored. Values of q greater than $\frac{1}{2}r$ are inadmissible, because a product for which $q = q' > \frac{1}{2}r$ would be identical, except for a difference in sign, with a product already included in the class having the smaller number $(r - q')$ of negative components. For example, $A + B - C - D - E$ would be counted as a $q = 3$ product, although it would already have been counted as the $q = 2$ product $-A - B + C + D + E$.

Following the method used earlier for the second-order $q = 1$ products, i.e. the $(A - B)$ products, it will be seen that the distribution curve for the $q = 1$ products of order r can be found by multiplying the ordinates of the $q = 0$ curve by r and moving it bodily to the left a distance $(f_1 + f_2)$. Similarly, for any other value of q, the $q = 0$ curve is multiplied by $r(r-1)(r-2)\dots$ to q terms$/q!$ and moved a distance $q(f_1 + f_2)$ to the left. The only exception is in the case of $q = \frac{1}{2}r$ products, when the ordinate multiplier is $r(r-1)(r-2)\dots$ to q terms$/2q!$ Any parts of curves which appear in the negative frequency region are to be laterally inverted about the ordinate axis, and overlapping parts of curves can be added directly, provided that Group 1 and Group 2 products (see Section 9.6) are kept separate where necessary.

The sum of the numbers $1, r, r(r-1)/2!, \dots$ by which the ordinates of the $q = 0$ curve are to be multiplied to give the $q = 0, 1, 2, \dots$ curves, is always $2^{(r-1)}$, so that the whole of the $q = 0$ curve represents a fraction $(\frac{1}{2})^{(r-1)}$ of the total rth-order distortion power, and it can be shown that the proportion of the rth-order distortion power for $q = 0$, falling in a narrow band $(f - \frac{1}{2}b)$ to $(f + \frac{1}{2}b)$, is given by

$$y_r = \frac{b}{B}\,\frac{1}{(r-1)!\,2^{r-1}(f_2-f_1)^{r-1}}\Big\{[f-rf_1]^{(r-1)} - r[f-(r-1)f_1-f_2]^{(r-1)} + \frac{r(r-1)}{2!}[f-(r-2)f_1-2f_2]^{(r-1)} - \dots \text{ to } (r+1) \text{ terms}\Big\} \quad (16)$$

where $B = (f_2 - f_1)$. Frequencies in eqn. (16) can be expressed in any desired units, and the corresponding curve depends only on the ratio f_2/f_1, and not on the absolute values of f_1 and f_2. Corresponding expressions for $q = 1, 2, 3, \dots$ can be obtained by shifting the curve to the left and multiplying ordinates as before: for example, for $q = 1$,

$$y_r = \frac{b}{B}\,\frac{r}{(r-1)!\,2^{r-1}(f_2-f_1)^{r-1}}\Big\{[f-(r-1)f_1+f_2]^{(r-1)} - r[f-(r-2)f_1]^{(r-1)} + \frac{r(r-1)}{2!}[f-(r-3)f_1-f_2]^{(r-1)} - \dots \text{ to } (r+1) \text{ terms}\Big\} \quad (17)$$

A case of particular interest occurs when r is odd and $q = \frac{1}{2}(r-1)$. If the corresponding equation is obtained from (16) by increasing f to $f + \frac{1}{2}(r-1)(f_1 + f_2)$ and multiplying ordinates, and if the substitution $\sigma = (f - f_1)/(f_2 - f_1)$ is made, the resultant expression is found to be dependent only on r and σ, and a single curve can be used for all values of f_2/f_1. The positive part of such a curve contains Group 1 products only (Section 9.6).

The substitution $\sigma = (f - f_1)/(f_2 - f_1)$ can be made in any of the expressions (16), (17), etc., and for a given order a set of distribution curves can be plotted with a common scale of abscissae, the range $\sigma = 0$ to $\sigma = 1$ representing the working band in every case. These curves, when summed, give the distribution of the total distortion power for the particular order (Figs. 1–4).

Besides distribution curves, the general expressions of the type (16) can be used for other purposes, e.g. to calculate the proportion of distortion power falling in the working band (Table 4). The method can also be extended to deal with less simple distributions of fundamental power, and some results for second and third orders are illustrated in Fig. 5.

(9.5) Determination of P

Holbrook and Dixon carried out in the Bell Telephone Laboratories a most valuable practical and theoretical study of the summation of speech voltages, and their results are used freely below. In Fig. 6, curve 1, is shown their derived characteristic relating the peak voltage which will not be exceeded for more than 1% of the time during the busy hour, based on a measured mean talker power of 12·1 db below reference telephonic power. Measurements by the British Post Office on incoming calls at the London Trunk Exchange gave a mean talker power of 17·6 db below reference telephonic power, and curve 2 shows curve 1 reduced by 5·5 db to conform to these conditions. The C.C.I.F.[5] recommends a limiting noise value of 2 mV psophometric e.m.f. under conditions which are not precisely defined. In preference, therefore, to assigning this to the peak noise it is considered to be permissible and more economic to design for an average noise e.m.f. of 2 mV. If it is assumed that third-order intermodulation noise is the limiting factor, it is possible to derive a curve for P which, when used with the distortion formula, e.g. eqn. (6), will give an average noise equal to the peak noise obtained from curve 2. This value is shown in curve 3 and is calculated from Fig. 9 of Holbrook and Dixon's paper. For non-trunk speech systems the value of P will probably be greater by an amount which will be determined by the particular conditions.

The value of P for telegraph channels can be readily calculated, and by the use of Rayleigh's distribution formula[4] the power/probability function can be evaluated.

(9.6) Summation of Distortion Noises in a Multi-Repeater System

Consider a multi-repeater system comprising R identical repeater sections carrying a number of sine-wave tones of frequencies $A, B, C \ldots F, G, H, \ldots$, and let intermodulation occur at a particular point R_1 in one repeater, and the corresponding point R_2 in the next repeater, etc. At R_1 a number $(p + q)$ of the fundamental tones will combine to form an intermodulation product which can be represented by

$$\sin\left\{(2\pi At + 2\pi Bt + \ldots \text{ to } p \text{ terms}) - (2\pi Ft + 2\pi Gt + \ldots \text{ to } q \text{ terms})\right\}$$
$$= \sin 2\pi t\left\{(A + B + \ldots) - (F + G + \ldots)\right\} \quad (18)$$

This product will pass over a complete section of cable and repeater, arriving at R_2 with the same level but different phase. It will be assumed that the phase-shift/frequency characteristic of this section is a straight line of slope θ and zero-frequency intercept ϕ. A close approximation to this is usually obtained over the working band.

The term on the right-hand side of product (18) will therefore be changed to

$$\sin\left[2\pi t\left\{(A + B + \ldots) - (F + G + \ldots)\right\} + \phi + \theta\left\{(A + B + \ldots) - (F + G + \ldots)\right\}\right] \quad (19)$$

At R_2 the tones $2\pi At$, $2\pi Bt$, etc., will have changed to 2π $+ \phi + A\theta$, $2\pi Bt + \phi + B\theta$, etc., and repeater R_2 will produc new intermodulation product

$$\sin\left[\left\{(2\pi At + \phi + A\theta) + (2\pi Bt + \phi + B\theta) + \ldots \text{ to } p \text{ terms}\right\} - \left\{(2\pi Ft + \phi + F\theta) + (2\pi Gt + \phi + G\theta) + \ldots \text{ to } q \text{ terms}\right.\right.$$
$$= \sin\left[2\pi t\left\{(A + B + \ldots) - (F + G + \ldots)\right\} + (p - q)\phi + \theta\left\{(A + B + \ldots) - (F + G + \ldots)\right\}\right] \quad ($$

The intermodulation products (19) and (20), having the sa frequency, will add at R_2, and the magnitude of the result will depend on the relative phase. The important case is wh $p - q = 1$, which makes (19) and (20) identical, and the p ducts therefore add in-phase. Products of this type have b called "voltage-adding" products. It has been tacitly assum that eqn. (18) represents a positive frequency. If this frequen is negative, however, it follows that the condition for equality now $p - q = -1$.

Group 1 or "voltage-adding" products are therefore produ if the number of frequency components in the product prefi by a positive sign exceeds by $+1$ or -1 the number of f quency components prefixed by a negative sign. The frequen of the product in the former case must be positive and in latter case negative. Group 2 includes all products which not meet this criterion. Group 2 products can add on a volt basis if, in addition to the linear phase/frequency characteris over the working band, the extension of this characteristic zero frequency gives a phase intercept of zero or a multiple 2π radians. An approximation to this may be obtained practice if the product of the phase angle and the number repeaters is small, but this condition is usually not fulfilled practice, and in general it can be assumed that Group 2 produ add on a power basis.

From the definition of Group 1 products, it can be seen th these can be produced only by odd-order intermodulation. is also of interest to note from the analysis that for Group products, provided the phase/frequency curve is linear for ea section, the values of θ and ϕ are immaterial and can be differ for different sections of cable.

APPENDIX III

Report No. 552-1

Study on Intermodulation Analysis

and Recommendations

for the

UHF Multipurpose Satellite Transponder

Summary of Discussions

Held at

TRW, Hughes Aircraft, and Naval Research Laboratory

Under

DSS Contract PL 36100-4-2003

Serial OPL4-0184

April 28, 1975

Sinclair Radio Laboratories Limited
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SINCLAIR

Summary of Discussions

Held At

Naval Research Laboratory

Address: Washington D.C.

Date of Meetings: March 26-27, 1975

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1.0 General

Discussions with NRL were primarily concerned with methods of measurement of intermodulation occuring in UHF satellite system components, and problem areas in passive hardware generated intermodulation. The most urgent intermodulation problem areas under investigation with the NRL group visited concerns the Fleetsat Program. Quantitative information was relatively minimal due to security restrictions. Several important guidelines to UHF satellite development based upon experience with the Fleetsat program were stated, and are summarized as follows:

1) Simulate the transponder to establish the potential problem areas: the Test Simulator for Fleetsatcom development contains about eight relay racks of equipment comprising the transponder, power supplies, and necessary spacecraft equipment.

The test simulator was considered essential to the extent that, the complete communication package must be simulated in order to have a reason to go ahead with the spacecraft program.

2) Carry out knowledgeable testing to determine intermodulation and related problem areas: it was stated that all space-intermodulation sources will likely still not have been completely solved after simulator evaluations. The comment is assumed to apply particularly to the Fleetsat Program where transmit-receive band allocations permit low-order products to fall on the receive band.

3) Design the transponder first; then design the spacecraft.

With specific reference to the DOC-UHF satellite, simulation of the UHF-SHF-UHF system and a dual UHF antenna were recommended. In relation to Fleetsat experience, the design requirement is dependent upon frequency allocations and simulator test data.

2.0 Comments on High-Power Amplifiers

2.1 Experience with random multiple-access high-power amplifiers (TACSAT) has led to the present channelized amplifier approach used in the Fleetsatcom program. The problem is principally that of controlling the users' access power level. A single high carrier access to a hard-limiting mode amplifier, due to an indiscriminate or unauthorized user, reduces the signal-to-noise ratio to zero for small carriers present.

2.2 Signal to noise ratio for hard-limited amplifiers; 10 dB for single carrier to total intermods on carrier.

2.3 Communications Transistor Corp. 2N6439 Transistor, 60 watts (formerly C2M60-28); the transistor was stated to contain 400-500 discrete junctions. The junctions fail in a shorting-mode, where partial shorts will behave as non-linear resistors across the array. The intermod levels consequently increase with contact failures.

Variation in intermod levels was found in laboratory measurements on transistors. Harmonic distortion measurements are performed to determine contact failures.

2.4 Cascaded Stages of Amplification: Isolators are required between input-output cascaded stages of amplifiers for broad-band matching to reduce modulation of the mismatch due to input impedance dependency on power level.

2.5 Power level control of carriers was stated to be less difficult with a larger number (12 or more) channels. With a small number of channels (5 or 6) it was felt important to have separate (channelized) power control.

3.0 Solar Panels

3.1 The solar panels on Fleetsat satellite can be illuminated with an appreciable level of the transmitting antenna off-axis power. Panel rotation causes both forward and rear surfaces to be illuminated by antenna fields.

3.2 Intermodulation generated in the solar panels was determined to be due largely to steering diodes (efficient switching diodes) mounted on the panels. RF power coupling was by means of the short interconnecting lead-lengths on the diodes.

The solar cell arrays generated intermod levels approximately 10 dB lower than the diode generated intermod levels.

3.3 The equivalent circuit (capacity) of solar cells was noted to vary widely with light excitation.

4.0 Multipactor Effect

4.1 Multipacting breakdown was reviewed as a fundamental non-linear phenomenon. Multipaction is a precursor to plasma, and the plasma current frequency spectra contains the intermodulation frequency components.

Multipacting is associated with alternating current in pressure regions lower than that for ionization, when the mean-free path of the electrons is equal or larger than the electrode separation. The multipacting breakdown is primarily dependent on secondary emission from the gap walls as a source of free electrons. In a vacuum, the breakdown is related to frequency, spacing, and material at the gap boundaries.

For example, a 50 ohm coaxial line, (air filled and reflection-free) has a threshold power for multipactor breakdown of approximately 15 watts average at 100 (MHz-cm). Assuming a transmitter frequency (nominally 300 MHz) and line spacing of 0.6 cm., multipacting would be expected to occur at approximately 50 watts. The same transmission line would be subject to ionization breakdown at 50 watts at a pressure of 0.25 Torr (about 200,000 feet).

4.2 <u>Multipacting Problem Areas</u>

4.2.1 Multipacting occurs usually at transmission line discontinuities, initiated by microdischarge.

The following have been areas of difficulty in Fleetsat design.

1) 50 ohms line connection (transformer) to transmitting helix.

2) Transmitting antenna balun (present major problem).

3) High power filters (still to be tested at date of meeting).

4) Transmitting filters.

5) High power amplifiers.

6) TT&C antenna, due to the proximity of transmission lines in the mast.

   Note: The principal intermodulation problem was caused by transmitter energy coupled into the diode input telemetry receiver due to the common-mast mounting of the two antennas.

7) Thermal blanket material; subsequently removed from the transmitting helix mast. Charge build-up in space, associated with fracturing of the blanket surface relates to multipacting and intermodulation problems.

4.2.2 Component design for elimination of multipacting is primarily related to the PEP requirement of the circuit. Fleetsat is operating at high PEP (estimated greater than 4 KW) and presents a very definite design problem.

The PEP rating for the DOC-UHF satellite appears sufficiently low as to present major design problems due to multipacting.

Teflon-filling of the balun and pressurizing of all filters is employed in Fleetsat to solve multipacting design problems.

4.2.3 Multipacting is not expected to present a problem at SHF due to the increased breakdown potential for increased frequency.

Antenna intermodulation at UHF due to the close physical spacing of SHF and UHF antennas (constructed on axis of common mast) is a potential design problem.

4.3 Spacecraft Typical Operating Pressure

Space pressure has been the subject of satellite experiments. 10^{-5} Torr was suggested as a typical value. Actual pressure will vary due to venting and outgassing of components.

4.4 Multipactor Test Facilities

1) TRW employ an 18 foot diameter Bell jar for antenna system space simulation test.
2) A tank facility (company unknown) located in Ohio is capable of holding the entire spacecraft.

4.5 Pressurizing

Pressurizing for 5-7 year life expectancy in space can be accomplished. Beam welding techniques are employed in pressurizable filter construction. Glass spheres-potting in lieu of pressurizing for solving multipaction proved unsatisfactory, due to fracturing of the glass spheres during launch vibration.

5.0 Filter Types

5.1 Diplexing filters of the interdigital rod type are preferrable from the standpoint of employing low-Q circuits for solving multipactor and intermodulation designs.

Other favourable factors include mechanical simplicity, strength, size, weight, and relative ease of design for intermod reduction.

5.2 The transmit filters for Fleetsat are coaxial resonators, capacitive-end-loaded to approximately one-eighth wavelength, and probe-coupled. Dual resonators (approx. 4" diam.) are employed in a back-to-back configuration.

5.3 Stripline type filters are not presently favoured due to the intermodulation design difficulty. Rough edges and metal-fingers at the metal edge-substrate interface cause microdischarge generated intermodulation.

5.4 Filter Isolation Achievement

140 dB isolation has been achieved, and was considered to represent a practical limit for specifying filter performance.

5.5 Diplexer Third-Order Intermod Achievement

Third order intermodulation levels of -140 dBM have been achieved at NRL for two +50 dBM clean-carrier test. The result is obtained with great care in manufacturing and assembly. The level is not consistent on removal and reassembly of diplexer cover plates.

The expected space deterioration of the level is 20 to 30 dB.

Summary of Discussions

Held at

Systems Group of TRW, Inc.

One Space Park, Redondo Beach, California

Present

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Prepared by:

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1.0 General

1.1 The report summarizes discussions with TRW pertaining to intermodulation specifications and intermodulation problem areas encountered in the Fleetsatcom UHF transponder.

The original design approach for Fleetsatcom required twenty-three filtered transmit channels and the receiver diplexed into a common antenna.

Passive component intermodulation products, principally fifth-orders and to lesser extent third-orders, falling on the lower receive channels necessitated abondonment of the diplexed system in favour of a 50 dB isolated dual antenna system.

1.2 Program Schedule

Fleetsatcom developments were carried out to theffollowing time schedule:

Nov. 1973: Start of program

Feb. 1974: Engineering model of simulator racks completed. System testing was begun.

June 1974: Realized problem areas with transponder UHF section intermod. Decision to abondon diplexed single antenna in favour of separate transmit-receive antennas.

July 1974: Completed the preliminary design phase. Began transponder UHF system redesign.

Sept. 1974: Ran full tests on redesigned engineering model; demonstrated that the separate antennas design approach was feasible with regard to IM signal levels falling on the receive band.

Dec. 1974: Payload critical design review meeting. Redesign for IM due to multipactor effect (mechanical design for high-power components in a vacuum)

Apr. 1975: Completed the final engineering model of transponder. Completed payload simulator. Completed high-power IM tests in vacuum chamber.

July 1975: Expected completion of Qualification test model.

1.3 Transponder Specifications

1.3.1 Transponder Frequency Bands

a) UHF, typically 244-270 MHz transmit band
292-400 MHz receive band.

b) SHF

1.3.2 UHF Transmit Channels

a) Total number of transmit channels: 23

1) 10 single-carrier channels, fixed tuned, flat to within 0.1 dB over 25 KHz bandwidth.

2) 1 single-carrier channel, wideband, high-power.

3) 12 multicoupled channels, 5 KHz bandwidth, at approx. 25 dB adjacent channel (crossover) isolation.

b) Transmit Power Levels

1) Approx. 30 watt average power, unequal carriers; about 10 dB max. to min. ratio on carrier levels.

2) High power channel, approx. 250 watt PEP.

3) Total UHF channels average power, approx. 350 watts.

1.3.3 Intermodulation Spec. for Transmit Channel Filters

Third order IM products are required to be less than -115 dBM for two-tone +46 dBM signals test.

1.3.4 General Intermodulation Problem due to Tx-Rx Frequency Allocations

a) Third to eleventh-order IM products fall on the receive frequency band.

b) Third-order products restricted mainly to those products falling on channel 23 (near low-end of receive channels) were a main problem.

c) Fifth-order products were the greatest problem, affecting most receive channels. Approx. 10,000 fifth-order products fall on channel 2.

NOTE Two-carrier fifth-order product levels were found to be 20 to 30 dB less than five-carrier fifths.

d) Seventh-order products were sufficiently low to be of no problem.

e) Transmit-receive frequency allocations could not be changed to be more compatible with the IM spectra. A diplexed system employing a single 16 foot dish with a crossed-dipole feed, selected for good antenna efficiency, was attempted for the transponder original design.

A dual UHF antenna system requiring 50 dB of isolation between transmitting and receiving antennas, including 20 dB margin, was introduced in the intermodulation redesign phase.

f) Passive component intermod. performance produced main or major problem areas for transponder system redesign. Extensive investigations into manufacturing and testing methods for passive components (transmitting and receiving filters and antenna circuit components) and theoretical investigations in discrete problem areas were carried out.

2.0 Intermodulation Performance of Original UHF Transponder Components

2.1 Transmitting Multicoupler

The original model produced third-order IM at -70 dBM, for two 30 watt clean carrier mixing.

2.2 Diplexer

-60 dBM 3rd order, for two 30 watt carriers.

NOTE The best eventual result for third-order was -130 dBM.

2.3 Crossed-dipole Antenna, including Hybrid and 50Ω Termination.

-40 dBM 3rd order, for two 30 watt carriers.

NOTE Best eventual result for helical transmitting antenna, -130 dBM.

3.0 UHF Transponder IM Performance

3.1 Receiver Noise Requirement

a) Spurious and noise falling on receive channels was required to be -10 dB or less from receiver thermal noise, for 0.5 dB degradation. (i.e. Total noise in a 25 KHz BW shall not degrade more than 0.5 dB). The 0.5 dB S/N degradation specification specifically applies to the worst channel spec., which is one of the lower 25 KHz channels. The specification is similar for all channels of 5 KHz and 25 KHz BW.

b) Design Specification

Third-order intermods on receive channels are to meet -150 dBM for clean carrier test signals. A -130 dBM transponder level is required, with 20 dB margin allowed for additional degradation due to the spacecraft. The signal to intermod is 20 to 25 dB. for clean carrier test.

c) TRW feel that they have arrived at the best way to specify the system IM performance; then plus 20 dB margin. At the stated required level (-130 dBM), IM performance of the system was noted to be a risk. The system will handle the clean carrier, worst case, with a 10 dB margin.

d) Maximum fifth-orders at the receiver are -148 dBM.

3.2 Smallest Expected Received Carrier; -127 dBM

a) Receiver noise level; -127 dBM.

b) Narrow-band channels are 7 dB more sensitive, i.e. received signals are 7 dB lower.

c) It was stated that the ratio of IM spike power to uplink carrier power can be 5 to 10 dB on the worst channel.

3.3 IM Test Signals

3.3.1 a) Component tests are specified for two clean test carriers.

b) Transponder system tests specify 23 channel noise-loaded carriers.

3.3.2 Discussion on Two-channel versus 23-channel Testing.

a) A unit third-order IM specification of -150 dBM for two-tone 30 watt clean carrier test satisfies the system S/N degradation requirement of 0.5 dB degradation for 23 noise-loaded carriers. The technique for correlation of the two specifications was stated to be based upon analysis and measurement experience. Apparently the conversion of two tone third IM to 23 noise-loaded carriers third can be predicted within 5 dB.

The analysis ignores intermods. of eleventh-order and higher, and makes certain assumptions regarding seventh and ninth-orders.

Fifth-order intermods. were -10 dB from third-order for the analysis.

b) Clean carrier IM performance comparison specifications for two-tone versus 23-tone test was apparently not considered by TRW. A single (discrete component of) IM generated for the 23-tone clean carrier case decreases from the 2-tone case; the total number of products increases, and the amplitude sum of products falling on the discrete IM component frequency increases.

3.4 Test Diplexer and Equipment for Spacecraft IM Testing

3.4.1 Test diplexers comprised of interdigital bandpass transmit and receive filters with common antenna-port, are employed for IM testing of transponder antenna circuit components.

Single diplexers are machined of solid brass. (Outer ground plane T-configuration, very approximately 24"x18"x3" thick.) Cover mating surfaces are machined optically flat. Two diplexers are connected in a bridge circuit via coaxial switches, for testing forward and reflected intermods on the antenna circuit.

3.4.2 Diplexer Specification

Third-order intermodulation at -130 dBM was required for the test system. The design goal for diplexer third-order intermodulation was -145 dBM for two-tone test with 30 watt average carriers.

3.4.3 Diplexer; Mating Surfaces Machining

Approximately 8-20μ inch finish is performed on lapped joints. Surface contact pressures exceed 10,000 psi.

3.4.4 Test Receiver

Spectrum analyzer at -155 dBM sensitivity, operated at 10 Hz bandwidth with frequency-stabilized transmitting carriers.

3.4.5 Power Simulation for Spacecraft Testing

Eight noise-load carriers (signal generators) driving a one KW. broadband amplifier employed for simulator testing.

4.0 UHF Transponder Antenna System Redevelopment

4.1 The following components are now included in the dual antenna UHF system:

a) Transmit multicoupler; same as diplexed system.

b) Transmit filter; same transmit section as utilized in original diplexer.

c) Receive filter; same receive section as utilized in original diplexer.

d) Transmit helical antenna; triple-taper bi-filar back-fire type, approximately 9 turn total length. Circumferential dimensions of the helix are triple-tapered. The helix mast is rigid-mounted at the dish vertex.

e) Receive helical antenna; bifilar, approximately 18 turn, 10 inch diameter, turns consisting of approximately 1½ inch x 1/8 inch conductor. The helix mast is hinge-mounted to the spacecraft.

f) Transmitting antenna balun.

The TT&C antenna (2.5 GHz) occupies the same mast as the transmit helix, and is a significant intermod. source. A balun (mounted in the spacecraft body) is required to reduce the mast excitation and consequent unsymmetrical currents to the TT&C antenna. Multipacting requires that the balun and preferably also the antenna transformer (two-wire transmission line inside mast) be dielectric-filled.

g) Parabola and Mast Dimensions

1) Dish diameter, 16 feet; stainless-steel silver-loaded wire mesh construction.

2) Mast height (overall) to helix reflector, 15 feet.

3) Mast height to helix base, approx. 6 feet; (f/D = 0.4).

h) Dish Illumination

Approx. 10 dB edge illumination; structure directly behind the antenna is isolated 100 dB.

i) Mast Construction

The transmitting mast proper is fabricated of spun aluminum, polished and chem. etched. Helix support mast; graphite-filled epoxy.

j) Dish Fabrication

All contacting wires in the mesh are individually welded by means of an automatic welding machine. No random contacts are permitted.

k) Hinges

Hinges were main problem areas in IM reduction. Hinges employed at dish (mesh) fold-down points are fiberglass insulated to prevent random contact to the mesh.

l) Graphite Paint

Paint coatings are applied to antenna support masts (internal to helix), antenna mast (stripping), and reflector solid surface. Aluminized mylar was avoided.

Graphite coating produces a lossy surface, and was stated to be one of the best methods for controlling multipacting on antenna surfaces.

4.2 Transmit-Receive Antenna Isolation

Transmit power at the receiver terminal is reduced 50 dB due to antenna physical separation and antenna bandwidth.

4.3 Intermodulation Requirements for Present System

4.3.1 The following are system requirements for third-order two-tone 30 watt test:

1) Transmitting Filter: -100 dBM

2) Antenna and Reflector: -100 dBM

4.3.2 Degradation

The -100 dBM two-tone spec can be degraded to -80 dBM without greater than 0.5 dB reduction of S/N.

4.4 Measured Transmit Antenna Intermods

1) Transmitting antenna Intermod; -150 dBM third-order; antenna measured on antenna farm, with receiver on receive antenna terminal.

2) With the receiver at the receiving antenna terminal, transmit antenna system intermod need only be -100 dBM since 50 dB of path isolation is provided. -110 dBM has been achieved.

3) Five-carrier fifth orders were -10 dB from third-orders.

4) Two-tone system test results:

   3rd order: -100 dBM

   5th order: -110 dBM

   7th order: dropped much more than -10 dB from 5th.

   The reason for a 10 dB discrepancy in the 3rd order result was not ascertained.

5) <u>Back-off Rate on Third-Order</u>

2 dB/dB reduction in carrier power.

4.5 <u>Measured Receive Antenna Intermod</u>

1) Third-order intermod generated in the receive antenna due to transmit power; approx. -200 dBM, due to 50 dB isolation.

4.6 <u>Antenna Test Facility</u>

Antenna system tests are performed in an anechoic chamber. It was stated that the chamber has a <u>quiet spot</u> for placement of the antenna under test.

5.0 <u>Main Intermodulation Contributors in System</u>

5.1 The following were noted as hardware problem areas:

1) Thermal Wrap; contact problems.
2) Heater and ordnance lines.
3) Solar array; deployment mechanism
4) Antenna cables and connections.
5) Antenna mesh; less problem than contemplated.
6) Multicoupler connectors.
7) Ceramic Seals
8) Ferromagnetic material; Kovar seals.
9) Metal-joining techniques.
10) Micro-discharge mechanisms; burrs, metal chips, cracks, material edges, hairs of metallic materials.

11) Flat metal surfaces in contact; interdigital filter covers.

12) Antenna hybrid dummy load; in original diplexed antenna.

13) Corrosion problems; dissimilar metal contacts, oxidized surface interfaces.

14) Multipactor breakdown.

15) Outgassing; dielectrics.

5.2 Material Finish

1) Bare aluminum was stated to be one of the most suitable materials.

2) Gold Plating: TRW were not convinced that gold plating is satisfactory. Gold purity is likely the matter to be resolved. High-purity metal surfaces are required.

6.0 Pressurizing

6.1 All transmitting filters (multicoupler and output filters) are pressurized.

6.2 Pressurizing (dry nitrogen or helium) is to 2 atmospheres.

6.3 Safety factor on leakage rate; 20 over spacecraft lifetime.

7.0 Multipacting

7.1 The main problem areas for multipacting were the following:

1) Transmitting antenna balun, and antenna transformer.
2) TT&C Antenna.
3) Proximity of TT&C and UHF transformer transmission lines inside the mast.
4) Transmitting multicoupler and diplexer.

7.2 Parameters Stated for Multipacting

1) Pressure range for multipacting; 10^{-2} torr to 10^{-9} torr.
2) Power Levels; multipacting can occur at 40 watt levels.
3) Measurements indicate that greater than 10 to 20 cycles may be required for secondary emission charge build up.

7.3 Test for Multipacting

Intermod testing is a suitable test for multipactor breakdown IM levels were noted to increase 30 to 50 dB with multipacting.

Summary of Discussions

Held at

Hughes Aircraft

Address: 90 9 North Sepulveda Blvd., El Segundo, California

Date: April 2, 1975

Present:

Mr. Hal Smith -	Hughes Aircraft Asst. Program Mgr. - Marisat Tel. 213-648-1766
Mr. Jerry Dutcher -	Hughes Aircraft System Engr. Marisat Program Tel. 213-648-4755
Mr. H. Werstiuk -	Dept. of Communications, Ottawa
Mr. M.A. Stott -	Canadian Astronautics Ltd. Ottawa.
Mr. F.G. Buckles -	Sinclair Radio Laboratories Ltd.

Prepared by:

F.G. Buckles,
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1.1 General

The discussions were limited to data and performance specifications applicable to the Marisat Program.

The transponder provides for:

a) Three channelized UHF transmitting channels:
   - one at +47 dBM, 500 KHz BW
   - two at +42 dBM, 25 KHz BW
b) L-Band
c) Telemetry, 4-6 GHz band.

1.2 The UHF amplifiers are solid state, Class C, hard limited, 60% efficiency; with a measured signal-to-intermod (S/IM = 9.5 dB) generated in the hard limiter. The corresponding S/IM for the amplifier separate from the limiter had not been measured.

A TWT L-Band amplifier (not hard limited) is employed.

1.3 UHF Receiver thermal noise power (kTB): -170 dBW/hz BW.

1.4 Spurious Specification

a) The specification for spurious inputs to the receiver was -160 dBM, for a -120 dBM lowest expected received signal and 40 dB signal/spurious specification.

   The transponder performance does not meet the specification. Only spurious falling in the receive band was of importance to transponder performance.

1.5 Transponder Performance

a) Specified input signal levels to the transponder are -80 dBM to -120 dBM (minimum).

b) The transponder carrier to noise performance was stated to be:
-9 dB on Channel 1, at -120 dBM input signal
+4 dB on other channels, at -120 dBM input.

2.0 Intermodulation Performance

2.1 What is considered to be a useable system having suitably low IM levels has been achieved for the transponder without a major effort directed toward state-of-art components and latest of fabrication techniques. Antenna intermodulation presented the main problem, and was always determined to be due to poor RF contacts. Mechanical stability of contacts was noted to be essential. The remaining IM problem area to be solved for the Marisat antenna is the helical antennas feed three-way splitter, (strip-line structure is typical).

2.2 IM Levels in Receive Band

2.2.1 The 13th order is the lowest order IM falling in the receive band, determined for 3-carrier excitation (one band-pass carrier per each of three transmit channels). All passive IM tests were three-tone tests. Only one two-tone frequency components was observed in the receive band.

2.2.2 The total passive IM's at the receiver are at levels -120 to -140 dBM for order 13, 15, 17, measured for a three clean carrier case.

2.2.3 The 13th order antenna intermod (measured) is -150 dBM, with 3-carrier (+46 dBM each CW) excitation.

Antenna testing was performed in an anechoic chamber, stated to be non-contributary at the thirteenth-order.

2.2.4 A single-carrier IM (broadband noise phenomena) was encountered in atmospheric tests on the antenna, where at a threshold of approximately -115 dBM, the broadband noise contribution increases the IM approximately 20 to 30 dB. It was recommended that one look for broadband noise when spurious levels exceed -115 dBM. Single carrier IM was stated to be well below threshold for antenna intermods at the -140 to -150 dBM level. (Note: Applies only to 13th order; no data as regards lower orders.)

2.2.5 High-order IM drop-off rate was less than 1 dB/dB change of excitation.

2.2.6 Measurement Data to be Supplied by Hughes

Measured IM data for the first two repeaters (total IM in receive band) for 13th to 22nd orders up to -155 dBM detectability limit, is to be supplied by Hughes.

2.2.7 Detector Bandwidth for IM Testing

100 hz BW was used for all measurements.

2.2.8 Diplexer, Rotary Joint, and Antenna IM Levels

a) Diplexer and rotary joint; -125 dBM measured at 13th order, 3 carrier test.

b) Rotary joint and antenna mounted on spacecraft; -117 dBM was the approximate highest 13th order level for three saturated carrier test.

2.2.9 Spacecraft Intermodulation Tests

Spacecraft tests were three-channel noise-loaded carrier tests. It is stated that spurious signals drop out dramatically with carrier power back-off, before predictions indicated.

2.3 Hughes (military) Program IM Level Achievement

Another development program (diplexed antenna system comprising filters, antenna, etc.) for which data is not available has achieved low-order IM levels of -110 to -130 dBM for 10-100 watt carriers.

2.4 Lifetime Stability of IM Level

The transponder was subjected to a six-month environmental test period additionally involving transportation and vibration test. The spurious levels were stated to be relatively stable, with residuals probably due to dissimilar metal junctions and tunneling-contact non-linearity.

3.0 Multipacting

3.1 Multipacting occurred in a standard switch at L-band due to incorrect dimensions in the component. The effect can be obviated by careful design of the component in regard to voltage stress and physical separation of conductors.

3.2 The original switch, with SMA connectors is employed for L-band output switching into the quad-helix antenna array.

Multipactor effect was solved by replacing the original switch with an enlarged version, not hermetrically-sealed.

4.0 UHF Diplexer

4.1 A set of four discrete filters are employed in the UHF section

a) Three transmit filters comprising two-section cavities, with 70 dB isolation at receive frequencies and approximately 110 dB isolation at Tx-Tx frequencies.
b) Nine-section receive filter, assumed to be interdigital type.

4.2 All UHF filters are parallel connected to the antenna port. The filters with residual at -145 dBM at 13th order are employed as the intermod test set for spacecraft tests.

4.3 UHF Diplexer Weight

10 lbs. for four filters plus output junction.

4.4 Plating

Filters are silver-plated. Connectors are gold-plated.

4.5 Isolators

Isolators are employed between the power amplifier final stage and the transmit filters.

5.0 UHF Antenna Array

5.1 Type and Gain

Three-helix array.

Array gain: 14 dB peak at transmit frequencies
15 dB peak at receive frequencies
Approx. 9.5° beamwidth.

Single helices are bifilar, approximately 4½ turn, aluminum.

5.2 Isolation

The L-band quad-helix array is located on axis with the UHF array. The UHF and L-band arrays are isolated 20-25 dB at UHF frequencies.

6.0 High-Power Amplifiers

6.1 UHF channels power output vs. amplifier stages:

a) 50 to 55 watts, four devices operated in parallel.
b) 20 to 25 watts, two devices in parallel.

The devices are input and output hybrid coupled, with wide-band matching between stages.

6.2 Redundancy is provided by switched input-output separate hybrid-coupled amplifiers.

6.3 Power transistor procurement, and availability of repeatable devices was noted as a difficulty. Manufacturer screening yields approximately 10 useful amplifiers per 6000 run.

Amplifier heat dissipation was stated a main problem area for the devices, and may create a thermal problem for the repeater. All heat is removed by radiation coupling to the sun shield. The maximum junction temperature for the UHF transistors is $< 200^{0}C$, and amplifiers are de-rated considerably. Good thermal-band continuity, and heat transfer tests on new design was recommended.

6.4 The L-band tubes are the main heat dissipators within the repeater. The tubes are three-level output; 7, 30, and 60 watts by beam current control. Efficiency of tubes less power supply is 50% in higher power mode to 25% in lower power mode.

7.0 <u>Thermal Wrap</u>

7.1 Thermal blanket is stretched drum-taut (not wrapped) and mechanically stable over the spacecraft end. The antenna structure is wrapped only at the base. The spacecraft and blanket is subject to high illumination (everywhere) by UHF currents. Crazing of the material was of concern in respect to static-charge build-up.
Aerospace Corporation has investigated plasma-thermal wrap related problems.